April 2018 meeting of the American Physical Society, Columbus, Ohio. Poster presented on 14th April, 2018.
http://meetings.aps.org/Meeting/APR18/Session/F01.62ost

Gravitational Waves: Propagation Speed is <u>Co-ordinate Dependent</u>

Stephen J. Crothers

1. Proof: Linearise Einstein's field equations by,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

where the $h_{\mu\nu} << 1$ and $\eta_{\mu\nu}$ represents the Galilean values (1,-1,-1,-1). The $h_{\mu\nu}$ and their derivatives are small, so all products of them are neglected. Contract the Riemann-Christoffel curvature tensor $R_{\rho\mu\nu\sigma}$ to obtain the Ricci tensor:

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right) + \Gamma_{\beta\rho\sigma}\Gamma^{\beta}_{\mu\nu} - \Gamma_{\beta\rho\nu}\Gamma^{\beta}_{\mu\sigma}\right] \tag{2}$$

The Γ terms are products of the $g_{\mu\nu}$ and their first derivatives, therefore neglected, so,

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right)\right] \tag{3}$$

which can be broken into two parts,

$$R_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}g_{\mu\nu,\sigma\rho} + \frac{1}{2}g^{\sigma\rho}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right) \tag{4}$$

Choose co-ordinates x^{α} so that the second part of eq.(4) vanishes:

$$R_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}g_{\mu\nu,\sigma\rho} = \frac{1}{2}g^{\sigma\rho}\frac{\partial^2 g_{\mu\nu}}{\partial x^{\rho}\partial x^{\sigma}} \tag{5}$$

$$g^{\sigma\rho}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right) = 0 \tag{6}$$

According to eq.(1), $g_{\mu\nu,\beta} = h_{\mu\nu,\beta}$ and so on for higher derivatives, hence,

$$R_{\mu\nu} = \frac{1}{2}\eta^{\sigma\rho}h_{\mu\nu,\sigma\rho} = \frac{1}{2}\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^{\rho}\partial x^{\sigma}} \tag{5b}$$

$$\eta^{\sigma\rho}\left(h_{\sigma\rho,\mu\nu} - h_{\nu\rho,\sigma\mu} - h_{\mu\sigma,\nu\rho}\right) = 0 \tag{6b}$$

Contracting eq.(5b) yields the Ricci scalar:

Crothers, S.J., A Critical Analysis of LIGO's Recent Detection of Gravitational Waves Caused by Merging Black Holes, *Hadronic Journal*, n.3, Vol. 39, 2016, pp.271-302, http://vixra.org/pdf/1603.0127v5.pdf

$$R = \eta^{\nu\mu} R_{\mu\nu} = \frac{1}{2}\eta^{\nu\mu}\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^\sigma \partial x^\rho} = \frac{1}{2}\eta^{\sigma\rho}\frac{\partial^2 h}{\partial x^\sigma \partial x^\rho} \tag{7}$$

Einstein's field equations (without cosmological constant) are,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} \tag{8}$$

In terms of $h_{\mu\nu}$ these become, using eq.(5b) and eq.(7),

$$\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} - \frac{1}{2}\eta^{\sigma\rho}\frac{\partial^2 h}{\partial x^\sigma \partial x^\rho} = -2\kappa T_{\mu\nu} \tag{8b}$$

The d'Almbertian operator $\Box$ is defined as,

$$\Box \equiv -\eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} \tag{9}$$

Hence,

$$\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{10}$$

where ∇ is the differential operator *del* (or *nabla*), defined as,

$$\nabla \equiv \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle .$$

Taking the dot product of *del* with itself gives the Laplacian operator ∇^2,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} .$$

Setting $x^0=ct$, $x^1=x$, $x^2=y$, $x^3=z$, eq.(8b) can be written as,

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu} h\right) = 2\kappa T_{\mu\nu} \tag{11}$$

These are the linearised field equations. They are subject to the condition (6b), which can be condensed to the following condition,

$$\frac{\partial}{\partial x^\alpha}\left(h^\alpha_\mu - \frac{1}{2}\delta^\alpha_\mu h\right) = 0 \tag{6c}$$

Using eq.(9), eq.(5b) can be written as,

$$\Box\, h_{\mu\nu} = -2R \tag{12}$$

For empty space this becomes,

$$\Box\, h_{\mu\nu} = 0 \tag{13}$$

which by eq.(10) describes a wave propagating at the speed of light *in vacuo*. But the speed of the waves is co-ordinate dependent, as the constraint at eq.(6) attests: different co-ordinates, different speeds. Einstein chose a set of co-ordinates that yields his presumed speed of propagation: an example of the logical fallacy of assuming as premise that which is to be demonstrated (*petitio principii*).

2. General Relativity: Its Violation of the Usual Conservation Laws for a Closed System.

"We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter'. Our use of the word therefore includes not only matter in the ordinary sense, but the electromagnetic field as well." **Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916), §14**

"It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum of matter alone." **Einstein, A., The Meaning of Relativity, expanded Princeton Science Library Edition, (2005)**

$T_\sigma^\alpha \equiv$ energy-momentum of Einstein's matter alone

$t_\sigma^\alpha \equiv$ energy-momentum pseudotensor of Einstein's gravitational field alone, defined,

$$\kappa t_\sigma^\alpha = \frac{1}{2}\delta_\sigma^\alpha g^{\mu\nu}\Gamma_{\mu\beta}^\lambda\Gamma_{\nu\lambda}^\beta - g^{\mu\nu}\Gamma_{\mu\beta}^\alpha\Gamma_{\nu\sigma}^\beta \tag{2.1}$$

"*The quantities t_σ^α we call the 'energy components' of the gravitational field*". **Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916), §15**

But t_σ^α is not a tensor and is co-ordinate dependent, contrary to the basic tenet of General Relativity.

"*It is to be noted that t_σ^α is not a tensor.*" **Einstein, A., the Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916), §15**

Allegedly, t_σ^α acts 'like a tensor', for linear transformations of co-ordinates under certain conditions. Einstein then takes an ordinary (not tensor) divergence,

$$\frac{\partial t_{\sigma}^{\alpha}}{\partial x_{\alpha}} = 0 \tag{2.2}$$

and proclaims,

"*This equation expresses the law of conservation of momentum and of energy for the gravitational field.*" **Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916), §15**

Einstein's total energy-momentum equation for his field and sources is,

$$E = \left(t_{\mu}^{\sigma} + T_{\mu}^{\sigma}\right) \tag{2.3}$$

This is not a tensor equation, so again only an ordinary divergence,

$$\frac{\partial\left(t_{\mu}^{\sigma} + T_{\mu}^{\sigma}\right)}{\partial x_{\sigma}} = 0 \tag{2.4}$$

by which Einstein proclaimed:

"*Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied. ... here, instead of the energy components* t_{μ}^{σ} *of the gravitational field, we have to introduce the totality of the energy components of matter and gravitational field.*" **Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916), §17**

The mathematical error is profound. Contract Einstein's pseudotensor:

$$\kappa t_{\alpha}^{\alpha} = \kappa t = \frac{1}{2}\delta_{\alpha}^{\alpha} g^{\mu\nu}\Gamma_{\mu\beta}^{\lambda}\Gamma_{\nu\lambda}^{\beta} - g^{\mu\nu}\Gamma_{\mu\beta}^{\alpha}\Gamma_{\nu\alpha}^{\beta} \tag{2.4}$$

Thus the invariant t is *a first-order intrinsic differential invariant,* i.e. t depends solely upon the components of the metric tensor and their first derivatives. But the pure mathematicians proved in 1900 that first-order intrinsic differential invariants do not exist (**Ricci-Curbastro, G., Levi-Civita, T., Méthodes de calcul différentiel absolu ET leurs applications, *Matematische Annalen*, B. 54, p.162, 1900**). Thus, by *reductio ad absurdum*, Einstein's pseudotensor is a meaningless concoction of mathematical symbols. It cannot therefore be assigned to anything in physics or to make any calculations. Nevertheless, Einstein and cosmologists assign and calculate.

Consider the following two equivalent forms of Einstein's field equations,

$$R_{\nu}^{\mu} = -\kappa\left(T_{\nu}^{\mu} - \frac{1}{2}Tg_{\nu}^{\mu}\right) \tag{2.6}$$

$$T_{\nu}^{\mu} = -\frac{1}{\kappa}\left(R_{\nu}^{\mu} - \frac{1}{2}Rg_{\nu}^{\mu}\right) \tag{2.7}$$

By eq.(2.6), according to Einstein, if $T_\nu^\mu = 0$ then $R_\nu^\mu = 0$. But by eq.(2.7), if $R_\nu^\mu = 0$ then $T_\nu^\mu = 0$. In other words, R_ν^μ and T_ν^μ must vanish identically: 0 = 0. If there are no material sources then there is no gravitational field, and no universe. Consequently, Einstein's field equations must take the following,

$$\frac{G_\nu^\mu}{\kappa} + T_\nu^\mu = 0 \tag{2.8}$$

$$\text{where } G_\nu^\mu = R_\nu^\mu - \frac{1}{2} R g_\nu^\mu$$

Comparing this to eq.(2.3) the G_ν^μ / κ constitute the energy-momentum components of Einstein's gravitational field alone: after all, G_ν^μ describes Einstein's gravitational field. Equation (2.8) also constitutes the total energy-momentum equation for Einstein's gravitational field and its material sources combined.

"*space as opposed to 'what fills space', which is dependent on the coordinates, has no separate existence*" **Einstein, A., 'Relativity and the problem of space', Appendix 5 in the 15th edition of Relativity: the Special and the General Theory, Methuen, London, (1954), pp. 135-157**

"*I wish to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality.*" **Einstein, A., Preface to the 15th edition of Relativity: the Special and the General Theory, Methuen, London, (1954)**

Unlike eq.(2.3), eq.(2.8) is a tensor equation. Its tensor divergence is zero and therefore constitutes the only conservation law for Einstein's gravitational field and its material sources combined. Since the total energy-momentum eq.(2.8) is always zero, and the G_ν^μ / κ and the T_ν^μ must vanish identically because spacetime and matter have no separate existence in General Relativity, gravitational energy cannot be localised, i.e. no possibility of gravitational waves. Since the total energy-momentum is always zero the usual conservation laws for energy and momentum of a closed system cannot be satisfied. General Relativity is therefore in conflict with a vast array of experiments on a fundamental level. The so-called 'cosmological constant' can be easily included as follows,

$$\frac{\left(G_\nu^\mu + \lambda g_\nu^\mu\right)}{\kappa} + T_\nu^\mu = 0 \tag{2.9}$$

In this case the energy-momentum components of Einstein's gravitational field are $\left(G_\nu^\mu + \lambda g_\nu^\mu\right)/\kappa$. When G_ν^μ or T_ν^μ is zero, all must vanish identically.

The mysterious 'dark energy' is arbitrarily attributed to λ by cosmologists. But according to Einstein, λ is not a material source for his gravitational field, instead vaguely implicated in his gravitational field alone:

"*... by introducing into Hamilton's principle, instead of the scalar of Riemann's tensor, this scalar increased by a universal constant*" **Einstein, A., Cosmological Considerations on the General Theory of Relativity, Sitzungsberichte der Preussischen Akad. d. Wissenschaften, (1917), §4**

By eq.(2.9) λg_ν^μ is part of the energy-momentum of the gravitational field, which however necessarily vanishes when $T_\nu^\mu = 0$. Expand eq.(2.9):

$$\frac{\left[R_\nu^\mu - \frac{1}{2}(R - 2\lambda)g_\nu^\mu\right]}{\kappa} + T_\nu^\mu = 0 \qquad (2.9b)$$

Einstein's "*scalar increased by a universal constant*" is the term –(R–2λ)/2. Again, Einstein's field equations "*in the absence of matter*", $R_\nu^\mu = 0$, have no physical meaning. Therefore the 'Schwarzschild solution' also has no physical meaning. The 'cosmological constant' also falls afoul of de Sitter' empty universe $\left(R_\nu^\mu = \lambda g_\nu^\mu\right)$, which possesses spacetime curvature (gravity) but contains no matter ($T_\nu^\mu = 0$), and is therefore physically meaningless. Consequently, the theories of black holes and gravitational waves are invalid ([1] **Crothers, S.J., On Corda's 'Clarification' of Schwarzschild's Solution, *Hadronic Journal*, Vol. 39, 2016, http://vixra.org/pdf/1602.0221v4.pdf** [2] **Crothers, S. J., General Relativity: In Acknowledgement Of Professor Gerardus 't Hooft, Nobel Laureate, 4 August, 2014, http://vixra.org/pdf/1409.0072v9.pdf**).

3. The Mathematical Theory of Black Holes: its Violation of the Rules of Pure Mathematics and Logic

Presumably the gravitational waves reported by LIGO-Virgo are present inside some Big Bang expanding universe as there has been no report that Big Bang cosmology has been abandoned. The LIGO-Virgo Collaborations have stated of first 'discovery':

"*It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole.*" **Abbott, B.P. *et al.*, Observation of Gravitational Waves from a Binary Black Hole Merger, *PRL* 116, 061102 (2016)**

All black hole metrics are solutions to corresponding specific forms of Einstein's non-linear field equations. Gravitational waves are however obtained from a linearisation of Einstein's field equations, combined with a selection of co-ordinates that produce the presumed propagation speed c. Because General Relativity is a nonlinear theory, the Principle of Superposition does not hold. Let $\boldsymbol{X}$ be some black hole universe and $\boldsymbol{Y}$

be some Big Bang universe. Then the linear combination (i.e. superposition) $\boldsymbol{X} + \boldsymbol{Y}$ is not a universe. Indeed, $\boldsymbol{X}$ and $\boldsymbol{Y}$ pertain to completely different sets of Einstein field equations having absolutely nothing to do with one another. Similarly if $\boldsymbol{W}$ and $\boldsymbol{Z}$ are both black hole universes, be they the same or not, and if $\boldsymbol{U}$ and $\boldsymbol{V}$ are Big Bang universes, be they the same or not. A black hole universe cannot co-exist with any other black hole universe or with any Big Bang universe.

Black Hole Universes	**Big Bang Universes**
(1) Have no k-curvature.	(1) Have a k-curvature.
(2) Are spatially infinite.	(2) Are either spatially finite ($k = 1$) or spatially infinite ($k = -1$ or $k = 0$).
(3) Are eternal.	(3) Not eternal (~13.8 billion years old)
(4) Contain only one mass.	(4) Contain arbitrarily many masses.
(5) Are not expanding (*not* non-static).	(5) Are expanding (non-static).
(6) Are asymptotically flat.	(6) Are not asymptotically anything.
(7) Cannot be superposed with anything.	(7) Cannot be superposed with anything.

Since a black hole universe is a solution to a specific set of Einstein's nonlinear field equations it is not possible to extract from it any gravitational waves produced from linearised field equations. No gravitational waves can in fact be extracted from Einstein's nonlinear field equations. Superposing solutions obtained from the nonlinear system with those from the linearised system violates the mathematical structure of General Relativity. Accordingly, gravitational waves cannot exist in any black hole universe. Neither can they exist in any Big Bang universe because all Big Bang models are in fact single mass universes by mathematical construction (uniform macroscopic density and pressure). Nonetheless the LIGO-Virgo Collaborations superpose everything.

In can be shown that the mathematical theory of black holes latently requires that the absolute value of a real number must take on values less than zero, which is impossible ([1] **Crothers, S.J., The Painlevé-Gullstrand 'Extension' - A Black Hole Fallacy, *American Journal of Modern Physics,* 5, Issue 1-1, 2016, pp.33-39, http://vixra.org/pdf/1512.0089v1.pdf** [2] **Crothers, S. J., On the Generation of Equivalent 'Black Hole' Metrics: A Review, *American Journal of Space Science*, v.3, i.2, pp.28-44, July 6, 2015, http://vixra.org/pdf/1507.0098v1.pdf**).

4. The Secret Methods of LIGO

For their 'discovery' the LIGO-Virgo teams numerically manufactured 250,000 model waveforms ('templates') from meaningless equations. A 'generic' noise was initially reported by LIGO, after which LIGO's computers extracted GW150914 from the LIGO manufactured meaningless database as a curve of best fit to the noise:

"The initial detection was made by low-latency searches for generic gravitational-wave transients [41] and was reported within three minutes of data acquisition [43]. Subsequently, matched-filter analyses that use relativistic models of compact binary waveforms [44] recovered GW150914 as the most significant event from each detector for the observations reported here." **Abbott, B.P. *et al.*, Observation of Gravitational Waves from a Binary Black Hole Merger, *PRL* 116, 061102 (2016)**

With such powerful computing resources and so many degrees of freedom it is possible to best fit just about any LIGO instability with an element of its numerically manufactured database of curves. The LIGO-Virgo Collaborations have managed to best fit a numerically manufactured curve for and to entities that do not exist. This amplifies the futility of applying numerical and perturbation methods to generate desired outcomes.

There are no known Einstein field equations for two or more masses and hence no known solutions thereto. There is no existence theorem by which it can even be asserted that Einstein's field equations contain latent capability for describing configurations of two or more masses. General Relativity cannot account for the simple experimental fact that two fixed suspended masses approach one another upon release. For precisely these reasons all Big Bang models make the universe a single mass: an ideal indivisible fluid of uniform macroscopic density and pressure that permeates the entire universe. (**[1] Crothers, S. J., 'Flaws in Black Hole Theory and General Relativity', *Proceedings of the XXIXth International Workshop on High Energy Physics*, Protvino, Russia, 26-28 June 2013, http://vixra.org/pdf/1308.0073v1.pdf [2] Crothers, S. J. "On the Invalidity of the Hawking-Penrose Singularity 'Theorems' and Acceleration of the Universe with Negative Cosmological Constant", *Global Journal of Science Frontier Research Physics and Space Science*, Volume 13 Issue 4, Version 1.0, 2013, http://vixra.org/pdf/1304.0037v3.pdf [3] Crothers, S. J., On the Generation of Equivalent 'Black Hole' Metrics: A Review, *American Journal of Space Science*, v.3, i.2, pp.28-44, 2015, http://vixra.org/pdf/1507.0098v1.pdf [4] Crothers, S.J., LIGO at the University of Queensland, http://vixra.org/pdf/1612.0209v1.pdf**