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ARE ELECTRONS WAVES? *

BY

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I MUST tell you to begin with that Dr. Germer and I are keenly aware of the very great compliment you have paid us in asking me to come here this evening and describe to you our experiments with electrons. It is a compliment which we appreciate very much indeed.

The title which I have chosen for my address, "Are electrons waves?," suggests that some doubt has arisen in regard to the nature of electrons. And this is true. The fact is that circumstances have been found in which electrons try to make out that they are not particles at all, but are instead waves.

As an example of this perverseness I shall describe a simple type of experiment that Dr. Germer and I have been making for the past several months. We direct a narrow stream of electrons against the face of a nickel crystal, and observe that under certain conditions a sharply defined stream of electrons leaves the crystal in the direction of regular reflection—angle of reflection equal to angle of incidence.

At first thought there may seem to be nothing so very strange in this. Why should not electrons be regularly reflected from a metal surface? We know that Newton and

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other adherents of the corpuscular theory of light were not embarrassed by the fact that light is regularly reflected from a plane mirror. The phenomenon is one that they could explain quite easily. It is well known that in an elastic encounter between a particle and a plane surface the particle rebounds from the surface in the direction of regular reflection —hence the regular reflections of light on the corpuscular theory, and why not also the regular reflection of electrons?

Well, the adherents of the corpuscular theory of light had certain advantages over us in picturing reflection in this way. They had not committed themselves in regard to the size of the light corpuscle, and they knew nothing about the structure of metallic surfaces. We have reasons for believing that the electron is about 10^{-13} cm. in diameter. We know that atoms have diameters of the order 10^{-8} cm. and we know also that the least distance between atoms in the nickel crystal is 2.48×10^{-8} cm. If we take 10^{-13} cm. as a unit of length, then the diameter of the electron is one of these units, the diameter of the nickel atom is one hundred thousand, and the least distance between atoms in the nickel crystal is nearly 250,000.

The difficulty of picturing the regular reflection of particles as small as electrons from a surface made up of bodies as large as atoms is at once evident. If we were to fire a load of bird shot against a pyramid of cannon balls, we should not expect to find a little cloud of shot moving off in the direction of the regular reflection from the face of the pyramid. A surface made up of cannon balls is much too coarse grained to serve as a regular reflector for particles as small as bird shot.

The analogy is not such a good one really, for we do not think of electrons rebounding from the surface of an atom in the way that shot rebound from a cannon ball. We have been accustomed to think of the atom as rather like the solar system—a massive nuclear sun surrounded by planetary electrons moving in closed orbits. On this view the electron which strikes into a metal surface is like a comet plunging into a region rather densely packed with solar systems.

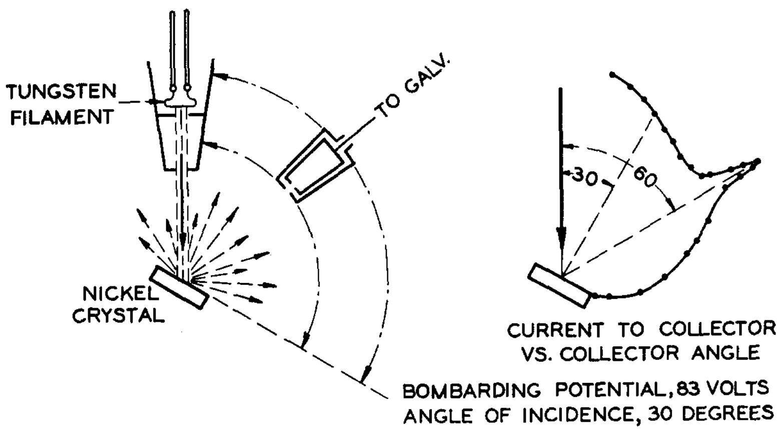
There is a certain small probability, or at least there might seem to be, that the electron will strike into an atom in or near the surface of the metal, be swung about comet-wise, and sent flying out of the metal without loss of energy. The direction taken by such an electron as it leaves the metal should be a matter of private treaty between the electron and the individual atom. One does not see how the neighboring atoms could have any voice in the matter. And yet we find that the high-speed scattered electrons have a preference for moving off in the direction of regular reflection, a direction which is related to the plane of the surface. Three atoms at least are required to fix this plane, so that the direction taken by the electron is determined not by one atom, but by three atoms at least.

One may say without qualification that in terms of atoms and electrons and their interaction as we have been accustomed to picture them the regular reflection of electrons from a metal surface is quite incomprehensible.

Of course, if electrons were waves there would be no difficulty. We think we understand the regular reflection of light and of x-rays—and we should understand the reflection of electrons as well if electrons were only waves instead of particles. This observation though true does not seem a particularly valuable one. It is rather as if one were to see a rabbit climbing a tree, and were to say, "Well, that is rather a strange thing for a rabbit to be doing, but after all there is really nothing to get excited about. Cats climb trees—so that, if the rabbit were only a cat, we would understand its behavior perfectly." Of course, the explanation might be that what we took to be a rabbit was not a rabbit at all, but was actually a cat. Is it possible that we are mistaken about electrons? Is it possible that we have been wrong all this time in supposing that they are particles, and that actually they are waves? Well, I do not need to enumerate to you the many reasons we have for believing—I may say for knowing—that electrons are actually particles.

As if these reasons were not numerous enough—the very method by which we detect the regular reflected beam supplies still another. The regularly reflected beam is found by moving a small bucket about in front of the crystal and observing that more electrons are caught when the bucket stands in the direction of regular reflection than in any other.

FIG. 1.



Experimental arrangement for investigating the scattering of electrons by a crystal, and a typical curve showing beam of regularly reflected electrons.

A diagram of the experimental arrangement is shown in Fig. 1.

This arrangement of filament and box is an electron gun which supplies us with a steady stream of electrons. The speed of the electrons is under our control and can be given any desired value by maintaining a suitable potential difference between the filament and the box. This stream is directed against the crystal, and electrons of various speeds move off in all directions from the bombarded area.

To find how many are moving off in different directions we move the collector, which is really a bucket, and find how many electrons we catch in different positions. To get into the inner box an electron must pass through the opening in the outer box. Those that succeed in doing this flow off through a galvanometer, and the deflection of the galvanometer is a measure of the rate at which they are being caught.

The method is one which with some slight modification might be used to find how bird shot are scattered by a pile of cannon balls. It is not in principle a method we would employ to investigate the scattering of light or of x-rays.

In making observations the collector is moved about in front of the crystal, and curves are constructed showing the current received by the collector as a function of angle.

Such a curve for angle of incidence 30 degrees and for bombarding potential 83 volts is shown on the right, and you see the sharp spur protruding from the curve exactly in the direction of regular reflection. You will want to know about the electrons leaving the crystal in other directions. Well, those are, almost all of them, low-speed secondary electrons—while the electrons responsible for this spur have, most of them, the same speed as the incident electrons.

There is no doubt that the incident electrons recognize the surface of the crystal, and prefer to move off in the direction of regular reflection.

The next experiment I shall describe is more simple even than the first. We direct a stream of electrons against a target of ordinary nickel—a target made up of many small crystals instead of one large one—and we never under any circumstances find any indication of regular reflection. Electrons are not regularly reflected from a target of ordinary polycrystalline nickel.

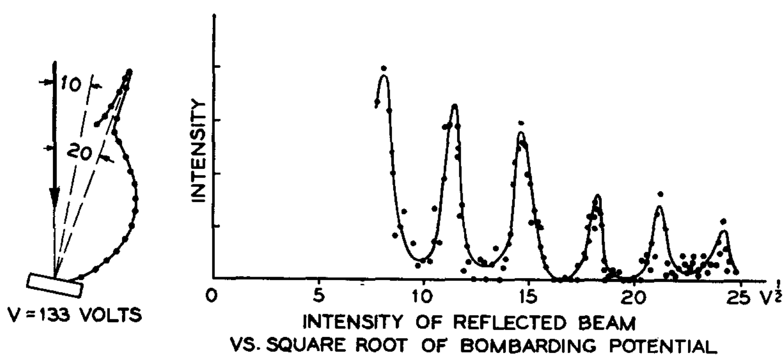
It seems curious that electrons should be reflected only from a crystal-face—and then we remember that this is true also of x-rays. X-rays may be regularly reflected from the face of a crystal, but not from a polycrystalline mirror. The difference between light and x-rays in this respect is due, as we know, to a difference in order of wave-lengths. The lengths of light waves are great compared to the distance between atoms in solids while the x-ray wave-lengths are comparable with these distances.

We may say then that both of these results—the regular reflection of electrons from a crystal-face and the absence of such reflections from a polycrystalline surface—would be comprehensible if electrons were trains of waves of wave-lengths comparable to distances between atoms in solids.

Now it will be remembered that x-ray reflection is characterized by a marked selectivity. If a beam of monochromatic x-rays is directed against a crystal face, the intensity of the beam reflected at a certain angle is very nearly zero unless the wave-length of the beam happens to lie at, or very near to, one or another of a series of discrete values. It is as if we had a mirror which would reflect red light of a certain wave-length and also blue light of a certain wave-length, but which would not reflect light of any of the intermediate wave-lengths.

This suggests an interesting experiment. If electrons resemble x-rays in being reflected from a crystal, but not from a polycrystalline surface, do they also resemble x-rays in exhibiting selective reflection? We might expect, for example,

FIG. 2.



Showing selectivity of electron reflection—angle of incidence 10 degrees.

that if electron reflection is really like x-ray reflection it would be selective in speed of bombardment. Well, the astonishing thing is that it *is* selective in speed. When we measure the intensity of the reflected beam as a function of speed of bombardment, we find that it passes through one maximum after another as the speed is increased. A curve exhibiting this behavior is shown in Fig. 2.

The ordinate is the intensity of the reflected beam, and the abscissa is the square root of the bombarding potential, which is proportional to the speed of the electrons in the incident beam. These observations are for angle of incidence ten degrees, and the curve on the left shows the reflected beam at the second maximum of the intensity curve.

Now the selective reflection of x-rays is a phenomenon which is very thoroughly understood. In explaining it we make very definite and explicit use of the idea that x-rays are waves. In fact, the phenomenon supplies us with our most reliable means of measuring x-ray wave-lengths. It is very significant indeed that electron reflection resembles x-ray reflection in this particular respect.

FIG. 3.

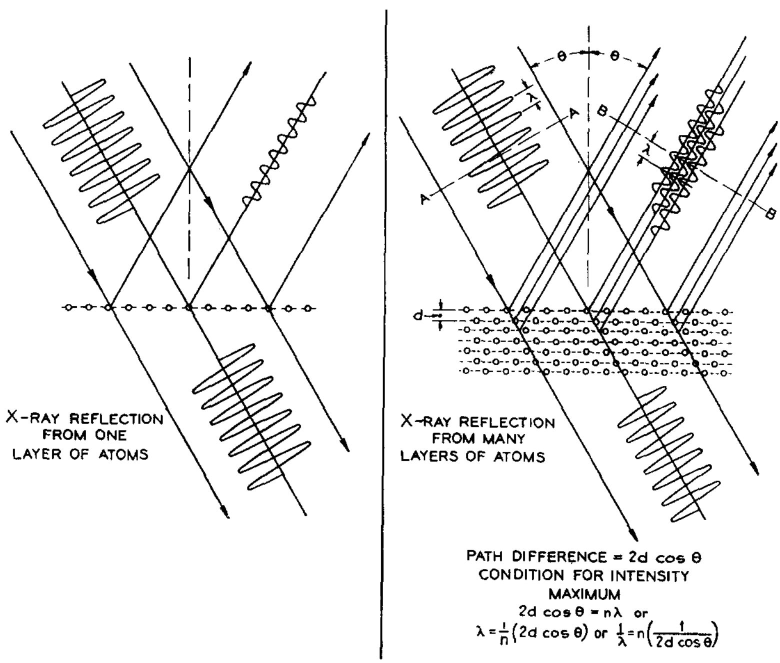


Diagram illustrating the selective reflection of x-rays from a crystal.

I shall take just a few minutes to review with you the theory of x-ray crystal reflection and the explanation of the selectivity.

When a beam of x-rays is incident upon a single layer of atoms as illustrated on the left in Fig. 3, the beam passes through the layer with only very slight diminution in its intensity. It does, however, set up forced vibrations in the atoms which it irradiates, and these send forth trains of spherical waves which are related in phase, and which combine to form a beam of waves moving off from the plane in the direction of regular reflection. The reflection of x-rays from a single layer of atoms is not selective.

Selectivity develops when reflection occurs from a number of parallel layers of atoms such as we have in a crystal. The case is illustrated in the figure on the right. The reflection beams proceeding from the different layers are superposed and the resultant beam exhibits a strong intensity maximum when the elementary wave trains proceed from the crystal in phase as they do in the figure. The condition for such a maximum is clearly that the path-lengths from a plane AA to a plane BB via successive atom layers shall differ by a whole number of wave-lengths. This path difference is given by twice the distance between successive atom layers multiplied by the cosine of the angle of incidence, so that the intensity of the reflected beam is at a maximum when

$$2d \cos \theta = n\lambda.$$

The condition may be stated this way: The intensity of the reflected beam will be at a maximum when the wave-length of the incident beam has any one of the values

$$\lambda = \frac{1}{n}(2d \cos \theta),$$

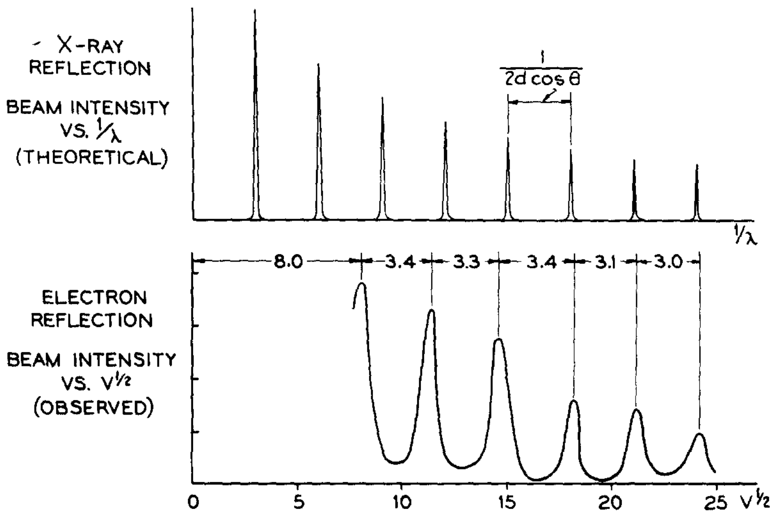
or when the reciprocal of the wave-length has any one of the values

$$\frac{1}{\lambda} = n \frac{1}{2d \cos \theta}.$$

Thus, if we plot the intensity of the reflected beam against the reciprocal of the wave-length, we should obtain a curve characterized by a series of equally spaced maxima. What we should find is illustrated by the curve at the top of Fig. 4.

In the lower half of the figure I show again, for comparison, the intensity of the electron reflection beam as a function of $V^{1/2}$, the square root of the bombarding potential.

FIG. 4.



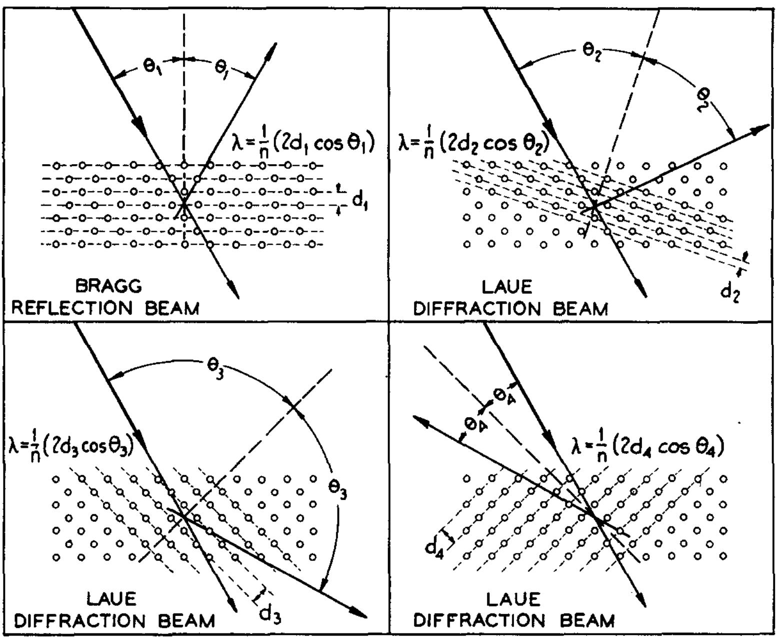
Showing the selective reflection of x-rays, and the selective reflection of electrons.

The maxima in the electron curve fall, as you see, at almost equal intervals. We may say, in fact, that we could understand electron reflection fairly well, including its selectivity, if electrons were waves of wave-length inversely proportional to the square root of the bombarding potential—inversely proportional, that is, to their speed. Apparently this would not be a perfect interpretation, for the maxima in the electron curve do not fall at exactly equal intervals. But it would do fairly well.

Well, here we are almost on the point of calculating electron wave-lengths—knowing perfectly well that electrons are particles. It is time for us to take some definite stand in regard to this matter, and I propose that we hold to our knowledge that electrons are particles, but admit that they are behaving as if they were waves—at least, that we can describe what we observe by pretending that they are waves, and that we do not see how the observations can be described in terms of particles. We take this point of view, and see how long it can be maintained.

Now, when x-rays are scattered by a crystal, beams issue from the crystal not only in the direction of regular reflection, but in other directions as well. One way of understanding this is that the atoms in the crystal may be regarded as

FIG. 5.



Reflection of x-rays from various planes of atoms in a crystal—illustrating the formation of Laue diffraction beams.

arranged in planes parallel to the surface of the crystal, but that they may also be regarded as arranged in planes that are not parallel to the surface—and that as far as the x-rays are concerned one set of atom planes is just as good as another. The situation is illustrated in Fig. 5.

If a beam of x-rays is incident upon a crystal face at angle θ_1, a regularly reflected beam issues from the crystal when the wave-length has any of the values

$$\lambda = \frac{1}{n}(2d_1 \cos \theta_1).$$

This beam is due to regular reflection from atom planes parallel to the surface and is ordinarily referred to as the Bragg reflection beam. But the atoms may also be regarded as arranged in other sets of planes as indicated in the other figures, and these also give rise to reflected beams. We find, for example, a beam issuing from the crystal in the direction of regular reflection from the atom planes shown in the second figure at the top when the wave-length has any of the values

$$\lambda = \frac{1}{n}(2d_2 \cos \theta_2).$$

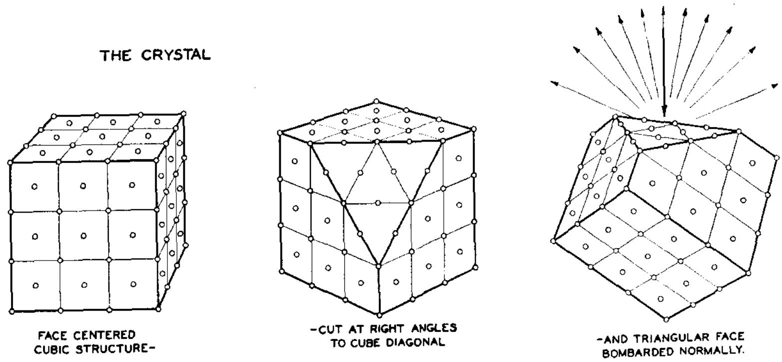
Such beams are ordinarily known as Laue diffraction beams.

We have seen that electrons resemble x-rays in being regularly reflected from the face of a crystal and in exhibiting selectivity. The question now is, do they also resemble x-rays in giving rise to diffraction beams. Well, as it happens, we observed these diffraction beams first—more than a year before we got around to looking for the reflection beams.

It is not going to be so easy to describe these diffraction beams as it has been to describe the reflection beams; there are differences between the characteristics of the x-ray and electron diffraction beams from which one might think that after all electrons are really not so very good at passing themselves off as waves. And yet we will find that these differences can be explained in a reasonable way, and that the diffraction data lead to quite definite values of electron wave-lengths. In describing these experiments I shall try first to give you a clear idea of the conditions under which the observations were made, next what would have been observed had the experiments been made with x-rays, and finally what was actually observed with electrons.

To begin with we shall need to understand the arrangement of atoms in the nickel crystal. Nickel forms crystals of the face centered cubic type. The unit of structure is a cube—3.51 Å. on the edge—with an atom at each corner and one in the center of each face. The large cube on the left in Fig. 6 is built up of 27 of these unit cubes. The only

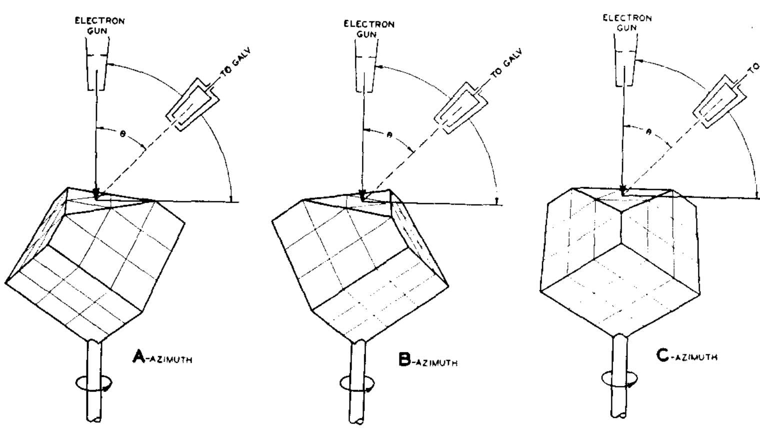
FIG. 6.



Schematic representations of the face centered cubic crystal of nickel.

atoms shown are those in the surface of the large cube. Henceforth I shall use this large cube as a symbol to represent the nickel crystal with which we began our experiments. We first cut through this structure at right angles to one of the cube diagonals, forming the triangular faces shown in the

FIG. 7.



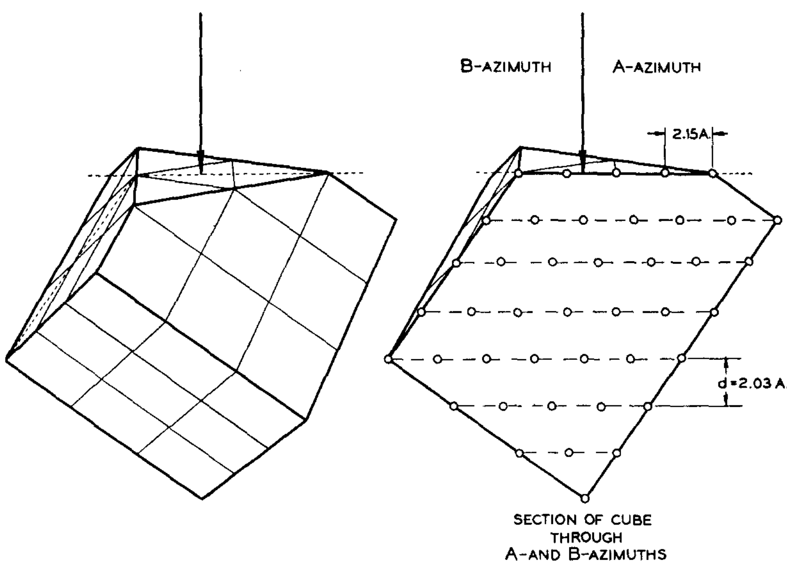
Schematic representations of experimental arrangementf or investigating electron diffraction.

central figure. A beam of electrons was then directed against this face at normal incidence, as indicated in the figure on the right, and measurements were made of the number of electrons leaving the crystal as a function of direction and of speed of bombardment.

The experimental arrangements for making these measurements are indicated in Fig. 7. The collector could be moved about in a single plane—the plane of the drawing and the crystal could be rotated about a vertical axis so that any azimuth of the crystal could be brought into the plane of rotation of the collector.

It is clear that the crystal has a three-fold symmetry. If we find a beam issuing from the crystal when one of the apexes of the triangle is in the plane of the collector, we will expect, of course, to find a similar beam when the crystal

FIG. 8.



has been turned through 120° to bring another of the apexes into the collector plane, and again when it has been turned through 240 degrees. We will call the azimuths of the crystal that include the apexes of the triangle the *A*-azimuths; those including the midpoints of the sides of the triangles the *B*-azimuths; and those parallel to the sides of the triangle the *C*-azimuths.

In Fig. 8 we show a cross section of the crystal through the plane of the A–B-azimuths. The circles represent lines of atoms extending through the crystal at right angles to this plane. The crystal may be regarded as built up of planes of atoms lying parallel to the surface of the crystal. The distance between these planes is 2.03 Å., and the distance between the lines of atoms in each plane is 2.15 Å. It will be noticed that the lines of atoms in a given plane are not directly below the lines of atoms in the next higher plane, but are shifted to the right by an amount equal to one-third of the distance between lines.

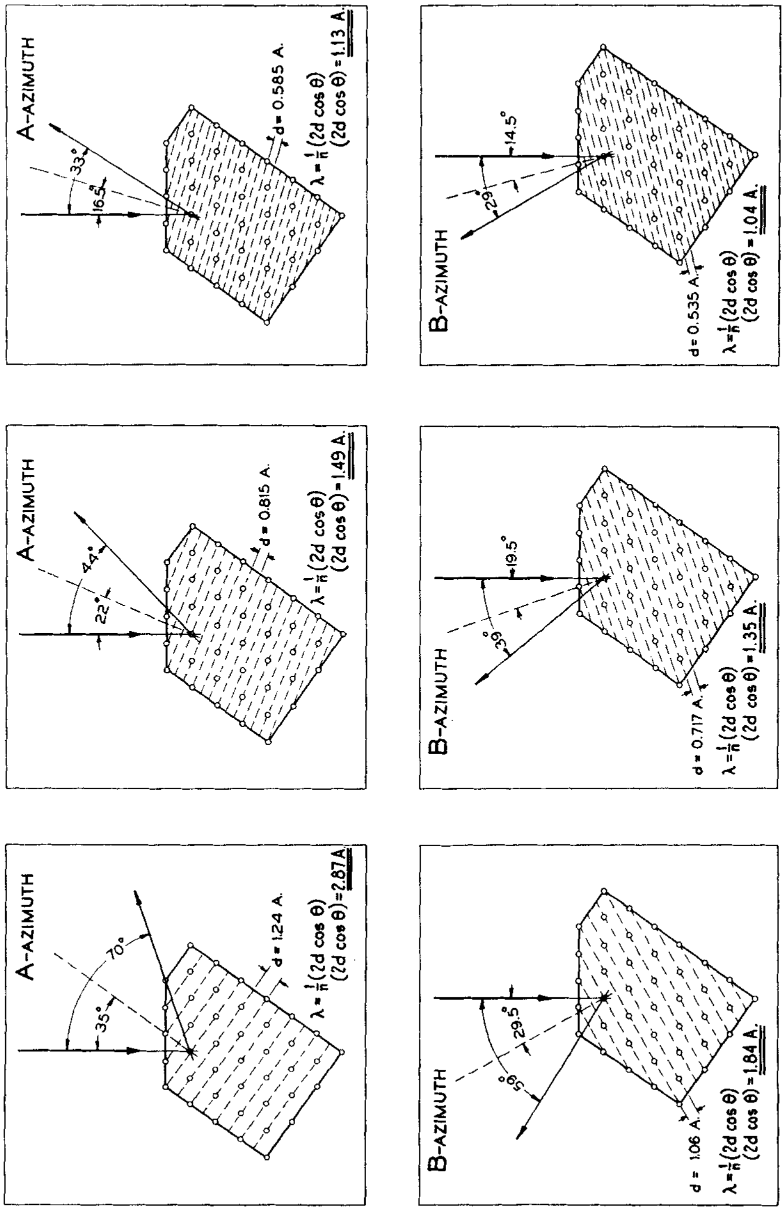
The crystal is now sufficiently specified to enable us to calculate the wave-lengths and positions of all the x-ray diffraction beams that can appear in the A- and B-azimuths. Thus, the atoms may be regarded as arranged in planes as shown in the upper left-hand diagram in Fig. 9. The distance between the successive atom planes is 1.24 Å. The angle of incidence is 35 degrees, and an x-ray diffraction beam will issue from the crystal in the direction $\theta' = 70$ degrees when the wave-length has any of the values

$$\frac{1}{n}(2d \cos \theta) = \frac{1}{n}(2 \times 1.24 \times \cos 35°) = \frac{2.87}{n} \text{ Å}.$$

The three A-azimuth beams shown in the upper diagrams are the three for which the modulus $(2d \cos \theta)$ has the greatest values, that is, they are the three A-azimuth beams of longest wave-length. Those shown below are the three B-azimuth beams of longest wave-length. What we need to get from this figure particularly is that as the wave-length of the incident x-ray beam is decreased from some large value diffraction beams appear in the following order: first, a beam at 70 degrees in the A-azimuth; next, a beam at 59 degrees in the B-azimuth; then, a beam at 44 degrees in the A-azimuth, followed by a B-azimuth beam at 39 degrees.

This is what would occur if the incident beam were a beam of x-rays—and from what we have seen of the regular reflection of electrons from the atom planes lying parallel to

FIG. 9.

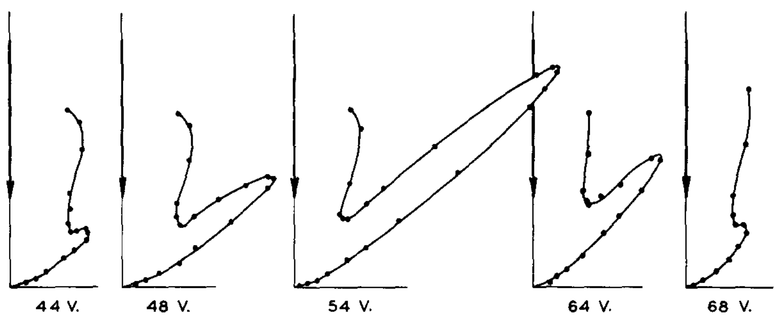


The principal diffraction beams that would appear in the *A*- and *B*-azimuths if the incident beam were a beam of x-rays.

the surface it seems not unreasonable to expect that at particular speeds of bombardment electron beams will be found issuing from the crystal in these same directions. Well, electron beams issue from the crystal in its principal azimuths at certain critical speeds of bombardment, but they do not coincide in directions with any of these principal Laue beams. They appear not to be regularly reflected from any of the principal planes of atoms.

As the speed of bombardment is increased from zero the first of these beams appears in the *A*-azimuth, not however at 70 deg. or at 44 deg. but at 50 deg.; and the second appears in the *B*-azimuth, not at 59 deg. or at 39 deg. but at 44 deg. Curves for the first of these beams are shown in Fig. 10. The beam first appears at about 40 volts; it disappears at 70 volts, and is most intense when the bombarding potential is 54 volts. In Fig. 11 we have the same beam in a weakened condition owing to gas adsorbed onto the surface of the crystal. The curve in the lower diagram was obtained by setting the collector in the axis of the beam at its maximum, and then measuring the collector current as the crystal was rotated. You see that there is a strong maximum in each of the *A*-azimuths, as, of course, there should be.

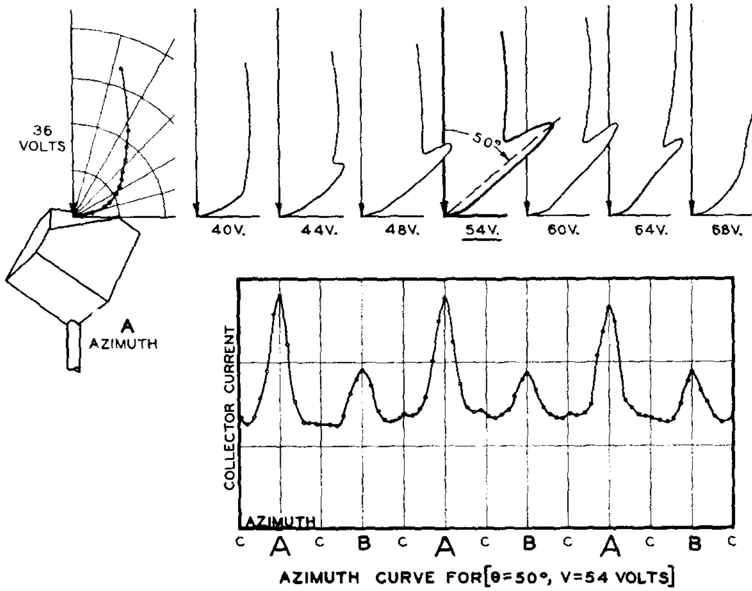
FIG. 10.



Showing the growth and decay of the "54-volt" electron diffraction beam in the *A*-azimuth—surface of crystal clean.

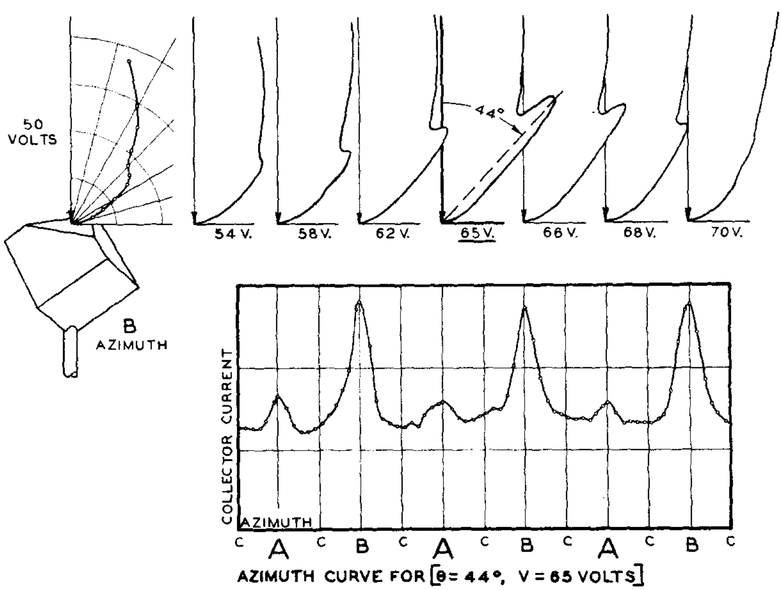
The corresponding curves for the first beam in the *B*-azimuth are shown in Fig. 12. Maximum intensity is attained at 65 volts, with the beam at 44 deg.

FIG. 11.



Showing "54-volt" diffraction beam weakened by contamination of the crystal surface—azimuth curve showing maxima in *A*-azimuths.

FIG. 12.



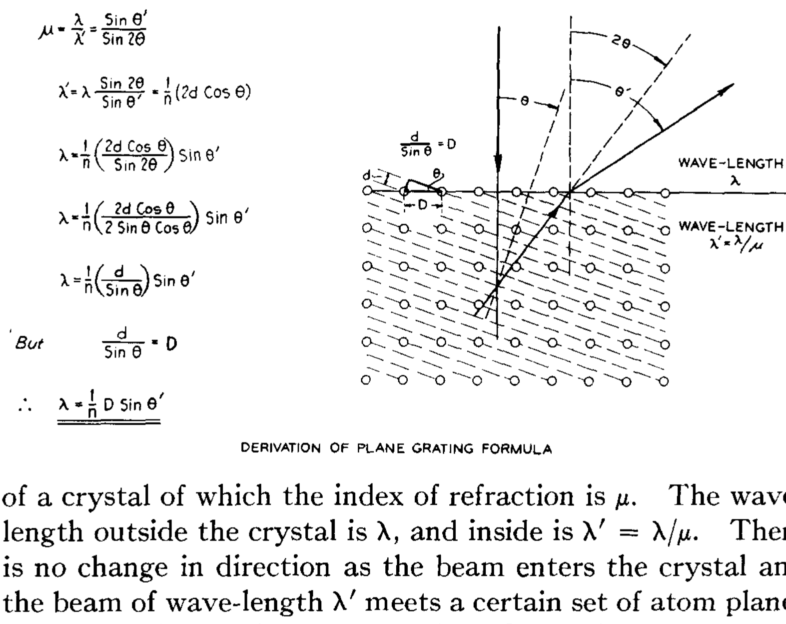
Growth and decay of the "65-volt" beam in the *B*-azimuth—surface of crystal contaminated by gas. Azimuth curve showing maxima in *B*-azimuths.

Now there is an interesting possibility in regard to this discrepancy between the directions taken by the electron beams and those taken by the x-ray beams. It may be that the crystal should be regarded as a refracting medium for electrons—that is, as a medium characterized by an index of refraction different from unity.

I shall take a few minutes to develop this idea. Let us imagine a beam of radiation incident normally on the surface

FIG. 13.

DIFFRACTION BY REFRACTING CRYSTAL



DERIVATION OF PLANE GRATING FORMULA

of a crystal of which the index of refraction is μ. The wave-length outside the crystal is λ, and inside is $\lambda' = \lambda/\mu$. There is no change in direction as the beam enters the crystal and the beam of wave-length λ' meets a certain set of atom planes at angle θ (Fig. 13). A regularly reflected beam moves off from these planes provided

$$\lambda' = \frac{1}{n}(2d\cos\theta).$$

This beam meets the crystal surface at an angle 2θ, but is refracted on passing through the surface and leaves the crystal in the direction θ'. The beam issuing from the crystal appears not to be regularly reflected from any of the principal atom planes.

And now I am going to ask you to follow through the brief mathematical deduction on the left in Fig. 13 as it leads to an interesting and important relation. We treat electron radiation as if it were light, and write $\mu = \lambda/\lambda' = \sin\theta'/\sin 2\theta$. Solving this for λ', we have $\lambda' = \lambda \sin 2\theta/\sin\theta'$, but by Bragg's law $\lambda' = (2d\cos\theta)/n$. Equating these expressions for λ', and solving for λ, we obtain

$$\lambda = \frac{1}{n}\left(\frac{2d\cos\theta}{\sin 2\theta}\right)\sin\theta'.$$

And writing in $2\sin\theta\cos\theta$ for $\sin 2\theta$ and eliminating $2\cos\theta$, this reduces to

$$\lambda = \frac{1}{n}\left(\frac{d}{\sin\theta}\right)\sin\theta'.$$

But from the construction on the left of the diagram in Fig. 13 we see that $d/\sin\theta$ is equal to D, the distance between adjacent lines of atoms in the surface of the crystal.* We have, therefore,

$$\lambda = \frac{1}{n}D\sin\theta'.$$

This is an extremely useful relation, as it enables us to calculate the wave-length of a diffraction beam (provided we knew its order n) from the distance between lines of atoms in the surface of the crystal and the angle at which the beam emerges. We do not need to know with what set of atom planes a given beam is associated, and neither do we need to know the index of refraction of the crystal.

The idea of regarding the crystal as a refracting medium for electrons is due to Dr. Eckart of the California Institute of Technology, although we had already assumed that the wave-lengths of diffraction beams could be calculated from this formula.

We therefore apply this formula to the electron diffraction beams for which we have data. The distance between the lines of atoms lying normal to the A- and B-azimuths is 2.15 Å. The beam which is observed in the A-azimuth when the

* In general $d/\sin\theta = D/m$, where m represents an integer. The conclusion is unaffected, however, by this circumstance.

bombarding potential is 54 volts lies at $\theta' = 50$ degrees. The wave-length of "54-volt electrons" should then be given by

$$\lambda = \frac{2.15}{n} \sin 50^\circ = \frac{1.65}{n}.$$

But since this is the first beam that appears in the *A*-azimuth it is certainly a first order beam, and therefore $\lambda = 1.65$ Å. When we make a similar calculation for the 65-volt beam in the *B*-azimuth, we find $\lambda = 1.50$ Å.

That one can calculate the wave-length of a stream of electrons in a straightforward and simple way seems, of course, very surprising, and yet it is much less surprising today than it would have been, say, five years ago. During the last four or five years there has been a rapidly growing conviction that the principles of mechanics, in their various formulations, as we have known them, are really only first approximations to what we may call the true principles of mechanics. They are remarkably close approximations for most purposes—for the purposes of mechanical engineering and astronomy it is unlikely that they can be improved upon either for convenience or reliability—and yet there is the conviction that classical mechanics is in a sense a degenerate form of the true mechanics, and therefore of limited applicability. It is a first approximation to the true mechanics applicable only in those cases in which the products of the momenta and linear dimensions used in describing the system are large compared to the Planck constant of action *h* —large, that is, compared to 10^{-26} erg sec. We are to feel no hesitation for example in using classical mechanics in dealing with the mechanics of the solar system; the linear dimensions involved are of the order of the major axes of the orbits of the planets, the momenta involved are the momenta of the planets, and the products of these quantities are enormously great compared to the value of *h*. On the other hand, we are not to suppose that the approximate form of the true mechanics which is applicable to the solar system is applicable also to a system such as the Bohr atom. Here the products of linear dimensions and momenta are not large compared to h. They are in fact of the same order as h, and laws of mechanics as we have known them are therefore of no service. They are of limited applicability, and do not apply in this case.

This conviction has grown out of dissatisfaction with the ever-mounting artificiality of the Bohr atom model as means for describing and correlating the data of spectroscopy. It has led to attempts in various quarters to discover or invent a new system of mechanics which will degenerate to our ordinary mechanics in the case of gross systems, but which will be applicable as well and without forcing to systems involving atoms and electrons. One of these attempts was made by L. deBroglie who put forward the idea, a little more than three years ago, that every mechanical phenomenon is in some sense a wave phenomenon—that every problem in mechanics is in a way a problem in optics—that in the rigorous solution of all such problems one must always concern himself with the propagation and interference of waves. This idea has been taken up with great enthusiasm by theoretical physicists, notably by Schroedinger, and has enjoyed a rapid and remarkable development. This development is known as the undulatory or wave theory of mechanics, and it gives promise of being perhaps the sorely needed true mechanics. It is yet in a state of flux—no one, I think, knows what its final form will be. Ideas regarding its form and interpretation are constantly changing, and yet from its inception in the hands of deBroglie to the present time there has persisted the idea that a freely moving particle of momentum (mv) is equivalent to, or has associated with it, a train of waves of wave-length h/mv. Whether the particle is itself a group of waves—whether the waves are to be thought of as real physical waves such as we have supposed light waves to be, or whether the waves are purely analytical—a mathematical convenience only—no one, I think, knows.

The development of this theory has so far been directed almost entirely toward giving us a new picture of the atom, and a new and less arbitrary set of rules for correlating and interpreting the data of spectroscopy. Not so much attention has been paid to aperiodic phenomena such as we are here concerned with, and yet a few months after we had begun our experiments, but more than a year before we had obtained any of the results I have described, the prediction was made by Elsasser in Germany that evidence for the wave nature of mechanics would be found in the interaction between a stream of electrons and a crystal.

Most of you know already, of course, that the electron wave-lengths obtained by the measurements I have been describing are in good agreement with the values of h/mv of this new theory of mechanics. We have

$$\lambda = h/mv,$$

and for electrons of moderate speeds such as we have used we have also

$$\frac{mv^2}{2} = \frac{Ve}{300},$$

where e represents the charge of the electron in electrostatic units and V the bombarding potential in volts.

Eliminating the velocity v between these equations, we obtain

$$\lambda = \left(\frac{h^2}{me}\right)^{1/2}\left(\frac{150}{V}\right)^{1/2} \text{cm.}$$

The value of $(h^2/me)^{1/2}$ differs from 1×10^{-8} by about 2 parts in a thousand so that to a close approximation

$$\lambda = \left(\frac{150}{V}\right)^{1/2} 10^{-8} \text{ cm.}$$

or

$$= \left(\frac{150}{V^{1/2}}\right)^{1/2} \text{Å.}$$

$$= \frac{12.25}{V^{1/2}}.$$

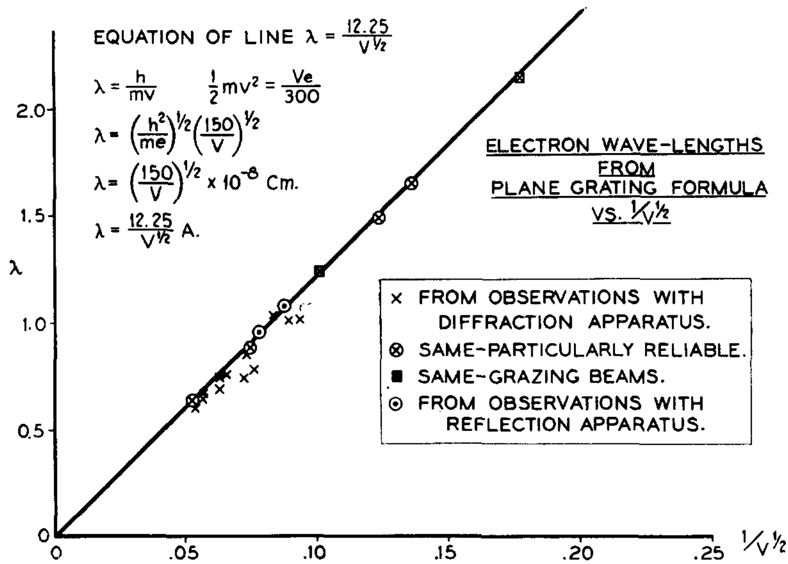
The value of h/mv for electrons that have been accelerated from rest through a potential difference of 54 volts is

$$12.25/(54)^{1/2} = 1.67 \text{ Å.},$$

which is in good agreement with the value 1.65 Å. which we find by our measurements. For "65-volt" electrons the theoretical wave-length is 1.52 Å. and our observed value is 1.50 Å.

We have made in all twenty or more determinations of electron wave-lengths. All of these values are plotted in Fig. 14 against the reciprocal of the square-root of the accel-

FIG. 14.



Graphical comparison of all experimentally determined values of electron wave-lengths with the relation $\lambda = h/mv$.

erating or bombarding potential. The values which we have reason to believe are the most reliable are enclosed in circles or in squares. The straight line through the origin is the graph of the equation

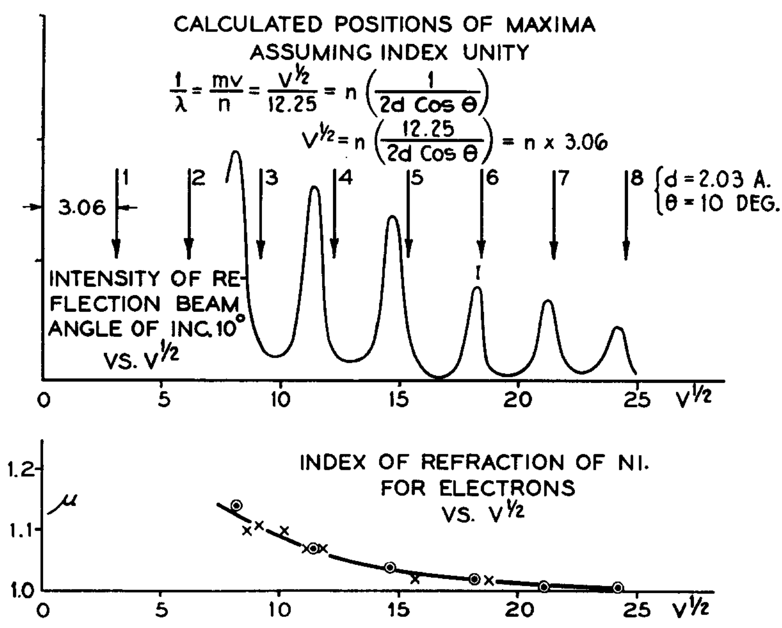
$$\lambda = 12.25/V^{1/2}.$$

If the experimentally determined wave-lengths agreed exactly with the values of h/mv, all of these points would fall accurately on this line. The departures are none of them greater than can be reasonably accounted for by the uncertainty of the measurements. We may say, I think, that in certain circumstances a stream of electrons of speed v behaves as if it were a beam of waves of wave-length h/mv, in accordance with the postulates of the wave mechanics.

And now I would like to return for a few minutes to a further consideration of the regularly reflected beam.

You will remember that we plotted the intensity of the electron reflection beam for angle of incidence ten degrees against the square root of the bombarding potential and obtained a curve characterized by a series of maxima at nearly equal intervals.

FIG. 15.



We can now calculate the positions these maxima would occupy if the index of refraction of nickel for electrons were unity. We have

$$\frac{1}{\lambda} = \frac{mv}{h} = \frac{V^{1/2}}{12.25} = n\frac{1}{2d\cos\theta},$$

that is, we assume the wave-length of the electrons to be $h/mv = 12.25/V^{1/2}$ and apply the Bragg formula to find the values of $V^{1/2}$ at which the reflected beam should exhibit intensity maxima. Substituting for d and θ their values, 2.03 Å. and 10 deg., we find

$$V^{1/2} = n \times 3.06.$$

If the index were unity, the maxima should be found at these values of $V^{1/2}$. These positions are indicated in Fig. 15. The observed maxima lie to the left of the calculated positions, the displacement decreasing, however, toward the higher orders. This is the type of displacement to be expected if the index of refraction of the nickel for electrons is greater than unity. We can in fact use these displacements to calculate values of the refractive index. The more general form of the Bragg formula applicable to the case in which the index is not unity is

$$V^{1/2} = n\left(\frac{12.25}{2d(\mu^2 - \sin^2\theta)^{1/2}}\right).$$

Using this formula and the data now available, we have indices of refraction for electrons of various speeds and these are plotted in the lower diagram in Fig. 15. The values indicated by circles are from the observations at ten degrees incidence, and the others are from observations at other angles.

We see that the index approaches unity as the speed of the electrons is increased; that is, the results indicate that the purely geometrical differences between the reflection and diffraction of x-rays on the one hand and of electrons on the other disappear for electrons of high speed. That this is true is evident from the very interesting experiments recently reported by Prof. G. P. Thomson of the University of Aberdeen who has studied the diffraction of beams of high-speed electrons (15,000 to 60,000 volts) by extremely thin metal foils. What Prof. Thomson observes is that electrons of these speeds are diffracted by the polycrystalline metal in the way familiar to us in the powder method of x-ray diffraction devised independently by Hull, and Debye and Scherrer. The cross-section of the transmitted beam consists of a central spot surrounded by a number of concentric rings. These rings seem to correspond exactly to the rings that would be observed if the incident beam were a beam of x-ray of wave-length equal to h/mv. For electrons of such high speeds the index of refraction of metals is apparently very nearly unity.

Are electrons waves? The easiest way of answering this question is to ask another. *Are x-rays waves?* If x-rays are waves, then so also are electrons. But we are no longer so certain as we used to be that x-rays are waves. The Compton effect and the photoelectric effect are most simply described by supposing that there is some sense in which x-rays are particles.

It is all rather paradoxical and confusing. We must believe not only that there is a certain sense in which rabbits are cats, but that there is also a certain sense in which cats are rabbits.

I would not have you think, however, that confusion in these matters is universal. There are plenty of theoretical physicists to whom these matters are not at all confusing. I am sure that Professor Swann here would not admit to the least confusion. I may cite also Professor C. G. Darwin of the University of Edinburgh. In a recent article on this subject Prof. Darwin writes:

"The central difficulty of the quantum theory has always been the conflict between waves and particles. On the one hand, we have the theorems of conservation of matter, energy, etc.; these tell us that matter keeps together, and endow a particle or a quantity of energy with individuality, so that we can trace its history. On the other hand, we have the theorems of interference—of light and now of matter as well, which as definitely tell us that the things which we before regarded as particles must spread, and so must lose their individuality. The recent work of Bohr explains, at any rate in outline, how the apparent contradiction is to be reconciled. The two lines of thought are not contradictory, but complementary. They do not come into conflict because they never meet. To verify conservation we must obviously have an enclosed system, and this excludes observation of what happens in the enclosure. If nothing is observable, it is only proper to say that nothing is happening; the system is settled into a spaceless and timeless stationary state outside our intuitions. On the other hand, if we want to observe what happens, we must make a hole in the enclosure and see what leaks out. By the very act conservation is destroyed, but in exchange we get interference phenomena, and these introduce geometry and so a connection with space and time. This very inadequate description shows that we are entitled, when we want to discuss happenings in space and time, to make full use of the wave theory and to pay no attention to the conservation difficulties, because in fact these do not arise."

As you see, it is really all very simple.