The Sagnac effect in a rotating ring with Dirac fermions

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The observation of the Sagnac effect for massive material particles offers a significant enhancement in sensitivity when compared to optical interferometers with equal area and angular rotation velocity. As a result, there have been suggestions to employ solid-state interferometers that rely on semiconductors and graphene. However, in the case of monolayer graphene, its quasiparticles exhibit a linear dispersion, thus making the Sagnac effect in graphene comparable to that of for light. We investigate the Sagnac effect in the Dirac materials governed by the relativistic dispersion law and find the value of the fringe shift. The analysis reveals that optimal sensitivity is achieved in materials featuring a reduced value of Fermi velocity. Notably, the sign of the fringe shift depends on the nature of the charge carriers – whether they are electrons or holes.

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Introduction. Physical phenomena associated with rotation possess a captivating allure that spans multiple levels. One of the fundamental illustrations of this allure lies in the impossibility of establishing a standard clock synchronization procedure along a closed curve when the metric is non-static, as is the case of a rotating frame of reference (see the textbooks [1, 2] and [3]). The Sagnac effect refers to the phenomenon where a phase shift is observed between two coherent beams that travel along opposite paths within an interferometer situated on a rotating disk (see Refs. 4–7 for the reviews). This phase shift, which was first demonstrated for light by Sagnac [8] in 1913 is the intrinsically relativistic effect. Thus, it can be essentially viewed as a consequence of the impossibility of synchronization of clocks along the circumference of the rotating disk. It is not restricted to light waves, but is observed for electron waves in vacuum [9], neutrons [10] and atoms [11] (see also the latest work [12]). Moreover, the observation of the Sagnac phase shift for massive particles in solids, specifically superconducting Cooper pairs, dates back to as early as 1965 [13].

While practical applications of the Sagnac effect currently rely on light waves, there is a compelling physical explanation as to why massive particles are significantly more advantageous for its realization. The Sagnac fringe shift, denoted as $\Theta_{\rm S}$, with respect to the fringe position for the stationary interferometer, reads [6, 9, 14]

$$\Theta_{\rm S} = \frac{4EA\Omega}{\hbar c^2}. \tag{1}$$

This formula is applicable to waves comprising both massless and massive particles, and E represents the total energy of a corresponding particle. The value A denotes the area enclosed by the light or particle beams in the interferometer, Ω is the angular velocity of the interferometer's rotation within an inertial frame, $\hbar$ is the reduced Planck constant, and c is the the free-space velocity of light. This equation is written neglecting a small relativistic correction and under the assumption that the plane of the interferometer is perpendicular to the axis of rotation. Substituting in Eq. (1) the energy $E = \hbar\omega$, where ω is either the frequency of the light or the frequency of the de Broglie wave of a material particle, we recover the standard formula for the Sagnac phase shift [4–7, 9] $\Theta_{\rm S} = 4\omega A\Omega/c^2$.

When considering light and using the dispersion relationship $\omega = 2\pi c/\lambda$, where λ represents the vacuum wavelength of light, we arrive at another commonly used form for the Sagnac phase shift [4–6],

$$\Theta_{\rm S} = \frac{8\pi A\Omega}{c\lambda}. \tag{2}$$

In the case of slow massive particles (nonrelativisitic case), the energy $E = mc^2$ is associated with their rest mass m, and the phase fringe acquires the following form

$$\Theta_{\rm S} = \frac{4mA\Omega}{\hbar}. \tag{3}$$

Comparing the phase shift for the matter-wave and optical interferometers, one finds that for the equal area and angular velocity the phase shift is enhanced by a factor $mc^2/(\hbar\omega)$ [15]. For atoms, the matter-wave interferometer is significantly more sensitive to rotation, with a factor reaching the value of 10^{10}. This constitutes the primary reason why the existing optical gyroscopes necessitate the utilization of either several kilometers of optical fiber or a substantial area to achieve the necessary sensitivity. Conversely, the sensitivity enhancement for matter-waves has led to proposals to utilize cold atom interferometers in the search for smaller signals beyond Earth rotation [12].

For free electrons, this factor also reaches the value 10^6 and, as mentioned earlier, the Sagnac effect was observed using an electron interferometer in vacuum [9]. There is a possibility of realizing the Sagnac effect in solid-state by employing a serial array of mesoscopic ring-shaped electron interferometers, which was discussed in Refs. 16–19.

It has to be noted that the simulations for ring arrays discussed in [16–18] are conducted with the assumption that electrons in solids have an effective mass, m^*. Consequently, the enhancement factor $m^* c^2/(\hbar\omega)$ is slightly reduced, estimated to be in the range of 10^5 to 10^6. Moreover, as highlighted in [18, 19], graphene emerges as a promising material for electron interferometry, attributed to its extraordinary electronic properties. Indeed, recent experiment [20] on Aharonov-Bohm oscillations in ring-shaped gated bilayer graphene provides further confirmation of this potential.

However, at least monolayer graphene belongs to the new class of Dirac materials with a zero effective carrier mass m^* and linear dispersion relation as for light. At first glance, due to the assertion discussed e.g. in Ref. [6] that the phase shift remains independent of the phase velocity of the wave, the Sagnac effect in graphene seems to be analogous to the case of light.

The objective of the present work is to examine this issue also taking into account that the density of carriers in graphene is finite. More broadly, the goal is to investigate the Sagnac effect in relativistic Dirac materials and to clarify the distinctions from the previously known cases.

Model and formalism. We consider the Dirac materials characterized by the following dispersion relation:

$$\epsilon(\mathbf{k}) = \pm\sqrt{\hbar^2 v^2 k^2 + \Delta^2} - \mu, \tag{4}$$

where $\mathbf{k}$ represents the wave vector counted from the Dirac points, which can be either 2D or 3D. However, since we are examining a ring on the plane geometry, its z-component, denoted as k_z, can be effectively disregarded by setting it to zero. Also in Eq. (4) v is the Fermi velocity, μ is the chemical potential and Δ is the gap in the quasiparticle spectrum. In the case of graphene, the value of μ (including the change of the character of carriers, either electrons or holes) is tunable by applying the gate voltage to the devices, and $\mu > 0$ corresponds to electrons. The gap term Δ is present, in particular, in the Hamiltonian derived by Wolf (see Ref. [21] for a review) for Bi and similar effective Hamiltonians describing other 3D Dirac materials. It can also be induced in graphene monolayer by placing it on top of hexagonal boron nitride (G/hBN).

As already mentioned, the quasi-relativistic spectrum (4) follows, for example, from the Wolf Hamiltonian as well as other effective low-energy either 2D or 3D Dirac Hamiltonians used to describe Dirac materials (for graphene see, e.g. the review [22]).

To focus on the Sagnac effect for the quasiparticles with the relativistic-like dispersion we restrict ourselves by considering the squared Dirac Hamiltonians neglecting the coupling between pseudospin degree of freedom with the rotation of the frame [23]. Thus we assume that a free quasiparticle in the electron subsystem of the Dirac material in the inertial frame of reference obeys the following wave equation

$$\left(\Box' + \frac{\Delta}{\hbar^2}\right)\psi(t', \mathbf{r}') = 0, \qquad \Box' \equiv \frac{1}{v^2}\frac{\partial^2}{\partial t'^2} - \triangle'. \tag{5}$$

Here $\Box'$ and $\triangle'$ are the d'Alembertian and Laplace operators, respectively, $\psi(t', \mathbf{r}')$ is the electron wave function in the inertial frame of reference denoted as primed. The chemical potential should also be included in Eq. (5) by the standard prescription for the relativistic systems, $i\hbar\partial_t \to i\hbar\partial_t + \mu$.

Seeking for a solution of Eq. (5) in the form $\psi \sim \exp(-i\epsilon t'/\hbar + i\mathbf{k}\mathbf{r}')$, one reproduces the spectrum (4). Clearly, relating the energy gap Δ to the mass, $m = \Delta/v^2$, and setting $v = c$ and $\mu = 0$, the equation (5) reduces to the usual Klein-Gordon-Fock (KGF) equation.

We shall consider the Sagnac effect for the quasisparticles characterized by the wave equation (5) from the point of view of a co-rotating observer employing the approach of Ref. [14]. It can be traced back to the explanation of the Sagnac effect proposed by Langeven in the framework of general relativity in 1921 [24] (see also Refs. [4, 7] for the reviews and the textbook [1]). The invariant interval in polar coordinates (t', r', ϕ') in the inertial rest frame is

$$ds'^2 = c^2 dt'^2 - dr'^2 - r'^2 d\phi'^2, \tag{6}$$

where as mentioned above we restricted ourselves by the planar geometry. The transformation to a new non-primed frame of reference rotating about the z-axis with angular velocity Ω is done by $t' = t$, $r' = r$ and $\phi' = \phi + \Omega t$, so the invariant interval reads

$$ds^2 = c^2\left(1 - \frac{\Omega^2 r^2}{c^2}\right)^2 dt^2 - \frac{2r^2\Omega}{c} c dt d\phi - dr^2 - r^2 d\phi^2. \tag{7}$$

The corresponding contravariant metric tensor ($\mu, \nu = 0, 1, 2 = t, r, \phi$) is

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & -\frac{\Omega}{c} \\ 0 & -1 & 0 \\ -\frac{\Omega}{c} & 0 & -\frac{1}{r^2} + \frac{\Omega^2}{c^2} \end{pmatrix}. \tag{8}$$

Note that $g = \det[g_{\mu\nu}] = r^2 > 0$ because $2 + 1$ dimensional space is considered.

To elucidate the propagation of electron waves within a rotating coordinate frame, characterized by the metric (8), it becomes necessary to employ the following equation

$$\left(\Box + \frac{\Delta}{\hbar^2}\right)\psi(t, \mathbf{r}) = 0, \tag{9}$$

with the generalized d'Alembertian operator [25]

$$\Box\psi = \nabla_\mu \nabla^\mu \psi = \frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g} g^{\mu\nu}\partial_\nu\psi). \tag{10}$$

Recall that by the definition the covariant derivative $\nabla_\mu \equiv \partial_\mu = \frac{\partial}{\partial x^\mu}$, where $x^\mu = (vt, r, \phi)$. It easy to obtain that the operator $\Box = \Box^0 + \Box^\Omega$ with

$$\begin{aligned} \Box^0 &= \frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}, \\ \Box^\Omega &= -\frac{2\Omega}{vc}\frac{\partial^2}{\partial t \partial \phi} + \frac{\Omega^2}{c^2}\frac{\partial^2}{\partial \phi^2}. \end{aligned} \tag{11}$$

Obviously, for $\Omega = 0$ Eq. (9) reduces to Eq. (5) written in the rest frame in the polar coordinates. Also for $v = c$ and $\mu = 0$ Eq. (9) reduces to the KGF equation in the rotating frame considered in Ref. [14] to investigate the Sagnac effect and in Ref. [26] to study the relativistic superfluidity. There is, however, an essential difference in the approaches used in [14] and [26]. The KGF equation within the rotating frame was obtained in [14] by replacing $\partial t \to \partial t - \mathbf{V}\cdot\nabla_{\mathbf{r}}$, where $\mathbf{V} = \boldsymbol{\Omega}\times\mathbf{r}$ is the local rotating velocity. Consequently, its extension to the case $v \neq c$ is obscure because it is not enough to replace c by v in the KGF equation in the rest frame. On the other hand, the derivation in Ref. [26] utilizes the operator (11) as done in this work. The first term of $\Box^{\Omega}$ with the mixed derivative, which is sensitive to the rotational direction, corresponds to the Coriolis force. Meanwhile, the second one is related to the centrifugal force [26].

The $v = c$ case. Before going ahead with the analysis of Eq. (9) for the general $v \neq c$ and finite μ case, it is instructive to recapitulate the derivation of Eqs. (1) and (3) made in Ref. [14]. The signals designated as $\pm$, which propagate in the clockwise and counterclockwise directions around a circle of the radius R, are considered. These signals are described by the solutions of Eq. (9) which depend on t and the angular variable ϕ with $\Delta = mc^2$. The solutions which have the frequency ω in the local Lorenz frame at the source are

$$\psi_{\pm}(t,\phi) = \exp\left[i\gamma\left(\pm k + \frac{\Omega\omega R}{c^2}\right)R\phi - i\omega t/\gamma\right] \qquad (12)$$

with

$$\gamma = \left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{-1/2}. \qquad (13)$$

The wave number in Eq. (12) is determined by the KGF dispersion relationship $\omega^2 = c^2k^2 + m^2c^4/\hbar^2$. Let's assume that the two counter-propagating signals originate in phase from the same source at $\phi = 0$. Subsequently, these signals are detected after completing a full round-trip, with phases the counterclockwise signal is detected at $\phi = 2\pi$, and the clockwise at $\phi = -2\pi$. The phase difference between the two detected signals is therefore $\Theta_{\rm S} = 4\pi\gamma\omega\Omega R^2/c^2$. Considering that the circular interferometer has an area of $A = \pi R^2$, one finds that the last expression reproduces Eq. (1) rewritten via the frequency ω up to the relativistic factor γ. This can be further elaborated upon by examining whether Eq. (1) is written for the rest or rotating frame (as discussed in the review by Post [4]). Nevertheless, this distinction is not crucial to our discussion, as $\Omega R \ll c$ by several orders of magnitude, rendering the corrections to the fringe shift arising from γ (which are of the order Ω^2R^2/c^2) practically indiscernible in experimental observations.

In the nonrelativistic limit, the phase fringe shift characterized by Eq. (3) emerges as the KGF equation reduces to the Schrödinger equation. This can be obtained by transforming away quickly oscillating rest energy dependence, $\psi = \chi\exp(-imc^2t/\hbar)$ which results in the Schrödinger equation in the rotating frame:

$$i\frac{\partial\chi}{\partial t} = \frac{1}{2m}\left(-i\hbar\nabla - m\mathbf{V}\right)^2\chi - \frac{1}{2}m\mathbf{V}^2\chi. \qquad (14)$$

Its solution for the circularly propagating waves [14] results in the phase fringe (3).

It follows from Eq. (14) that a profound analogy exists between Aharonov-Bohm (AB) oscillations in mesoscopic rings and the Sagnac effect in the nonrelativistic case, as described by Eq. (3). The rotation of a thin ring, and thus the Sagnac effect, can be associated with the AB effect occurring in a uniform (Larmor) magnetic field, $\mathbf{H} = 2m\boldsymbol{\Omega}/e$, or expressed in terms of the effective AB flux penetrating the ring as $\Phi_\Omega = 2mc\Omega A/e$ [14, 27] (for additional references and discussion see the reviews [5, 6]). Here $-e < 0$ is the electron charge.

One can also observe that Eq. (3) can be derived directly by substituting the nonrelativistic limit of the KGF dispersion relation, given by $\omega \approx mc^2/\hbar + \hbar k^2/(2m)$, into the solution (12). It is worthwhile to remind that the original consideration by de Broglie of the matter waves relied on the special relativity and contained the de Broglie frequency, $\omega = mc^2/\hbar$ associated with a resting particles. Although the Schrödinger theory does not contain explicitly such quantity, it appears when one calculates the phase difference between the two counter-propagating wave packages [28].

Since the de Broglie frequency is not a commonly used quantity, it is convenient to express the phase fringe for massive particles in the form (2). For example, one can reformulate Eq. (3) for electrons in the same manner as Eq. (2), but with the photon wavelength λ replaced by Compton wavelength, $\lambda_C = 2\pi\hbar/(m_ec) \approx 0.0243\,\text{Å}$, where m_e represents the electron mass. This representation simplifies the comparison between interferometers using light and matter, both having the same area and angular velocity. For example, when comparing blue light with a frequency ω_B and a wavelength of $\lambda_B = 450\,\text{nm}$ to electrons, one can estimate the enhancement factor as $m_ec^2/\hbar\omega_B = \lambda_B/\lambda_C \approx 1.85\times10^5$, in agreement with the aforementioned estimation.

The case of the Dirac materials, $v < c$. Now we may return to the consideration of a rotating ring made of the Dirac material. The Dirac electron subsystem in the rotating ring is described by Eq. (9) with a finite chemical potential introduced by the prescription given below Eq. (5). We neglect the effect of deformation of the ring due to rotation and consider one dimensional rigid ring of the radius R, so the wave function $\psi(t,\phi)$ is independent of the radial coordinated as before. Then the solutions of Eq. (9) for the two counter-propagating electron waves [cp. Eq. (12)] reads

$$\psi_{\pm}(t,\phi) = \exp\left[i\gamma\left(\pm k + \frac{\Omega(\epsilon+\mu)R}{\hbar cv}\right)R\phi - i\frac{\epsilon}{\hbar}\frac{t}{\gamma}\right] \qquad (15)$$

with the relativistic factor γ given by Eq. (13) and the energy $\epsilon = \epsilon(\mathbf{k})$ and wave number k obeying the dispersion law (4) for the Dirac quasiparticles. The phase difference between the two counter-propagating electron waves is therefore $\Theta_{\rm S} = 4\gamma(\epsilon + \mu)\Omega R^2/(\hbar c v)$.

Let's begin by examining the case where $\mu = \Delta = 0$, which corresponds to a quasiparticle exhibiting a linear dispersion relationship given by $\epsilon = \pm\hbar v k$. Expressing the wave vector k via the wavelength $\lambda_{\rm gr}$ by the relation $k = 2\pi/\lambda_{\rm gr}$, one again arrives at the expression (2) with λ replaced by $\lambda_{\rm gr}$. As was anticipated, the phase shift remains independent of the phase velocity v of the wave. As pointed out in Ref. 6, the Sagnac effect holds holds not only for quantum-mechanical particles (photons, electrons, etc.) but also for ordinary acoustic waves.

The physical meaning of Eq. (2) considered for light and material particles is further elucidated when one rewrites it as follows [5], $\Theta_{\rm S} = 4\pi(V/c)(P/\lambda)$ with $P = 2\pi R$. The fringe shift is proportional to the number of the corresponding wavelengths along the wave path. This interpretation of the phase shift $\Theta_{\rm S}$ is always valid, although the meaning of the wavelength λ has to be clarified for each case.

Let us proceed with the cases of a finite μ or/and Δ. If the temperature T is much smaller than $|\mu|$, only the quasiparticle excitations near the Fermi surface contribute to the transport. For the spectrum given by Eq. (4) the Fermi surface is determined by the condition $\epsilon(k_F) = 0$, where k_F represents the Fermi wave vector. Then neglecting the factor $\gamma \approx 1$, the Sagnac fringe shift for the Dirac fermions reads

$$\Theta_{\rm S} = \frac{4\mu A\Omega}{\hbar v c} = \frac{4\,{\rm sgn}\,(\mu) m_c A\Omega}{\hbar}\frac{v}{c}, \qquad m_c = \frac{|\mu|}{v^2}. \quad (16)$$

Here in the second equality $\Theta_{\rm S}$ is written in terms of a fictitious "relativistic" mass, m_c, which plays the role of the cyclotron mass in the Lifshits-Kosevich formula [29] and also allows to rewrite Eq. (16) in the form resembling the nonrelativitic expression (3). As was already mentioned, in graphene μ and thus m_c are easily tunable by the gate voltage. Interestingly, the value of $\Theta_{\rm S}$ turns out to be sensitive to the sign of μ or character of the carriers. At first glance, this seemingly paradoxical result is actually sensible, as it arises from the fact that the motion of hole carriers corresponds to the rotation of electrons in opposite directions.

The representation of $\Theta_{\rm S}$ in terms of m_c also turns out to be very convenient for an estimate of the phase fringe. Indeed, one finds in Ref. 29 that for the carrier density $n \approx 7 \times 10^{12}\,{\rm cm}^{-2}$ the mass $m_c \approx 0.06 m_e$. Also considering that in graphene, $c/v \approx 300$, one can estimate that $\Theta_{\rm S}^{\rm e}/\Theta_{\rm S}^{\rm gr} = c m_e/(v m_c) \approx 5 \times 10^3$. Alternatively, when comparing with blue light, $\Theta_{\rm S}^{\rm gr}/\Theta_{\rm S}^{B} \approx 37$.

This enhancement factor is not significant enough to make graphene attractive for applications. However, a more substantial value could be potentially achieved by further increasing the carrier density. In contrast, there is a 3D topological insulator, specifically Bi_2Te_3, exhibiting a low Fermi velocity [30]. While the Fermi energy in Bi_2Te_3 is approximately 10 times lower than in graphene, the Fermi velocity is $v \approx 3260\,{\rm m/s}$ which is over 10^4 times smaller than in graphene. This significant difference makes Bi_2Te_3 and similar materials attractive for electron interferometry.

Equation (16) is valid for both gapless and gapped cases. In the former, $\Delta = 0$ case, the Fermi wave vector, k_F, is determined by the relationship $\hbar v k_F = |\mu|$. Once again, it is evident that Eq.(16) can be reformulated in the manner of Eq.(2), utilizing the Fermi wavelength, $\lambda_F = 2\pi/k_F$. Using the relationship between the carrier imbalance n and chemical potential for graphene, $|n| = (\mu^2 - \Delta)/(\pi\hbar^2 v^2)$ (see e.g. Ref. [22]), one obtains for $\Delta = 0$ that $\lambda_F = 2\sqrt{\pi/|n|}$. Taking $n = 7 \times 10^{12}\,{\rm cm}^{-2}$ one gets $\lambda_F \approx 13.5\,{\rm nm}$. This corresponds to the enhancement factor $\lambda_B/\lambda_F \approx 33$ which is quite close to the previous estimate made in terms of m_c.

The opening of the gap Δ in graphene can be taken into account by expressing the chemical potential via the carrier imbalance, $\mu^2 = \Delta^2 + \pi\hbar^2 v^2 |n|$. This relation proved to be useful for analyzing the dependence $\mu(n)$ in G/hBN structures [31]. In particular, for $n \approx 0$ one obtains that the fringe shift $\Theta_{\rm S} = 4\,{\rm sgn}\,(\mu)\Delta A\Omega/(\hbar v c)$. To produce the same phase fringe as light with a photon energy $\hbar\omega$, the value of the gap is $\Delta = \hbar\omega(v/c)$. Accordingly, for the blue light with $\hbar\omega_B \approx 2.76\,{\rm eV}$ to reach the same sensitivity in graphene the gap has to be $\Delta \approx 9\,{\rm meV}$, while in the experiment [31] on G/hBN $\Delta \approx 3.4\,{\rm meV}$. However, the inclusion of the gap, along with a finite carrier density, and more notably, the reduction in the Fermi velocity, v, definitely enhances the sensitivity.

Conclusion. To conclude, we have obtained analytic expressions for the Sagnac fringe shift in the Dirac materials. The direction of the shift relies on the charge carriers' nature – whether they are electrons or holes. When considering graphene, the enhancement factor is not as substantial as that achievable in conventional semiconducting materials with finite effective carrier masses. Our analysis illustrates that the most significant enhancement factor values are attainable in materials characterized by a reduced Fermi velocity.

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