Special relativity and Generalized Sagnac effect

Alexandro Palacios

On 1913, the French scientist George Sagnac conducted an experiment showing that it is possible to get velocities higher than light speed in an accelerated frame. Sagnac himself described the effect as a violation to special relativity because it was in accordance with Galilean mechanics. However, it was shown that the effect is, indeed, possible in the frame of Einstein theory because special relativity does not apply to accelerating frame, so it must be described in the frame of general relativity. Nowadays, the Sagnac effect is very important. It is useful for laser gyroscopes and fiber optic gyroscopes, based on the phase shift of two light beams travelling clockwise and counter-clockwise.

The Sagnac effect

The Sagnac effect is the phenomenon in which two light beams travel in a rotating interferometer. The detector of the apparatus is co-moving with the interferometer clockwise. In theory, the light beam that is travelling counter-clockwise would need to travel less distance than the light beam travelling clockwise, causing a time difference in the arrival to the detector (figure 2). It is important to remark that the light source and the detector are in the same place along the experiment (figure 1), so if *M* is the initial position of the source, it will be the initial position of light beams too. In theory, if the interferometer were not rotating, there would not be time difference in the arrival, as both light beam would travel the same distance (figure 1). Nevertheless, if there exists rotation, one light beam would travel a larger distance to reach the detector than the other would (figure 2). In fact, experiments had demonstrated that the time difference depends of the angular velocity of the interferometer, the higher the angular velocity, the higher the time difference and the other way around. Therefore, if the apparatus has an angular velocity Ω, the tangential or linear speed will be $V = \Omega R$, where R is the radius of the interferometer (figure 2). So, from the reference frame of a the detector moving clockwise, the light beam moving clockwise, (figure 2 - green light) is moving at a speed $C + V$ or if want to use angular velocity: $C + \Omega R$, and the light beam that is travelling counter-clockwise, (figure 2 - red light) will travel at a speed $C - V$ or if want to use angular velocity: $C - \Omega R$ from the detector reference frame, which is in accordance with Galilean mechanics.

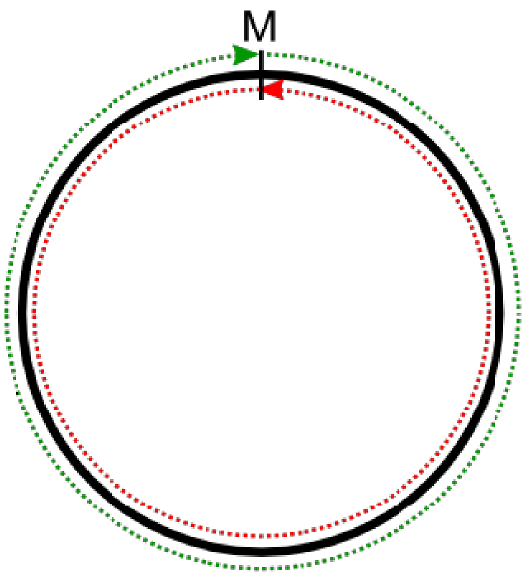

Figure 1: the interferometer is not rotating so there is not travel time difference. M is the initial position of both light beams and the detector

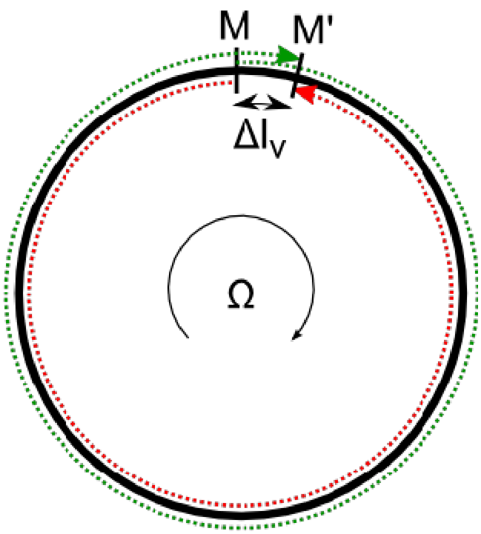

Figure 2: the interferometer is rotating at an angular velocity Ω. The green light beam is moving with the detector and the red light beam is travelling opposite to the detector. M is the initial position of the source/detector, and M' is the final position.

This is important because it means that we can imagine the Sagnac interferometer as a linear interferometer (figure 3). The length of the interferometer is 16 m. and it is moving at a linear speed of 5 m/s. The initial position of the detector is 8 m., in the middle of the interferometer. The initial position of green light beam is 0 m. and for red light beam is 16 m.

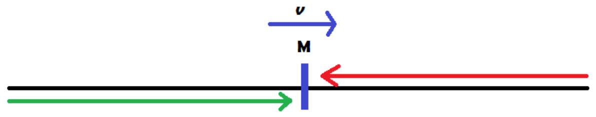

Figure 3: M is the initial position of the detector, v is the constant linear speed of the detector.

Therefore, we can use Newtonian rectilinear uniform movement formula to calculate the time difference between both light beams as is shown here:

$$x' = x_0 + vt$$

- x_0 is the initial position.
- v is the linear velocity of the light beam measured, which is C.
- x' is the target position.

So, the formula for the clockwise light beam, (figure 3 - green light) would be:

$$x' = 0\,m + 299792458\frac{m}{s}.t$$

In addition, for the counter-clockwise light beam (figure 3 - red light) would be:

$$x' = 16\,m - 299792458\frac{m}{s}.t$$

Moreover, for the detector, the above formula would be:

$$x' = 8\,m + 5\frac{m}{s}.t$$

Thus, if we want to find out the time in which the green light beam would reach the detector, we need to do the following calculation:

As both distances x' are the same for the detector and green light when the light beam reach the detector:

$$0\,m + 299792458\frac{m}{s}.t = 8\,m + 5\frac{m}{s}.t$$

$$299792458\frac{m}{s}.t - 5\frac{m}{s}.t = 8\,m$$

$$t_1 = \frac{8\,m}{299792458\frac{m}{s} - 5\frac{m}{s}}$$

Therefore, we can write that:

$$t_1 = \frac{L}{C - V}$$

Where L is the length of the interferometer, C is light speed and V is the velocity of the interferometer.

In addition, if we want to find out the time in which the red light beam would reach the detector, we need to do the same calculation, but for a counter-clockwise beam:

$$16\,m - 299792458\frac{m}{s}.t = 8\,m + 5\frac{m}{s}.t$$

$$16\,m - 8\,m = 299792458\frac{m}{s}.t + 5\frac{m}{s}.t$$

$$8\,m = 299792458\frac{m}{s}.t + 5\frac{m}{s}.t$$

$$t_2 = \frac{8\,m}{299792458\frac{m}{s} + 5\frac{m}{s}}$$

Therefore, we can write that:

$$t_2 = \frac{L}{C + V}$$

Moreover, the only thing we need to do to calculate the time difference between both light beams is

$$\Delta t = |t_1 - t_2|$$

Alternatively:

$$\Delta t = \left|\frac{L}{C - V} - \frac{L}{C + V}\right|$$

$$\Delta t = \left|\frac{(C + V).L}{(C + V).(C - V)} - \frac{(C - V).L}{(C - V).(C + V)}\right|$$

$$\Delta t = \left|\frac{(C + V).L - (C - V).L}{(C - V).(C + V)}\right|$$

$$\Delta t = \left| \frac{\cancel{LC} - LV - \cancel{LC} - LV}{(C - V).(C + V)} \right|$$

$$\Delta t = \left| \frac{-2LV}{(C - V).(C + V)} \right|$$

$$\Delta t = \left| -\frac{2LV}{C^2 - V^2} \right|$$

As $C^2 - V^2 \approx C^2$ we obtain:

$$\Delta t \approx \frac{2LV}{C^2}$$

On the other hand, if we want to have the formula in the frame of angular velocity:

As $V = \Omega R$ and $L = 2\pi R$ we obtain:

$$\Delta t \approx \frac{2.2\pi R.\Omega R}{C^2}$$

$$\Delta t \approx \frac{4\pi R^2 \Omega}{C^2}$$

Alternatively:

$$\Delta t \approx \frac{4A\Omega}{C^2}$$

Where A is the area of the circumference (πR^2).

Generalized Sagnac effect

Although it is believed that the Sagnac effect only applied to rotational motion, experiments conducted by Wang et al. [1] [2] showed that in fact, the effect does not disappear in linear motion. They built a modified Sagnac interferometer, which includes rotational and linear motion in the same experiment. The results were that every part of the loop contribute to the total time difference between both light beams.

The interferometer they built has the following shape (figure 4), and as the traditional interferometer, the detector and the source are moving with the interferometer.

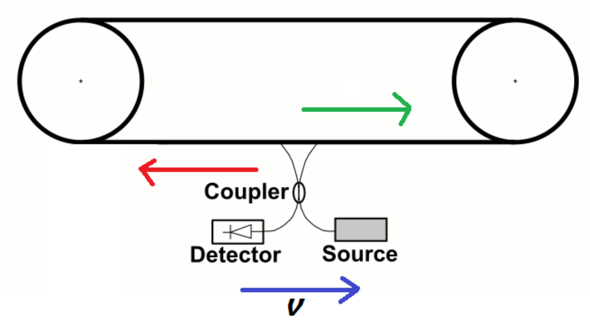

Figure 4: modified Sagnac interferometer for measured linear Sagnac effect. The initial position of the detector and the source is the same, and they move at a velocity V.

Their experiments have shown that **total time difference** is in accordance with the traditional formula of Sagnac effect for rotational motion, which is:

$$\Delta t = \frac{2LV}{C^2}$$

The formula derivation is the same we do in **"The Sagnac effect"** section, where L is **the total length of the fiber** conveyor, and V is the conveyor speed. Therefore, the generalized Sagnac effect is in conflict with Special relativity, because the Galilean velocity addition applied.

What Special relativity predicts in generalized Sagnac effect

The second postulate of Special relativity says that light speed must be C independent of the source speed and observer/detector speed. Thus, if a person were travelling at 0.9 C, in his reference frame light speed still would be C. Therefore, we can make the following assumption: if an observer or detector is travelling at a constant speed V in a linear interferometer as shown in figure 5, both light beams would travel at speed C from the detector´s reference frame.

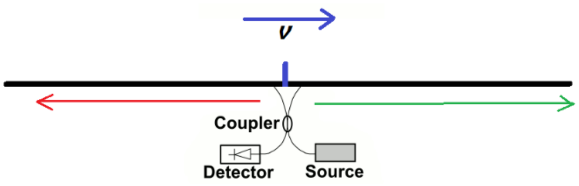

Figure 5: the detector is moving at a speed V. From the detector's reference frame, both light beams (green and red) are travelling at C.

In addition, in special relativity, it is impossible to detect which frame is moving and which frame is stationary, so we can imagine that the detector is stationary and the interferometer is moving at speed *V* (figure 6). This is the same interferometer we have drawn in figure 3, but in this apparatus, the detector is stationary, and the source of light beams is in the middle of the conveyor, as shown in figure 5. Therefore, from the detector´s frame, both light beams travel at speed C. Now, it is easy to imagine there must be a beam that will reach the end of the interferometer faster than the other will, because the interferometer would be travelling towards one beam, and the other would need to travel a longer distance to reach the extreme of the fiber at speed C. Consequently, we can say that the light beam travelling to the left in the interferometer shown in figure 6 would be travelling at speed *C – V*, where *V* is the velocity of the conveyor because the left extreme of the interferometer is moving away from the left light. Moreover, the light beam travelling to

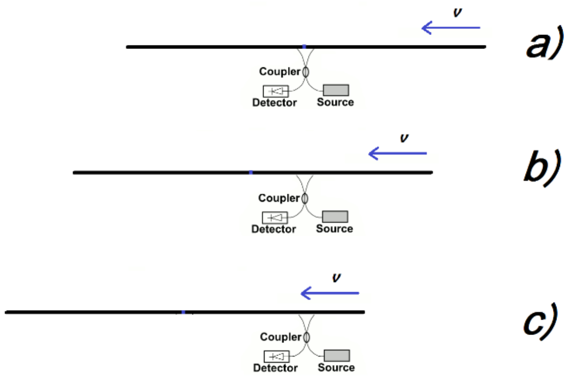

Figure 6: it is the same interferometer shown in figure 5, but the detector is stationary and the fiber interferometer is moving at speed V.

the right in the interferometer shown in figure 6 would be travelling at speed *C + V*, because the right extreme of the interferometer would be approaching to the right light beam. Thus, we can affirm that from the reference frame of the detector, both light beams would reach their respective extremes with a time difference. It is important to remark that the addition of velocity *C ± V* we are analysing here is in accordance with relativity, because what we are actually measuring is the distance difference both beams need to travel to reach the extreme of the fiber when the interferometer is moving favouring one of them. We can calculate it using the same formula we used above for the classical Sagnac effect:

$$x' = x_0 + vt$$

Assuming that the length of the interferometer is 16 m., the initial position of the source/detector is 8 m., and the interferometer is moving at speed *V = 5 m/s* , for the light beam moving to the right of the interferometer in figure 6, the formula would be:

$$x' = 8\,m + (C + V)t$$

$$x' = 8\,m + (299792458\frac{m}{s} + 5\frac{m}{s})t$$

$$x' = 8\,m + 299792463\frac{m}{s}.t$$

Therefore, if we want to find out the time when the beam reach the right extreme, we need to replace x' with the right extreme of the conveyor, which is 16 m:

$$16\,m = 8\,m + 299792463\frac{m}{s}.t$$

$$16\,m - 8\,m = 299792463\frac{m}{s}.t$$

$$t_1 = \frac{8\,m}{299792463\frac{m}{s}}$$

Alternatively:

$$t_1 = \frac{L}{C + V}$$

In addition, for the light beam moving to the left of the interferometer in figure 6, the formula would be:

$$x' = 8\,m - (C - V)t$$

$$x' = 8\,m - (299792458\frac{m}{s} - 5\frac{m}{s})t$$

$$x' = 8\,m - 299792453\frac{m}{s}.t$$

Therefore, if we want to find out the time when the beam reach the left extreme, we need to replace x' with the left extreme of the conveyor, which is 0 m:

$$0\,m = 8\,m - 299792453\frac{m}{s}.t$$

$$8\,m = 299792453\frac{m}{s}.t$$

$$t_2 = \frac{8\,m}{299792453\frac{m}{s}}$$

Alternatively:

$$t_2 = \frac{L}{C - V}$$

Consequently, if we do the same calculation we did above in **"The Sagnac effect"** section, we obtain that:

$$\Delta t = |t_1 - t_2|$$

$$\Delta t = |\frac{L}{C+V} - \frac{L}{C-V}|$$

$$\Delta t \approx \frac{2LV}{C^2}$$

At this part of the reading, it may sounds confusing that an experiment that disproves relativity is in accordance with the formula that Special relativity predicts. However, what we have just calculated is the time difference only on the bottom arm. If we make an analysis of the complete interferometer, we will see that Special relativity is no appropriate to describe the generalized Sagnac effect phenomenon.

Let take just one beam to make the calculation. For example, the green light which is moving to the right (figure 7). Assuming that the **total conveyor length** is 40 m, the wheel radius is 1,2732 m, the conveyor speed is 5 m/s, and the initial position of the detector is the middle of the bottom-arm.

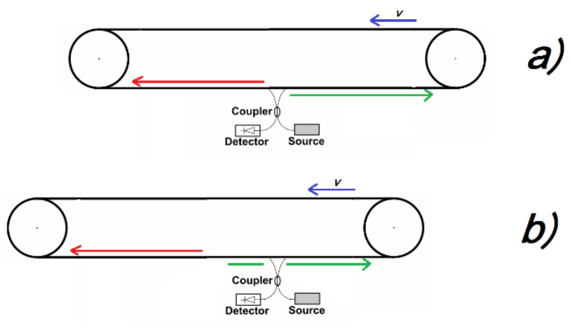

Figure 7: Same configuration shown in figure 6, but here we have the complete interferometer as in figure 4. The conveyor is mobbing at constant speed V. Both light beams are travelling at speed C from the detector´s reference frame

Therefore, to make a correct analysis we must separate the interferometer in five parts for each light beam: the bottom-arm (a), left-perimeter (b), top-arm (c), and right-perimeter (d). Thus, if we want to know the length of each segment of the loop, we need to do this calculation:

For (b) and (d):

$$L_b = L_d = \frac{2\pi R}{2}$$

$$L_b = L_d = \frac{2\pi \times 1{,}2732\text{ m}}{2}$$

Consequently:

$$L_b + L_d = 2\pi R$$

Here we have the formula for both perimeters (b) and (d). Now we need to find out the length of the bottom-arm (a), and top-arm (c). We can affirm that $L_t = L_a + L_b + L_c + L_d$, where L_t is the **total conveyor length**. Therefore:

$$L_t = L_a + L_c + 2\pi R$$

If L_t = 40 m, we have:

$$40\,m = L_a + L_c + 2\pi R$$

$$40\,m - 2\pi R = L_a + L_c$$

$$40\,m - 2\pi \times 1{,}2732\text{ m} = L_a + L_c$$

$$32\,m = L_a + L_c$$

As the bottom-arm (a) in equivalent to the top-arm (c), we obtain:

$$L_a = L_c$$

$$L_a = L_c = \frac{32\,m}{2}$$

$$L_a = L_c = 16\,m$$

The result we obtained above is equivalent to the example we have used for figure 6, therefore, if the detector's initial position is at the middle of the bottom-arm, we can say that it is equal to $\frac{16\,m}{2} = 8\,m$, and we can apply the same formula we have made in figure 6 analysis, which is:

$$\Delta t = \frac{2LV}{C^2}$$

As we saw before, let take just one light beam, for example the green light (figure 7). If the initial time of the experiment is $t_0 = 0\ s$, from the reference frame of the detector the **green light beam** will reach the right extreme of the conveyor at time given by this formula:

$$t_1 = \frac{L_{a1}}{C+V}$$

Where La_1 is equal to 8 m, because the ΔL of the green beam at the bottom-arm is 16 m – 8 m.

Then, the light beam would enter to the right perimeter (d) at time t_1 from the detector´s reference frame. As we said before, the (d) length is equal to:

$$L_d = \frac{2\pi R}{2}$$

Consequently, we can say that the green light beam would reach the top-arm (c) at time:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{\frac{2\pi R}{2}}{C-V}$$

Alternatively:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{L_d}{C-V}$$

It is important to remark that in this case, it is $\frac{\frac{2\pi R}{2}}{C-V}$ and not $\frac{\frac{2\pi R}{2}}{C+V}$ because the green light beam passing through (d) is travelling at the same direction of the fiber, thus, as we saw at the first section **"The Sagnac effect"**, the light beam moving *with* the detector would have to travel a longer distance to reach it again.

After that, the light beam would enter to the top arm (c) at time t_1 from the detector´s reference frame. In this part of the loop, the light beam would be moving towards the detector (figure 8), not with the detector, therefore, the time difference while the green light beam is passing through (c) is given by:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{L_d}{C-V} + \frac{L_c}{C-V}$$

Where L_c is equal to 16 m.

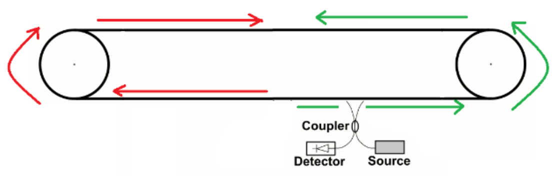

Figure 9: Both light beam passing through segment (a), (d) and (c) of the loop.

Then, the green light beam will pass through segment (b) of the loop (figure 9). The time difference formula is the same we use for segment (d), that is:

$$L_d = L_b = \frac{2\pi R}{2}$$

Therefore:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{L_d}{C-V} + \frac{L_c}{C-V} + \frac{L_b}{C-V}$$

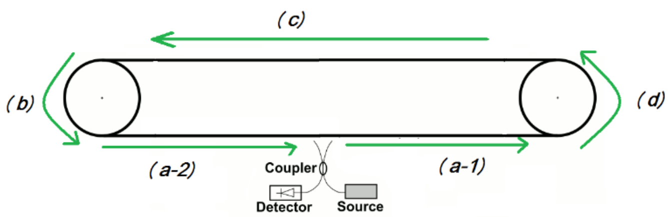

Figure 8: complete travel of the green light beam.

Lastly, the green light beam will pass through segment (a-2) of the loop (figure 9). The time difference formula when the green light would reach the detector again is given by:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{L_d}{C-V} + \frac{L_c}{C-V} + \frac{L_b}{C-V} + \frac{L_{a2}}{C+V}$$

- It is important to remark that L_{a2} is not equal to L_{a1} because we should remember that when the light beam is passing through L_{a2} the detector is going to be a little bit further from its initial point. Thus:

$$L_{a2} = L_{a1} + L_{aux1}$$

- Consequently:

$$t_1 = \frac{L_{a1}}{C+V} + \frac{L_d}{C-V} + \frac{L_c}{C-V} + \frac{L_b}{C-V} + \frac{L_{a1} + L_{aux1}}{C+V}$$

- As:

$$\frac{L_{a1}}{C+V} + \frac{L_{a1} + L_{aux1}}{C+V} = \frac{2L_{a1} + L_{aux1}}{C+V}$$

- We obtain:

$$t_1 = \frac{2L_{a1} + L_{aux1}}{C+V} + \frac{L_d}{C-V} + \frac{L_c}{C-V} + \frac{L_b}{C-V}$$

- Then:

$$t_1 = \frac{2L_{a1} + L_{aux1}}{C+V} + \frac{L_d + L_c + L_b}{C-V}$$

$$t_1 = \frac{(C-V)(2L_{a1} + L_{aux1})}{(C-V)(C+V)} + \frac{(C+V)(L_d + L_c + L_b)}{(C+V)(C-V)}$$

$$t_1 = \frac{(C-V)(2L_{a1} + L_{aux1}) + (C+V)(L_d + L_c + L_b)}{(C-V)(C+V)}$$

- As $L_d = L_b$

$$t_1 = \frac{(C-V)(2L_{a1} + L_{aux1}) + (C+V)(L_c + 2L_b)}{C^2 - V^2}$$

$$t_1 = \frac{2L_{a1}C + L_{aux1}C - 2L_{a1}V - L_{aux1}V + L_cC + L_cV + 2L_bC + 2L_bV}{C^2 - V^2}$$

- We organize the formula:

$$t_1 = \frac{2L_{a1}C + L_cC + 2L_bC + L_{aux1}C + L_cV - 2L_{a1}V + 2L_bV - L_{aux1}V}{C^2 - V^2}$$

- As $L_c = 2L_{a1}$ we obtain:

$$t_1 = \frac{L_cC + L_cC + 2L_bC + L_{aux1}C \cancel{+ L_cV - 2L_{a1}V} + 2L_bV - L_{aux1}V}{C^2 - V^2}$$

$$t_1 = \frac{2L_cC + 2L_bC + L_{aux1}C \cancel{+ L_cV - 2L_{a1}V} + 2L_bV - L_{aux1}V}{C^2 - V^2}$$

$$t_1 = \frac{2L_cC + 2L_bC + L_{aux1}C + 2L_bV - L_{aux1}V}{C^2 - V^2}$$

$$t_1 = \frac{C.(2L_c + 2L_b + L_{aux1}) + V.(2L_b - L_{aux1})}{C^2 - V^2}$$

In addition, if we want to do the same calculation for red light, we obtain that t_2 has this value (**appendix[a]**):

$$t_2 = \frac{L_{a3}}{C-V} + \frac{L_d}{C+V} + \frac{L_c}{C+V} + \frac{L_b}{C+V} + \frac{L_{a4}}{C-V}$$

The only thing that changes is that where for t_1 it was $\frac{L}{C+V}$ now it is $\frac{L}{C-V}$ because the light beam is travelling in the opposite direction. In addition, we include two new variables, L_{a3} (which is equal to L_{a1}) and L_{a4} (which is not equal to L_{a2}). Therefore, we obtain:

$$t_2 = \frac{C.(2L_c + 2L_b + L_{aux2}) + V.(2L_b - L_{aux2})}{C^2 - V^2}$$

Now we have found out the travel time of both light beams that is expected by Special relativity for the entire loop, we can obtain the travel time difference, which is:

$$\Delta t = |t_1 - t_2|$$

Alternatively:

$$\Delta t = \left| \frac{C.(2L_c + 2L_b + L_{aux1}) + V.(2L_b - L_{aux1})}{C^2 - V^2} - \frac{C.(2L_c + 2L_b - L_{aux2}) + V.(-2L_b - L_{aux2})}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{\big(C.(2L_c + 2L_b + L_{aux1}) + V.(2L_b - L_{aux1})\big) - \big(C.(2L_c + 2L_b - L_{aux2}) + V.(-2L_b - L_{aux2})\big)}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{(2L_cC + 2L_bC + L_{aux1}C + 2L_bV - L_{aux1}V) - (2L_cC + 2L_bC - L_{aux2}C - 2L_bV - L_{aux2}V)}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{2L_cC + 2L_bC + L_{aux1}C + 2L_bV - L_{aux1}V - 2L_cC - 2L_bC + L_{aux2}C + 2L_bV + L_{aux2}V}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{2L_cC \cancel{+ 2L_bC} + L_{aux1}C + 2L_bV - L_{aux1}V - 2L_cC \cancel{- 2L_bC} + L_{aux2}C + 2L_bV + L_{aux2}V}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{\cancel{2L_cC} + L_{aux1}C + 2L_bV - L_{aux1}V \cancel{- 2L_cC} + L_{aux2}C + 2L_bV + L_{aux2}V}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{L_{aux1}C + 2L_bV - L_{aux1}V + L_{aux2}C + 2L_bV + L_{aux2}V}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{L_{aux1}C + 4L_bV - L_{aux1}V + L_{aux2}C + L_{aux2}V}{C^2 - V^2} \right|$$

- We organize the formula:

$$\Delta t = \left| \frac{L_{aux1}C + L_{aux2}C + L_{aux2}V + 4L_bV - L_{aux1}V}{C^2 - V^2} \right|$$

$$\Delta t = \left| \frac{C.(L_{aux1} + L_{aux2}) + V.(L_{aux2} + 4L_b - L_{aux1})}{C^2 - V^2} \right|$$

- As $C^2 - V^2 \approx C^2$:

$$\Delta t \approx \left| \frac{C.(L_{aux1} + L_{aux2}) + V.(L_{aux2} + 4L_b - L_{aux1})}{C^2} \right|$$

The formula above can be used by Special relativity to find out the total time difference between the red and green light to reach the detector. We can notice that it is longer than the formula that Wang et al. have used for their interferometer, which is:

$$\Delta t \approx \frac{2LV}{C^2}$$

Consequently, Special relativity is not appropriate to describe the generalized Sagnac effect.

Conclusion

After the analysis we did above we can conclude that Special relativity is not appropriate to describe the generalized Sagnac effect. Wang et al. have made many experiments in order to verify if the classic formula of Sagnac effect can be applied to the generalized version of the effect. The results indicate that it is true. If apply relativity clock synchronization we obtain a total different formula.

References

[1] Wang, R., Zheng, Y., Yao, A, & Langley, D. Physics Letters A, 312 7–10 (2003).

[2] Wang, R., Zheng, Y., & Yao, A. Physical Review Letters, 93 (14), 143901 (2004).

[3] Wang, R. Galilean Electrodynamics, 16 (2), 23-30 (2005).

[4] Georges Sagnac, “Sur la propagation de la lumière dans un système en translation et sur l'aberration des étoiles,” C. R. Acad. Sci. Paris **141**, 1220–1223 (1913).

[5] A. Ori and J. E. Avron, "Generalized Sagnac-Wang-Fizeau formula". PHYSICAL REVIEW A94, 063837 (2016).

[6] Angelo Tartaglia and Matteo Luca Ruggiero, "The Sagnac effect and pure geometry". American Journal of Physics 83, 427 (2015); https://doi.org/10.1119/1.4904319.

[7] Kevin Brown, "Reflections on Relativity" MathPages.com.

Appendix

[a] Look at figure 10 for a better understanding of the formula

$$t_2 = \frac{L_{a3}}{C-V} + \frac{L_d}{C+V} + \frac{L_c}{C+V} + \frac{L_b}{C+V} + \frac{L_{a4}}{C-V}$$

- It is important to remark that L_{a3} is not equal to L_{a4} because we should remember that when the light beam is passing through L_{a4} the detector is going to be a little bit further from its initial point, in other words, closer to the red light beam. Thus:

$$L_{a4} = L_{a3} - L_{aux2}$$

- Consequently:

$$t_2 = \frac{L_{a3}}{C-V} + \frac{L_d}{C+V} + \frac{L_c}{C+V} + \frac{L_b}{C+V} + \frac{L_{a3} - L_{aux2}}{C-V}$$

- As:

$$\frac{L_{a3}}{C-V} + \frac{L_{a3} - L_{aux2}}{C-V} = \frac{2L_{a3} - L_{aux2}}{C-V}$$

- We obtain:

$$t_2 = \frac{2L_{a3} - L_{aux2}}{C-V} + \frac{L_d}{C+V} + \frac{L_c}{C+V} + \frac{L_b}{C+V}$$

- Then:

$$t_2 = \frac{2L_{a3} - L_{aux2}}{C-V} + \frac{L_d + L_c + L_b}{C+V}$$

$$t_2 = \frac{(C+V)(2L_{a3} - L_{aux2})}{(C+V)(C-V)} + \frac{(C-V)(L_d + L_c + L_b)}{(C-V)(C+V)}$$

$$t_2 = \frac{(C+V)(2L_{a3} - L_{aux2}) + (C-V)(L_d + L_c + L_b)}{(C-V)(C+V)}$$

- As $L_d = L_b$

$$t_2 = \frac{(C+V)(2L_{a3} - L_{aux2}) + (C-V)(L_c + 2L_b)}{C^2 - V^2}$$

$$t_2 = \frac{(2L_{a3}C - \ L_{aux2}C + 2L_{a3}V - L_{aux2}V) + (L_cC - L_cV + 2L_bC - 2L_bV)}{C^2 - V^2}$$

$$t_2 = \frac{2L_{a3}C - \ L_{aux2}C + 2L_{a3}V - L_{aux2}V + L_cC - L_cV + 2L_bC - 2L_bV}{C^2 - V^2}$$

- We organize the formula:

$$t_2 = \frac{2L_{a3}C + L_cC + 2L_bC - \ L_{aux2}C + 2L_{a3}V - L_cV - 2L_bV - L_{aux2}V}{C^2 - V^2}$$

- As $L_c = 2L_{a3}$ we obtain:

$$t_2 = \frac{L_cC + L_cC + 2L_bC - \ L_{aux2}C \cancel{+ 2L_{a3}V - L_cV} - 2L_bV - L_{aux2}V}{C^2 - V^2}$$

$$t_2 = \frac{2L_cC + 2L_bC - \ L_{aux2}C \cancel{+ 2L_{a3}V - L_cV} - 2L_bV - L_{aux2}V}{C^2 - V^2}$$

$$t_2 = \frac{2L_cC + 2L_bC - \ L_{aux2}C - 2L_bV - L_{aux2}V}{C^2 - V^2}$$

$$t_2 = \frac{C.(2L_c + 2L_b - \ L_{aux2}) + V.(-2L_b - L_{aux2})}{C^2 - V^2}$$

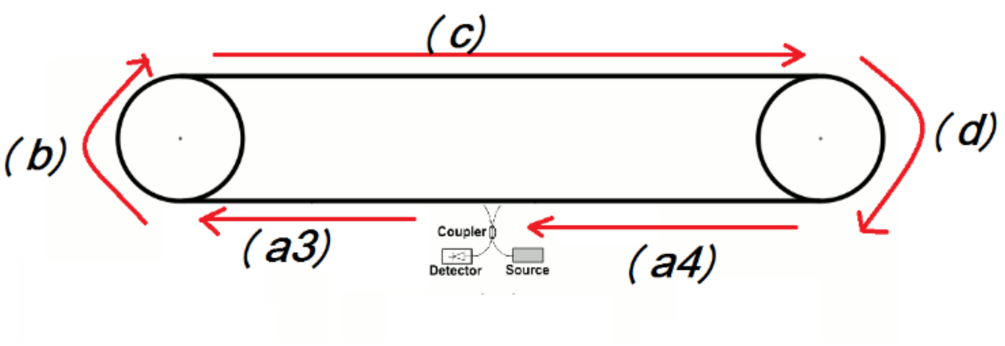

Figure 10: complete travel of the red light beam.