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New Analysis of the Interferometer Observations of Dayton C. Miller

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For nearly thirty years the results of the Michelson-Morley experiment obtained by Dayton C. Miller on Mount Wilson have stood at variance with all other trials of this experiment. As interest in Miller's results has continued to the present time, and since the original data sheets are available to the present writers, it has seemed appropriate that the observations be subjected to a new analysis. It is now shown that the small periodic fringe displacements found by Miller are due in part to statistical fluctuations in the readings of the fringe positions in a very difficult experiment. The remaining systematic effects are ascribed to local temperature conditions. These were much more troublesome at Mount Wilson than those encountered by experimenters elsewhere, including Miller himself in his work done at Case in Cleveland. As interpreted in the present study, Miller's extensive Mount Wilson data contain no effect of the kind predicted by the aether theory and, within the limitations imposed by local disturbances, are entirely consistent with a null result at all epochs during a year.

INTRODUCTION

THE null result obtained in the Michelson-Morley experiment at Cleveland in 1887[1] was the culmination of the long nineteenth-century search for an aether and proved to be a major incentive for the theoretical developments made by Lorentz, FitzGerald, Larmor, Poincaré, and especially Einstein. The crucial significance of the Michelson-Morley result stimulated many repetitions of this experiment during the next fifty years, especially as the implications of the theory of relativity unfolded. All trials of this experiment except those carried out at Mount Wilson by Dayton C. Miller yielded a null result within the accuracy of the observations. Miller's observed fringe displacements also were very small, being on the average only about 1/13 of those predicted by the aether theory for the 30 km/sec velocity of the earth in its orbit. However, as these small residuals have received no adequate explanation, and since interest in them has continued, it has seemed desirable to make a rather detailed analysis of the observational material.[2]

Michelson and Morley had originally planned to repeat their experiment at intervals of about three months throughout a year and thus to allow for any possibility that the earth's orbital velocity at one epoch might fortuitously combine with the motion of the entire solar system through space and give a small resultant velocity to their apparatus. However, after completing the July, 1887 observations, they did not return to this problem, and the completion of the aether-drift experiment for all epochs was finally carried out by Miller during the years from 1921 through 1926 in Cleveland and at Mount Wilson. Miller's Mount Wilson experiments[3] were conducted at intervals from April, 1921, through February, 1926, being interspersed with and followed by extensive trials at Cleveland.

In February, 1921, the interferometer was set up at Mount Wilson, and Miller made numerous observations during the period April 8–21, 1921. These data indicated a possible small periodic effect, with an average second-harmonic amplitude of about 0.04 fringe, but Miller suspected that temperature effects or magnetostriction in the steel base of the interferometer as it rotated in the earth's magnetic field might be the cause. Accordingly he had designed a concrete base for the interferometer, which was cast at Mount Wilson in October of 1921. Undoubtedly the greatest incentive to continue the experiments came from Professor Albert Einstein who visited Miller at Case on May 25, 1921, and urged that

[1] A. A. Michelson and E. W. Morley, Am. J. Sci. **34**, 333 (1887).

[2] A rather complete report on the early experiments was made in this journal, hence the present analysis of the data is published here. See D. C. Miller, Revs. Modern Phys. **5**, 203 (1933).

[3] D. C. Miller: (a) Proc. Natl. Acad. Sci. **11**, 306 (1925); (b) Science **61**, 617 (1925); (c) **63**, 433 (1926); (d) Astrophys. J. **68**, 341 (1928); (e) see reference 2; (f) Nature **133**, 162 (1934).

TABLE I. Trials of the Michelson-Morley experiment.

Observer	Year	Place	D	$2D/\lambda(v/c)^2$	A	Ratio
Michelson[a]	1881	Potsdam	120 cm	0.04 fringe	0.01 fringe	2
Michelson and Morley[b]	1887	Cleveland	1100	0.40	0.005	40
Morley and Miller[c]	1902–04	Cleveland	3220	1.13	0.0073	80
Miller[d]	1921	Mt. Wilson	3200	1.12	0.04	15
Miller[e]	1923–24	Cleveland	3200	1.12	0.015	40
Miller (sunlight)[f]	1924	Cleveland	3200	1.12	0.007	80
Tomaschek (starlight)[g]	1924	Heidelberg	860	0.3	0.01	15
Miller[h]	1925–26	Mt. Wilson	3200	1.12	0.044	13
Kennedy[i]	1926	Pasadena and Mt. Wilson	200	0.07	0.001	35
Illingworth[j]	1927	Pasadena	200	0.07	0.0002	175
Piccard and Stahel[k]	1927	Mt. Rigi	280	0.13	0.003	20
Michelson, *et al.*[l]	1929	Mt. Wilson	2590	0.9	0.005	90
Joos[m]	1930	Jena	2100	0.75	0.001	375

[a] A. A. Michelson, Am. J. Sci. **22**, 120 (1881); Phil. Mag. **13**, 236 (1882).
[b] A. A. Michelson and E. W. Morley, Am. J. Sci. **34**, 333 (1887); Phil. Mag. **24**, 449 (1887).
[c] E. W. Morley and D. C. Miller, Phil. Mag. **9**, 680 (1905); Proc. Am. Acad. Arts Sci. **41**, 321 (1905).
[d] D. C. Miller, Data sheets of Observations December 9 to 11, 1921 (unpublished).
[e] D. C. Miller, Observations, August 23 to September 4, 1923; June 27 to July 26, 1924 (unpublished).
[f] D. C. Miller, "Observations with Sunlight on July 8 to 9, 1924," Proc. Natl. Acad. Sci. **11**, 311 (1925).
[g] R. Tomaschek, Ann. d. Physik **73**, 105 (1924).
[h] D. C. Miller, Revs. Modern Phys. **5**, 203 (1933).
[i] R. J. Kennedy, Proc. Natl. Acad. Sci. **12**, 621 (1926); Astrophys. J. **68**, 367 (1928).
[j] K. K. Illingworth, Phys. Rev. **30**, 692 (1927).
[k] A. Piccard and E. Stahel, Compt. rend. **183**, 420 (1926); **184**, 152, 451 (1927); **185**, 1198 (1927); J. phys. radium **8**, 56 (1927).
[l] Michelson, Pease, and Pearson, Nature **123**, 88 (1929); J. Opt. Soc. Am. **18**, 181 (1929).
[m] G. Joos, Ann. Physik **7**, 385 (1930); Naturwiss. **38**, 784 (1931).

further trials be made to remove any possible doubts concerning the earlier results obtained in this experiment.

Miller conducted a series of observations with the concrete base interferometer from December 4–11, 1921, but with rather discouraging results. Much of the time the interferometer behaved poorly, the fringes were often unsteady, temperature variations near the instrument were troublesome, and vibrations caused by high winds often made observation entirely impossible. However, at last on December 9–11, 1921, thirteen sets of readings comprising 153 complete turns of the interferometer were made under favorable conditions, some with "excellent seeing." These data gave an average periodic amplitude of only 0.04 fringe, nearly the same as the April, 1921 results with the steel base interferometer, and Miller concluded that "all effects are probably due to the instrument. This is the end!"[4] So, shortly thereafter he dismounted his apparatus for its return to Cleveland where it was reassembled in a basement room of the Case Physics Laboratory.

Since the 1921 experiments with both the steel and the concrete bases for the interferometer gave essentially the same results, Miller concluded that magnetostriction was not the cause of the small periodic effects observed. However, since the mechanical rigidity of the concrete base instrument was less than that for the steel, a conclusive argument could not be made. The magnitude of the magnetostriction effect is marginal. Nevertheless it cannot be ignored, since it would produce a second harmonic in the fringe displacements. Professor G. Joos,[5] in his repetition of these experiments in 1930 at Jena, used a fused quartz base for his interferometer because he concluded that magnetostriction in a steel or Invar base would be excessive for the precision he expected to attain.

After the 1921 trials at Mount Wilson, Miller probably would have abandoned further work on the problem except for a visit to Case made by H. A. Lorentz during the following spring. During his visit he discussed the Mount Wilson observations and encouraged Miller to continue the experiments.

From the time the steel base interferometer was set up again at Case in 1922 until it was returned to Mount Wilson near the end of the summer of 1924, Miller carried out many experiments designed to test and improve the performance of the apparatus. He established the order of magnitude of the fringe shifts produced by unequal heating of the air in the optical paths of the interferometer, by varying the speed and reversing the direction of rotation of the instrument, and by various adjustments of the centering pin. He also used different light sources, including sunlight.

In many of the Cleveland trials of 1923 and 1924, especially those for which the data sheets record the best "seeing" of the interference fringes, the periodic effects are very small. These data were not analyzed in detail by Miller as he considered them only preliminary to the later work at Mount Wilson. However, 19 sets of observations made from August 23 to September 4, 1923, and 42 sets of data taken from June 27 to July 26, 1924, including those made with sunlight, for which the interferometer was in the best adjustment and the fringes were often noted as "good" or "excellent," constitute some of the best data obtained with Miller's interferometer. These 1923–1924 Cleveland results are included in Table I which summarizes the important trials of the Michelson-Morley experiment.

[4] D. C. Miller, Research Notebook, December 11, 1921.
[5] G. Joos, Ann. phys. **7**, 385 (1930) and personal communication in a letter of May, 1954.

After completion of the July, 1924 trials, the apparatus was returned to Mount Wilson where it was again set up in the observation hut, which was moved back some distance from the canyon edge to reduce the effects of wind. After a considerable number of preliminary trials, extensive series of observations were made by Miller at four epochs: March 27–April 10, 1925; July 24–August 8, 1925; September 10–23, 1925; and February 3–12, 1926.

The readings of fringe position were taken at 16 azimuth positions starting from the north and measured clockwise. A data sheet usually contained readings for 20 turns of the interferometer, although some contained a greater or lesser amount of data. For his analyses Miller averaged the 20 readings at each azimuth and then applied a linear correction for fringe pattern drift so that the averages closed as a periodic function in the 360° rotation. Furthermore, a constant was then subtracted from these averages so that their mean was zero. These adjusted means were plotted in azimuth and connected by straight lines to give a graph for harmonic analysis with the Henrici machine. A sample data sheet which presents details of his reduction method is published in Miller's paper in reference 2.

Miller's harmonic analyses of these curves yielded amplitudes and phases for the first five harmonics. The second harmonic provides the parameters needed for computing the presumed aether drift. The phases obtained from these analyses were never capable of being fitted into a logical relationship corresponding to an oscillation about the north point during the course of a sidereal day. As reported by Miller,[2] the axes were displaced from the meridian as follows: "for February 10° to the west of north; for April the displacement is 40° east; for August 10° east; and for September 55° east." This azimuth anomaly has been the greatest obstacle to the acceptance of the small periodic amplitudes reported by Miller as having relevance to an aether-drift effect. Synge and Gardner[6] developed a theory of the Michelson-Morley experiment, including the effects of acceleration, which was not inconsistent with the occurrence of azimuth anomalies, such as found by Miller, but further predictions of this theory have been disproved in experiments by Ditchburn.[7]

STATISTICAL ANALYSIS OF MILLER'S DATA

The data collected by Miller have recently been reexamined with a view to establishing the relative importance of statistical fluctuations and physical causes in the small periodic effects which he obtained.

Consider first the individual data sheets. It is generally agreed that an experienced observer with keen eyesight, such as Miller had, can estimate the fringe position to 0.05 fringe, which may be regarded as the least count of the instrument. This personal factor will introduce variations in the data in addition to those arising from other causes. Prominent among the latter possibilities are mechanical vibrations and bending as the interferometer rotates, temperature disturbances producing both a drift of the fringes and periodic changes, and magnetostriction which also would cause a periodic effect.

A typical data sheet contains entries in 16 columns for the azimuth points of the instrument, and 20 rows, one for each complete turn of the interferometer. Under ideal conditions with no aether drift nor other systematic or random effects present, all entries in a table should be identical. In a few sets of data taken at Cleveland in 1924, this is nearly the case. Usually, however, the readings show considerable variation, having the general character of random fluctuations superimposed on an irregular drift. A standard analysis of variance technique may be used to examine the degree of randomness in the entries on a data sheet.

Let x be a reading of fringe position and let $\bar{x}_i(i=1, 2, \cdots 16)$ be the average of the ith column of data (azimuth position on the interferometer); furthermore, let $\bar{x}$ be the average of all 320 entries, x_{ij}, on a data sheet, where the x_{ij}, and hence the averages, have already been freed of the linear drift of the whole fringe system in the same manner as Miller did it. Suppose that these 320 values be considered as random sample values drawn from a normal population. Furthermore, suppose the arrangement into columns and rows is a random one. Then statistical theory shows that the variance of x may be resolved into a contribution due to variation of the individual entries in the columns about the column means and a contribution due to differences between column means and the mean of the whole assembly. A measure of the relative significance of these is the statistic F defined by

$$F=\frac{320\times 19}{15}\frac{\sum_{i=1}^{16}(\bar{x}_i-\bar{x})^2}{\sum_{i=1}^{16}\sum_{j=1}^{20}(x_{ij}-\bar{x}_i)^2}.$$

Under the hypotheses of randomness and normality of population, the probability that F for a given sample will exceed a given value has been tabulated. For the number of degrees of freedom involved in the present data, theory shows that the probability of obtaining by pure sampling fluctuation an $F>1.71$ is only 0.05; the probability of obtaining an $F>2.21$ is only 0.01. These probabilities are accepted limits for rejecting the hypothesis that the array could have arisen by sampling fluctuations in a normally distributed population. When a large number of data sheets are analyzed, one would expect only one out of twenty to exhibit an $F>1.71$ if the population of which the sheets are samples consists only of randomly fluctuating data. If many of the data sheets lead to an F-value greater than the limits quoted,

[6] J. L. Synge and G. H. F. Gardner, Nature **170**, 243 (1952); also Proc. Royal Dublin Soc. **26**, 45 (1952).
[7] R. W. Ditchburn and O. S. Heavens, Nature **170**, 705 (1952).

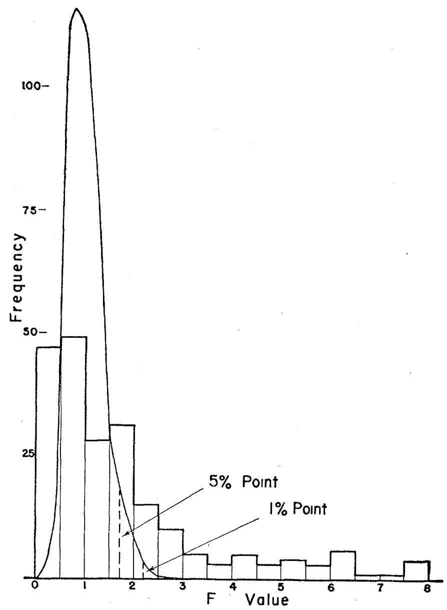

FIG. 1. The distribution of F-values for 216 sets of Mount Wilson data. The smooth theoretical curve is normalized so that the area under this curve is equal to the area of the histogram.

it is highly probable that some systematic effects are present in the data.

Figure 1 shows the frequency distribution of the F-values computed from the data sheets together with a theoretical distribution of F appropriate to the number of degrees of freedom involved. It is apparent immediately that the observed number of large F-values far exceeds the number to be expected on the basis of a random sampling from a normal population. In fact, 36.5 percent of the F-values exceed the critical value 1.71, and 25.3 percent exceed the value 2.21. The analysis indicates that the fluctuations in the column means cannot be attributed entirely to random effects, but that systematic effects are present to an appreciable degree.

A second method also shows that the periodic effects observed by Miller cannot be accounted for entirely by random statistical fluctuations in the basic data. Five representative sheets with original unprocessed data were subjected to an autocorrelation analysis.[8] These sheets were selected as typical of the Mount Wilson data. Miller had deduced for them second-harmonic amplitudes of 0.021, 0.045, 0.059, 0.082, and 0.123 fringe respectively. Thus they embrace nearly the entire range of the second-harmonic effect which is under examination.

In three sets of data (Nos. 42, 75, and 79 in Table II) a strong period of 8 (second harmonic) was found relative to those of periods 4 through 12. However, in the other two sets of data analyzed by this method, the period of 8 was no more prominent than the other periodic components. This indicates that the true magnitude of the systematic effect varies greatly with the conditions of observation, as would be the case if caused by local disturbances. A summary of the pertinent results regarding the period of 8 is given in Table II. The units in the table are fringes.

Column 2 gives the amplitude of the second harmonic as obtained from the Henrici machine by harmonic analysis; column 3 gives the amplitude deduced from the correlogram and contains whatever random fluctuations are in the data; column 4 gives the amplitude with the random effects removed. It is again apparent that random statistical processes contribute considerably to the periodic effect when it is small but that the larger amplitudes are relatively unaffected and cannot be explained in this manner.

Consider next the actual harmonic analyses of the Mount Wilson data. Miller's harmonic analysis records give amplitude distributions for the first five harmonics for which the relevant statistical parameters are summarized in Table III. The values of the mean amplitude $\bar{A}_k (k=1, 2, \cdots 5)$ and the standard deviation $s(A_k)$ have been computed directly from his data cards. Altogether, for the four seasons, 306 sets of data were used. There is no significant seasonal variation in the mean amplitude of the second harmonic, the values of $\bar{A}_2$ being 0.042, 0.049, 0.038, and 0.045 fringes for April, 1925, July, 1925, September, 1925, and February, 1926 epochs respectively.

Table III indicates clearly the dominance of the first and second harmonics over the others in the harmonic analyses. While the third, fourth, and fifth harmonics have an order of magnitude equal to the standard deviations, the first and second harmonics are considerably larger. However, the average $\bar{A}_2$ here is only 1/13 of the value to be expected on the basis of the usual aether-drift hypothesis. It is interesting to note that among the trials which yielded unusually large amplitudes of the second harmonic ($A_2 > 0.08$ fringe), 14 out of 17 were made either at the beginning or the end of a series of observations, or were made under adverse conditions noted on the data sheets.

The question now is, "What part of the observed average amplitude for the second harmonic, deduced by harmonic analysis, may reasonably be attributed to random statistical fluctuations in the data?" Let sets of

TABLE II. Summary of autocorrelation analysis.

Sheet	Miller's observed A_2	Correlogram Uncorrected A_2	Correlogram Corrected for random effects
15	0.021	0.032	0.013
23	0.045	0.054	0.043
79	0.059	0.062	0.060
75	0.082	0.079	0.078
42	0.123	0.129	0.126

[8] We are indebted to R. L. Stearns, Dr. E. F. Shrader, and Dr. L. L. Foldy for making the autocorrelation analysis; the MS thesis of R. L. Stearns, Case Institute of Technology, 1952, gives details of the analysis and description of machine used.

observations be selected at random from a population which is normally distributed and be plotted as ordinates with the order numbers of drawing as abscissae. Suppose a harmonic analysis is made of each set. Then the probability that an amplitude of a kth harmonic, A_k, chosen at random among the sets, will lie between A and $A+dA$ is

$$P(A)dA=nh^2\exp(-h^2nA^2/2)AdA,$$

where n is the number of observations analyzed and h is the measure of precision of the population. From this, the mean amplitude $\bar{A}_k$, often designated $M(A_k)$, and the standard deviation $\sigma(A_k)$ are found to be, respectively,

$$\bar{A}_k=\left(\frac{\pi}{2nh^2}\right)^{\frac{1}{2}} \quad \text{and} \quad \sigma(A_k)=\left(\frac{4-\pi}{2nh^2}\right)^{\frac{1}{2}},$$

independent of k.

A tabulation of the values of $\bar{x}_i$ irrespective of the azimuth i from which Miller made his harmonic analyses indicates an approximately normal population with a standard deviation 0.05 fringe. Thus in this extreme case, if all of the variation in the data used for analysis were attributable to random fluctuations, one would expect

$$\bar{A}_k=0.023 \text{ fringe} \quad \text{and} \quad \sigma(A_k)=0.012 \text{ fringe}.$$

On the usual assumption that the squares of the random and systematic contributions to amplitude add to produce the observed $(\bar{A}_2)^2$, it appears that not more than 15 percent of the second-harmonic amplitude can be due to statistical causes. This is in accord with the result of the autocorrelation analysis and leaves an average residual amplitude of $\bar{A}_2=0.038$ fringe to be explained in other ways.

Thus there can be little doubt that statistical fluctuations alone cannot account for the periodic fringe shifts observed by Miller. On the other hand, the presence of a periodic effect in the column means $\bar{x}_i$ may be clearly demonstrated in a way which again exposes the incompatibility between the phase anomalies already noted and the usual kinematic accounting in aether-drift experiments.

When the $\bar{x}_i$ for twenty sheets of the July–August, 1925 Mount Wilson data are plotted as a function of azimuth index i, the result is as shown in Fig. 2. These sheets have been chosen *so as to span the 24 hours of the sidereal day.* There is obviously considerable scatter in the $\bar{x}_i$ at each azimuth position, but the average values $\langle\bar{x}_i\rangle$, *calculated for each index, show a marked second harmonic effect.* The large filled circles represent the $\langle\bar{x}_i\rangle$. Similar results have been found for the other three epochs of the Mount Wilson observations.

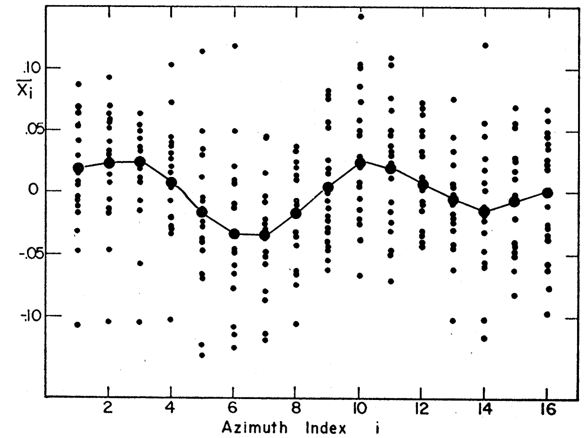

FIG. 2. The individual column means $\bar{x}_i$ are plotted as a function of azimuth position for the July, 1925 observational data. Large circles and the connecting curve show the second harmonic effect exhibited by the averages, $\langle\bar{x}_i\rangle$, due to ordering in azimuth. Units for the ordinate are fringes.

TABLE III. Summary of harmonic analyses—Mount Wilson data.

Harmonic (k)	Mean amplitude $\bar{A}_k$ (fringes)	Standard deviation $s(A_k)$ (fringes)
1	0.046	0.033
2	0.044	0.022
3	0.011	0.008
4	0.007	0.005
5	0.005	0.004

Now let the values of $\langle\bar{x}_i\rangle$ for $i=9, 10\cdots16$ be averaged with those for $i=1, 2, 3\cdots8$. When this is done and a smooth curve is drawn through the points, the result is the curve labelled "July" in Fig. 3. Similar curves for the other epochs are also shown in the figure. Twenty data sheets, representative of all sidereal times, were used for each sample.

Note that all of the resulting periodic functions have amplitudes between 0.02 and 0.03 fringe, but that they differ in phase. Particularly striking is the difference between the February, 1926 result and the others.

It is, on the other hand, possible to *calculate* in a rather straightforward way the fringe shift pattern, *averaged over all sidereal times,* to be expected on the assumption of an absolute motion of the earth through an aether on the basis of the usual kinematic considerations. Let $\mathbf{V}$ be the vector velocity of the earth at a given epoch toward an apex whose celestial coordinates are α, δ; let $\mathbf{V}_p$ be the projection of $\mathbf{V}$ in the plane of the interferometer, φ be the latitude of the observer, and ψ be the azimuth of the telescope arm of the interferometer measured from the north point through the east. Then calculation shows that the value $\langle\Delta n\rangle_{\text{Av}}$ of the fringe shift, averaged over all sidereal times, is given by

$$\langle\Delta n\rangle_{\text{Av}}=V^2F(\delta,\varphi)\cos2\psi. \tag{1}$$

Since φ is constant and δ is fixed at a specified epoch, an average fringe shift of second-harmonic nature results. Clearly, the angle ψ equals $2\pi(i-1)/16$ and the computed $\langle\Delta n\rangle_{\text{Av}}$ corresponds to the observational $\frac{1}{2}[\langle\bar{x}_i\rangle+\langle\bar{x}_{i+8}\rangle]$. Thus there is a direct correspondence between the curves of Fig. 3 and Eq. (1), and no phase

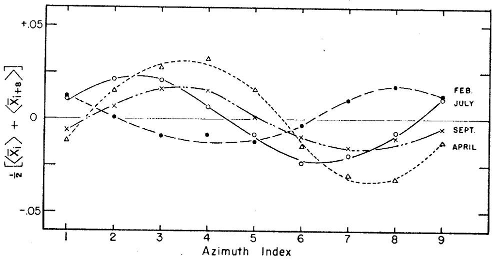

FIG. 3. Second harmonics in the $\frac{1}{2}[\langle\bar{x}_i\rangle+\langle\bar{x}_{i+8}\rangle]$ for the four epochs of the Mount Wilson data.

anomalies of the kind observed in Fig. 3 are permissible. The four curves should have a *common* maximum (or minimum) at $i=1$; only the amplitude may be different at different epochs.

This criticism is, of course, only different in form from the objections raised by v. Laue[9] and Thirring.[10] Both authors point out that the Lorentz criterion, which requires the mean value of $\mathbf{V}_p$ over a sidereal day to be a vector in the meridian, is not satisfied by the results of the Mount Wilson observations of 1921 and 1925, and Thirring shows for a single daily pattern of these observations that it cannot be made to coincide with any one of the daily patterns derivable from the usual kinematic theory upon variation of δ. The present form of the criterion, if it may be so designated, is perhaps more convenient when a large amount of data is to be checked for consistency.

In the interpretation of his observations, Miller hypothesized that, for unknown reasons, the daily swing of the vector $\mathbf{V}_p$ might be symmetric about some line other than the meridian, the angular deviation from the meridian being permitted to vary from epoch to epoch. In terms of Fig. 3, this is simply the deviation of the first maximum from the north point, but the angles in our figure differ considerably from those found by Miller.[2] Actually, Miller drew the curves A *vs* θ ($A=$azimuth of $V_{p,\,\max}$; $\theta=$sidereal time) for each of the four epochs and determined his anomalies from what he called the "own axis" of the plot. The meaning of that term is obscure. Contrary to expectation, it does *not* designate the horizontal axis through the centroid of the observational curve, since the ratio of the upper to the lower area is found to be 1.3, 1.7, 1.4, and 0.6 respectively for the four epochs.

Under these circumstances, little significance can be attached to the remarkable agreement between the values of α and δ, computed from the V_p *vs* θ curves on the one hand and from the A *vs* θ curves on the other hand, as presented in Tables I and II in reference 2, p. 230. The second method of computing uses the θ values at which A passes "its own axis" and the quantity $A_{\max}$, both taken from the curves. Both parameters depend, of course, on the choice of the "own axis," but the first of them is *extremely sensitive* to that choice and, furthermore, is directly equal to the right ascension of the apex, α. Hence, even if one were to accept the azimuth anomalies as an unknown effect to be explained by a more refined theory, any numerical results based on the A *vs* θ curves should be accepted with reservation.

Whether a reliable determination of the other pair of parameters, $V_{p,\,\max}/V_{p,\,\min}$ and $\theta v_{p,\,\min}$, from the V_p *vs* θ curves is at all possible has not been decided. The method of averaging small groups of V_p-values (each V_p corresponding to one set of twenty turns) in order to obtain twenty V_p-averages roughly equidistant in θ, and the use of these twenty points as true representatives of the curve sought cannot be considered statistically satisfactory because of the enormous scatter of the observational results. However, a more refined redetermination of these parameters that enter the first method of computation did not seem justified to us for the following reasons.

If the azimuth anomalies are accepted, then Eq. (1) establishes the simple connection,

$$\langle\Delta n\rangle_{\text{Av, max}}=V^2F(\delta,\varphi_0) \tag{2}$$

between the maximum of the fringe shift average observed during the epoch and the function $F(\delta,\varphi_0)$, whose derivative with respect to δ can be shown to vanish for $\delta=90°$. Now, $\langle\Delta n\rangle_{\text{Av, max}}$ changes only slightly from epoch to epoch (cf. Fig. 3), which makes it feasible to satisfy (2) approximately by the same constant vector $\mathbf{V}$ for all epochs. However, the vector $\mathbf{V}$ must contain the variable contribution of the orbital motion of the earth. This difficulty may be overcome by choosing a cosmic $\mathbf{V}_0$ such that $V_0\gg V_{\text{orbital}}$ and δ_0 is not far from 90°; but a further obstacle arises here, namely, at $\delta=90°$, $F(\delta,\varphi_0)$ becomes simply $a^2\cos^2\varphi_0$, where a^2 is a constant completely determined by the geometry and wavelength of the interferometer. Thus, the magnitude of $\mathbf{V}_0+\mathbf{V}_{\text{orbital}}$ cannot be chosen freely. A reduction factor becomes necessary so as to make $\langle\Delta n\rangle_{\text{Av, max}}=kV^2F(\delta,\varphi_0)$ where k is about 1/20 in Miller's cosmic solution.

It seems to us on the basis of this discussion that the internal consistency of the cosmic solution is not so great a surprise as it appears at first glance. It certainly is not cogent enough to serve as a logical support of the claim that the half-period effect observed is a true aether-drift effect. We therefore did not embark on a statistically sound recomputation of the cosmic solution, but rather concentrated our efforts on an interpretation of Miller's observations in terms of systematic local disturbances such as may be caused by mechanical effects or by nonuniform temperature distributions in the observation hut.

POSSIBLE MECHANICAL EFFECTS

The question may be raised whether some of the local causes which are responsible for Miller's results can be found in the mechanical performance of his apparatus.

[9] M. v. Laue, *Handbuch d. Experimentalphysik* (1926), Vol. XVIII, pp. 95, 101.

[10] H. Thirring, Z. Physik **35**, 723 (1926); Nature **118**, 81 (1926).

Miller's papers and notebooks do not offer much pertinent material but it has been possible to extract some information from his report,[2] from the originals of the photographs published there, and from interviews with former co-workers. These data make it possible to investigate the motion and deformation of the steel cross carrying the optics of the interferometer.

The cross consists of four arms bolted together, each arm being a box structure held together by rivets. The central portion of the cross rests squarely on a wooden support which floats on mercury.

Consider the beams B_1 and B_2, each formed by one pair of opposite arms. If the supporting float is horizontal, the ends of the beams will sag under the influence of gravity somewhat below the level of the supported central portion. Now let the cross turn about the beam B_2 through a small angle α_1. This will cause the beam B_1 to unbend slightly, since only the component of gravity normal to the beam contributes to the bending moment. The mirrors rigidly connected to the ends of B_1 will therefore suffer small additional rotations in opposite directions about axes parallel to B_2. A very small change in the length of the light path will be the consequence of this change in the relative position of the mirrors on B_1.

In reference 2, p. 215, Miller supplies direct evidence of the effect of bending, when he states that an end load of 282 g placed on one end of the four arms produces a shift of one fringe.[11] We may reasonably assume that half this load, $\delta=141$ g, placed simultaneously on two opposite arms would have produced the same effect; a simple argument now gives the fringe shift due to an angular rotation α_1 about B_2 in units of one fringe as

$$n_1=(1-\cos\alpha_1)q/\delta, \tag{3}$$

where q depends on the weight of the beam and on its distribution, and the analogous formula holds for a rotation α_2 about B_1. We obtained[12] for q the value 48×10^3 g.

From Eq. (3) we can find the angle α_1 necessary to produce a total fringe shift $n_1=2\times0.044$, which is the mean of the observations on Mount Wilson. The result is $\alpha_1=1.3°$, which seems too large to permit an interpretation of the observed fringe shifts in terms of a wobbling motion of the slowly rotating cross (one turn in 50 sec). But smaller libratory irregularities, perhaps of the order of a few tenths of a degree, must undoubtedly have been present in Miller's experiments. In order to find out about their possible contributions to the results, the dynamic constants of the apparatus in rotation were determined, and a simple analysis of its motion was made.

[11] This figure is not strictly a constant; according to a check on February 4, 1926, it is 313 g.

[12] It will be noticed that the quantity δ is determined when the structural properties of the cross, the geometry of the light path, and the wavelength are known. The calculated δ came out about twice as large as δ given by Miller, but an overestimation of the stiffness was to be expected, since no correction for incomplete fixity of rivets and bolts was made. We wish to thank E. Krasnoff for his assistance in these computations.

Assuming the apparatus strictly symmetrical about the vertical, one obtains for the moments of inertia C and A about the vertical symmetry axis and about a horizontal axis through the center of mass, respectively,

$$C=1.67\times10^{10}\text{ g-cm}^2,\quad A=9.05\times10^9\text{ g-cm}^2.$$

The center of mass is 48.5 cm above the centroid of the displaced mercury, and the metacenter is 210.5 cm above the center of mass. The restoring moment produced by a small angle θ between the symmetry axis of the apparatus and the vertical is θD, where

$$D=2.48\times10^{11}\text{ dyne-cm}.$$

The period of roll about a horizontal axis through the center of mass is therefore only 1.20 sec.

The motion of the system may be considered as that of a (hanging) top. The restoring couple is θD. Depending on the fixity of the centering pin (which was not the same in all experiments), the fixed point may be identified either with the center of mass itself or with the bearing of the pin, which is about 51.5 cm below the center of mass. The equatorial moment of inertia is in this latter case $A=1.22\times10^{10}$ g-cm^2; C and D are, of course, unchanged.

Let us consider the motion in the neighborhood of steady rotation about the vertical axis of symmetry. The values of the constants A, C, D, together with the very small spin $r=2\pi/50$ sec^{-1}, are such as to preclude regular precession, whichever fixed point is chosen. The motion is of the "rosette-type," the instantaneous position of the symmetry axis being given by[13]

$$\begin{aligned}\theta&=\eta\sin\omega_1 t, & \omega_1&=[(C^2r^2+4AD)/4A^2]^{\frac{1}{2}},\\ \psi&=\omega_2 t, & \omega_2&=(C/2A)r.\end{aligned} \tag{4}$$

Here θ and ψ are the polar coordinates of the axis of symmetry, and η is an amplitude constant characteristic of the asymmetry of the initial impulse.

It is now possible to find an approximate expression for the fringe shift due to angles α_1 and α_2 subtended simultaneously by B_1 and B_2 with the horizontal plane, when the symmetry axis moves according to Eq. (4). One obtains

$$\Delta n=n_1-n_2=(pW/2\delta)\eta^2\sin^2[\omega_1 t]\cos[2(r-\omega_2)t]. \tag{5}$$

The circular frequency ω_1 is very close to $(D/A)^{\frac{1}{2}}$ and corresponds to the roll period mentioned before. Thus Δn oscillates with twice the roll frequency, the amplitude being modulated with the circular frequency $2(r-\omega_2)$. The value of ω_1 is not very sensitive to a change of the fixed point; the roll period increases from 1.20 sec to 1.40 sec if the fixed point is assumed at the pin bearing. The period of modulation, however, decreases from 39 sec to 10 sec if, instead of the center of mass, the pin bearing is assumed fixed.

Obviously, these rapid oscillations cannot account for

[13] F. Klein and A. Sommerfeld, *Ueber die Theorie des Kreisels* (Leipzig, 1898), p. 331.

the true second-harmonic period (25 sec) which strongly dominates such runs as shown by sheets 79, 75, and 42 in Table II. Beyond that, one may in general expect the contributions of these oscillations to cancel each other in the average throughout a set of 20 runs. In order to estimate the efficiency of the averaging process, assign to the amplitude constant η the (improbably large) value of 0.5°; compute a table of 20 rows according to Eq. (5) by setting $t=50k/16$, where $k=1, 2, \cdots 320$; and determine the column averages $\langle\Delta n\rangle_{\text{Av}, i}$. It turns out that the maximum value of $|\langle\Delta n\rangle_{\text{Av}, i}|$ stays well below 0.005 fringe.

These conclusions would not be strongly affected by a slight asymmetry of the apparatus about its vertical axis as must undoubtedly have been present.

EFFECTS OF TEMPERATURE

Miller's 1923 laboratory tests of the effects of thermal variations on the interference fringes have been studied with the view of establishing their relationship to his Mount Wilson observations, where temperature effects due to intense sunlight during the day and to canyon air currents at night were often troublesome. Miller's own suspicions that thermal effects might be important are shown by the entry in his laboratory notebook for April 14, 1921: "Sun shining full on side of house. There was a very large drift which seems to be in the direction of the sun; indicating possibility that the entire effect is due to temperature!"

In the laboratory tests, electric heaters were placed at the level of the mirrors and about three feet from the circle travelled by them. The altered refractive index of the heated air and the thermal effects on the mirror supports change the optical path lengths of the interferometer, and when these are affected unequally in the two arms, the fringes shift in position. On the assumption that the four arms of the interferometer all have the same thermal insulation, a localized temperature anomaly or a temperature gradient across the room will produce a second-harmonic term in the fringe positions as the interferometer rotates, similar to that anticipated for an aether drift.

Localized heating will also produce a first harmonic in the fringe displacements when one of the four interferometer arms, for example that containing the observing telescope, has different thermal insulation properties from the others. The effects of heat on the 3rd, 4th, 5th, and higher harmonics of the fringe displacements should be small for any temperature conditions likely to be encountered.

The laboratory tests of 1923 were conducted with various amounts of thermal insulation protecting the light paths and mirror supports. In some trials the air was directly exposed to the heaters; in many cases the glass and wood casing that served at Mount Wilson as insulation for the light path was employed; and in certain experiments additional corrugated paper was placed over the vertical glass walls of the casing, the

TABLE IV. Laboratory heating trials (unit: fringes).

Periodic amplitudes	Controls Set 17	Controls Set 28	Heat Set 18	Heat Set 29
A_1	0.006	0.006	0.010	0.021
A_2	0.015	0.010	*0.049*	*0.052*
A_3	0.006	0.005	0.009	0.005
A_4	0.004	0.003	0.006	0.011
A_5	0.001	0.001	0.003	0.002

mirror mountings, and in some cases over the steel base of the interferometer as well. In the experiments where the air in the optical paths was directly exposed to heat, large second harmonics ($A_2=0.35$ fringe for one heater, and about twice this value for two heaters) were always observed in the fringe displacements, and with the expected phase. Shifting the heaters to a different azimuth produced a correspondlng change in the phase of the second harmonic. When the optical paths and mirror supports were thermally insulated, the second harmonics were greatly reduced; for example, with the glass coverings as used at Mount Wilson in place, the amplitudes were reduced to about 0.07 fringe.

Among the laboratory trials made by Miller at Case in 1923 are four sets of observations which reveal rather clearly the character of the temperature effects. In these sets the optical paths of the interferometer were enclosed with the glass thermal insulation as used at Mount Wilson, and, in addition, corrugated paper was placed over all arms of the interferometer. In two of these trials no artificial heating was used, but in each of the two sets immediately following these controls, the heater, in the position mentioned above, was in operation. The results of these experiments are given in Table IV. It is evident that the heater produced disturbances which increased the amplitudes of all five harmonic components. However, the effect on the second harmonic A_2 is much the largest, as is to be expected on physical grounds. Furthermore, the phases of the second harmonic in sets 18 and 29 have values consistent with the position of the heater. The first harmonic A_1 is somewhat increased by heat, and for this also there is some physical justification. The increases in the amplitudes A_3, A_4, and A_5 are much smaller.

It must be emphasized that the foregoing analysis of these tests reveals small but certain temperature effects, in contrast to Miller's statement that he had shown the absence of periodic effects caused by artificial heating when the light path was thermally insulated as previously described.[14] Similar conclusions regarding the danger of spurious second harmonics due to thermal conditions were reached by Joos in his elaborate preparations for a repetition of the Michelson-Morley experiment at Jena. In fact, Joos concluded from his laboratory trials that temperature disturbances would be so serious that photographic recording of fringe positions

[14] See reference 2, p. 220.

would be impossible except in a well-insulated basement laboratory.[15]

Thus Miller's experiments in 1923 do not rule out the possibility of attributing the remaining systematic effects in the Mount Wilson data, which are most prominent in the second harmonic A_2, and to a lesser degree in the first harmonic A_1, to temperature causes. In what follows, we shall interpret the systematic effects on this basis, but must admit that a direct and general quantitative correlation between amplitude and phase of the observed second harmonic on the one hand and the thermal conditions in the observation hut on the other hand could not be established. The reason for this failure lies in the inherent inadequacy, for our purpose, of the temperature data available.

Let us first discuss the physical consequences of the weak radiation field maintained across the hut by the temperature differences of the walls. Since periodic temperature variations of only 0.001°C in the air of the optical arms would produce fringe shifts as large as the average effects observed at Mount Wilson,[16] a very conservative estimate was made of the wall temperature differences necessary to produce temperature oscillations of that magnitude. For this purpose it was assumed that (1) the ambient air temperature in the hut was essentially constant, (2) radiation was absorbed only by the vertical glass plates of the casing, and (3) heat transfer from the glass to the air inside the casing was by conduction only.[17] The resulting wall temperature differences are about ten times as large as those usually recorded by the four thermometers located on the walls of the observation hut. There is no doubt, however, that this factor ten would be very considerably reduced if convection of the air inside the casing were taken into account and if the contribution of the cover of this casing, facing the roof of the hut, could be evaluated.

In reality, however, the effects of temperature on the apparatus must have been very complex, being mixed contributions of changes in density of the air in the optical paths, angular deflection of the mirror supports, and thermal expansion of the steel frame, the latter effect introducing a long time lag. It is practically impossible to carry through calculations which would predict the over-all behavior of the interferometer due to temperature anomalies, since hardly any of the necessary data for such calculations exist. In fact, the readings of the four thermometers constitute all of the available information about the temperature (and radiation) pattern in the hut. They give essentially the air temperature along the wall (but not the wall temperature), and say nothing about the temperature distribution along the roof, itself a low-gable construction with ridge at 20° to the N–S direction, and with its under side only a few feet above the thin wooden cover of the casing of the light paths. We conclude from the foregoing estimate that an interpretation of the systematic effects in terms of the radiation field established by the nonuniform temperatures of the roof, the walls and the floor of the observation hut is not in quantitative contradiction with the physical conditions of the experiment.

In view of the local factors affecting the temperature conditions of the interferometer, we may now ask whether the epoch averages shown in Fig. 3 should not in some way correlate with what is known about the mean temperature conditions at the several epochs. Thus it has been noted that in Fig. 3 the curve of the February, 1926 observations differs considerably in phase from those for the other epochs. We believe this behavior is correlated with the fact that throughout the February experiments the thermometers on the north and west walls of the house consistently registered temperatures from 1°C to 2°C lower than the south and east wall thermometers. This situation resulted primarily from a blanket of snow which covered the ground on the N–W side of the hut. Furthermore, the west wall of the hut was water-soaked throughout the February runs. As a result the average temperature gradient through the hut was in the general SE→NW direction in February, in contrast to that in July and September, 1925 when the temperature gradient averaged throughout a day was more nearly along the N–S line.

Turning now to single sets of observations, we may first try to correlate the smallest fluctuations f occurring[18] in the Mount Wilson sets with the thermometer readings. There are eleven sets with f-values smaller than 0.015 fringe. If we introduce $\langle \Delta T \rangle_{\mathrm{Av}} = \mathrm{av.}|T_i - \bar{T}|\ (i=1, 2, 3, 4)$ as an indicator of uniformity in temperature conditions, where T_i denotes a thermometer reading, then $\langle \Delta T \rangle_{\mathrm{Av}}$ ranges from 0.02°C to 0.25°C with a mean value of 0.11°C for these observations.

Thus small f-values usually go with small $\langle \Delta T \rangle_{\mathrm{Av}}$ values, but the correlation is very incomplete, as $\langle \Delta T \rangle_{\mathrm{Av}}$ values of 0.2°C are very often found in the Mount Wilson sets. Furthermore, sets with small f-values usually do not compose a normal population. For example, if $(\bar{x}_i + \bar{x}_{i+8})/2$ is computed for half of one of these sets, then usually a much larger f-value results. This agrees with the existence of periods other than 8 found in the autocorrelation analysis of the two samples having small A_2 shown in Table II and indicates that many of the small f-values observed are the result of chance cancellations.

If, on the other hand, the largest f-values (>0.08 fringe) are considered, one is left, after elimination of

[15] According to the personal communication from Professor G. Joos (1954); see also reference 5.

[16] This figure is in agreement with similar estimates made by R. J. Kennedy, Proc. Natl. Acad. Sci. **12**, 621 (1926); and by G. Joos, Phys. Rev. **45**, 114 (1934).

[17] We are indebted to our colleague Dr. H. G. Elrod for these calculations.

[18] Let averages $\bar{x}_i$ be calculated as on p. 169 and consider $(\bar{x}_i + \bar{x}_{i+8})/2$ as an approximation to the amplitude of the second harmonic. The maximum absolute difference between any of these numbers will be denoted by $2f$, where the fluctuation f now corresponds to A_2.

sets where remarks such as "sun shines on interferometer" occur, with a selection that appears to consist of two different groups. In one group, including sets 103 and 106 of March 28 around 5:00 P.M.; 128 of April 8, 5:00 P.M.; 135 of April 9, 3:00 P.M.; 45 and 47 of September 17, before noon, 68 and 69 of September 19, around 8:00 A.M.; and sets 4, 5, 6 of February 4, around 3:00 P.M.; and 12 and 13 of February 5, around noon, $\langle \Delta T \rangle_{Av}$ is large (>0.4°C). In the other group, which includes sets 72 to 75 of August 6, around noon; sets 80 to 82, soon after sunrise on August 7; 88 to 90 near midnight of August 7 and 8; and 91 to 94 following sunrise on August 8, $\langle \Delta T \rangle_{Av}$ is about 0.2°C or smaller.

Thus in the April, September, and February epochs the largest occurring f-values are associated with large $\langle \Delta T \rangle_{Av}$ values, but not so in the July epoch. We have no certain explanation for the existence of large effects when the wall thermometers give reasonably small $\langle \Delta T \rangle_{Av}$ values, but would point out that no temperature data are available to reveal thermal conditions at the roof, which may be responsible for the large fringe displacements at the times of highest altitude of the sun.

Although the thermometer readings cannot be used to describe the thermal conditions in the observation hut in all cases, they should nevertheless provide an indication of the stability of the thermal pattern in the hut affecting the fringe positions, particularly during the night hours. Accordingly the Mount Wilson data for each of the four epochs of observation have been searched for sets of readings taken during this part of the day and which exhibit "similar" temperature patterns.

Ideal observing conditions could obtain if all four thermometers read the same and remained constant throughout a series of observations. Departures from the ideal always occur, and the extent to which temperature disturbances are revealed by the recordings of the thermometers may be estimated by applying the following criteria:

1. The magnitude of the average absolute temperature difference $\langle \Delta T \rangle_{Av} = \langle (T_i - \bar{T}) \rangle_{Av} (i = 1, 2, 3, 4)$.
2. The time rate of change of the temperature pattern in the room as indicated by the readings of the four thermometers.
3. The drift in mean temperature, $\bar{T}$, with the time.
4. The linearity of the drift of the fringe pattern and the nonreversal of sign of this drift during a series of observations.

The last two criteria are particularly important because of the complex time lag in the thermal behavior of the heavy steel interferometer base and the mirror supports.

Thus, on the hypothesis that the second harmonics of the fringe displacements are due primarily to temperature conditions, the observed fringe behavior throughout a set of midnight-dawn experiments should be the same within the experimental uncertainties on nights when the temperature conditions remained rather constant, as

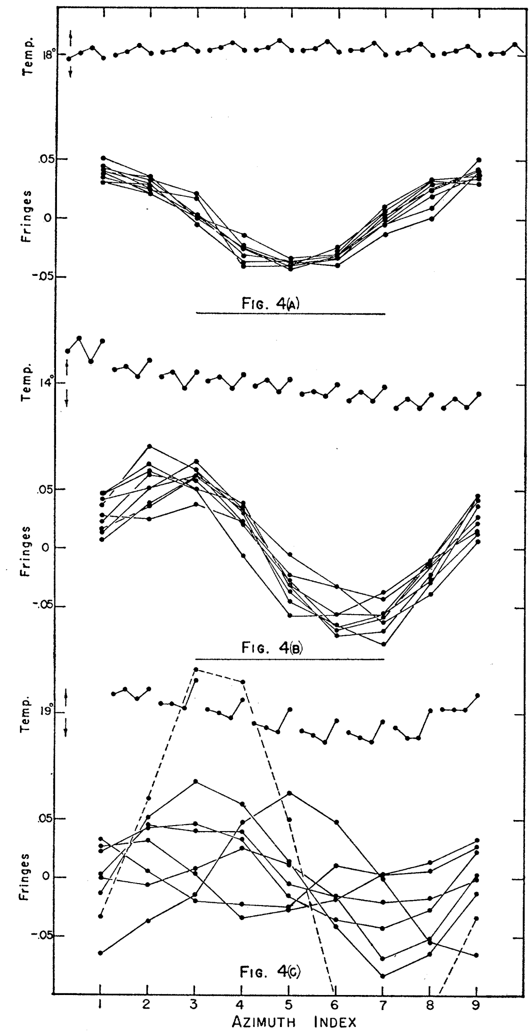

FIG. 4. The temperature pattern in the observation hut and the observed fringe shift is shown here for three dates: Fig. 4(A), August 30, 1927 (Cleveland); 4(B), September 23, 1925 (Mount Wilson); 4(C), July 30, 1925 (Mount Wilson). Each group of four points in the upper parts of the figures represents the thermometer readings on the four walls of the hut at a given time between midnight and dawn (a range of about 5 hours in sidereal time). Arrows at the left of the temperature scales indicate a 1° range. The lower part of each figure shows the fringe shift as a function of azimuth index for the times corresponding to the temperature patterns. The lower part of each figure shows the fringe shift as a function of azimuth index for the times corresponding to the temperature patterns. The dashed curve in Fig. 4(C) shows the fringe displacement when direct sunlight fell on the interferometer at dawn.

defined by the criteria above. This, in fact, is the case. Figure 4 illustrates the observed influence of several temperature conditions on the second harmonics of the fringe displacements. Figure 4(A) shows ten sets of observations, Nos. 31 to 40 inclusive, made in the hut on the Case campus between midnight and 5:00 A.M. on August 30, 1927. During this entire interval the readings of the four thermometers were remarkably constant, the average temperature changing by only 0.4°C. Likewise the second harmonics are almost identical in both phase and amplitude throughout the entire series. This behavior persists throughout almost five hours of sidereal time as the earth makes nearly $\frac{1}{4}$ of a revolution and would be extremely unlikely if the fringe shifts were due to any cosmic effect. On the contrary, it strongly supports our hypothesis that local temperature conditions are the dominant factor producing the observed second harmonics.

Similar correlations between the observed second harmonics and the temperature conditions existing from midnight to dawn can also be made in each of the four epochs during the Mount Wilson experiments. In all but one case where the criteria given above indicated good temperature conditions, the second harmonics were nearly alike during the time interval encompassed by the observations. A typical example of Mount Wilson results taken under rather good temperature conditions is shown in Fig. 4(B) for runs 75 to 83 inclusive taken from 12:18 A.M. to 6:00 A.M. on September 23, 1925. Figure 4(C) will be discussed later.

Other sets of Mount Wilson data taken under steady temperature conditions and which show constancy of both amplitude and phase in the second harmonics of the fringe displacements are the following: April 2, 1925, sets 113, 114, 115, 116, 117, and 118, taken between 1:52 A.M. and 4:58 A.M.[19] On August 8, 1925, sets 88, 89, 90, 91, 92, and 93 were made between midnight and 6:26 A.M. and with similar room temperature patterns throughout, and again the amplitudes and phases of the second harmonics are nearly alike. A final example is provided by data taken between 2:30 A.M. and 6:38 A.M. on February 11, 1926, in sets 84 to 91 inclusive. Here again the temperature patterns during all runs are similar, and within the experimental errors, all the second harmonics are alike.

The April 2 and September 23 runs show another important property, namely that the maxima of the second harmonic curves are definitely removed from the north point (the maxima of the other two night runs being close to it). This behavior throughout nearly six hours of sidereal time *conclusively* rules out cosmic effects. It also removes magnetostriction as the possible cause of the fringe displacements, since the interferometer steel cross was not dismantled during the four epochs of the Mount Wilson observations; hence any magnetostriction effect should not change its directional character during the four epochs.

In addition to the night runs cited above, daytime conditions occasionally gave steady temperature conditions as judged by the aforementioned criteria for three or more consecutive sets, which then usually showed similar second harmonics. When temperature conditions departed markedly from the ideal as defined by our criteria, widely varying second harmonics, both as regards amplitude and phase, were observed in consecutive sets.

Particularly remarkable among the daytime observations are sets 56, 57, and 58, made in 1924 in a basement room of the Case physics laboratory between noon and 6:00 P.M. on July 8, which give almost identical null results. In fact, throughout most of the turns of the interferometer the fringes showed no change in position whatever. The temperature conditions were ideal; all four thermometers gave identical readings throughout each set of observations, and the drift in average room temperature was only 0.1°C throughout the entire afternoon. If our hypothesis is correct, then this group of observations is the best ever made with Miller's interferometer and shows the zero effect consistent with the results of other experimenters.[20]

Among the Mount Wilson night sets there is, however, *one* unusual series of observations, Nos. 21 to 28 inclusive, made between 1:43 A.M. and 6:04 A.M. on July 30, 1925. Here the temperature criteria are in part violated (but not very strongly), and in part satisfied, yet the run shows an extremely erratic behavior [Fig. 4(C)]. We have no ready explanation for this apparent departure from the four other night runs with "steady thermometric conditions." Perhaps the fact that the canvas over the roof was not yet installed is the reason; or again canyon winds may have been unusually troublesome. It is certainly not possible to blame the lack of regularity of this run on the statistical incompleteness of its single sets. If one reduces half of a set by Miller's method, one obtains essentially the same result as that obtained by reduction of the whole set, the agreement between the two results becoming progressively closer during the course of the night's observations.[21]

The run of Fig. 4(C) differs in the following respect from the other night runs. One may define some measure of the "noise" of a set of observations by taking the average, for the whole set, of the absolute values of the first differences of the single readings. In the run shown in Fig. 4(A), one so finds 0.079 and 0.063 fringe for sets 31 and 32 around midnight, and this figure drops steadily to 0.035 for No. 40 at 5:00 A.M. In the Mount Wilson night runs, with the exception of that of Fig. 4(C), the corresponding "noise" drops from about 0.110 to about 0.060 fringe, while it decreases from 0.190 to

[19] This run was followed by sets 119 and 120 taken after sunrise, when the temperature pattern in the room had changed due to heating of the east wall of the hut. These latter sets exhibit considerably increased amplitudes and different phases in the second harmonics.

[20] Sets 57 and 58 were made with sunlight, rather than the usual acetylene source employed in set 56. The null results obtained with sunlight are of especial interest in that they disprove the Ritz emission theory of light. Trials of the Michelson-Morley experiment with sunlight or starlight had been urged by Tolman and also by LaRosa to test this point, see R. C. Tolman, Phys. Rev. **35**, 136 (1912); M. LaRosa, Nuovo cimento **3**, 345 (1912); Phys. Z. **13**, 1129 (1912).

[21] This holds for all of these night runs; in particular, the sets shown in Fig. 4(A) from No. 35 on have a surprisingly high degree of statistical completeness in the above sense. Together with the autocorrelation results of Table II for sets 42, 75, and 79, this seems to justify Miller's method of correcting for drift by assuming it to be linear.

0.100 in the night run of Fig. 4(C), if the last set, No. 28, is omitted.[22] This indicates an irregularity in the conditions of the exceptional run which is not present in the other night runs and is hardly reflected in the thermometer readings.

CONCLUSIONS

We believe that this discussion of the effect of temperature permits the following inference: Under the most favorable experimental circumstances the second harmonics in the Mount Wilson data remain essentially constant in phase and amplitude through periods of several hours and are then associated with a constant temperature pattern in the observation hut. This, together with the statistical and mechanical analyses, forces us to conclude that the observed harmonics in the fringe displacements are not due to a cosmic phenomenon (aether drift), nor to magnetostriction, nor to mechanical causes, but rather to temperature effects on the interferometer. These disturbances were much more severe at Mount Wilson than those encountered by other observers in their repetitions of the Michelson-Morley experiment performed in laboratory rooms.

Table I summarizes the results of the significant trials of the Michelson-Morley experiment. The first three columns of the table record the experimenter, year, and place of observation. The fourth column gives the optical path length D of an interferometer arm, as used in each of the several experiments. Column 5 gives the maximum anticipated fringe displacement corresponding to a 30-Km/sec aether drift velocity when the apparatus is rotated through 90° in its optical plane. Column 6 lists the amplitude A of the second harmonic of the fringe shifts actually found by each observer. In every case the observed double amplitudes $2A$ are much smaller than the expected fringe displacements listed in column 5. To provide a basis for comparison of the several trials of the experiment, the ratios of the expected fringe displacements in column 5 to the values of $2A$ actually found in each experiment are listed in the last column.

[22] Set 28 (indicated by the dashed line) was taken at sunrise, and Miller noted, "Sun shines on interferometer; fringes becoming unsteady." The very large increase in the second harmonic in this set clearly reveals the effect of radiant heat on the interferometer, and similar effects occur at dawn in other sets. In judging the sensitivity of the interferometer toward radiation, it should be kept in mind that it was exposed to the direct sun rays only through cracks in the beaverboard wall of the hut or leaks around windows and the door. On July 31, a large canvas cover was put over the roof (apparently at some distance above it) and over most of the walls of the hut in order to protect the interferometer from light leaking through and to get rid of the effects of the direct irradiation of roof and walls. The canvas cover was at least partly in place also in the September and February epochs.