

# Gravitation and electromagnetism

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Maxwell's equations comprise both electromagnetic and gravitational fields. The transverse part of the vector potential belongs to magnetism, the longitudinal one is concerned with gravitation. The Coulomb gauge indicates that longitudinal components of the fields propagate instantaneously. The delta-function singularity of the field of the divergence of the vector potential, referred to as the dilatation center, represents an elementary agent of gravitation. Viewing a particle as a source or a scattering center of the point dilatation, the Newton's gravitation law can be reproduced.

## 1. MAXWELL'S EQUATIONS IN THE KELVIN-HELMHOLTZ REPRESENTATION

The general form of Maxwell's equations is given by

$$\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} + \mathbf{E} + \nabla \varphi = 0, \quad (1.1)$$

$$\frac{\partial \mathbf{E}}{\partial t} - c \nabla \times \nabla \times \mathbf{A} + 4\pi \mathbf{j} = 0, \quad (1.2)$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho. \quad (1.3)$$

The Helmholtz theorem: a vanishing at infinity vector field  $\mathbf{u}$  can be expanded into the sum of solenoidal  $\mathbf{u}_r$  and potential  $\mathbf{u}_g$  components. We have for the electric field:

$$\mathbf{E} = \mathbf{E}_r + \mathbf{E}_g, \quad (1.4)$$

where

$$\nabla \cdot \mathbf{E}_r = 0, \quad (1.5)$$

$$\nabla \times \mathbf{E}_g = 0. \quad (1.6)$$

The respective expansion for the vector potential can be written as

$$\mathbf{A} = \mathbf{A}_r + \frac{c}{c_g} \mathbf{A}_g, \quad (1.7)$$

where

$$\nabla \cdot \mathbf{A}_r = 0, \quad (1.8)$$

$$\nabla \times \mathbf{A}_g = 0, \quad (1.9)$$

and  $c_g$  is a constant. Substitute (1.4) and (1.7) into (1.1):

$$\frac{1}{c} \frac{\partial \mathbf{A}_r}{\partial t} + \mathbf{E}_r + \frac{1}{c_g} \frac{\partial \mathbf{A}_g}{\partial t} + \mathbf{E}_g + \nabla \varphi = 0. \quad (1.10)$$

Taking the curl of (1.10), we get through (1.6) and (1.9)

$$\nabla \times \left( \frac{1}{c} \frac{\partial \mathbf{A}_r}{\partial t} + \mathbf{E}_r \right) = 0. \quad (1.11)$$

On the other hand, by (1.5) and (1.8) we have

$$\nabla \cdot \left( \frac{1}{c} \frac{\partial \mathbf{A}_r}{\partial t} + \mathbf{E}_r \right) = 0. \quad (1.12)$$

If the divergence and curl of a field equal to zero, then the very field is vanishing. Hence (1.11) and (1.12) imply that

$$\frac{1}{c} \frac{\partial \mathbf{A}_r}{\partial t} + \mathbf{E}_r = 0. \quad (1.13)$$

Subtracting (1.13) from (1.10) we get also

$$\frac{1}{c_g} \frac{\partial \mathbf{A}_g}{\partial t} + \mathbf{E}_g + \nabla \varphi = 0. \quad (1.14)$$

Similarly, expanding as well the density of the current

$$\mathbf{j} = \mathbf{j}_r + \mathbf{j}_g, \quad (1.15)$$

$$\nabla \cdot \mathbf{j}_r = 0, \quad (1.16)$$

$$\nabla \times \mathbf{j}_g = 0, \quad (1.17)$$

(1.2) can be broken up in two equations

$$\frac{\partial \mathbf{E}_r}{\partial t} - c \nabla \times \nabla \times \mathbf{A}_r + 4\pi \mathbf{j}_r = 0, \quad (1.18)$$

$$\frac{\partial \mathbf{E}_g}{\partial t} + 4\pi \mathbf{j}_g = 0. \quad (1.19)$$

Through (1.4) and (1.5) equation (1.3) will be

$$\nabla \cdot \mathbf{E}_g = 4\pi \rho. \quad (1.20)$$

## 2. WAVE EQUATIONS

Let us derive from (1.13), (1.14), (1.18), (1.19) and (1.20) the wave equations for the solenoidal (transverse) and potential (longitudinal) components of the fields. In what follows we will use the general vector relation

$$\nabla (\nabla \cdot \mathbf{u}) = \nabla^2 \mathbf{u} + \nabla \times \nabla \times \mathbf{u}. \quad (2.1)$$

The wave equation for  $\mathbf{A}_r$  can be found thus. Differentiate (1.13) with respect to time:

$$\frac{1}{c} \frac{\partial^2 \mathbf{A}_r}{\partial t^2} + \frac{\partial \mathbf{E}_r}{\partial t} = 0. \quad (2.2)$$

Substitute (1.18) into (2.2). With the account of (2.1) we get

$$\frac{\partial^2 \mathbf{A}_r}{\partial t^2} - c^2 \nabla^2 \mathbf{A}_r = 4\pi c \mathbf{j}_r. \quad (2.3)$$

The wave equation for  $\mathbf{E}_r$  can be found as follows. Differentiate (1.18) with respect to time

$$\frac{\partial^2 \mathbf{E}_r}{\partial t^2} - c \nabla \times \nabla \times \frac{\partial \mathbf{A}_r}{\partial t} + 4\pi \frac{\partial \mathbf{j}_r}{\partial t} = 0. \quad (2.4)$$

Substitute (1.13) into (2.4). With the account of (2.1) we get

$$\frac{\partial^2 \mathbf{E}_r}{\partial t^2} - c^2 \nabla^2 \mathbf{E}_r = -4\pi c \partial_t \mathbf{j}_r. \quad (2.5)$$

In order to find the wave equations for the potential fields we need a gauge relation. Let us postulate for the potential part of the vector potential the specific Lorentz gauge

$$\nabla \cdot \mathbf{A}_g + \frac{1}{c_g} \frac{\partial \varphi}{\partial t} = 0, \quad (2.6)$$

where in general  $c_g \neq c$ . The solenoidal part of the vector potential meets automatically the Coulomb gauge (1.8). The wave equation for  $\mathbf{A}_g$  can be found as follows. Differentiate (1.14) with respect to time:

$$\frac{1}{c_g} \frac{\partial^2 \mathbf{A}_g}{\partial t^2} + \frac{\partial \mathbf{E}_g}{\partial t} + \frac{\partial \nabla \varphi}{\partial t} = 0. \quad (2.7)$$

Take the gradient of (2.6):

$$\nabla (\nabla \cdot \mathbf{A}_g) + \frac{1}{c_g} \nabla \frac{\partial \varphi}{\partial t} = 0. \quad (2.8)$$

Combine (2.7), (2.8) and (1.19). With the account of (2.1) we get

$$\frac{\partial^2 \mathbf{A}_g}{\partial t^2} - c_g^2 \nabla^2 \mathbf{A}_g = 4\pi c_g \mathbf{j}_g. \quad (2.9)$$

Next, we will find the wave equation for  $\varphi$ . Take the divergence of (1.14):

$$\frac{1}{c_g} \frac{\partial \nabla \cdot \mathbf{A}_g}{\partial t} + \nabla \cdot \mathbf{E}_g + \nabla^2 \varphi = 0. \quad (2.10)$$

Combine (2.10), (2.6) and (1.20):

$$\frac{\partial^2 \varphi}{\partial t^2} - c_g^2 \nabla^2 \varphi = 4\pi c_g^2 \rho. \quad (2.11)$$

The wave equation for  $\mathbf{E}_g$  we will find from the wave equations of  $\mathbf{A}_g$  and  $\varphi$ , using (1.14). Differentiate (2.9) with respect to time

$$\frac{\partial^2}{\partial t^2} \frac{\partial \mathbf{A}_g}{\partial t} - c_g^2 \nabla^2 \frac{\partial \mathbf{A}_g}{\partial t} = 4\pi c_g \frac{\partial \mathbf{j}_g}{\partial t}. \quad (2.12)$$

Take the gradient of (2.11)

$$\frac{\partial^2 \nabla \varphi}{\partial t^2} - c_g^2 \nabla^2 \nabla \varphi = 4\pi c_g^2 \nabla \rho. \quad (2.13)$$

Summing (2.12) and (2.13), we get with the account of (1.14)

$$\frac{\partial^2 \mathbf{E}_g}{\partial t^2} - c_g^2 \nabla^2 \mathbf{E}_g = -4\pi \left( c_g^2 \nabla \rho + \frac{\partial \mathbf{j}_g}{\partial t} \right). \quad (2.14)$$

Thus, Maxwell's equations (1.1)-(1.3) with the specific Lorentz gauge (2.6) imply that the solenoidal and potential components of the fields propagate with different velocities. Solenoidal components propagate with the speed  $c$  of light. Their wave equations are (2.3) and (2.5). Potential components and the electrostatic potential propagate with a speed  $c_g$ . Their wave equations are (2.9), (2.11) and (2.14).

### 3. QUASIELASTICITY

Equations (2.3) and (2.9) have the character of the elastic equations. In this connection, the vector potential  $\mathbf{A}$  can be correspondent<sup>1</sup> with a certain displacement field  $\mathbf{s}$ , and the density  $\mathbf{j}$  of the current – with the density  $\mathbf{f}$  of an external force. The gauge relation (2.6) is interpreted as a linearized continuity equation, in which the constant  $c_g$  has directly the meaning of the speed of an expansion-contraction wave<sup>2</sup>. We are interested in the interaction of two

external forces  $\mathbf{f}_1$  and  $\mathbf{f}_2$ , which produce elastic fields  $\mathbf{s}_1$  and  $\mathbf{s}_2$ , respectively. The energy of the elastic interaction is given by the general relation

$$U_{12} = -\varsigma \int \mathbf{f}_1 \cdot \mathbf{s}_2 d^3x = -\varsigma \int \mathbf{f}_2 \cdot \mathbf{s}_1 d^3x, \quad (3.1)$$

where the sign minus in (3.1) corresponds to conditions of the Clapeyron theorem<sup>3</sup>,  $\varsigma$  is a constant.

The energy of the static interaction can be found substituting into (3.1)

$$\mathbf{s} \sim \frac{1}{c} \mathbf{A}, \quad (3.2)$$

$$\mathbf{f} \sim 4\pi c \mathbf{j}, \quad (3.3)$$

$$\varsigma \sim \frac{1}{4\pi c}. \quad (3.4)$$

We have for the transverse interaction

$$U_r = -\frac{1}{c} \int \mathbf{j}_r \cdot \mathbf{A}_r d^3x. \quad (3.5)$$

Suppose that

$$c_g \gg c. \quad (3.6)$$

Then, through (1.7) relations (2.6) and (1.8) turn to the Coulomb gauge

$$\nabla \cdot \mathbf{A} = 0. \quad (3.7)$$

We have according to (1.16) and (1.17)

$$\mathbf{j}_r = \nabla \times \mathbf{R}, \quad \mathbf{j}_g = \nabla G, \quad (3.8)$$

where  $\mathbf{R}$  and  $G$  are vector and scalar fields. Using (1.15), (3.8), (3.7), (1.7) and (1.9) take the following integral by parts:

$$\begin{aligned} \int \mathbf{j} \cdot \mathbf{A} d^3x &= \int (\mathbf{j}_r + \nabla G) \cdot \mathbf{A} d^3x = \int \mathbf{j}_r \cdot \mathbf{A} d^3x \\ &= \int (\nabla \times \mathbf{R}) \cdot (\mathbf{A}_r + c \mathbf{A}_g / c_g) d^3x = \int \mathbf{j}_r \cdot \mathbf{A}_r d^3x. \end{aligned} \quad (3.9)$$

From (3.9) and (3.5) we get the regular expression for the energy of magnetostatic interaction

$$U_r = -\frac{1}{c} \int \mathbf{j} \cdot \mathbf{A} d^3x. \quad (3.10)$$

Elementary sources of the magnetic field correspond to the two forms of the external force density  $\mathbf{f}$  (3.3). The point force at  $\mathbf{x}'$ :

$$\mathbf{f} = 4\pi c q \mathbf{v} \delta(\mathbf{x} - \mathbf{x}'), \quad (3.11)$$

and the torsion center at  $\mathbf{x}'$ :

$$\mathbf{f}_r = 4\pi c a \nabla \times [\boldsymbol{\mu} \delta(\mathbf{x} - \mathbf{x}')], \quad (3.12)$$

where  $q\mathbf{v}$  and  $a\boldsymbol{\mu}$  are constant vectors,  $|\boldsymbol{\mu}| = 1$ . They describe a moving electric charge and a point magnetic dipole, respectively<sup>1</sup>. Substituting (3.11) and (3.12) into the right-hand part of the equation (2.3) we can find the fields  $\mathbf{A}$  produced by these forces. Then, substituting these fields into (3.10) and (3.5), we arrive at the well-known expressions for the interaction energies of electric currents and point magnetic dipoles.

The elementary source of the longitudinal part  $\mathbf{A}_g$  of the vector potential is given by the density of the external force of the form

$$\mathbf{f}_g = -4\pi c_g b \nabla \delta(\mathbf{x} - \mathbf{x}'), \quad (3.13)$$

where  $b$  is the strength of the dilatation center (3.13)<sup>4</sup>. Substitute (3.13) into the right-hand part of the static variant of the equation (2.9):

$$c_g \nabla^2 \mathbf{A}_g = 4\pi b \nabla \delta(\mathbf{x} - \mathbf{x}') . \quad (3.14)$$

With the account of (2.1) and (1.9) we get from (3.14)

$$c_g \nabla \cdot \mathbf{A}_g = 4\pi b \delta(\mathbf{x} - \mathbf{x}') . \quad (3.15)$$

Following (3.1)-(3.4) we have for the energy of longitudinal interaction:

$$U_g = -\frac{1}{c_g} \int \mathbf{j}_g \cdot \mathbf{A}_g d^3x . \quad (3.16)$$

Substitute (3.13) with the account of (3.3) into (3.16):

$$U_{12} = \frac{b_1}{c_g} \int \nabla \delta(\mathbf{x} - \mathbf{x}_1) \cdot \mathbf{A}_2 d^3x = -\frac{b_1}{c_g} \int \delta(\mathbf{x} - \mathbf{x}_1) \nabla \cdot \mathbf{A}_2 d^3x . \quad (3.17)$$

Substituting (3.15) into (3.17), we get

$$U_{12} = -\frac{4\pi b_1 b_2}{c_g^2} \int \delta(\mathbf{x} - \mathbf{x}_1) \delta(\mathbf{x} - \mathbf{x}_2) d^3x \quad (3.18)$$

$$= -\frac{4\pi b_1 b_2}{c_g^2} \delta(\mathbf{x}_1 - \mathbf{x}_2) . \quad (3.19)$$

Expression (3.19) implies, that two dilatation centers (3.15) interact with each other only if they are in a direct contact. The sign of (3.18), or (3.19), indicates that this is the attraction.

Take notice that solenoidal and potential fields are orthogonal to each other in the sense of (3.1). Indeed, using (3.8), (1.8) and (1.9), we find that

$$\int \mathbf{j}_g \cdot \mathbf{A}_r d^3x = \int \nabla G \cdot \mathbf{A}_r d^3x = -\int G \nabla \cdot \mathbf{A}_r d^3x = 0 , \quad (3.20)$$

$$\int \mathbf{j}_r \cdot \mathbf{A}_g d^3x = \int \nabla \times \mathbf{R} \cdot \mathbf{A}_g d^3x = \int \mathbf{R} \cdot \nabla \times \mathbf{A}_g d^3x = 0 . \quad (3.21)$$

#### 4. GRAVITATION

We consider dilatation centers distributed with the volume density  $bp(\mathbf{x})$ . Then equation (3.15) becomes

$$c_g \nabla \cdot \mathbf{A}_g = 4\pi bp(\mathbf{x}) . \quad (4.1)$$

The interaction energy of the two clusters, or clouds, of dilatation centers can be found substituting delta-functions in (3.18) by the reduced densities  $p(\mathbf{x})$  of the distributions. This gives

$$U_{12} = -\frac{4\pi b_1 b_2}{c_g^2} \int p_1(\mathbf{x}) p_2(\mathbf{x}) d^3x . \quad (4.2)$$

Consider a weak source at  $\mathbf{x}^*$ , which emits dilatation centers with a sufficiently high linear velocity  $v_g$ . Such a source will create a quick-formed stationary distribution of the point dilatation with the reduced density

$$p(\mathbf{x}) = \frac{g}{4\pi v_g (\mathbf{x} - \mathbf{x}^*)^2} , \quad (4.3)$$

where  $g$  is a universal constant. Substituting (4.3) into (4.2), we find the interaction energy for two sources of the point dilatation

$$U_{12} = -\frac{g^2 b_1 b_2}{4\pi c_g^2 v_g^2} \int \frac{d^3x}{(\mathbf{x} - \mathbf{x}_1)^2 (\mathbf{x} - \mathbf{x}_2)^2} = -\frac{\pi^2 g^2}{4c_g^2 v_g^2} \frac{b_1 b_2}{|\mathbf{x}_1 - \mathbf{x}_2|} . \quad (4.4)$$

We will assume that each particle is a weak source of the point dilatation (3.15) or a scattering center in a dynamic sea of the point dilatation, the strength  $b$  of the source being proportional to the particle's mass. Then relation (4.4) will be a model of the Newton's law of gravitation.

Notice that in the model thus constructed we must distinguish the speed  $v_g$ , with which the gravitational interaction is transmitted, and the speed  $c_g$  of the longitudinal wave. The latter can be interpreted as the gravitational wave.

Thus, gravitation enters into the general structure of Maxwell's equations. A gravitating center is formally modeled by a potential component of the current having the form

$$\mathbf{j}_g = -\frac{gb}{4\pi v_g} \nabla \frac{1}{(\mathbf{x} - \mathbf{x}^*)^2}. \quad (4.5)$$

And the gravitational interaction is calculated by means of the general relation (3.16), where the longitudinal component  $\mathbf{A}_g$  of the vector potential is found substituting (4.5) into the longitudinal part (2.9) of Maxwell's equations.

## 5. CONCLUSION

Maxwell's equations (1.1)-(1.3) describe both electromagnetic and gravitational fields. The transverse part of the vector potential belongs to magnetism, and longitudinal one is concerned with gravitation. Transverse fields propagate with the speed of light. The Coulomb gauge (3.7) indicates that longitudinal waves propagate in effect instantaneously, comparing with transverse waves. Choosing properly expressions for the current density, magnetic and gravitational interactions can be modeled. An elementary agent of the gravitational interaction corresponds to the dilaton, which is a delta-function singularity (3.15) of the field of the divergence of the vector potential. The sources of longitudinal and transverse fields do not interact with each other. This signifies that gravitation can not be detected with the aid of light.

In the end it should be noted that some of the questions considered here and in<sup>1</sup> were recently approached in<sup>5</sup>.

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<sup>1</sup> V.P.Dmitriyev, "The elastic model of physical vacuum", *Mech.Solids*, **26**, No 6, 60-71 (1992).

<sup>2</sup> V.P.Dmitriyev, "Mechanical analogies for the Lorentz gauge, particles and antiparticles", *Apeiron*, **7**, No 3/4, 173-183 (2000); <http://xxx.arXiv.org/abs/physics/9904049>.

<sup>3</sup> W.Nowacki, *The theory of elasticity* (Warsaw, 1970), §4.16.

<sup>4</sup> J.D.Eshelby, "The continuum theory of lattice defects", *Solid State Physics*, **3**, 79-144 (New York 1956).

<sup>5</sup> David M.Drury, "Irrotational and solenoidal components of Maxwell's equations", *Galilean Electrodynamics*, **13**, No 4, 72-75 (2002).