Chapter 1

Introduction: Principles of Dynamics

Principles of Dynamics is a subset of Mechanics that deals with bodies in motion under the action of forces. The subject of Dynamics is completely captured by *Newton's Second Law*, $\sum \vec{F} = m\vec{a}$. To study Dynamics, we must be able to handle correct force analysis. In the book, "*Difficult Engineering Concepts Better Explained: Statics and Applications*,"[1] a vigorous method of force analysis, named the ABCC method for constructing correct free-body diagrams, is presented. We will present the key elements of the ABCC method in Chapter 2.

The right side of *Newton's Second Law* involves two important elements. One is the system, represented by m, and the other is the motion of the system, represented by $\vec{a}$. The motion analysis of different systems, particles and rigid bodies, will be discussed in Chapters 3 and 4, for both 2D and 3D motions, respectively.

The applications of *Newton's Second Law* to different systems will be presented in Chapter 5. In Chapter 6, we reformulate *Newton's Second Law* again to a different form, which leads to the concept of momentum. The concept of momentum is important in explaining problems related to impacts and collisions. Different forms of energy are discussed in Chapter 7. The concepts of energy and work are derived by converting the vector equation of *Newton's Second Law* into a scalar equation. In Chapters 8 and 9, we discuss the

[1]Tu, J.F. *Difficult Engineering Concepts Better Explained: Statics and Applications*. World Scientific, 2020. ISBN 978911213786.

dynamics analysis for many interesting real world problems related to automotive vehicles and other useful engineering applications in manufacturing. In Chapter 10, we extend the analysis into dynamics of non-rigid body systems. Finally, in Chapter 11, we provide detailed solutions to 32 difficult problems in Dynamics.

As stated, the main objective of studying Dynamics is to conduct correct force and motion analyses for mechanical or integrated systems. To this end, we, as engineers, must approach the analysis quantitatively, with proper knowledge about precision and uncertainties. We will discuss key concepts and practices to ensure confidence in the calculations.

Finally, in Chapter 1, we will suggest a general problem-solving protocol to promote a systematic and creative thought process. Again, the idea is that we can do a little thinking to solve new problems, not limited to the problems we have studied before.

1.1 Space and Time

Time keeping is critical in describing motion, which is related to the right side of *Newton's Second Law*. Space and time (or spacetime) are fundamental metrics we need for solving problems. To quantify a space, typically, we will define a reference coordinate. Typically, in Mechanics, we deal with a 1D, 2D, or 3D space, defined commonly by a suitable coordinate such as the $O{-}x{-}y{-}z$ Cartesian coordinate, shown in Figure 1.1. Each axis of the $O{-}x{-}y{-}z$ coordinate is typically fixed in direction in Statics, but not necessarily so in dynamics.

We need to define metrics along each axis. In other words, how do we define length, for example, the length of one meter in the SI system? In the old days, there was a "National Prototype Metre Bar No. 27, made in 1889 by the international Bureau of Weights and Measures (BIPM) and it was given to the United States to serve as the standard for defining all units of length in the US from 1893 to 1960."[2] "The bars were to be made of a special alloy, 90% platinum and 10% iridium, which is significantly harder than pure platinum,

[2]https://en.wikipedia.org/wiki/History_of_the_metre#International_prototype_metre.

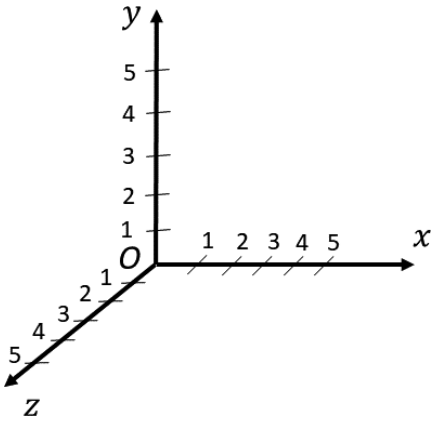

Figure 1.1: A Cartesian coordinate with metrics defined along each axis.

and have a special X-shaped cross section (a 'Tresca section,' named after French engineer Henri Tresca) to minimize the effects of torsional strain during length comparisons."[3]

Today, the standard of length is no longer defined by a meter bar, but by time, based on Einstein's theory of special relativity, which assumes that the speed of light in vacuum is constant. The official definition is[4]

> *The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m/s, where the second is defined in terms of the caesium frequency* $\Delta\nu Cs$.

Therefore, length is based on time. The question now is how do we achieve precise time-keeping? According to NIST,[5] "since 1967, the International System of Units (SI) has defined a second as the period equal to 9,192,631,770 cycles of the radiation, which corresponds to the transition between two energy levels of the ground state of the

[3]Nelson, R.A. Foundations of the international system of units (SI). *The Physics Teacher* **19**(9) (1981) 596–613.

[4]BIPM *SI Brochure*, 9th edition, 2019, p. 131.

[5]https://www.nist.gov/pml/time-and-frequency-division/timekeeping-and-clocks-faqs.

Cesium-133 atom. This definition makes the cesium oscillator (sometimes referred to generically as an atomic clock) the primary standard for time and frequency measurements." In other words, we rely on counting to establish time and thus the standard of length.

1.2 Concept of Precision

Being quantitative is one important requirement for engineers. We need to handle numbers properly.

In Section 1.1, we talked about using counting for time-keeping, precisely, at least down to $1/9,192,631,770$ seconds, or 0.10873 nanoseconds, based on the definition of 1 second.

As we count, we could make mistakes or simply reach different counts because the event has changed. In other words, there are errors and uncertainties. Errors are due to mistakes. The uncertainties are due to factors out of our control, no matter how carefully we try to avoid mistakes. The subject of precision deals with errors and uncertainties. There are three main concepts in precision, which are accuracy, repeatability, and resolution.[6]

For example, if we use a ruler to measure the length of a rod, the accuracy is the deviation of the measurement from the true value of the length. The problem is that we usually do not know the true value. When we take measurements with a ruler, the ruler has its limit of resolution, which is 1 mm for an office ruler. We can get a 0.01 mm resolution using a caliper.

With a simple ruler, we align one end of the rod to the zero mark of the ruler and check where the other end ends. If the other end lands between the marks of 11 and 12 mm, we can only guess that the true length is, judging where it lands between the marks, as 11.4 mm, for example. In fact, we are not sure if it is 11.3, 11.4, or 11.5 mm, and different people might guess it differently. As a result, we have an uncertainty, which is at least 0.2 mm, in this case. This uncertainty affects how repeatable the measurement could be.

[6]Slocum, A. *Precision Machine Design*. Society of Manufacturing Engineers, Dearborn, Michigan, 1992. ISBN-13: 978-0872634923.

One practical way of achieving a higher precision is to make multiple measurements. For example, if we want to estimate the value of an antique piece, we can ask many experts to estimate its value. Because we cannot trust one expert more than others, the best practice is to calculate the average of all the estimates. The average, thus, is the best estimate to incorporate all the inputs from all the experts. To determine the accuracy, we should use the best estimate (the average) and calculate its deviation from the true value (if we know it). The variation among the estimates from different experts represents repeatability. The resolution is related to the smallest money unit used in the estimates for this case.

1.2.1 *Significance of figures*

When we make a measurement or present a number, we often do not explicitly state its precision in terms of accuracy, repeatability, and resolution. Instead, we use the concept of significant figures to indicate the precision of a measurement. This is the subject of arithmetic precision. "A significant figure is any one of the digits 1 to 9; zero is a significant figure except when it is used to fix the decimal point or to fill the places of unknown or discarded digits."[7] "Thus in the number 0.000532, the significant figures are 5, 3, and 2, while in the number 2,076, all the digits, including the zero, are significant. For a number such as 2,300, the zeros may or may not be significant. To convey which figures are significant, we write this as 2.3×10^3 if two significant figures are intended, 2.30×10^3 if three, 2.300×10^3 if four, and so forth (see footnote 7)."

We often have to compute a variable based on several measurements with different significant figures. When we are doing hand calculation, it is preferred to round the number with more significant figures to be the same as the one with less significant figures. There are practical reasons for this rounding practice such as: (1) the final results cannot have a higher number of significant figures; and (2)

[7]Doebelin, E.O. (1990). *Measurement Systems Application and Design.* McGraw-Hill Publishing Company, New York, NY. pp. 58-67, 19990 ISBN 0-07-017338-9.

handling a long string of digits can incur more mistakes. However, in today's computing practice using computers, the second concern is no longer an issue. We should still round the number of the final result to recognize its limited significant figures and to enable easy reading.

A widely used rounding practice is as follows (see footnote 7):

> *To round a number to n significant figures, discard all digits to the right of the nth place. If the discarded number is less than one-half a unit in the nth place, leave the nth digit unchanged. If the discarded number is greater than one-half a unit in the nth place, increase the nth digit by 1. If the discarded number is exactly one-half a unit in the nth place, leave the nth digit unchanged if it is an even number and add 1 to it if it is odd.*

Therefore, if we are given a number, 12.35, we should know that the actual number could range from 12.346 to 12.355, or $12.35^{+0.005}_{-0.004}$. The uncertainty range could be larger than 0.009 because most people may not follow the rounding rule consistently.

In some applications, we need to emphasize the number of decimal places (the number of significant figures following the decimal point). For example, in your bank statement, the balance of your checking account is rounded to two places after the decimal point. This is of course for practical reasons because we do not have a coin smaller than one cent.

In general, when one is performing addition and/or subtraction, it is better to round the result to the same number of decimal places as the number with the lowest number of decimal places. When the computation involves multiplication and/or division, it is better to round the result to be the same as the one with the lowest significant figures.

In this book, armed with modern computing power, all numbers of final results will be rounded to two decimal places unless a higher precision is required. If a number is very small, such as 0.0017345, it would be rounded to 0.00173, to contain three significant figures, unless a higher precision is required.

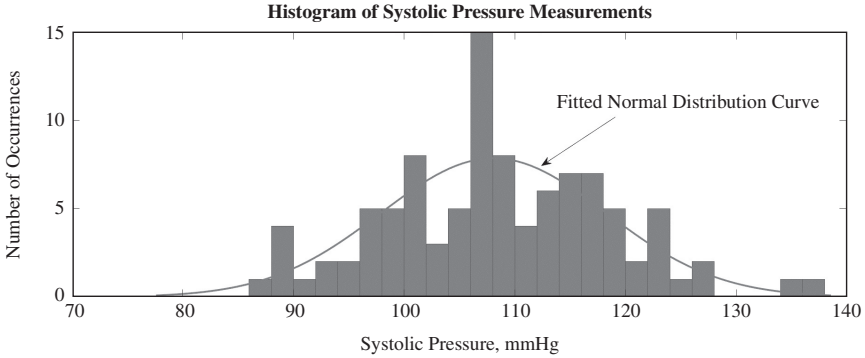

Figure 1.2: Histogram of 100 measurements of systolic pressure.

1.2.2 *Statistical nature of uncertainty*

In the above discussion, we specified the range of uncertainty of a measurement using the plus and minus expression. If a number is expressed as $12.35^{+0.005}_{-0.004}$, we only specify that all the measurements would be between 12.346 and 12.355. To understand this uncertainty more, we should also define the probability distribution within the range of uncertainty. Essentially, there are no fundamental laws to help us decide how often a number would appear within the range of uncertainty.

To discuss issues related to the probability distribution, a basic discussion on statistics is in order. If we measure the blood pressure of a person several times, it is highly unlikely we will get the same reading. Let us say that we take the measurement 100 times, consecutively (hypothetically, not advised to do so), and obtained the measurements of the systolic pressure as shown in Figure 1.2 (this figure is denoted as a histogram).

The measurements are grouped into 26 bins and each bin covers 2 mmHg. From Figure 1.2, we found that, for example, there are 15 readings between 106 and 108 mmHg, only 2 readings over 130 mmHg, 10 readings between 120 and 130 mmHg, and 5 readings below 90 mmHg. Therefore, is the blood pressure of the person normal? There are 83 readings within the normal range between 90 and 120 mmHg, but 12 times when the pressure is in the pre-high blood pressure range. What is causing all these variations? The actual

blood pressure could fluctuate, and the measurement device could have measurement errors. The question is still "Is the blood pressure of this person normal?" The same scenario can be related to the strength of a structure or the quality of a part after we obtain multiple measurements.

One common statistical practice is to assume that the probability distribution of the reading is a normal distribution. We can fit a normal distribution curve to the histogram, as shown in Figure 1.2. This assumption is typically valid due to the Central Limit Theorem when we have a large number of samples. Often, we simply assume the normal distribution for convenience. There are, of course, many exceptions. Based on the fitted normal distribution, we assume that if we repeat the measurement an almost infinite number of times, the histogram will approach a normal distribution curve (not necessarily the same as the fitted one in Figure 1.2). Based on the fitted curve, we estimate the average of the reading, m, is 108.0 mmHg, and the standard deviation, σ, is 10.2 mmHg. From the normal distribution curve, we know that 68.27% of the readings will fall in the range of 108.0 ± 10.2 mmHg (97.8 − 118.2 mmHg), 95.45% in the range of $108.0 \pm 2 \times 10.2$ mmHg (87.6 − 128.4 mmHg), and 99.73% in the range of $108.0 \pm 3 \times 10.2$ mmHg (77.4 − 138.6 mmHg).

From these values, we are pretty confident that the person is not suffering from high blood pressure over 138.6 mmHg because there is only 0.14% chance for it to be so.

The average of 108.0 mmHg provides us with the best guess of the true blood pressure, while we are 99.73% confident that the true blood pressure is between 77.4 and 138.6 mmHg, which represents the repeatability or the range of the uncertainty. Unfortunately, we can never know what the true blood pressure is; we can only give our best guess and indicate how confident we are with our guess.

When we have additional information about a measurement, the implied uncertainty of using the significant figures and number of decimal places is no longer adequate. We should represent the measurement in the format of plus and minus, such as 108.0 ± 30.6 mmHg. If not specifically stated, we will assume that this is defined with 99.73% confidence. In many cases, a 95.45% confidence is sufficient; therefore, the measurement should be expressed as 108.0 ± 20.4 mmHg with 95.45% or simply 95% confidence. The confidence should be explicitly stated if it is not 99.73%.

1.2.3 *Computation with uncertainties*

We will use an excellent example and discussion from Doeblin (1990) (see footnote 7) to demonstrate how to handle uncertainties in computation. Let a variable $y = f(x_1, x_2, x_3, \ldots, x_n)$, where x_i's denote the measured variables with the uncertainties $\Delta x_1, \Delta x_2, \Delta x_3, \ldots, \Delta x_n$. What will be the uncertainty Δy?

Using the Taylor series, we expand function f as

$$\begin{aligned} y = y_0 + \Delta y &= f\left(x_1 + \Delta x_1, x_2 + \Delta x_2, x_3 + \Delta x_3, \ldots, x_n + \Delta x_n\right) \\ &= f\left(x_{10}, x_{20}, x_{30}, \ldots, x_{n0}\right) + \Delta x_1 \frac{\partial f}{\partial x_1} + \Delta x_2 \frac{\partial f}{\partial x_2} + \Delta x_3 \frac{\partial f}{\partial x_3} \\ &\quad + \cdots + \Delta x_n \frac{\partial f}{\partial x_n} + \text{H.O.T.} \end{aligned} \tag{1.1}$$

where all the partial derivatives are to be evaluated at the known values of x_{i0}'s and the values of Δx_i's could be positive or negative. The variable y_0 is the nominal value of f with respect to x_{i0}'s. We can ignore the Higher Order Terms (H.O.T.) if the values of Δx_i's are small.

To determine Δy, we usually consider the worst-case and the best-case scenarios. The real case most likely falls somewhere in between.

For the worst-case scenario, we define the range of Δx_i as $R\Delta x_i$; therefore, $-\frac{1}{2}R\Delta x_i < \Delta x_i < \frac{1}{2}R\Delta x_i$ and the range of Δy becomes,

$$\begin{aligned} R\Delta y = \Bigg(&\left| R\Delta x_1 \frac{\partial f}{\partial x_1} \right| + \left| R\Delta x_2 \frac{\partial f}{\partial x_2} \right| + \left| R\Delta x_3 \frac{\partial f}{\partial x_3} \right| \\ &+ \cdots + \left| R\Delta x_n \frac{\partial f}{\partial x_n} \right| \Bigg) \end{aligned} \tag{1.2}$$

We should express y as

$$y = y_0 \pm \frac{1}{2} R\Delta y \tag{1.3}$$

If we consider Δx_i's as random variables with normal distributions, then we can compute the standard deviation of Δy as

$$\sigma_{\Delta y} = \sqrt{\left(\sigma_{\Delta x_1} \frac{\partial f}{\partial x_1}\right)^2 + \left(\sigma_{\Delta x_2} \frac{\partial f}{\partial x_2}\right)^2 + \left(\sigma_{\Delta x_3} \frac{\partial f}{\partial x_3}\right)^2 + \cdots + \left(\sigma_{\Delta x_n} \frac{\partial f}{\partial x_n}\right)^2} \tag{1.4}$$

From Equation (1.4), if we present the uncertainty of y with 99.73% confidence, we have

$$y = y_o + \Delta y = y_o \pm 3\sigma_{\Delta y} \tag{1.5}$$

Equation (1.5) represents the best-case scenario.

What if the uncertainties are not in normal distribution? In that case, there is no easy analytical way to determine Δy. However, with today's computing power, we can use the Monte Carlo simulation to determine the probability distribution of Δy and plot a histogram similar to Figure 1.2.

Let us consider an example from Doeblin (1990) regarding the measurement obtained from a dynamometer. The output power of a dynamometer can be written as

$$P = \frac{2\pi RF\left(\frac{L}{12}\right)}{550t} = \frac{2\pi}{550 \times 12}\frac{RFL}{t} \tag{1.6}$$

where P is the power in hp, R is the revolution of the shaft during the time period t, F is the measured force in lbf at the end of the torque arm, L is the length of the torque arm in inches, and t is time in second, s.

For a specific run, if the data are

$$F = 10.12 \pm 0.040\,\text{lbf} \tag{1.7}$$

$$R = 1202 \pm 1.0\,\text{rev} \tag{1.8}$$

$$L = 15.63 \pm 0.050\,\text{in} \tag{1.9}$$

$$t = 60.0 \pm 0.50\,\text{s} \tag{1.10}$$

the uncertainties of these measurements are determined from the sensors' calibration. Note that all the uncertainty terms are expressed with one extra decimal place than the nominal value.

The nominal value of P is computed based on Equation (1.1). We then have

$$P_0 = \frac{2\pi}{550 \times 12}\frac{1202.0 \times 10.12 \times 15.63}{60.0} = 3.01668 \cong 3.02 \tag{1.11}$$

We round the above value to two decimal places.

Now, we compute the partial derivatives in Equation (1.1) to three significant figures as

$$\left.\frac{\partial f}{\partial x_1}\right|_0 = \left.\frac{\partial P}{\partial F}\right|_0 = \left.\frac{2\pi}{550\times 12}\frac{LR}{t}\right|_0 = \frac{2\pi}{550\times 12}\frac{15.63\times 1202}{60.0}$$
$$= 0.298\,(\text{hp/lbf}) \tag{1.12}$$

$$\left.\frac{\partial f}{\partial x_2}\right|_0 = \left.\frac{\partial P}{\partial R}\right|_0 = \left.\frac{2\pi}{550\times 12}\frac{FL}{t}\right|_0 = 0.00251\,(\text{hp/rev}) \tag{1.13}$$

$$\left.\frac{\partial f}{\partial x_3}\right|_0 = \left.\frac{\partial P}{\partial L}\right|_0 = \left.\frac{2\pi}{550\times 12}\frac{FR}{t}\right|_0 = 0.193\,(\text{hp/in}) \tag{1.14}$$

$$\left.\frac{\partial f}{\partial x_4}\right|_0 = \left.\frac{\partial P}{\partial t}\right|_0 = \left.\frac{-2\pi}{550\times 12}\frac{FLR}{t^2}\right|_0 = -0.0500\,(\text{hp/s}) \tag{1.15}$$

For the worst-case scenario, the ranges of uncertainties are based on Equations (1.7)–(1.10)

$$R\Delta F = 0.080\,\text{lbf} \tag{1.16}$$

$$R\Delta R = 2.0\,\text{rev} \tag{1.17}$$

$$R\Delta L = 0.100\,\text{in} \tag{1.18}$$

$$R\Delta t = 1.00\,\text{s} \tag{1.19}$$

From Equation (1.2), we have

$$R\Delta y = \left(\left|R\Delta F\frac{\partial P}{\partial F}\right| + \left|R\Delta R\frac{\partial P}{\partial R}\right| + \left|R\Delta L\frac{\partial P}{\partial L}\right| + \left|R\Delta t\frac{\partial P}{\partial t}\right|\right)$$
$$= 0.098\,\text{hp} \tag{1.20}$$

From Equation (1.3), we have

$$P = P_0 \pm \frac{1}{2}R\Delta y = 3.02 \pm 0.049\ (\text{hp}) \cong 3.02 \pm 0.05(\text{hp}) \tag{1.21}$$

For the best-case scenario, we first assume that the expressions of Equations (1.7)–(1.10) are based on 99.73% confidence level of normal distributions. Therefore, we have

$$\sigma_{\Delta F} = \frac{1}{3}(0.040)\,\text{lbf} \tag{1.22}$$

$$\sigma_{\Delta R} = \frac{1}{3}(1.0)\,\text{rev} \tag{1.23}$$

$$\sigma_{\Delta L} = \frac{1}{3}(0.050)\,\text{in} \tag{1.24}$$

$$\sigma_{\Delta t} = \frac{1}{3}(0.50)\,\text{s} \tag{1.25}$$

From Equation (1.4), the standard deviation of ΔP is

$$\begin{aligned}\sigma_{\Delta P} &= \sqrt{\left(\sigma_{\Delta F}\frac{\partial P}{\partial F}\right)^2 + \left(\sigma_{\Delta R}\frac{\partial P}{\partial R}\right)^2 + \left(\sigma_{\Delta L}\frac{\partial P}{\partial L}\right)^2 + \left(\sigma_{\Delta t}\frac{\partial P}{\partial t}\right)^2} \\ &= 0.00977\,\text{hp}\end{aligned} \tag{1.26}$$

Finally,

$$P = P_0 \pm 3\sigma_{\Delta P} = 3.017 \pm 3\,(0.00977) \cong 3.017 \pm 0.029\,(\text{hp}) \tag{1.27}$$

In other words, we are confident (99.73% sure) that the actual value of P will lie between 2.988 and 3.046 hp. With a lower confidence of 95.45%, the true value of P will lie between 2.997 and 3.036 hp. There is only a 4.28% chance that the true value lies between 3.036 and 3.046 hp or between 2.988 and 2.997 hp. We use three decimal places for the best-case scenario.

1.2.4 *Precision consideration in differential quantities*

As shown in Equation (1.1), the H.O.T. can be neglected if the values of Δx_i's are small. This is the case when we discuss the differential terms. For example, the Taylor expansion of $\sin\theta$ is

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots \tag{1.28}$$

For a differential angle $d\theta$, it becomes

$$\sin d\theta = d\theta - \frac{d\theta^3}{3!} + \frac{d\theta^5}{5!} - \cdots \cong d\theta \tag{1.29}$$

when the H.O.T are neglected. What is the error if we neglect the H.O.T.? In this case, the error ratio, ε, defined as the error to the true value, is

$$\varepsilon = \frac{|\sin\, d\,\theta - d\theta|}{\sin\, d\,\theta} = \frac{\frac{d\theta^3}{3!} - \frac{d\theta^5}{5!} + \frac{d\theta^7}{7!} - \cdots}{\sin\, d\,\theta} < \frac{\frac{d\theta^3}{3!}}{\sin\, d\,\theta} \cong \frac{\frac{d\theta^3}{3!}}{d\theta} = \frac{d\theta^2}{3!} \tag{1.30}$$

which is infinitesimal. Similarly, we found that $\cos d\theta \cong 1$ and $\tan d\theta \cong d\theta$.

In engineering analysis, there will be cases where integration will need to be done over different geometric shapes to determine the surface area, volume, and moments of inertia. Often, we have to decide on the inclusion or exclusion of differential terms. We will present a few cases for illustration.

As shown in Figure 1.3, we want to determine the volume of a straight cone with a height h and a base radius r. We can slice the cone into infinitesimally thin disks and add up the volume of all the disks. The top surface of the thin disk in Figure 1.3(b) has a radius x and the bottom surface has a radius $x + dx$. Let us say that we do not know how to determine the differential volume, dV, of the thin disk of (b), but we know it will be between those two cylinders with a radius x and $x + dx$, respectively, as shown in Figure 1.3(c). Therefore, we have

$$\pi x^2 dy < dV < \pi(x + dx)^2 dy \tag{1.31}$$

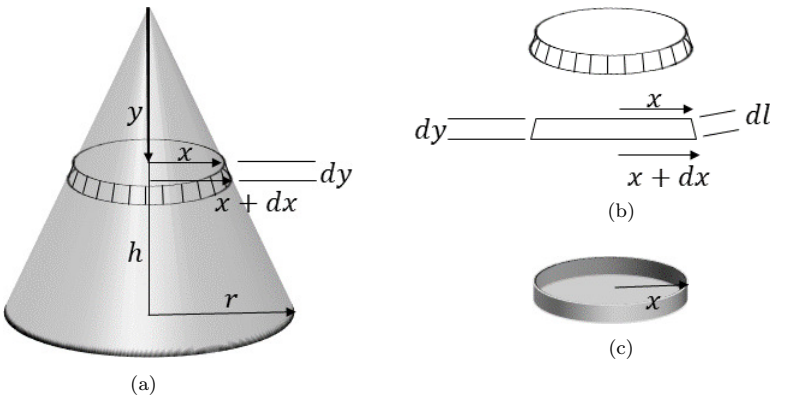

Figure 1.3: (a) Volume calculation of a straight cone; (b) an infinitesimal conical layer; and (c) an infinitesimal cylindrical layer.

If we decide to set $dV = \pi x^2\, dy$, the error ratio, ε, will be

$$\varepsilon < \frac{\pi(x+dx)^2\, dy - \pi x^2\, dy}{\pi x^2\, dy} = \frac{2\, dx}{x^2} + \frac{dx^2}{x^2} \tag{1.32}$$

which is still negligible.

Now, integrating dV over y, we have the well-known result

$$\int_0^V dV = \int_0^h \pi x^2\, dy = \int_0^h \pi \frac{r^2}{h^2} y^2\, dy = \frac{\pi}{3} r^2 h \tag{1.33}$$

For the same straight cone, if we would like to determine the exterior cone surface, can we use the exterior surface of the thin cylindrical disk and carry out similar integration?

If we do so, then the exterior cone surface becomes

$$\hat{S} = \int_0^{\hat{S}} d\hat{S} = \int_0^h 2\pi x\, dy = \int_0^h 2\pi \frac{r}{h} y\, dy = \pi r h \tag{1.34}$$

which is incorrect. So what went wrong?

The actual exterior surface of the thin disk in Figure 1.3(b) should be

$$dS = 2\pi x\, dl = 2\pi x \sqrt{dx^2 + dy^2} = 2\pi \frac{r}{h} \sqrt{\frac{r^2 + h^2}{h^2}}\, y\, dy \tag{1.35}$$

The error ratio between dS and $d\hat{S}$ is

$$\varepsilon = \frac{dS - d\hat{S}}{dS} = \frac{\sqrt{r^2 + h^2} - h}{\sqrt{r^2 + h^2}} = \frac{l - h}{l} \tag{1.36}$$

where $l = \sqrt{r^2 + h^2}$, which is the length along the cone exterior surface. As shown in Equation (1.36), the error ratio is finite, not negligible. As a result, the error will accumulate, leading to mistakes. The correct answer is

$$S = \int_0^S dS = \int_0^h 2\pi x\, dl = \int_0^h 2\pi \frac{r}{h} \sqrt{\frac{r^2 + h^2}{h^2}}\, y\, dy = \pi r l \tag{1.37}$$

Between $\hat{S}$ and S, we have the same error ratio as in Equation (1.36). A similar consideration should be applied when calculating the exterior surface of a paraboloid disk, as discussed in Problem 11.19 of Chapter 11 in the book of Difficult Engineering Concepts Better Explained (see footnote 1).

1.3 Problem-Solving Protocol in Dynamics

To solve a problem in Statics, we should follow a set of systematic steps as follows:

(a) Problem statements;
(b) Force analysis;
(c) Motion analysis;
(d) Governing equations;
(e) Solving equations;
(f) Answer verifications;
(g) Extensions.

In the problem statements, one should make proper sketches and define proper reference coordinates. List all the given conditions and the unknowns to be found.

For force analysis, we will review the systematic method, denoted the ABCC method, in Chapter 2 so that the force analysis can be conducted correctly every time.

Chapters 3 and 4 discuss systematic ways of motion analysis. As regards governing equations, we have several choices based on *Newton's Second Law* and its different variations (Chapters 5–7).

As regards solving equations, after we have established relevant governing equations, we should always count the number of the unknowns associated with the governing equations. If the number of unknowns is greater than the number of the governing equations, we are not able to solve for all the unknowns. When this occurs, we should carry out the following steps to overcome this problem. First, we should check if there is an equation involving only one unknown. If so, this particular unknown can be solved. As a result, the number of unknowns is reduced by one, but the number of equations is also reduced by one. We should also check if there is a subset of the governing equations involving the same number of unknowns. If so, we can solve for those unknowns. Once we exhaust all the equations that can be solved, we still need to find additional equations.

To find additional equations, we should examine the unknowns and consider if they have specific relationships among them so that we can establish additional equations.

If we cannot find additional equations, then we have to make assumptions to define the values of some unknowns. For example, in the case of a smooth surface the friction can be assumed to be zero.

Finally, as engineers, we should know that we can conduct measurements using sensors to determine the values of some unknowns. Once we reduce the number of unknowns to be the same as that of the governing equations, we can proceed to solve for all the unknowns.

In solving equations, we can use computing aids for help. Today, there are equation solvers useful for solving complicated equations, numerically or symbolically. We provide a few programming examples in this book.

After we have solved the equations and obtained the values of unknowns, we should conduct answer verifications. Do they look reasonable? We could have made some mistakes along the way. Do they violate the assumptions? Do they violate basic laws?

Finally, we should think about the implications of the results for extensions.

1.4 Concluding Remarks

Practice makes perfect. It applies to both honing the problem-solving skill and deepening the understanding of the concept. In Chapter 11, we provided detailed solutions to 32 difficult problems based on Appendix A of the classic Dynamics textbook by Meriam.[8] These problems, along with the examples presented in Chapters 1–10, are useful to learn how to think properly to solve difficult Dynamics problems through proper understanding of difficult concepts in Dynamics.

[8]Meriam, J.L. *Dynamics*, 2nd Edition. John Wiley & Sons, Inc., Hoboken, NJ, 1975.