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THE PROBLEMS OF THE TIDE

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AS a phenomenon of every-day occurrence the regular rise and fall of the tide must have been noted early in the history of mankind. It so happens, however, that the maritime people of antiquity whose history has come down to us lived close to the shores of the Mediterranean Sea where the tide is very small. As a consequence, the tide received scant attention from these people, since it was of little importance in their every-day affairs, and the tidal knowledge possessed by them was not very extensive.

Not only did the maritime people of antiquity disregard the tide for practical purposes, but even as a subject of study and speculation tidal phenomena received little attention. In consequence of the small range of the tide in the Mediterranean the tidal phenomena were not very impressive, and the regularity of their occurrence was frequently masked by the disturbing effects of wind and atmospheric pressure. So far as biblical literature is concerned, there appears no direct reference to the tide either in the Old or New Testaments. And even in classical literature the passages dealing with tides are relatively few in number.

In common with the early explanations advanced for other physical phenomena, the earlier attempts at explaining the tides were based largely on fanciful notions. Some of the ancient philosophers believed the earth to be an animal; it therefore appeared entirely logical to ascribe the rise and fall of the tide to the breathing of this animal or to its drinking in and spouting out a certain portion of the water. Another explanation based on the same belief regarded the water as constituting the blood of the earth and the tide as the beating of its pulse.

With the growth of a more critical spirit more rational theories were advanced, and we find the tide ascribed to differences in the level of the sea, to the discharge of rivers into the sea, to whirlpools and eddies, and finally to sun and moon. Just how early the connection between moon and tide had been recognized we do not know; we do however have record of the fact that Pytheas of Massilia who lived about 325 B. C., and who navigated the ocean from the Strait of Gibralter to the British Isles, noted a relationship existing between moon and tide.

When we come toward the end of the first century of the Christian era, we find the tides ascribed definitely to the action of sun and moon. In his Historia Naturalis, which appeared in the year 77 A. D., Pliny, the Elder, speaks of the tides in the following words:

> Much has been said about the nature of waters; but the most wonderful circumstance is the alternate flowing and ebbing of the tides, which exists, indeed, under various forms, but is caused by the sun and moon.

In succeeding passages, Pliny describes some of the principal phenomena of the tides. He is aware that there are two high and two low waters in a day; that the tides at any given place follow the moon's meridian passage by an approximately constant interval; that the extent of rise and fall varies with the changing phases of the moon, and that the tides are higher at the times of the sun's equinoxes than at the solstices. He does not however advance any explanation for the relationship of moon and tide except that in the concluding passage of the section devoted to tides he says:

> Hence we may certainly conjecture, that the moon is not unjustly regarded as the star of our life. This it is that replenishes the earth; when she approaches it, she fills all bodies, while when she recedes, she empties them. From this cause it is that shell-fish grow with her increase, and that those animals which are without blood more particularly experience her influence; also, that the blood of man is increased or diminished in proportion to the quantity of her light; also, that the leaves and vegetables generally, as I shall describe in the proper place, feel her influence, her power penetrating all things.

That the tide is brought about by the combined action of sun and moon, Pliny definitely states; but he likewise definitely ascribes the leading rôle to the moon. The early formulation of the problem of the tide may therefore be stated as follows: The moon governs the rising and falling of the surface of the sea; how does the moon do this?

Progress to the Time of Newton

This problem of the agency by means of which the moon exerts its influence on the waters of the earth appears to have engaged the attention of many of the leading philosophers in the centuries following Pliny. Thus at the beginning of the eighth century, some six hundred years after Pliny, we find the noted English scholar, the Venerable Bede, devoting to the tides a chapter in his De Temporum Ratione. And while on the English coast the tide is a much more impressive phenomenon than on the shores of the

Mediterranean, there appears no progress in Bede's remarks toward a solution of the problem of the tide.

Indeed, virtually no progress can be recorded for sixteen hundred years following the time of Pliny. While we find Kepler and others attributing the tides to an attractive force of the moon analogous to magnetic attraction, there were not wanting others, among whom must be mentioned Gallileo, who contended that the idea of the moon being the principal cause of the tides was preposterous. The state of knowledge regarding the tide about the middle of the seventeenth century is well summarized in the Geographia Generalis of Bernhardus Varenius which appeared in 1650. The following quotation is from an English translation by Dugdale in 1733:

> There is no Phænomenon in Nature that has so much exercised and puzzled the Wits of Philosophers and learned Men as this. Some have thought the Earth and Sea to be a living Creature, which, by its Respiration, causeth this ebbing and flowing. Others imagined that it proceeds, and is provoked, from a great Whirlpool near Norway, which, for Six Hours, absorbs the Water, and afterwards, disgorges it in the same space of Time. Scaliger, and others, supposed that it is caused by the opposite Shores, especially of America, whereby the general Motion of the Sea is obstructed and reverberated. But most Philosophers, who have observed the Harmony that these Tides have with the Moon, have given their Opinion, that they are entirely owing to the Influence of that Luminary. But the Question is, what is this Influence? To which they only answer, that it is an occult Quality, or Sympathy, whereby the Moon attracts all moist Bodies. But these are only Words, and signify no more than that the Moon does it by some means or other, but they do not know how: Which is the Thing we want.

Newton's Contribution

With the discovery of the law of gravitation by Newton, the connection between moon and tide received a rational explanation. In his Principia, which appeared in 1687, Newton proved that the tides were a necessary consequence of the law of gravitation. The sun and moon in their varying positions relative to the earth bring about attractive forces which, with regard to the solid earth and the overlying waters, are unequal. And it is these differences of attraction which give rise to the tides.

Newton treated the problem as a static one. Simplifying matters by supposing the sea to cover the whole earth and to assume at each instant a surface of equilibrium, he was able to deduce the principal phenomena of the tides in terms of the theory of gravitation. Thus, from this method of treatment it followed that two high and two low waters should occur each day and that the range of the tide should be greatest about the times of new and full moon and least when the moon was in quadrature. Furthermore, morning and afternoon tides should be unequal except when the sun and moon are in the plane of the equator.

Having shown the adequacy of the theory of gravitation to account for the principal phenomena of the tides, Newton did not push his investigations further, but left the development of the theory of the tides to subsequent investigators. And it appeared, at first, as if the problem of the tide were nearing a complete solution.

But it soon became evident that Newton's theory of the tide could not be made to explain a number of important features. On the assumption of the surface of the sea being a surface of equilibrium in response to the tide-producing forces of sun and moon, the range of the tide should not be over three feet, and should vary from the equator to the poles. As we actually find the tide in nature, however, the rise and fall varies from less than a foot to more than forty feet, but without the slightest relation to latitude. Furthermore, according to this theory, the daily inequality in the tides, that is, the inequality in the two high or two low waters of a day, should be zero at the equator and very considerable in the higher latitudes. Yet we find this inequality quite negligible on the coasts of Europe and very marked in the equatorial regions of the Pacific.

There are other features of the tide which this theory leaves unexplained. In fact, the basis of this static theory is in one respect completely at variance with the actual condition of things, for the surface of the sea in response to the tide-producing forces does not even approximate toward a surface of equilibrium. Nevertheless, this theory of the tide, as formulated by Newton, furnished the foundation on which all subsequent work was based; and in the hands of Daniel Bernoulli (1700-1782) this static theory, known as the Equilibrium Theory, was developed sufficiently to give it practical value in the prediction of tides for any particular port when based on tidal observations made at that port.

It may be of interest to note here that in 1738—some fifty years after Newton's formulation of the law of gravitation—the Académie des Sciences at Paris proposed the problem of the tides as the subject of a prize essay. Two years later this prize was divided among four contestants: Daniel Bernoulli, professor of anatomy and botany at Basel; Colin Maclaurin, professor of mathematics at Edinburgh; Leonard Euler, professor of mathematics at St. Petersburg and the Jesuit Antoine Cavalleri. The three first mentioned based their essays on the principle of gravitation and on Newton's theory of the tide, but Cavalleri based his on Descartes' theory of vortices. This appears to have been the last honor paid to Descartes' theory which had already been abandoned by most philosophers in favor of Newton's more rational theory.

With the discovery of the law of gravitation, the formulation of the problem of the tide became somewhat changed, for it was now no longer a question as to the agency by means of which the moon controlled the tide. The problem now resolved itself to deriving a formula which would express completely the relation between the rise and fall of the sea and the tide-producing forces brought about by the gravitational attraction of moon and sun. This, as we found, Newton's static theory of the tide did not accomplish.

THE DYNAMIC THEORY

Toward the close of his prize essay on the tides, Euler attempted to treat the problem as one of fluid motion. However, the equation he derived to express the tidal conditions is regarded as not expressing the true tidal conditions, but merely somewhat analogous ones. And it is to Laplace that we must credit the first attempt at a solution of the tidal problem as one of fluid motion. In other words, he approached the problem from the standpoint of dynamics and his theory is known as the dynamic theory of the tides.

Laplace's theory of the tides is contained in his Mécanique Céleste, and his contribution has been of profound importance in the development of the subject. He determined the fundamental tidal equations and expressed the tide-producing forces in the form of the potential, from which the actual forces upon any point of the ocean can readily be obtained. He showed further that these forces could be put in the form of a trigonometric series in which the angle varied with the time. But the solution of the equations resulting from the dynamic theory, after introducing the complex conditions of the existing oceans, either surpasses the power of analysis or entails such enormous labors as to be practically impossible. So that Laplace's theory, although very profound, does not succeed in expressing by means of a formula the rise and fall of the tide as we actually find it in nature.

A different approach to the dynamic solution of the problem was made by Airy, who treated the rise and fall of the tide as the movement of waves in canals. While expressly stating that this theory was imperfect, since this mode of treatment would not apply to every part of the ocean, he nevertheless derived a number of important results which serve to explain many of the observed phenomena of the tides in rivers and channels to which none of Laplace's results is strictly applicable.

Following Airy, a number of eminent mathematicians—Ferrel and Harris in America; Stokes, Kelvin, Darwin, Rayleigh, Lamb and Hough in Great Britain; Lévy and Poincaré in France; Börgen in Germany—have added to the further development of the theory of the tides, either by dealing with the matter directly or by investigating some of the mathematical and physical questions involved. In the meantime there had also occurred a very notable increase in our knowledge of the geographical distribution of the tides brought about, in part, by the use of the automatic tide gauge for securing continuous observations over a considerable period of time.

SUBSTITUTION OF "PROBLEMS" FOR "THE PROBLEM"

With the extension of our knowledge of the rise and fall of the tide as it actually takes place in the various oceans, it became evident that the use of a simple mathematical formula to express the phenomenon was becoming increasingly difficult. For the coordination of the material at hand necessitated such an overloading with corrections of the simpler formulæ previously in use that the unity and simplicity assumed became altogether fictitious.

There thus came to be a tacit recognition of the fact that instead of being confronted with a problem of the tide, the phenomenon involves a number of problems. In other words, the tides as they manifest themselves in the various oceans constitute, not a single phenomenon, but a number of phenomena united only by the bond of a common sustaining force in the gravitational action of sun and moon.

The earliest formulation of the problem of the tide involved the determination of the agency whereby the moon controlled the tide. With the announcement of the law of gravitation, the problem shifted to deriving a mathematical formula to express completely the rise and fall of the tide at any point in response to the tide-producing forces of sun and moon, this involving the assumption that the tide represents a world phenomenon. Now we come to a further shift in the recognition that the phenomena of the tides as we find them in nature involve a number of problems. As matters stand now we may formulate the problems of the tide as follows: Given the tide-producing forces of sun and moon and the form, size, depth and location of an ocean basin or other body of water; required the resulting tidal phenomena.

It is to be noted that in the present formulation of the old "problem of the tide" there is a tacit recognition of the fact that the tide may not constitute a single world phenomenon, and that the tides in any given ocean basin may be independent, to a very large extent, of the tides in the other oceans. This change of view resulted directly from the increased knowledge of the behavior of the tides at various places. The tides of the north Atlantic were the ones with which the first investigators were familiar, and the ones with which they compared their theories. The tides of the

Pacific were found to be considerably different, and the tides in the Gulf of Mexico differed still further. And as accurate observations for the lesser known regions increased, further differences in the tides were brought to light.

This increase in the store of accurate information regarding the tides of the seven seas, while disastrous to the elegance of the solution of the problem of the tide, permitted a mechanical conception of the movement of the tide. By a synthesis of the results of these widely scattered tidal observations it had become possible to construct a theory, based on the observed times and heights of the tide, as to the mechanism whereby the tides along the various coasts are brought about by the tide-producing forces of sun and moon.

Whewell's Progressive Wave Theory

In 1833 William Whewell presented before the Royal Society of London a memoir entitled "Essay Towards a First Approximation to a Map of Cotidal Lines." Included in this memoir was a map of the world on which were drawn so-called cotidal lines, that is, lines joining points at which high water occurs at the same time. On this cotidal map, the tide is shown progressing from south to north in the Atlantic, Pacific and Indian Oceans, while in the Southern Ocean, the belt of water that completely encircles the globe southward of the great land masses, the tide is shown as progressing westward.

It is to be remembered that tidal observations have been confined almost without exception to the immediate vicinity of the coast, and that over the wide expanses of the open ocean the time of high water is not known from direct observations. The joining by a cotidal line of two points separated by a wide expanse of water, can only be made in accordance with certain assumptions, and the entire character of a cotidal map depends on these assumptions. Whewell expressly emphasized this by stating in his conclusion to the memoir "I shall be neither surprised nor mortified if the lines which I have drawn shall turn out to be in many instances widely erroneous: I offer them only as the simplest mode which I can now discover of grouping the facts which we possess."

The name of Whewell and also that of Sir John Lubbock (1803-1865) should have been included in the list of those whose work contributed considerably to the advancement of our tidal knowledge. Dealing largely with observational results these two investigators analyzed and coordinated enormous masses of tidal data at various ports. And it was at Whewell's suggestion that in 1835 the United States and several European countries cooperated in securing simultaneous tidal observations covering a period of about three weeks at a number of points.

As represented by Whewell the tide has its origin in the Southern Ocean. Here, it was argued, the tidal forces have almost uninterrupted sway and the moon in its journey around the earth compels the tide in this ocean to keep time with its own motion. And it is from this tide wave, which is constrained to keep step with the moon, that tides are propagated to the north through the three great channels of the Atlantic, Pacific and Indian Oceans.

Whewell's progressive wave theory, or, as it is frequently called, the Southern Ocean theory, therefore sets up the forced tide wave in the Southern Ocean as dominating the tides of the world. From this primary forced tide wave, progressive waves set northward through the various oceans at a rate dependent on the depth of the tidal waterway. And the differences in the times, ranges and types of the tide are accounted for as being due to differences in depth and width of channel, to changes in the configuration of the shore line and to interferences of tide waves coming from different directions.

This progressive wave theory has many things in its favor: it is very plausible and explains certain features of the tides as they are found in nature. And it has had many distinguished proponents, notably Sir George Darwin, son of the great Darwin and himself a mathematician of the highest rank. To quote Darwin, "It is interesting to reflect that our tides to-day depend even more on what occurred yesterday or the day before in the Southern Pacific and Indian Oceans than on the direct action of the moon to-day."

Too many of the characteristics of the tides however, are left by the progressive wave theory to be explained by changes in cross section of channel, configuration of coast line and by interferences of tide waves coming from different directions. Moreover, a number of investigators had from time to time suggested stationary waves or oceanic oscillation as a probable explanation of the very considerable rise and fall of the tide at many places on the open coast. And at the beginning of the present century the stationary wave was made the basis of a new theory of the tide.

Harris' Stationary Wave Theory

This newer theory is diametrically opposed to the ideas advanced by the Southern Ocean theory of the making of the tide. It does away with the conception of a single world phenomenon and substitutes regional oscillating areas as the origin of the dominant tides of the various oceans. It may be of interest to note here that the older theory is due to European mathematicians and tidal workers, while the newer theory is the outgrowth of American genius. Almost entirely, the stationary wave theory is the work of one man, the late R. A. Harris of the United States Coast and Geodetic Survey. Before taking up this newer theory, it will be of advantage to digress for a moment to a consideration of progressive and stationary waves.

Along the coast we are familiar with the waves that come in from the ocean, the crests of which progress uniformly from point to point. If for the moment we call the crest of such a wave high water and its trough low water, it is evident that when this wave travels over a body of water, the times of high and low water will progress uniformly from one end to the other of the body of water. This kind of wave is known as a progressive wave, and such a wave travels with a speed depending on the depth of water.

A wave of a totally different kind may also be made to travel through a body of water. Suppose we have a vessel, say a rectangular tank, partly filled with water. If we raise and then immediately lower one end, a wave will be started which puts into oscillation the whole body of water. But it will be noticed that high water will occur at one end when it is low water at the other end, and that for the body of water as a whole, high water will occur simultaneously for one half at the same instant that it is low water for the other half. This type of wave is known as a stationary wave.

If we start stationary waves in tanks of various lengths filled with water to different depths, we will find that the time taken for a wave to travel from one end to the other and back, or the period of the wave, depends only on the length of the tank and the depth of the water. And if it is desired to maintain a wave of this kind in a tank, it is only necessary to apply a slight force to the tank at regular intervals; but it will be found that if this force is applied at intervals that coincide with the period of the wave we will have the maximum results.

Now to come back to the tides, the Stationary Wave theory states that the dominant tides of the world are caused by stationary waves which are set up and maintained in various portions of the oceans by the periodic tidal forces of sun and moon. According to this theory therefore, the tides do not constitute a general world phenomenon, but are local phenomena, the tides of any given region being due primarily to the stationary wave oscillation of that region.

The principal tidal forces of sun and moon have a period of about half a day. A stationary wave of the same period, in the deep waters of the ocean, has a length of approximately five thousand miles. On a map of the world that shows soundings we can therefore locate regions which have the requisite lengths and depths to support a stationary wave having the same period as the principal tidal forces. Dr. Harris has done this and has outlined the systems of oscillating areas for the various oceans; and furthermore, by theoretical considerations, he has connected the phases of oscillation of these systems with the phases of the tide-producing forces.

The stationary wave theory thus makes of the tides of any given body of water a separate and distinct problem. If the body of water is small and sufficiently deep, we shall have equilibrium tides, that is, the surface will arrange itself normal to the direction of terrestrial gravity as disturbed by moon and sun. If the body of water is situated along the coast, the tide may be due either to a progressive wave from an oscillating system of the open sea or to a dependent stationary wave excited in the body of water itself. But in the open ocean the dominant tide is due to a stationary wave oscillation brought about by the tide-producing forces of sun and moon acting upon such portions of the ocean basin as are susceptible of sustaining stationary waves having the same period as the tide-producing forces.

It was unfortunate for the stationary wave theory that at its birth it met with adverse criticism at the hands of Sir George Darwin, who dissented absolutely from the views advanced by Harris. In his well known book on The Tides and Kindred Phenomena in the Solar System, Darwin further disparaged this feature of Harris's work by stating that "One cannot but admire his courage in attacking so formidable a problem; but I do not propose to explain his conclusions because I cannot bring myself to believe in the trustworthiness of the principles on which he relies."

Darwin's adverse criticism, in view of his well-deserved reputation as an authority on tidal matters, together with the weight carried by the name of Darwin, resulted in bringing Harris' theory into disfavor in Europe for some time. But in 1910 there appeared the third volume of Poincaré's Leçons de Mécanique Céleste in which after subjecting the various tidal theories to searching analysis, the great master states "Il est vraisemblable que la théorie définitive devra emprunter à celle de Harris, une part notable de ses grandes lignes." Due to Poincaré's exposition of Harris' work and also to the ease with which a number of otherwise baffling questions can be answered by the aid of the stationary wave theory, recent tidal researches have come more and more to be based on this newer theory of the tide.

The Prediction of Tides

In the development of the theory of tides, a number of interesting collateral problems have been brought to light. As examples we may mention the determination of the mass of the moon from the observed heights of the tide; the prediction of the times and heights of high and low water; the determination of the rigidity of the earth from tidal observations; the effects of tidal friction; the variations in mean sea level. Even the briefest discussion of these collateral problems would not be possible in the present paper and we shall therefore limit ourselves to summarizing the work on two of these problems, namely, the prediction of tides and the effects of tidal friction.

An advance knowledge of the times and heights of high and low water is obviously of considerable importance to the mariner in entering or leaving a harbor; and the practical value of such knowledge led, early, to the prediction of tides for the construction of tide tables. It may perhaps be of interest to note here that the oldest tide table of which there is record is one now in the library of the British Museum. It is a manuscript table that appears to have been written in the thirteenth century, and gives the time of "flod at london brigge," that is, the time of high water at London Bridge. The time of high water as shown in this old tide table is made to increase by a constant difference of 48 minutes from day to day and is given not for calendar days of the month, but only with reference to the age of the moon.

To predict the tides two different methods have been employed. The older one, technically known as the nonharmonic method, is based on the close relationship existing between the time of high or low water at any given place and the moon's meridian passage. It begins by determining, from tidal observations made at the port for which predictions are desired, the time intervals elapsing between the moon's meridian passage and the occurrence of high and low water. These time intervals, known respectively as the high-water and low-water lunitidal intervals, have an approximately constant value for any given place and after having been once determined from a month or more of observations, may be used for making a rough tide table for that place by adding to the times of the moon's meridian passage as given in a nautical almanac.

As stated above, the lunitidal intervals for any given place are only approximately constant. During a lunar month they undergo periodic changes, depending principally on the variations in phase and declination of the moon. From long series of tidal observations these periodic changes may be determined, and by using these as corrections to the lunitidal intervals, satisfactory predictions of the times of high and low water may be secured. And for many years the tide tables issued in the various countries were constructed substantially as here outlined.

The height of the tide was predicted in a similar manner. The average heights of high and low water at the port for which predictions were desired were determined from observations. To these average heights there were then applied corrections for changes in the phase and parallax of the moon, these corrections likewise being derived from observations. And the tide tables produced by this method worked quite satisfactorily for Europe and for the Atlantic coast of the United States, where the tide is of a simple type. But when the nonharmonic method is used for the prediction of tides of a more complex type, such as found on the shores washed by the Pacific and Indian Oceans, it necessitates so many corrections as to become prohibitive. Before the need for accurate tide tables covering the whole maritime world became pressing, the mathematician had introduced a more powerful method for the prediction of tides, known as the harmonic method.

In the harmonic method the tide is conceived as being made up of a number of simple harmonic waves, each of which may be referred to some motion of sun or moon. In other words for sun and moon as tide-producing agencies this method substitutes a number of hypothetical tide-producing bodies which, with respect to the earth, have circular orbits in the plane of the equator. Each of these simple tide-producing bodies is assumed to give rise to a tide of its own kind, and the tide as it actually occurs in nature is thus considered as being made up of a number of simple harmonic tide waves each of which has a period corresponding to the period of its particular hypothetical tidal body.

The periods of revolution of the assumed tidal bodies and hence the periods of the simple constituent tides, are determined once for all from the known motions of sun and moon. These periods being independent of local conditions are therefore the same all over the earth; what remains to be determined for the various simple constituent tides is their phases and amplitudes which vary from place to place and which can be determined accurately only from observations. The mathematical process by which these phases and amplitudes are disentangled from the tidal observations at any place is a very ingenious one known as the harmonic analysis and is due to that versatile British mathematician, William Thomson, better known as Lord Kelvin, who first proposed it in 1867. Since that time the harmonic analysis has been extended and perfected chiefly by Darwin, Ferrel and Harris.

Now it is obvious that when the period, phase and amplitude of a simple harmonic constituent tide is known, it is not a difficult matter to find the height of the tide due to that constituent at any given future time. To predict, therefore, the tide that will actually occur at some future time, it is only necessary to add together the heights of the constituent tides at that time. The labor involved in doing this by ordinary methods of computation, however, is so great as to be prohibitive, and it was only after Lord Kelvin had devised a machine which mechanically effects the summation of all the various tidal components, that the prediction of tides by the harmonic method was put on a practical basis.

Since Kelvin's first tide predictor, made in 1872, there have been introduced improved mechanical tide predictors, notably two devised in the U. S. Coast and Geodetic Survey, the earlier one in 1883 and the later one in 1910. In the tide tables for our own country, issued annually in advance by the Coast and Geodetic Survey, all the predictions are made by means of the mechanical tide predictor and this is also true to a large extent of the tide tables published by the other leading maritime nations. And the accuracy of these predictions as determined by comparison with the actual times and heights of the tide is all that one can expect in view of the disturbing influences of wind and weather. The problem of the prediction of tides, in so far as this is based on previous observations made at any given port, may therefore be considered as completely solved.

The Effects of Tidal Friction

In the motion of the waters brought about by the tides, friction is produced by the movements of the water particles against each other, by the movement of the water over the beds of seas and rivers and by the movement of the water on the shelving shores along the coast. There is, furthermore, friction produced in the yielding of the solid earth to the tide-producing forces of sun and moon. This tidal friction consumes energy which can come only from the rotational energy of the earth. In other words, tidal friction acts as a sort of brake on the rotating earth, tending to reduce its rotational velocity and as a consequence tending to make the day longer. The stock of energy possessed by the earth, however, is so enormously great as compared with the friction produced by the tides that it is only by a minute quantity that the day is lengthened by tidal friction even over a period of years. And while attempts at an accurate numerical estimate of the amount of this lengthening of the day has thus far been unsuccessful it appears probable that it is of the order of something like the thousandth part of a second in a century.

The effect of tidal friction is not confined to the earth alone, but makes itself felt on the moon. A mathematical investigation proves that besides decreasing the rotational velocity of the earth, tidal friction also tends to increase the period of revolution of the moon and to increase the distance between earth and moon. All these effects of tidal friction are exceedingly small now, but they have been operating for untold ages, so that by this time the cumulative effect must be considerable.

With these conclusions as to the effects of tidal friction, suppose we go backward in time and attempt to trace the early history of the earth and moon. Let us go so far back that the whole life of man on this earth is but a day in the reckoning, back to the time when according to the nebular hypothesis our earth was a molten mass. As we travel backward through time we are undoing the effects of tidal friction and the day is becoming shorter, the moon is approaching nearer the earth and the month is becoming shorter.

But it is to be noted that when the moon was nearer the earth the tides raised were much greater than now, for the tide-producing power of a heavenly body varies inversely as the cube of its distance from the earth. And aside from the increased friction due to the greater tides, the tidal friction varies inversely as the cube of the moon's distance from the earth. Hence the efficiency of tidal friction in increasing the length of day and month and the distance between earth and moon varies inversely as the sixth power of that distance. So that when the moon was one tenth her present distance from the earth, the effects of tidal friction were one million times as great as they are now. It follows therefore that although the effects of tidal friction now are excessively small, they were enormously greater in the remote past when the moon was nearer the earth.

Based on considerations as outlined above, Sir George Darwin has investigated the subject mathematically and developed an exceedingly interesting and very plausible theory as to the early history of earth and moon, from which it appears probable that the moon was at one time part of our earth.

The effect of tidal friction is to make both the day and month longer, but the increase in the length of the day is greater than the increase in the length of the month. It follows therefore, disregarding any counteracting influences that may intervene, that a time will come in the distant future when the day and the month will be of the same length. At this time moon and earth will be presenting the same faces to each other all the time and the moon will have ceased producing any tides on the earth, although the sun will still bring about a rise and fall of the surface of the sea.

The above is but a very meagre outline of this exceedingly interesting problem. But taken with what precedes it is probably sufficient to indicate the nature of some of the problems of the tide. Prior to the formulation of the law of gravitation, the study of the tide had engaged the attention of the leading philosophers; and since Newton's time many distinguished mathematicians have contributed to its development. It still constitutes a fertile field for research, offering to the investigator a number of interesting problems.