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THE SAGNAC VORTEX OPTICAL EFFECT

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It is shown that the Sagnac phenomenon observed experimentally is in conflict with special (and also general) relativity. The calculation of the magnitude of the Sagnac effect within the framework of a relativistic theory results in a paradox. Thus, for the moving relativistic observer, the simultaneous events of meeting of ± rays with the radiant are nonsimultaneous for the events of meeting of the radiant with ± rays. Here, the index "+" denotes a ray propagating in the direction of motion of the observer, while the index "–" denotes a ray propagating against the motion. The paradox is resolved within the framework of the theory of a luminiferous ether at rest.

INTRODUCTION

The Sagnac vortex effect is the phenomenon whereby, in a rotating ring interferometer, the counterwaves acquire a mutual phase shift proportional to the angular velocity of rotation and to the area covered by the interferometer. The light ray propagating in the direction of rotation of the interferometer platform comes to the radiant later than the ray propagating in the opposite direction. From the viewpoint of special (and general) relativity, the effect is quite mysterious and, thus, defies a consistent explanation. Sagnac himself, to explain this effect, used the theory of an ether at rest and has obtained a theoretical value of the effect's magnitude by the classical addition of the velocity of light and the linear rotational velocity of the platform. The discrepancy of this result with experiment is about one percent.

Such a "correct" explanation of the experiment survived in part later, and, as was validly noted by S. I. Vavilov [1]: "...If the Sagnac effect was discovered earlier than the first results of the second-order tests, it, certainly, would be considered a brilliant proof of the existence of the ether... ." However, as the Sagnac effect takes place in a rotating frame of reference, it is customary to assume that the observed effect is also a relativistic effect of general relativity. It is necessary, however, to note that in the literature it is possible to provide an explanation of the effect in terms of not only general, but also special relativity [2]. Also it is necessary to accept that some authors [3, 4] adhere this viewpoint. For example, in [4] we read: "...As mentioned above, the Sagnac effect is an effect of general relativity. Nevertheless, calculations carried out on the basis of special relativity give the same expression for the phase shift of counterwaves in a ring interferometer. Here, we have a rather rare case: for nonrelativistic velocities of motion of the mirrors of a ring interferometer, the predictions of the theory of ether at rest, special relativity, and general relativity give identical results... ."

However, this is perhaps the rarest case in science where erroneous predictions of special and general relativity coincide, merely by chance, with "correct" results of the theory of an ether at rest, which is a misunderstanding. Actually, the Sagnac vortex optical effect is not in the competence of either special or general theory of relativity – the theory leads to a paradox, and the paradox is completely resolvable within the framework of the theory of ether at rest. Therefore, the author has found it pertinent, to avoid misunderstanding, to stress once again the purely ethereal nature of the Sagnac effect that defies a consistent explanation within the framework of both special and general theory of relativity.

1. NEGATION OF THE EXISTENCE OF THE SAGNAC EFFECT IN SPECIAL RELATIVITY

Thus, following [2], we shall consider once again a circular trajectory of propagation of rays in the Sagnac experiment, corresponding to the case of an infinite number of mirrors in a ring interferometer. Let there be an interferometer rotating in a laboratory frame of reference S with angular velocity $+\omega$ clockwise (say, in the plane of this

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page). The rays of light emitted by a moving radiant in both directions propagate in vacuum in the plane of the interferometer platform (with the z-coordinate of the ray being constant: $z = 0$) over a circle of radius r with linear velocity c. The rays (recall that thay are emitted at $t_0 = 0$), in view of the initial state $\varphi_{\pm}(0) = 0$, will meet at time t_1 at which $\varphi_{\pm}(t_1) = \pm\pi$. (Here, $\varphi(t)$ is the current coordinate of the ray, and it is supposed that the angular coordinate of the ray varies continuously and can take any large positive or negative value). Next time the rays will meet at t_2 at which $\varphi_{\pm}(t_2) = \pm 2\pi$, i.e., at a multiple point of the initial state, etc. Here, the index "+" denotes a ray propagating in the direction of rotation of the platform, and the index "–" denotes a ray propagating in the opposite direction. Obviously, the result does not depend on the velocity of the radiant. The velocity of light in the frame of reference S is equal to c by definition.

However, as the angular coordinate of the radiant varies as $+\omega t$, the meeting of the ± rays with the radiant (or, what is same in this case, the meeting of the radiant with the ± rays) will happen at the coordinate (i.e., laboratory) time $t_{\pm}$ determined by the condition $\pm ct_{\pm} = \pm 2\pi r + \omega t_{\pm} r$:

$$t_{\pm} = \frac{2\pi r}{c \mp \omega r}. \tag{1}$$

From the viewpoint of the immobile observer S, the events under consideration are nonsimultaneous. The ray for which the direction of bypass coincides with the direction of rotation comes to the radiant later than the ray for which the direction of bypass is opposite (the radiant has time to move over distance $+\omega t_{\pm} r$). Therefore, the distances covered by the ± rays by the instant they meet with the radiant will also be different:

$$l_{\pm} = \pm 2\pi r + \omega t_{\pm} r = \pm 2\pi r\left(1 \pm \frac{\omega r}{c \mp \omega r}\right). \tag{2}$$

It is easy to be convinced that here the coordinate velocity of light indeed does not depend on the velocity of the radiant, and, thus, the relation $c_{\pm} = l_{\pm} / t_{\pm} = \pm c$ is fulfilled. We stress that this velocity is a physical velocity, though, for example, in [2], for some reason, a difference is made between the so-called physical velocity of light and the coordinate one, having, so to speak, not a physical, but mathematical sense. Actually, the coordinate velocity of light (i.e., by definition, the velocity of light in the laboratory frame of reference S) is just the physical velocity of light observed in the experiment, and it is found by division of the distance covered by light by the time interval during which the given closed travel is executed. It is not clear to the author what is the reason for this difference.

Of interest to us, however, are the instants of meeting of the ± rays with the radiant (which are not identical by definition) not from the viewpoint of the laboratory observer S, but from the viewpoint of the relativistic observer S' moving in a circle. It appears that from the viewpoint of the observer S', the ± rays meet with the radiant simultaneously in complete correspondence with the Einstein idea of the relativity of simultaneity of events. Actually, substitution of the coordinates of events (1) and (2) into the Lorentz transform (written in cylindrical coordinates)

$$l' = \gamma(l - \omega r t),\; r' = r,\; z' = z,\; t' = \gamma\left(t - \frac{\omega r}{c^2} l\right), \tag{3}$$

where $\gamma = 1/\sqrt{1 - \frac{\omega^2 r^2}{c^2}}$ and l is the length of the circle arc, gives simultaneous events:

$$l'_{\pm} = \pm\gamma 2\pi r,\; t'_{\pm} = \gamma \frac{2\pi r}{c}. \tag{4}$$

Thus, the meetings of the ± rays with the radiant (not to be confused with the meetings of the radiant with the ± rays; later we shall see that the latter are not simultaneous), from the viewpoint of the relativistic observer S', are simultaneous. One can check that Lorentz transforms do not alter the velocity of light, i.e., $c'_\pm = l'_\pm / t'_\pm = \pm c$. Thus, we see that special relativity leads to negation of the existence of the Sagnac effect in the framework of the theory itself.

Certainly, the inverse course of reasoning gives the same negative results. Indeed, from the viewpoint of the observer S', the laboratory frame of reference S rotates counterclockwise with angular velocity $-\omega$, while the platform of the interferometer is at rest. The rays emitted at time $t'=0$ from a point $\varphi'_\pm(0)=0$ will meet at the time t'_1 at which $\varphi'_\pm(t'_1)=\pm\pi$. Next time the rays will meet at t'_2 at which $\varphi'_\pm(t'_2)=\pm 2\pi$, i.e., again at a point multiple with respect to the initial one. As the radiant is at rest in the "frame of reference of the platform ", the ± rays will meet with the radiant (or, what is the same, the radiant will meet with the ± rays) simultaneously, i.e., $t'_\pm = \gamma \frac{2\pi r}{c}$ (determined from the equation $\pm c t'_\pm = \pm\gamma 2\pi r - 0$).

This consideration, naturally, also agrees with the Einstein principle of the relativity of simultaneity of events. Actually, if we consider, for example, a rod moving in a frame of reference *S* with some velocity *V*, the light signal having emitted from the middle of the rod and having reached simultaneously, in "the frame of reference of the rod," the equidistant ends of the rod, reaches, in *S*, the approaching end of the rod moving towards this signal earlier than the end moving away (the velocity of the signal is considered the same in both frames of reference). That is, this consideration is completely similar to that with a rotating ring interferometer.

However, on the other hand, according to the clock of the radiant (being at rest at the origin of S'), the events of meeting of the radiant with the ± rays (not to be confused with the events of meeting of the rays with the radiant; the former, as we were convinced, have taken place simultaneously) are nevertheless associated with different times:

$$t'_\pm = \frac{1}{c}\int_0^{t_\pm} ds = t_\pm\sqrt{1-\frac{\omega^2 r^2}{c^2}} = \frac{2\pi r}{c \mp \omega r}\sqrt{1-\frac{\omega^2 r^2}{c^2}}\,, \tag{5}$$

where $ds = c\,dt\sqrt{1-\frac{\omega^2 r^2}{c^2}}$ is the value of the interval at the location of the radiant. The result (5) can be obtained immediately from the Lorentz transform (3) if we substitute into the equation, instead of the current parameter *l*, not the current coordinates of the rays, $\pm 2\pi r + \omega t_\pm r$, but the current coordinates of the moving radiant, $+\omega r t_\pm$, which differ from the coordinates of the rays by $\pm 2\pi r$. Solution (5) is directly opposite to solution (4). It is this second solution (5) that is considered, for example, by the authors of [2] as a special relativistic effect of the theory.

It should, however, be noted that in the general case the invariance of the interval ($s^2 = s'^2$) is not the necessary and sufficient condition for the invariance of the velocity of light ($c'=c$). In actuality, the definition of the interval involves the product of an unknown velocity of light by an unknown time, and this product can generally have an infinite number of solutions, including the solution $c' \neq c$ (see [5]), for example:

$$x' = \frac{x-Vt}{\sqrt{1-V^2/c^2}}, \quad y'=y, \quad z'=z, \quad t'=\sqrt{1-V^2/c^2}, \quad c' = \frac{c - Vx/ct}{1-V^2/c^2}\,. \tag{6}$$

This transform is preferable to the Lorentz transform also for the reason that the Lorentz transform does not satisfy the Sagnac effect nor the principle of correspondence. Actually, for $V/c \ll 1$, the Lorentz transform of time, $t' = \gamma\left(t - Vx/c^2\right)$, goes over not to the classical transform $t' \approx t$, as it should be, but to a transform of the form $t' \approx t - Vx/c^2$ containing a term of the first order of smallness, V/c. For $x/c \gg 0$, the last term in the expression cannot be neglected. Thus, even by this criterion only, the Lorentz transform of time can be considered erroneous.

2. NEGATION OF THE EXISTENCE OF THE SAGNAC EFFECT IN GENERAL RELATIVITY

Thus, we have shown that the special theory of relativity is inconsistent with the Sagnac vortex effect. Certainly, Einstein's general theory of relativity also gives the same inconsistencies. The effect is calculated based on the Newtonian rather than relativistic transforms of coordinates and time. Actually, if we take, for example, the book [6], it appears that the Sagnac effect is considered there in the Newtonian approximation, which is specially mentioned on page 34 of this book. A reference to earlier publications is also given [7], where we see that V. A. Fock, indeed, makes the same note in the monograph [7] on page 164. After a relevant example of evaluating the metric of a rotating frame of reference, he writes: "...Let's stress once again that the above examples have physical sense only in that field where Newtonian mechanics is applicable... ."

Thus, the authors of [6], when calculating the components of the metric tensor of a rotating frame of reference, used classical rather than relativistic transforms of coordinates and time:

$$x = x'\cos\omega t' - y'\sin\omega t', \quad y = x'\sin\omega t' + y'\cos\omega t', \quad z = z', \quad t = t' \tag{7}$$

(though, strictly speaking, it is necessary to consider differential transforms of coordinates and time). It is clear that in this case the squared differential of the interval is not retained (the coefficient of $c^2 dt'^2$ is not equal to unity):

$$\begin{aligned} ds^2 &= \left[1 - \frac{\omega^2}{c^2}\left(x'^2 + y'^2\right)\right]c^2 dt'^2 - 2\omega\left(x'dy' - y'dx'\right)dt' - dx'^2 - dy'^2 - dz'^2 \\ &= \left[1 + 2\frac{\omega}{c^2}\left(y'\frac{dx'}{dt'} - x'\frac{dy'}{dt'}\right) - \frac{\omega^2}{c^2}\left(x'^2 + y'^2\right)\right]c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 \\ &\neq ds'^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 . \end{aligned} \tag{8}$$

Therefore, this transformation is a mere misunderstanding. We, for example, in calculating the components of the metric tensor of a moving frame of reference, use not the classical transforms

$$dx = dx' - Vdt', \quad dy = dy', \quad dz = dz', \quad dt = dt' , \tag{9}$$

but the relativistic ones

$$dx = \frac{dx' - Vdt'}{\sqrt{1 - V^2/c^2}}, \quad dy = dy', \quad dz = dz', \quad dt = \frac{dt' - Vdx'/c^2}{\sqrt{1 - V^2/c^2}} , \tag{10}$$

as we realize that in the first case the squared differential of the interval would be expressed in the same way as, for example, in [6] or [7], that is, with an improper metric, and, thus, its wholeness and invariance would be broken:

$$ds^2 = \left(1 - \frac{V^2}{c^2}\right)c^2 dt'^2 + 2Vdx'dt' - dx'^2 - dy'^2 - dz'^2 \tag{11}$$

$$= \left(1 + 2\frac{V}{c^2}\frac{dx'}{dt'} - \frac{V^2}{c^2}\right)c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 \neq ds'^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 .$$

Only in the second case did we obtain a proper relation satisfying simultaneously the principle of invariance of the velocity of light, the principle of relativity, and the principle of invariance of the differential of the interval.

The relativistic transform for the differential of time in general relativity can be found in a very simple way. It is necessary, first, to form the sum (negative in our case) of squared differentials of the spatial coordinates of the transform

$$-dx^2 - dy^2 - dz^2 = -\left[-2\frac{\omega}{c^2}\left(y'\frac{dx'}{dt'} - x'\frac{dy'}{dt'}\right) + \frac{\omega^2}{c^2}\left(x'^2 + y'^2\right)\right]c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2, \qquad (12)$$

and then solve this equation in combination with the equation

$$c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 \qquad (13)$$

to find the sought-for transform of the differential of time. Thus, we have

$$dt = dt'\sqrt{1 - 2\frac{\omega}{c^2}\left(y'\frac{dx'}{dt'} - x'\frac{dy'}{dt'}\right) + \frac{\omega^2}{c^2}\left(x'^2 + y'^2\right)} \qquad (14)$$

instead of $dt = dt'$, as in [6]. (Recall that in [6], silently and without any reasoning, the transform of the differential of time is taken merely as $dt = dt'$). Thus, the Sagnac effect is a casual mathematical error rather than an effect of general relativity, and, in actuality, no distortion of the space-time in a rotating frame of reference takes place. The rotary motion is as relative as is the translational one. Therefore, if the authors of [6] would use in their calculations purely relativistic (i.e., leaving the squared differential of the interval invariant) transforms of coordinates and time, they should also obtain for a rotating frame of reference an undisturbed metric, i.e., $ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 = ds'^2$, which should result in negation of the existence of the Sagnac effect in general relativity, as was to be shown.

Thus, the key feature of the Sagnac experiment is that the time in a rotating frame of reference can be measured at its equidistant points by the same clock. That is, there is no need for syncing the run of the clock by the method proposed by Einstein, such that a remote clock is silently "tuned" to a negative result by light signals, assuming that their velocities of propagation are invariable (it is believed that the sent signals reach equidistant clocks simultaneously irrespective of their direction). Thus, there is an opportunity to check experimentally the Einstein principle of syncing equidistant clocks. Actually, if in the above example with a moving rod we turn the rod in a circle of radius r so that the ends of the rod come together, we shall obtain an example of a Sagnac rotating ring interferometer for checking this principle. From the viewpoint of both special and general relativity, the signal emitted by a radiant from the middle of the thus curved rod along the rod should reach the rod ends simultaneously. However, the Sagnac experiment shows that the light signal reaches the ends of the rod at different times. Thus, the Einstein principle of syncing a clock is not confirmed in the experiment. Naturally, the transform of time in Lorentz representation, in general, does not prove to be true.

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