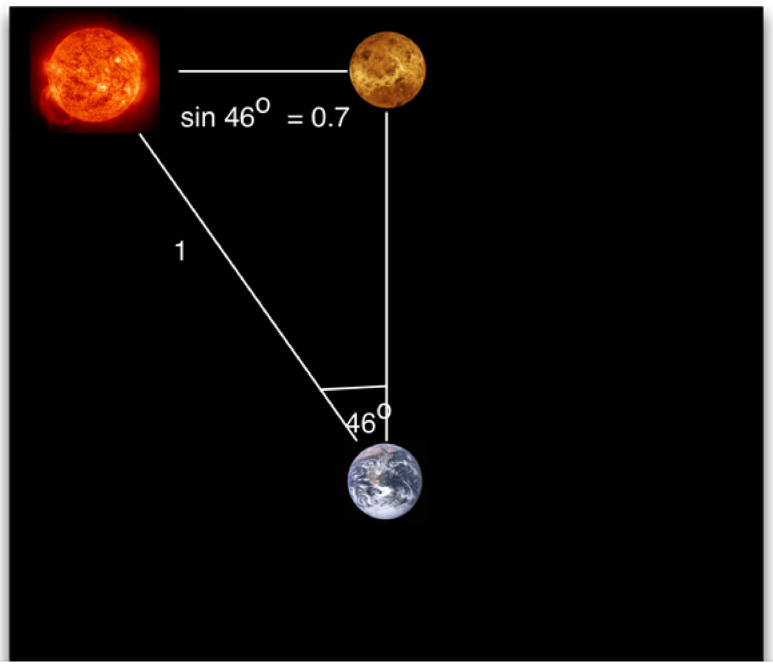
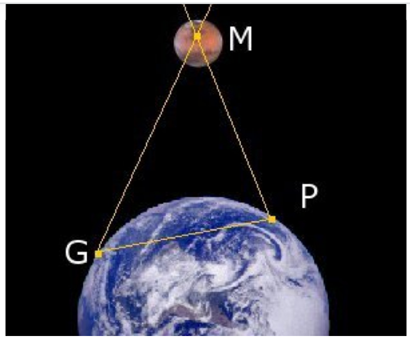
How did Newton calculate the distance between the Earth and the Sun to calculate the mass of the Earth?

Rick McGeer
Former Distinguished Technologist at Hewlett-Packard Labs (2003–2014) · Upvoted by Dimitris Mouratidis, MSc Civil Engineering, University of Portsmouth, UK (2011) · Updated 4y

- He didn't. Newton didn't know the mass of the Earth OR the mass of the Sun. Kepler knew the *relative* distances of the planets in 1619 — he needed that to derive his third law, the lovely Kepler's Harmonic Law — but he didn't know the absolute distance. The relative distances can be worked out by trigonometry. For example, when Venus is at its maximum height in the sky, it's about 46° degrees from the Sun. This means that the Earth, the Sun, and Venus make a right-angle triangle, where the hypotenuse is the Earth-Sun line (length 1 AU) and the base is the Sun-Venus line. The length of the Sun-Venus line is $\sin 46^\circ = 0.72$ AU. A similar procedure is used to find the relative distances to the outer planets. This is shown here:



The absolute distance of the Sun to the Earth was established in 1672 by Giovanni Cassini, using the method of parallax (the same method that gives you depth perception). A colleague in French Guiana took measurements of the position of Mars relative to background stars while he took the measurements in Paris. Mars shifts with respect to the background stars over distance, and by finding the angle between the two they had an angle of a triangle, where the length of the opposite side was known. Simple trigonometry gave the height, which was the distance between the Earth and Mars. Since the relative distances of all of the planets to each other and the Sun had been known since Kepler, this gave all planetary distances at a stroke. A picture of this method is given here:



On to masses. Newton knew the relative distances to the planets, and the absolute distance to the Moon. In fact, it was considering the Moon *falling around* the Earth that was the first indication of the law of Universal gravitation. Newton knew (again, from simple geometry) that an object moving at constant velocity v in a circle of radius r experienced a continuous acceleration towards the center of the circle of magnitude $\frac{v^2}{r}$, and therefore was subject to a continuous force of $\frac{v^2m}{r}$. He could (and Galileo already had) measure the force of gravity near the Earth and had discovered it followed an inverse-square relationship:

$$\frac{GM_em}{r^2}$$

Equating

$$\frac{GM_em}{r^2} = \frac{v^2m}{r}$$

Cancelling

$$\frac{GM_e}{r^2} = \frac{v^2}{r}$$

Now, the velocity of the Moon is just given by the circumference of its orbit divided by its period, so $v = \frac{2\pi r}{P}$

$$\frac{GM_e}{r^2} = \frac{4\pi^2 r}{P^2}$$

GM_e (the gravitational constant times the mass of the Earth) is just a constant; Newton didn't know the values of each, but Galileo had found the value of the product. So every term in that equation was now known as a number, and when Newton did the calculation he found that the sides matched ("pretty nearly"). Gravitation predicted the length of the month.

Newton then demonstrated, using calculus, that Kepler's Laws of planetary motion were predicted by the law of gravitation. This was the proof that the laws of gravitation were correct. While the full proof is somewhat complicated, because planetary orbits are ellipses, a simple version for circular orbits gives the flavor and requires only high-school algebra. Kepler had discovered, in the notation used here:

$$P^2 = kr^3$$

A planet's period squared was proportional to its distance from the Sun, cubed. k is just a constant, which will come in handy later. Again, from basic geometry, each planet was undergoing an acceleration

$$\frac{v^2}{r}$$

And, since $v = \frac{2\pi r}{P}$, the acceleration was just:

$$\frac{v^2}{r} = \frac{4\pi^2 r^2}{P^2 r} = \frac{4\pi^2 r}{P^2}$$

But Kepler had shown $P^2 = kr^3$. Substituting, the acceleration was

$$\frac{4\pi^2 r}{P^2} = \frac{4\pi^2 r}{kr^3} = \frac{4\pi^2}{kr^2}$$

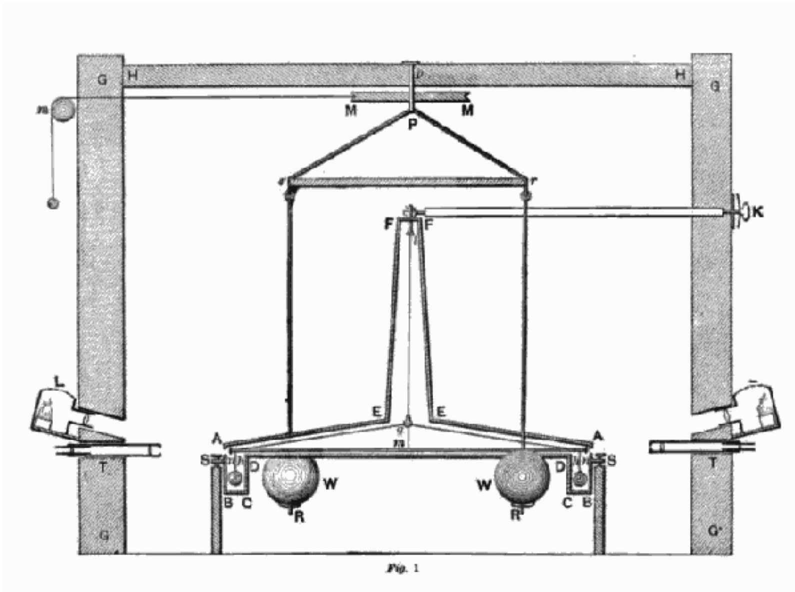
Now, $\frac{4\pi^2}{k}$ is just a constant, which we can take to be GM_c, where G is Newton's constant of gravitation and M_c is the mass of the central body, so the acceleration is:

$$\frac{GM_c}{r^2}$$

r^2

Ta da! Universal gravitation. Of course the real proof had to work for elliptical orbits, and also show Kepler's "equal area, equal time" law. That's why it needs calculus. But Newton had it, and so he knew the *product* GM for each of the Sun, the Earth, Jupiter, and Saturn, and so he knew that the mass of the Sun was about 333,000 times the mass of the Earth, Jupiter 318 times the mass of the Earth, and Saturn 95 times the mass of the Earth. But he didn't know the mass in kilograms of any of them. That had to come from experiment; the value of the Gravitational Constant, G had to be found.

This was done by Henry Cavendish in 1797. His ingenious Cavendish experiment measured the gravitational attraction between two iron balls of known mass. The balls, when released from a grip, caused a slight twist in a thread connecting them. By measuring how much the thread twisted Cavendish could calculate the force between the balls, and thus the value of the gravitational constant (since he knew all of the other terms in the expression for gravitational force). Thus, he could "weigh the Earth" and determine that its mass was 5.972×10^{24} kg. A picture of the Cavendish experiment is shown here:



My thanks to Michael Brenner, in an upvoted comment below, for pointing out that I added an incorrect detail, now removed.

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Carl Tucker · 5y
As an amature astronomer I love your wright up. I am a late learner, and only wish I could have had some of the info you have provide me (us), while in high school. Thank you

1 Reply

Rick McGeer · 4y
My pleasure

Reply

Michael Brenner · 4y
Another question: based on what line of reasoning do you introduce the term **v^2/r**? Newton didn't use this "anti-gravity" acceleration in his argument for the moon orbit, and neither did Kepler. **a=v^2/r** is hammer throw physics, which is *"involuntary"* whereas Newton was awarded the title of genius because he "realized" that the orbits of planets would be *"voluntary"*. **a=v^2/r** is incompatible with the notion of ***"free fall"***.

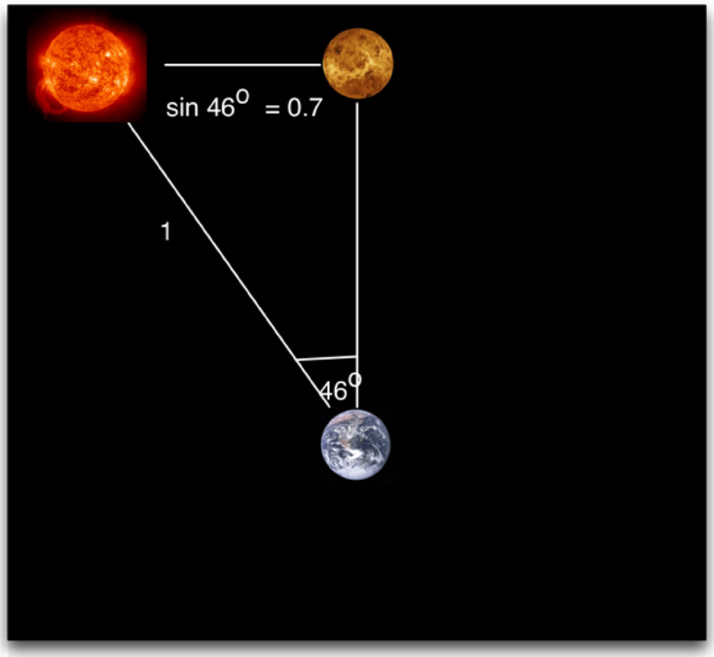
1 Reply

argument I gave is correct for circular orbits. Newton's argument was more complex, because it solved for elliptical, not circular, orbits. That argumen (more)

Reply

Michael Brenner My question was "what is your line of reasoning" and you answer: "I'm...

Michael Brenner · 4y



Why do you call these 46˚ *"above horizon at sunset"*, when you clearly depict Venus at 46˚ from *"noon"*?

Reply

Rick McGeer · 4y
It's still 46° at sunset; it's sunset for the places on the Earth's surface on the line 90° from the Earth-Sun line. But the detail about sunset is irrelevant in any case.

Reply

Steven Spielman · 4y
Who cares whether he said sunset or noon? If they're separated by 46° at noon, they'll still be separated at 46° at sunset. Celestial objects don't reposition themselves as they move across the sky.

Reply

Richard J Lawrence Well there is a slight change in parallax in viewing a celectial object at...

Chris John · 1y
Thanks a lot this is incredibly fascinating. I know this answer was written a while ago, but if anyone sees this who knows, could they explain how, as mentioned in the first paragraph of the answer, we would know when Venus was at its highest point (more)

Reply

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