

# Fresnel drag formula derived from Hertz EM

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## Abstract

The Fresnel drag formula was verified by Fizeau's water-channel test.

$$\text{SoL} = c/n + v(1-1/n^2) \quad [1]$$

The theoretical derivation\* offered herein supports:

- Full aether dragging by fluids – speed of the fluid medium equals the speed of the entrained aether in the lab -  $V_m = V_a$
- The Hertz EM laws, which replace Maxwell's partial derivatives with total time derivatives, introducing thereby laws that are Galilean covariant(not Lorentzian)...and an additional parameter, a convective velocity ...of aether,  $V_a$ .
- The kinematic invariants of proper time, using distance and velocity to first order in  $v/c$ .
- The Fresnel drag equation, as confirmed by Fizeau.

The following uses Robert's Rules of reasoning, based on the scientific method and philo-realism.

Kinematics is a science of mathematical abstraction that uses kinematic variables to model generic patterns of motion. T. Phipps Jr has used the Hertz EM laws and kinematic invariants of first power of ten in  $v/c$  to develop a general solution to EM tests of motion, such as the Fresnel formula described here.

### **Proper time: a key kinematic invariant.**

A key component of the derivation is the use of proper time, the time a clock measures in the rest frame of an observer, and the coordinate time as measured by a clock in motion relative to the rest frame observer. The two are related as:

$$d^2\tau = dt^2 - (dx^2 + dy^2 + dz^2)/c^2$$

Einstein generalized this proper time to be the space-time interval,  $\tau \Rightarrow \sigma$ , the fundamental metric of motion in special and general relativity.

For finite clock ticks,

$$t = \tau / \sqrt{1 - v^2/c^2} \quad [2]$$

where  $v = \sqrt{x^2 + y^2 + z^2}/t = r/t$ .

In the application,  $\tau_{ph}$  will be the proper time computed for the photon  $ph$  frame of reference;  $v$  will equal either  $V_{ph,L}$  or  $V_{ph,m}$ , the relative speed of the photon frame  $ph$  to either the medium  $m$  or lab  $L$  reference frames, where coordinate time  $t$  can be measured.

The Fresnel formula predicts the speed of light,  $SoL$ , or speed of photons  $ph$ , in a fluid medium  $m$ , by adding velocity of photons relative to a medium,  $V_{ph,m}$ , plus the velocity of the medium relative to the lab,  $V_{m,L}$ .

This aether model of a fluid recognizes that aether motion can exist unstructured in the free unbound state as  $V_a$ , or as motion of structured motion of bound aether particles formed from the unbound state,  $V_m$ . This is similar to the phase difference between water and ice, where ice is formed as a solid from water, not as a separate type of object.

### **Tests of $V_a = V_m$ .**

The prevalent concept of dragging in physics resembles a log floating in a river, being dragged by a different material, water. The concept promoted here is that particles/objects moving in free aether are actually the collective motion of bound and unbound aether, as when ice floats in a river.

Tests consistent with full dragging,  $V_a = V_m$ , are the tests of Sagnac, Dufour&Prunier and Ruyong Wang, which all found that solids moving at  $V_m$  cause the ambient speed of light to become  $SoL = c + V_a$ , where the light speed is boosted by  $V_m$  by exactly the same amount, so  $V_a = V_m$ . What is true for solid dragging is asserted as true for fluids,  $V_a = V_m$ .

For example, if aether and medium were separate states of being, then  $SoL$  in medium  $m$  measured in the lab frame would be

$$SoL = V_{ph,a} + V_{a,m} + V_{m,L}$$

But full dragging implies that

$$V_a = V_m \text{ and so } V_{a,m} = 0.$$

This assumption will be supported when the assumption is used to correctly derive the Fresnel formula.

### **Derivation of the Fresnel Formula**

The speed of light in medium  $m$  measured in the lab frame is

$$SoL = V_{ph,m} + V_{m,L} \text{ to first order in } v/c. \quad [3]$$

If the distance the photons(light) travels in the lab is  $D_{ph,L}$ , then

$$D_{ph,L} = V_{ph,L} * t \quad [4]$$

Combining Eqs 2 and 4,

$$D_{ph,L} = V_{ph,L} * \tau_{ph} / \sqrt{1 - v^2/c^2}$$

Just as with velocity, the first order equation for  $D_{ph,L}$  is

$$D_{ph,L} = D_{ph,L}/t_L \cdot \sqrt{1 - v^2/c^2} = D_{ph,m} + D_{m,L} \quad [5]$$

Dividing  $D_{ph,L}$  by the invariant  $\tau_{ph}$ ,

$$(D_{ph,m} + D_{m,L})/\tau_{ph} = D_{ph,m}/t_m \cdot \sqrt{1 - v^2/c^2} + D_{m,L}/t_L \cdot \sqrt{1 - v^2/c^2}$$

or

$$\underline{V_{ph,L}}/\sqrt{1 - v^2/c^2} = V_{ph,m}/\sqrt{1 - v^2/c^2} + V_{m,L}/\sqrt{1 - v^2/c^2}$$

The solution for  $\underline{V_{ph,L}}$ , highlighted above, leads to the quadratic equation :

$$(1)\underline{V_{ph,L}} + \underline{V_{ph,L}}[2V_{m,L} \cdot v/c^2 - 2V_{m,L}] + [(1 - v^2/c^2)V_{m,L}^2 - V_{ph,m}^2] = 0$$

When simplified the solution is

$$\underline{V_{ph,L}} = V_{m,L} (1 - v^2/c^2) \pm V_{ph,m} \sqrt{[1 - v^2/c^2](1 - v^2/c^2)} \quad [6]$$

All velocities are positive; the result could be negative if the negative root were included... so only the + root matches the physical situation.

### Application of the light speed in the lab, $V_{ph,L}$ , to Fresnel's formula

Let's simplify the variables to common (Phipps) notation:

The medium  $m$  has index of refraction  $n$  so  $V_{ph,m} = c/n$

The medium speed in the lab is  $V_{m,L} = v$ ...also  $v = V_a$ , the aether speed.

The photon speed in the lab is  $V_{ph,L} = u$

Variable substitution into [6] yields

$$\begin{aligned} u &= v (1 - (c/n)^2/c^2) + (c/n) \sqrt{[1 - v^2/c^2](1 - (c/n)^2/c^2)} \\ &= v (1 - 1/n^2) + (c/n) \sqrt{[1 - (v^2/c^2)](1 - 1/n^2)} \end{aligned} \quad [7]$$

The second term differs from the Fresnel form...but remember, the derivation is valid to first order in  $v/c$ . The sqrt has a binomial expansion of the form

$$(1-x)^k = 1 - kx + k(k-1)x^2/2! - k(k-1)(k-2)x^3/3! + ..$$

When  $k = 1/2$  and  $x = (v^2/c^2)(1 - 1/n^2)$  in the binomial series, [7] is replaced by

$$u = c/n + v (1 - 1/n^2) - (v^2/2cn)(1 - 1/n^2) - (v^4/8c^3n)(1 - 1/n^2)^2 + .....$$

or

$$u = c[1/n + (1 - 1/n^2)v/c - (v^2/c^2 2n)(1 - 1/n^2) - (v^4/c^4 8n)(1 - 1/n^2)^2 + .....$$

To order  $v/c$ ,  $u$  is the Fresnel formula.... eq [1] = eq[8]

$$u \approx c/n + v (1 - 1/n^2) \quad [8]$$

### Comparison of aether/Hertz model with Special Relativity's prediction for Fresnel dragging

Note: this is only of historical interest; both axioms of SR have been refuted by Ruong Wang's Linear Sagnac test in 2005.

In SR:

$$u = (c/n + v) / (1 + v/cn) = c/n + v(1 - 1/n^2) - (v^2/cn)(1 - 1/n^2) + (v^3/c^2n^2)(1 - 1/n^2) + \dots$$

or

$$u = c[1/n + (v/c)(1 - 1/n^2) - (v^2/c^2n)(1 - 1/n^2) + (v^3/c^3n^2)(1 - 1/n^2) + \dots] \quad [9]$$

The difference in the two models,  $U$  is only  $(v^2/2cn)(1 - 1/n^2)$  to second order in  $v/c$ , which tests the threshold of detecting the difference with current precision. And again, the issue is purely academic, since SR fails experimental testing and rational analysis.

## Summary

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The claim of Special Relativity to derive the Fresnel Drag equation is a consequence of Galilean relativity's replacement of  $t' = t$  with the proper time,  $\tau = t \cdot \sqrt{1 - v^2/c^2}$ ...corresponding to time dilation in the Lorentz transform. But in the Galilean model,  $d' = d$ , at odds with the length contraction of Lorentz relativity.

The time components explains the similarity in the Fresnel formula; the difference in the space components accounts for the difference in second order of  $v/c$ .

\*The derivation here follows from the research of T. Phipps, Jr. into the kinematic invariants of Hertz's ElectroMagnetism. Ref: *Heretical Verities*, 1986, P152-156