abundances. The major source of the heavy cosmic rays is supernova explosions in our galaxy.

(6) In order to study the extremely important charge region $Z>92$, experiments with area-time factors much greater than 100 m^2 days will be required.

ACKNOWLEDGMENTS

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1970 Solar Eclipse as "Seen" by a Torsion Pendulum

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During the solar eclipse of 7 March 1970, readings were taken and recorded electronically of the times required for the torsion pendulum to rotate through a given fixed part of its path, involving both clockwise and counterclockwise motions, on its first swing from rest. Significant variations in these times were observed during the course of the eclipse as well as in the hours just preceding and just following the eclipse itself. Between the onset of the eclipse and its midpoint there is a steady increase in the observed times. After the midpoint the times decrease suddenly and level off promptly to values considerably greater than those observed before the eclipse. Furthermore, before the eclipse there is a periodic variation in these times. This strange periodicity was essentially repeated two weeks later at the same hours, though the actual values were somewhat greater than the earlier ones. These increases in actual values exceed by a factor of 10^6 those that can be explained by the attraction of the moon due to its change in position relative to the sun and earth. All this leads to the conclusion that classical gravitational theory needs to be modified to interpret these experimental facts.

INTRODUCTION

In this paper a study is made of the variations in behavior of a torsion pendulum during the solar eclipse of 7 March 1970. The torsion pendulum is essentially the same as that used in studying the effect of added weights on its period,[1,2] but with modifications discussed later in this paper. With this setup, when the pendulum is started reproducibly from rest in a precisely defined release position, the time of traversing a constant portion of its path is timed accurately and recorded automatically. To do this, light from a fixed source is reflected from a mirror attached to the torus to fall on a photocell. Over a preamplifier the latter starts a crystal-controlled counter as the light beam travels clockwise across the photocell and stops it on its return counterclockwise trip, to record the times between these two passages of the light beam across the photocell. This gives recordings of the times used in traversing a constant fixed part of the total vibration path of the oscillating torus on the first swing from rest.

Some improvements had been made in the apparatus since the previous work, which increased its precision still further. Notably, a stronger light source was used which made it possible to narrow the vertical slit limiting the width of the light beam falling on the photocell. Preamplification of the signal received by the photocell and other minor changes (such as a constant voltage transformer for a uniform power supply and a nonferrous and nonmetallic manual release mechanism) were made to assure safe and reliable action of the electronic timing and printout mechanism. Earlier observations during other eclipses, taken before these needed improvements, agree qualitatively with the present results, but are not good enough for quantitative comparison.

Furthermore, it was possible to keep the temperature around the isoelastic Ni-Span "*C*" suspension wire at 21.7°C with a fluctuation of only ±0.6°C. This suspension has been kept under the given load for some 17

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[1] E. J. Saxl and M. Allen, J. Appl. Phys. **40**, 2499 (1969).

[2] M. Allen and E. J. Saxl, J. Appl. Phys. **40**, 2505 (1969).

years so that possible creep should have reached equilibrium. Moreover, the operation of the pendulum was started at 4 a.m. EST while the critical readings did not begin until 10:15 a.m. This long running-in period should eliminate for the suspension wire any lack of stability due to repeated twisting or malfunction arising from mechanical hysteresis or sudden slippage in its metallic structure or lack of temperature stability inside the tube surrounding it. In another prolonged set of observations taken to study the effect of electric charge on the period of the pendulum, there was no comparable change in the grounded period. Moreover, the torsional elasticity of the suspension wire studied statically did not change with the position of the moon. Furthermore, the wire was checked statically to see that it followed Hooke's law exactly as well as calculations made to show that the margin of safety for operation within the elastic limit was substantial (1:7). To avoid slipping, not only was the wire tightly clamped at both ends but three pointed set screws were driven, one above the other, solidly into the wire.

Observations in the present case were recorded alternately with the pendulum grounded and with it charged to ±4900 V, but only the grounded results are presented in this paper. Somewhat different, and at times unexpected, effects were noted when the pendulum was charged electrically.

PROCEDURE

Under these carefully guarded conditions, automatic recordings of the times during which the torus rotated through a constant angle were made from 10:15 a.m. until 3:40 p.m. EST on the day of the eclipse in the town of Harvard, Mass. The eclipse in Boston, some 20 m distant, was about 96.5% total.

Significant variations in the recorded times were observed during the course of the eclipse, as is shown by the full line in Fig. 1. Each point in this figure is the average of five consecutive grounded readings. The limited vertical lines indicate the average deviations of the five readings from the average circled values. The beginning of the eclipse at 12:31 p.m., its midpoint at 1:40 p.m. and its end at 2:58 p.m. are also indicated on the graph. It is to be noted that these observed time intervals level off to about 29.581 sec after the end of the eclipse, whereas in the morning they had started at about 29.570 sec, an appreciable difference inasmuch as as the times can be read to 0.00001 sec and are significant to about 0.0001 sec. The precision of the quartz-crystal–controlled oscillator in the Beckman EPUT (events per unit time) counter is one part in 10^8.

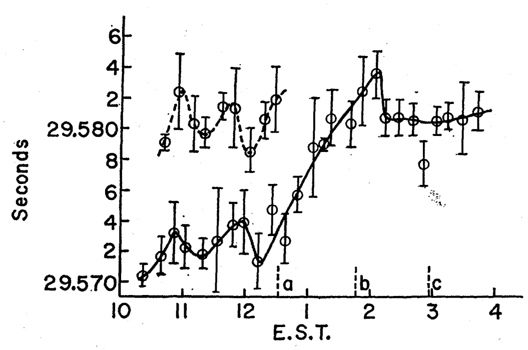

FIG. 1. Times required to traverse the fixed part of the path of oscillation (ordinates) vs the hour at which the observations were made, from about 10 a.m. until nearly 4 p.m. (abscissas). The full line shows the observations made on 7 March 1970, the day of the total eclipse. The short vertical dashed lines, *a*, *b*, and *c*, show the times of onset, midpoint, and endpoint of the eclipse. The curved dashed line shows the data taken two weeks later, 21 March, when the sun and moon were on opposite sides of the earth.

The irregularities occurring before the start of the eclipse might be considered accidental, except that data taken two weeks later at the same hour of the day (dashed curve) show corresponding humps—an indication, by the way, that the observations are reproducible. These maxima and minima may indicate a kind of gravitational fine structure which is reproducible even when the positions of the sun and moon relative to the earth are quite different. This apparent wavelike structure has been observed over the course of many years at our Harvard laboratory. It cannot be predicted on the basis of classical gravitational theory nor has it been observed in the quasistationary experiments underlying this theory (e.g., spring-operated gravimeters, seismographs, and interferometer devices).

Furthermore, the actual values of these observed times are greater at the later date. On that occasion the sun and moon were on the opposite sides of the earth, whereas during the eclipse they were in conjunction on the same side. This difference in relative position might well explain an increase in the observed times. These times are known to increase with increase in tension on the wire and therefore with gravitational attraction. Thus the moon pulling in the same direction as the earth could be expected to increase the observed times.

The difficulty is that this *relative* increase of about 2.7×10^{-4} recorded here would require an increase in tension of 1.2 kg, as calculated from the results of our paper on the period of a torsion pendulum[1] (see Fig. 5 therein). This is 5% of the total weight of the pendulum bob, 23.4 kg (51.5 lb), and is far greater than classical theories of gravitation can explain. Results of this order of magnitude have been consistently observed in Harvard over a period of 17 years. The greatest possible variation in g computed according to the older theories[3] for a given site on the earth's surface is 0.00016 cm/sec^2 or 1.6×10^{-5}%, so that *our results are about 10^5 times as great.* As shown in Fig. 1, the maximum average deviation of our results (which is a measure of our uncertainty) is about 2.5×10^{-2}%.

It is further to be noted that the greatest change

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[3] W. A. Heiskanen and F. A. Vening Meiness, *The Earth and Its Gravity Field* (McGraw-Hill, New York, 1958), p. 120.

occurs between the onset of the eclipse and its midpoint. This agrees qualitatively with the work of Allais[4] with a paraconical pendulum, where the change of azimuth increased substantially in the first half of the eclipse of 30 June 1954. Both these effects would seem to have a gravitational basis which cannot be explained by accepted classical theory.

Both our experimental findings and those of Allais cause one to question whether the classical laws of gravitation hold without modification.

CONCLUSION

Quantitative observations made with a precise torsion pendulum show, in agreement with many earlier, less precise recordings made at Harvard since 1953, that the times required to traverse a fixed fraction of its total angular path vary markedly during the hours before the eclipse and during its first half, i.e., up to its midpoint. Also the significant changes in these times do not coincide exactly with the astronomically determined onset, midpoint, and endpoint of the eclipse.

These variations are too great to be explained, on the basis of classical gravitational theory, by the relative change in position of the moon with respect to the earth and sun. This leads to the same conclusion arrived at by Allais—that classical gravitational theory needs to be modified to interpret his (and our) experimental results. Moreover, the findings with the torsion pendulum, the significant mass of which moves perpendicularly to the geogravitic vector, seem to indicate the possibility of a fine structure in these observations neither predicted nor recorded using the orthodox methods of quasi-stationary gravitational investigations.

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[4] Maurice F. C. Allais, Aerospace Eng. **18**, 46 (1959).

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Symmetry, Unitarity, and Geometry in Electromagnetic Scattering*

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Upon defining vector spherical partial waves $\{\boldsymbol{\psi}_n\}$ as a basis, a matrix equation is derived describing scattering for general incidence on objects of arbitrary shape. With no losses present, the scattering matrix is then obtained in the symmetric, unitary form $S=-\hat{Q}'^*\hat{Q}^*$, where (perfect conductor) $\hat{Q}$ is the Schmidt orthogonalization of $Q_{nn'}=(k/\pi)\int d\boldsymbol{\sigma}\cdot[(\nabla\times\mathrm{Re}\boldsymbol{\psi}_n)\times\boldsymbol{\psi}_{n'}]$, integration extending over the object surface. For quadric (separable) surfaces, Q itself becomes symmetric, effecting considerable simplification. A secular equation is given for constructing eigenfunctions of general objects. Finally, numerical results are presented and compared with experimental measurements.

INTRODUCTION

In earlier work, a matrix description of acoustic scattering was given, based on the *full* Helmholtz-Kirchhoff integral formula plus interior continuation arguments.[1] The present work constitutes the sequel for the vector electromagnetic case. Close parallels between the scalar and vector formalism are evident; we have attempted to accentuate them by using the same notation whenever possible.

Section I deals with derivation of the basic equations for the transition matrix. Incident illumination is constrained only to have no singularities in the interior volume of the scatterer; both volume- and surface-type scattering are considered for objects of general geometry, the surface of which need not be smooth (i.e., have continuous-turning normal). In Sec. II the scattering matrix is defined, and symmetry and unitary constraints are introduced into the original equation to obtain the solution in exactly symmetric, unitary form at any truncation. A secular equation is also discussed, from which one could alternatively proceed by constructing eigenfunctions appropriate to the given object. Our approach to the problem in terms of the scattering matrix in a spherical-wave basis is not new, incidentally, and has been described in some detail by Newton, for example.[2]

In Sec. III. a closer look is taken at matrix elements required in the computation. Constraints arising from the object geometry are discussed, and an important reduction found for objects bounded by quadratic surfaces, i.e., coordinate surfaces in one of the 11 systems in which the *scalar* Helmholtz equation is separable. Finally, numerical results are presented in Sec. IV for bodies with rotational symmetry, and compared with experimental measurements, as well as the Rayleigh and geometrical-optics approximations.

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[1] P. C. Waterman, J. Acoust. Soc. Am. **45**, 1417 (1969).

[2] R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill, New York, 1966), Chap. 2, pp. 101–104, pp. 189–190.