A hypothesis relating the entropy to the inhomogeneity of the universe

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(Submitted 9 August 1972)
Zh. Eksp. Teor. Fiz. **64**, 58-66 (January 1973)

A hypothesis is proposed which assumes that at the instant of the cosmological singularity the Universe was filled with cold baryons. The averaged evolution is described by an isotropic homogeneous expansion according to the Friedmann solution and the equation of state of cold baryons. On this evolution are superimposed scale-independent perturbations of the metric of the order of 10^{-4}. The short-wave part of the perturbations explains the entropy of the hot (big bang) Universe and the long-wave part describes the formation of galaxy clusters and the approximate homogeneity of the Universe as a whole.

1. INTRODUCTION AND BRIEF SUMMARY OF RESULTS

A hypothesis is proposed which assumes that at the initial instant of the cosmological singularity the Universe was filled with cold baryons. The averaged evolution is described by an isotropic homogeneous expansion according to the Friedmann solution and the equation of state of the cold baryonic matter[1]. On this averaged picture are superimposed initial fluctuations of the baryon density and related fluctuations of the metric. It turns out that a single value (of the order 10^{-4}) of the dimensionless fluctuation amplitude of the metric, practically not depending on the scale, describes two types of phenomena which are of completely different nature and were never before related to each other, viz. the structure and entropy of the Universe.

To the indicated fluctuations of the metric correspond density fluctuations with an amplitude which is inversely proportional to the square of the size at a given (initial) instant. Density fluctuations of the order of the average distance between the baryons first increase and are then transformed into acoustic waves, i.e., into phonons. The damping of short-acoustic waves means a transformation of their energy into all different forms of excitations. A relaxation to a thermodynamic equilibrium state occurs, where the specific entropy per baryon is large. In the final count this line of reasoning leads (by appropriate choice of the perturbation parameter) to a ratio of the photon and baryon numbers which is characteristic for the theory of the hot Universe ("big bang"). The approach to thermodynamic equilibrium takes place at a very early stage (for $t \ll \hbar/mc^2$). Therefore all the usual consequences relating to the evolution of hot matter are preserved, including a hadronic stage with an abundance of baryon-antibaryon pairs,[1)] the stage of nucleogenesis with the formation of 25—30% He^4 and the radiation-dominated stage, ending with the formation of neutral H at $T \sim 4000°$K.

The second line of reasoning deals with the long-wave perturbations of the metric. For these perturbations the wavelength is comparable to the horizon $\lambda = ct$ at the late stage of radiation-dominated plasma or (for the longest wavelengths) the stage of a neutral gas. The theory of such perturbations was developed in detail in recent years.

The amplitude of the perturbations of the metric is of the order of 10^{-4}—10^{-5} for long waves (i.e., is practically the same as the one assumed above for the short waves) and leads to the formation of galaxy clusters of characteristic size 10^{13} $M_\odot$ [2)] in the recent past (for a redshift $z \sim 2$—20). On a large scale the density fluctuations are at present smaller than one, i.e., are insufficient for the separation (individualization) of appropriately large masses. The hypothesis agrees with the smallness of the fluctuations of the background microwave radiation.

It seems a remarkable fact that a unique flat spectrum of perturbations of the metric with a single constant (not calculated theoretically at the present time), unifies the description of the "contents" (entropy, photon/baryon ratio, chemical composition) and the "structure" (characteristic size of clusters, their density, depending on the instant of individualization) of the Universe.

The natural question arises: can one not develop the theory further, can one not derive from first principles that spectrum of perturbations on the "initial" Planckian instant which is "necessary" in the above indicated sense? Two approaches to the solution of this problem are known. The first consists in considering quantum fluctuations of the metric, taking into account their renormalization. These fluctuations of the metric should be accompanied by a nonuniformity in the distribution of baryons. In the course of evolution the fluctuations of the metric do not increase and give no observable effects (if renormalization is taken into account), but the nonuniformity of the baryons increases. However, this approach yields too low an entropy (by 3—4 orders of magnitude) and incredibly small (a factor of 10^{26} is missing!) long-wave perturbations. Another approach is constructed by analogy with the theory of equilibrium fluctuations. The calculations, which yield $\Delta N = N^{1/2}$ for the ideal gas and similar expressions for the zero-point oscillations of a liquid, correspond to a Fourier expansion of the perturbations with the assumption that the energy of individual oscillations is kT or $\hbar\omega/2$. But the cosmological problem is nonstationary; as a consequence of the gravitational interaction in place of the oscillations there appears an instability and growth of the amplitudes; one cannot define an "energy" of the perturbation. Therefore this approach is not convincing. In addition, a naive calculation, without taking into account these objections, yields too large fluctuations (10^4 times larger in the quantum case, 10^{15} times in the thermal case) not in agreement with the observations. Thus, no consistent theoretical solution has been found to this time.

2. AN ESTIMATE OF THE ENTROPY

We give here some order-of-magnitude estimates. We shall use a system of units with $\hbar = c = 1$; let m denote the proton mass, G the newtonian gravitational constant, $g = Gm^2$ ($= Gm^2/\hbar c = 10^{-38}$) is a dimensionless number characterizing the smallness of the gravitational interaction. In this system of units m has the dimension cm^{-1}, and G has the dimension cm^2.

The subscript zero denotes quantities referring to the planckian instant of time

$$t_0 = G^{1/2} (= G^{1/2}\hbar^{1/2}c^{-5/2} = 10^{-43}\ \text{sec}). \qquad (1)$$

The amplitude of the perturbations of the metric will be denoted by b. We systematically omit dimensionless factors of the order of one.

For cold baryonic matter we make use of the "extremely rigid" equation of state[1]

$$\rho = p = \varepsilon = n^2m^{-2} + n^{4/3} + nm, \qquad (2)$$

where n is baryon density (cm^{-3}); the second and third terms will be omitted in the sequel. The averaged solution has the form

$$\rho = G^{-1}t^{-2}, \qquad (3)$$

whence

$$n = mG^{-1/2}t^{-1},\ \rho_0 = G^{-2},\ n_0 = mG^{-1}. \qquad (4)$$

The shortest waves have a wavelength of the order of the mean distance between the baryons:

$$\lambda_{min} = n^{-1/3}. \qquad (5)$$

At the instant t_0 we find

$$\lambda_{min,0} = G^{1/2}m^{-1/3} = G^{1/2}g^{-1/6}. \qquad (6)$$

For these waves the condition $\lambda = t$ is attained at the instant

$$t_1 = G^{1/4}m^{-1/2} = t_0 g^{-1/4} \gg t_0. \qquad (7)$$

Let us find the fluctuations of the density $\delta\rho = \psi$ and the corresponding acoustic energy ϵ_{ac}, by specifying the fluctuation b of the metric[3], which remained unchanged for $t < t_1$. We have the order of magnitude (M is the mass in the volume λ^3, $M = \rho\lambda^3$, $\delta M = \psi\lambda^3$)

$$b = G\lambda^{-1}\delta M = \psi G\lambda^2. \qquad (8)$$

Hence at the instant t_1

$$\psi = bG^{-1}\lambda^{-2} = bG^{-1}t_1^{-2} = b\rho_1,\quad (\psi/\rho)_1 = b; \qquad (9)$$

$$\varepsilon_{ac} = \psi^2\rho^{-1},\quad \varepsilon_{ac,1} = b^2\rho_1 = b^2G^{-3/2}m. \qquad (10)$$

After thermodynamic equilibrium is established we obtain for the density of thermal energy ϵ_{th}, the temperature T and the entropy density S:

$$\varepsilon_{th,1} = T_1^4 = \varepsilon_{ac,1},\ S_1 = T_1^3 = b^{3/2}G^{-9/8}m^{3/4}. \qquad (11)$$

The specific entropy s per baryon is obtained dividing S by $n_1 = \lambda_1^{-3} = t_1^{-3}$. We find

$$s = b^{3/2}G^{-3/8}m^{-3/4} = b^{3/2}g^{-3/8} = 10^{14}b^{3/2}. \qquad (12)$$

Consequently $b = 10^{-3}$ yields $s = 3\times10^9$, $b = 10^{-4}$ yields $s = 10^8$. The observed entropy lies within these limits. In letters, adopting for s the empirical formula[4] $s = g^{-1/4}$, we obtain the required $b = g^{1/12}$.

3. ESTIMATES OF THE LARGE-SCALE PERTURBATIONS

We now turn to the other side of the problem: the long-wave perturbations and the appearance of structure.

As is well known[3], perturbations corresponding to masses smaller than $10^{13}\ M_\odot$ are damped out before recombination, owing to dissipative processes related to photon transfer within the hot plasma. A mass of $10^{13}\ M_\odot$ contains $N = 10^{70}$ baryons, so that $\lambda = \lambda_{min} \times 10^{70/3} = 2\times10^{23}\ n^{-1/3}$. The condition $\lambda = ct$ for such a perturbation is attained in the radiation-dominated period at the instant t_2. At that instant $(\psi/\rho_2) = b_2$. In the sequel the perturbation is transformed into an undamped acoustic wave with $\psi/\rho = u/c = b_2 = \text{const.}$, where u is the hydrodynamic velocity, whereas the perturbation of the metric decreases, $b < b_2$ for $t > t_2$.

At the instant of recombination the acoustic wave transforms into a superposition of an increasing and decreasing perturbation in neutral matter. Let us derive the effective amplitude of the increasing mode (i) after the recombination. The quantities referring to the instant of recombination will be denoted with the subscript r. We have

$$(\psi/\rho)_i = b_2ct_r\lambda_r^{-1}(1+z_r)(1+z)^{-1}\ \text{for}\ z+1 > \Omega^{-1}. \qquad (13)$$

For $\Omega = 0.2$ (open Universe) we obtain $t_r = 10^{13}$ sec and for $M = 10^{13}\ M_\odot$ we have $\lambda_r = 10^{22}$ cm, $ct_r\lambda_r^{-1} = 30$. The condition $(\psi/\rho)_i = 1$ for $z + 1 = 5$[5] yields $b_2 = 10^{-4}$.

The flat spectrum b = const (M) leads to the result that for a given z, e.g., z = 4, for $M > 10^{13}\ M_\odot$ $(\psi/\rho)_i$ decreases inversely proportional to λ_r, i.e., like $M^{-1/3}$, so that

$$(\psi/\rho)_{z=4} = (10^{13}\ M_\odot/M)^{1/3}. \qquad (14)$$

For masses which exceed $M_r = \rho_r(ct_r)^3 = 3\times10^{17}\ M_\odot$ the decrease becomes faster

$$(\psi/\rho)_{z=4} = 30^{-1}(3\cdot10^{17}\ M_\odot/M)^{2/3}. \qquad (15)$$

For a mass corresponding to the present radius of the Universe $R = cH^{-1} = 10^{28}$ cm, $\rho = 2\times10^{-30}$, $M_U = \rho R^3 = 10^{21}\ M_\odot$, we obtain

$$(\psi/\rho)_{U,z=4} = 10^{-4}.$$

For $\Omega = 0.2$ the perturbations do not increase for $z < 4$. The flat spectrum of the metric with $b = 10^{-4}$ gives a picture which does not exhibit sharp, unsolvable, contradictions with the observations. One should keep in mind, however, that by unifying with a single flat spectrum the entropy and the theory of structure, we have committed a giant extrapolation from λ_{min}, corresponding to one baryon to a λ 10^{23} times larger, corresponding to a galactic cluster, or to a λ 10^{26} times larger, which corresponds to the whole observable Universe. Therefore we are entitled to carry out the numerous rough estimates mentioned (or not) above.

The flat spectrum of perturbations of the metric leads in a radiation-dominated world ($p = \epsilon/3$) to the conclusion that the density perturbations are equal for different masses at the instant $t_M = \lambda_M c^{-1}$, but that instant itself is different for different masses, $t_M \sim M^{2/3}$. Reducing everything to the same early instant ($\lambda > ct$) we obtain for different masses $(\psi/\rho)_M \sim t_M^{-1} \sim M^{-2/3}$. In this form the result was noted earlier by Harrison[4]; it seems that no one has noticed before the relation of the spectrum of perturbations of the metric to the entropy.

Let us make more precise the definition of the quantities considered above. We use the dimensionless amplitude of the perturbations of the density, ψ/ρ or the perturbation of the metric, b; these amplitudes are considered as a function of the scale λ or of the corresponding mass $M = \lambda^3 nm$, or of the wave vector $k = \lambda^{-1}$. Thus, we consider the dimensionless amplitude b(M) or b(k) and the analogous amplitude of the quantity ψ/ρ.

In the theory of random functions one considers spectral resolutions and the spectrum is characterized by the Fourier-amplitude b_k which is not dimensionless, even if the amplitude itself is dimensionless. The quantity we consider is related to the Fourier-amplitude by the relation

$$b(k) = (b_k^2 k^3)^{1/2}. \tag{16}$$

For our quantity b(k) the completeness theorem can be written in the form

$$V^{-1}\int_V b^2(x)\,dV = \int b^2(k)\,d\ln k = \int b^2(M)\,d\ln M. \tag{17}$$

These equations show that the introduction of dimensionless b(k) or b(M) is indeed indicated. The proposed hypothesis, with a b(M) independent of M and $\psi(M) \sim M^{-2/3}$ yields for the Fourier-components $b_k \sim k^{-2/3}$, $\psi_k \sim k^{1/2}$. We recall that a random distribution of independent particles with $\Delta N = N^{1/2}$ would have yielded Fourier components ψ_k which do not depend on k.

4. QUANTUM FLUCTUATIONS OF THE METRIC AND THE ROLE OF BARYONS

We return to the original assumptions. The basis of everything that was said above is the principle of considering only those perturbations of the metric, to which are related displacements of the baryons in the initially cold matter. Just because of that principle we have restricted our attention above to waves with $\lambda \geq \lambda_{min}$, so that the total number of degrees of freedom considered (normal modes, separate modes) in a given volume is approximately equal to the number of baryons in the same volume. This principle strongly restricts the number of degrees of freedom under consideration. It would seem that at the planckian instant t_0 one must consider all waves, at least[6)] down to $\lambda \geq t_0$. This yields a total number of degrees of freedom $n_0^{-1} t_0^{-3} = g^{-1/2} = 10^{19}$ times larger than the one considered before.

In order to motivate our refusal to consider the huge number of nonbaryonic degrees of freedom we note that the fluctuations corresponding to these degrees of freedom exist, in a certain sense, everywhere and at all times, and not only near the singularity. If one investigates the structure of the electromagnetic field or of the metric of space at the present time, i.e., far enough away (10^{10} years) from the singularity, then the transition to ultrasmall scales will exhibit these fluctuations. At the same time Nature (and consequently also the correct theory) is arranged in such a way that these fluctuations do not contribute either to the density or to the entropy of matter filling a given volume. If one wishes to talk in the language of vacuum fluctuations one should not forget about their renormalization to zero in quantities which refer to the vacuum (or to small corrections in the properties of real particles interacting with such fields).

Further, it has been shown in[5,6] that the fields corresponding to vacuum fluctuations do not produce the creation of real particles and vacuum polarization for the conformally flat metric[7)]. Therefore the vacuum fluctuations which renormalize to zero today do not yield observable effects even if the zero fluctuations are introduced near the singularity with a subsequent homogeneous isotropic expansion of the Universe.

What then distinguishes the baryons and their displacements? The motion of baryons in ultradense matter can be described by means of phonons; for a rigid equation of state the velocity of the phonons equals the speed of light, but the equations for the phonons are not conformally invariant, and even not Lorentz-invariant. In ultradense matter, for $p = \epsilon$, $T_{0\alpha} = 0$ only in one coordinate system, namely the system where the baryons are on the average at rest and there is no flux of baryon number. It is just because the equations for phonons are not conformally-invariant that phonons can be produced in a Friedmann universe.

We consider perturbations of the metric corresponding to zero-oscillations of the gravitational waves, recalling that the corresponding energy density is renormalized to zero. From dimensional considerations it is clear that the energy density is $\epsilon = \lambda^{-4}$. On the other hand the energy density of a gravitational wave is expressed in terms of the dimensionless perturbation b of the metric and the wavelength as follows

$$\varepsilon = G^{-1}\lambda^{-2}b^2. \tag{18}$$

Consequently, the perturbations of the metric related to zero-oscillations of the gravitational waves depend on the wavelength in the following way

$$b = G^{1/2}\lambda^{-1} = t_0\lambda^{-1}. \tag{19}$$

This expression has a natural form: for $\lambda = G^{1/2} = t_0$ (at the Planck wavelength) the perturbations of the metric are exactly of the order of unity. One could not expect anything else, since in the consideration of gravitational waves the answer does not contain either the particle properties or other dimensional constant, with the exception of G, which characterizes the elasticity of the vacuum. Less trivial is only the character of the dependence of the dimensionless fluctuations on λ.

We now apply the principle of equipartition of perturbations[10] and we assume that the longitudinal fluctuations of the metric are of the same order and have the same spectrum (dependence on λ) as the gravitational waves. We carry this out at the instant t_0, then substitute into the expression (19) the minimal phonon wavelength $\lambda_{min,0} = n_0^{-1/3} = G^{1/2}g^{-1/6}$. Thus we obtain

$$b(\lambda_{min}) = g^{1/6}. \tag{20}$$

This value of b leads according to (12) to

$$s = b^{3/2}g^{-3/8} = g^{-1/8} = 10^{4.5}. \tag{21}$$

As the perturbation of the metric expands the corresponding gravitational waves vary in such a way that the graviton vacuum remains a vacuum. The longitudinal perturbations of the metric and the phonons related to them evolve in such a way that they yield an entropy which is considerably smaller than the observed value. Consequently, in an approach containing the consideration of zero oscillations of the gravitational waves a lowered value of the entropy is obtained for matter; one also obtains a special conclusion, that in the big-bang Universe there are less short-wavelength gravitational waves than in the equilibrium Universe. The perturbation spectrum obtained falls off rapidly in the direction of long waves; the long-wave perturbations which give rise to galaxies are not explained, are not unified with the perturbations which produce entropy and the discrepancy at long wavelengths is particularly large. Thus, at present one does not obtain the observed perturbations from first principles, starting with the quantum fluctuations of the metric[8)].

5. DIFFICULTIES OF THE THEORY OF QUANTUM FLUCTUATIONS OF THE DENSITY

The standard approach to the problem of the amplitude of fluctuations consists in considering the motion of the fluid as a superposition of excitations with different wave vectors **k**, i.e., one performs a spatial Fourier transform $\rho = \bar{\rho} + \Sigma\psi_k e^{ikx}$. In the usual "laboratory" system one considers a stationary situation and does not take into account gravity. In this case, taking into account the elasticity of the fluid (speed of sound a), one obtains for each amplitude ψ_k an oscillator equation

$$\psi_k + k^2a^2\psi_k = 0, \quad \varepsilon_k = a^2\rho^{-1}\psi_k{}^2 + k^{-2}\rho\dot{\psi}_k{}^2. \tag{22}$$

It is natural to assume that the energy of each oscillator is either 1) $\epsilon_k = \omega = ka$ in the quantum case, or 2) $\epsilon_k = T$ for temperature T (measured in units of energy) which is sufficiently high, $T > \omega$, so that classical formulas apply. The first case leads to $\psi_k = (k\rho/a)^{1/2}$, the second, to $\psi_k = (T\rho/a^2)^{1/2}$ (independent of k).

We pass from the amplitude of a separate harmonic (having a dimension different from the dimension of density when one considers random fluctuations and a Fourier integral over infinite space) to the r.m.s. density fluctuation at a given scale $\psi(k)$ (cf. supra, end of Section 3). Omitting numerical factors we have in the system with $\hbar = c = 1$:

$$\psi(k) = k^2\rho^{1/2}a^{-1/2}, \quad \psi(k) = k^{3/2}T^{1/2}\rho^{1/2}a^{-1} \tag{23}$$

respectively for the quantum and high-temperature (classic) cases. We apply the latter formula to the ideal gas: $a^2 = Tm^{-1}$, where m is the particle mass. We obtain

$$\psi(k) = \rho^{1/2}k^{3/2}m^{1/2}, \quad \psi(k)/\rho = k^{3/2}n^{-1/2} = V^{-1/2}n^{-1/2} = N^{-1/2}; \tag{24}$$

here $V = k^{-1}$ is the volume and N is the average number of particles in the volume in which fluctuations are considered. The fluctuations in the number of independent particles can be considered as a result of the addition of acoustic vibrations, which are in thermal equilibrium. Eq. (24) is valid when the temperature is larger than the phonon energy for a phonon of wavelength of the order of the scale under consideration. In a small volume in which the wavelength is small and the phonon energy is larger than T the quantum formula is valid.

A simple rule can be formulated as follows: for a given temperature and volume v one must calculate the fluctuations according to the quantum and the classical formulas and use the larger of the two resulting quantities.

We now turn to the cosmological problem. Assuming the cold model $T \equiv 0$ we use the quantum formula. For cold matter we assume a = c = 1 and apply the formula to the planckian instant of time $t_0 = G^{1/2}$:

$$\psi_0(k) = k_0{}^2\rho_0{}^{1/2}, \quad \rho_0 = G^{-2}, \quad \psi_0(k) = k^2G^{-1}. \tag{25}$$

The corresponding fluctuations of the metric will be

$$b_0(k) = Gk^{-2}\psi(k) = 1. \tag{26}$$

This result is a natural consequence of the assumptions. In the approximation when the fluid is described only by its density and its pressure the nucleon mass does not appear in any way in the theory, and neither does the unique dimensionless parameter g.

As is known from the classical theory of Lifshitz[8], and also follows from considerations about the independent evolution of remote parts of the Universe for $\lambda > t$ (cf.[9]), the quantity b(k) does not depend on time for $t < k^{-1}$. But a dimensionless quantity which does not depend on time and does not contain g, can only be equal to one, as was shown above.

Thus, a naive application of the ideas of quantum fluctuations to cosmology yields a correct form for the spectrum $b \sim k^0$, $\psi \sim k^{-2}$. However, the absolute magnitude of the amplitude of the fluctuations in this conception is 10^4-10^5 times larger than the observed one; thus, the agreement with observations of the type of the spectrum is illusory. For the thermal fluctuations of a fluid with the equation of state (2), temperature T and parameters taken at the planckian instant, we obtain for large masses and wavelengths:

$$(\psi/\rho) = N^{-1/2}g(T/m)^{1/2}, \quad b = (gN)^{1/3}(T/m)^{1/2}.$$

For a mass of $10^{13}\,M_\odot$, $N = 10^{70}$, T = m we obtain $\psi = 10^{-3}\rho$, $b = 10^{11}$. Thus, $b \gg 1$, which seems to signal that the initial assumptions are wrong.

The bases of the naive application of the theory of quantum fluctuations also does not withstand criticism. We enumerate some objections.

1) In the region of very large wavelengths (important for the birth of galaxies) a phantastically low temperature is necessary in order that the temperature fluctuations be smaller than the quantum fluctuations; at the same time the "naive" temperature fluctuations at the planckian instant are too large and absolutely inadmissible: the large-scale perturbations of the metric would be $b \gg 1$.

2) The cosmological situation is nonstationary, according to the uncertainty relation $\Delta E \Delta t = \hbar$ there is no reason to restrict the energy of the fluctuations by $\hbar\omega$ per degree of freedom.

3) If one takes into account the gravitational interaction (or, what is the same, the equations of general relativity) the density perturbations with $\lambda > t$ increase as t^ν with a ν depending on the equation of state. Consequently, at this stage there are no acoustic vibrations and the equation of evolution of the perturbation has the form[9)]:

$$\ddot{\psi}_k + k^2a^2\psi_k - rG\rho\psi_k = 0, \tag{27}$$

so that one cannot introduce the frequency and a positive definite energy for each mode of the perturbation (this difficulty has been overcome, but at the cost of changing the equation of state, in[7]).

One must admit that at the present time there is no logically satisfactory theory, which describes the observable Universe, is based on first principles and our knowledge of elementary particles. It remains to look for semiempirical laws, which remain unreliable, as long as they are not derived from a fundamental theory.

I am grateful to V. N. Gribov, I. D. Novikov, L. B. Okun', R. A. Sunyaev and particularly to A. A. Starobinskiĭ for discussions and help.

1)However, the baryon excess is automatically conserved everywhere.

2)$M_\odot$ is the solar mass, equal to 2×10^{33} g.

3)The metric is $g_i^k = \delta_i^k + b_i^k$; we omit the tensor indices in our order-of-magnitude calculations.

4)Cf. [2], where an attempt was made to derive s theoretically; we do not defend the justification of the expression for s given in that paper, but as an empirical formula it retains its validity.

5)$z \sim 4$ is the assumed instant of individualization of galaxies.

6)We recall that by setting c = 1, we do not distinguish the dimensions of length and time. In centimeters $t_0 = 10^{-33}$.

7)Strictly speaking this assertion refers to fields and particles of zero rest mass $m_0 = 0$, but for particles of mass $m_0 = m$ one obtains an effect proportional to a power of g, i.e., an extremely small effect.

8)We note that in the paper of Sakharov [7] the computation of the quantum fluctuations was based on a substantial change of the equation of state of matter.

9)The dimensionless r is related to ν, $G\rho = t^{-2}$.

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Translated by M. E. Mayer
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