Rotational Doppler shift of the phase-conjugated photon.

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The rotational Doppler shift of a photon with orbital angular momentum $\pm\ell\hbar$ is shown to be an even multiple of the angular frequency Ω of the reference frame rotation when photon is reflected from the phase-conjugating mirror. The one-arm phase-conjugating interferometer is considered. It contains N Dove prisms or other angular momentum altering elements rotating in opposite directions. When such interferometer is placed in the rotating vehicle the $\delta\omega = 4(N+1/2)\ell\cdot\Omega$ rotational Doppler shift appears and rotation of the helical interference pattern with angular frequency $\delta\omega/2\ell$ occurs. The accumulation of angular Doppler shift via successive passages through the N image-inverting prisms is due to the phase conjugation, for conventional parabolic retroreflector the accumulation is absent. The features of such a vortex phase conjugating interferometry at the single photon level are discussed.

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I. INTRODUCTION.

Single photon interferometry utilizes the superposition of a mutually coherent (phase-locked) quantum states Ψ_j [1] to whom photon belongs simultaneously. The interference pattern depends on a method of Ψ_j preparation. The double-slit Young interferometer creates two free-space wavefunctions Ψ_1,Ψ_2, whose interference pattern produced by detection of the individual photons is recorded by an array of detectors or a photographic plate located in the near or far field. In Mach-Zehnder configuration [2, 3] two wavefunctions separated by entrance beamsplitter recombine at the output beamsplitter. The Michelson interferometer recombines at the input beamsplitter two *retroflected* quantum states provided these states are phase-locked and their path difference δL is smaller than the coherence length L_c. Thus interference pattern is simply $\sim[1+V(\delta L)\cdot\cos(2k\cdot\delta L)]$, where $V(\delta L)$ is a visibility or second-order correlation function and $k = 2\pi/\lambda$. When retroflection is accompanied by wavefront reversal (PC) realized with phase-conjugating mirrors (PCM) [4] based upon Stimulated Brillouin scattering [5, 6], photorefractivity [7, 8] or holographic PCM's, the optical path δL difference is almost entirely compensated due to PC. Noteworthy the small phase lag due to the relatively small frequency shift $\delta\omega = \omega_f - \omega_b$ arising due to the excitation of internal waves inside PCM volume [9], where ω_f and ω_b are the carrier frequencies of incident and PC-reflected photon respectively. This leads to the interference term $1 + V(\delta L)\cdot\cos(\delta k\cdot\delta L)$, where $\delta k = \delta\omega/c$ [6].

We study the photon in the optical vortex quantum state [2, 3] with topological charge ℓ, where the angular momentum $L_z = \pm\ell\cdot\hbar$ is due to the phase singularity located at propagation axis Z (hereafter the spin component of angular momentum [10] is supposed to be zero due to the linear polarization). It is convinient to use the single-photon wavefunctions which coincide with the positive frequency component of the electric field envelope $|\mathbf{\Psi}>=\sqrt{2\epsilon_0}\cdot E(t,\vec{r})$ [11]. The square modulus of Ψ is proportional to the energy density of the *continuous wave* laser beams (CW) and to the photons count rate in a different fringes of the interference pattern for the single-photon experiments [12]. We will assume Ψ to have the form of the Laguerre-Gaussian beam (LG) with $\ell\hbar$ orbital angular momentum (OAM) per photon [9] but any other isolated vortex solutions, e.g. Bessel vortices [13, 14] will demonstrate the same final results:

$$\mathbf{\Psi}_{(f,b)}(z,r,\theta,t)\sim\sqrt{2\epsilon_0}\cdot\frac{\exp[i(-\omega_{(f,b)}t\pm k_{(f,b)}z)\pm i\ell\theta]}{(1+iz/z_R)}$$
$$E_{(f,b)}(r/D_0)^{|\ell|}\exp[-\frac{r^2}{{D_0}^2(1+iz/z_R)}], z_R = k_{(f,b)}{D_0}^2 \quad (1)$$

where the cylindrical coordinates (z,r,θ) are used, D_0 is the vortex radius, z_R is Rayleigh range, Ψ_f, E_f stands for the forward wave, propagating in positive Z-direction, Ψ_b, E_b stands for the wave, propagating in the negative one. Of special interest is the sub-Hz - order angular frequency splitting $\delta\omega = c(k_f - k_b)$ which appears due to the slow mechanical rotation of the setup [3, 15]. It was already shown that rotation of the $\lambda/2$ waveplate with angular frequency $\Omega\sim 2\pi(1-100)rad/s$ in a one arm of the Mach-Zehnder interferometer induces the rotational Doppler shift (RDS) $\delta\omega = 2\Omega\ell$ for circularily polarized broadband CW with linewidth $\Delta\omega/2\pi\simeq 10^{10}Hz$. In this configuration the broadband spectrum was shifted *as a whole* via mechanical rotation (by angular Doppler effect) at $\delta\omega/2\pi = \pm 2\cdot 7Hz$ and the beats at the output mirror induced an appropriate rotation of the interference pattern [16].

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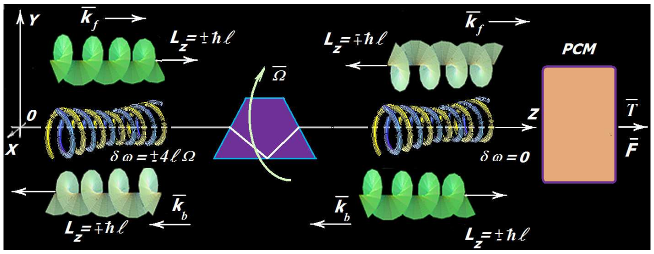

FIG. 1: (Color online)Additivity of RDS for the PCM-reflected photon in the *rest frame*. Rotation of the Dove prism (positive Ω) decreases frequency to $-2\ell \cdot \Omega$ of the co-rotating incident photon with $L_z = +\ell\hbar$. Reflection from PCM alters L_z projection to the opposite one and clockwise rotation of Dove prism (as seen to backward photon) again decreases the frequency of co-rotating photon to $-2\ell \cdot \Omega$. Helical interference pattern is static between prism and PCM (where $\delta\omega = 0$) and rotates *before* prism with angular velocity $\delta\omega = -2\Omega$.

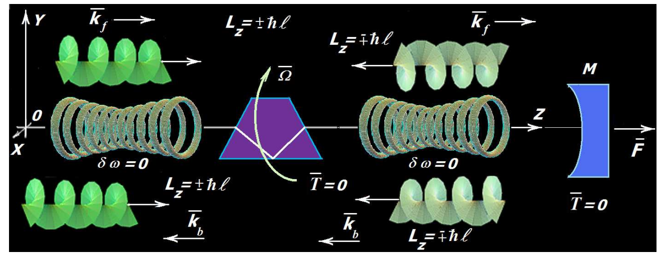

FIG. 2: (Color online)Mutual cancellation of RDS for retroreflected photon. Rotation of the Dove prism again decreases frequency of the co-rotating forward photon with $L_z = +\ell\hbar$ by $-2\ell{\cdot}\Omega$. In backward propagation the Dove prism is counter rotating with respect to photon. Backward RDS is positive thus resulting $\delta\omega$ is zero hence toroidal interference pattern is static for all Z.

II. PHASE-CONJUGATING MIRROR IN A REST FRAME.

Let us consider first the single-arm phase-conjugating vortex interferometer (PCVI) when PC-mirror is in the rest frame. Due to the reflection from PCM the helical photon with a *linear* polarization proves to be in a superposition of the two counter propagating quantum states $\Psi_{f,b}$ (fig.1). Currently the best candidate for the *ideal* single-photon PCM is a thick hologram written with sufficiently high diffraction efficiency ($R \sim 0.9$) for the ℓ -charged optical vortex [3, 17, 18]. In such a case the amplitudes of forward and backward fields are close to each other and visibility of the interference pattern $V(\delta L)$ is close to 1, provided that coherence length $L_c \sim 2\pi c/\Delta\omega$ is bigger than the doubled length of PCVI.

The *ideal* PCM ensures the perfect coincidence of the helical phase surfaces of the counter propagating optical vortices $\Psi_{f,b}$ and zeros of their electric field amplitudes on Z axis. In contrast to the speckle fields whose interference pattern is composed of intertwined Archimedean screws [21] in PCVI the isolated Archimedean screw pattern appears both for the *single* photon with LG wavefunction and for CW resulting in the intensity profile I_{tw} composed of 2ℓ twisted *fringes* [7, 19, 20]:

$$z' = z - z_{pc}, |\Psi|^2 = |\Psi_f + \Psi_b|^2 \sim I_{_{tw}}(z', r, \theta, t) = \frac{2\epsilon_0 c |E_{(f,b)}|^2 2^{(|\ell|+1)} (r/D_0)^{2|\ell|}}{\pi \ell! {D_0}^2 (1 + z'^2/{z_R}^2)} \cdot \exp[-\frac{2r^2}{{D_0}^2(1 + z'^2/{z_R}^2)}] [1 + R^2 + 2R \cdot \cos[\delta\omega \cdot t - (k_f + k_b)z' + 2\ell\,\theta]], \quad (2)$$

where z_{pc} is location of PCM entrance window. The angular speed of pattern rotation $\dot{\theta} = \delta\omega/2\ell$ is given by the differentiation of the self similar argument $2\theta(t){\cdot}\ell{+}\delta\omega{\cdot}t{-}(k_f + k_b)z'$ [16] vs time t. Consider the origin of RDS $\delta\omega$ [3, 22–24] for the photon with topological charge ℓ after the double passage through a Dove prism rotating with angular velocity $\vec{\Omega}$ and reflection from PCM. OAM projection on propagation axis Z is $< \Psi_{f,b}|\hat{L}_z|\Psi_{f,b} >= \pm\ell\hbar$, where $\hat{L}_z = -i\hbar \cdot \partial/\partial\theta$.

The RDS occurs because the optical torque on a slowly rotating element changes the angular momentum of the prism [9, 10]. In its turn this changes the prism's angular velocity $\vec{\Omega}$ and such a change requires the energy supply. Because typical optical elements including prisms are macroscopic classical objects having the *continuous spectrum* of energies, in such case the energy $\hbar\omega_{f,b}$ hence the frequency of the photon may be changed continuously [16].

Noteworthy that without PCM the rotation of Dove prism with respect to the other components of the optical setup would be highly sensitive to misalignments and this would require a very accurate tuning [3]. The phase conjugation facilitates the adjustments and provides interference pattern with good visibility [6].

In the following phase-conjugating optical interferometer the photon's OAM direction is altered as well [9, 26] (fig.1). Let the optical vortex $E_f(t, \vec{r})$ of charge ℓ to pass through a rotating Dove prism and to be reflected with $E_b(t, \vec{r})$ from a some *ideal* (PCM). The non-rotating PCM is supposed to produce *no* frequency shift as it happens in some cases in photorefractive crystals [7, 8], degenerate four-wave mixing [5, 26], and holographic PC couplers [3, 17]. Noteworthy that a small $10^{-1} - 10Hz$ frequency shifts in $BaTiO_3$ photorefractive PCM may mask the RDS. These additional frequency shifts due to slow internal charge waves and filamentation effects were reported in early 1980's yet [27, 28].

Because space is homogeneous and isotropic the conservation of energy, momentum and angular momentum is expected [29]. Reproducing the Dholakia's symmetry arguments [16] adapted to the current case we have the following conservation laws for the angular momenta L_z with respect to $z - axis$ and the energies of the incident photons and those transmitted through the Dove prism, when the latter rotates with the angular velocity $\vec{\Omega}$:

$$I_{zz} \cdot \Omega + L_z = I_{zz} \cdot \Omega^{'} + L_z^{'}$$
$$\hbar\omega_f + \frac{I_{zz}\Omega^2}{2} = \hbar\omega^{'} + \frac{I_{zz}\Omega^{'2}}{2}, \qquad (3)$$

where I_{zz} is the moment of inertia around Z-axis, left hand sides of this system correspond to the incident photon and the right hand sides correspond to the transmitted one. The co-rotation of the prism and the photon corresponds to the same sign of projections $(\Omega, \Omega^{'})$ of $\vec{\Omega}$ and L_z on z-axis before and after the photon's passage. For the incident $L_z = +\ell\hbar$ and passed $L_z^{'} = -\ell\hbar$ the eq. (3) gives the difference of the angular velocities of the prism before and after the photon passage:

$$\Omega - \Omega^{'} = -\frac{2\ell \cdot \hbar}{I_{zz}}. \qquad (4)$$

This means that co-rotation increases the angular velocity of prism, because the energy is transmitted to prism by a virtue of the optical torque $|\vec{T}| = 2\ell \cdot P/\omega_f$, where P is total power carried by LG [3] hence Doppler frequency shift for the photon $\omega^{'} - \omega_f$ is negative:

$$\delta\omega = \omega^{'} - \omega_f = \frac{I_{zz}}{2\hbar}(\Omega - \Omega^{'})(\Omega^{'} + \Omega) = -2\ell \cdot \Omega - \frac{2\ell \cdot \hbar}{I_{zz}}. \qquad (5)$$

Obviously in the counter rotating case, when projections of $\vec{\Omega}$ and L_z are in the opposite directions, the rotational Doppler shift is positive. The net RDS during total forward-backward passage is additive due to PCM and this results in the net OAM change $\delta\omega = \pm 4\ell \cdot \Omega$. The interference pattern and RDS will be the same for all PC-mirrors close to an ideal one, including just proposed *linear loop* PCM, which uses flat optical surfaces without any holographic element [20]. The frequency shift $\delta\omega$ is zero in between Dove prism and PCM and helical pattern is static there. In the region before Dove prism, the frequency shift causes the clockwise ($\delta\omega = -4\ell \cdot \Omega$) or counterclockwise ($\delta\omega = +4\ell \cdot \Omega$) rotation of optical helix pattern [9]. As stated before (2) the angular speed of the optical helix rotation $\dot{\theta} = \pm 2\Omega$ is smaller than $\delta\omega$.

Because of the angular momentum conservation the PCM feels the *rotational recoil* which is proportional to topological charge ℓ: $|\vec{T}_{pc}| = \ell \cdot P(\omega_f^{-1} + \omega_b^{-1})$. The Dove prism feels doubled torque : $|\vec{T}_{Dove}| = 2\ell \cdot P(\omega_f^{-1} + \omega_b^{-1})$. Note that Lebedev radiation pressure force is always directed in positive Z direction and $|\vec{F}_{pc}| = 2 \cdot P/c$ is independent of ℓ [30].

In the absence of the truly phase-conjugating mirror when the forward beam is retroreflected by spherical mirror (fig.2) without altering the angular momentum the interference pattern is a toroidal one I_{tor} [7, 9, 13, 31]:

$$|\Psi|^2 = |\Psi_f + \Psi_b|^2 \sim I_{tor}(z', r, \theta, t) =$$
$$\frac{2\epsilon_0 c|E_{(f,b)}|^2 2^{(|\ell|+1)}(r/D_0)^{2|\ell|}}{\pi\ell! {D_0}^2(1 + z'^2/{z_R}^2)} \cdot \exp[-\frac{2r^2}{{D_0}^2(1 + z'^2/{z_R}^2)}]$$
$$[1 + R^2 + 2R \cdot \cos[\delta\omega \cdot t - (k_f + k_b)z']] . \qquad (6)$$

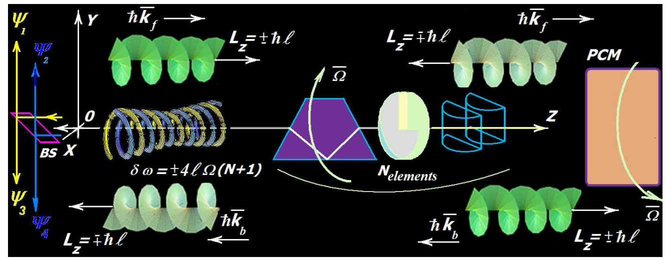

FIG. 3: (Color online)Additivity of RDS in PCVI inside rotating vehicle. PC-mirror counter rotating around Z-axis changes the carrier frequency of reflected photon to $\delta\omega = \pm 2\ell \cdot \Omega$. The sign of $\delta\omega$ is positive when optical torque $\vec{T}$ produced by a bunch of the rotating photons upon mirror has the opposite direction compared to the rotation frequency of PCM $\vec{\Omega}$. When Dove prism rotates in opposite direction vs PCM the net RDS reaches six-fold value $\delta\omega = \pm 6\ell \cdot \Omega$ due to the additional OAM alternation. The sequence of N counter rotating OAM altering elements (including helical waveplates and cylindrical lenses) will produce net RDS of $\delta\omega = 4\ell \cdot \Omega(N + 1/2)$ value. $|\Psi_{1,2,3,4} >$ designates antibunching of photons, which belongs to the *two* (for $\ell = 1$) helical wavefunctions, separated by $\lambda/2$ interval, deflected by entrance beamsplitter BS [7].

The RDS is not accumulated here (the Doppler shifts for the forward and backward photons cancel each other $\delta\omega = 0$) because the truly phase-conjugation is absent hence toroidal interference pattern is static. The mechanical torques on prism $\vec{T}$ induced by OAM alternation will cancel each other too: $|\vec{T}| = 2\ell \cdot P(\omega_f^{-1} - \omega_b^{-1}) \cong 0$.

III. PHASE-CONJUGATING MIRROR IN A ROTATING FRAME.

The more fundamental case is a rotation of the all setup as a whole around a some axis and this general case is relevant to the detection of the slow rotations of the reference frame [1]. In contrast to Sagnac interferometer where frequency splitting and running interference pattern appears in active loop only the PCVI produces frequency splitting $\delta\omega$ even in the passive configuration (fig.3). Apparently the RDS $\delta\omega$ will be the same when CW laser is placed in both inertial (rest) frame and in the noninertial frame associated with rotating vehicle. For the simplest case when the sole PCM rotates around propagation axis Z of a twisted photon with charge ℓ the RDS appears due to the alternation of the photon's OAM. The eqs. (3) give again the frequency shift $\delta\omega = \omega_b - \omega_f$ due to the reflection from rotating PCM:

$$\delta\omega = \omega_b - \omega_f = \pm 2\ell \cdot \Omega + \frac{2\ell \cdot \hbar}{(I_{zz})_{PCM}}. \qquad (7)$$

The second term in the right-hand side of (7) is negligible for typical masses ($m \sim g$) and sizes ($r \sim cm$) of a prisms and mirrors $\hbar/I_{zz} \sim \hbar/(m\cdot r^{-2}) \cong 10^{-27}Hz$. as in Beth's [10] and Dholakia's [16] experiments for the interaction of circularily polarized photons with the macroscopic object (half-wavelength plate).

The angular speed of rotation of the interference pattern proves to be $\dot{\theta} = \delta\omega/2\ell = \Omega$ thus the pattern rotates synchronously with the reference frame. Consequently the sole PCM cannot detect frame rotation. The helical interference pattern outside PCM will be dragged by helical diffraction grating [9] within the phase-conjugating mirror. No atomic coherence [32] is required in our case.

Nevertheless there exists a possibility to accumulate the RDS by means of a chain of OAM alternating elements. To achieve the accumulation of RDS the adjacent components of PCVI must rotate in opposite directions $\vec{\Omega}_n = (-1)^n\vec{\Omega}$, where $n = 0$ stands for PCM, $n = 1$ for the adjacent Dove prism to PCM, $n = N$ for the last Dove prism(DP) near BS. This is necessary because OAM is altered after the passage of the Dove prism and the mutual orientation of the angular momenta of the photon and the next prism should be maintained throughout the chain. When *even* elements (the PCM itself and N/2 Dove prisms) of PCVI are fixed in $\vec{\Omega}$ rotating frame and the rest N/2 *odd* elements ought to rotate there with angular velocity $-2\vec{\Omega}$. The chain of the N rotating OAM-alternating elements will produce the net rotational Doppler shift amounting to $\delta\omega = 4\ell\cdot\Omega(N+1/2)$. Thus PCVI interference pattern (fig.3) will revolve with enhanced angular speed $\dot{\theta}$ of the frame rotation by the factor $\dot{\theta} = \pm 2\cdot(N+1/2)\cdot\Omega$.

For example PCVI (fig.3) may be used for demonstration of the possibility of detection of the sub-Hertzian rotation of the reference frame with the Earth ($\Omega_\oplus \sim 2\pi/86400$). The helical interference pattern will rotate much faster than Earth itself. Namely the equation $4\ell\cdot(N+1/2) = 24$ [16] have the only one solution for integer ℓ, N ($\ell = 4, N = 1$). Hence the optical vortex with charge $\ell = 4$ passed through single Dove prism rotating with $\vec{\Omega}_\oplus$ and PCM rotating in opposite direction $-\vec{\Omega}_\oplus$ will produce the $2\cdot\ell$ spots of interference pattern. The reflection from entrance beamsplitter BS will cause one pass per hour of the spot of interference pattern across the detector window, despite the Earth rotates once in 24 hours only.

The further accumulation of RDS in PCVI might be achieved due to installing the $N = 60$ counter rotating image-inverting elements and $\ell = 6$ optical vortex. In such configuration the $2\cdot\ell$ helices of interference pattern eq.(2) will produce 2ℓ spots at the PCVI output (entrance beamsplitter BS) with a one pass through detector window within approximately each 60 seconds. For this purpose the even components (PCM and N/2 Dove prisms) may be fixed at setup rotating with velocity $\vec{\Omega}_\oplus$ while the others N/2 prism should rotate with "bias" angular velocity $-2\vec{\Omega}_\oplus$ with respect to rotating setup (rotating table). This enhancement will model the detection Earth rotation, and this will alter the $\ell\hbar$ OAM of the *each* photon 121 times during one passage through PCVI. Noteworthy the Dove prism is not the sole element capable to alter the photon's OAM. This can be done as well with helical waveplates and cylindrical lenses [17, 33]. The helical interference pattern within PCVI might also be written by means of atomic coherence effects in a solid-state resonant medium [32].

IV. SINGLE-PHOTON OPERATION OF THE PHASE-CONJUGATING VORTEX INTERFEROMETER.

The single-photon operation [12] is based upon the superposition of the forward and backward quantum states with $\ell\hbar$ OAM:

$$|\Psi>_{helix} = \frac{1}{\sqrt{2}}(|\Psi_{\pm\ell\hbar}>_f + |\Psi_{\mp\ell\hbar}>_b) = \frac{1}{\sqrt{2\ell}}\sum_{j_h}|\Psi_{j_h}>. \tag{8}$$

The detection of this superposition is not a trivial two-detector procedure, because the interference pattern is composed of 2ℓ twisted helices $|\Psi_{j_h}>$. The entrance beamsplitter BS will reflect both upward and downward the interference pattern [7, 9] composed of the 2ℓ spots located on an ellipse, rather than independent forward $|\Psi_{\pm\ell\hbar}>_f$ and backward $|\Psi_{\mp\ell\hbar}>_b$ photon states. For the simplest case $\ell = 1$ the photon will be in the superposition state of the two *helical* wavefunctions designated by appropriate colors at fig.(3):

$$|\Psi>_{helix} = \frac{1}{\sqrt{2}}(|\Psi_{Blue}> + |\Psi_{Yellow}>). \tag{9}$$

This means that two detectors (for $|\Psi_1>$ and $|\Psi_2>$) placed above the entrance beamsplitter BS [7] and two detectors located below BS (for $|\Psi_3>$ and $|\Psi_4>$) can indicate the *antibunching* of the photons [1], belonging to either of the two helices composing the interference pattern. As in a double-slit Young interference experiment the crude attempt of the eavesdropping the *which way* photon moves (the forward or backward one) will destroy the helical interference pattern. On the other hand when single-photon quantum state is prepared as a toroidal pattern (fig.2) the photon belongs to the sequence of the equidistantly spaced toroidal Wannier wavefunctions $|\Psi_{j_{tor}}>$ separated by $\lambda/2$ intervals:

$$|\Psi>_{tor} = \frac{1}{\sqrt{2}}(|\Psi_{\pm\ell\hbar}>_f + |\Psi_{\pm\ell\hbar}>_b) = \frac{1}{\sqrt{N_{tor}}}\sum_{j_{tor}}\Psi_{j_{tor}}. \tag{10}$$

V. CONCLUSION.

In summary we analyzed the phase conjugating vortex interferometer for the both single photon [12] and the *cw* laser output. Because of the alignment of the all optical components along photon Z propagation axis PCVI looks promising from the point of view of rotation sensing [1].

In PCVI the RDS $\delta\omega$ enhances the noninertial frame rotation $\vec{\Omega}$ by a factor of the even multiple of the photon's topological charge ℓ and of the number of angular momentum inverting elements N in PCVI chain. Noteworthy that in the proposed measurement of the Earth rotation $\delta\omega = \pm 4\ell \cdot (N + 1/2)\Omega_{\oplus}\cos(\phi)$ will show dependence on geographical latitude ϕ as it known for the Foucault pendulum [34]: on the poles $\delta\omega$ will be equal to the maximum value when the angle ϕ between normal and PCVI axis is 0 or π, while at equator $\delta\omega$ might reach maximum value when PCVI axis is parallel to the Earth rotation axis. The preliminary analysis have shown that the laser linewidth of the order $\Delta\omega/2\pi \sim 10^3 Hz$ might be sufficient for Earth rotation detection by PCVI (fig.3). We hope to consider the above issues including *entanglement* of the helical photons in PCVI due to mixing counter propagating photon vortex states via entrance beamspliter in a more details in the subsequent work [35].

[1] M.O.Scully, M.S.Zubairy,*"Quantum optics"*, Ch.4, (Cambridge University Press) (1997).

[2] J.Leach,M.J.Padgett,S.M.Barnett,S.Franke-Arnold, and J.Courtial, "Measuring the Orbital Angular Momentum of a Single Photon," Phys.Rev.Lett. **88**, 257901 (2002).

[3] A. Bekshaev, M.Soskin and M. Vasnetsov, *"Paraxial Light Beams with Angular Momentum",* Nova Science(2008).

[4] R.W.Boyd, *"Nonlinear Optics"*, Academic Press (2003).

[5] B.Y.Zeldovich, N.F.Pilipetsky and V.V.Shkunov, *"Principles of Phase Conjugation"*, (Berlin:Springer-Verlag)(1985).

[6] N.G.Basov, I.G.Zubarev, A.B.Mironov, S.I.Mikhailov and A.Y.Okulov, "Laser interferometer with wavefront reversing mirrors", JETP, **52**, 847 (1980) .

[7] M.Woerdemann, C.Alpmann and C.Denz, "Self-pumped phase conjugation of light beams carrying orbital angular momentum," Opt. Express, **17**, 22791 (2009).

[8] A.V.Mamaev, M.Saffman and A.A.Zozulya, "Time dependent evolution of an optical vortex in photorefractive media,"Phys.Rev.A, **56**, R1713 (1997).

[9] A.Yu.Okulov,"Angular momentum of photons and phase conjugation," J.Phys.B., **41**, 101001 (2008).

[10] R.A. Beth,"Mechanical detection and measurement of the angular momentum of light," Phys.Rev., **50**, 115(1936).

[11] J.E.Sipe,"Photon wave functions," Phys.Rev.A, **52**, 1875 (1995).

[12] M. Baier, S. Watanabe, E. Pelucchi, E. Kapon, S. Varoutsis, M. Gallart, I. Robert-Philip and I. Abram, "Single-Photon Emission from Site-Controlled Pyramidal Quantum Dots," Appl. Phys. Lett. **84**, 648-650 (2004).

[13] K.Volke-Sepulveda and R.Jauregui, "All-optical 3D atomic loops generated with Bessel light fields," J.Phys.B., **42**, 085303 (2009).

[14] J.Durnin, and J.J.Miceli, Jr., J. H. Eberly, "Diffraction-free beams," Phys.Rev.Lett.,**58**, 1499 (1987).

[15] M. P. MacDonald, K. Volke-Sepulveda, L. Paterson, J. Arlt, W. Sibbett and K. Dholakia. "Revolving interference patterns for the rotation of optically trapped particles", Opt.Comm.,**201**(1-3),21-28 (2002).

[16] J. Arlt, M. MacDonald, L. Paterson, W. Sibbett,K. Volke-Sepulveda and K. Dholakia,"Moving interference patterns created using the angular Doppler-effect," Opt. Express, **10**(19),844(2002).

[17] E. Abramochkin, V. Volostnikov, "Spiral light beams," Phys.Usp., **47**, 1177(2004).

[18] Ch.V.Felde, P.V.Polyanski and H.V.Bogatyryova, "Comparative analysis of techniques for diagnostics of phase singularities," Ukr. J. Phys. Opt. , **9**, 8290 (2008).

[19] M.Bhattacharya,"Lattice with a twist: Helical waveguides for ultracold matter", Opt.Commun. **279** (1), 219-222 (2007).

[20] A.Yu.Okulov, "Phase-conjugation of the isolated optical vortex using a flat surfaces," J. Opt. Soc. Am. B , **27**,2424-2427 (2010).

[21] A.Yu.Okulov, "Twisted speckle entities inside wavefront reversal mirrors,"Phys.Rev.A , **80**, 163907 (2009).

[22] B. A. Garetz, "Angular Doppler effect," J. Opt. Soc. Am. **71**,609(1981).

[23] I. Bialynicki-Birula and Z. Bialynicka-Birula, "Rotational Frequency Shift," Phys.Rev.Lett. **78**, 2539 (1997).

[24] Courtial J., Robertson D. A., Dholakia K., Allen L., Padgett M. J., "Rotational Frequency Shift of a Light Beam," Phys.Rev.Lett., **81**,4828(1998).

[25] M.V.Vasnetsov, V.A.Pas'ko and M.S.Soskin, "Analysis of orbital angular momentum of a misaligned optical beam," New.J.Phys.,**7**, 46 (2005).

[26] D.M.Pepper,*"Phase conjugate optics"*, Ph.D. Thesis, Caltech, p.37 (1980). (http:// thesis.library.caltech.edu/4044/1/ Pepper /dm/1980.pdf)

[27] A.V. Nowak, T. R. Moore, and R.A. Fisher, "Observations of internal beam production in barium titanate phase conjugators" JOSA B, **5**, 1864 (1988).

[28] S. Sternklar, S. Weiss and B. Fischer, "Tunable frequency shift of photorefractive oscillators," Opt. Lett., **11**, 165 (1986).

[29] E.M.Lifshitz, L.P.Pitaevskii and V.B.Berestetskii, *"Quantum Electrodynamics"*,(Butterworth-Heineman, Oxford) § 6,8 (1982).

[30] P.N.Lebedev,"Experimental examination of the light pressure", Annalen der Physik, **6**, 433(1901).

[31] T. Puppe, I. Schuster, A. Grothe, A. Kubanek, K. Murr, P.W.H. Pinkse, and G. Rempe, "Trapping and Observing Single Atoms in a Blue-Detuned Intracavity Dipole Trap," Phys.Rev.Lett., **99**, 013002 (2007).

[32] S. Franke-Arnold,G. Gibson, R. W. Boyd, M. J. Padgett, "Rotary Photon Drag Enhanced by a Slow-Light Medium",Science **333**, 6038 (2011).

[33] L.Allen, M.W.Beijersbergen, R.J.C.Spreeuw and J.P.Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," Phys.Rev. A **45**,8185-8189 (1992).

[34] L. Foucault, "D'emonstration physique du mouvent de rotation de la Terre, au moyen dun pendule", Comptes rendus hebdomadaires des seances de l'Academie des Sci-

ences (Paris), vol. 35, p. 135-138 (1851).
[35] M. Shirasaki, H. A. Haus, and D. Liu Wong, "Quantum theory of the nonlinear interferometer," J. Opt. Soc. Am. B , **6**,82-88 (1989).