RADIATION OF TRANSVERSE ELECTROMAGNETIC WAVES DUE TO SCATTERING OF CHARGED PARTICLES BY PLASMA WAVES

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It is shown that the emission of transverse waves by epithermal electrons takes place in the field of a plasma wave (in the classical limit) as the result of dipole radiation due to the oscillations of the electron in the wave (Compton effect on plasma waves), as well as a result of passage of the electron through density inhomogeneities created by the plasma wave. The emission of transverse waves by electrons is forbidden in the nonrelativistic case v = 0 ($\hbar = c = 1$) by interference of these two effects. The forbiddenness does not hold for particles whose masses differ from that of the electron. The radiation spectrum of electrons and ions is calculated in the broad energy range from nonrelativistic to relativistic energies. The graph technique is used to calculate quantum effects that become significant for secondary quantum energies close to the energy of the charged particles. Possible astrophysical applications are discussed, as well as the possibility of determining particle energy and mean energy density of the plasma waves on the basis of the radiation intensity. It is also shown that the frequencies of transverse waves produced in the scattering of cosmic ray electrons by plasma waves may considerably exceed the frequency of waves generated by the synchrotron mechanism.

INTRODUCTION

1. The problem of the conversion of longitudinal waves of a plasma into transverse waves is of interest from the viewpoint of the study of nonlinear effects in plasma,[1] and also for possible astrophysical applications (see[2,3]).

In the present work we consider the conversion of a plasma wave into a transverse wave by scattering from an isolated, epithermal charged particle. It is necessary to keep in mind that any plasma particle can be regarded as a test particle. If the analysis is generalized by taking into account spatial dispersion, then the resultant probabilities will enter directly into the "kinetic equation" that describes the nonlinear effects of wave conversion in the plasma. Here we are interested in elementary processes and carry out a detailed study, including the ultrarelativistic limit. The study of an isolated fast charged particle in a medium is essential also for questions of the passage through matter of fast particles whose temperature differs from zero. Under these conditions, there are excitations in the medium (longitudinal waves in the plasma), scattering from which produces additional radiation which is superimposed on the other radiation. In Sec. 4, some possibilities are discussed for the observation of such radiation. The problem here is of interest as to the presence of transverse waves in a beam of fast particles as the result of scattering of the plasma waves by the beam particles. For a low-density beam, the result can be obtained by means of the scattering probabilities from single particles as found below.

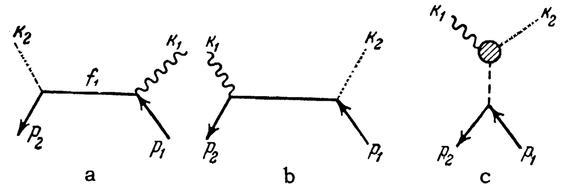

FIG. 1

2. We shall consider a set of weakly interacting charged particles, i.e., the so-called collision-free plasma. The effects of interaction of waves in such a system with charged particles can be considered by perturbation theory. If the nonlinear effects in a vacuum correspond to closed electron loops,[4] then in a plasma, in the presence of real particles, the conversion of waves comes about in the first approximation of perturbation theory as the result of scattering from real particles.[1)] For a fast

[1)]In what follows, in the graphical representations of the process, the plasma longitudinal wave is pictured as a wavy line and the transverse wave by a dotted line, while the virtual quantum is represented by the dashed line.

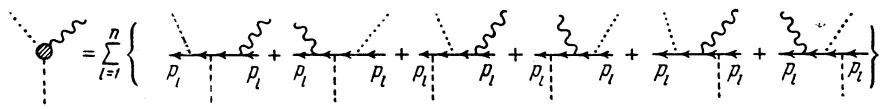

FIG. 2. Interaction between waves by means of plasma electrons. Summation is carried out over all electrons of the plasma. p_l is the momentum of the test electron which does not change as the result of the interaction.

epithermal plasma particle, the processes of conversion of longitudinal waves into transverse are represented by Fig. 1.

Processes a and b are similar to the Compton effect for a fast particle, while process c describes the nonlinearity of the plasma, the circle corresponding to the nonlinear interaction between the incident plasma wave, which is scattered by the transverse wave, and the field of the electron. In the final analysis, the indicated nonlinearity is connected with scattering processes on the plasma electrons and is determined by the set of graphs shown in Fig. 2.[2)] Account of the graph c is very important, since in the nonrelativistic limit it completely compensates the effects arising from a and b.

3. The conservation laws for graphs a, b, and c of Fig. 1 are:

$$f_1 = p_1 + k_1 = p_2 + k_2 \qquad (1.1)$$

These connect the frequency $\omega_2 = \omega^t(\mathbf{k}_2)$ and the direction of the scattered photon with the initial momenta of the electron $\mathbf{p}_1$ and the momentum of the plasma quantum $\mathbf{k}_1$ of frequency $\omega_1^l = \omega^l(\mathbf{k}_1)$ (the angles are defined in Fig. 3). For frequencies ω_2 that are much larger than the plasma frequency ω_0, we have

$$|\mathbf{k}_2| = \omega_2 = \left|\frac{(\varepsilon_{\mathbf{p}_1}\omega_1 - \mathbf{k}_1\mathbf{p}_1) + {}^1/_2\,(\mathbf{k}_1^2 - \omega_1^2)}{\varepsilon_{\mathbf{p}_1} + \omega_1 - |\mathbf{p}_1 + \mathbf{k}_1|\cos\vartheta_2}\right| \qquad (1.2)$$

(ϵ_{p_1}, $\mathbf{v}_1 = \mathbf{p}_1/\epsilon_{p_1}$ and, m are the energy, velocity, and mass, respectively, of the electron before scattering). For not very energetic electrons,

$$\varepsilon_{\mathbf{p}_1}/m \ll \min\{m/\omega_1,\, m/|\mathbf{k}_1|\}, \qquad (1.3)$$

(1.2) becomes simplified:

$$\omega_2 = \frac{|\omega_1 - k_1 v_1\cos\vartheta_1|}{1 - v_1\cos\vartheta_2}. \qquad (1.4)$$

The maximum frequency for $v \to 1$ corresponds to $\vartheta_2 = 0$ and $\vartheta_1 \to \pi$ (Fig. 3) and is equal to

$$\omega_{max}^t \approx 2\varepsilon_{\mathbf{p}_1}^2 m^{-2}\,(\omega^l(k_1) + k_1). \qquad (1.5)$$

Upon satisfaction of the inequality (1.3), the quantum effects become negligibly small. The intensity of scattering, with (1.3) satisfied, is calculated in the next section. The opposite limiting case is considered in Sec. 4.

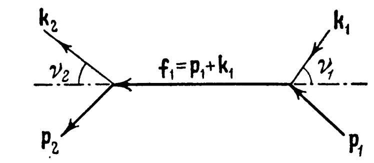

FIG. 3. Scattering kinematics. $\vartheta_1(\vartheta_2)$ – angle between the longitudinal (transverse) quantum and the total momentum.

2. CLASSICAL LIMIT

It follows from what has been said above that the quantum effects do not play an important role when (1.3) is satisfied. Therefore, in the present section we carry out a purely classical calculation of the radiation of transverse waves by an electron scattered from a plasma wave.

Let the electric field of the plasma wave have the form

$$\mathbf{E}^l = \mathbf{E}_0\cos(\mathbf{k}_1\mathbf{r} - \omega_1 t) \qquad (\mathbf{E}_0 \parallel \mathbf{k}_1). \qquad (2.1)$$

The method of successive approximations is used to find the interaction between the wave (2.1) and the electron. In the zeroth approximation, we shall consider the electron to be moving uniformly and rectilinearly with velocity $\mathbf{v}$. A force acts on it from the wave (2.1) of the form $e\mathbf{E}_0\cos(\mathbf{k}_1\cdot\mathbf{v} - \omega_1)t$. In the first approximation to uniform motion, the small oscillations shown below are added[3)]

$$\mathbf{r} = \mathbf{v}t + \mathbf{R}\cos\Omega t, \qquad (2.2)$$

$$\mathbf{R} = -\frac{e}{\varepsilon_{p_1}}\frac{E_0}{\Omega^2 k_1}(\mathbf{k}_1 - \mathbf{v}(\mathbf{k}_1\mathbf{v})), \qquad (2.3)$$

$$\Omega = \omega_1 - \mathbf{k}_1\mathbf{v} = \omega_1 - v k_1\cos\vartheta_1. \qquad (2.4)$$

If the electron moved in a vacuum according to the law (2.2), then for calculation of the resultant radiation it would be sufficient to find the oscilla-

[2)]The electron line p_l represents any of the electrons of the plasma ($l = 1, 2, \ldots, n$). To obtain the vertices c, the graphs on the right side of Fig. 2 are summed over all the electrons of the plasma (denoted by a summation sign). As a result of summation of a large number of diagrams of very high order, the graph c becomes of the same order as a and b.

[3)]Equations (2.2)–(2.6) were obtained as the result of solution of the equation $\frac{d}{dt}\frac{m\mathbf{u}}{\sqrt{1-u^2}} = e\mathbf{E}^l$ under the assumption that E_0 is small so that $|\mathbf{u} - \mathbf{v}| \ll v$.

tory part of the dipole moment cR with the aid of (2.3) and to calculate of the intensity of the dipole radiation; in the language of graphs, this would mean a restriction to diagrams a and b (Fig. 1). However, the electron under consideration moves in an essentially inhomogeneous plasma. Its inhomogeneity is brought about by oscillations of the electron density n in the wave (2.1), connected with the field of the wave by the equation

$$\operatorname{div} \mathbf{E} = 4\pi e (n - \bar{n}) \qquad (2.5)$$

($\bar{n}$ is the mean value of the electron density). The dielectric constant, which depends on n, changes simultaneously with the change in density:

$$\varepsilon(\omega) = 1 - 4\pi e^2 n / m\omega^2. \qquad (2.6)$$

In addition to the electron in the plasma, the polarization produced by it also moves. Because of the inhomogeneity of ϵ, a dipole moment is generated which partially (for a nonrelativistic electron, completely) cancels the dipole moment of the oscillations of the electron.

A charge moving in a medium with ϵ that is variable in time and space radiates transverse waves. The graph c of Fig. 1 corresponds to such a mechanism of radiation. We note that the mechanism of radiation corresponding to graph c in Fig. 1 has a well-known analog in the radiation of a charge in a layered medium (see [5-8]). We limit ourselves to the case in which the frequencies of the radiated transverse waves appreciably exceed the frequencies of the longitudinal waves that create the density inhomogeneities. This case is the simplest. Of course, the graph c also describes the case of comparable frequencies.

Of considerable significance is the fact that the phases of the oscillations of the electron and of the change in ϵ are not independent. The total scattered radiation is not the sum of the radiation produced by the oscillations of the electron (graphs a, b) and the radiation due to the inhomogeneities of the medium (graph c). Interference of these radiations is appreciable. A charge moving according to (2.2) radiates (at an angle ϑ_2 to the velocity v) a wave of frequency

$$\omega_2 = \Omega / (1 - \sqrt{\varepsilon^t(\omega_2)}\, v \cos\vartheta_2). \qquad (2.7)$$

For simplicity, we restrict ourselves to plasma waves whose phase velocity is much less than the velocity of light ($k_1 \gg \omega_1$), and also to the condition (see above)

$$\omega_2 \gg \omega_1 \gtrsim \omega_0 \qquad (2.8)$$

or

$$v k_1 \cos\vartheta_1 / (1 - v\cos\vartheta_2) \gg \omega_1. \qquad (2.9)$$

The last condition is satisfied if the velocity of the charge is appreciably larger than the phase velocity of the plasma wave and also the mean thermal velocity of the electrons of the plasma, and if the angle ϑ_1 is not close to $\pi/2$. The condition (2.9) is necessary so that Eq. (2.6) can be used; this equation does not take spatial dispersion into account.[4)]

In accord with (2.5), (2.6), and (2.9), ϵ changes in the plasma wave according to the law

$$\varepsilon = 1 + \frac{e k_1 E_0}{m\omega_2^2} \sin(\mathbf{k}_1\mathbf{r} - \omega_1 t). \qquad (2.10)$$

Let us find the power which an electron moving according to (2.2) radiates in a medium with ϵ of the form (2.10). This power Q is equal to the mean work per unit time performed by the electron in motion in the electric field E created by it:

$$Q = -\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int d^3r\, \mathbf{E}(\mathbf{r}, t)\, \mathbf{j}(\mathbf{r}, t)$$

$$= -\lim_{T\to\infty} \frac{(2\pi)^4}{T} \int d^3k\, d\omega\, \mathbf{j}(-\mathbf{k}, -\omega)\, \mathbf{E}(\mathbf{k}, \omega), \qquad (2.11)$$

$$j(\mathbf{r}, t) = e(\mathbf{v} - \Omega\mathbf{R}\sin\Omega t)\, \delta(\mathbf{r} - \mathbf{v}t - \mathbf{R}\cos\Omega t), \qquad (2.12)$$

and $\mathbf{E}(\mathbf{k}, \omega)$ and $\mathbf{j}(\mathbf{k}, \omega)$ are the Fourier components of the electric field and current density, respectively.

To find the field $\mathbf{E}(\mathbf{k}, \omega)$, we use the Maxwell equation

$$\Delta\mathbf{E} + \operatorname{grad}\operatorname{div}(\varepsilon - 1)\mathbf{E} - \partial^2\varepsilon\mathbf{E}/\partial t^2$$

$$= 4\pi(\partial\mathbf{j}/\partial t - \operatorname{grad}\rho).$$

By substitution of (2.10), we go over to the Fourier representation

$$iE_i(\mathbf{k}, \omega) = \frac{k_i k_j - \omega^2\delta_{ij}}{(\omega + i\cdot 0)^2 - k^2} \left\{ \frac{E_0 k_1}{2m\omega^2} [E_j(\mathbf{k} - \mathbf{k}_1, \omega - \omega_1) - E_j(\mathbf{k} + \mathbf{k}_1, \omega + \omega_1)] - \frac{4\pi}{\omega} j_j(\mathbf{k}, \omega) \right\} \qquad (2.13)$$

and then solve by the method of successive approximations.

To find $\mathbf{j}(\mathbf{k}, \omega)$, we use (2.2) and (2.12). After substituting the solution (2.13) in (2.11) and keeping only terms proportional to E_0^2, we find

$$Q = \frac{e^4 E_0^2 k_1^2}{4\pi m^2} \int d^3k_2\, |\omega_2|\, d\omega_2\, \delta(k_2^2 - \omega_2^2)\, \delta(\omega_2(1 - v\cos\vartheta_2) + k_1 v\cos\vartheta_1)\, \{\beta^2 - (\mathbf{k}_2\beta)^2/\mathbf{k}_2^2\}, \qquad (2.14)$$

[4)]This is not fundamental, and the results are easily generalized to the case where one must take into account spatial dispersion.

where

$$\beta = \frac{k_1}{\omega_2}\left[\frac{\sqrt{1-v^2}}{(1-v\cos\vartheta_2)\,k_1^2} - \frac{1}{(k_2-k_1)^2-k_2^2}\right]$$

$$+ v\left[\frac{\sqrt{1-v^2}\,(k_1k_2 - k_1 v\omega_2)}{(k_1 v\cos\vartheta_1)^2\,k_1^2} - \frac{1}{(k_2-k_1)^2-k_2^2}\right]. \quad (2.15)$$

The integrand function (2.15) is the power radiated in the frequency range $d\omega_2$ and with wave vectors d^3k_2. Account of the change in the plasma density would give second terms in the square brackets of Eq. (2.15). They are appreciable in the scattering of a plasma wave by a nonrelativistic electron ($v \ll 1$). As is seen from Eq. (2.15), the first term in it approaches zero simultaneously with v. Therefore, for $v \ll 1$, we have $\beta \sim v/\omega_2$ and $Q \sim v^2$. The two mechanisms of radiation cancel one another. It must be noted that this refers only to electrons. For ions, because of their large mass M, the conversion of the longitudinal wave to the transverse takes place only as the result of the inhomogeneous density in the plasma wave and is determined by Eq. (2.14) with the replacement of β by

$$\beta_i = \frac{k_1/\omega_2 + v}{(k_1-k_2)^2 - k_2^2}. \quad (2.16)$$

Therefore, a nonrelativistic ion in a plasma wave produces transverse waves that are much larger than those produced by the nonrelativistic electron. Only for the limit of ultrarelativistic ions, for which $\sqrt{1-v^2} \sim m/M$, is there a dependence of the oscillations of the ion itself, and (2.16) is violated.

In the limit $v \ll 1$, we obtain the following expressions for the intensity of radiation of electrons and ions, respectively:[5)]

$$Q = \frac{e^4E_0^2}{15m^2}v^2[3 + 13\cos^2\vartheta_1], \quad (2.17)$$

$$Q_i = e^4E_0^2/3m^2; \quad (2.18)$$

both ions and electrons radiate at the frequency $\omega_2 = k_1 v\,|\cos\vartheta|$ when $v \ll 1$.

3. SCATTERING OF CHARGED PARTICLES BY ISOTROPICALLY DISTRIBUTED PLASMA WAVES

If a continuous spectrum of plasma waves is incident on the electron, rather than the plane monochromatic wave considered in the preceding section, the scattered radiation can be found by integrating (2.14) over all the scattered waves.

[5)]For small $v \lesssim m/M$, the effect of ions which disturb the compensation is important. Compensation is also destroyed for failure of the relation (2.9).

We assume that the directions of motion of the plasma waves are distributed isotropically. In astrophysical applications, the latter is not obvious, inasmuch as the magnetic field and the directional character of the discontinuities can have an effect on the distribution of the plasma waves. We note that the inhomogeneous distribution of plasma waves can lead to polarization of the scattered radiation. We now limit ourselves to consideration of the isotropic distribution.

To find the frequency spectrum of the transverse radiation $Q(\omega_2)$, we average Q over all angles ϑ_1:

$$\overline{Q} = \frac{1}{2}\int_{-1}^{+1} d\cos\vartheta_1\, Q = \int_0^\infty Q(\omega_2)\,d\omega_2, \quad (3.1)$$

$$Q(\omega_2) = \frac{e^4E_0^2k_1^2\omega_2^2}{16m^2\pi}\int d\cos\vartheta_1\, d\cos\vartheta_2\, d\varphi\,\{\beta^2 - (k_2\beta)^2/k_2^2\}$$
$$\times [\delta(\omega_2(1 - v\cos\vartheta_2) + k_1 v\cos\vartheta_1)$$
$$+ \delta(\omega_2(1 - v\cos\vartheta_2) - k_1 v\cos\vartheta_1)] \quad (3.2)$$

(φ is the angle between the planes in which the vectors $\mathbf{k}_1$, u and $\mathbf{k}_2$, u lie).

Calculation of the integral (3.2) leads to the expression

$$Q(\omega_2) = \frac{e^4E_0^2\varepsilon_{p_1}^2}{4m^4\omega_2}\Phi(\gamma, q), \quad (3.3)$$

$$\gamma = \frac{\varepsilon_{p_1}}{m}, \qquad q = \frac{\omega_2(1-v)}{k_1 v},$$

$$\Phi(\gamma, q) = 0 \text{ for } q \geqslant 1, \quad (3.4)$$

where q is the ratio of the frequency of the radiated waves to the maximum possible frequency for given k_1 and v.

The function $\Phi(\gamma, q)$ is given by a cumbersome expression given in the appendix. Here we limit ourselves to graphs of $\Phi(\gamma, q)$ for a number of values of γ, obtained with the help of (3.3). The presence of two maxima in the curves of Fig. 4 is brought about by the presence of the two mechanisms of scattering considered. At ultrarelativistic energies ($\gamma \gg 1$), the principal role is played by radiation from the oscillations of the electron. This radiation results in a broad smooth maximum. The narrow maximum at small q is brought about by the mechanism that is similar to the mechanism of transition radiation (radiation from inhomogeneities of the density produced by the plasma wave). Inasmuch as in the limit $\gamma \to \infty$ the latter radiation has a constant intensity which does not increase with γ, and a mean frequency, while in radiation from the electronic oscillations, these quantities increase with increase in γ, in proportion to γ^2, the left maximum decreases as $\gamma \to \infty$ and shifts

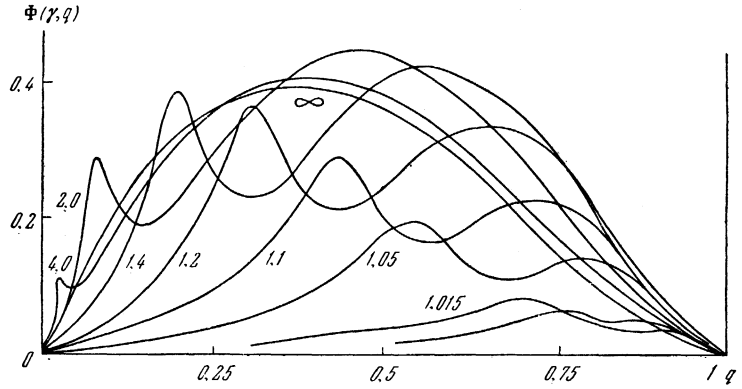

FIG. 4

to the left, and $\Phi(\gamma, q)$ approaches the limiting expression, given analytically by the formula

$$\Phi(\infty, q) = \frac{8}{3} q [(1-q)^3 - 3q^2(1 - q + \ln q)]. \quad (3.5)$$

In the nonrelativistic limit ($\gamma \to 1$), the two maxima merge and both radiation mechanisms suppress each other. For comparison, the curves for $\Phi(\gamma, q)$ are plotted in Fig. 5 without account of the density inhomogeneities in the plasma wave. These curves have the same limiting curve (3.5) as $\gamma \to \infty$, but behave quite differently as $\gamma \to 1$.

In scattering of plasma waves by ions, Eq. (3.3) holds for the resulting transverse radiation; therein, $\Phi_2(\gamma, q)$ has the form shown in Fig. 6. These curves were computed with the help of Eq. (2.16).

We find the total power radiated by the electron by integrating (3.3) numerically:

$$\overline{Q} = \frac{2e^4 E_0^2 \gamma^2}{9m^2} \Pi(v). \quad (3.6)$$

A plot of $\Pi(v)$ is shown in Fig. 7. The coefficient for $\Pi(v)$ in Eq. (3.6) is determined by the condition $\Pi(v) \to 1$ as $v \to 1$.

4. QUANTUM EFFECTS

In the process under study, the quantum effects begin to play a role when the energy of the secondary quantum becomes of the order of the initial energy of the electron: $\omega_2 \sim \epsilon_{p_1}$. For this case, the latter must be sufficiently large [see (1.3)]:

$$\varepsilon_{p_1} \gtrsim \varepsilon_{qu} = m^2 / |\mathbf{k}_1| \quad (4.1)$$

(for example, for a plasma wavelength ~ 1 cm, $\epsilon_{qu} \sim 10^{15}$ eV). Because of what was pointed out above (Sec. 2), only the graphs a and b (Fig. 1) are of importance in scattering of such energetic elec-

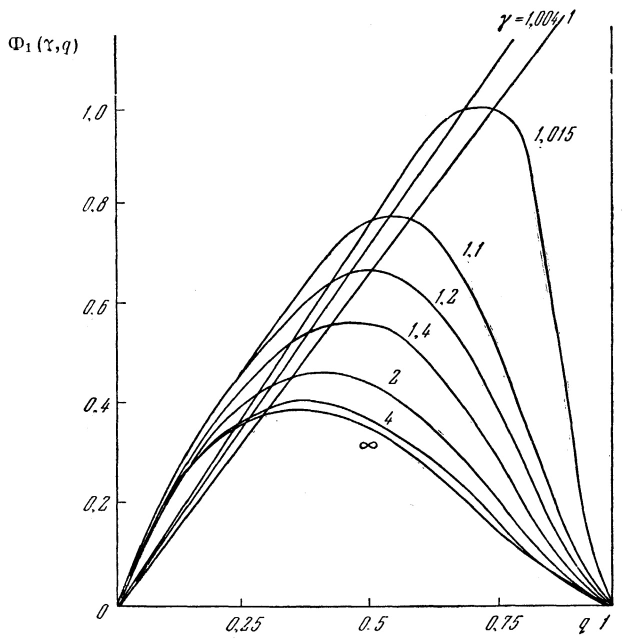

FIG. 5

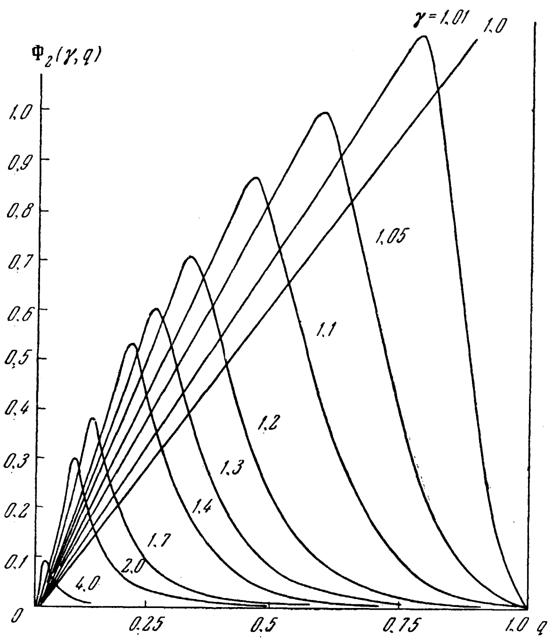

FIG. 6

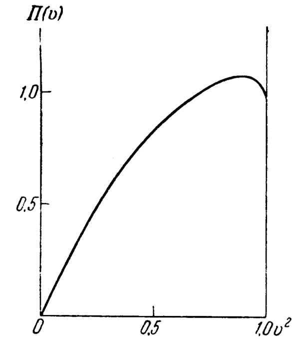

FIG. 7

trons. Because of the large energy of the secondary quanta, their dispersion can be neglected, if we consider $\omega_2 \sim |\mathbf{k}_2|$; therefore, these graphs differ from the graphs of the Compton effect in vacuum only by the replacement of the initial transverse quantum by the longitudinal plasma quantum, which has a momentum $k_1 = (\mathbf{k}_1, i\omega_1)$. One can show that replacement of the vacuum photon in the external photon line by the plasma photon leads to the result that in the matrix element (computed in accord with the rules set forth in [4]) it is formally necessary to replace the unit polarization vector of the vacuum photon by the non-unit vector [6)]

$$e_1 = \frac{\sqrt{2\omega_1}\,(\omega_1\mathbf{k}_1,\ i\mathbf{k}_1^2)}{(\mathbf{k}_1^2 - \omega_1^2)\,|\mathbf{k}_1|\,(\partial\varepsilon^l(\omega_1, \mathbf{k}_1)/\partial\omega_1)^{1/2}}. \tag{4.2}$$

We carry out the indicated substitution in the matrix elements represented by the graphs a and b. Calculation by standard methods (see [4]) of the scattering probability dw, averaged over the initial spins of the electron and summed over the finite spins of the electron and the polarizations of the secondary photon, leads to the result

$$dw = \frac{e^4\omega_2^2\, d\Omega_{k_2}}{2\varepsilon_{p_1}^2 k_1 |\varkappa_1|\, \partial\varepsilon^l/d\omega_1}\Big\{2\Big(\frac{\varkappa_1}{\varkappa_2} + \frac{\varkappa_2}{\varkappa_1}\Big) - 2k_1^2\Big(\frac{1}{\varkappa_1} + \frac{1}{\varkappa_2}\Big)$$
$$+ m^2k_1^2\Big(\frac{1}{\varkappa_1} + \frac{1}{\varkappa_2}\Big)^2 - \frac{k_1^2}{\varkappa_1\varkappa_2}\,[(\varepsilon_{p_1} + \varepsilon_{p_2})^2 + \omega_2^2 - k_1^2]$$
$$- 4m^2\Big(\frac{\varepsilon_{p_1}}{\varkappa_1} + \frac{\varepsilon_{p_2}}{\varkappa_2}\Big)^2\Big\}. \tag{4.3}$$

Here $d\Omega_{k_2}$ is the element of solid angle in the direction of flight of the secondary quantum,

$$\varkappa_1 = p_1k_1 - k_1^2/2, \qquad \varkappa_2 = -\,p_1k_2. \tag{4.4}$$

As in the classical calculation, we have limited ourselves in the derivation of (4.3) to the consideration of plasma waves having a phase velocity appreciably less than the velocity of light ($|\mathbf{k}_1| \gg \omega_1$).

Multiplying (4.3) by the energy of the secondary quantum ω_2 and by the number of plasma quanta

$$N = \frac{E_0^2}{16\pi}\,\frac{\partial\varepsilon^l}{\partial\omega_1}, \tag{4.5}$$

we obtain the intensity of the radiation in an element of solid angle:

$$dQ = \frac{\omega_2 E_0^2}{16\pi}\,\frac{\partial\varepsilon^l}{\partial\omega_1}\,dw. \tag{4.6}$$

The integration of (4.6) over the directions of flight of the secondary quantum of frequency ω_2 and averaging over the angle of collision between $\mathbf{k}_1$ and $\mathbf{p}_1$ gives us the spectral distribution of the radiation in the scattering of the electron by the isotropically distributed plasma waves:

$$Q(\omega_2) = \frac{e^4E_0^2\varepsilon_{p_1}^2}{4m^4\omega_2}\,\Phi_k(q, \sigma);$$

$$\sigma = \frac{2\varepsilon_{p_1}k_1}{m^2}, \qquad q = \frac{\omega_2}{\sigma(\varepsilon_{p_1} - \omega_2)} = \frac{1}{(1+\sigma)\,\omega_{max}/\omega_2 - \sigma}, \tag{4.7}$$

$$\Phi_k(q, \sigma) = \frac{2}{3}\,q\,\Big\{(1 - q)\Big[3(1+q)\Big(1 + \frac{1}{(1+q\sigma)^2}\Big)$$
$$- \frac{2}{1+q\sigma}(1 + q + 4q^2)\Big] - \frac{12q^2}{1+q\sigma}\ln q\Big\}; \tag{4.8}$$

here $\omega_{max} = \epsilon_{p_1}/(1 + 1/\sigma)$ is the maximum radiated frequency.

In obtaining (4.8) from (4.3), it was taken into account that the energy of the electrons was ultrarelativistic ($\epsilon_{p_1} \gg m$) while the plasma wavelength was large in comparison with the Compton wavelength ($k_1 \ll m$). In the non-quantum region ($\sigma \ll 1$), (4.8) undergoes a transition to (3.5). By integration of (4.7) and (4.8), we find the total radiation in the ultraquantum case ($\sigma \gg 1$):

$$\overline{Q} = \int_0^{\omega_{max}} Q(\omega)\,d\omega \approx \frac{e^4E_0^2\varepsilon_{p_1}}{4m^2k_1}\ln\frac{2\varepsilon_{p_1}k_1}{m^2}$$
$$(\varepsilon_{p_1}k_1 \gg m^2). \tag{4.9}$$

By comparison of (4.9) with the non-quantum formula (3.6), we establish the fact that the quantum effects impede the growth of radiation with increase in energy ($\overline{Q} \sim \epsilon_p^2$ in the non-quantum case, $\overline{Q} \sim \epsilon_{p_1}\ln\epsilon_{p_1}$ in the ultraquantum case).

The quantum effects are most important in a relatively dense plasma, for example, the electron plasma in metals, where they play a role at low energies. We have to deal here with the additional "bremsstrahlung" generated in the medium, which is superimposed on the long-wavelength part of the ordinary bremsstrahlung. We estimate the relative

[6)]Equation (4.2) is obtained from the requirement $\mathbf{e}_1 \parallel \mathbf{k}_1$, $\mathbf{e}_1\mathbf{k}_1 = 0$ (if a Lorentz gauge is chosen). The normalizing factor in the quantization assures the equality of the energy of the plasma quantum with its frequency.

role of both radiations in the region of application of (4.9).

In the continuous spectrum of plasma waves, (4.9) transforms to

$$\overline{Q} \approx \frac{e^4 \varepsilon_{p_1}}{4m^2} \int \frac{d^3 k_1}{k_1} E_{0k_1}^2 \ln \frac{2\varepsilon_{p_1} k_1}{m^2}. \quad (4.10)$$

For estimation purposes, we assume that the plasma waves possess an effective temperature T_{eff} in the region of weak damping ($k_1 < 1/R_D = \sqrt{4\pi e^2 n/T}$); this temperature generally differs from the electron temperature T. In accord with (4.10),

$$\overline{Q} \sim \frac{e^6 n \varepsilon_{p_1}}{m^2} \frac{T_{eff}}{T} \ln 2 \frac{\varepsilon_{p_1}}{m^2 R_D}. \quad (4.11)$$

By comparing (4.11) with the bremsstrahlung of the electron

$$Q_b = \frac{4e^6 Z^2 \varepsilon_{p_1}}{m^2} n_i \ln \frac{2\varepsilon_{p_1}}{m}, \quad (4.12)$$

we find

$$\overline{Q}/Q_b \sim (n/n_i Z^2)\, T_{eff}/T; \quad (4.13)$$

Z is the nuclear charge of the material, n/n_i is the number of free electrons in the nucleus. We note that $\overline{Q}/Q_b$ is smaller in the non-quantum region by a factor $m^2/k_1 \epsilon_{p_1}$ than in (4.12). For $T_{eff}/T \gg Z^2 n_i/n$ in the quantum region, $\overline{Q} \gg Q_b$. In the non-quantum region, $\overline{Q}$ is smaller than Q_b for energies of the electron[7)]

$$\varepsilon_{p_1} < \varepsilon^* \sim \frac{m^2}{k_1} \frac{n_i Z^2}{n} \frac{T}{T_{eff}}. \quad (4.14)$$

5. SOME APPLICATIONS TO THE DIAGNOSTICS OF PLASMA TURBULENCE

All the methods of plasma diagnostics (see Artsimovich[9]) are in some degree associated with the obtaining of definite information on the plasma parameters, either with the aid of different radiations (in the broad sense of the word) arising in a gas-discharge plasma, or with the aid of changes in radiation from the outside. Here and in most cases, an important loss of information takes place on the boundary of the plasma, which has a complicated structure. The diagnostics of beams of charged particles has an important advantage in the sense that the generated transverse radiation considered in the present work can have a sufficiently high frequency, such that the plasma boundary and the plasma itself are in practice no different than the vacuum for these frequencies. Thus this radiation, which is generated in the plasma, and which carries information on the plasma parameters, in practice loses no information on the boundary of the plasma. The problem touched upon here is also of interest for the so-called "turbulent" plasma (see[10-12]), which is characterized by the high intensity of the plasma waves in it (epithermal noise).

The intensity and the distribution of plasma waves can be determined in principle from the intensity and the spectrum of the transverse radiation which arises in the passage of a beam of charged particles through the plasma.[8)] We write down the formula which connects $N(\omega, \mathbf{k})$ with $Q(\omega, \mathbf{k})$ for each individual charged particle [compare (2.14), (2.15)]:

$$Q(\omega, \mathbf{k}) = \frac{4e^4 \mathbf{k}_1^2 \omega}{m^2} \frac{N(\omega_1, \mathbf{k}_1)}{\partial \varepsilon^l(\omega_1, \mathbf{k}_1)/\partial \omega_1} \delta(\mathbf{k}^2 - \omega^2)\, \delta(k_1 v \cos \vartheta_1 + \omega(1 - v\cos\vartheta_2)) \{\beta^2 - (\mathbf{k}\beta)^2/\mathbf{k}^2\}. \quad (5.1)$$

Attention should also be paid to the fact that if $Q(\omega, \mathbf{k})$ and $N(\omega, \mathbf{k})$ are known, then measurement of the energy of ultrarelativistic particles, $\epsilon_{p_1}/m \gg 1$, is possible in principle.

6. RADIATION PRODUCED BY THE ELECTRONS OF COSMIC RAYS IN COSMIC PLASMA

At the present time, it is generally accepted that cosmic plasma is in a state of turbulent motion (it is not quiescent). One of the manifestations of turbulence is the presence of plasma waves capable of accelerating the cosmic rays by the mechanism considered by one of the authors in[13,14]. Information on the presence of cosmic rays, for example, in the Galaxy,[15] is given us by radio emission[16] and by the optical radiation of cosmic objects. The cyclotron-radiation nature of the cosmic radiation, i.e., the radiation of the electrons in the cosmic rays in cosmic magnetic fields, is also generally accepted. Yet the nature of the cosmic radiation at very high frequencies is still not clear. We would like to direct attention here to the fact that a definite contribution to cosmic radiation can be brought about by the transformation of longitudinal plasma waves which exist in a turbulent plasma, into transverse waves as the result of scattering by the cosmic rays.

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7)In a medium at thermodynamic equilibrium, even under the most favorable conditions ($\hbar\omega_0 \ll T$, where $T_{eff} = T$) the radiation studied by us does not exceed the bremsstrahlung in intensity, in accord with (4.13). In a recently published paper,[17] a contradictory conclusion is reached, which appears to be the result of an error made in the derivation of Eq. (2.4) of Ryazanov's paper.[17]

8)The beam should be so weak that no instabilities develop during the time of observation.

Although the fluctuating electric fields of the plasma waves are presumed to be weak in comparison with the magnetic field, and consequently only a small fraction of all the cosmic electromagnetic radiation ($\sim E^2/H^2$) is associated with the presence of the plasma waves; the frequency of the radiation arising from the mechanism under consideration is much larger than the frequency of the cyclotron radiation (by $\sim 1/v_T^2$, where v_T is the velocity of the turbulent motion in the plasma). Therefore, the appearance of transverse electromagnetic waves in the scattering of electrons by plasma waves can create new bands of non-thermal radiation of cosmic objects. For example, sources having a maximum of radio emission in the region of centimeter waves and $v_T^2 \sim 10^{-5} - 10^{-6}$ must also emit in the optical region as the result of the scattering of the electrons by plasma waves.

Actually, let us compare the mechanism considered here, radiation of transverse electromagnetic waves, with cyclotron radiation. Losses of energy by the electron as the result of the cyclotron radiation are equal to

$$Q_{\rm cr} = 2e^4H^2v^2/3m^2\,(1 - v^2). \tag{6.1}$$

The loss ratio in the ultrarelativistic limit tends to

$$\overline{Q}/Q_{\rm cr} = \tfrac{2}{3}\langle E^2\rangle/H^2. \tag{6.2}$$

Here $\sqrt{\langle E^2\rangle}$ is the mean fluctuating electric field of the plasma waves.

In accord with (3.3) [see also (1.5)], the radiation in scattering by plasma waves is concentrated in the region of frequencies of the order

$$\omega_\pi \sim k_1\varepsilon_{p_1}^2/m^2. \tag{6.3}$$

The cyclotron radiation is chiefly concentrated in the region

$$\omega_{\rm cr} \sim eH\varepsilon_{p_1}^2/m^3. \tag{6.4}$$

Then the frequency ratio has the order

$$\omega_\pi/\omega_{\rm cr} \sim k_1m/eH = k_1R_Dm/eHR_D. \tag{6.5}$$

Taking it into account that the Debye radius R_D, k_1, and the spontaneous magnetic field are connected with the plasma temperature T by the relations

$$H \sim \sqrt{8\pi Tn}, \qquad k_1R_D \lesssim 1, \qquad R_D = \sqrt{T/4\pi ne^2}, \tag{6.6}$$

we find that the scattering by the plasma waves produces radiation that is

$$\omega_\pi/\omega_{\rm cr} \sim m/T \sim 1/v_T^2 \tag{6.7}$$

greater in frequency than the cyclotron radiation mechanism.

In a number of cases, the ultrarelativistic electrons have a power-law spectrum

$$n(\gamma) = K\gamma^{-\alpha}. \tag{6.8}$$

We now find the frequency spectrum of the radiation of the electrons with the distribution (6.8). In accord with (3.3) and (3.5),

$$I(\omega) = \int d\gamma\, n(\gamma)\, Q_\gamma(\omega)$$
$$= b(\alpha)\frac{e^4E_0^2K}{m^2}\left(\frac{1}{2k_1}\right)^{(3-\alpha)/2}\omega^{(1-\alpha)/2}, \tag{6.9}$$

$$b(\alpha) = \frac{2}{3}\left(\frac{1}{\alpha-1} - \frac{3}{\alpha+1} + \frac{2}{\alpha+3} + \frac{6}{(\alpha+1)^2}\right). \tag{6.10}$$

The radiation also possesses a power spectrum, with the same spectral index $\frac{1}{2}(\alpha - 1)$ as the cyclotron radiation. A different dependence on α of the coefficient $b(\alpha)$ and the corresponding coefficient in the formulas of the cyclotron radiation[15] leads to the result that when the spectrum of the electrons deviates from the power-law, the spectra of the cyclotron radiation and the mechanism studied by us do not appear entirely similar.

APPENDIX

FORMULAS FOR THE FUNCTIONS $\Phi(\gamma, q)$

By integration of (3.2), we get the following expressions for the functions $\Phi(\gamma, q)$ that enter into Eq. (3.3):

$$\Phi(\gamma, q) = \Phi_1(\gamma, q) + \Phi_2(\gamma, q) + \Phi_3(\gamma, q); \tag{A.1}$$

$$\Phi_1(\gamma, q) = \psi_1(\gamma, q, x_{max}) - \psi_1(\gamma, q, x_{min}),$$
$$\Phi_2(\gamma, q) = \psi_2(\gamma, q, x_{max}) - \psi_2(\gamma, q, x_{min}),$$
$$\Phi_3(\gamma, q) = \psi_3(\gamma, q, x_{max}) - \psi_3(\gamma, q, x_{min}); \tag{A.2}$$

$$\psi_1 = \frac{1}{\gamma^6v^3(1+v)z^4q}\left\{-x - \frac{z^2}{x} - \frac{1}{2v^4\gamma^2}\right.$$
$$\times\left[3x + \frac{1}{x}\left(z^2v^2 - \frac{3-2v^2}{\gamma^2}\right)\right.$$
$$\left.\left. - \frac{v^2z^2}{x^2}\left(1 - \frac{1}{3x\gamma^2}\right) - 2(3-2v^2)\ln x\right]\right\}; \tag{A.3}$$

$$\psi_2 = \frac{1}{v^3(1+v)\gamma^4q}\left\{-\frac{x}{4} + \frac{v^2}{32}(2x + 3z^2 - 12)\varphi(x)\right.$$
$$+ \frac{v}{32}\,[(5-3v^2)z^4 - (16-8v^2)z^2$$
$$+ 24 - 8v^4]\ln|v\varphi(x) + 2x - z^2|$$
$$- \frac{(z^2-2)^2}{8(4-z^2)\varphi(x)}\left[(2x - z^2)\left(\frac{z^2v^2}{2} + 2 - v^2\right)\right.$$
$$\left.\left. - (1-z^2)\left(v^3z^2 + \frac{4}{\gamma^2} - 2x\right)\right]\right\}; \tag{A.4}$$

$$\psi_3 = -\frac{1}{z^2v^3(1+v)q\gamma^5}\left\{\frac{x - \ln x}{v^2\gamma^2} - \left(\frac{z^2}{2} + \frac{1}{\gamma^2}\right)\frac{1}{x}\right.$$
$$- v^2\varphi(x)\left[\frac{1}{2} - \frac{z^2/2 + \gamma^{-2}}{[v^2z^2 + 4/\gamma^2]x}\right]$$

$$+\frac{2v}{z\sqrt{v^2z^2+4/\gamma^2}}\left[\frac{1}{\gamma^2}-\frac{z^2}{2}+\frac{z^2(z^2/2+\gamma^{-2})}{v^2z^2+4/\gamma^2}\right] \times \ln\left|\frac{1}{x}\left(v^2z^2+\frac{4}{\gamma^2}-2x+\frac{v\varphi(x)}{z}\sqrt{v^2z^2+4/\gamma^2}\right)\right|\Bigg\}. \qquad (A.5)$$

In the formulas given above, the following notation is used:

$$v=\sqrt{\gamma^2-1}/\gamma, \quad z=(1-v)/vq, \quad x_{min}=1-v,$$

$$x_{max}=\min(vz,\ 1+v)$$

$$\varphi(x)=\left[z^4+\frac{4z^2}{v^2\gamma^2}-\frac{4z^2}{v^2}x+\frac{4x^2}{v^2}\right]^{1/2}. \qquad (A.6)$$

The graphs of the functions $\Phi(\gamma, q)$ computed by these formulas, are shown in Fig. 4. $\Phi_1(\gamma, q)$ corresponds to a neglect of the oscillations of the density in the plasma wave, and its graphs are given in Fig. 5. The function $\Phi_2(\gamma, q)$ characterizes the radiation by ions and is shown in Fig. 6.

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