seems justifiable to assume that the transition to the B^{12} ground state is allowed in the usual sense.

The next question is that of transitions to excited, bound states of B^{12} in the μ-capture process. The fact that only 13% of all absorptions lead to bound states of B^{12} implies that high excitations are favored. Appreciable formation of excited states would wash out the orientation in the ground state because of the smearing over magnetic quantum numbers that occurs in the process of de-excitation by γ-ray emission. Fortunately, the situation here seems favorable. There are only four known excited states below the threshold for particle emission.[9] While no firm arguments can be made, what is known of the spins and parities of these states makes it seem probable that the large majority of μ-capture events leading to *bound* states of B^{12} actually go directly to the ground state.

Another effect which must be considered is possible depolarization of the B^{12} nucleus due to hyperfine interaction with the atomic electrons. Rough estimates indicate that the atom is probably ionized due to recoil at the instant of absorption of the μ meson. If the atom is always ionized and then re-forms again after it stops, we can calculate the depolarization under the assumption that the fine-structure substates are populated statistically. This gives, for the resultant B^{12} polarization,

$$\langle \mathbf{J} \rangle = \tfrac{2}{3}(0.54)\langle \boldsymbol{\sigma} \rangle = 0.36\langle \boldsymbol{\sigma} \rangle. \quad (2)$$

Thus, if $|\langle \boldsymbol{\sigma} \rangle|$ equals 15%, the final polarization $|\langle \mathbf{J} \rangle|$ of the B^{12} is probably closer to 5% than to the value of 10% given above.

There is an additional depolarization due to the environment in which the B^{12} atom finds itself. But the relaxation time for this effect in graphite is presumably longer than the mean life of B^{12} since metals show relaxation times of the order of tens of milliseconds. In any event, such solid-state effects can be essentially eliminated by a suitable choice of organic material as target.

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[1] T. D. Lee and R. P. Feynman, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics*, April, 1957 (to be published).

[2] R. L. Garwin, L. Lederman, and co-workers at Columbia have observed the longitudinal polarization of the electrons in μ decay (L. Lederman, in reference 1). On the basis of a theory of μ decay the direction of the μ meson's polarization can then be inferred.

[3] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[4] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).

[5] T. N. K. Godfrey, Princeton University thesis, 1954 (unpublished).

[6] J and J' are the final and initial nuclear spins respectively, while $\lambda_{J'J}$ is a numerical factor defined in the appendix of Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957). For a transition with $\Delta J=0$, the polarization of the daughter nucleus is of the form of Eq. (1) with the factor $\lambda_{J'J}$ replaced by $N/(1+b)$, the coefficients N and b being given in the above reference (with $E_e=m$, and the sign appropriate for electrons).

[7] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[8] Since the larger fraction of μ mesons bound in carbon decay before nuclear capture, the directional asymmetry of the prompt electrons can be used to measure the magnitude of $\langle \boldsymbol{\sigma} \rangle$ directly, while the asymmetry of the delayed electrons will determine $\langle \mathbf{J} \rangle$.

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Magnetic Dipole Moment of the Electron

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THE fourth-order radiative corrections to the magnetic dipole moment of the electron were calculated by Karplus and Kroll in 1949.[1] Their result is contained in the complete expression for the moment,

$$\mu_e/\mu_0 = 1+(\alpha/2\pi)-2.973(\alpha^2/\pi^2) = 1.0011454, \quad (1)$$

where μ_0 is the Bohr magneton.

The calculation has been redone in the present instance using the mass-operator formalism of Schwinger.[2] We consider a single electron moving in a constant (in space and time) electromagnetic field. The expectation value of the mass operator in the lowest state represents the self or proper energy of the electron. The magnetic moment is identified from that part of the self-energy which is linear in the external field.

The electron Green's function G, the photon Green's function $\mathcal{G}$, and the interaction operator Γ, which appear in the symbolic expression for the mass operator,

$$M = m_e + ie^2 \operatorname{Tr}\gamma G \Gamma \mathcal{G},$$

are computed in the presence of (as functions of) the external field. To do this it is sufficient to replace the electron's momentum operator, p, where it occurs, by by the combination $\Pi = p - eA$, provided that full account is taken of the commutation properties of Π. Units are such that $\hbar = c = 1$. Renormalized quantities are used throughout the perturbation calculation.

The fourth-order contribution to the moment is found to be

$$\frac{\mu_e^{(4)}}{\mu_0} = \frac{\alpha^2}{\pi^2}\left(\frac{197}{144}+\frac{\pi^2}{12}+\tfrac{3}{4}\zeta(3)-\tfrac{1}{2}\pi^2\ln 2\right) = -0.328\frac{\alpha^2}{\pi^2}, \quad (2)$$

where $\zeta(3)$ is the Riemann zeta function of 3. Thus

$$\mu_e/\mu_0 = 1.0011596.$$

The discrepancy between (1) and (2) has been traced to the term $\mu^{\mathrm{I}}+\mu^{\mathrm{II}c}$ of Karplus and Kroll. In other words, terms $\mu^{\mathrm{II}e}$ and $\mu^{\mathrm{II}a}+\mu^{\mathrm{II}d}$ appear unchanged in the new result. A further point-by-point comparison of the two answers is not readily accomplished because the grouping of the terms differs markedly in the two cases. The present calculation has been checked several times and all of the auxiliary integrals have been done in at least two different ways.

The theoretical magnetic moment may be compared with the experimental moment; it is also used in determining the fine-structure constant α; and it contributes to the Lamb shift. The magnetic moment is measured by determining μ_e/μ_p and μ_p/μ_0, where μ_p is the proton moment. The measurements of μ_e/μ_p have been quite accurate.[3] On the other hand, there are two conflicting experimental determinations[4,5] of μ_p/μ_0, which result in two different values for the magnetic moment:

References	μ_e/μ_0
3 and 4	1.001146±0.000012
3 and 5	1.001165±0.000011.

The theoretical value[6] for the hyperfine splitting in hydrogen is proportional to the quantity

$$\alpha^2(\mu_p/\mu_0)(\mu_e/\mu_0)=\alpha^2(\mu_p/\mu_e)(\mu_e/\mu_0)^2.$$

Since there is agreement on the experimental value of μ_p/μ_e, we use the second form, together with the present value of α,[7] to determine a new value. This turns out to be

$$1/\alpha=137.039.$$

The theoretical Lamb shifts in hydrogen, deuterium, and singly ionized helium are affected by the changes in both α and μ_e. Incorporating these changes into the calculations of Salpeter,[8] along with the proton-recoil recoil corrections of Fulton and Martin,[9] and the proton-structure corrections of Aron and Zuchelli,[10] we obtain the following results in Mc/sec:

	Theoretical	Experimental	Reference
S_H	1057.99±0.13	1057.77± 0.10	11
S_D	1059.23±0.13	1059.00± 0.10	11
S_D-S_H	1.24±0.04	1.23± 0.15	11
S_{He}	14055.9 ±2.1	14043 ±13.0	12

The experimental values[11,12] have been listed for comparison. There remain several uncomputed theoretical effects which are expected to be of the same order of magnitude as the indicated theoretical uncertainties.

The magnetic moment of the μ meson, as computed by Suura and Wichmann, and Petermann,[13] would be changed to read

$$\mu_\mu=\left(1+\frac{\alpha}{2\pi}+0.75\frac{\alpha^2}{\pi^2}\right)\frac{e\hbar}{2m_\mu c}.$$

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Note added in proof.—Petermann[14] has placed upper and lower bounds on the separate terms of Karplus and Kroll. He finds that their value for μIIc does not lie within the appropriate bounds. Assuming the other terms to be correct, he concludes that $\mu^4/\mu_0=(-0.53\pm0.37)\alpha^2/\pi^2$, which is consistent with the value presented above.

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Allowed Capture-Positron Branching Ratios

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IN a previous paper,[1] tables of allowed K capture-positron branching ratios were presented. However, it was pointed out by Wapstra[2] and Perlman[3] that numerical errors existed in the table. These errors appear in the first, third, and fifth columns of Table II of reference 1, each entry of which should be multiplied by the factors of 0.5018, 1.2244, and 0.6462, respectively. In Table I of this communication, the corrected table of allowed K to positron branching ratios is given. In this work, the effect of the finite nuclear size on the bound electron wave functions, which was ignored in reference 1, was taken into account.[4] This effect, which is negligible for low Z, reduces the branching ratio by about 10% for $Z=84$ and by about 15% for $Z=92$. Effects of finite size on the positron wave functions was ignored, since it is a considerably smaller effect.[5]

As in reference 1, the bound electron wave functions were taken from Reitz's thesis[6] except for $Z=16$, for

TABLE I. Allowed K to positron branching ratios.

W_0/mc^2 \ Z	16	29	49	84	92
1.28	46.6	707	1.208×10^4	4.56×10^5	8.92×10^5
1.44	8.65	112	1.58×10^3	4.50×10^4	8.41×10^4
1.60	2.83	33.6	425	1.03×10^4	1.84×10^4
1.76	1.24	13.9	164	3.57×10^3	5.01×10^3
1.92	0.641	6.91	77.6	1.57×10^3	2.67×10^3
2.08	0.373	3.91	42.3	807	1.36×10^3
2.40	0.190	1.60	16.4	289	479
2.88	0.0613	0.597	5.86	96.4	158
3.84	0.0169	0.160	1.51	23.6	39.0
4.80	7.00×10^{-3}	0.0648	0.603	9.10	15.7
5.76	3.56×10^{-3}	0.0328	0.302	4.82	8.05
6.72	2.06×10^{-3}	0.0188	0.173	2.82	4.75
7.68	1.30×10^{-3}	0.0118	0.109	1.80	3.06
8.64	8.85×10^{-4}	7.93×10^{-3}	0.0729	1.23	2.10
9.60	6.29×10^{-4}	5.60×10^{-3}	0.0513	0.879	1.52
10.56	4.48×10^{-4}	4.09×10^{-3}	0.0377	0.652	1.13
11.52	3.37×10^{-4}	3.07×10^{-3}	0.0281	0.498	0.869
12.48	2.60×10^{-4}	2.37×10^{-3}	0.0219	0.393	0.685