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DID EINSTEIN STUMBLE? THE DEBATE OVER GENERAL COVARIANCE

ABSTRACT. The objection that Einstein's principle of general covariance is not a relativity principle and has no physical content is reviewed. The principal escapes offered for Einstein's viewpoint are evaluated.

1. INTRODUCTION

. . . the general theory of relativity. The name is repellent. Relativity? I have never been able to understand what that word means in this connection. I used to think that this was my fault, some flaw in my intelligence, but it is now apparent that nobody ever understood it, probably not even Einstein himself. So let it go. What is before us is Einstein's theory of gravitation. (Synge 1966, p. 7)

The magnitude of Einstein's success with his theories of relativity brought its own peculiar problem. His success attracted legions of cranks to his work, all determined to show where Einstein had blundered and anxious to accuse him of the most fundamental of misconceptions. On first glance, you might well imagine that the sentiments quoted above were drawn from this tiresome crank literature. However you would be mistaken. These remarks were made by J. L. Synge, one of this century's most important and influential relativists. They reflect the growth of a tradition of criticism of Einstein's views on the foundations of general relativity. The tradition began with the theory's birth in the 1910s as a minority opinion. Over the decades following, it refused to die out, instead growing until it is now one of the major schools of thought, if not the majority view amongst relativists.

The deep reservations of this tradition do not apply to the theory itself. The general theory of relativity is nearly universally hailed as our best theory of space, time and gravitation and a magnificent intellectual achievement – although followers of Synge might prefer a different name for the theory. What is questioned is the account that Einstein gave of its fundamental postulates. His account has been criticized in many of its aspects. The one that has attracted the most criticism is the prominence he accorded the requirement of general covariance, which Einstein saw as the crowning achievement of his theory. Through it, Einstein proclaimed, the theory had extended the principle of relativity to accelerated motion. Einstein's critics responded that general covariance had nothing to do with a generalization of the principle of relativity. Worse, general covariance was physically vacuous, a purely mathematical property.

My purpose in this paper is to review some of the principal positions advanced in this debate.[1] I will pursue two themes: whether covariance

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principles have physical content and whether they express a relativity principle. First, in Sections 2 and 3, I will review the role Einstein claimed for covariance principles in the foundations of relativity theory and the ensuing objection, originating with Kretschmann in 1917, that the principle of general covariance is physically vacuous. Then, in Section 4, I will outline the stratagems that have been proposed to restore physical content to the principle. I will conclude that they succeed only in the degree to which they deviate from a simple reading of the original principle. In Section 5, I will review the development of the modern view that covariance principles are not relativity principle and that relativity principles express a symmetry of a spacetime. Finally, in Section 6, I will review Anderson's notion of absolute object. This notion provides our best attempt to reconcile Einstein's view of the connection between covariance and relativity principle and the modern view of relativity principles as symmetry principles.

2. COVARIANCE IN EINSTEIN'S ACCOUNT OF THE FOUNDATIONS OF RELATIVITY THEORY

For Einstein, covariance principles were the essence of his theories of relativity. For a theory to satisfy a principle of relativity, the equations expressing its laws needed to have a particular formal property. They needed to remain unchanged in form – covariant – under a group of coordinate transformations characterizing the principle of relativity at issue. This was the clear moral of his famous 1905 special relativity paper (Einstein 1905). The emphasis in that paper was to discover the correct form of the group of coordinate transformations associated with the relativity of inertial motion. These, he argued, were the Lorentz transformation equations. It then followed that Maxwell's electrodynamics satisfied the principle of relativity of inertial motion since the basic equations of Maxwell's theory remained unchanged in form under Lorentz transformation. Einstein (1940, p. 329) later summarized his approach:

> The content of the restricted relativity theory can accordingly be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations.

That the laws of a theory have the appropriate covariance is something that must be demonstrated by calculation, often by quite arduous manipulation. The mechanical exercise of establishing the Lorentz covariance of Maxwell's theory occupies a significant part (§§6, 9) of Einstein's 1905 paper.

Einstein's algebraic approach to the principle of relativity was quite different from that soon introduced by Minkowski (1908, 1909). He formulated the special theory of relativity in terms of the geometry of what we now know as a Minkowski spacetime. Satisfaction of the principle of relativity of inertial motion followed automatically provided one used only the natural geometric structures of the Minkowski spacetime to formulate one's theory. Sommerfeld's (1910, p. 749) capsule formulation of what it took to satisfy the principle of relativity was quite unlike Einstein's:

According to Minkowski, as is well known, one can formulate the content of the principle of relativity as: Only *spacetime vectors* may appear in physical equations . . . .

Instead of tedious calculation to verify preservation of form of equations under transformation, one could verify that a theory formulated in a Minkowski spacetime satisfied the principle of relativity by inspection.

In 1907, Einstein began the long series of investigations that would ultimately lead to his general theory of relativity. Einstein's goal was to construct a relativistically acceptable gravitation theory by extending the principle of relativity to acceleration.[2] Throughout the entire project Einstein's emphasis remained on covariance principles and the associated algebraic viewpoint. His first step was to unveil (Einstein 1907, Part V) what he hoped would be the key to the extension, the hypothesis of the complete physical equivalence of uniform acceleration in a gravitation free space and rest in a homogenous gravitational field. The significance of this hypothesis – soon to be called the "principle of equivalence" – lay in the fact that it allowed Einstein to include transformations to accelerating coordinate systems in the covariance group of his theory. That is, it took the first step in extending the covariance of his special theory.[3]

This is the gist of the principle of equivalence: In order to account for the equality of inert and gravitational mass within the theory it is necessary to admit non-linear transformations of the four coordinates. That is, the group of Lorentz transformations and hence the set of "permissible" coordinate systems has to be extended. Einstein (1950, p. 347)

The completion of the project lay in the further extension of the covariance of his gravitation theory.

Even though Minkowski's spacetime approach would provide the formal basis for the final theory, Einstein was very slow to adopt Minkowski's methods. He did not use Minkowski's spacetime methods in developing his static theory of gravitation in the 1911 and 1912 (Einstein 1911, 1912a, b, c). The spacetime approach entered his analysis only with the publication of Einstein and Grossmann (1913), the first sketch of the general theory of relativity. This paper was distinctive in using the absolute differential calculus of Ricci and Levi-Civita (1901), later known as the tensor calculus. Yet Einstein's emphasis remained on the covariance properties of the laws of his theory, rather than its intrinsic geometric properties. Here Einstein and Grossmann were actually following the approach of Ricci and Levi-Civita. The latter preferred to think of their methods as providing an abstract calculus for manipulating systems of variables; the natural application in the geometry of curved surfaces was just one of many applications and was not to be allowed to dominate the method.

With the writing of Einstein and Grossmann (1913), Einstein's quest for his general theory of relativity should have been completed, for he had virtually the entire theory in hand. Their theory differed only from the final theory in its gravitational field equations. For reasons that have been dissected extensively elsewhere (Norton 1984), Einstein and Grossmann considered generally covariant field equations but rejected them in favor of a set of equations of restricted covariance. This disastrous decision dominated the next three years of Einstein's work on gravitation as he struggled to reconcile himself with his misshapen theory. These efforts led Einstein to his ingenious "hole argument" which purported to show that generally covariant field equations would be physically uninteresting. To show this, Einstein assumed that the gravitational field equations were generally covariant and considered a matter free region of spacetime, the "hole". He then showed that general covariance allowed him to construct two solutions of the gravitational field equations g_{ik} and g'_{ik} in the *same* coordinate system such that g_{ik} and g'_{ik} agreed outside the hole but came smoothly to disagree within the hole. This, Einstein felt, was a violation of the requirement of determinism, for the fullest specification of both gravitational field and matter distribution outside the hole must fail to fix the field g_{ik} within the hole. This violation was deemed fatal by Einstein to generally covariant gravitational field equations.[4]

In November 1915, Einstein finally emerged victorious from his struggle. He had returned to the quest for generally covariant gravitational field equations and had found the equations now routinely associated with his theory. Early the following year he celebrated his achievement with the well-known review of his new theory, Einstein (1916). Its early sections laid out the motivation and physical basis of the new theory, culminating in the association of the generalized principle of relativity with the general covariance of the theory (§3):

> *The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally covariant).*
>
> It is clear that a physical theory which satisfies this postulate will also be suitable for the general postulate of relativity. For the sum of *all* substitutions in any case includes those which correspond to all relative motions of three-dimensional systems of co-ordinates. (Einstein's emphasis)

The passage continued to state what John Stachel has labelled the "point-coincidence argument".

> That this requirement of general co-variance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences

between the hands of a clock and points on the clock dial, and observed point-events happening at the same place and the same time.

The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences. We allot to the universe four space-time variables x_1, x_2, x_3, x_4, in such a way that for every point-event there is a corresponding system of values of the variables $x_1 \ldots x_4$. To two coincident point-events there corresponds one system of values of the variables $x_1 \ldots x_4$, i.e. coincidence is characterized by the identity of the co-ordinates. If, in the place of the variables $x_1 \ldots x_4$, we introduce functions of them, x'_1, x'_2, x'_3, x'_4, as a new system of co-ordinates, so that the system of values are made to correspond to one another without ambiguity, the equality of all four co-ordinates in the new system will also serve as an expression for the space-time coincidence of the two point-events. As all our physical experience can be ultimately reduced to such coincidences, there is no immediate reason for preferring certain systems of co-ordinates to others, that is to say, we arrive at the requirement of general co-variance.

The purpose of this argument at this point in Einstein's exposition had become completely obscured until rediscovered by Stachel (1980). The point coincidence argument was Einstein's answer to the hole argument – although Einstein only explained this in private correspondence. Readers of Einstein (1916) would have to figure this out for themselves and fill in the details alone. In brief, the argument's basic assumption was that the physical circumstance described by any field g_{ik} was exhausted by the catalog of spacetime coincidences that it allowed. It turned out that, by construction, the two fields g_{ik} and g'_{ik} of the hole argument agreed on all such coincidences. Therefore any difference between them could not be physical. It was a purely mathematical effect, one we would now describe as a gauge freedom. Thus the indeterminism of the hole argument provided no physical ground for rejecting generally covariant field equations. Finally, to forgo general covariance and the use of arbitrary coordinate systems is to restrict the theory in a way that goes beyond its physical content, for that content is exhausted by the catalog of spacetime coincidences, which cannot pick between different coordinate systems.

There was an ease in the writing of other parts of Einstein's accounts of the foundations of this theory that proved deceptive to readers who took Einstein's discussion to represent a register of uncontroversial postulates or consequences of the theory. In his Section 2, for example, Einstein had urged that the inertial properties of a body must be fixed completely by the other bodies of the universe. This was a result that Einstein had so far only been able to recover in weak field approximation. Therefore its inclusion in the discussion was more a statement of what Einstein hoped to recover from his theory than a report of what he had recovered. When he later tried to derive the full effect, he ran into very serious problems. He initially sought to contain these problems in his cosmological work by augmenting his field equations with the notorious "cosmological term". He later abandoned as wrongheaded these attempts to recover what had then become known as "Mach's Principle".

3. IS GENERAL COVARIANCE PHYSICALLY VACUOUS?

A more immediate shock awaited Einstein. Erich Kretschmann (1917) had read and understood all too clearly the fragility of Einstein's account of the foundations of his theory. He began his paper with the remarks (pp. 575–576).[5]

The forms in which different authors have expressed the postulate of the Lorentz–Einstein theory of relativity – and especially the forms in which Einstein has recently expressed his postulate of general relativity – admit the following interpretation (in the case of Einstein, it is required explicitly): A system of physical laws satisfies a relativity postulate if the equations by means of which it is represented are covariant with respect to the group of spatio-temporal coordinate transformations associated with that postulate. If one accepts this interpretation and recalls that, in the final analysis, all physical observations consist in the determination of purely topological relations ("coincidences") between objects of spatio-temporal perception, from which it follows that no coordinate system is privileged by these observations, then one is forced to the following conclusion: By means of a purely mathematical reformulation of the equations representing the theory, and with, at most, mathematical complications connected with that reformulation, any physical theory can be brought into agreement with any, arbitrary relativity postulate, even the most general one, and this without modifying any of its content that can be tested by observation.

Einstein had used the point-coincidence argument to establish the requirement of general covariance. If coincidences are all that matters physically, then we ought to be able to use any coordinate system, since all coordinate systems will agree on spacetime coincidences. Kretschmann now objects that this argument works too well. If we accept the point-coincidence argument, then we ought to be able to use arbitrary coordinates in any theory, for the physical content of any theory ought to remain unchanged with the adoption of new coordinate systems. The challenge is merely a mathematical one: find a generally covariant formulation of the theory. Kretschmann later remarked (p. 579) that this task ought to be perfectly manageable for any physical theory given the power of such methods as Ricci and Levi-Civita's.

Kretschmann's original objection was conditioned on acceptance of the point-coincidence argument. That condition was soon dropped when his remarks were cited. Kretschmann's objection is now routinely recalled as the observation that general covariance is a purely mathematical property of the formulation of a theory. Any spacetime theory can be given generally covariant formulation; general covariance has no physical content. In this form, Kretschmann's objection has become one of the most cited and endorsed objections to Einstein's account of the foundations of the general theory of relativity.

4. GENERAL COVARIANCE HAS PHYSICAL CONTENT IF YOU . . .

If the caution and awkwardness of Einstein's (1918) reply is any guide Einstein must have been quite seriously troubled by Kretschmann's assault. He began by carefully stating the three principles upon which his theory was based: the (a) principle of relativity, (b) the principle of equivalence and (c) Mach's principle. That list already contained concessions to Kretschmann, for, in a footnote, Einstein confessed that he had not previously distinguished (a) and (c) and that this had caused confusion. Further, his statement of the principle of relativity had been reduced to its most circumspect core, far removed from the vivid thought experiments usually surrounding the principle:

(a) *Principle of Relativity*: The laws of nature are only assertions of timespace coincidences; therefore they find their unique, natural expression in generally covariant equations.

The principle was now merely a synopsis of the point-coincidence argument itself.

However this cautious reorganization was an exercise in relabelling. It had still not escaped Kretschmann's objection that general covariance is physically vacuous. Einstein's options were extremely limited, for Kretschmann had come to his conclusion by using Einstein's own point-coincidence argument and that was an argument Einstein was unable to renounce. Einstein's response was the first of a series which conceded that general covariance *simpliciter* is physically vacuous after all. In this tradition it is urged that general covariance has physical content if it is supplemented by a further requirement. The problem is to decide what that further requirement should be. Einstein's choice is best known.

4.1. *Add the Requirement of Simplicity*

Einstein wrote

I believe Herr Kretschmann's argument to be correct, but the innovation proposed by him not to be commendable. That is, if it is correct that one can bring any empirical law into generally covariant form, then principle (a) still possesses a significant heuristic force, which has already proved itself brilliantly in the problem of gravitation and rests on the following. Of two theoretical systems compatible with experience, the one is to be preferred that is the simpler and more transparent from the standpoint of the absolute differential calculus. Let one bring Newtonian gravitational mechanics into the form of absolutely covariant equations (four-dimensional) and one will certainly be convinced that principle (a) excludes this theory, not theoretically, but practically!

That is, Einstein responds that the requirement of general covariance has physical content if we augment it with the additional constraint that generally covariant formulations of theories must be simple.

Einstein chose to illustrate his point by challenging readers to seek a generally covariant formulation of Newtonian theory, which Einstein supposed would be unworkable in practice. The choice proved a poor one, for it was discovered shortly by Cartan (1923) and Friedrichs (1927) that it was quite easy to give Newtonian theory an entirely workable generally covariant formulation using essentially the same techniques as Einstein. Nonetheless Einstein's escape became one of the most popular in later literature. Misner, Thorne and Wheeler (1973, pp. 302–303), for example, complete their discussion of generally covariant formulations of Newtonian theory with a recapitulation of Einstein's escape in characteristically colorful language:

> But another viewpoint is cogent. It constructs a powerful sieve in the form of a slightly altered and slightly more nebulous principle: "Nature likes theories that are simple when stated in coordinate-free, geometric language" . . . . According to this principle, Nature must love general relativity, and it must hate Newtonian theory. Of all theories ever conceived by physicists, general relativity has the simplest, most elegant geometric foundations . . . . By contrast, what diabolically clever physicist would ever foist on man a theory with such a complicated geometric foundation as Newtonian theory?

Einstein's escape works in a straightforward but limited sense. The requirement of simplicity in generally covariant formulation induces an ordering on empirically equivalent theories. The physical content arises in the assumption that the simpler theory in the ranking is more likely to be true. However this success is limited by two qualifications. First, we have no objective scheme for comparing the simplicity of two formulations.[6] In practice in individual cases, judgments of relative simplicity can be made with wide agreement. However that judgment rests on the intuitive sensibilities of the people evaluating the formulations and not on explicitly stated rules. This is hardly a comfortable basis for underwriting the physical content of a fundamental physical principle. Second, the requirement of simplicity can only direct us if we have empirically equivalent theories. In this regard, Einstein and Misner, Thorne and Wheeler's comparison of general relativity and Newtonian theory is somewhat misleading and inflates the significance of the requirement. The decision between these two theories is not based on the simplicity of the generally covariant formulations. Had the celebrated tests of general relativity failed and all experiments favored Newtonian theory, could we justify our current enthusiastic support for general relativity no matter how simple it may be?

There is a deeper worry. It is possible to give an entirely innocent explanation of the empirical success of the requirement of simplicity of generally covariant formulations. We know from other grounds that general relativity is our preferred theory of space, time and gravitation. Since it happens to have an especially simple generally covariant formulation, we naturally prefer theories with such simple formulation just as a rather cumbersome way of expressing our preference for general relativity. Our preference for them, then, is merely an accidental by- product of our preference for general relativity. This innocent views seems not to be that of Einstein. He sought to elevate the requirement of simplicity to fundamental metaphysics.[7] Elsewhere he made the celebrated proclamation (Einstein 1933):

Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and laws connecting them with each other, which furnish the key to the understanding of natural phenomena. . . . the creative principle resides in mathematics.

Thus Einstein's view reverses the innocent explanation of the success of a simple generally covariant theory. The success of general relativity derives from the fact of its simple generally covariant formulation.

The difficulty with Einstein's proposal is that there is scant evidence to justify the move from the innocent explanation to Einstein's deeper metaphysics. Our experience does not justify what Einstein claims, that is, that the canon of mathematical simplicity provides the decisive heuristic guide in our search for physical theories. Our experience is that major changes in physical theory require new mathematical languages and only in that new mathematical language does the theory appear simple. In the earlier languages, the theory may be extraordinarily complicated or even inexpressible. An obvious example is Einstein's own general theory of relativity, which found simple expression only by reviving a mathematical method that had lain stagnant for over a decade. A second example is the advent of the modern quantum theory in the 1920s. The theory finally found its simplest general mathematical expression in the mathematics of operators on Hilbert spaces. Yet this new mathematics emerged quite painfully as a synthesis of many explorations: the matrix methods of Born, Heisenberg and Jordan, the wave mechanics of Schrödinger, Dirac's *c* and *q* numbers as well as major contributions from group theory.

The moral of experience is that our best theories find simple mathematical expression because of the special efforts of mathematicians and physicists to find simple mathematical expression of our best theories. There is no single, natural mathematical language in which to judge the simplicity of theories. Throughout the history of science, physicists have drawn on many different mathematical tools. Sometimes it is the then standard method; sometimes it is a revival of one that has languished; and sometimes it is one that is developed hand-in-hand with the theory or even after the new theory is well established. Because of the wide range of choice of mathematical method, any successful theory choice can be cast as the choice of the mathematically simplest. In so far as mathematical simplicity has heuristic value, it is entirely dependent on the choice of the right mathematical tool. How to make that choice is only apparent *after* the success of the theory. As a heuristic guide, mathematical simplicity is entirely dependent on the fortuitous choice of the right mathematical language, that is, on being lucky by guessing correctly.

Nature is not tuned into our mathematics. Our mathematics is adjusted painstakingly to fit nature as out understanding of nature deepens. Mathematics hardly seems to provide us with a fixed and elevated vantage point from which to direct the development of new physical theories. The vantage point mathematics provides is as mutable as our physics.

4.2. *Restrict the Addition of New Structures*

The difficulty with Einstein's escape from Kretschmann's objection is that it leads us towards a problematic metaphysics of simplicity. A more empirically motivated escape originated with several authors. Kretschmann had urged that one can take any spacetime theory and find generally covariant formulation for it. Fock (1959, p. xvi) considers this transition. He points out that one can always find a generally covariant formulation of a theory if one is allowed to introduce new auxiliary quantities arbitrarily. This easy achievement of a generally covariant formulation can be blocked if we insist that any new quantity must have proper physical basis and not be a purely mathematical artifice. Trautman (1964, pp. 122–123) and Wald (1984, p. 57) use the same example to illustrate this escape. If A_a is a covector field, then the equation that merely requires the vanishing of its first component

$$A_1 = 0$$

is clearly not generally covariant. We could, however, render this equation generally covariant by explicitly introducing the coordinate basis vector field u^a associated with the x^1 coordinate, rewriting the equation as

$$u^a A_a = 0$$

This transition is blocked by the requirement that any new quantity introduced must reflect some element of physical reality according to the theory. A coordinate basis vector field, however, just reflects some arbitrary choice of coordinate system.

Pauli (1921, p. 150) gives a more realistic example of this escape. In his transition to general relativity, Einstein took a Lorentz covariant formulation of special relativity and expanded the coordinate systems it used to include those associated with uniformly accelerated motion. In this expansion, new quantities appear, the coefficients g_{ik} of the metric tensor. What makes this transition acceptable is that these coefficients do have a physical meaning. Einstein's principle of equivalence enjoins us to interpret these coefficients as describing a gravitational field.

There is a difficulty with this proposal. The problem is that it is not so much a well articulated principle as a rule of thumb with a few suggestive examples. In particular, just how are we to distinguish between new quantities that properly reflect some element of reality and those that are merely mathematical artifices? Pauli and Weyl (1921, pp. 226–227) stress that the coefficients of the metric are distinguished by the fact that they are not given a priori. They are influenced and even determined by the metric field. But this cannot be the only criterion for identifying elements of reality.[8] If we give just special relativity a generally covariant formulation, then the Minkowski metric of spacetime is represented in arbitrary coordinate systems by components g_{ik}. What principle could deny that the Minkowski metric is properly an element of reality? But if we allow this, then we must also allow generally covariant formulation of special relativity. Similarly, we can give Newtonian spacetime theory generally covariant formulation if we are allowed to treat explicitly several familiar geometric structures in spacetime: the degenerate spacetime metric h^{ab}, the absolute time one form t_a and the affine structure ∇_a. Once again each of these structures seems to represent some proper element of physical reality. But if the augmented requirement of general covariance succeeds only in ruling out contrived examples but cannot rule out generally covariant formulations of both Newtonian theory and special relativity, then it is hard to see what interesting physical content lies in the principle.[9]

4.3. *Require That There is Also No Formulation of Restricted Covariance*

The last escape attempted to make more precise the intuition that there is something unnatural in forcing generally covariant formulations onto theories that are more familiar in formulations of restricted covariance. Bergmann sought to give content to the principle of general covariance by building this intuition directly into its definition. The principle becomes the injunction to prefer spacetime theories that cannot be given formulations of restricted covariance. He wrote (1942, p. 159)[10]

> The hypothesis that the geometry of physical space is represented best by a formalism which is covariant with respect to general coordinate transformations, and that a restriction to a less general group of transformations would not simplify that formalism, is called *the principle of general covariance*.

Thus neither special relativity nor standard Newtonian theory is admissible in generally covariant formulation. Each also admits formulations of restricted covariance, the former is Lorentz covariant, the latter Galilean covariant.

This proposal comes closer to achieving the desired criterion for deciding between theories. However the division is not perfect. First, it is not clear that general relativity is irreducibly generally covariant. As Bondi (1959, p. 108) had pointed out, Fock (1959) has been prominent in calling for a restriction to the covariance of general relativity by augmenting it with the harmonic coordinate condition. Second, the theories that this proposal will select as generally covariant seem unrelated to those satisfying a general principle of relativity. A simplified formulation of reduced covariance is available for a theory if that theory's spacetime structures admit symmetry transformations. Since the Lorentz boost is a symmetry of a Minkowski spacetime, special relativity can be written in simplified, Lorentz covariant formulation. Thus Bergmann's proposal directs us to associate a *smaller* covariance group with a theory the *larger* the group of its spacetime symmetries. We shall see below that symmetry groups of a spacetime theory are those that are now associated with the relativity principle a theory may satisfy. Therefore, under Bergmann's proposal, the larger the group associated with a theory's relativity principle the smaller the theory's covariance group. That is, the more relative, the less covariant. For example, consider some spacetime theory that posits a completely inhomogeneous but otherwise fixed background spacetime structure. Such a theory satisfies no relativity principle at all. All events, let alone states of motion, are distinct. Yet it will be generally covariant. Special relativity and versions of Newtonian theory are more relativistic in the sense that they at least satisfy a principle of relativity of inertial motion. Yet they are less covariant.

4.4. *Contrive to Add the Result You Want*

In general, I think all these escapes contrived. What has failed is Einstein's original vision. The principle of general covariance was to have formed the core of general relativity in the way that the requirement of Lorentz covariance was central to special relativity. This hope has been dashed. The escapes discussed here all appear to be attempts to force an outcome that the unsupplemented principle could not deliver. Clearly, by sufficiently ingenious supplements, one can force the revised principle to deliver any result we choose. However we delude ourselves if we think that general covariance delivered the result. It comes from the supplement. In some cases, the supplements carry a burden extraordinarily specific to general relativity, so that it becomes hardly surprising the principle points directly to Einstein's theory. For example, Weinberg's (1972, pp. 91–92) principle of general covariance has two parts, the second only resembling Einstein's original principle. His principle

> ... states that a physical equation holds in a general gravitational field, if two conditions are met:
>
> 1. The equation holds in the absence of gravitation; that is, it agrees with the laws of special relativity when the metric field $g_{\alpha\beta}$ equals the Minkowski tensor $\eta\alpha\beta$ and when the affine connection $\Gamma^{\alpha}_{\beta\gamma}$ vanishes.
> 2. The equation is generally covariant; that is, it preserves its form under a general coordinate transformation $x \rightarrow x'$.

One might well wonder if the second condition is needed at all.

5. ARE COVARIANCE PRINCIPLES RELATIVITY PRINCIPLES?

Whatever may the outcome of the attempts to restore physical content to the principle of general covariance, a second problem remains. Is the principle a relativity principle? A long tradition of criticism maintains that covariance principles are not relativity principles. Its origins lay in a problem apparent from the earliest moments of Einstein's general theory. On the level of simple observation, there was a significant gap between the relativity principles of the special and general theory. Under the relativity of inertial motion in the special theory, an observer in a closed chamber such as a railway car could do no experiment within the car to determine whether the car was in uniform motion or at rest. Were the car to accelerate, however, that motion would be entirely apparent to the occupants of the car through inertial effects. They would hardly need to carry out delicate experiments to detect even quite modest acceleration. Yet according to Einstein an extended principle of relativity was supposed to cover both uniform and accelerated motion of the car. It was supposedly equally admissible to imagine the accelerated car still at rest but temporarily under the influence of a gravitational field.[11] Whatever may be the covariance of special and general relativity, their relativity principles seemed to be quite different and that of general relativity quite suspect.

In the 1920s it was possible to dismiss this type of skepticism about the generalized principle of relativity as shallow or even willfully obstructive – especially given its association with the politically motivated anti-relativity movement. But it became harder to dismiss the tradition of criticism that sought to give deeper expression to this worry. Kretschmann (1917) was again one of the earliest voices of this tradition of criticism. The major part of his paper had been devoted to understanding what were the relativity principles of special and general relativity. His analysis was based on a geometric characterization of a relativity principle. The relativity principle of a spacetime theory of the type of relativity was fixed by the group of transformations that mapped the lightlike and timelike worldlines of spacetime back into themselves. In the case of special relativity, this criterion gave the expected answer: the group identified was the Lorentz group. Since motions connected by a Lorentz transformation were governed by a relativity principle, the analysis returned the relativity of inertial motion as advocated by Einstein. Kretschmann's analysis gave quite different results in the case of general relativity. Lightlike and timelike worldlines were, in general, mapped back into themselves by the identity map only. It followed that general relativity satisfied no relativity principle at all. It is a fully absolute theory.

Where Kretschmann's analysis was geometric in spirit, another approach was more algebraic and closer to Einstein's own methods. The basic worry was that the form invariance of laws required by general covariance was too weak to express a relativity principle. Sesmat (1937, pp. 382–383) for example pointed out that the Lorentz transformation stood in a quite different relation to the special theory than did general transformations to the general theory. Lorentz transformations left unchanged the basic quantities describing spacetime such as the components of the metric tensor. They remained the same functions of the coordinates.

The equations of general relativity, however, could only be preserved in form under general transformations since the components of the metric tensor were adjusted by the tensor transformation law. Under arbitrary coordinate transformation, the functional dependence of these components on the coordinates would change.

Fock (1957) synthesized both geometric and algebraic approaches. A relativity principle expressed the geometrical notion of a uniformity of spacetime, such as the lack of privileged points, directions and states of motion. In a spacetime theory of the type of special and general relativity, the group associated with such uniformity is not the group under which the metric tensor transforms tensorially. It is the group under which the components of the metric tensor remain the *same functions* of the coordinates. That is, if a transformation takes coordinates x_σ to x'_σ and the components of the metric tensor $g_{\mu\nu}$ to $g'_{\mu\nu}$, then the transformation is associated with a relativity principle if it satisfies

$$g'_{\mu\nu}(x'_\sigma) = g_{\mu\nu}(x_\sigma)$$

where the equality is read as holding for equal numerical values of x_σ and x'_σ. This is the algebraic condition that expresses the uniformity of a spacetime. It is satisfied by the Lorentz transformation in special relativity but not by the general transformations of general relativity.

Fock's analysis is essentially the one now standard in both philosophical and physical literatures, although it is now expressed in the modern intrinsic geometrical language.[12] In the case of a spacetime theory which employs a semi-Riemannian spacetime, we represent a spacetime by the pair $\langle M, g_{ab}\rangle$ where M is a four-dimensional manifold and g_{ab} a Lorentz signature metric. Fock's condition picks out a group of transformations that correspond with the symmetry group of $\langle M, g_{ab}\rangle$.[13] That symmetry group is the group of all diffeomorphisms $\{h\}$ which map the metric tensor back onto itself according to

$$g_{ab} = h^* g_{ab}$$

One quickly sees that this group is the one that is naturally associated with the relativity principle. Consider, for example, two frames of reference, each represented by a congruence of timelike worldlines, in some spacetime $\langle M, g_{ab}\rangle$. If the two frames map into each other under members of the symmetry group, then any relation between the frame and background spacetime will be preserved under the map. Thus any experiment done in each frame concerning the motion of the frame in spacetime must yield the same result. Informally, spacetime looks exactly the same from each frame. If the two frames of reference are associated with railroad cars in inertial motion in a Minkowski spacetime, then any experiment concerning the motion of one car must yield an identical result if carried out in the other. But if the frames of the two cars are not related by a symmetry transformation, then the experiments will yield different results.

This is the case in which one moves inertially and the other accelerates. The experiments reveal inertial effects which distinguish the frames.

What it is for a theory to satisfy a relativity principle is understood most simply by the geometric approach to spacetime theories initiated by Minkowski, rather than the algebraic approach favored by Einstein in which one concentrates on formal properties of the equations defining the theory. If the spacetime admits a symmetry group then the theory satisfies a relativity principle for the transformations of that group.

6. ABSOLUTE AND DYNAMICAL OBJECTS

The association of relativity principles with the symmetry group of a spacetime theory seems to leave little hope for recovering a generalized principle of relativity within Einstein's general theory whatever may be its covariance. However an avenue for doing this remains. The spacetimes of special relativity are the Minkowski spacetimes $\langle M, \eta_{ab}\rangle$ where η_{ab} is a Minkowski metric. If there are matter fields present in spacetime, their presence is encoded by adding further members to this tuple. For simplicity, we can represent these further fields by their stress-energy tensor T_{ab} A theory with models $\langle M, \eta_{ab}, T_{ab}\rangle$ satisfies a relativity principle associated with the Lorentz group exactly because a Lorentz transformation is a symmetry of the pair $\langle M, \eta_{ab}\rangle$ This is the crucial point. The Lorentz transformation need not be a symmetry of the matter fields represented by T_{ab}. More generally, we divide the model $\langle M, \eta_{ab}, T_{ab}\rangle$ into two parts: $\langle M, \eta_{ab}\rangle$ which represents the background spacetime and T_{ab} which represents the matter contained in the spacetime. Figuratively, we might write this as

$$\langle M, \eta_{ab}|T_{ab}\rangle$$

where "|" represents the cut between the spacetime container and the matter contained. The group associated with the relativity principle of the theory is the symmetry group of everything to the left of this cut |.

In general relativity, the corresponding models are $\langle M, g_{ab}, T_{ab}\rangle$ It is natural to place the cut as $\langle M, g_{ab}|T_{ab}\rangle$. But that is disastrous for Einstein's approach, for the symmetry group of $\langle M, g_{ab}\rangle$ in general relativity is, in general, the identity group. The situation would be quite different if a reason could be found for relocating the cut as

$$\langle M|g_{ab}, T_{ab}\rangle$$

Then the background spacetime would merely be the bare manifold itself. If we set aside global topological issues and consider just R^4 neighborhoods of M, then the symmetry transformations are just the of C^∞ diffeomorphisms, in effect, the arbitrary transformations Einstein associated with general covariance. If the cut were moved in this way for general relativity but not special relativity, then the modern association of symmetry groups with relativity principles would pick out the Lorentz group in special relativity and the general group in general relativity.

One might seek to justify the different placing of the cut by observing that the metric tensor g_{ab} of general relativity now represents the gravitational field. Therefore it carries mass-energy and deserves to be considered as belonging on the right side of the cut as a matter field within spacetime. As it turns out there is a related way of justifying the moving of the this cut that draws directly on Einstein's own pronouncements concerning the fundamental difference between special and general relativity. Einstein insisted that a major achievement of the transition from special to general relativity was the elimination of a causal defect in the structure of special relativity. As he explained in his text *Meaning of Relativity* (1922, pp. 55–56)

> . . . from the standpoint of the special theory of relativity we must say, *continuum spatii et temporis est absolutum.* In this latter statement *absolutum* means not only "physically real," but also "independent in its physical properties, having a physical effect, but not itself influenced by physical conditions."

and these absolutes are objectionable since

> . . . it is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum) which acts itself, but which cannot be acted upon.

This theme of a causal defect in earlier theories is a stable feature of Einstein's accounts of general relativity, from his earliest[14] to his latest.[15] The theme was entangled with Einstein's fascination with what became Mach's principles. Yet it survived in his writings after Einstein had abandoned this principle.

Einstein's notion of the causal defect of earlier theories finds clear expression through the work of Anderson. Anderson (1967, pp. 83–87) divides the geometric object fields of a spacetime theory into the absolute $A_1, A_2, \ldots$ and dynamical $D_1, D_2, \ldots$ so that we might write the models as $\langle M, A_1, A_2, \ldots, D_1, D_2, \ldots\rangle$ The absolute objects are, loosely speaking, those that affect other objects but are not in turn affected by other objects. In special relativity, the Minkowski metric would be an absolute object since it determines the inertial trajectories of matter fields, without itself being affected by the matter fields. In general relativity, the spacetime metric is dynamical. It fixes the inertial trajectories of matter fields and at the same time its disposition is affected by the mass-energy of the matter field through the gravitational field equations. Because the manifold M together with the absolute objects $A_1, A_2, \ldots$ form an immutable arena, it seems natural to place the cut between spacetime container and matter contained between the absolute objects and the dynamical objects $D_1, D_2, \ldots$

$$\langle M, A_1, A_2, \ldots | D_1, D_2, \ldots\rangle$$

We now have a principled reason for placing the Minkowski metric η_{ab}

of special relativity to the left of the cut and the spacetime metric of general relativity to its right. Anderson's "principle of general invariance" identifies the symmetry group of a spacetime theory with the symmetry group of its absolute structure. This is the group associated with its relativity principle. If the theory has no absolute objects, the symmetry group is the group of symmetries of the manifold itself. Under this principle, the symmetry group of special relativity is the Lorentz group; the symmetry group of general relativity is the general group.

While this analysis offers the most promising explication of Einstein's claims concerning relativity principles, several problems remain. The first is a technical problem. Absolute objects are introduced informally as those objects which act but are not acted upon. Anderson gives a formal definition in which the absolute objects are picked out as those which are the same in all the models of the theory. Friedman (1973) identified the sense of sameness as diffeomorphic equivalence.[16] The definition is too broad for, as pointed out by Geroch (in Friedman 1983, p. 59) all non-vanishing vector fields are diffeomorphically equivalent. Therefore any non-vanishing velocity field in a spacetime theory will be deemed an absolute object. Conversely, as Torretti (1984, p. 285) has pointed out, the definition is too narrow. One can conceive spacetime theories with absolute background structures that fail to be picked out by the criterion of diffeomorphic sameness in all models. For example, one might consider a theory which posits that the background space has some fixed curvature, however the theory does not know what that curvature might be. Its value is located within some range, circumscribed, perhaps, by the reach of observational test. Such a theory would admit models with the relevant curvature drawn anywhere from this range. However the different curvatures of the different models would not be a dynamical response to the amount of matter present in spacetime, as it is in standard relativistic cosmologies. Rather it would merely reflect our ignorance of the correct value of the curvature. The true value of this curvature, whatever it may be, would not vary with a change in the matter content of the spacetime. The background structure would be absolute in the intuitive sense that it acts without being acted upon. However it would not be picked out as absolute by the definition because it fails to be diffeomorphically the same in all the models. Curvature is an invariant property, so structures of different curvature cannot be mapped diffeomorphically onto oneanother. It would seem that the criterion of diffeomorphic sameness in all models falls well short of the intuitive notion of things that act without being acted upon.

A deeper problem is that there remains good reasons for leaving the spacetime metric of general relativity on the left side of the cut. The background spacetime has traditionally been that structure that fixes lengths and times of processes as well as inertial motions. That this structure now responds dynamically to matter does not deprive it of these quintessentially spatio-temporal properties that mark it as belonging to the spacetime container. More seriously, prior to the advent of general relativity and with Einstein's urging, the principle of relativity was understood as expressing an experimental result about the impossibility of distinguishing certain states of motion through space. If we allow that the bare manifold M is the spacetime background, then the principle of relativity ceases to have any direct meaning in terms of experiment. The equivalence of all frames of reference is merely captured in the assertion of the equivalence of suitable congruences of curves with respect to the manifold. There is no direct experimental translation of this equivalence that is akin to special relativity's prediction of failure of all the 19th century aether drift experiments. The principle of relativity of the special theory links the theory directly to its empirical base in experiment. The corresponding principle in general relativity, as it arises in Anderson's analysis, plays no such role. Given all these disanalogies, it is hard to see what in Einstein's general theory ought to be labelled a "generalized principle of relativity", especially if we are interested maintaining some continuity of meaning for the term "principle of relativity".[17]

Finally, Einstein's discussion of the absolutes that act but are not acted upon contains a mysterious element. It was not just that the transition from special to general relativity happened to be accompanied by the elimination of the absolutes. Einstein depicted them as intrinsically defective and demanded their elimination. Anderson (1967, p. 339) expresses this requirement quasi-formally as a "generalized law of action and reaction". The difficulty is to see what compels us to this law. At best one can see loose analogies, perhaps to Newton's third law of motion. However the case of Newton's third law is significantly different. Those who deny it find themselves violating the law of conservation of momentum with all its attendant difficulties. There seems to be no analogous compulsion in the case of Anderson's generalized principle. Newton's mechanics violates the principle without precipitating any obvious problems. Here I agree with Schlick (1920, p. 40) who observed that "Newton's dynamics is quite in order as regards the principle of causality". The problem with Newton's mechanics is that it *happens* to be false. Let us not try to erect a dubious metaphysics merely to convince ourselves that it *has* to be false.[18]

7. CONCLUSION

This modern analysis offers an all too easy diagnosis of Einstein's error concerning relativity and covariance principles. In the Lorentz covariant formulation of special relativity, groups associated with covariance and relativity principles happen to coincide. With the transition to general relativity, the covariance group grew to the general group. What Einstein missed was that the group associated with the principle of relativity did not grow with it. It shrank to the identity group. Had Einstein pursued the geometrical approach of Minkowski rather than his own algebraic approach, he would have been far less likely to confuse covariance and relativity.

This analysis does provide, in my view, a perfectly satisfactory answer to the philosophical question of whether general relativity generalized the principle of relativity of the special theory. As an historical account of Einstein's work, however, it supplies at best a grossly oversimplified caricature. This is already suggested by Anderson's discussion of absolute and dynamical objects. His discussion provides the best modern explication of Einstein's account of the foundations of general relativity and shows how his ideas can be given more precise form. With care, as Stachel (1986, §§5, 6) has shown, Anderson's account can be extended to give precise meaning to Einstein's pronouncement that spacetime cannot exist without the gravitational field.

However another puzzle remains. All this work is focused on taking what Einstein actually said and translating it into a form in which Einstein's original statements are barely discernible. If Einstein's viewpoint was sound, why does it need such dramatic transformation in order for us to see its soundness? I believe there is a better approach that solves this essentially historical puzzle. Einstein's own pronouncements are incoherent to us when read literally only if we fail to take into account the enormous developments in mathematical techniques since the time Einstein wrote. If we account for these changes properly, we find that Einstein can be read literally. His pronouncements on general covariance turn out to be directed at solving a problem peculiar to his simpler mathematical apparatus. This problem remains opaque to us in the modern context since, in part due to Einstein's own efforts, it has been solved automatically and almost completely by the newer mathematical methods. This story is told in Norton (1989, 1992a).[19]

NOTES

[1] See Norton (1993a) for a more detailed survey of the many variant positions that emerged during this debate and how they developed over time as the debate unfolded.

[2] Einstein had decided that no Lorentz covariant gravitation theory could do justice to gravitation. As it turned out, his decision was too hasty. See Norton (1992, 1993b) for Einstein's reasons and the discovery of his error.

[3] Thus Einstein's version of the principle of equivalence is quite distinct in both statement and purpose from the later infinitesimal version of the principle of equivalence which is now universally but incorrectly attributed to Einstein. Einstein's version of the principle did not allow an arbitrary gravitational field to be transformed away infinitesimally. For discussion, see Norton (1985, 1993a, §4.1).

[4] For further discussion, see Norton (1984, §5; 1987) and Howard and Norton (1993).

[5] Kretschmann's footnotes to related literature have been suppressed. The selection of literature cited suggests obliquely that Kretschmann's own earlier work may have been an unacknowledged source for Einstein's point-coincidence argument. See Howard and Norton (1993, §7).

[6] Do we reduce the evaluation to a count of equations and quantities employed? How do we handle alternative formulations of the same theory? How do we count quantities? Is the metric tensor g_{ab} of general relativity one quantity or ten? Is its unique compatible derivative operator ∇_a not to be counted since ∇_a is a derivative quantity fixed once g_{ab} is fixed. Or are we to count g_{ab} and ∇_a as two quantities and add their compatibility relation $\nabla_a g_{bc} = 0$ to the count of equations?

[7] Do Misner, Thorne and Wheeler intend the same when they say "Nature likes . . ."?

[8] Pauli and Weyl's criterion anticipates the later distinction of absolute and dynamical objects. I will remark below on the difficulty of finding a formal characterization of this distinction.

[9] We may even wonder about its success with contrived examples. In the example $A_1 = 0$, the fact of its lack of general covariance means that the equation picks out preferred coordinate systems. Therefore, implicit in the equation is the selection of the relevant coordinate basis vector field u^a. So if $A_1 = 0$ is offered as a physical law, then the basis vector u^a reflects an element of the physical reality depicted by the law and its explicit appearance cannot be ruled out.

[10] I pass over the vagueness of "simplify" in Bergmann's definition. Does it mean "reduce the number of mathematical structures present"?

[11] For this objection, see, for example, Lenard (1921, p. 15) and Einstein (1918a) for the reply.

[12] See for example Earman (1974), Friedman (1973, 1983, Chap. IV), Jones (1981) and Wald (1984, pp. 58, 60, 438).

[13] The correspondence is through the connection between passive coordinate transformations and active boosts. For further discussion, see Norton (1989).

[14] See for example Einstein (1913, pp. 1260–1261). Notice that here as in other places Einstein specifically identifies the inertial system of special relativity as causally objectionable.

[15] See for example Einstein's (1954, pp. 139–140) appendix to Einstein (1922).

[16] Definitions of absolute objects akin to Anderson's but with slight variations are given by Friedman (1973, 1983, pp. 58–60) and Earman (1974, p. 282).

[17] Friedman (1983, Chap. III) has pointed out that we can come close in Newtonian spacetime theory in the following sense. Consider those versions of the theory which combine the gravitational field Φ and the flat affine structure ${}^0\nabla_a$ into a single curved affine structure ∇_a. It is possible to decompose this curved ∇_a into many distinct pairs of affine structure ${}^\Phi\nabla_a$ and associated field Φ, all of which are empirically equivalent. Since the various ${}^\Phi\nabla_a$ designate different motions as inertial, one arrives at the kind of extension of the relativity principle that Einstein associated with the principle of equivalence.

[18] I mention briefly the problem of the vagueness of the notion of "acting" in the context of these absolutes. Do universal constants such as Planck's constant h and the gravitational constant G act without being acted upon? One might be tempted to say they are not absolutes in this sense, for there is no physical process connecting h and G with systems in the world. There are no exchanges of energy and momentum, for example. We might wonder, however, whether Einstein would have categorized them as absolutes to be eliminated, for he did hold out the hope for a physics free of arbitrary constants like h and G.

[19] I am grateful to the Research Group: Semantical Aspects of Spacetime Theories (1992/93), Center for Interdisciplinary Research, University of Bielefeld, for helpful discussion.

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