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GENERAL RELATIVITY THEORY OF GRAVITATION AND EXPERIMENTS

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1. *Relativity and Experiment.*—The so-called "restricted relativity" has been carefully checked in a variety of experiments. There is, however, still some uncertainty about the role played by potential energies in the famous Einstein relation:

$$E = Mc^2. \tag{1}$$

Conservation of energy requires not only kinetic energy but also potential energy; nevertheless, the corresponding mass is usually ignored. These questions were discussed in two recent papers[1] and some possible experimental checks were suggested.

The difficulty is closely related to the problem of the junction between classical mechanics and relativity. "This is very simple," says the mathematician. "Just imagine that the velocity of light c becomes infinite, and relativity reduces to classical mechanics." Here, the physicist strongly objects. "Never," says he, "can velocity c become infinite. Your method is meaningless. If c could be infinite, then the energy E in equation (1) would also be infinite!"

Another question comes up immediately. In classical mechanics, the mass is always definite and positive. The energy, on the contrary, can be either positive or negative; it is defined but for an arbitrary constant: only the differences in energy have a meaning, but the absolute value of the energy does not play a direct role. Shall we admit the possibility of *negative masses*, which would correspond to negative energies?

2. *Gravitation and General Relativity.*—When it comes to "general relativity," many new difficulties should not be overlooked, and the experimental meaning of the theory is far from clear. P. W. Bridgman[2] has already written that "Einstein did not carry over into his general relativity theory the lessons and insights which he himself had taught us in his special theory."

Operational analysis was first applied by Einstein in his famous discussions about the meaning of length and time measurements. This was the basis of "special relativity." But when he attacked "general relativity," Einstein did not follow a similar procedure; he attempted to guess how to introduce gravity laws in relativity and to obtain a finite velocity of propagation for gravity forces. Some examples of operational discussion were first used to prove the equivalence of gravitation and acceleration fields, but later on, Einstein introduced a very heavy mathematical structure that goes much beyond any physical need, and the experimental proofs of the theory are very few. Similar remarks were presented by R. H. Dicke in some recent papers.[3]

This brings us to the problem of finding out what is essential in general relativity, and what may be considered as unnecessary embellishments! The role played by arbitrary "frames of reference," by "rigid yardsticks," or "exactly similar clocks" is extremely confusing. Here again, this author's recent thinking brings him in agreement with Bridgman (ref. 2, pp. 319 *passim*) whose discussion can be used as a program for future research. A painful and complete reappraisal is absolutely needed.

In order to be able to attack these problems, the first step is to investigate carefully what can be predicted for conditions where general relativity introduces only a small correction to classical physics. This was attempted in a recent paper,[4] where the role of Einstein's "energy-mass" relation was investigated and led to a number of interesting suggestions.

Let us also cite from a very remarkable book by V. Fock.[5]

This book written by a famous Russian scientist contains a brilliant discussion of Einstein's ideas, and rebuilds the usual Einstein theory from a very original viewpoint. Fock's discussion is rather similar to our own, although not identical; some of the points we raised have not been noted by Fock, but on other questions he was able to obtain a practical solution of many difficulties. His most interesting result is the fact that it is *unreasonable* to keep the theory as *completely general as Einstein did*, and that some simple rules lead to a great simplification of the mathematical structure; at the same time he obtains a much better physical explanation of the practical meaning of the theory.

The theory developed by Fock requires careful examination. His method is certainly brilliant, but it is not obvious whether his solution is the only possible one. He selects a certain class of frames of reference that simplify the resolution, but there may perhaps be other classes to be considered and compared with Fock's classes. It also remains to be proved that Fock's selection of "preferred" frames corresponds to practical experimental conditions.

All this is both of theoretical interest and of great practical importance.

3. *Schwarzschild's Problem.*—Some special examples may be helpful for a better understanding of the difficulties. Let us first consider the static problem of a particle at rest, with a field of spherical symmetry;[6] we use coordinates x^1, x^2, x^3, and $x^4 = ct$ and obtain Schwarzschild's solution:

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + \frac{2m}{r^2(r - 2m)} \times [x^1dx^1 + x^2dx^2 + x^3dx^3]^2 - \left(1 - \frac{2m}{r}\right)(dx^4)^2 \quad (2)$$

with

$$m = \frac{\gamma M}{c^2} \qquad \begin{array}{ll} M, & \text{mass} \\ \gamma, & \text{gravitation constant of Newton} \end{array}$$

and $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$.

This solution becomes singular for

$$r = 2m. \tag{3}$$

One should immediately note the possibility of deriving other mathematical solutions by suitable changes in the four coordinates. For instance, one may avoid the queer second group of terms in equation (2) and obtain isotropic space with

$$ds^2 = \left(1 + \frac{m}{2r}\right)^4 [(dx^1)^2 + (dx^2)^2 + (dx^3)^2] - \left[\frac{1 - m/2r}{1 + m/2r}\right]^2 (dx^4)^2, \tag{4}$$

a new solution that collapses for

$$r = \frac{m}{2}. \tag{5}$$

Both solutions behave similarly at infinity. Which one should we compare with experimental measurements? Should we select x^1, x^2, x^3, and ct from equation (2) or from equation (4)? The question remains open. It is even worse than that. Any arbitrary change of coordinates can be applied and an infinite number of solutions obtained! Einstein's methods are much too general and do not yield any precise answer!

This is where Fock comes in. He gets rid of this unphysical generality and selects what he thinks is the best set of coordinates by assuming that the Riemann-Christoffel symbols Γ^α are zero (ref. 5, pp. 4, 193, and 215).

$$\Gamma^\alpha = 0 \qquad \textit{harmonic coordinates.} \tag{6}$$

These four additional conditions completely determine a "preferred" frame of reference, for which no correction is needed in the four-dimensional operation of wave propagation:

$$\Box \psi = 0. \tag{7}$$

This means isotropic wave propagation with *c playing* the role of *an absolute constant. Fock's assumption* is that this frame of reference corresponds to actual physical observations. It yields:

$$ds^2 = \frac{r + m}{r - m} dr^2 + (r + m)^2(d\theta^2 - \sin^2 \theta d\varphi^2) - c^2 \frac{r - m}{r + m} dt^2 \tag{8}$$

and condition for collapse reads

$$r = m. \tag{9}$$

The comparison of equations (2), (4), and (8) clearly shows the trouble with Einstein's overgeneralization. And we may add: Does equation (8) represent the last word? How can we prove that this solution actually corresponds to our length and time measurements in a laboratory at rest in a gravitational field? This cannot result from mathematical considerations but only from a careful discussion of actual experimental conditions. Such a detailed "operational analysis," according to Bridgman, is absolutely needed, and it seems to be still missing. Looking at the preceding formulas, one feels that equation (2) looks awkward, and attempts at explaining it in physical terms were not too good. So we are left with equation (4) exhibiting local isotropy in space, or with equation (8) characterized by local isotropy in wave propagation. We shall discuss these possibilities in a later paper.

4. *Gravitational Red Shift and Connected Problems.*—In the relativistic theory of Einstein, there is obviously a weak point: the lack of definition of "ideal clocks." This, however, was impossible at the beginning of the century, before quantum theory and Bohr's atom were discovered. We now have a definition, based on Bohr's second condition,

$$\Delta E = h\nu, \tag{10}$$

relating the frequency ν measured in a frame of reference where the atom is at rest to the energy transition ΔE in the atom. Next to this relation we rewrite equation (1)

$$\Delta E = \Delta(mc^2). \tag{11}$$

Energy, mass, and frequency are just one single physical entity. A perfect clock is assumed to be stabilized on the frequency ν, for which we select the most stable atom structure we know from experiment, namely, an atom maintained at rest in a heavy solid structure, by Mössbauer effect. This is the standard of frequency used by R. V. Pound and R. L. Snider,[7] who were able to check a formula of Einstein's about the red shift due to gravity, and verified the theoretical prediction with a remarkable accuracy.

This represents, with the mass-energy relation, the best (and almost the only) experimental check of Einstein's gravitational theory: deflection of light rays near the sun is extremely inaccurate, and Mercury's rotation of the perihelion is seriously in doubt after Dicke's experiments.[3]

Let us investigate carefully the problem of gravitational red shift from an experimental and "operational" point of view. Our assumptions are explained in Figure 1: a spherical body of mass M, at rest, yields a gravitation potential V at a distance r.

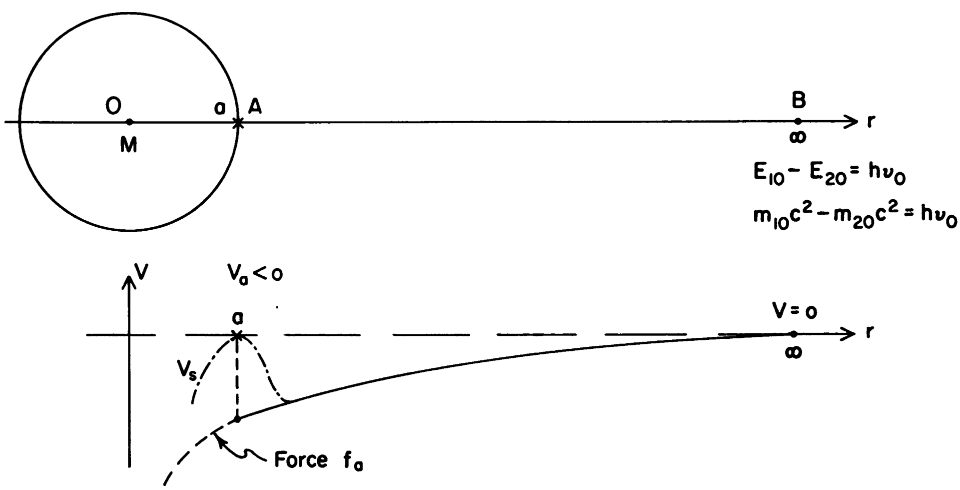

FIG. 1.

This potential is zero at infinity and takes a negative value V_a on the surface of the sphere, where we have (at $r = a$) an experimental laboratory at rest. We compare a clock located at infinity (potential and field both zero) with a similar clock at rest at point A (*Mössbauer effect*) under gravitational potential V_a and a force f_a

$$V_a < 0 \qquad f_a = -m\frac{\partial V_a}{\partial a} \qquad m,\ \text{mass of the atom.} \tag{12}$$

At zero potential, the atom has two energy levels E_{10} and E_{20}, whose difference yields a radiation frequency:

$$E_{10} - E_{20} = h\nu_0 = m_{10}c^2 - m_{20}c^2, \tag{13}$$

where m_{10} and m_{20} are the masses of the atom on both levels. Let us now move the atom (on level 1) from infinity to a, where it arrives with a velocity v_1. Conservation of energy requires

$$E_{10} = E_{1a} + m_1V_a + {}^1/_2\, m_1v_1^2 = E_{1a}. \tag{14a}$$

Since a mass m_1 in a gravity potential V_a obtains a negative potential energy m_1V_a, that compensates exactly for the kinetic energy.

Similarly,

$$E_{20} = E_{2a} \quad \text{and} \quad v_1 = v_2. \tag{14b}$$

From both equations (14) we see that the emitted frequency is unchanged. There is, however, still one difficulty: the new energy levels E_{1a} and E_{2a} refer to a clock falling freely in the gravity field. Our clock is supposed to be staying at rest at point A. To bring it to rest without perturbation we must use *another type of force* (elastic force, for instance) decelerating the clock. We actually put the clock on a table whose elastic strains and stresses keep the clock at rest despite its weight. The superposition of elastic forces and gravity forces is represented by the curve V_s and brings the clock to rest (V_s being maximum and zero) at a. This deceleration does not alter the energy levels E_1 and E_2, and the frequency ν remains unchanged for a local observer at rest. This is what happens in the Mössbauer effect, where elastic forces in a crystal lattice keep the atom at rest and compensate the gravity forces. The elastic forces of Mössbauer will absorb the recoil $h\nu/c$ due to emission of radiation.

The atom at rest at point A emits a photon $h\nu_0$ identical with the one emitted at infinity (eq. 13). This photon is observed at infinite distance, at point B, and one must notice that the photon is not sensitive to elastic forces, while its μ mass in motion makes it sensitive to gravity.

$$\mu c^2 = h\nu. \tag{15}$$

While climbing the gravity field from A to B, it loses energy, mass, and frequency. For a displacement dr, let the *potential increase* be $dV > 0$

$$d(h\nu) = -\mu dV = -\frac{h\nu}{c^2}\,dV \qquad \text{or,} \qquad \frac{d\nu}{\nu} = -\frac{dV}{c^2}, \tag{16a}$$

assuming c constant despite changing gravity. We may assume the potential increase from A to B to be a small quantity and write:

$$\frac{d\nu}{\nu} = \frac{V_a}{c^2} \qquad V_a < 0. \tag{16b}$$

The photon's frequency is decreasing (red shift), and formulae (16*a*) and (16*b*) correspond to Einstein's prediction. We felt it necessary to discuss everything in detail, especially the role of elastic forces in the Mössbauer effect, because most authors omitted one point or another, often reaching the correct result through incomplete reasoning.

The curious point is that the local frequency, observed near the atomic clock at rest, is constant and does not depend upon the gravitational potential; but the frequency observed at a distance depends upon this potential V and does not depend upon other sorts of potential energies. Let us emphasize that many gravity fields may not derive from a potential V, thus leaving the question open.

The discussion may apply to photons or gravitons $h\nu$, all being uncharged and reacting only to gravity changes. Let us note that Einstein's original discussion, using a free-falling clock and paying no attention to bringing it to rest, is not complete.

5. *The Possibility of a "Gravi-Spectral" Effect.*—The preceding discussion predicts a frequency change due to the gravitation potential. Let us note here the difficulty in defining a potential energy in a field where propagation of waves proceeds with a finite velocity c. This brings us back to the problems discussed in previous papers.[1]

We may also be surprised not to find anything similar to the Zeeman or Stark effects, where frequency of radiation depends upon the *magnitude of the field, not the potential.* This simply means that such effects have been *ignored and overlooked,* but they should exist. We intend to show it, using an oversimplified model that may have to be corrected later when the effect is observed.[8] Let us assume that the Mössbauer elastic forces act directly upon the nucleus of the atom and not upon the surrounding electrons. These electrons are still feeling the local gravity field f_a (eq. 12), giving equal and parallel forces on all individual electrons, just as a constant local electric field f_e would do

$$ef_e = -m' \frac{\partial V_a}{\partial a} = f_a \qquad e,\ m',\ \text{electronic charge and mass,} \tag{17}$$

and these forces must result in a very weak Stark-like multiplet. The order of magnitude of Stark multiplet splitting is

$$\Delta\nu_e = \frac{3ef_e nh}{8\pi^2 m' Ze^2} = 137\,\frac{3nef_e}{4\pi m' Zc} \qquad +Ze,\ \text{charge of the nucleus} \tag{18}$$

with n integer and $hc/2\pi e^2 = 137$. Let us now use equation (17) and consider the situation at point A of Figure 1, where the atom is at a distance a from the center O of attraction M,

$$ef_e = f_a = -\frac{m'\gamma M}{r^2} \qquad \gamma,\ \text{Newton's constant,} \tag{19}$$

hence

$$\Delta\nu_g = -137 \frac{3n}{4\pi Zc} \frac{\gamma M}{r^2} \quad \text{for the "gravi-spectral" Stark-like effect.} \tag{20}$$

Let us compare this new splitting due to gravity forces with the red shift produced by gravity potential (eq. 16),

$$V_g = -\frac{\gamma M}{r} \qquad \frac{\delta\nu_{\text{rel}}}{\nu} = \frac{V_g}{c^2} = -\frac{\gamma M}{rc^2}. \tag{21}$$

The orders of magnitude can readily be compared

$$\frac{\Delta\nu_g}{\delta\nu_{\text{rel}}} = 137 \frac{3n}{4\pi Z} \frac{\lambda}{r} \qquad \lambda = \frac{c}{\nu} = \text{wavelength.} \tag{22}$$

The gravi-spectral effect $\Delta\nu_g$ is of the order of 137 $\frac{\lambda}{r}$ times the relativistic red shift $\delta\nu_{\text{rel}}$. It could be observed only for very short distances r from the center of attraction M.

This short discussion proves that direct effects of the *gravity field* should exist, but may be very difficult to observe practically. Einstein's theory and quantized atomic theories both ignored this possibility, and experimental checks would be important.

Of all epoch-making discoveries of Einstein, we invoked only two: mass-energy relation and photon; strangely enough, we did not need the curved space-time universe!

[1] Brillouin, L., these PROCEEDINGS, **53,** 475–482, 1280–1284 (1965); these papers require some complements and corrections, to be given later.

[2] Bridgman, P. W., *Reflections of a Scientist* (New York: Philosophical Library, 1955), p. 309.

[3] Dicke, R. H., "Gravitational theory and observations," *Phys. Today,* **20** (1), 55–70 (1967), where many references may be found.

[4] Brillouin, L., and R. Lucas, "La relation Masse-Energie en gravitation," *J. Phys. Radium,* **27,** 229 (1966).

[5] Fock, V., *The Theory of Space, Time and Gravitation*: (New York: Pergamon Press, Macmillan, 1964).

[6] Pauli, W., *Theory of Relativity* (New York: Pergamon Press, 1958), p. 166, eq. (421. *a*) and (421.*b*).

[7] Pound, R. V., and R. L. Snider, *Phys. Rev.,* **140,** B788 (1965).

[8] Brillouin, L., *Compt. Rend.,* **263,** 755 (1966).