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Conference Paper · February 2000

DOI: 10.1109/PLANS.2000.838298 · Source: IEEE Xplore

1 author:

Ruyong Wang
St. Cloud State University

Proceedings of IEEE 2000 Position Location and Navigation Symposium *(IEEE Catalog Number: 00CH37062), San Diego, California, March 13-16, 2000, pp. 612-619.*

Re-examine the Two Principles of Special Relativity and the Sagnac Effect Using GPS' Range Measurement Equation

Ruyong Wang
St. Cloud State University, St. Cloud, Minnesota 56301, USA (e-mail: ruwang@stcloudstate.edu)

Abstract - Successful GPS operations are based on a basic equation: the range measurement equation, $|\mathbf{r}_r(t_r) - \mathbf{r}_s(t_s)| = c(t_r - t_s)$, in an Earth-centered inertial system. The calculations based this equation show that the correctness of the equation leads to the incorrectness of the two principles of Special Relativity, the principle of relativity and the principle of the constancy of the speed of light. The Sagnac effect can be fully interpreted by this equation and therefore, the Sagnac effect is a non-relativistic effect. It is indicated that the relativity of simultaneity of Special Relativity contradicts the basic operational principle of GPS. Moreover, based on the range measurement equation, it is expected that a practical and crucial experiment that does not require any clock synchronization will give a result contradicting the two principles of Special Relativity. The crucial experiment can be further simplified by using GPS. We should conduct the crucial experiment, and re-examine and re-construct Special Relativity starting from its foundations.

I. INTRODUCTION

GPS is a timing-ranging system. The operations of GPS are based on the range measurement equation in an Earth-centered inertial system, ECI (in an idealized, error-free situation) [1]:

$$|\mathbf{r}_r(t_r) - \mathbf{r}_s(t_s)| = c(t_r - t_s).$$

Here t_s is the instant of transmission of the signal from the source, and t_r is the instant of reception at the receiver; $\mathbf{r}_s(t_s)$ is the position of the source at the transmission time, and $\mathbf{r}_r(t_r)$ is the position of the receiver at the reception time.

Highly successful practices of GPS, especially its unprecedented precision of measuring distance up to the order of millimeters (equivalent to measuring the time to the order of 0.01ns), have proved the correctness of GPS' range measurement equation with high accuracy. Based on that, we re-examine a fundamental problem in physics – the two principles of Special Relativity, the principle of relativity and the principle of the constancy of the speed of light, and an unsolved fundamental problem in physics – the Sagnac effect [2], which is extensively used in GPS.

Here, we should emphasize three important points first:

1) The correct coordinate frame for the range measurement equation is the Earth-centered inertial (ECI) frame;

2) Most GPS receivers are moving relative to ECI, e.g., receivers on ground stations or on airplanes and cars. Obviously, the definition of the position of the receiver at the reception time is important for a moving receiver, since at the reception time, the position of the moving receiver is different from the position of a stationary receiver.

3) The range ρ, $|\mathbf{r}_r(t_r) - \mathbf{r}_s(t_s)|$, is not the distance between two points in its traditional meaning. Traditionally, the distance between two points is the distance between two points at a given time, e.g., $D_1 = |\mathbf{r}_A(t_1) - \mathbf{r}_B(t_1)|$. The range ρ corresponds to the distance between the position of the satellite at transmission time t_s and the position of the receiver's antenna at reception time t_r. That is, the range ρ is the distance between two points at two different times, reception time t_r and transmission time t_s. The traditional distance is an invariant for the coordinate transformation between two systems moving relative to each other, but the range ρ is not an invariant even for the simplest Galilean transformations.

Suppose we have two coordinate systems, S and S', and the transformation between S and S' is $x' = x - vt$, $y = y'$, $z = z'$ and $t = t'$ (fig. 1).

In S, the distance between two points, A and B, at two different times, t_1 and t_2, is $x_B(t_2) - x_A(t_1)$.

In S', we have $x'_B(t'_2) = x_B(t_2) - vt_2$ and $x'_A(t'_1) = x_A(t_1) - vt_1$. Then, we have $x'_B(t'_2) - x'_A(t'_1) = x_B(t_2) - x_A(t_1) - v(t_2 - t_1)$. Therefore, the range ρ, the distance between two points at two different times, is not an invariant for the Galilean transformations. For more complicated transformations, like the transformation between ECI and ECEF, the range ρ is more likely not an invariant.

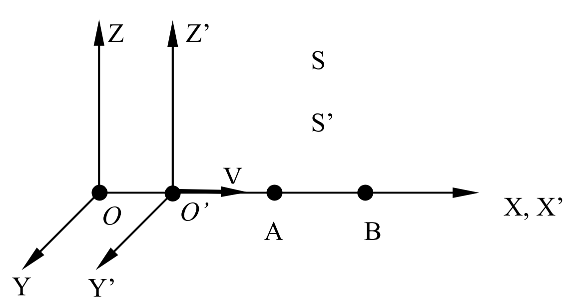

Fig. 1 Coordinate transformation between S and S'

II. TWO PRINCIPLES OF SPECIAL RELATIVITY

Here, we will discuss the implication of GPS to one of the foundations of modern physics, Special Relativity. GPS has in fact provided a big 'laboratory' and 'instruments' for tests of Special Relativity: moving sources and moving receivers; accurate atomic clocks in both sources and (some) receivers; precise knowledge of the positions of the sources and receivers and long distances between sources and receivers; signals carrying the information of positions and times; etc., almost all the needs for tests of Special Relativity. Although one would think that based on the successful practices and unprecedented precision of GPS, a conclusion about Special Relativity could have been made, so far, a common opinion has not yet been reached, and sometimes, completely opposite opinions are held by different people.

Some people [1] think that the range measurement equation is based on the constancy of the speed of light. On the surface, this may appear to be true: c, the speed of light, is the only velocity term that appears within the equation. Expressions such as c-v and c+v, which are often seen in discussions of Special Relativity and classical physics, do not exist in the equation. Therefore, some people would conclude that if this equation is correct, Special Relativity is correct; if this equation has been proved with a high degree of accuracy, Special Relativity has been proved with a high degree of accuracy. For example, it has been concluded [3] that Special Relativity had been confirmed to the limit of $\delta c/c < 5\times10^{-9}$.

But we should not judge things by their appearance; we must try to grasp their essences. If we analyze the implication of the range measurement equation carefully, we will find that, contrary to what its appearance tells us and what some people think, the correctness of the GPS' range measurement equation actually leads to the incorrectness of the principle of the constancy of the speed of light, and furthermore, the principle of relativity. This may seem unexpected, but it is quite understandable if we compare it with Sonar systems. Recall that in underwater navigation, Sonar uses the same range measurement equation in a reference frame based on water to calculate the distance traveled by sound even though the sound receiver is moving relative to water. The difference there is that the speed of sound in water, a, is used instead of the speed of light in vacuum, c. However, no one would emphasize the constancy of the speed of sound, and contrarily, every one thinks the speed of sound is dependent on the motion of the sound receiver.

The range measurement equation and moving source – the speed of light is independent of the translational motion of the source.

Most GPS sources move in circular motion, e.g., GPS satellites and DGPS sources moving with the rotation of the earth. But from the theoretical point of view, we will only investigate sources that are moving translationally and uniformly.

Let us suppose that we have two sources, one stationary and one moving translationally with a speed of v, and a stationary receiver. The distance between the stationary source and the receiver is L and the moving source passes the stationary source at t_0 (fig. 2). When will the receiver receive the signals emitted at t_0 from the two sources according to the range measurement equation?

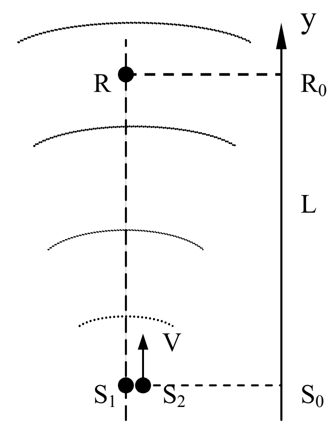

Fig. 2 Moving and stationary sources

For the stationary source, S_1, and the receiver, R, we have (using one-dimensional expression)

$$\begin{cases} |y - y_1(t_0)| = c(t - t_0) \; (\textit{range measurement equation}) \\ y = R_0 \; (\textit{the equation of the position of the receiver}) \end{cases}$$

Hence, $t - t_0 = [R_0 - y_1(t_0)]/c = (R_0 - S_0)/c = L/c$.

For the moving source, S_2, and the receiver, R, we have

$$\begin{cases} |y - y_2(t_0)| = c(t - t_0) \\ y = R_0 \end{cases}$$

Hence, $t - t_0 = [R_0 - y_2(t_0)]/c = (R_0 - S_0)/c = L/c$.

That means, at the same time instant t, receiver R will receive the two signals emitted at t_0 from the two sources, one stationary and one moving, which are at the same distance from the receiver at t_0. Therefore, we can conclude that the speed of light is independent of the translational motion of the source.

The range measurement equation and moving receiver – the speed of light is dependent on the translational motion of the receiver.

Most GPS receivers are moving too. Some of them are in circular motion, e.g., the receivers on the ground stations and the receivers fixed on the earth; Some of them are in translational motion, e.g., on missiles, on airplanes and on cars.

Let us suppose that we have a stationary source and two receivers, one stationary and one moving translationally. The distance between the source and the stationary receiver is L. The moving receiver passes the stationary receiver at t_0 (fig.

3). When will the two receivers receive the signal emitted at t_0 from the source according to the range measurement equation?

For the source, S, and the stationary receiver, R_1, we have

$$\begin{cases} |y - y_s(t_0)| = c(t - t_0) \\ y = R_0 \end{cases}$$

Hence, $t - t_0 = [R_0 - y_s(t_0)]/c = (R_0 - S_0)/c = L/c$.

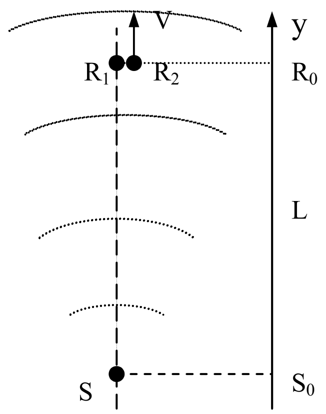

Fig. 3 Moving and stationary receivers

For the source, S, and the moving receiver, R_2, we have

$$\begin{cases} |y - y_s(t_0)| = c(t - t_0) \\ y = R_0 + v(t - t_0) \end{cases}$$

Hence, $t - t_0 = [R_0 - y_s(t_0)]/(c - v) = (R_0 - S_0)/(c - v) = L/(c - v)$.

For a signal transmitted from the source at t_0, the two receivers, one stationary and one moving, will receive it at different instants, although the distances between them and the source at t_0 are the same. Therefore we can conclude that the speed of light is dependent on the translational motion of the receiver.

Contrary to the appearance of the range measurement equation that the speed of the receiver, v, does not appear explicitly, the speed of the receiver is implied in the definition of the position of the receiver, i.e., the position of the receiver at the reception time. Compared with the position of the stationary receiver at the reception time, the position of the moving receiver at the reception time is different, and the difference is proportional to the speed of the moving receiver, v.

Global simultaneity vs. the relativity of simultaneity.

In any debate about the speed of light, the problem of simultaneity is always a focus. Special Relativity claims the relativity of simultaneity which states that two events occurring at two different places which are viewed as simultaneous for an observer in a system, usually will not be simultaneous if viewed for an observer in another system. But contrary to this, simultaneity is the key to GPS operations. GPS is a Timing – Ranging system: it does not directly measure the distance between two places where two events, i.e. signals transmitting and receiving, occur. It measures the difference of the two instants when these two events happen and then, the distance is calculated using the range measurement equation. GPS, especially its space segment and control segment, makes a huge effort to establish and maintain a GPS system time, or simply, GPS time [4]. In a scope where GPS is applied, roughly a scope with diameter of 50,000 km or bigger, if one is using GPS, one is using GPS time and therefore the concept of simultaneity of GPS: two events happened at two different places, (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2), are simultaneous if $t_1 = t_2$. This is true no matter who the observer (receiver) is, where the receiver is, what its status is, or what its speed is. This is the basic operational principle of GPS. We can call it Global Simultaneity.

In the books about Special Relativity, the most commonly cited example about the relativity of simultaneity is the example about the railway platform and the moving train [5]. It says that two events (e.g., the two strokes of lightning A and B) which are simultaneous with reference to the platform are not simultaneous with respect to the moving train and vice versa. But now GPS receivers have been utilized extensively on railway platforms and moving trains, and lightning at two different places, A and B, conceptually is the same as the emissions of GPS signals from two satellites or two DGPS stations. In fact, if two signals are emitted from two satellites or two DGPS stations at the same GPS time, both the GPS receiver on the railway platform and the GPS receiver in the moving train would acknowledge the two events, the emissions of the signals, to be simultaneous. Without this basic acknowledgement, the GPS receivers can not function at all.

The range measurement equation and the crucial experiment of Special Relativity.

We have shown that the correctness of the range measurement equation contradicts the principle of the constancy of the speed of light which asserts that light in vacuum always has a definite speed of propagation that is independent of the state of the motion of the observer [6]. We have also indicated that the relativity of simultaneity contradicts the purpose of GPS system time and the basic operational principle of GPS. Due to the popularity of Special Relativity, a lot of people still will not accept these. Therefore, we would examine a crucial experiment, in which the result can be used to refute or verify Special Relativity from everybody's point of view. More importantly, in this experiment, simultaneity, or the synchronization of the clocks, is not a concern.

We mount two atomic clocks with the same construction, signal transmitter, reflector, and receivers on the two ends, points A and B, of a vehicle, with distance L between A and B. First (fig. 4a), the vehicle is stationary relative to the earth (facing due South or due North, so when the vehicle moves, the direction of the velocity is due South or due North, eliminating the effect of the rotation of the earth.) The two clocks are not synchronized with each other. A signal is transmitted from A at $t_1(A)$ (according to clock A) to B (arriving at $t_1(B)$ according to clock B) and reflected back to A (arriving at $t'_1(A)$ according to clock A). By the readings of clocks, we can calculate the difference of the nominal travelling times for two directions, $\Delta t_1 = [t'_1(A) - t_1(B)] - [t_1(B) - t_1(A)]$. (We say that the travelling times $t_1(B) - t_1(A)$ and $t'_1(A) - t_1(B)$ are nominal because the two clocks are not synchronized. For example, $t_1(B) - t_1(A)$ could be negative if clock B is too much behind clock A.) Then we repeat the same measurement when the vehicle moves uniformly due North at speed of v (fig. 4b). We will obtain $\Delta t_2 = [t'_2(A) - t_2(B)] - [t_2(B) - t_2(A)]$. If the readings of the clocks show that Δt_1 is different from Δt_2, we think everybody would agree that the experiment refutes the principle of the constancy of the speed of light, and the principle of relativity (because we now find a difference between two uniform motion states), especially noting that the relativity of simultaneity is not a problem here, because the synchronization of clocks is not required. If Δt_1 is equal to Δt_2, then the experiment verifies Special Relativity.

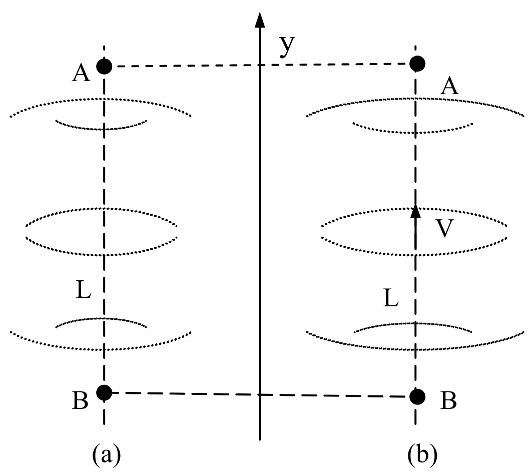

Fig. 4 Crucial experiment

Let us calculate Δt_1 and Δt_2 according to the range measurement equation. First, we assume that both clocks have GPS time for convenience. Later we will show that this assumption is not necessary to obtain the same result.

In case 1, when the vehicle is stationary, for the signal transmitted from A to B, we have

$$\begin{cases} |\,y_B - y_A[t_1(A)]\,| = c[t_1(B) - t_1(A)] \\ y_B = y_B[t_1(A)] \end{cases}$$

Hence, $[t_1(B) - t_1(A)] = \{y_A[t_1(A)] - y_B[t_1(A)]\}/c = L/c$.

For the signal reflected back from B to A, we have

$$\begin{cases} |\,y_A - y_B[t_1(B)]\,| = c[t'_1(A) - t_1(B)] \\ y_A = y_A[t_1(B)] \end{cases}$$

Hence, $[t'_1(A) - t_1(B)] = \{y_A[t_1(B)] - y_B[t_1(B)]\}/c = L/c$.

In this case, we have $\Delta t_1 = [t'_1(A) - t_1(B)] - [t_1(B) - t_1(A)] = 0$.

In case 2, when the vehicle is uniformly moving North with a speed of v, for the signal emitted from A to B, we have

$$\begin{cases} |\,y_B - y_A[t_2(A)]\,| = c[t_2(B) - t_2(A)] \\ y_B = y_B[t_2(A)] + v[t_2(B) - t_2(A)] \end{cases}$$

Hence, $[t_2(B) - t_2(A)] = \{y_A[t_2(A)] - y_B[t_2(A)]\}/(c + v) = L/(c + v)$

For the signal reflected back from B to A, we have

$$\begin{cases} |\,y_A - y_B[t_2(B)]\,| = c[t'_2(A) - t_2(B)] \\ y_A = y_A[t_2(B)] + v[t'_2(A) - t_2(B)] \end{cases}$$

Hence, $[t'_2(A) - t_2(B)] = \{y_A[t_2(B)] - y_B[t_2(B)]\}/(c - v) = L/(c - v)$

Therefore, we have $\Delta t_2 = [t'_2(A) - t_2(B)] - [t_2(B) - t_2(A)] = L/(c - v) - L/(c + v) \approx 2Vl/c^2$, neglecting the quantities of the second and higher order of v/c.

Now let us eliminate the assumption of having GPS time in both clocks and find the result. Let us suppose that neither clock A nor clock B uses GPS time, and the two clocks are not synchronized with each other: clock A would be δt_A ahead of GPS time and clock B would be δt_B ahead of GPS time, Then, in case 1, where the vehicle is stationary, we will record a $\Delta t_1 = 2\delta t_A - 2\delta t_B$, in stead of recording $\Delta t_1 = 0$. In case 2, when the vehicle is moving, we will record a $\Delta t_2 = 2Vl/c^2 + 2\delta t_A - 2\delta t_B$. Therefore, when we calculate the time difference between two cases, we will find the same $\Delta t = \Delta t_2 - \Delta t_1 = 2Vl/c^2$, whether clock B is synchronized with clock A or not, and whether the clocks are synchronized with GPS time or not.

Hence, according to the range measurement equation, when we conduct this experiment, we will find a time difference of $\Delta t = \Delta t_2 - \Delta t_1 = 2Vl/c^2$ between the two cases. It is a first-order effect and Lorentz contraction, which is a second-order effect, is irrelevant here. Time dilation, and hence, the effect of moving clocks are relevant here. However, since both clocks move in exactly the same way, there will be no net effect on the time difference from the two moving clocks. Therefore, the range measurement equation's correctness has lead to the prediction that the crucial experiment will refute the two principles of Special Relativity.

It is suggested in [7] that this experiment can be implemented by mounting the two clocks not in one moving object, but in two separate objects that move in a straight line, one after another, with the same velocity. This way, L, the distance between the two clocks can be increased substantially, and hence, the predicted time difference can reach up to 1 nanosecond, a value that is relatively easy to detect with current technology. Also, the effect of moving clocks, including time dilation, and the effect resulting from the fact that L is not strictly constant are discussed there in detail, and it has been indicated that these effects will not prevail over the time difference we are trying to detect.

III. THE SAGNAC EFFECT

In 1913, Sagnac [8] conducted an experiment that is now named after him. This experiment consists of a beam splitter and several mirrors mounted on a disk (fig. 5). The beam splitter divides the light beam into two portions; one traverses clockwise along the quadrilateral formed by the mirrors, the other counterclockwise. An interference pattern is formed when the beams unite. When the disk rotates clockwise with an angular velocity ω around its axis, there is a shift of fringes $\Delta N = 4\omega S_{ABCD}/c\lambda$ (S_{ABCD}, the area of the quadrilateral ABCD and λ, the wavelength of light). If the disk rotates counterclockwise, the shift is in the opposite direction. The Sagnac experiment indicated that, for the observer at A, the two light beams travelling in opposite directions do not return to the starting point at the same instant when the disk rotates and the general expression of the Sagnac effect for the time difference is $\Delta t = \Delta N\lambda/c = 4\omega S/c^2$, where S is the area enclosed by the light path and can be of any shape.

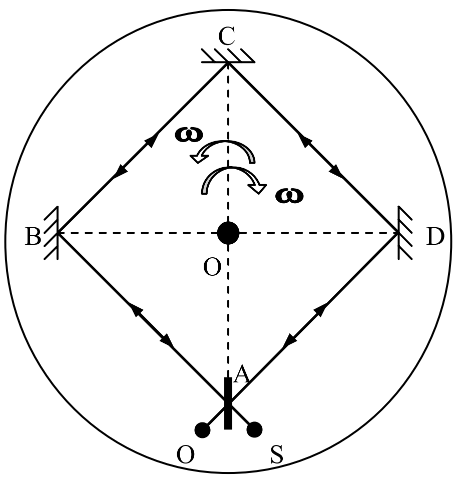

Fig. 5 The Sagnac experiment

Since then, the Sagnac experiment has been conducted in many different ways. For example, the Michelson-Gale experiment[9] examined the effect of the rotation of the earth instead of the rotation of the disk, and the around-the-world Sagnac experiment[10] recorded signal arrival times with atomic clocks instead of the interferometers. The Sagnac effect has been applied to many systems ranging in size from a few centimeters, e.g., in fiber-optic gyroscopes[11], to the Global Positioning System [1]. Especially, there will be a positioning error of 30 meters in GPS if the Sagnac effect caused by the rotation of the Earth is not considered correctly. Although everybody understands the existence and the importance of the Sagnac effect, the interpretation of the Sagnac effect is still a very controversial topic and there are more than a dozen ways to interpret the Sagnac effect [12]. As mentioned above, it is stated that the Sagnac effect is an unsolved fundamental problem in physics [2]. Here, we would show that the Sagnac effect could be fully interpreted by the range measurement equation of GPS.

Interpretation of the Sagnac effect with the range measurement equation of GPS.

First, we need to simplify the light path in the Sagnac experiment, from a quadrilateral to a circle (fig. 6), so we can identify the light path better when the disk is rotating. However, when the light path is a circle, we can not directly utilize the range measurement equation in its current form which is suitable for straight line light path, instead, we should reform the range measurement equation. When a GPS signal is transmitted from A to B and then consecutively from B to C, we have $|\mathbf{r}_B(t_B) - \mathbf{r}_A(t_A)| = c(t_B - t_A)$ and $|\mathbf{r}_C(t_C) - \mathbf{r}_B(t_B)| = c(t_C - t_B)$. We notice that $\mathbf{r}_B(t_B)$ in both expressions is the same, we can re-write them in $|\Delta\mathbf{r}_{BA}| = c\Delta t_{BA}$ and $|\Delta\mathbf{r}_{CB}| = c\Delta t_{CB}$, or $\Sigma|\Delta\mathbf{r}| = \Sigma(c\Delta t)$, here $|\Delta\mathbf{r}_{BA}|$, $|\Delta\mathbf{r}_{CB}|$ and $\Sigma|\Delta\mathbf{r}|$ are the real paths of the signal in ECI. Now we can convert them to a differential form $|d\mathbf{r}| = cdt$, and then utilize its integral form $\int_s |d\mathbf{r}| = \int cdt$, where s is the real propagation path from the source to the receiver in ECI. Obviously, $\int_s |d\mathbf{r}|$ is the length of the light path s, L_S.

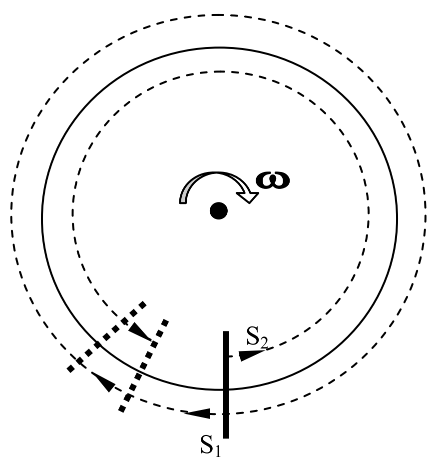

Fig. 6 The Sagnac experiment with a circular light path

Now, let us examine the Sagnac experiment. For the clockwise light path, we have

$$\int_{S1} |\mathbf{dr}| = c\int_{t_0}^{t_1} dt$$ and the real path $L_{S1} = 2\pi R + \omega R\,(t_1 - t_0)$.

For the counterclockwise light path, we have

$$\int_{S2} |\mathbf{dr}| = c\int_{t_0}^{t_2} dt$$ and the real path $L_{S2} = 2\pi R - \omega R\,(t_2 - t_0)$.

Therefore, we have $2\pi R + \omega R\,(t_1 - t_0) = c(t_1 - t_0)$, $2\pi R - \omega R\,(t_2 - t_0) = c(t_2 - t_0)$ and $t_1 - t_0 = 2\pi R/(c - \omega R)$, $t_2 - t_0 = 2\pi R/(c + \omega R)$. The difference of two travelling times is $\Delta t = (t_1 - t_0) - (t_2 - t_0) = 2\pi R/(c - \omega R) - 2\pi R/(c + \omega R) \approx 4\pi R^2\omega/c^2 = 4S\omega/c^2$, neglecting the quantities of the second order of $\omega R/c$ or higher. The last expression is the same as the result of the Sagnac experiment. This tells us that the range measurement equation of GPS in ECI can interpret the result of the Sagnac experiment perfectly. It also tells us that if we utilize the real propagation paths in ECI to calculate the travelling times of the signals, the Sagnac effect is included automatically.

The Sagnac effect and the Sagnac correction.

Then, a reasonable question is when do we need to add the Sagnac effect into the calculation of the travelling time of the signal. Here, it is useful that we identify the difference between two terms: Sagnac effect and Sagnac correction.

The Sagnac effect is an effect or a phenomenon: compared with a stationary receiver in ECI, a receiver moving relative to ECI will receive the signal from the source at a different time. The time difference depends on the distance between the source and the receiver and the relationship between two directions, the direction of the propagation and direction of the motion of the receiver.

The Sagnac correction is a correction added in the calculation when we use the range measurement equation. In fact, if we use the range measurement equation in its original meaning, i.e., range being the distance in ECI between the source at the transmission time and the receiver at the reception time, no correction is needed. The Sagnac effect is included automatically. That is, although there is a Sagnac effect, there is no need for Sagnac correction. However, if we utilize the range measurement equation differently from its original meaning, we need to add a correction. Usually, there are two cases: 1) the range is not in ECI, but in other coordinate systems because the range is not an invariant of coordinate transformations as we mentioned before, and 2) the range is not the distance between the source at the transmission time and the receiver at the reception time, but the distance between the source and the receiver at the same time, the transmission time. Therefore, a Sagnac correction must be added in order to compensate for the difference caused by the incorrect definition of the range.

We can see these two cases very clearly in the Sagnac experiment. First, if the length of the signal propagation path is not in ECI, but in a coordinate system rotating with the disk, the lengths of the propagation paths in the two directions will be the same, $2\pi R$. Therefore, if we investigate the signal propagation using the range measurement equation in this rotating system, we will get an incorrect result, unless we add a positive Sagnac correction, $2\omega\pi R^2/c$, to the clockwise propagation length and a negative Sagnac correction, $-2\omega\pi R^2/c$, to the counterclockwise propagation length.

Second, some times we have difficulties deciding the propagation length between the source at the transmission time and the receiver at the reception time because the receiver is moving and it is easier to decide the length between the source and the receiver at the same time. In this case, for the latter, both lengths are $2\pi R$. Therefore, if we use the definition of propagation length like this, we should also add the corrections. Just like the previous example, we need to add a positive Sagnac correction, $2\omega\pi R^2/c$, to the clockwise propagation length and a negative Sagnac correction, $-2\omega\pi R^2/c$, to the counterclockwise propagation length. Notice that the reasons for adding Sagnac correction in these two examples are different, although here their values are the same.

Two examples of the Sagnac correction in GPS.

The transformation from ECI to ECEF when investigating the propagation from a satellite to a ground station. With the around-the-world Sagnac experiment [10], it is well known that the signal propagation eastward from a satellite to a ground station will take a longer time compared to the signal propagation westward because of the rotation of the earth. As we mentioned above, this Sagnac effect will be included automatically and we do not need a Sagnac correction if we utilize the range measurement equation correctly, i.e., utilize the range as the distance in ECI between the source at the transmission time and the receiver at the reception time. If we define the range not as the distance in ECI, but as the distance in ECEF, then we need to add a Sagnac correction, like in the rotating disk case mentioned above.

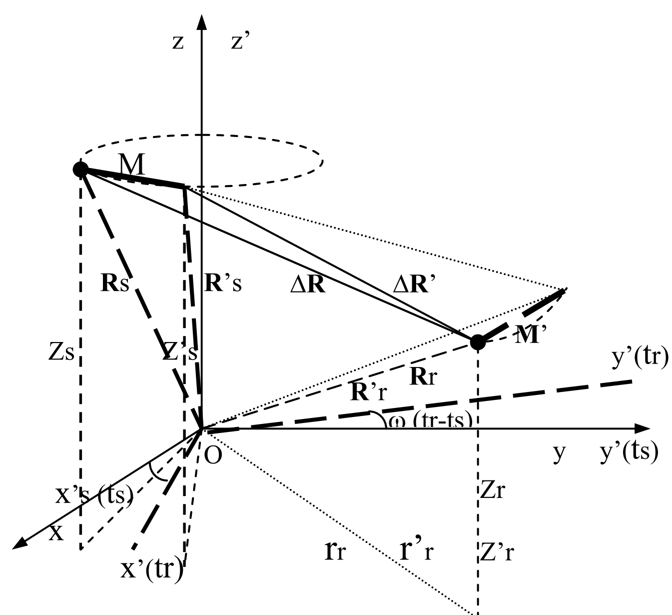

Fig.7 The Sagnac correction in ECEF

Let us calculate the difference between two ranges, one defined in ECI and one defined in ECEF. We can utilize both the rectangular coordinates and the cylindrical coordinates for ECI (x, y, z and r, θ, z) and for ECEF (x', y', z' and r', θ', z') and the transformation between ECI and ECEF can be described in cylindrical coordinates easily as $r' = r$, $\theta' = \theta - \omega(t - t_0)$, $z' = z$, where ω is the Earth angular rotation rate. To simplify the calculation here, we define $t_0 = t_s$. That means, at transmission time t_s, ECEF coincides with ECI. Fig. 7 has shown the signal propagation from $\mathbf{R}_s(r_s, \theta_s, z_s)$ to $\mathbf{R}_r(r_r, \theta_r, z_r)$ viewed from ECI (**R**, instead of **r**, is used here because r is a coordinate of the cylindrical coordinates.) The same propagation can be viewed from ECEF as that from $\mathbf{R'}_s(r'_s, \theta'_s, z'_s)$ to $\mathbf{R'}_r(r'_r, \theta'_r, z'_r)$ and we have $r_s = r'_s$, $\theta_s = \theta'_s$, $z_s = z'_s$ and $r_r = r'_r$, $\theta_r = \theta'_r - \omega(t_r - t_s)$, $z_r = z'_r$. Fig. 7 has also shown the signal propagation from $\mathbf{R'}_s(r'_s, \theta'_s, z'_s)$ to $\mathbf{R'}_r(r'_r, \theta'_r, z'_r)$ viewed from ECEF at the reception time t_r. Notice that the axes of ECEF at the reception time t_r have rotated an angle of $\omega(t_r - t_s)$ compared with the axes of ECI, and therefore, $\mathbf{R'}_s$ is obtained from $\mathbf{R}_s$ by rotating $\mathbf{R}_s$ around z(z') by $\omega(t_r - t_s)$ and $\mathbf{R'}_r = \mathbf{R}_r$.

Obviously, $|\Delta\mathbf{R}| = |\mathbf{R}_r - \mathbf{R}_s|$ is different from $|\Delta\mathbf{R'}| = |\mathbf{R'}_r - \mathbf{R'}_s|$ and their difference is the Sagnac correction. Since $|\Delta\mathbf{R'}| = |\Delta\mathbf{R}| - \mathbf{M}$ and $\mathbf{M} = \boldsymbol{\omega} \times \mathbf{R}_s(t_r - t_s) \approx \boldsymbol{\omega} \times \mathbf{R'}_s(t_r - t_s)$, we have

$|\Delta\mathbf{R'}| = [|\Delta\mathbf{R}|^2 - 2\Delta\mathbf{R} \bullet \mathbf{M} + |\Delta\mathbf{M}|^2]^{1/2} \approx |\Delta\mathbf{R}| - \Delta\mathbf{R} \bullet \mathbf{M}/|\Delta\mathbf{R}|$.

$\Delta\rho = |\Delta\mathbf{R}| - |\Delta\mathbf{R'}| = \Delta\mathbf{R} \bullet \mathbf{M}/|\Delta\mathbf{R}| = \Delta\mathbf{R} \bullet (\boldsymbol{\omega} \times \mathbf{R}_s)(t_r - t_s)/|\Delta\mathbf{R}| = \Delta\mathbf{R} \bullet (\boldsymbol{\omega} \times \mathbf{R}_s)/c = (\mathbf{R}_r - \mathbf{R}_s) \bullet (\boldsymbol{\omega} \times \mathbf{R}_s)/c = \mathbf{R}_r \bullet (\boldsymbol{\omega} \times \mathbf{R}_s)/c = (x_r y_s - y_r x_s)\omega/c$, where $\mathbf{R}_r = x_r i + y_r j + z_r k$, $\mathbf{R}_s = x_s i + y_s j + z_s k$ and $\boldsymbol{\omega} = \omega k$.

Or utilizing ECEF, $\Delta\rho = |\Delta\mathbf{R}| - |\Delta\mathbf{R'}| = \mathbf{R'}_r \bullet (\boldsymbol{\omega} \times \mathbf{R'}_s)/c = (x'_r y'_s - y'_r x'_s)\omega/c$, where $\mathbf{R'}_r = x'_r i + y'_r j + z'_r k$, $\mathbf{R'}_s = x'_s i + y'_s j + z'_s k$.

Sometimes, it is convenient to add a Sagnac correction not as a range correction, but as a coordinate correction, i.e., modifying the position of the receiver to compensate for the range correction. We understand that $\mathbf{R'}_s$ is obtained from $\mathbf{R}_s$ by rotating $\mathbf{R}_s$ around z(z') by $\omega(t_r - t_s)$, therefore, rotating $\mathbf{R'}_r$ around z(z') by the same $\omega(t_r - t_s)$, we could have the same range. That means adding $\mathbf{M'} = \boldsymbol{\omega} \times \mathbf{R'}_r(t_r - t_s)$ to the position of the receiver.

We have $\mathbf{M'} = \boldsymbol{\omega} \times \mathbf{R'}_r(t_r - t_s) = -\omega y'_r \Delta t i + \omega x'_r \Delta t j$. Hence, the coordinate correction of the position of the receiver is $(-\omega y'_r \Delta t, \omega x'_r \Delta t, 0)$.

Utilize the definition of the range as the distance between the source and the receiver at the same time, the transmission time, when investigating the propagation between two satellites. GPS IIR satellites implement the communications between two satellites. Therefore, we should calculate the propagation time between two satellites correctly. Correct usage of the range measurement equation utilizes the distance between the two satellites, source and the receiver, at two different times, the transmission time and the reception time. But this distance is not easy to decide. It is much easier to decide the distance between two satellites at the same time, e.g., the transmission time and it is an invariant of the coordinate transformation. If we utilize this definition of the distance, we need to add a correction, Sagnac correction, to compensate for the difference between the position of the receiver at the transmission time and position of the receiver at the reception time.

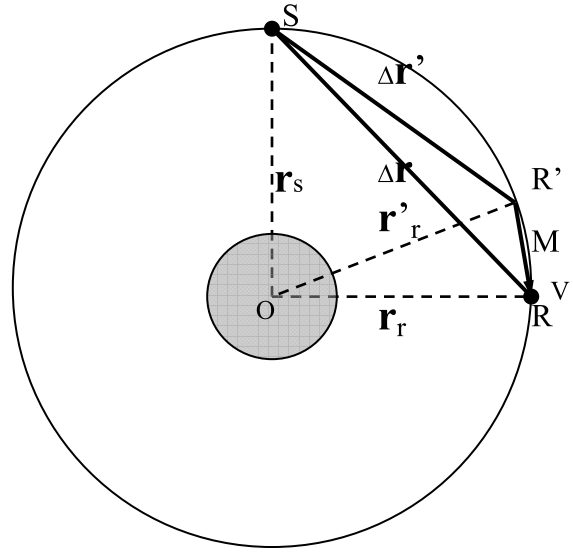

Fig. 8 The Sagnac correction for using the position of the second satellite at the transmission time

Suppose in ECI, the position of the first satellite at the transmission time is $\mathbf{r}_s$ (fig. 8). The position of the second satellite is $\mathbf{r}_r$ at reception time t_r and is $\mathbf{r'}_r$ at transmission time t_s. The difference between $\mathbf{r}_r$ and $\mathbf{r'}_r$ is that during signal propagation, the second satellite has moved **M** where $\mathbf{M} = \mathbf{v}\,\Delta t$ if Δt is short. Obviously, $|\Delta\mathbf{r}| = |\mathbf{r}_r - \mathbf{r}_s|$ is different from $|\Delta\mathbf{r'}| = |\mathbf{r'}_r - \mathbf{r}_s|$ and the difference is the Sagnac correction.

$\Delta\rho = |\Delta\mathbf{r}| - |\Delta\mathbf{r'}| = \Delta\mathbf{r} \bullet \mathbf{M}/|\Delta\mathbf{r}| = \Delta\mathbf{r} \bullet \mathbf{v}\,\Delta t/|\Delta\mathbf{r}| = \Delta\mathbf{r} \bullet \mathbf{v}/c$.

As we mentioned before, $|\Delta\mathbf{r'}| = |\mathbf{r'}_r - \mathbf{r}_s|$ is an invariant of coordinate transformation, we can utilize $\Delta\rho = \Delta\mathbf{r'} \bullet \mathbf{v}/c$, instead of $\Delta\mathbf{r} \bullet \mathbf{v}/c$, because $\Delta\mathbf{r} \bullet \mathbf{v}/c - \Delta\mathbf{r'} \bullet \mathbf{v}/c = \mathbf{M} \bullet \mathbf{v}/c \ll \Delta\mathbf{r} \bullet \mathbf{v}/c$.

The Sagnac effect, the Sagnac correction and the translational motion including uniform motion.

There is a misconception that the Sagnac effect, a first order effect, uniquely belongs to the rotational motion. Although it is true that for an interference experiment, the necessary condition of the Sagnac effect is the rotation. It is because for an interference experiment, the light path is a closed one and for this closed light path, the possible Sagnac effects caused by the translation motion are cancelled with each other like in the Michelson-Morley experiment which only detected the second order effect. Now the signal propagation in GPS is a one-way propagation, there will not be a cancellation and therefore, rotation is not a necessary condition for a first order Sagnac effect anymore. Besides, we should notice that the motions of the ground station and the satellite in the previous examples are not purely rotational, but circular motions, which are the combination of the translational motion and the rotational motion. Here, we will show that for purely translation motions, including uniform motions, all the discussions about the Sagnac effect and the Sagnac correction are suitable too. That is, there is a Sagnac effect when the receiver is in a uniform motion, and we need to add a Sagnac correction if we use the range measurement equation not in its original meaning.

The Sagnac correction with a range defined in a system U instead of the range defined in ECI. Suppose there is a system U which is moving uniformly with a velocity **v** in ECI. At transmission time t_s, it is located at o_u and at reception time t_r, it is located at o'_u (fig. 9). Viewed from the system U at the reception time t_r, the propagation from $\mathbf{r}_s$ to $\mathbf{r}_r$ becomes the propagation from $\mathbf{r}'_s(t_s)$ to $\mathbf{r}'_r(t_r)$. Obviously, viewed from the system U, the range $|\Delta\mathbf{r}'|$ is different from the range $|\Delta\mathbf{r}|$, and based on the same calculation made before, we have $\Delta\rho = |\Delta\mathbf{r}| - |\Delta\mathbf{r}'| = \Delta\mathbf{r} \bullet \mathbf{v}/c \approx \Delta\mathbf{r}' \bullet \mathbf{v}/c$.

If the system U is based on the receiver, the velocity **v** is the velocity of the receiver.

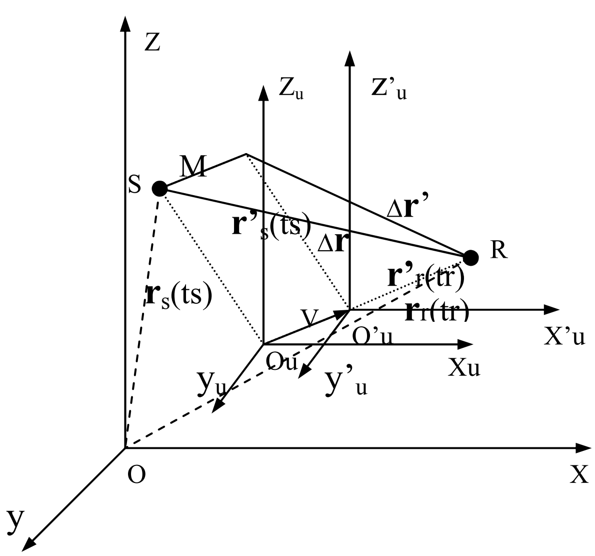

Fig. 9 The Sagnac correction in system U

The Sagnac correction with the position of the receiver defined at transmission time t_s instead of reception time t_r. Suppose in ECI, the position of the receiver is $\mathbf{r}_r$ at reception time t_r and is $\mathbf{r}'_r$ at transmission time t_s (fig. 10). The difference between $\mathbf{r}_r$ and $\mathbf{r}'_r$ is that during signal propagation, the receiver has moved **M** where $\mathbf{M} = \mathbf{v}\,\Delta t$ if Δt is short. The Sagnac correction in this case is $\Delta\rho = |\Delta\mathbf{r}| - |\Delta\mathbf{r}'| = \Delta\mathbf{r} \bullet \mathbf{M}/|\Delta\mathbf{r}| = \Delta\mathbf{r} \bullet \mathbf{v}/c \approx \Delta\mathbf{r}' \bullet \mathbf{v}/c$.

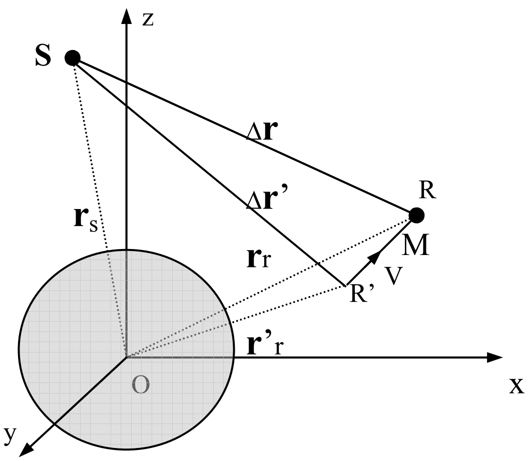

Fig. 10 The Sagnac correction for using the position of the receiver at the transmission time

This result is the same as the result of the Sagnac correction for signal propagating from a satellite to another satellite mentioned above. In fact, when Δt (= tr – ts) is short, there is no difference between the motion of the satellite which is a circular motion and the motion of the receiver which is a translational motion.

Finally, we would summarize the following for the Sagnac effect and the Sagnac correction: (1) The Sagnac effect is caused by the motion of the receiver in ECI. (2) The Sagnac effect will be automatically included or considered if we use the range measurement equation correctly, i.e., use the real propagation path in ECI. (3) We should add the Sagnac correction if we use the range measurement equation in a coordinate frame that is moving relative to ECI or if the position of the receiver is its position not at the instant of reception, but at the instant of the transmission, whether the motion of the receiver is circular or purely translational. (4) Therefore, the Sagnac effect is not a relativistic effect. Contrarily, it is a non-relativistic effect.

IV. Simplifying the Crucial Experiment Using GPS

When we say that the correctness of the range measurement equation leads to the incorrectness of the two principles of Special Relativity, we mean it not only qualitatively as we mentioned before, but also quantitatively. The difference between what Special Relativity predicts and what the range measurement equation calculates is an item of vL/c (for length) or vL/c^2 (for time). This item is 'big' in GPS applications because L is about 20,000km. Therefore, vL/c reaches 200m when v = 3 km/s (speed of missiles), it reaches 20m when v = 300 m/s (speed of airplanes), and it reaches 2 m when v = 30 m/s (speed of cars). GPS has reached unprecedented precision of positioning up to the order of millimeters which is much smaller than the values listed above. Therefore, quantitatively, GPS practices have proved the correctness of the range measurement equation and the incorrectness of the two principles of Special Relativity.

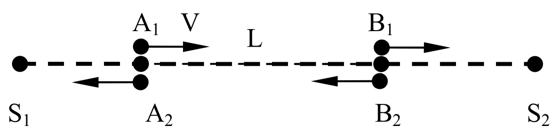

Fig. 11 Simplifying the crucial experiment (ideal case)

As we mentioned before, because of the popularity of Special Relativity, a lot of people, especially relativistic physicists, would not accept this unless a crucial experiment is conducted with a result refuting Special Relativity. The crucial experiment we proposed before is a practical one. But we can simplify it further using GPS: assume that two satellites, S_1 and S_2, are located on the extension of line AB (fig. 11), then it is not necessary to have a signal transmitter at A and a reflector at B. What we need are only GPS receivers at A and B, and we can calculate the times needed as $t(A \rightarrow B) = t(S_1 \rightarrow B) - t(S_1 \rightarrow A)$ and $t(B \rightarrow A) = t(S_2 \rightarrow A) - t(S_2 \rightarrow B)$. We can conduct the experiment in two parts. In part 1, A and B move south (to eliminate the effect caused by the rotation of the earth) with a speed of v and in part 2, A and B move north with a speed of v. Special Relativity predicts that there is no time difference between the two parts, but the calculation of the range measurement equation tells us that we will find a time difference in the experiment, $\Delta t = 4vL/c^2$ (all the discussions about clock synchronization of the crucial experiment can be applied here too). It reaches 4 ns when L = 3,000 km and v = 30 m/s (speed of cars).

Realistically, the assumption that GPS satellites are exactly located on the extension of line AB is unpractical and almost unachievable. Any small shift of the positions of satellites from the extension of line AB will cause big errors because the distances between satellites and A or B are much longer than the distance between A and B. Then, is a way to simplify the crucial experiment still possible? It is possible if we recall how Galileo overcame the seemly inevitable difficulty that there was no a perfectly frictionless and perfectly horizontal track when he conducted the experiment that led him to his Law of Inertia. He used two inclined planes set end-to-end and changed the tilt of the second track. The ball always reached a vertical height that was almost the same as it started from. Then Galileo argued that if the second track were perfectly frictionless and perfectly horizontal, the ball would roll forever. We can gain a good deal of enlightenment from this famous experiment. We can conduct the experiment with different positions of the satellites, then, different inclinations to line AB (fig. 12). According to the range measurement equation, we will have the result as

$$[(t_{S1B1} - t_{S1A1}) - (t_{S2A1} - t_{S2B1})] - [(t_{S1B2} - t_{S1A2}) - (t_{S2A2} - t_{S2B2})] = 2v(D_{1B} - D_{1A})\cos\theta_1/c^2 + 2v(D_{2A} - D_{2B})\cos\theta_2/c^2.$$

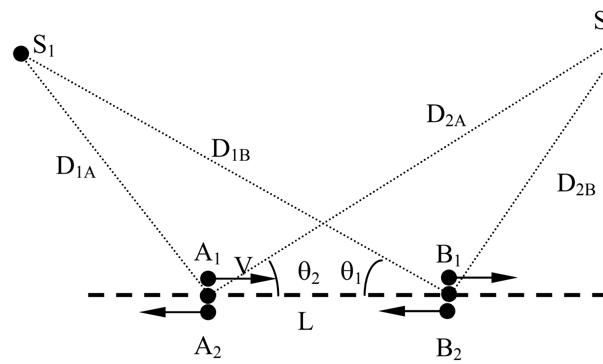

Fig. 12 Simplifying the crucial experiment (real case)

If we can find, from the results of the experiments, that this is true for different $\theta_1$and θ_2, then we can conclude that it will be true for $\theta_1 = 0$ and $\theta_2 = 0$ also. It means $(t_{A1B1} - t_{B1A1}) - (t_{A2B2} - t_{B2A2}) = 4vL/c^2$, and therefore, the two principles of Special Relativity will be falsified.

V. Conclusion

Up to now all experiments used to verify Special Relativity have been done with the Earth as the reference frame, and for the speed of light, there has never been an experiment in which the receiver is linearly moving relative to the Earth. Therefore, the two principles of Special Relativity, the principle of relativity and the principle of the constancy of the speed of light, have never been fully verified by experiment [13]. The calculations based on GPS's basic equation, the range measurement equation, show that the correctness of the equation leads to the incorrectness of the two principles of Special Relativity. The Sagnac effect can be fully interpreted by this equation and therefore, the Sagnac effect is a non-relativistic effect. It is indicated that the relativity of simultaneity of Special Relativity contradicts the basic operational principle of GPS. Moreover, based on the range measurement equation, it is expected that a practical and crucial experiment that does not require any clock synchronization will give a result contradicting the two principles of Special Relativity. The crucial experiment can be further simplified by using GPS. Therefore, if scientific contention can not settle the dispute, this crucial experiment definitely will. We should conduct this crucial experiment, and re-examine and re-construct Special Relativity starting from its foundations.

ACKNOWLEDGEMENTS

The author would like to thank Dr. A. Kelly and Dr. D. Langley for useful discussions.

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