

KINEMATICAL TEST THEORIES FOR SPECIAL RELATIVITY: A COMPARISON

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A comparison of certain kinematical test theories for Special Relativity including the Robertson and Mansouri–Sext test theories is presented and the accuracy of the experimental results testing Special Relativity are expressed in terms of the parameters appearing in these test theories. The theoretical results are applied to the most precise experimental results obtained recently for the isotropy of light propagation and the constancy of the speed of light.

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1. Introduction

In this note we give an overview of existing kinematical test theories designed to describe in a consistent manner tests of the foundations of Special Relativity such as the Michelson–Morley or the Kennedy–Thorndike experiment. Such an overview appears to be useful because different test theories also use different parameters in order to make quantitative statements about the validity of Special Relativity. We compare the parameters which are used in the different test theories. These considerations are stimulated by a renewed interest in this topic due to new theoretical and experimental developments. Recent success in the field of ultrastable cavities already has led to an improvement of the Kennedy–Thorndike test [1] and, in the near future, will enable more precise Michelson–Morley tests [2]. There are also new proposals for space missions (SUMO [3] and OPTIS [4]) to perform new tests of Special Relativity which are expected to give an increase in accuracy of several orders of magnitude, see also [5]. The validity of Special Relativity may also

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be of importance for laser ranging missions like ASTROD. In fact, Lunar Laser Ranging, LLR, has been used for a test of Special Relativity [7]. On the theoretical side, loop gravity [8, 9] and string theory [10, 11, 12] predict modified Maxwell and Dirac equations which violate Lorentz invariance. Violations of Lorentz invariance also arise in extensions of the standard model [13] and in non-commutative field theories [14]. Improved tests of Lorentz invariance may be of great importance because even a hint of a tiny violation of Lorentz invariance will be a strong argument in favour of a quantum gravity theory.

To describe tests of basic principles underlying a theory and to quantitatively express the degree of agreement between experiment and these principles, a theory which allows violations of these principles is required. In the case of Special Relativity such test theories are the Robertson [15] and the Mansouri–Sexl [16, 17, 18] test theories. In these test theories a non-constant speed of light and a violation of the Relativity Principle is possible, implying a violation of Special Relativity. They lead to transformations which are more general than Lorentz transformations and thus do not fit into the framework of Special Relativity.

In these test theories a violation of the principles of Special Relativity is represented by certain non-vanishing parameters or non-vanishing combinations of parameters. Only if a certain set of parameters vanishes, does the corresponding test theory reduce to the Theory of Special Relativity. The vanishing of these parameters corresponds directly to a particular outcome of experiments. The distinguishing feature of these test theories is that the conditions for the validity of Special Relativity are fulfilled if and only if a certain number of experiments give particular results [15]. In the framework of the Robertson or Mansouri–Sexl test theories these experiments are

- (i) the Michelson–Morley experiment, testing the spatial isotropy of the velocity of light,
- (ii) the Kennedy–Thorndike experiment, testing the independence of the velocity of light from the velocity of the laboratory, and
- (iii) the Ives–Stilwell experiment determining the amount of time dilation.

Therefore, a test theory provides us with criteria for a unique test of the validity of theories (within the experimental accuracy, of course).

These remarks are true for the test theories of Robertson and Mansouri–Sexl. While the Robertson test theory is valid for all velocities, the Mansouri–Sexl test theory is specialized for small velocities but relaxes the condition of Einstein synchronization. Since physical results (for example, whether an atom absorbs a photon or not) do not depend on the chosen synchronization, both test theories are physically equivalent for small velocities. For a thorough theoretical analysis of these two test theories with respect to the synchronization, see [19, 20]. Here we do not consider generalizations which include gravity effects into the Mansouri–Sexl test theory [21].

These kinematical test theories should be contrasted with dynamical test theories. While in the first class of theories the transformations between inertial frames are generalized (they are no longer Lorentz transformations) the latter class sets up dynamics for matter and for the electromagnetic field. One such kind of dynamical test theory is the $TH\epsilon\mu$ formalism [22, 23] which reduces to the c^2 formalism as far as Special Relativity is concerned [24]. The distinguishing feature in this test theory is that the velocity of light is different from the maximum velocity of matter. Therefore we have two different velocities. This clearly violates the relativity principle and thus Special Relativity. In this dynamical test theory there is a single parameter which characterizes the various violations of Special Relativity. This means that *within this test theory* one experiment is enough in order to “prove” (in an idealized sense) Special Relativity. Each test theory defines its own set of experiments needed to verify Special Relativity. Furthermore, each test theory defines its own set of experiments which may be used in order to verify SR: For example, in the $TH\epsilon\mu$ framework more experiments than within the Robertson or Mansouri–Sexl formalism may be used to constrain the Special Relativity violating coefficients: in addition to the Michelson–Morley, Kennedy–Thornike and Ives–Stilwell experiments, the much more precise Hughes–Drever experiments.

Recently, more dynamical test theories have been developed. One of these [25] is based on modified Maxwell equations which are used to describe a modified propagation of light violating the constancy of the speed of light. In addition, the same Maxwell equations are used in order to describe the anomalous behaviour of ionic crystals which are taken as a model for the interferometer arms or for the cavity. One result is that, under certain non-exotic circumstances, which are, fortunately, not realised in the experiments carried out so far, the anisotropic light propagation is compensated by the anomalous behaviour of the crystal length. Another dynamical test theory [26] starts from the anisotropic velocity of light within the Mansouri–Sexl formalism and modifies the Maxwell equations in such a way that, in the eikonal approximation, one recovers this Mansouri–Sexl velocity of light. Again, with the same Maxwell equations the behaviour of solids and atoms is calculated which leads to a modified description of tests of Special Relativity and gives improved estimates on the validity of Special Relativity as contrasted to kinematical test theories. Both test theories can specify different sets of experiments needed in order to verify Special Relativity.

However, in this article we do not comment on these dynamical test theories. The purpose of the present work is to collect all descriptions of the three classes of classical tests defined by the kinematical test theories and to compare the various sets of parameters connected with the different formulations of the test theories. That means that we describe the three classes of tests mentioned above

- (i) within the Robertson formalism,
- (ii) within the Mansouri–Sexl formalism, and finally
- (iii) within the linearized Robertson and Mansouri–Sexl formalisms, which is the low-velocity approximation of the general formalism.

In doing so, we describe the tests of the isotropy of the speed of light as well as the tests of the independence of the speed of light from the velocity of the laboratory using interferometers and cavities. In the conclusion we give an overview of the current experimental status and present the estimates from the best experiments in terms of the various parameters connected with the different test theories. Our considerations are extended consistently to second-order parameters. The scope of the present article does not extend to Doppler shift experiments which will be covered elsewhere.

2. The Robertson test theory

2.1. Generalities

In the Robertson frame for the description of tests of Special Relativity [15] one starts with a preferred frame Σ with coordinates (T, \mathbf{X}) and with the line element $ds^2 = dT^2 - d\mathbf{X}^2$. Light propagation is given by $ds^2 = 0$, that is, we have an isotropic speed of light in this frame. In another frame S with coordinates (t, \mathbf{x}) moving with a velocity \mathbf{v} with respect to the preferred frame the line element is given by ($v = \|\mathbf{v}\| = \sqrt{v^2}$)

$$\begin{aligned} ds^2 &= g_0^2(v)c^2t^2 - g_{\parallel}^2(v)\frac{(\mathbf{x} \cdot \mathbf{v})^2}{v^2} - g_{\perp}^2\mathbf{x}_{\perp}^2 \\ &= g_0^2(v)c^2dt^2 - \left(g_{\parallel}^2(v)dx^2 + g_{\perp}^2(v)(dy^2 + dz^2)\right) \end{aligned} \quad (1)$$

where

$$\mathbf{x}_{\perp} = \mathbf{x} - \frac{\mathbf{x} \cdot \mathbf{v}}{v^2}\mathbf{v} \quad (2)$$

is the part of the spatial coordinate which is orthogonal to \mathbf{v} . In the right hand side of this equation, the velocity is chosen to be in x -direction. The propagation of light is given by the condition $ds^2 = 0$. In Special Relativity $g_0(v) = g_{\parallel}(v) = g_{\perp}(v) = 1$. Comparison with the line element in the preferred frame gives $\lim_{v \rightarrow 0} g_0(v) = \lim_{v \rightarrow 0} g_{\parallel}(v) = \lim_{v \rightarrow 0} g_{\perp}(v) = 1$

Here we would like to include two general statements about the role of the preferred frame in these kinematical test theories.

- (1) The only preferred frame one may think of is of course the cosmological frame in which the microwave background radiation is isotropic [27]. Though it is not very probable, this may change as our knowledge of the creation of the universe improves. As a hypothesis, one may think of a gravitational background radiation which may single out a frame different from the frame defined by the microwave background. Since all estimates on the validity of Special Relativity are based on the relative velocity of the laboratory with respect to the preferred frame, our characterization of the validity of Special Relativity depends on our knowledge of the universe. If the preferred frame changes, the estimates will

change. Therefore, kinematical test theories provide no *intrinsic* means to characterize the validity of Special Relativity. Only dynamical test theories make it possible to characterize the validity of Special Relativity without a need to refer to external physical knowledge which is not provided by the theory itself.

- (2) The particular form of the line element in the preferred frame has nothing to do with the relative velocity of the laboratory to the preferred frame. This line element fixes the zeroth order terms in a Taylor expansion of the coefficients $g_0(v)$, $g_{\parallel}(v)$ and $g_{\perp}(v)$. Furthermore, if in the preferred frame the line element is different from that of Special Relativity (like a line element of Finslerian type, for example), we will no longer have a test theory for Special Relativity but rather for a new theory with the other line element.

If light propagates a distance dl in a direction which encloses the angle ϑ with the \mathbf{v} -direction, $dl \cdot \mathbf{v} = v dl \cos \vartheta$, then it needs the time

$$g_0^2(v)c^2 dt^2 = (g_{\parallel}^2(v) \cos^2 \vartheta + g_{\perp}^2(v) \sin^2 \vartheta) dl^2. \quad (3)$$

The corresponding speed of light is given by

$$c(\vartheta, v) = \frac{dl}{dt} = c \frac{g_0(v)}{g_{\parallel}(v)} (1 + \delta g(v) \sin^2 \vartheta)^{-\frac{1}{2}}. \quad (4)$$

where we introduced $\delta g(v) = \frac{g_{\perp}^2(v) - g_{\parallel}^2(v)}{g_{\parallel}^2(v)}$ which vanishes in the case of isotropic light propagation. The speed of light depends on the velocity \mathbf{v} and on the orientation ϑ ; this clearly violates the postulates of Special Relativity. Since any anisotropy is very small, we can expand the square root and get

$$c(\vartheta, v) = c \frac{g_0(v)}{g_{\parallel}(v)} (1 - \frac{1}{2} \delta g(v) \sin^2 \vartheta). \quad (5)$$

For a change of the direction of propagation the relative variation of the velocity of light is given by

$$\begin{aligned} \frac{\Delta_{\vartheta} c}{c} &= \frac{c(\vartheta, v) - c(0, v)}{c(0, v)} = (1 + \delta g(v) \sin^2 \vartheta)^{-\frac{1}{2}} - 1 \\ &\approx -\frac{1}{2} \delta g(v) \sin^2 \vartheta. \end{aligned} \quad (6)$$

If one considers variations of the velocity one cannot take as reference the preferred frame with $\mathbf{v} = 0$ because under usual laboratory conditions we are not in the position to be at rest in the preferred frame. The only feasible variation of the velocity is a change according to $\mathbf{v} \rightarrow \mathbf{v} + \delta \mathbf{v}$. This yields as relative variation of the speed of light (with $\delta' v = \|\mathbf{v} + \delta \mathbf{v}\| - \|\mathbf{v}\|$)

$$\begin{aligned} \frac{\Delta_v c}{c} &= \frac{c(\vartheta, v + \delta' v) - c(\vartheta, v)}{c(\vartheta, v)} \\ &= \frac{g_{\parallel}(v)}{g_{\parallel}(v + \delta' v)} \frac{g_0(v + \delta' v)}{g_0(v)} \sqrt{\frac{1 + \delta g(v) \sin^2 \vartheta}{1 + \delta g(v + \delta' v) \sin^2 \vartheta}} - 1. \end{aligned} \quad (7)$$

Note that this quantity depends on three parameters, namely ϑ , \mathbf{v} , and $\delta\mathbf{v}$. If light propagates isotropically, $c(\vartheta, v) = c(v)$, then this variation reduces to

$$\frac{\Delta_v c}{v} = \frac{c(\vartheta, v + \delta'v) - c(\vartheta, v)}{c(\vartheta, v)} = \frac{g_{\parallel}(v)}{g_{\parallel}(v + \delta'v)} \frac{g_0(v + \delta'v)}{g_0(v)} - 1. \tag{8}$$

We can replace two of the three parameter functions $g_0, g_{\parallel}, g_{\perp}$ by the velocity of light c_{\parallel} in direction of the velocity \mathbf{v} and the velocity of light c_{\perp} orthogonal to \mathbf{v} , compare [19]:

$$c_{\parallel}(v) = c(\vartheta = 0, v) = c \frac{g_0(v)}{g_{\parallel}(v)}, \quad c_{\perp}(v) = c(\vartheta = \pi/2, v) = c \frac{g_0(v)}{g_{\perp}(v)}. \tag{9}$$

Then we have

$$ds^2 = g_0^2(v)c^2 \left(t^2 - \frac{1}{c_{\parallel}^2(v)} \frac{(\mathbf{x} \cdot \mathbf{v})^2}{v^2} - \frac{1}{c_{\perp}^2(v)} \mathbf{x}_{\perp}^2 \right) \tag{10}$$

and

$$c(\vartheta, v) = c \frac{c_{\parallel}(v)c_{\perp}(v)}{\sqrt{c_{\parallel}^2(v) \sin^2 \vartheta + c_{\perp}^2(v) \cos^2 \vartheta}} \tag{11}$$

and

$$\frac{\Delta_{\vartheta} c}{c} \approx -\frac{1}{2} \left(1 - \frac{c_{\perp}^2(v)}{c_{\parallel}^2(v)} \sin^2 \vartheta \right) \tag{12}$$

$$\frac{\Delta_v c}{c} = \frac{c_{\parallel}(v + \delta v)}{c_{\parallel}(v)} - 1 \tag{13}$$

However, the best way to analyze the experiments using these expressions is to expand the functions $g_i(v)$ for $i = 0, \perp, \parallel$ with respect to velocity. If we assume that $g_i(v) = 1 + \delta g_i(v)$, then we have $\delta g_i(v) \ll 1$ which certainly is satisfied because up to now all experiments are in agreement with Special Relativity. Since $\delta g_i(0) = 0$, the lowest order contributions are $\delta g_i(v) = g_i^0 \widehat{v}^2 + g_i^1 \widehat{v}^4 + \mathcal{O}(\widehat{v}^6)$, that is,

$$g_i(v) = 1 + g_i^0 \widehat{v}^2 + g_i^1 \widehat{v}^4 + \mathcal{O}(\widehat{v}^6). \tag{14}$$

where we introduced $\widehat{v} := v/c$. The first term is due to matching the line element for the preferred frame. Then we get from (4)

$$\begin{aligned} c(\vartheta, v) = c \left[1 + (g_0^0 - g_{\parallel}^0) \widehat{v}^2 + (g_{\parallel}^0 - g_{\perp}^0) \widehat{v}^2 \sin^2 \vartheta \right. \\ \left. + (g_{\parallel}^0 (g_{\parallel}^0 - g_0^0) + g_0^1 - g_{\parallel}^1) \widehat{v}^4 \right. \\ \left. - \frac{1}{2} \left((2g_0^0 + g_{\perp}^0 - 5g_{\parallel}^0) (g_{\perp}^0 - g_{\parallel}^0) + 2 (g_0^1 - g_{\parallel}^1) \right) \widehat{v}^4 \sin^2 \vartheta \right. \\ \left. + \frac{3}{2} (g_{\perp}^0 - g_{\parallel}^0)^2 \widehat{v}^4 \sin^4 \vartheta + \mathcal{O}(\widehat{v}^6) \right]. \tag{15} \end{aligned}$$

Now we have two ordering parameters, \widehat{v} and $\delta\widehat{v}$. Usually, $\delta\widehat{v} \ll \widehat{v} \ll 1$ so that for combinations (only even orders of velocities occur) $\widehat{v}^2 \gg \widehat{v} \delta\widehat{v} \gg \widehat{v}^4 \gg \widehat{v}^3 \delta\widehat{v} \gg$

$\widehat{v}^2(\delta\widehat{v})^2$. In the following we neglect terms of order $(\delta\widehat{v})^3$ and $(\delta\widehat{v})^3\widehat{v}$. The general result for the relative variation of the speed of light for a combined variation of orientation and velocity turns out to be

$$\begin{aligned}
& \frac{c(\vartheta, v + \delta v) - c(\vartheta, v)}{c(\vartheta, v)} \\
&= \left[g_{\parallel}^0 - g_{\perp}^0 \right] \widehat{v}^2 \sin^2 \vartheta \\
&+ \left[2 \left(g_0^0 - g_{\parallel}^0 \right) + \left(g_{\parallel}^0 - g_{\perp}^0 \right) (1 - \cos(2\vartheta)) \right] \widehat{v} \cdot \delta\widehat{v} \\
&+ \left[g_0^0 - g_{\parallel}^0 + \left(g_{\parallel}^0 - g_{\perp}^0 \right) \sin^2 \vartheta \right] (\delta\widehat{v})^2 \\
&+ \left[g_{\parallel}^1 - g_{\perp}^1 - \frac{1}{4} \left(g_{\parallel}^0 - g_{\perp}^0 \right) \left(3g_{\parallel}^0 + g_{\perp}^0 \right) - \frac{3}{4} \left(g_{\parallel}^0 - g_{\perp}^0 \right)^2 \cos(2\vartheta) \right] \widehat{v}^4 \sin^2 \vartheta \\
&+ \left[2 \left(g_0^1 - g_{\parallel}^1 \right) - (g_0^0)^2 + (g_{\parallel}^0)^2 - \left((g_0^0 + g_{\perp}^0 - 4g_{\parallel}^0)(g_{\perp}^0 - g_{\parallel}^0) + 2(g_{\perp}^1 - g_{\parallel}^1) \right) \sin^2 \vartheta \right. \\
&\quad \left. + 3(g_{\parallel}^0 - g_{\perp}^0)^2 \sin^4 \vartheta \right] (2\widehat{v}^2\widehat{v} \cdot \delta\widehat{v} + \widehat{v}^2(\delta\widehat{v})^2) \\
&+ 2 \left[2 \left(g_0^1 - g_{\parallel}^1 - g_0^0 g_{\parallel}^0 + (g_{\parallel}^0)^2 \right) - \left((2g_0^0 + g_{\perp}^0 - 5g_{\parallel}^0)(g_{\perp}^0 - g_{\parallel}^0) + 2(g_{\perp}^1 - g_{\parallel}^1) \right) \sin^2 \vartheta \right. \\
&\quad \left. + 3 \left(g_{\parallel}^0 - g_{\perp}^0 \right)^2 \sin^4 \vartheta \right] (\widehat{v} \cdot \delta\widehat{v})^2. \tag{16}
\end{aligned}$$

We discuss three special cases which are of interest for applications: (i) the case of pure rotation, $\delta v = 0$, (ii) a change of the relative velocity of light for the case where there is no orientation dependent change of the velocity of light, and (iii) the relative change of the velocity of light if the direction of the change of the velocity δv is correlated with the direction of the light ray.

(i) Pure rotation. The relative change of the velocity of light is

$$\begin{aligned}
\frac{\Delta_{\vartheta} c}{c} &= \left[\left(g_{\parallel}^0 - g_{\perp}^0 \right) \widehat{v}^2 + \left(g_{\parallel}^1 - g_{\perp}^1 - \frac{1}{4} \left(g_{\parallel}^0 - g_{\perp}^0 \right) \left(g_{\perp}^0 + 3g_{\parallel}^0 \right) \right) \widehat{v}^4 \right. \\
&\quad \left. - \frac{3}{4} \left(g_{\parallel}^0 - g_{\perp}^0 \right)^2 \widehat{v}^4 \cos(2\vartheta) \right] \sin^2 \vartheta + \mathcal{O}(\widehat{v}^6) \\
&= \frac{\Delta_{\vartheta} c}{c} \sin^2 \vartheta + \mathcal{O}(\widehat{v}^4) \tag{17}
\end{aligned}$$

If we observe no change then we conclude from the first order terms that, $g_{\parallel}^0 = g_{\perp}^0$. Using the second order terms, we furthermore get $g_{\parallel}^1 = g_{\perp}^1$.

(ii) If we use the results $g_{\parallel}^0 = g_{\perp}^0$ and $g_{\parallel}^1 = g_{\perp}^1$ in (16), then we get

$$\begin{aligned}
\frac{c(\theta, v + \delta v) - c(\theta, v)}{c(\theta, v)} &= 2 \left(g_0^0 - g_{\parallel}^0 \right) \mathbf{v} \cdot \delta \mathbf{v} + \left(g_0^0 - g_{\parallel}^0 \right) (\delta\widehat{v})^2 \\
&+ \left(2(g_0^1 - g_{\parallel}^1) - (g_0^0)^2 + (g_{\parallel}^0)^2 \right) (2\widehat{v}^2\widehat{v} \cdot \delta\widehat{v} + \widehat{v}^2(\delta\widehat{v})^2) \\
&+ 2 \left(2(g_0^1 - g_{\parallel}^1 - g_0^0 g_{\parallel}^0 + (g_{\parallel}^0)^2) \right) (\widehat{v} \cdot \delta\widehat{v})^2 \tag{18}
\end{aligned}$$

which no longer depends on the orientation θ . Here, the velocity of light depends on v , δv and the angle between v and δv .

- (iii) In the case that the velocity δv is in direction of the speed of light, then we have $\mathbf{v} \cdot \delta \mathbf{v} = v \delta v \cos \vartheta$. In this case the relative change of the velocity of light gives

$$\begin{aligned}
 & \frac{c(\vartheta, v + \delta v) - c(\vartheta, v)}{c(\vartheta, v)} \\
 &= \left[g_{\parallel}^0 - g_{\perp}^0 \right] \widehat{v}^2 \sin^2 \vartheta \\
 &+ \left[2(g_0^0 - g_{\parallel}^0) + (g_{\parallel}^0 - g_{\perp}^0) (1 - \cos(2\vartheta)) \right] \widehat{v} \delta \widehat{v} \cos \vartheta \\
 &+ \left[g_0^0 - g_{\parallel}^0 + (g_{\parallel}^0 - g_{\perp}^0) \sin^2 \vartheta \right] \frac{(\delta v)^2}{c^2} \\
 &+ \left[g_{\parallel}^1 - g_{\perp}^1 - \frac{1}{4} (g_{\parallel}^0 - g_{\perp}^0) (3g_{\parallel}^0 + g_{\perp}^0) - \frac{3}{4} (g_{\parallel}^0 - g_{\perp}^0)^2 \cos(2\vartheta) \right] \widehat{v}^4 \sin^2 \vartheta \\
 &+ \left[2(g_0^1 - g_{\parallel}^1) - (g_0^0)^2 + (g_{\parallel}^0)^2 \right. \\
 &\quad \left. - (g_0^0 + g_{\perp}^0 - 4g_{\parallel}^0)(g_{\perp}^0 - g_{\parallel}^0) + 2(g_{\perp}^1 - g_{\parallel}^1) \right] \sin^2 \vartheta \\
 &\quad \left. + 3(g_{\parallel}^0 - g_{\perp}^0)^2 \sin^4 \vartheta \right] (2\widehat{v}^3 \delta \widehat{v} \cos \vartheta + \widehat{v}^2 (\delta \widehat{v})^2) \\
 &+ 2 \left[2(g_0^1 - g_{\parallel}^1 - g_0^0 g_{\parallel}^0 + (g_{\parallel}^0)^2) \right. \\
 &\quad \left. - 2 \left((2g_0^0 + g_{\perp}^0 - 5g_{\parallel}^0)(g_{\perp}^0 - g_{\parallel}^0) + (g_{\perp}^1 - g_{\parallel}^1) \right) \sin^2 \vartheta \right. \\
 &\quad \left. + 3 \left(g_{\parallel}^0 - g_{\perp}^0 \right)^2 \sin^4 \vartheta \right] \widehat{v}^2 (\delta \widehat{v})^2 \cos^2 \vartheta. \tag{19}
 \end{aligned}$$

For a circular motion we have $\vartheta = \omega t$.

The expression (16) becomes very simple if we consider the first nontrivial effects only. Then we get from (6) and (7)

$$\begin{aligned}
 \frac{\Delta_{\vartheta} c}{c} &= (g_{\parallel}^0 - g_{\perp}^0) \widehat{v}^2 \sin^2 \vartheta + \mathcal{O}(\widehat{v}^4) \\
 &= \frac{\Delta_{\vartheta}^0 c}{c} \sin^2 \vartheta + \mathcal{O}(\widehat{v}^4) \tag{20}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\Delta_v c}{c} &= 2 \left(g_0^0 - g_{\parallel}^0 - (g_{\perp}^0 - g_{\parallel}^0) \sin^2 \vartheta \right) \widehat{v} \delta \widehat{v} + \mathcal{O}(\widehat{v}^4) \\
 &= 2 \left((g_0^0 - g_{\parallel}^0) \widehat{v}^2 - (g_{\perp}^0 - g_{\parallel}^0) \widehat{v}^2 \sin^2 \vartheta \right) \frac{\delta v}{v} + \mathcal{O}(\widehat{v}^4) \\
 &= 2 \left(\frac{\Delta_v^0 c}{c} - \frac{\Delta_{\vartheta}^0 c}{c} \sin^2 \vartheta \right) \frac{\delta v}{v} + \mathcal{O}(\widehat{v}^4), \tag{21}
 \end{aligned}$$

where we introduced the amplitude for an orientation dependence and for a velocity dependence of the relative variation of the speed of light

$$\frac{\Delta_{\vartheta}^0 c}{c} = (g_{\parallel}^0 - g_{\perp}^0) \widehat{v}^2, \quad \frac{\Delta_v^0 c}{c} = (g_0^0 - g_{\parallel}^0) \widehat{v}^2. \quad (22)$$

It is obvious that, besides experimental factors, the accuracy of estimates of $|g_0^0 - g_{\parallel}^0|$ is less than that of estimates of $g_{\parallel}^0 - g_{\perp}^0$ by a factor $2\delta v/v$.

Note that the parameters g_i^0 and g_i^1 do not depend on the velocity v . This means that any experimental values for these parameters give an absolute estimate of the validity of Special Relativity.

We infer from (21) that a change in the velocity of the source will lead to a change in the velocity of the light which is emitted by this source according to

$$c(\vartheta, v + \delta v) = c(\vartheta, v) + \kappa(\vartheta, v) \delta v \quad \text{with} \quad \kappa(\vartheta, v) = 2 \left(g_0^0 - g_{\parallel}^0 - (g_{\perp}^0 - g_{\parallel}^0) \sin^2 \vartheta \right) \widehat{v}^2. \quad (23)$$

This simplified version of a test theory has been taken as an ansatz for the analysis of astrophysical observations which can be used for searches of a variation of the speed of light due to a change of the velocity of the light emitting stars [28].

We use the above results in order to derive the phase shift and frequency shift in interferometric and cavity experiments desinged to search for violations of Special Relativity. If no deviation from the isotropy of the velocity of light is found, then we get $g_{\perp}(v) = g_{\parallel}(v)$. If no dependence of the velocity of light on the velocity of the laboratory is found, then $g_{\parallel}(v) = g_0(v)$. Finally, the function $g_0(v)$ can be fixed by means of Doppler experiments and gives $g_0(v) = 1$. This means that, within the present accuracy of experiments, in all moving frames the line element underlying the propagation of light rays has the Special Relativistic form $ds^2 = c^2 dt^2 - dx^2$.

2.2. Test of the isotropy of light propagation with interferometers

In a Michelson–Morley interference experiment (compare Fig.1) we measure the phase difference $\Delta\phi(v, \vartheta) = \omega(t_2 - t_1)$ where t_1 (t_2) is the time light needs to propagate back and forth along the first (second) interferometer arm. Since $t_i = l_i/c(\vartheta_i, v)$ we get

$$\begin{aligned} \Delta\phi(v, \vartheta) &= \omega \left(\frac{l_1}{c(\vartheta, v)} + \frac{l_1}{c(\vartheta + \pi, v)} - \frac{l_2}{c(\vartheta + \frac{\pi}{2}, v)} - \frac{l_1}{c(\vartheta + \frac{3}{2}\pi, v)} \right) \\ &= 2 \frac{\omega g_{\parallel}(v)}{c g_0(v)} \left(l_1 \sqrt{1 + \delta g(v) \sin^2 \vartheta} - l_2 \sqrt{1 + \delta g(v) \cos^2 \vartheta} \right) \end{aligned} \quad (24)$$

This is the exact phase. Since any angular dependence is small, this can be expanded to give

$$\Delta\phi(\vartheta, v) = 2 \frac{\omega g_{\parallel}(v)}{c g_0(v)} \left(l_1 - l_2 + \frac{1}{2} \delta g(v) (l_1 \sin^2 \vartheta - l_2 \cos^2 \vartheta) \right). \quad (25)$$

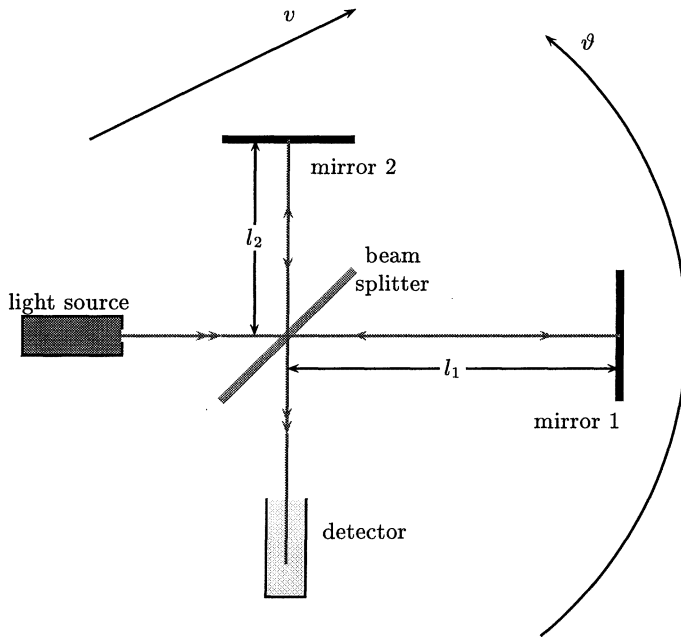


Fig. 1. The Michelson–Morley interferometer with unequal arm lengths $l_1 \neq l_2$. In order to test the isotropy of the speed of light, one has to turn the interferometer, and in order to test the constancy of the speed of light, one has to vary the velocity of the apparatus.

For Michelson–Morley experiments it is most convenient to choose $l_1 = l_2 = l$. Then the exact phase shift is given by

$$\Delta\phi(\vartheta, v) = 2 \frac{\omega l}{c} \frac{g_{\parallel}(v)}{g_0(v)} \left(\sqrt{1 + \delta g(v) \sin^2 \vartheta} - \sqrt{1 + \delta g(v) \cos^2 \vartheta} \right). \quad (26)$$

If the experiments show no effects from rotation for *all* velocities v , then one can easily infer that $|\delta g(v)| = 0$ must hold. For a small angular dependence we get

$$\Delta\phi(\vartheta, v) = -\frac{l\omega}{c} \frac{g_{\parallel}(v)}{g_0(v)} \delta g(v) \cos(2\vartheta). \quad (27)$$

If the phase can be measured with an accuracy $\delta\Delta\phi$, then, for a null experiment, we get the estimate

$$\left| \frac{g_{\parallel}(v)}{g_0(v)} \delta g(v) \right| \leq \frac{c\delta\Delta\phi}{\omega l}. \quad (28)$$

Since all experiments until now are compatible with Special Relativity, we know that $\frac{g_{\parallel}(v)}{g_0(v)} \approx 1$, so that we have the more explicit estimate

$$|\delta g(v)| \leq \frac{c(\delta\Delta\phi)}{\omega l}. \quad (29)$$

Since the left hand side can be written as $\delta g(v) = \frac{g_{\perp}^2(v)}{g_{\parallel}^2(v)} - 1 = \left(\frac{g_{\perp}(v)}{g_{\parallel}(v)} - 1\right) \left(\frac{g_{\perp}(v)}{g_{\parallel}(v)} + 1\right)$ and since, as far as we know, $\frac{g_{\perp}(v)}{g_{\parallel}(v)} \approx 1$, we get the final estimate

$$\left| \frac{g_{\perp}(v)}{g_{\parallel}(v)} - 1 \right| \leq \frac{c(\delta\Delta\phi)}{2\omega l}. \tag{30}$$

With the expansion (14) we get to the fifth order

$$\Delta\phi(\vartheta, v) = 2\frac{l\omega}{c} \left((g_{\parallel}^0 - g_{\perp}^0) \hat{v}^2 + (g_{\parallel}^0 (g_{\perp}^0 - g_{\parallel}^0) + g_{\parallel}^1 - g_{\perp}^1) \hat{v}^4 \right) \cos(2\vartheta) + \mathcal{O}(\hat{v}^6). \tag{31}$$

In terms of $\Delta_g^0 c/c$ this reads, to the third order,

$$\Delta\phi = 2\frac{l\omega}{c} \frac{\Delta_g^0 c}{c} \cos(2\vartheta) + \mathcal{O}(\hat{v}^4). \tag{32}$$

The measured phase difference has the structure $\Delta\phi = A(a\hat{v}^2 + b\hat{v}^4)$. The phase shift $\Delta\phi$ can be measured with a certain accuracy $\delta\Delta\phi$. In order to determine a and b one has to perform the experiment for two different velocities. However, if we assume that no effect has shown up and that there occurs no unfortunate cancellation of terms of different orders, then we can conclude that $a \leq \frac{\delta\Delta\phi}{A} \frac{c^2}{v^2}$ and $b \leq \frac{\delta\Delta\phi}{A} \frac{c^4}{v^4}$. Since $\hat{v}^2 < 1$, the estimate of a is better than the estimate of b . If now b depends linearly on a , as it happens in our case, then, in the second order estimate, this a can be safely neglected. Therefore, if we apply this line of reasoning to our case (31), we can conclude that

$$\left| g_{\parallel}^0 - g_{\perp}^0 \right| \leq \frac{c\delta\Delta\phi}{2l\omega} \frac{c^2}{v^2} \quad \text{and} \quad \left| g_{\parallel}^1 - g_{\perp}^1 \right| \leq \frac{c\delta\Delta\phi}{2l\omega} \frac{c^4}{v^4}. \tag{33}$$

That is, the estimate for the second order term is just given by the estimate for the first order term multiplied with c^2/v^2 .

2.3. Test of isotropy of light propagation with cavities

For a cavity (compare Fig.2) the wave vector for an electromagnetic wave is given by $k = n\pi/l$, where l is the length of the cavity and n the mode number of the standing wave in the cavity. The measured frequency is given by $2\pi\nu(v, \vartheta) = c(v, \vartheta)k$ where we now again assume a direction- and velocity-dependent speed of light. (Here we would like to mention that the dispersion relation is not always of this form. There are dynamical test theories such as the theory with a hypothetical scalar photon mass, see, e.g., [29], or vectorial photon mass [25], where the dispersion relation contains a mass term so that the relation between frequency and wave vector is no longer linear. However, we use the simpler form given here because this still covers a wide class of theories, see, e.g., [30, 31], and is very well supported by many experiments on the photon mass.)

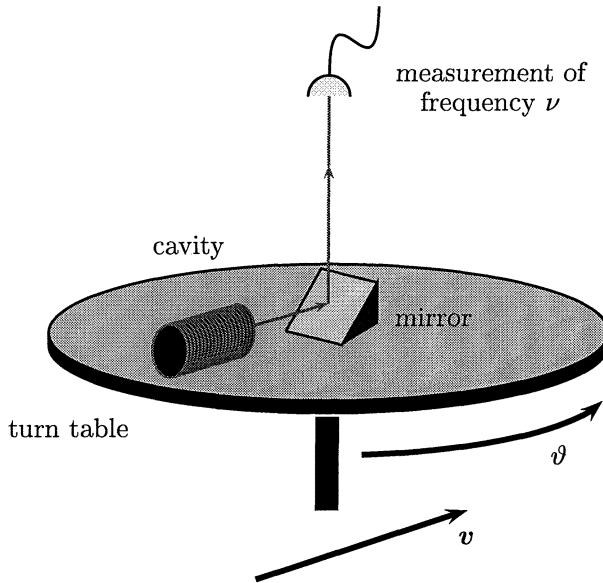


Fig. 2. Setup for a cavity experiment testing Special Relativity. If Special Relativity is violated, then the measured frequency depends on ϑ and v .

With the general result (16) for the speed of light, we get for the relative variation of the frequency

$$\begin{aligned}
 \frac{\Delta_{\vartheta}\nu}{\nu} &= \frac{\nu(v, \vartheta) - \nu(v, 0)}{\nu(v, 0)} \\
 &= \frac{c(\vartheta, v) - c(0, v)}{c(0, v)} \\
 &= \left[\left(g_{\parallel}^0 - g_{\perp}^0 \right) \hat{v}^2 + \left(g_{\parallel}^1 - g_{\perp}^1 + \frac{1}{4} \left(g_{\perp}^0 - g_{\parallel}^0 \right) \left(g_{\perp}^0 + 3g_{\parallel}^0 \right) \right) \hat{v}^4 \right. \\
 &\quad \left. - \frac{3}{4} \left(g_{\parallel}^0 - g_{\perp}^0 \right)^2 \hat{v}^4 \cos(2\vartheta) \right] \sin^2 \vartheta + \mathcal{O}(\hat{v}^6) \\
 &= \frac{\Delta_{\vartheta}^0 c}{c} \sin^2 \vartheta + \mathcal{O}(\hat{v}^4), \tag{34}
 \end{aligned}$$

so that the same combination of parameters is tested by turning the cavity as by turning the interferometer.

Along the same line of reasoning as for the interferometer result, we have that for an accuracy κ in the measurement of the relative frequency, the first and second order estimate turn out to be given by

$$\left| g_{\parallel}^0 - g_{\perp}^0 \right| \leq \kappa \frac{c^2}{v^2} \quad \text{and} \quad \left| g_{\parallel}^1 - g_{\perp}^1 \right| \leq \kappa \frac{c^4}{v^4}. \tag{35}$$

We get a similar result as for the interferometer. The difference lies in the overall accuracy ϵ which in actual experiments is much better than the corresponding

accuracy for measuring phase shifts in interferometers because of the much longer optical path length in a cavity compared to an interferometer.

2.4. Test of the velocity independence of the speed of light with interferometers

Since variations of the velocity of the laboratory are usually small, the results of Michelson–Morley experiments are much better than those of Kennedy–Thorndike experiments. Therefore, in order to describe Kennedy–Thorndike experiments, we assume isotropy of light propagation, that is, $g_{\parallel}(v) = g_{\perp}(v)$. In this case, we get from the exact result (24)

$$\Delta\phi(v) = 2\frac{\omega}{c}\frac{g_{\parallel}(v)}{g_0(v)}(l_1 - l_2). \quad (36)$$

Therefore, for a variation of the velocity v a change in the phase is expected if $g_{\parallel}(v)/g_0(v)$ is different from its Special Relativistic value of 1. If in a certain experiment the accuracy for the determination of the phase shift is given by $\delta\Delta\phi$, then we get the estimate

$$\left| \frac{g_{\parallel}(v + \delta v)}{g_0(v + \delta v)} - \frac{g_{\parallel}(v)}{g_0(v)} \right| \leq \frac{c\delta\Delta\phi}{2\omega(l_1 - l_2)}. \quad (37)$$

In the usual approximation (14) we get

$$\Delta\phi(v) = 2\frac{\omega}{c} \left(1 + (g_{\parallel}^0 - g_0^0) \hat{v}^2 + (g_{\parallel}^1 - g_0^1 + g_0^0 (g_0^0 - g_{\parallel}^0)) \hat{v}^4 \right) (l_1 - l_2) + \mathcal{O}(\hat{v}^6). \quad (38)$$

A variation in the velocity $\mathbf{v} \rightarrow \mathbf{v} + \delta\mathbf{v}$ gives finally

$$\begin{aligned} \Delta\phi &= 2\frac{\omega}{c} \left[(g_{\parallel}^0 - g_0^0) 2\frac{\mathbf{v} \cdot \delta\mathbf{v}}{c^2} + (g_{\parallel}^0 - g_0^0) (\delta\hat{v})^2 \right. \\ &\quad + 4 (g_{\parallel}^1 - g_0^1 + g_0^0 (g_0^0 - g_{\parallel}^0)) \hat{v}^2 \hat{v} \cdot \delta\hat{v} \\ &\quad \left. + 2 (g_{\parallel}^1 - g_0^1 + g_0^0 (g_0^0 - g_{\parallel}^0)) (2\hat{v}^2 (\delta\hat{v})^2 + (\hat{v} \cdot \delta\hat{v})^2) \right] (l_1 - l_2) + \mathcal{O}(\hat{v}^6) \\ &= 2\frac{\omega}{c} (l_1 - l_2) \frac{\Delta_v^0 c \delta v}{c v} + \mathcal{O}(\hat{v}^4), \end{aligned} \quad (39)$$

where we approximated to second order of δv . In the case that we observe a null result and that no unfortunate cancellations occur, we can again conclude from this result the estimates

$$\left| g_{\parallel}^0 - g_0^0 \right| \leq \frac{c\delta\Delta\phi}{\omega(l_1 - l_2)} \frac{c^2}{v\delta v} \quad \text{and} \quad \left| g_{\parallel}^1 - g_0^1 \right| \leq \frac{c\delta\Delta\phi}{\omega(l_1 - l_2)} \frac{c^2}{v^2} \frac{c^2}{v\delta v}. \quad (40)$$

It is obvious that a large change in the velocity δv will lead to better estimates.

2.5. Test of the velocity independence of the speed of light with cavities

For this kind of experiments, we again assume that light propagates isotropically, that is, that $g_{\parallel}(v) = g_{\perp}(v)$. Then we get from the dispersion relation $2\pi\nu = kc$ with $k = \frac{n\pi}{l}$,

$$\nu(v) = \frac{n}{2l} c \frac{g_0(v)}{g_{\parallel}(v)}. \quad (41)$$

From this, we get as the relative frequency variation, by a variation of the velocity of the laboratory,

$$\begin{aligned} \frac{\nu(v + \delta v) - \nu(v)}{\nu(v)} &= \frac{c(\vartheta, v + \delta v) - c(\vartheta, v)}{c(\vartheta, v)} \\ &= 2 \left(g_0^0 - g_{\parallel}^0 \right) \widehat{\mathbf{v}} \cdot \delta \widehat{\mathbf{v}} + \left(g_0^0 - g_{\parallel}^0 \right) (\delta \widehat{v})^2 \\ &\quad + \left(2(g_0^1 - g_{\parallel}^1) - (g_0^0)^2 + (g_{\parallel}^0)^2 \right) (2\widehat{v}^2 \widehat{\mathbf{v}} \cdot \delta \widehat{\mathbf{v}} + \widehat{v}^2 (\delta \widehat{v})^2) \\ &\quad + 2 \left(2(g_0^1 - g_{\parallel}^1 - g_0^0 g_{\parallel}^0 + (g_{\parallel}^0)^2) \right) (\widehat{\mathbf{v}} \cdot \delta \widehat{\mathbf{v}})^2 + \mathcal{O}(\widehat{v}^6) \\ &= 2 \frac{\Delta_v^0 c}{c} \frac{\mathbf{v} \cdot \delta \mathbf{v}}{v^2} + \mathcal{O}(\widehat{v}^4), \end{aligned} \quad (42)$$

so that, to lowest order, this experiment is sensitive to the same quantity $g_{\parallel}^0 - g_0^0$ as the Kennedy–Thorndike experiment. Again, a large δv is good for improved estimates of the quantity $g_{\parallel}^0 - g_0^0$.

Also, in this case, we get that the second-order estimates scale with a factor c^2/v^2 with respect to the first order estimates. If in a null result the accuracy of measuring the relative frequency is given by κ , then

$$\left| g_0^0 - g_{\parallel}^0 \right| \leq \kappa \frac{c^2}{2v\delta v} \quad \text{and} \quad \left| g_0^1 - g_{\parallel}^1 \right| \leq \kappa \frac{c^4}{2v^3\delta v}. \quad (43)$$

3. Mansouri–Sexl test theory

As in the Robertson frame we start with a preferred frame Σ with coordinates (T, \mathbf{X}) . In this reference frame the propagation of light is isotropic

$$ds^2 = c^2 dT^2 - d\mathbf{X}^2. \quad (44)$$

The most general transformation to another frame S' with coordinates (t', \mathbf{x}) , which can be described by means of a relative velocity \mathbf{v} , is given by

$$t' = a(v)T + \frac{e(v)}{c^2} \mathbf{v} \cdot \mathbf{X} \quad (45)$$

$$\mathbf{x} = d(v)\mathbf{X} + b(v) \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{X})}{v^2} + f(v)vT. \quad (46)$$

This is the most general linear transformation which can be described by means of a polar vector \mathbf{v} . The linearity of our ansatz can be based on the requirement that a force-free motion (straight line) in Σ should be a force-free motion in S' , too.

Since the velocity between the origins of S' and Σ is \mathbf{v} we have the additional condition

$$\frac{b(v)f(v)}{d(v)(d(v)+b(v))} - \frac{f(v)}{d(v)} = 1. \quad (47)$$

Using this in the inverse transformation, we have [16, 32]

$$T = \frac{1}{a(v)} \left(t' - \frac{e(v)}{c^2} \mathbf{v} \cdot \mathbf{x} \right) \quad (48)$$

$$\mathbf{X} = \frac{1}{d(v)} \mathbf{x} - \left(\frac{1}{d(v)} - \frac{1}{b(v)} \right) \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{x})}{v^2} + \mathbf{v}T. \quad (49)$$

There is a freedom to choose another synchronization in S' by $t = t' + \frac{1}{c} \boldsymbol{\epsilon}' \cdot \mathbf{x}$ [16] which defines the frame S with coordinates (t, \mathbf{x}) . If we combine the transformations $\Sigma \rightarrow S' \rightarrow S$, then we have

$$T = \frac{1}{a(v)} \left(t - \frac{1}{c} \boldsymbol{\epsilon} \cdot \mathbf{x} \right) \quad (50)$$

$$\mathbf{X} = \frac{1}{d(v)} \mathbf{x} - \left(\frac{1}{d(v)} - \frac{1}{b(v)} \right) \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{x})}{v^2} - \frac{1}{a(v)} \mathbf{v}(\boldsymbol{\epsilon} \cdot \mathbf{x}) + \frac{1}{a(v)} \mathbf{v}t, \quad (51)$$

where we introduced

$$\boldsymbol{\epsilon} := e(v)\hat{\mathbf{v}} + \boldsymbol{\epsilon}'. \quad (52)$$

with $\hat{\mathbf{v}} = \mathbf{v}/c$. This parameter $\boldsymbol{\epsilon}$ describes the chosen synchronization in S . Since for $v = 0$ we stay in the preferred frame, we have $\lim_{v \rightarrow 0} a(v) = 1$, $\lim_{v \rightarrow 0} b(v) = 1$, $\lim_{v \rightarrow 0} d(v) = 1$, and $\lim_{v \rightarrow 0} e(v) = 0$. In Special Relativity we have, with Einstein-synchronization, $a(v) = \sqrt{1 - \hat{v}^2}$, $b(v) = 1/\sqrt{1 - \hat{v}^2}$, $d(v) = 1$ and $\boldsymbol{\epsilon} = -\hat{\mathbf{v}}$.

With these transformations, the line element in a general frame is given by

$$\begin{aligned} c^2 T^2 - \mathbf{X}^2 &= \frac{1 - \hat{v}^2}{a^2} c^2 t^2 - 2 \left(\frac{1 - \hat{v}^2}{a^2} c \boldsymbol{\epsilon} + \frac{1}{ab} \mathbf{v} \right) \cdot \mathbf{x} t - \frac{x^2}{d^2} \\ &\quad + \frac{1 - \hat{v}^2}{a^2} (\boldsymbol{\epsilon} \cdot \mathbf{x})^2 + \frac{2}{ab} (\hat{\mathbf{v}} \cdot \mathbf{x})(\boldsymbol{\epsilon} \cdot \mathbf{x}) + \left(\frac{1}{d^2} - \frac{1}{b^2} \right) \frac{(\mathbf{v} \cdot \mathbf{x})^2}{v^2}. \end{aligned} \quad (53)$$

The velocity of light propagating in a direction with angle ϑ with respect to \mathbf{v} and angle ϑ' with respect to $\boldsymbol{\epsilon}$ can be calculated as

$$\begin{aligned} c(v, \boldsymbol{\epsilon}, \vartheta, \vartheta') &= c \frac{bd(1 - \hat{v}^2)}{bd\boldsymbol{\epsilon}(1 - \hat{v}^2) \cos \vartheta' + ad\hat{v} \cos \vartheta - a\sqrt{b^2(1 - \hat{v}^2) + (d^2 - b^2(1 - \hat{v}^2)) \cos^2 \vartheta}}. \end{aligned} \quad (54)$$

It is obvious that the velocity of light travelling in the opposite direction, $c(v, \boldsymbol{\epsilon}, \vartheta + \pi, \vartheta')$ is different. It is also clear that the expression under the square root is always greater than zero.

The two-way velocity of light in direction ϑ is defined through

$$\frac{2}{c_{(2)}(v, \epsilon, \vartheta, \vartheta')} = \frac{1}{c(v, \epsilon, \vartheta, \vartheta')} + \frac{1}{c(v, \epsilon, \vartheta + \pi, \vartheta')} \tag{55}$$

and can be calculated as

$$c_{(2)}(v, \epsilon, \vartheta, \vartheta') = c_{(2)}(\vartheta, v) = c \frac{bd(1 - \widehat{v}^2)}{a\sqrt{b^2(1 - \widehat{v}^2) + (d^2 - b^2(1 - \widehat{v}^2)) \cos^2 \vartheta}}. \tag{56}$$

This velocity no longer depends on the synchronization ϵ .

A general interference experiment with orthogonal interferometer arms with lengths l_1 and l_2 yields the phase shift

$$\Delta\phi(\vartheta, v) = 2\frac{\omega}{c} \frac{a}{bd(1 - \widehat{v}^2)} \left(l_2 \sqrt{b^2(1 - \widehat{v}^2) + (d^2 - b^2(1 - \widehat{v}^2)) \sin^2 \vartheta} - l_1 \sqrt{b^2(1 - \widehat{v}^2) + (d^2 - b^2(1 - \widehat{v}^2)) \cos^2 \vartheta} \right), \tag{57}$$

which is calculated in the same way as (24). It should be emphasized that the synchronization parameter ϵ completely dropped out. Of course, this is as expected because only two-way velocities of light are involved.

Two of the parameters a , b , and d can again be replaced by the two-way-velocity of light in direction of the velocity \mathbf{v} and in orthogonal direction [19]:

$$c_{\parallel}(v) = c_{(2)}(0, v) = c \frac{b(1 - \widehat{v}^2)}{a} \tag{58}$$

$$c_{\perp}(v) = c_{(2)}(\pi/2, v) = c \frac{d\sqrt{1 - \widehat{v}^2}}{a}. \tag{59}$$

We replace now the function b and d by c_{\parallel} and c_{\perp} and get for the transformations (50,51)

$$T = \frac{1}{a(v)} \left(t - \frac{1}{c} \boldsymbol{\epsilon} \cdot \mathbf{x} \right) \tag{60}$$

$$\mathbf{X} = \frac{\sqrt{1 - \widehat{v}^2}}{a(v)} \left(\frac{c}{c_{\perp}} \mathbf{x} - \left(\frac{c}{c_{\perp}} - \frac{c}{c_{\parallel}} \sqrt{1 - \widehat{v}^2} \right) \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{x})}{v^2} - \frac{1}{\sqrt{1 - \widehat{v}^2}} \mathbf{v} \left(t - \frac{1}{c} \boldsymbol{\epsilon} \cdot \mathbf{x} \right) \right), \tag{61}$$

and for the line element (53)

$$c^2 T^2 - \mathbf{X}^2 = \frac{1 - \widehat{v}^2}{a^2} \left(c^2 t^2 - 2c \left(\boldsymbol{\epsilon} + \frac{\mathbf{v}}{c_{\parallel}} \right) \cdot \mathbf{x} t - \frac{c^2}{c_{\perp}^2} x^2 + \left(\left(\boldsymbol{\epsilon} + \frac{\mathbf{v}}{c_{\parallel}} \right) \cdot \mathbf{x} \right)^2 + \left(\frac{c^2}{c_{\perp}^2} - \frac{c^2}{c_{\parallel}^2} \right) \frac{(\mathbf{v} \cdot \mathbf{x})^2}{v^2} \right). \tag{62}$$

In terms of c_{\parallel} and c_{\perp} we also get

$$c_{(2)}(\vartheta, v) = \frac{c_{\parallel}(v)c_{\perp}(c)}{\sqrt{c_{\parallel}^2(v) \sin^2 \vartheta + c_{\perp}^2(v) \cos^2 \vartheta}}. \tag{63}$$

From this it is clear that the parameter function $a(v)$ has nothing to do with the isotropy of the 2-way-velocity of light. This parameter is responsible for the relativistic time delay.

For an isotropic velocity of light, that is, for $c_{\parallel} = c_{\perp}$, $c_{(2)}(\vartheta, v)$ becomes constant, and the line element takes the usual form if we choose for the synchronization parameter $\epsilon = -\frac{v}{c_{\parallel}}$. For arbitrary synchronization parameter we have Special Relativity with arbitrary synchronization [33, 34, 35, 36, 37, 19].

Now we describe special cases of the general treatment.

3.1. Test of isotropy of light propagation with interferometers

In the case of a Michelson–Morley experiment, we choose $l_1 = l_2 = l$ and get a phase shift of

$$\Delta\phi(\vartheta, v) = 2\frac{l\omega}{c} \frac{a}{bd(1-\hat{v}^2)} \left(\sqrt{b^2(1-\hat{v}^2) + (d^2 - b^2(1-\hat{v}^2)) \sin^2 \vartheta} - \sqrt{b^2(1-\hat{v}^2) + (d^2 - b^2(1-\hat{v}^2)) \cos^2 \vartheta} \right). \quad (64)$$

If no phase shift during variation of ϑ is detected, that is, if $\Delta\phi(\vartheta, v) - \Delta\phi(0, v) = 0$ for all ϑ , then we can conclude from the independence of this expression from ϑ that

$$d^2 = b^2(1 - \hat{v}^2). \quad (65)$$

In principle, this has to be experimentally tested for all velocities v . If in an actual experiment the accuracy for measuring the phase shift is $\delta\Delta\phi$, then we get the estimate

$$a(v) \frac{d^2(v) - b^2(v)(1 - \hat{v}^2)}{b^2(v)d(v)(1 - \hat{v}^2)^{3/2}} \leq \frac{c\delta\Delta\phi}{2l\omega}. \quad (66)$$

For increasing accuracy this means $d^2(v) - b^2(v)(1 - \hat{v}^2) \rightarrow 0$.

3.2. Test of isotropy of light propagation with cavities

Again we use the usual dispersion relation $2\pi\nu = kc$ with $k = n\pi/l$. The frequency then is

$$\nu(\vartheta, v) = \frac{cn}{2l} \frac{b(v)d(v)(1 - \hat{v}^2)}{a(v)\sqrt{b^2(v)(1 - \hat{v}^2) + (d^2(v) - b^2(v)(1 - \hat{v}^2)) \cos^2 \vartheta}}. \quad (67)$$

It is clear that, by turning the cavity, the frequency changes if $d^2(v) - b^2(v)(1 - \hat{v}^2)$ is nonvanishing. If no orientation dependence of the frequency is detected, then the relation (65) for interferometer experiments is obtained here, too.

The relative variation of the frequency turns out to be

$$\frac{\nu(\vartheta, v) - \nu(\frac{\pi}{2}, v)}{\nu(\frac{\pi}{2}, v)} = \frac{1}{\sqrt{1 + \frac{d^2(v) - b^2(v)(1 - \hat{v}^2)}{b^2(v)(1 - \hat{v}^2)} \cos^2 \vartheta}} - 1, \quad (68)$$

where, for simplicity, the orientation $\pi/2$ as reference has been chosen. If in an experiment nothing is found with a given accuracy κ of the relative frequency ratio, then

$$1 - \frac{1}{\sqrt{1 + \frac{d^2(v) - b^2(v)(1 - \hat{v}^2)}{b^2(v)(1 - \hat{v}^2)} \cos^2 \vartheta}} \leq \kappa \tag{69}$$

for all ϑ . This means

$$\frac{d^2(v) - b^2(v)(1 - \hat{v}^2)}{b^2(v)(1 - \hat{v}^2)} \leq \frac{1}{1 - \kappa} - 1 \approx \kappa. \tag{70}$$

3.3. Test of the velocity independence of the speed of light with interferometers

In the case of isotropic light propagation, we get the general phase shift

$$\Delta\phi(v) = 2 \frac{(l_2 - l_1)\omega}{c} \frac{a(v)}{b(v)(1 - \hat{v}^2)}. \tag{71}$$

A search for a velocity dependence of the phase shift requires a change of the velocity: $v \rightarrow v + \delta v$ where, due to physical reasons, $\delta v \ll v$. The measured phase shift then is $\Delta\phi(v + \delta v) - \Delta\phi(v)$. If this difference does not depend on the velocity of the laboratory, then the right hand side of (71) is not allowed to depend on v . That means

$$\frac{a(v)}{b(v)} = K(1 - \hat{v}^2). \tag{72}$$

where K is a constant. If, again, in an actual experiment, the phase shift is determined to an accuracy $\delta\Delta\phi$, then we get from $\delta\Delta\phi \geq \Delta\phi(v + \delta v) - \Delta\phi(v)$ the estimate

$$\frac{a(v + \delta v)}{b(v + \delta v)(1 - (\hat{v} + \delta\hat{v})^2)} - \frac{a(v)}{b(v)(1 - \hat{v}^2)} \leq \frac{c\delta\Delta\phi}{2\omega(l_2 - l_1)}. \tag{73}$$

Since \hat{v}^2 as well as $(\hat{v} + \delta\hat{v})^2$ are of the order 10^{-6} we can safely approximate $1 - \hat{v}^2 \approx 1$ so that

$$\frac{a(v + \delta v)}{b(v + \delta v)} - \frac{a(v)}{b(v)} \leq \frac{c\delta\Delta\phi}{2\omega(l_2 - l_1)}. \tag{74}$$

3.4. Test of the velocity independence of the speed of light with cavities

Here we again assume that light propagates isotropically. Then we get as measured frequency

$$\nu(v) = \frac{cn}{2L} \frac{b(v)(1 - \hat{v}^2)}{a(v)}. \tag{75}$$

If ν does not depend on the velocity of the cavity, then we again arrive at the condition (72).

The relative frequency change is

$$\begin{aligned} \frac{\nu(v + \delta v) - \nu(v)}{\nu(v)} &= \frac{b(v + \delta v)a(v)(1 - (\widehat{v} + \delta\widehat{v})^2)}{b(v)a(v + \delta v)(1 - \widehat{v}^2)} - 1 \\ &\approx \frac{b(v + \delta v)}{a(v + \delta v)} \bigg/ \frac{b(v)}{a(v)} - 1, \end{aligned} \quad (76)$$

where we again neglected \widehat{v}^2 and $(\widehat{v} + \delta\widehat{v})^2$. If the experiment leads to a null result and if the accuracy in measuring the relative frequency change is given by κ , then we have the estimate

$$\left| \frac{b(v + \delta v)}{a(v + \delta v)} \bigg/ \frac{b(v)}{a(v)} - 1 \right| \leq \kappa. \quad (77)$$

4. The linearized Mansouri–Sexl test theory

In the limit for small velocities v the Mansouri–Sexl test theory described above reduces to the linearized theory of Mansouri and Sexl. We expand the parameters of our general theory with respect to v :

$$a(v) = 1 + \left(\alpha - \frac{1}{2}\right)\widehat{v}^2 + \left(\alpha_2 - \frac{1}{8}\right)\widehat{v}^4 + \dots = 1 + \alpha^{\text{MS}}\widehat{v}^2 + \alpha_2^{\text{MS}}\widehat{v}^4 + \dots \quad (78)$$

$$b(v) = 1 + \left(\beta + \frac{1}{2}\right)\widehat{v}^2 + \left(\beta_2 + \frac{3}{8}\right)\widehat{v}^4 + \dots = 1 + \beta^{\text{MS}}\widehat{v}^2 + \beta_2^{\text{MS}}\widehat{v}^4 + \dots \quad (79)$$

$$d(v) = 1 + \delta\widehat{v}^2 + \delta_2\widehat{v}^4 + \dots \quad (80)$$

$$\epsilon = (\epsilon - 1)\frac{v}{c} (1 + \epsilon_2\widehat{v}^2 + \dots), \quad (81)$$

where α^{MS} and β^{MS} are the original Mansouri–Sexl parameters. Our definition of the parameters, which we adopted from Will [32], is characterized by the fact that α , β and δ vanish in the case of the validity of Special Relativity. For Einstein synchronization, ϵ also vanishes.

In this approximation, the line element reads

$$\begin{aligned} s^2 &= [1 - 2\alpha\widehat{v}^2 + (-\alpha + 3\alpha^2 - 2\alpha_2)\widehat{v}^4] c^2 t^2 \\ &\quad - 2[\epsilon + (\alpha - \beta - 2\alpha\epsilon - \epsilon_2 + \epsilon\epsilon_2)\widehat{v}^2] \mathbf{v} \cdot \mathbf{x}t \\ &\quad - [1 - 2\delta\widehat{v}^2 + (3\delta^2 - 2\delta_2)\widehat{v}^4] x^2 \\ &\quad + [\epsilon^2 + 2(\beta - \delta) - (\beta + 2(\delta_2 - \beta_2) + 3(\beta^2 - \delta^2) \\ &\quad - 2\epsilon(\alpha - \beta - \epsilon_2) + 2\epsilon^2(\alpha - \epsilon_2))\widehat{v}^2] (\widehat{\mathbf{v}} \cdot \mathbf{x})^2. \end{aligned} \quad (82)$$

The velocity of light in this approximation is

$$\begin{aligned} \frac{c(\vartheta, v)}{c} &= 1 - \epsilon \widehat{v} \cos \vartheta - [\delta - \alpha + (\beta - \delta + \epsilon^2) \cos^2 \vartheta] \widehat{v}^2 \\ &+ [\beta - \alpha + \epsilon_2 - \epsilon(2(\delta + \alpha) + \epsilon_2) - \epsilon(2\beta - 2\delta + \epsilon^2) \cos^2 \vartheta] \widehat{v}^3 \cos \vartheta \\ &+ \frac{1}{2} [-2\alpha_2 + \alpha(-1 + 2\alpha - 2\delta) + 2\delta_2 \\ &+ \cos^2 \vartheta (-3\beta^2 + 2\beta_2 - 2\delta_2 - \beta(1 + 2\alpha - 6\delta + 4\epsilon) \\ &+ 2\alpha(\delta + (2 - 3\epsilon)\epsilon) - 3\delta(\delta - 2\epsilon^2) + 4(-1 + \epsilon)\epsilon\epsilon_2 \\ &+ (3(\beta - \delta)^2 + 6(\beta - \delta)\epsilon^2 + 2\epsilon^4) \cos^2 \vartheta)] \widehat{v}^4. \end{aligned} \tag{83}$$

where again ϑ is the angle between the light ray and the velocity \mathbf{v} . The two-way velocity of light as defined in (55) turns out to be

$$\begin{aligned} \frac{c_{(2)}(\vartheta, v)}{c} &= 1 + [\delta - \alpha + (\beta - \delta) \cos^2 \vartheta] \widehat{v}^2 \\ &+ \frac{1}{2} [\alpha(-1 + 2\alpha - 2\delta) + 2(\delta_2 - \alpha_2) \\ &- (\beta(1 + 2\alpha + 3\beta) - 2\beta_2 - 2(\alpha + 3\beta)\delta + 3\delta^2 + 2\delta_2) \cos^2 \vartheta \\ &+ 3(\beta - \delta)^2 \cos^4 \vartheta] \widehat{v}^4. \end{aligned} \tag{84}$$

Again, any reference to the synchronization has dropped out. The relative change of the 2-way velocity of light is

$$\begin{aligned} \frac{c_{(2)}(\vartheta, v) - c_{(2)}(0, v)}{c_{(2)}(0, v)} &= (\delta - \beta) \widehat{v}^2 \sin^2 \vartheta + \frac{1}{4} [3\delta^2 - \beta^2 + 4\beta_2 - 2\beta(1 + \delta) \\ &- 4\delta_2 + 3(\beta - \delta)^2 \cos(2\vartheta)] \widehat{v}^4 \sin^2 \vartheta \end{aligned} \tag{85}$$

With the result (83) for the velocity of light we can calculate the general phase shift for an interference experiment

$$\begin{aligned} \Delta\phi(\vartheta, v) &= \frac{\omega}{c} \{ 2(l_1 - l_2) + [(2\alpha - \beta - \delta)(l_1 - l_2) + (\delta - \beta)(l_1 + l_2) \cos(2\vartheta)] \widehat{v}^2 \\ &+ [(\alpha(1 - 2\delta) + 2(\alpha_2 + \delta^2 - \delta_2))(l_1 - l_2) \\ &+ (\beta - 2\beta_2 + 2\delta_2 - 2\alpha\beta + 3\beta^2 + 2\alpha\delta \\ &- 2\beta\delta - \delta^2 - (\beta - \delta)^2 \cos^2 \vartheta) l_1 \cos^2 \vartheta \\ &- (\beta - 2\beta_2 + 2\delta_2 - 2\alpha\beta + 3\beta^2 + 2\alpha\delta \\ &- 2\beta\delta - \delta^2 - (\beta - \delta)^2 \sin^2 \vartheta) l_2 \sin^2 \vartheta] \widehat{v}^4 \} \end{aligned} \tag{86}$$

As in the exact result, all synchronization parameters have dropped out.

Since we expanded the expression to the fifth order, we now have six parameters which need to be determined. Consequently, six experiments would be needed to fix these six parameters.

4.1. Test of isotropy of light propagation with interferometers

For a Michelson–Morley experiment we again choose $l_1 = l_2 = l$ and get from our general result

$$\Delta\phi(\vartheta, v) = 2\frac{l\omega}{c} [(\delta - \beta)\widehat{v}^2 \cos(2\vartheta) + (\frac{1}{2}\beta + \alpha(\delta - \beta) + \beta^2 + \delta_2 - \beta_2 - \delta^2)\widehat{v}^4 \cos(2\vartheta)] . \quad (87)$$

To lowest order, the phase shift is sensitive to the parameter combination $\delta - \beta = \delta^{\text{MS}} - \beta^{\text{MS}} + \frac{1}{2}$. In the next order, it is sensitive to $(\frac{1}{2}\beta + \alpha(\delta - \beta) + \beta^2 + \delta_2 - \beta_2 - \delta^2 = \frac{1}{2}(\alpha^{\text{MS}} + \delta^{\text{MS}} - 2\beta^{\text{MS}}) + \alpha^{\text{MS}}(\delta^{\text{MS}} - \beta^{\text{MS}}) + (\beta^{\text{MS}})^2 - (\delta^{\text{MS}})^2 + \delta_2^{\text{MS}} - \beta_2^{\text{MS}} + \frac{5}{8}$. If no signal is detected, then we can infer

$$\delta = \beta \quad \text{and} \quad \delta_2 = \beta_2 - \frac{1}{2}\beta . \quad (88)$$

In the case of a null result we get the estimates

$$|\delta - \beta| \leq \frac{c(\delta\Delta\phi)}{2\omega l} \frac{c^2}{v^2} \quad \text{and} \quad |\delta_2 - \beta_2 + \frac{1}{2}\beta| \leq \frac{c(\delta\Delta\phi)}{2\omega l} \frac{c^4}{v^4} . \quad (89)$$

4.2. Test of isotropy of light propagation with cavities

We use again the dispersion relation $\omega = ck$ and insert the 2-way-velocity of light. The result is

$$\begin{aligned} \nu(\vartheta, v) = & \frac{n}{L} \{ 1 + [\delta - \alpha + (\beta - \delta) \cos^2 \vartheta] \widehat{v}^2 \\ & + \frac{1}{2} [\alpha(-1 + 2\alpha - 2\delta) + 2(\delta_2 - \alpha_2) \\ & - (\beta(1 + 2\alpha + 3\beta) - 2\beta_2 - 2(\alpha + 3\beta)\delta + 3\delta^2 + 2\delta_2) \cos^2 \vartheta \\ & + 3(\beta - \delta)^2 \cos^4 \vartheta] \widehat{v}^4 \} \end{aligned} \quad (90)$$

From the null result of the experiment by Müller et al [38] we get that all coefficients connected with orientation dependent terms should vanish:

$$\beta - \delta = 0 \quad \text{and} \quad \delta_2 = \beta_2 - \frac{1}{2}\beta , \quad (91)$$

which are exactly the same conditions as derived for the interferometer experiment.

For the relative frequency variation we get

$$\begin{aligned} \frac{\nu(\vartheta, v) - \nu(0, v)}{\nu(0, v)} = & (\delta - \beta)\widehat{v}^2 \sin^2 \vartheta \\ & + \left(\frac{1}{2}\beta + \frac{5}{2}\beta^2 - \beta_2 - 4\beta\delta + \frac{3}{2}\delta^2 + \delta_2 \right) \widehat{v}^4 \sin^2 \vartheta \\ & + \frac{3}{2}(\beta - \delta)^2 (\cos^4 \vartheta - 1)\widehat{v}^4 . \end{aligned} \quad (92)$$

In the case of null experiments and of an accuracy κ of determining the relative frequency shift, we get the estimates

$$\delta - \beta \leq \kappa \frac{c^2}{v^2} \quad \text{and} \quad \frac{1}{2}\beta - \beta_2 + \delta_2 \leq \kappa \frac{c^4}{v^4}. \quad (93)$$

4.3. Test of the velocity independence of the speed of light with interferometers

In the case of isotropic light propagation we get from the general phase shift in the Mansouri–Sexl approximation

$$\Delta\phi(v) = 2 \frac{(l_1 - l_2)\omega}{c} \left(1 + (\alpha - \beta)\widehat{v}^2 + \left(\alpha_2 + \alpha \left(\frac{1}{2} - \beta \right) + \frac{1}{2}\beta + \beta^2 - \beta_2 \right) \widehat{v}^4 \right). \quad (94)$$

To lowest order, this kind of experiment is sensitive to the parameter combinations

$$\alpha - \beta = \alpha^{\text{MS}} - \beta^{\text{MS}} + 1 \quad \text{and} \quad \alpha_2 + \alpha - \beta_2. \quad (95)$$

In the case of a null result and an accuracy of the phase shift of $\delta\Delta\phi$ we get the estimates

$$|\alpha - \beta| \leq \frac{\delta\Delta\phi}{2\omega\Delta l} \frac{c^2}{v^2} \quad \text{and} \quad |\alpha_2 + \alpha - \beta_2| \leq \frac{\delta\Delta\phi}{2\omega\Delta l} \frac{c^4}{v^4}. \quad (96)$$

4.4. Test of the velocity independence of the speed of light with cavities

Here again we use the measured frequency (90) together with the conditions derived from the isotropy

$$\nu(v) = \frac{cn}{2L} \left\{ 1 + [\beta - \alpha]\widehat{v}^2 + \left[\alpha(\alpha - \beta - \frac{1}{2}) + \beta_2 - \frac{1}{2}\beta - \alpha_2 \right] \widehat{v}^4 \right\}. \quad (97)$$

The conditions for a velocity independence of the frequency as tested in the experiment by Braxmaier et al [1] are

$$\delta = \alpha, \quad \alpha = \beta_2 - \alpha_2, \quad (98)$$

which are again the same conditions as for the interferometric experiment. For the relative variation of the frequency we get

$$\frac{\nu(v) - \nu(0)}{\nu(0)} = [\beta - \alpha]\widehat{v}^2 + \left[\alpha(\alpha - \beta - \frac{1}{2}) + (\beta_2 - \frac{1}{2}\alpha - \alpha_2) \right] \widehat{v}^4 \quad (99)$$

and thus the following estimates in the case of a null experiment

$$|\beta - \alpha| \leq \kappa \frac{c^2}{v^2} \quad \text{and} \quad |\alpha - \beta_2 + \alpha_2| \leq \kappa \frac{c^4}{v^4}. \quad (100)$$

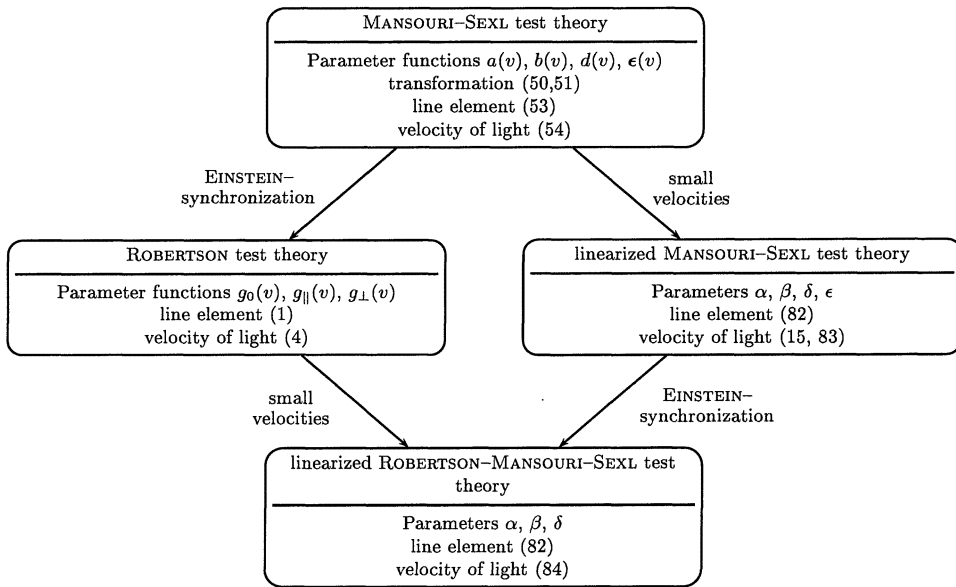


Fig. 3. The relation between the different kinetic test theories. The Mansouri-Sexl test theory and the Robertson test theory are physically equivalent because the latter is the Mansouri-Sexl test theory in the Einstein synchronization. Analogously, the linearized Mansouri-Sexl and the linearized Robertson-Mansouri-Sexl test theories are physically equivalent.

5. Summary and comparison with experiment

The relations between the various test theories discussed in this paper are depicted in Fig.3. The overall assumptions are that the transformation between frames (i) are linear, (ii) depend on the relative velocity only and (iii) that there exists a preferred frame with isotropic propagation of light. While the first assumption can be based on the physical requirement that a linear uniform motion (forceless motion) in one observer system is a uniform linear motion in any other observer system and that the transformations are reversible, the second requirement has no direct physical meaning. It is easy to think of examples where the second requirement does not hold. This is the case e.g. in Finsler-like structures.

In all expressions above, $v \approx 380$ km/s is the velocity of the laboratory or the apparatus with respect to the preferred frame which is identified with the cosmological reference system [27]. The variation of the velocity δv may be due to the rotation of the Earth around its own axis, $\delta v_{\max} \approx 0.3$ km/s (in planned space experiments, the variation δv is the velocity of the satellite which is about a factor 10 larger), or due to the rotation of the earth around the sun, $\delta v \approx 30$ km/s.

The estimates in table 1 are calculated using the various formulas derived above. We also have to know the absolute error in determining a phase shift. This is given by $\Delta\phi \leq 10^{-3}$.

Table 1. Summary of the estimates in the discussed test theories. The estimates are the same for the exact results. In the estimates for the coefficients introduced in the Taylor expansion with respect to the velocity, the first-order coefficients ($e_g, |g_{\perp}^0 - g_{\parallel}^0|, |\beta + \delta - \frac{\delta}{2}|$) are scaled with c^2/v^2 , the second-order coefficients ($e_g, |g_{\perp}^1 - g_{\parallel}^1|, |\delta_2 - \beta_2 + \frac{1}{2}\beta|$) with c^4/v^4 , and so on. The larger the order of the coefficient, the worse the estimate.

Experiment	Method	$\Delta^0 c/c$	Robertson (at $v = 370$ km/s)	Mansouri-Sextl (at $v = 370$ km/s)	Inertized Robertson and Mans.-Sextl
isotropy of velocity of light	interferometry [42]	$\Delta_g^0 c/c \leq 1.5 \cdot 10^{-11}$	$\left \frac{g_{\perp}(v)}{g_{\parallel}(v)} - 1 \right \leq 3 \cdot 10^{-11}$	$\left \frac{a^2(v) - b^2(v)(1-v^2)}{b^2(v)d(v)(1-v^2)^{3/2}} \right \leq 10^{-11}$	$ g_{\perp}^0 - g_{\parallel}^0 = \beta - \delta \leq 3 \cdot 10^{-6}$ $ g_{\perp}^1 - g_{\parallel}^1 = \delta_2 - \beta_2 + \frac{1}{2}\beta \leq 2$
	resonator [38]	$\Delta_g^0 c/c \leq 2 \cdot 10^{-15}$	$\left \frac{g_{\perp}(v)}{g_{\parallel}(v)} - 1 \right \leq 2 \cdot 10^{-15}$	$\left \frac{a^2(v) - b^2(v)(1-v^2)}{b^2(v)d(v)(1-v^2)^{3/2}} \right \leq 2 \cdot 10^{-10}$	$ g_{\perp}^0 - g_{\parallel}^0 = \beta - \delta \leq 2 \cdot 10^{-9}$ $ g_{\perp}^1 - g_{\parallel}^1 = \delta_2 - \beta_2 + \frac{1}{2}\beta \leq 3 \cdot 10^{-3}$
constancy of velocity of light	interferometry [43]	$\Delta_g^0 c/c \leq 10^{-10}$	$\left \frac{g_{\parallel}(v+\delta v)}{g_0(v+\delta v)} - \frac{g_{\parallel}(v)}{g_0(v)} \right \leq 10^{-10}$	$\left \frac{a(v+\delta v)}{b(v+\delta v)} - \frac{a(v)}{b(v)} \right \leq 10^{-10}$	$ g_{\perp}^0 - g_{\parallel}^0 = \alpha - \beta \leq 2 \cdot 10^{-2}$ $ g_{\perp}^1 - g_{\parallel}^1 = \alpha - \beta_2 + \alpha_2 \leq 2 \cdot 10^4$
	resonator [1]	$\Delta_g^0 c/c \leq 2 \cdot 10^{-13}$	$\left \frac{g_{\parallel}(v+\delta v)}{g_0(v+\delta v)} - \frac{g_{\parallel}(v)}{g_0(v)} \right \leq 2 \cdot 10^{-13}$	$\left \frac{b(v+\delta v)/a(v)}{a(v+\delta v)/a(v)} - 1 \right \leq 2 \cdot 10^{-13}$	$ g_{\perp}^0 - g_{\parallel}^0 = \alpha - \beta \leq 2 \cdot 10^{-5}$ $ g_{\perp}^1 - g_{\parallel}^1 = \alpha - \beta_2 + \alpha_2 \leq 2$

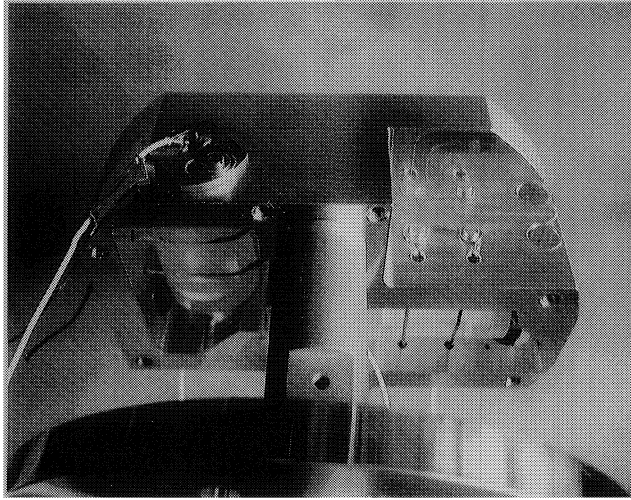


Fig. 4. Photograph of the two cryogenic optical sapphire resonators used in the Michelson–Morley experiment of ³⁸. The resonators operate inside a 4 K cryostat, whereas the lasers providing the interrogating light beams and the laser stabilization electronics are located outside at room temperature. Mounting the cavities inside a holder made from copper of high thermal conductivity reduces sensitivity to temperature changes. The excellent long-term stability of cryogenic sapphire resonators allows to utilize Earth’s rotation.

In comparison to interferometers, cavity experiments are of significantly higher sensitivity: whereas the arms of a classical interferometer are transversed by the interrogating light a few times only, in a modern high-finesse cavity this number can be up to $1 \dots 5 \cdot 10^5$. Thus, for the same level of phase-shift sensitivity, the mechanical length of an interferometer would have to be $> 10^5$ times longer than a cavity. The compact cavities, only a few cm in length, can be shielded from external influences significantly better, and can be operated at cryogenic temperatures ≤ 4 K to obtain low dimensional relaxation processes of the material, and extremely low thermal expansion [39, 40].

The precision of determination of relative variations of frequencies depends on the stability of the cavities. The present stability of the best cryogenic cavities at 4 K (made from sapphire crystals) is of the order $\Delta\nu/\nu \sim 3 \cdot 10^{-15}$ for $10 \dots 1000$ s, and $\sim 1 \cdot 10^{-14}$ for $10 \dots 20$ h. Over 190 days, $< 6 \cdot 10^{-12}$ has been achieved [40]. This leads to much better results than what can be obtained by interferometers. Up to now, the best results for tests of the isotropy of the speed of light were by Brillet and Hall [41], who used room temperature cavities. Compared to cryogenic cavities, these perform very well at short time scales of a few seconds, however over longer time scales are plagued by a relatively strong drift due to relaxation processes in the non-crystalline materials used. Preliminary results [38, 2] with cryogenic cavities indicate a threefold improvement of the test of the isotropy of light propagation where the rotation of the Earth has been used. These experiments aim for a total improvement of > 10 (see Fig. 4).

Further improvement should be possible by active rotation or by space-born operation, where systematic effects (e.g. gravitational bending of the cavities, temperature and air pressure changes) can be reduced or eliminated. For Kennedy–Thorndike tests, the best results were obtained recently by Braxmaier and coworkers [1]. This test may be also improved by space experiments [3, 4]. The performance of both Michelson–Morley and Kennedy–Thorndike tests can thus be expected to improve by a factor of 10 in the near future. Table 1 shows a complete comparison of the best estimates in the frame of the test theories discussed in this paper.

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