Generation of time-reversed wave fronts by nonlinear refraction*

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We describe a nonlinear method for generating, nearly instantaneously, a time-reversed replica of any monochromatic-beam wave pattern. The method employs the interaction of the incident beam, of arbitrary wave front, with counter-propagating plane "pump" waves in a homogeneous, transparent, nonlinear medium. Media are shown to exist in which time-reversed waves can be generated with high efficiency using available laser pump sources.

I. INTRODUCTION

For any electromagnetic wave that propagates through an inhomogeneous, nonabsorbing, medium (having no permanent magnetism), there can exist in principle a time-reversed replica of this wave. This means, for example, than an appropriately patterned but irregular wave front can travel through a randomly inhomogeneous medium and emerge as a coherent uniform wave front, providing it is a replica, reversed in time, of a coherent beam that is deformed by the same inhomogeneous medium. Here we propose a new method for generating, nearly instantaneously, the time-reversed replica of any monochromatic beam. Our method employs the nonlinear refraction present in any medium and is realizable with existing laser sources.

It is well known to be possible to generate a time-reversed wave by nonlinear effects. Zeldovich *et al.*[1] showed experimentally that a nearly "time-reversed" wave was produced by stimulated Brillouin scattering (SBS) in the backward direction of a ruby laser beam whose phase front had been deformed by an inhomogeneous medium. This wave was not perfectly time reversed as it was slightly downshifted in frequency by the acoustic frequency. Nosach *et al.*[2] used SBS to restore the coherence of a laser beam that had been amplified by an inhomogeneous amplifying medium. Recently, Yariv has proposed to "undo" the distortion of images transmitted by multimode optical fibers by parametric mixing in an acentric crystal.[3] He has shown that the mixed wave would be a time-reversed version of the propagated wave that, upon further propagation in the fiber, would evolve back into the original pattern at the entrance face of the fiber.[3] This mixing process could also be used to produce an unguided, time-reversed beam. In either case, limitations are placed on the beam-acceptance angles in this process by "phase-matching" requirements on waves that can mix efficiently in the crystal. In the case of the technique using nonlinear refraction, which we discuss below, neither a frequency shift nor phase-matching need play a role, thus allowing a more accurate time-reversed replication than is possible with SBS or parametric mixing. Also, on the basis of nonlinear optical coefficients known to date, the effect we discuss here can be produced with less laser pump power than either of the other effects.

In Sec. II we will show how, in the presence of counter-propagating pump waves, a beam will cause the generation of its time-reversed wave by the nonlinear refraction which exists in any medium. In Sec. III we show that the pump power levels required for efficient time-reversed generation are well within the capability of available sources. We also suggest a simple experimental arrangement for demonstrating the generation of a time-reversed wave by nonlinear refraction.

II. THEORY

Consider a monochromatic electromagnetic beam that has a complex wave front and is incident from the left on a transparent slab of nonlinear dielectric, as shown in Fig. 1. We assume that this beam has an electric field $\mathrm{Re}E_i(\mathbf{r})e^{-i\omega t}$ whose complex amplitude $E_i(i=x,y,z)$ is known at every point $\mathbf{r}$ in space. We will derive the amplitude $F_i(\mathbf{r})$ of the field radiated by the nonlinear electric polarization density $\mathrm{Re}P_i^{NL}(\mathbf{r})e^{-i\nu t}$ that is created in the nonlinear medium by the interaction of this beam with strong forward and backward plane waves at frequency ω_0 that also exist in the medium. That is, we assume the following electric field to be impressed in a homogeneous nonlinear medium:

$$\mathrm{Re}[E_i(\mathbf{r})e^{-i\omega t}+G_ie^{ik_0z-i\omega_0t}+H_ie^{-ik_0z-i\omega_0t}]. \quad (1)$$

By virtue of the (third-order) nonlinear susceptibility that exists in any medium, we have, at $\nu=2\omega_0-\omega$,

$$P_i^{NL}=X_{ij}E_j^*. \quad (2)$$

Here, as throughout, the summation convention is used for repeated space indices, and

$$X_{ij}=6c_{ijkl}(-\nu,\ -\omega,\omega_0,\omega_0)G_kH_l, \quad (3)$$

where the c_{ijkl} are the nonlinear susceptibility coefficients defined by Maker and Terhune[4] and which are known, at least approximately, for many materials.[5] We will call the oppositely traveling plane waves at ω_0

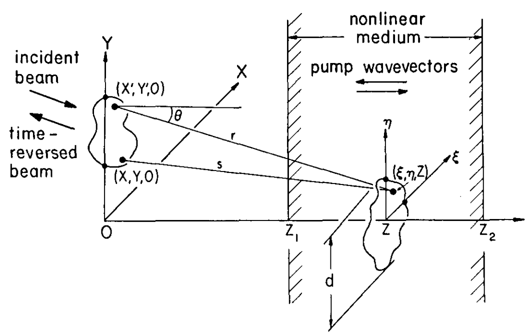

FIG. 1. Schematic and nomenclature for calculation of time-reversed wave fields.

the "pump" waves, as their amplitudes determine the magnitude of the nonlinear susceptibility X_{ij}.

Now we wish to calculate the electromagnetic field amplitude F_i generated at a point (x, y, z) far in front of the medium by P_i^{NL}, and show that, when $\omega = \omega_0$, F_i is proportional to the complex conjugate E_i^* of the incoming wave front at the same point. That is, the generated wave is the "time-reversed" wave. We can, without loss of generality, take the transverse frontal plane, in which we will demonstrate this relation, to be at $z = 0$, as shown in Fig. 1. For simplicity we will assume that, even though the nonlinear medium exists only between $z_1 < z < z_2$ (see Fig. 1), the linear refractive indices are the same inside and outside the medium. (Corrections for reflection and refraction at dielectric interfaces can be made later.) The incident wave amplitude at a point (ξ, η, z) inside the nonlinear medium may then be written simply in terms of its "initial" amplitude at the point $(x', y', 0)$. We assume these two points to be separated by a distance r much larger than a wavelength The Fresenel-Kirchoff diffraction formula gives

$$E_i(\xi, \eta, z) = -ik \int\int dx'\,dy' E_i(x', y', 0) K e^{ikr}/(2\pi r), \qquad (4)$$

where $k = n_1\omega/c$, n_1 being the refractive index for this wave, and $K = (1 + \cos\theta)/2$, θ being the angle between the ray along r and the z axis, as shown in Fig. 1. The nonlinear polarization density, which results from (4) substituted in (2), will radiate to give an electric field amplitude F_i at the point $(x, y, 0)$ in its far field:

$$F_i(x, y, 0) = q^2 \int_{z_1}^{z_2} dz \int\int d\xi\, d\eta\, T_{ij} X_{jk} E_k^*(\xi, \eta, z)\, e^{iqs}/s. \qquad (5)$$

Here $q = n_2\nu/c$, n_2 being the refractive index for the radiated wave at ν, and s is the distance between $(x, y, 0)$ and (ξ, η, z). The operator T_{ij} takes the transverse part of the vector source, which is essentially its projection on a plane perpendicular to the incident beam.

After substituting (4) into (5), one finds that the integral over ξ and η can be performed nearly exactly under conditions which are commonly obtained in practice. To see this, we expand the phase function in powers of [the transverse incident-beam coordinates (ξ, η) divided by the distance z between the radiating and initial planes]

$$qs - kr = \Delta k z + \tfrac{1}{2}(q\rho^2 - k\rho'^2)/z + (k\mathbf{x}' - q\mathbf{x}) \cdot \boldsymbol{\xi}/z + \tfrac{1}{2}\Delta k \sigma^2/z + O[k\sigma^4/z^3]. \qquad (6)$$

Here, $\Delta k \equiv q - k$ and $\mathbf{x}, \mathbf{x}'$, and $\boldsymbol{\xi}$ are two-dimensional vectors, whose coordinates are (x, y), (x', y'), and (ξ, η), and whose magnitudes are ρ, ρ', and σ, respectively. The first two terms in the rhs of (6) are not functions of ξ and η; the next is the important term. So that we may neglect the terms in (6) of order σ^2 and higher, we will assume that the incident beam is contained inside a circle of radius d in the ξ, η plane, and that both of the following conditions are satisfied:

$$z \gg d^2\Delta k \qquad (7)$$

and

$$z \gg kd^4/z^2. \qquad (8)$$

It is seen that, since $\Delta k < k$, these conditions require that the nonlinear interaction region be a minimum distance from the initial plane at $z = 0$, but not so far as the Fraunhofer diffraction region. In performing the integral over ξ and η we will assume that $(rs)^{-1}$ does not vary with ξ and η within d and replace it by its average, which we call $(\overline{rs})^{-1}$. We treat K similarly, replacing it by $\bar{K}$. Then, with (7) and (8), the integral over ξ and η in (5) gives a delta function. When (4) for E_i^* is substituted in (5), the (x', y') integral may be performed trivially to yield

$$F_i(x, y, 0) = 2\pi i q^2 k^{-1} \int_{z_1}^{z_2} dz\, e^{i\Delta k z}\, \xi(z)\, T_{ij} X_{jk} E_k^*\left(\frac{q}{k}x, \frac{q}{k}y, 0\right), \qquad (9)$$

where $\xi \equiv \bar{K}z^2/\overline{rs}$. We have assumed that the product $G_k H_l$ of the amplitudes of the strong, oppositely traveling, pump waves at ω_0 was independent of ξ and η at given z, at least over the area of the incident beam E_i. However, the amplitudes G_i and H_i (and hence, X_{ij}) may still vary with z and affect the integral in (9).

The important consequence of (9) is that F_i is proportional to E_k^* at the same point (if $k = q$), or at a nearby point (if $k \neq q$), in space. That is, if $k = q$, a time-reversed or phase-conjugate wave is generated in this process. When $k \neq q$, a magnified and displaced replica of this wave results. There are no phase-matching requirements here, so a quite divergent beam can be "time reversed." We proceed to estimate the beam powers that would be necessary to obtain a given efficiency of generation of the time-reversed wave.

III. NUMERICAL EXAMPLE

To estimate the pump-wave powers necessary to produce a desired time-reversed wave, consider the practical case where (1) the incident beam is nearly colinear with the pump beams ($\theta \ll 1$), (2) $k = q$, and (3) the pump amplitudes do not vary appreciably in the region of the nonlinear medium where they overlap the incident beam. Then $\xi \sim 1$, $\Delta k = 0$, $T_{ij} \sim \delta_{ij}$, and (9) reduces to

$$F_i(\mathbf{r}) = 2\pi i k L X_{ij} E_j^*(\mathbf{r}). \qquad (10)$$

where $L \equiv z_2 - z_1$ is the thickness of the nonlinear medium, and $\mathbf{r}$ is assumed to be far enough in front of the medium so that conditions (7) and (8) are satisfied. From (10) we see that the ratio R of conjugate-wave power to incident-wave power is

$$R = I_G I_H \beta^2 L^2, \qquad (11)$$

where

$$\beta = |96\pi^2 c^{-2} \omega c_{ijkl} f_i^* e_j^* g_k h_l|. \qquad (12)$$

Here, f_i, e_i, g_i, h_i are the complex, normalized, polarization vectors of the waves ($e_i \equiv E_i/|E_j^* E_j|^{1/2}$, etc.), and I_G and I_H are the intensities of the forward and backward pump waves. We have assumed that the time-reversed wave has an intensity small enough so as not to perturb appreciably the other wave intensities, i.e., $R \ll 1$.

For CS_2, one of the most optically nonlinear of

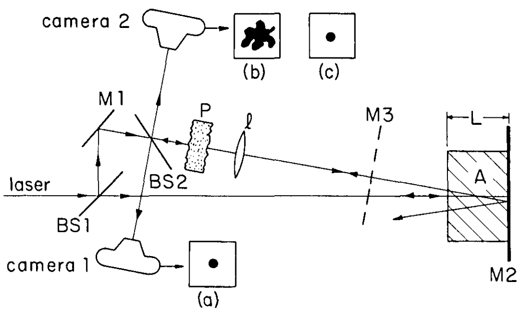

FIG. 2. Schematic diagram of apparatus described in text for observing generation of a time-reversed wave by nonlinear refraction in medium A.

liquids, $c_{xxxx}(-\omega, -\omega, \omega, \omega) \sim 2\times 10^{-13}$ esu.[5] At 6943 A this would imply by (12) that $\beta \sim 6\times 10^{-3}$ cm/MW. Grischkowsky and Armstrong have observed that for ω near the $^2P_{1/2}$ resonance in rubidium vapor, the c coefficient (and β) was nearly four orders of magnitude greater, implying $\beta \sim 30$ cm/MW near 7950 A.[6] With this latter figure as an example, we see from (11) that to obtain 10% conversion to a time-reversed wave ($R = 0.1$) in a 10 cm length of medium, the pump intensities required would be $I_G \sim I_H \sim 1$ kW/cm^2. The 2 mm diameter dye-laser beam of several kilowatts, which was used in Ref. (6) to observe the nonlinear index, would certainly satisfy this requirement.

In order to demonstrate the generation of a time-reversed wave with a single monochromatic laser source, one might employ an arrangement as shown in Fig. 2. A single-mode monochromatic laser source is directed into the nonlinear medium A. The mirror M2 reflects this beam back through the medium creating forward and backward plane-pump waves of nearly the same amplitude. Beam splitter BS1 and mirror M1 direct a portion of the laser source through a phase-distorting plate P and a focusing lens ℓ into the pumped region of the medium. Cameras 1 and 2 record the incident and backscattered beam patterns at beam splitter BS2. Camera 1 should show the "single-mode" pattern as in inset (a). If the nonlinear medium is removed and a back-reflecting mirror M3 placed at the focal point of lens ℓ, a distorted pattern, caused by double-passing P, will be recorded by camera 2 as indicated in inset (b). However, in the presence of the pumped nonlinear medium, camera 2 should record the same single-mode pattern (c) as recorded by camera 1. That is, the backscattered beam is re-formed by plate P to have a smooth phase front and appear to be the time-reversed image of the incident beam.

In an experimental arrangement such as in Fig. 2, the linear refractive index of the medium A may not match the index outside its front surface. In this case, the length L of the medium must be long enough so that condition (8) is met for most of the interaction volume when the comparison plane $z = 0$ is considered to be just inside the entrance surface of the medium (i.e., $z_1 = 0$ in Fig. 1). Then the time-reversed wave front will have become well formed before exiting the medium A and continue back toward lens ℓ as a time-reversed replica (except for the small reflections at the dielectric interface which can be eliminated by coatings).

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