

CARTOGRAPHY, PROJECTED COORDINATE SYSTEM

&

COSMOGRAPHY



CARTOGRAPHY,
PROJECTED
COORDINATE SYSTEMS

AND

COSMOGRAPHY

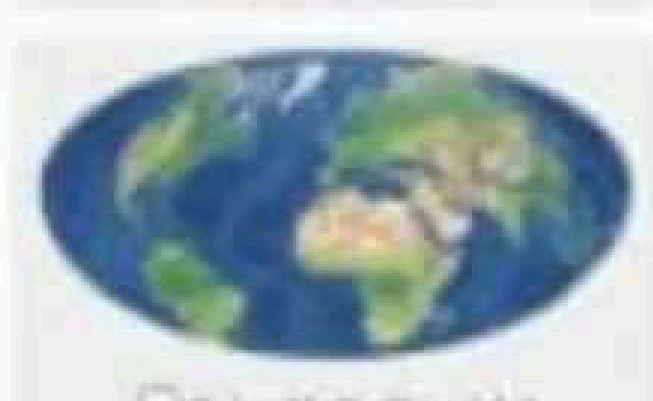
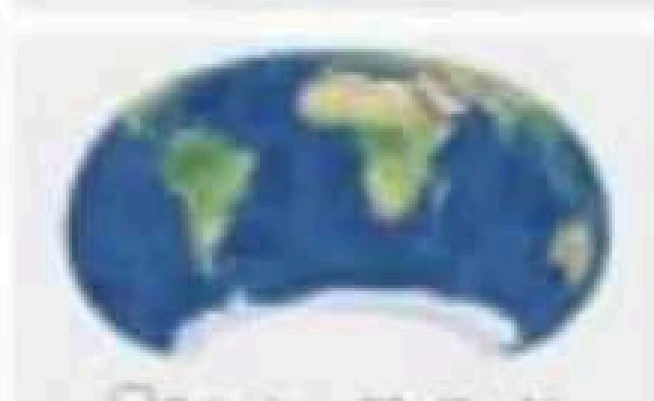
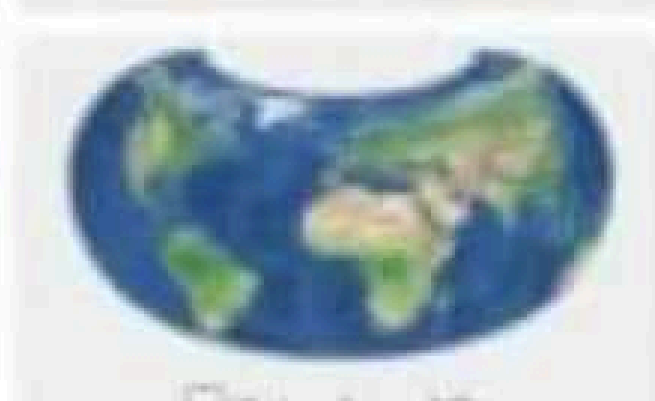
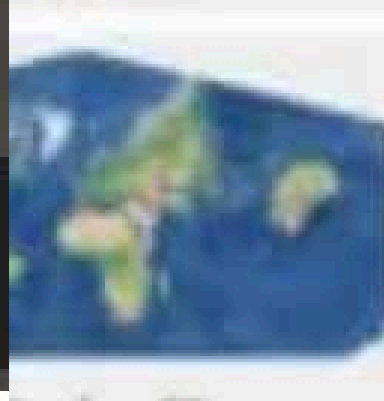
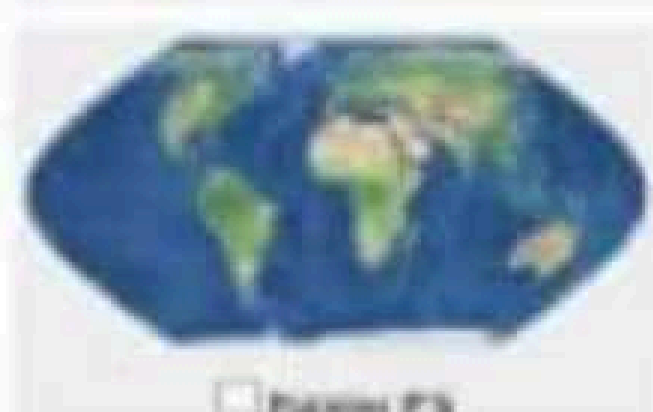
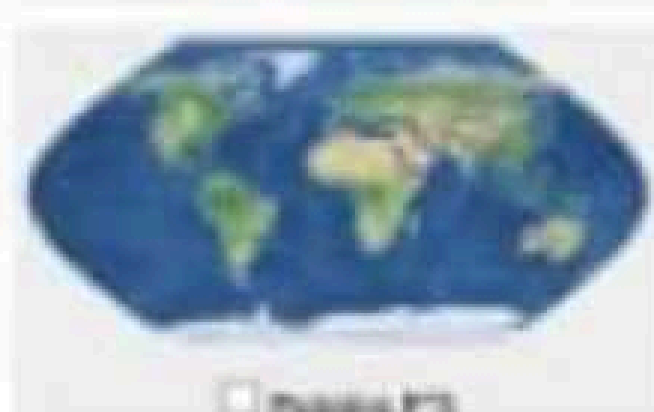
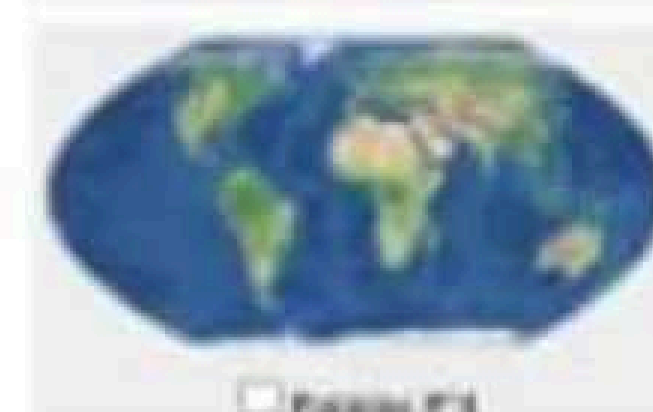
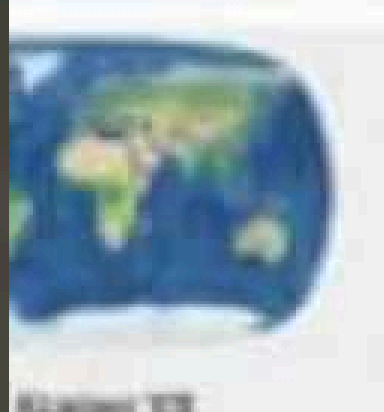
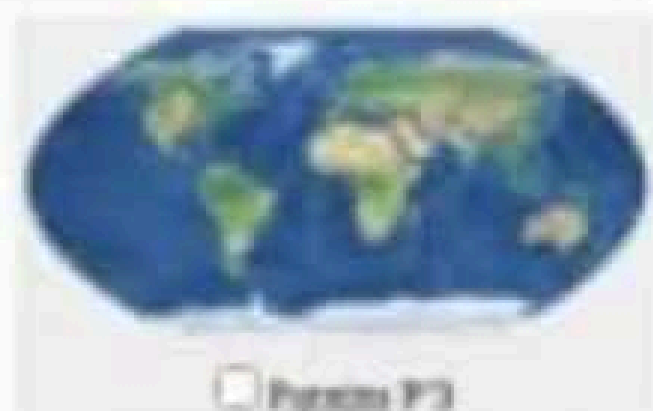
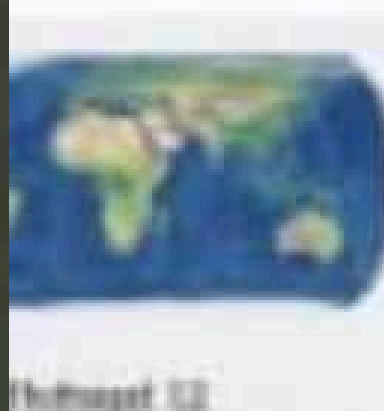
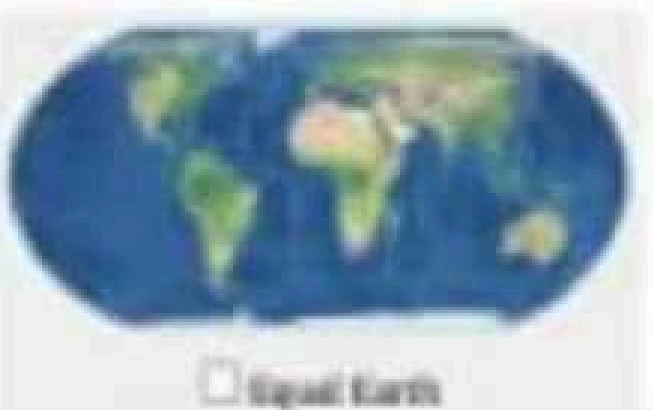
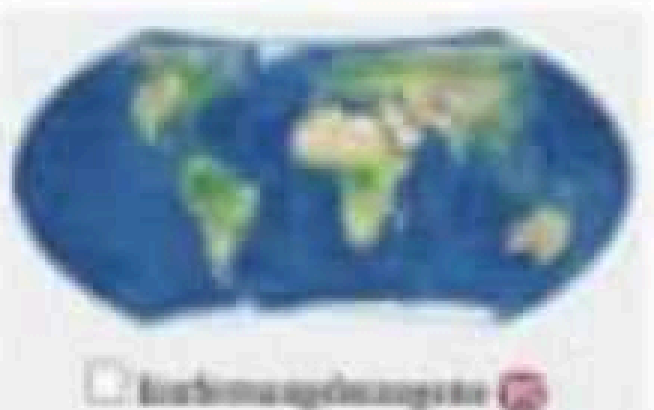
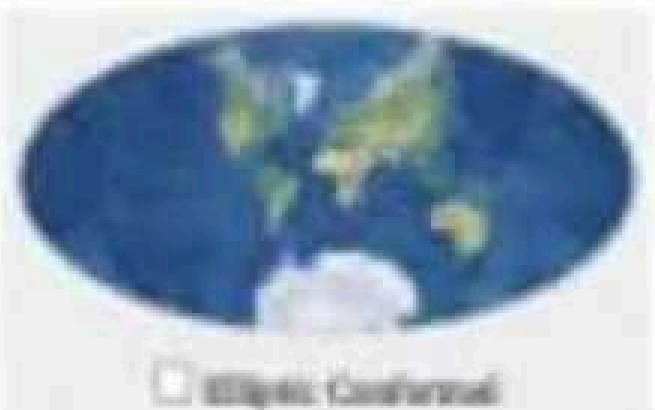
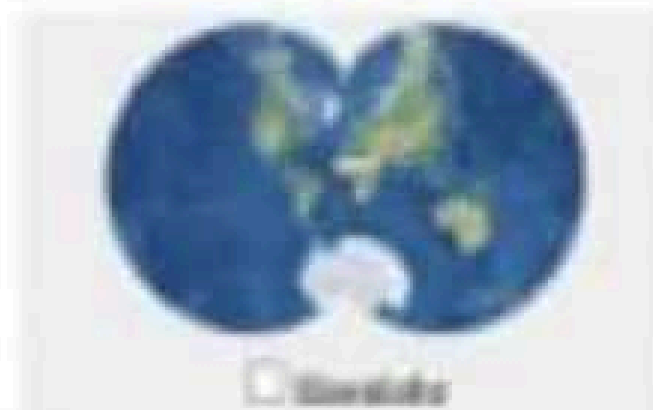
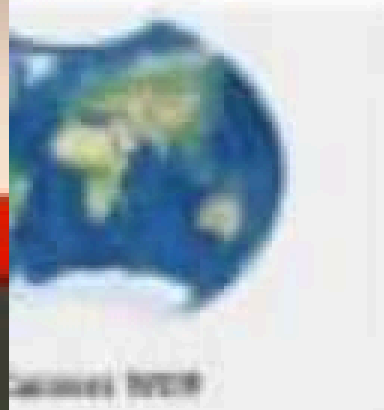
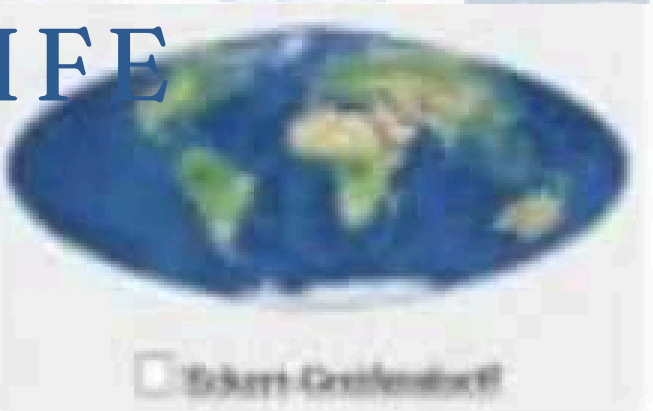
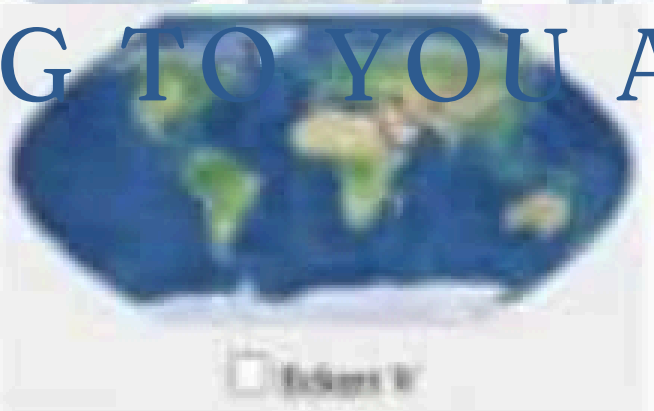
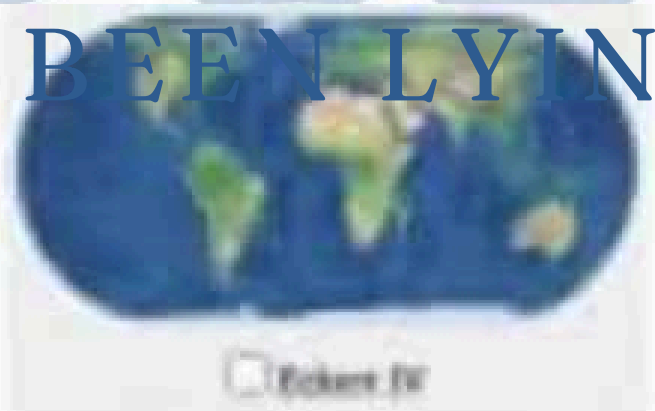
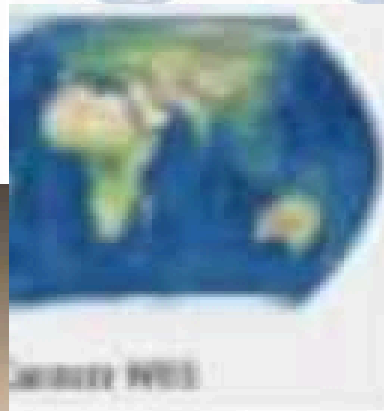
ITINERARY

120 ISH SLIDES (FAST)

- **MAP PROJECTION LIVE DEMO (10 MINS)**
- **DATABASE SUPPLAMENT**
- **PROJECTED COORDINATE SYSTEMS AND TYPES OF PROJECTIONS**
- **TYPES AND EXAMPLES.**
- **SPECIAL INTEREST ON AE**
- **CELESTIAL COORDINATES AND BASED ON ANGLES TO STARS**
- **CELESTIAL CURVATUE**
- **GEOGRAPHIC TO CELESTIAL 1 TO 1**
- **ASTROGEODEDICS**
- **SPHERE TO HEMISPHERE, AND CIRCUMFERENCES**
- **COSMOGRAPHY**

SHOOTING FOR 90 MINUTES

MAPS HAVE BEEN LYING TO YOU ALL YOUR LIFE



DO ALL MAPS PROJECT THE GLOBE?

Map projection

 54 languages ▼

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From Wikipedia, the free encyclopedia

(Redirected from [Map projections](#))

In [cartography](#), a **map projection** is any of a broad set of transformations employed to represent the curved two-dimensional surface of a globe on a plane.^{[1][2][3]} In a map projection, coordinates, often expressed as [latitude and longitude](#), of locations from the surface of the globe are transformed to coordinates on a plane.^{[4][5]} Projection is a necessary step in creating a two-dimensional map and is one of the essential elements of cartography.

All projections of a sphere on a plane necessarily distort the surface in some way and to some extent.^[6] Depending on the purpose of the map, some distortions are acceptable and others are not; therefore, different map projections exist in order to preserve some properties of the sphere-like body at the expense of other properties. The study of map projections is primarily about the characterization of their distortions. There is no limit to the number of possible map projections.^{[7]:1} More generally, projections are considered in several fields of pure mathematics, including [differential geometry](#), [projective geometry](#), and [manifolds](#). However, the term "map projection" refers specifically to a [cartographic](#) projection.



A medieval depiction of the [Ecumene](#) (1482, Johannes Schnitzer, engraver), constructed after the coordinates in Ptolemy's *Geography* and using his second map projection 

WHAT DO MAP PROJECTIONS ACTUALLY

'PROJECT'

- **ALL MAPS PROJECT COORDINATE SYSTEMS.**

- MOST COMMON IS THE *GEOGRAPHIC COORDINATE SYSTEM AKA (LATITUDE AND LONGITUDE)*

- IN ADDITION OTHER COORDINATE SYSTEMS ARE:

- UTM (UNIVERSAL TRANSVERSE MERCATOR)

- STATE PLANE COORDINATE SYSTEM (SPCS)

- CARTESIAN COORDINATE SYSTEM

- MILITARY GRID REFERENCE SYSTEM (MGRS)

- LOCAL COORDINATE SYSTEMS

- EARTH-CENTERED, EARTH-FIXED (ECEF) COORDINATE SYSTEM

- GEOCENTRIC COORDINATE SYSTEM

Projected coordinate system

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(Redirected from [Grid reference system](#))

"Easting and northing" redirects here. Not to be confused with [East north up](#).

For broader coverage of this topic, see [Spatial reference system](#).

A **projected coordinate system** – also called a **projected coordinate reference system**, **planar coordinate system**, or **grid reference system** – is a type of [spatial reference system](#) that represents locations on Earth using Cartesian coordinates (x, y) on a planar surface created by a particular map projection.^[1] Each projected coordinate system, such as "[Universal Transverse Mercator WGS 84 Zone 26N](#)," is defined by a choice of map projection (with specific parameters), a choice of geodetic datum to bind the coordinate system to real locations on the earth, an origin point, and a choice of unit of measure.^[2] Hundreds of projected coordinate systems have been specified for various purposes in various regions.

When the first standardized coordinate systems were created during the 20th century, such as the [Universal Transverse Mercator](#), [State Plane Coordinate System](#), and [British National Grid](#), they were commonly called *grid systems*; the term is still common in some domains such as the military that encode coordinates as alphanumeric *grid references*.

However, the term *projected coordinate system* has recently become predominant to clearly differentiate it from other types of [spatial reference system](#). It is used in international standards such as the [EPSG](#) and [ISO 19111](#) (also published by the [Open Geospatial Consortium](#) as [Abstract Specification 2](#)), and in most [geographic information system](#) software.^{[3][2]}

COORDINATE SYSTEMS

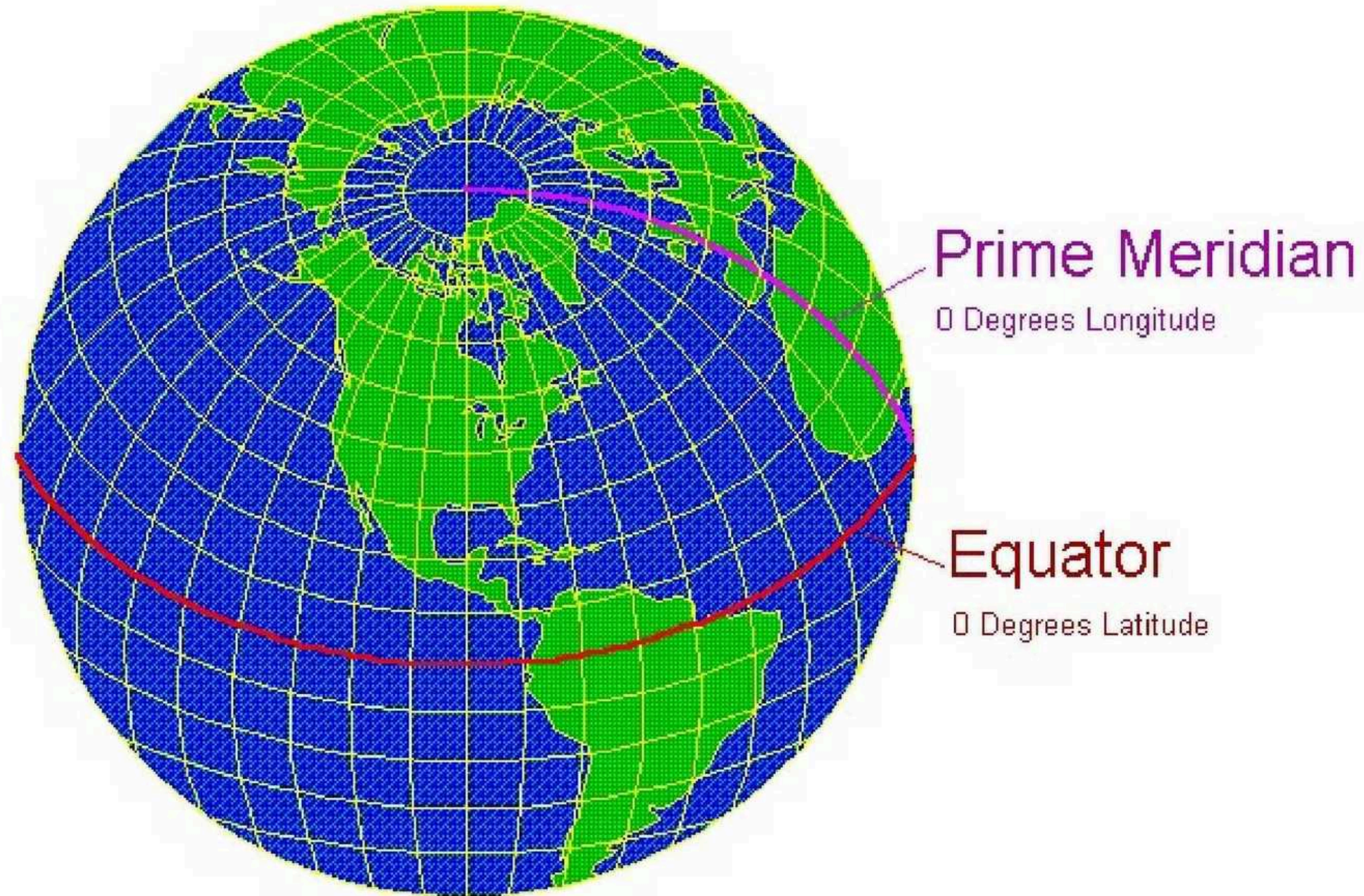
BASED ON A DATUM

Coordinate conversions and coordinate transformations change coordinate values in one coordinate reference system to coordinate values in another coordinate reference system. A **coordinate system** is a set of mathematical rules for specifying how coordinates are to be assigned to points. It includes the definition of the coordinate axes, the units to be used and the geometry of the axes. A coordinate system is an abstract concept, unrelated to the Earth. A coordinate system is related to the Earth through a **datum**. The combination of coordinate system and datum is a **coordinate reference system (CRS)**. If a different datum is used, the coordinates of a point to change. Colloquially the term coordinate system has historically been used to mean coordinate reference system.

Coordinates may be changed from one coordinate reference system to another through the application of a **coordinate operation**. Two types of coordinate operation may be distinguished:

- **coordinate conversion**, where no change of datum is involved and the parameters are chosen and thus error free.
- **coordinate transformation**, where the target CRS is based on a different datum to the source CRS. Transformation parameters are empirically determined and thus subject to measurement errors.

Geographical Coordinates



Peter H. Dana 9/1/94

A geographic coordinate system (GCS) is a spherical or geodetic coordinate system for measuring and communicating positions directly on Earth as latitude and longitude. It is the simplest, oldest and most widely used of the various spatial reference systems that are in use, and forms the basis for most others.

Although latitude and longitude form a coordinate tuple like a cartesian coordinate system, the geographic coordinate system is not cartesian because the measurements are angles and are not on a planar surface.

Geographic Coordinates

- ✦ **Latitude** (y) - north-south distance from the equator - the "origin". Also called parallels.
- ✦ **Longitude** (x) - east-west angular distance from a prime meridian - the "origin". Also called meridians.
- ✦ Not a map projection - a set of spherical coordinates used to reference positions on the curved surface of the Earth for use in map projections.
- ✦ Basis for projected coordinate systems

Geo-Referencing Systems

- ✦ **Geographic Grid**
 - a.k.a Latitude & Longitude
 - Continuous, 3-D
 - is geo-referenced... but not suitable for 2D mapping (not planar)

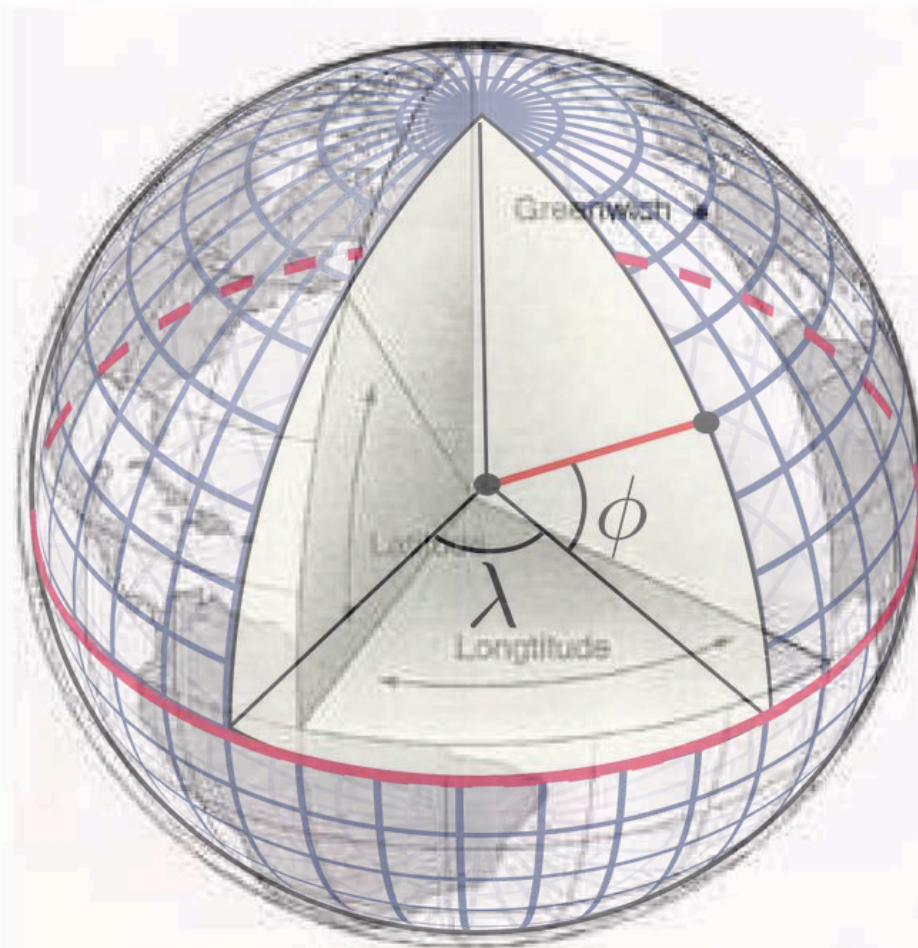
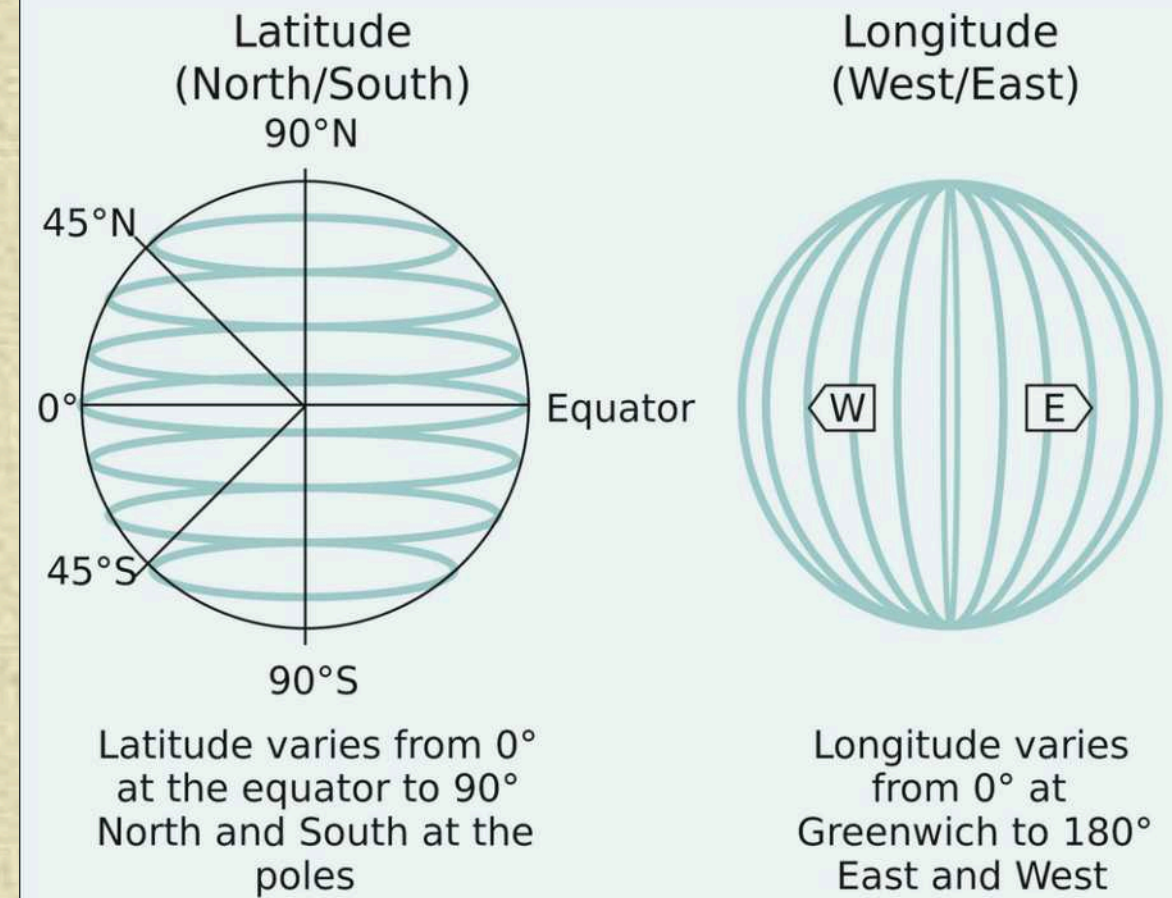


Figure 2.5 The geographic grid. Spherical grid system showing parallels and meridians. Parallels allow us to measure angular distance north and south (latitude) from the equator (0 degrees latitude) up to a maximum of 90 degrees north (North Pole) and 90 degrees south (South Pole). Meridians start at the prime meridian and allow us to measure angular distance east and west (longitude) up to a maximum of 180 degrees where they would meet at the international date line. Source: Robinson et al., *Elements of Cartography*, 6th ed., John Wiley & Sons, Inc., New York, © 1995, modified from Figure 4.4, page 47. Used with permission.



Map Projection

✦ **The systematic mathematical transformation of the three-dimensional curved surface of the CELESTIAL SPHERE to the two-dimensional flat surface of a map.**

– Basically, it is how you “show” the curved 3-D Earth on a flat map.

✦ **Attempt to accurately depict the following characteristics:**

1. Direction
2. Distance
3. Area
4. Shape

*** No Map Projection can accurately depict all four characteristics at once.**

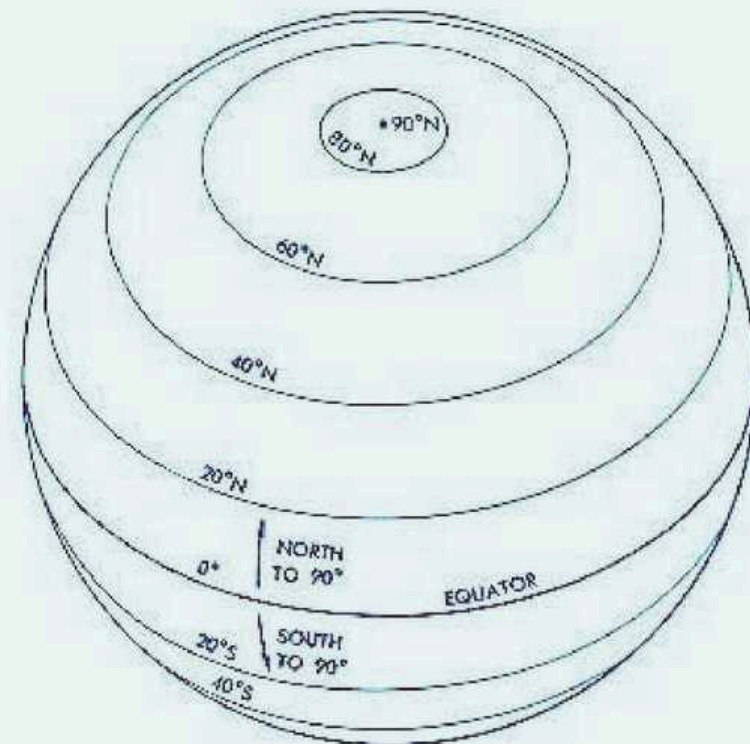
Every map projection will contain some distortion.

1. AREA-PRESERVING PROJECTION – ALSO CALLED EQUAL AREA OR EQUIVALENT PROJECTION, THESE PROJECTIONS MAINTAIN THE RELATIVE SIZE OF DIFFERENT REGIONS ON THE MAP.
2. SHAPE-PRESERVING PROJECTION – OFTEN REFERRED TO AS CONFORMAL OR ORTHOMORPHIC, THESE PROJECTIONS MAINTAIN ACCURATE SHAPES OF REGIONS AND LOCAL ANGLES.
3. DIRECTION-PRESERVING PROJECTION – THIS CATEGORY INCLUDES CONFORMAL, ORTHOMORPHIC, AND AZIMUTHAL PROJECTIONS, WHICH PRESERVE DIRECTIONS, BUT ONLY FROM THE CENTRAL POINT FOR AZIMUTHAL PROJECTIONS.
4. DISTANCE-PRESERVING PROJECTION – KNOWN AS EQUIDISTANT PROJECTIONS, THEY DISPLAY THE TRUE DISTANCE BETWEEN ONE OR TWO POINTS AND ALL OTHER POINTS ON THE MAP.

✦ Not a map projection - a set of spherical coordinates used to reference positions on the curved surface of the Earth for use in map projections. ✦ Basis for projected coordinate systems

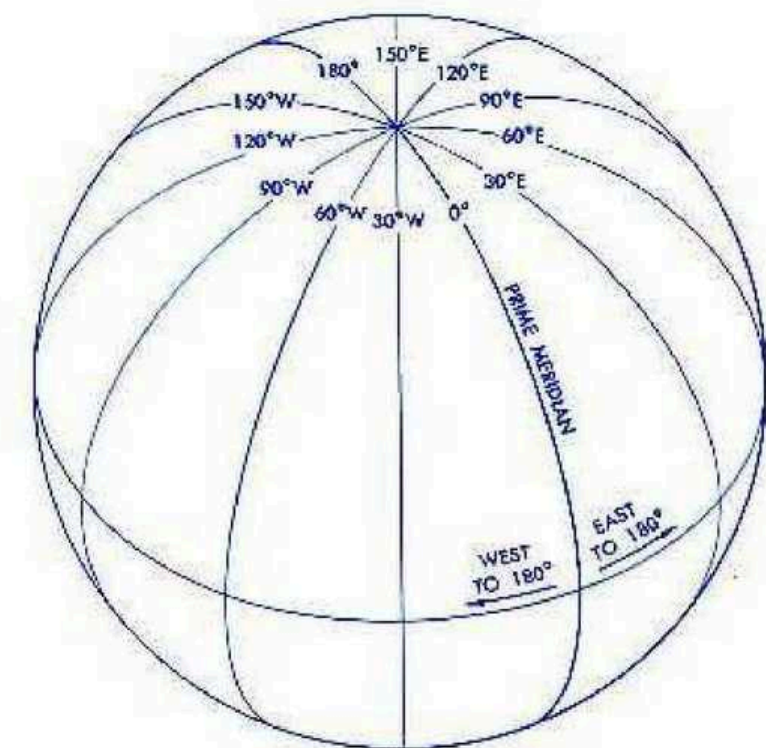
Latitude

- ✦ A function of the angle between the horizon and the North Star
- ✦ Range: 0 – 90 degrees
- ✦ Origin: The Equator
- ✦ Direction: North and South of the origin



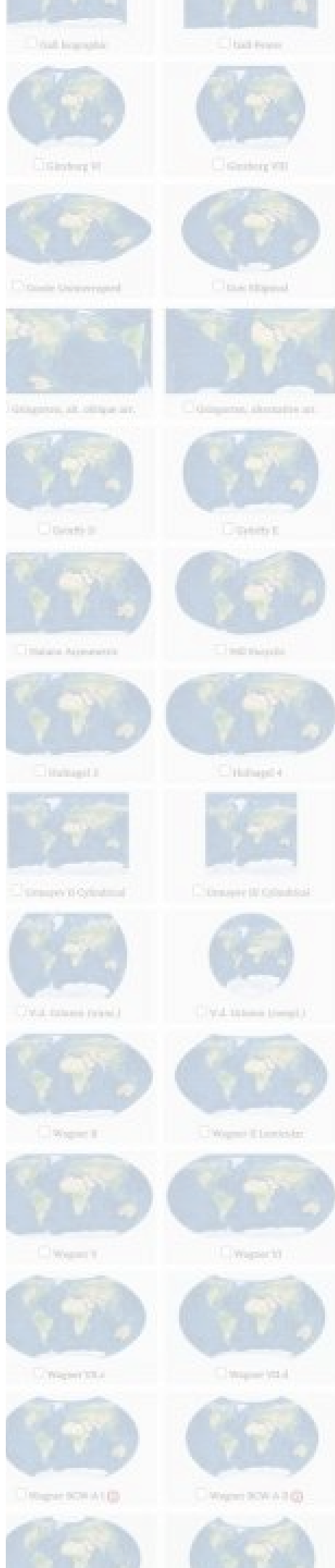
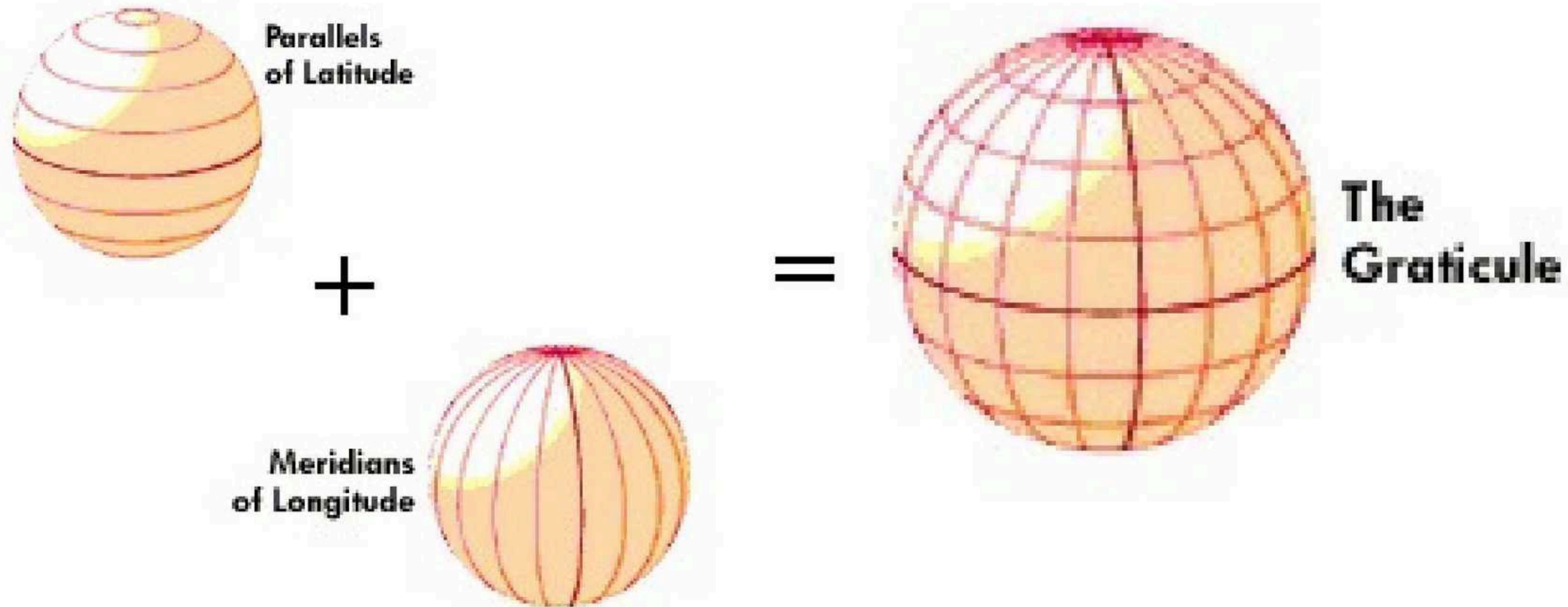
Longitude

- ✦ Comprises meridians (half circles)
- ✦ Range: 0 – 180 degrees
- ✦ Origin: The Prime Meridian
- ✦ Direction: East and West of the origin



The Graticule

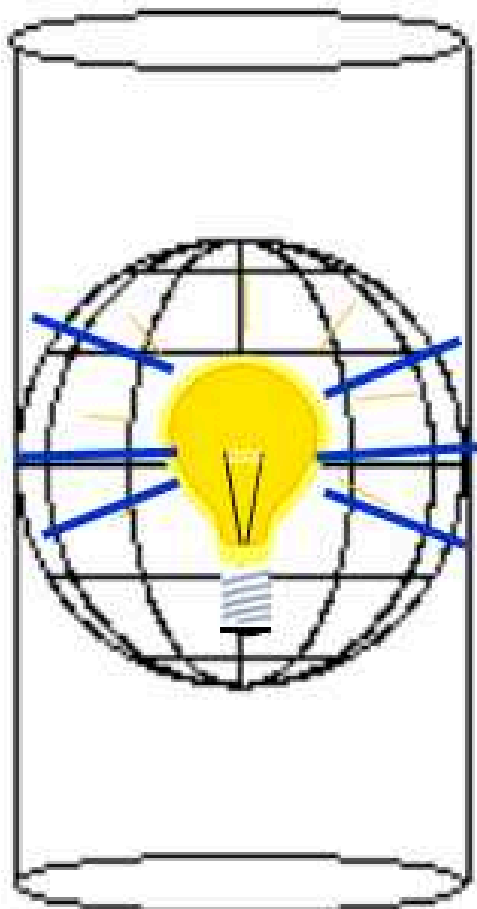
✦ The **graticule** is the gridded network of latitude and longitude, the pattern that the meridians of longitude and the parallels of latitude form on the surface of the earth.



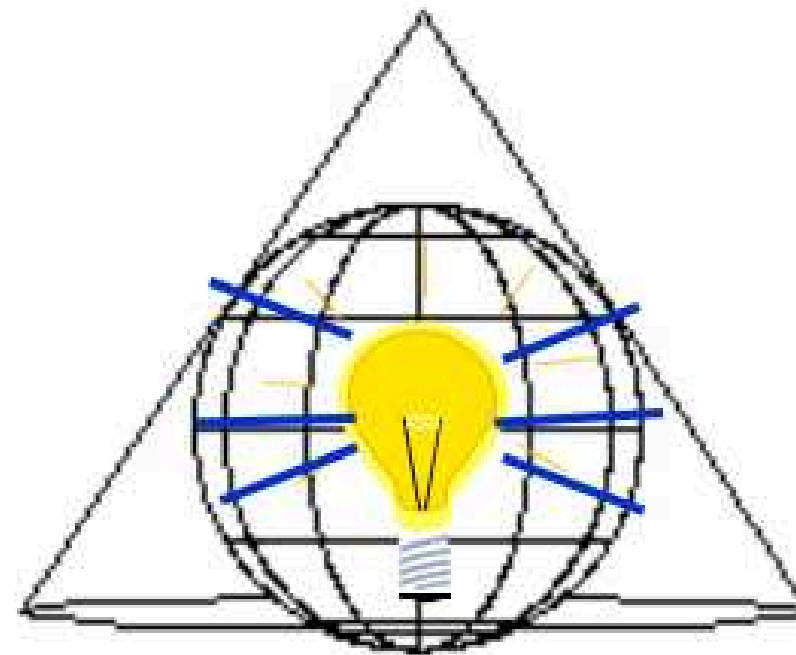
The Graticule

- Picture a light source **projecting** the shadows of the graticule lines on the surface of a transparent globe onto the developable surface ...

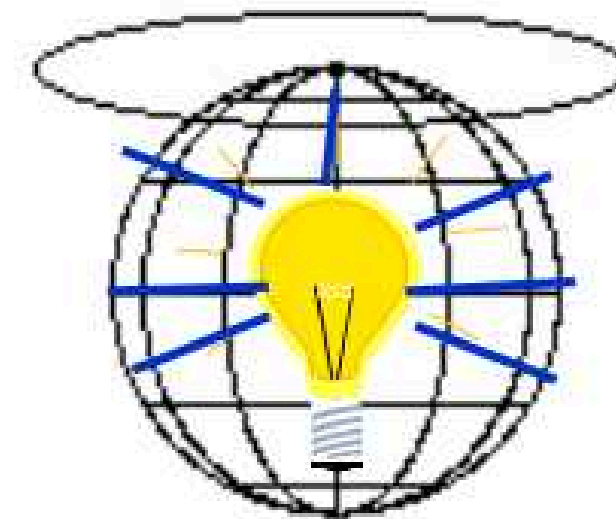
Cylinder



Cone



Plane

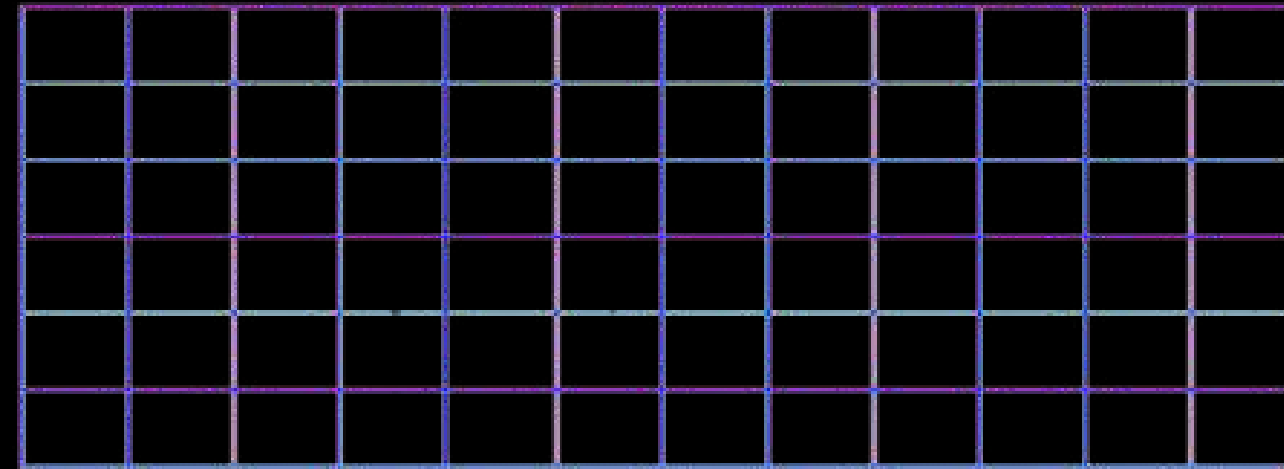


COORDINATE SYSTEM & TRANSFORMATION

- **Coordinate systems** are defined as a system used to represent a point in space
- Classified a Orthogonal and Nonorthogonal Coordinate system
- For **orthogonal coordinate system**, the coordinates are mutually perpendicular. The orthogonal coordinate systems include
 - **Rectangular or Cartesian coordinate system**
 - **Cylindrical or circular coordinate system**
 - **Spherical coordinate system**

The Graticule, Projected

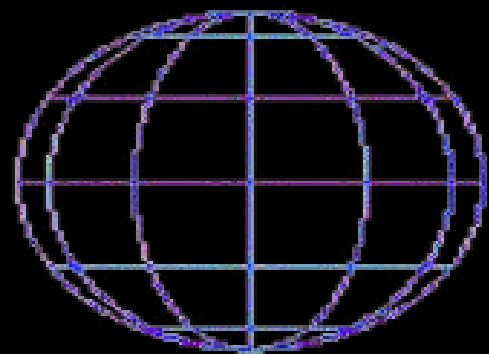
CYLINDRICAL GRATICULE



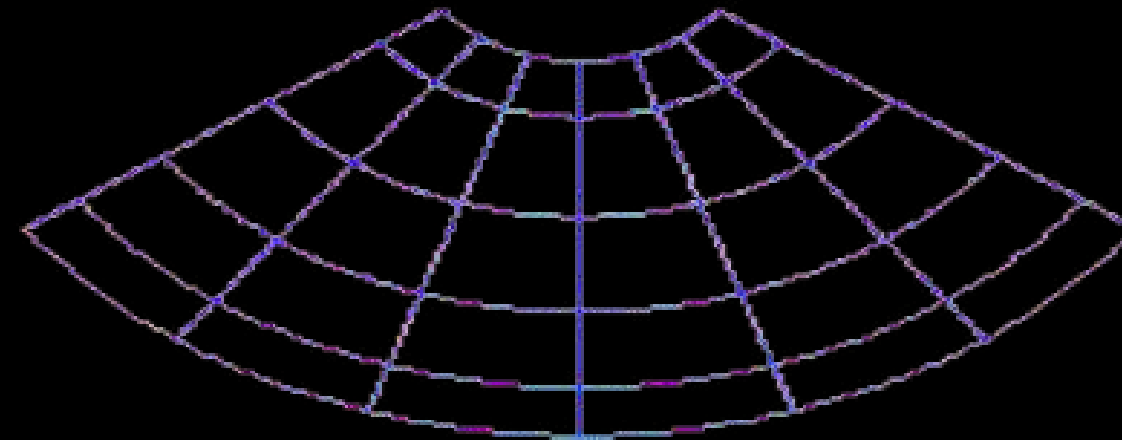
Cone



GLOBE GRATICULE



CONIC GRATICULE

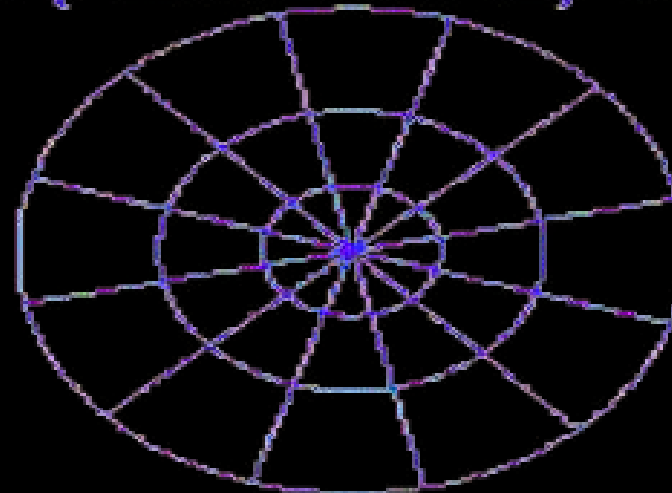


Cylinder



Plane

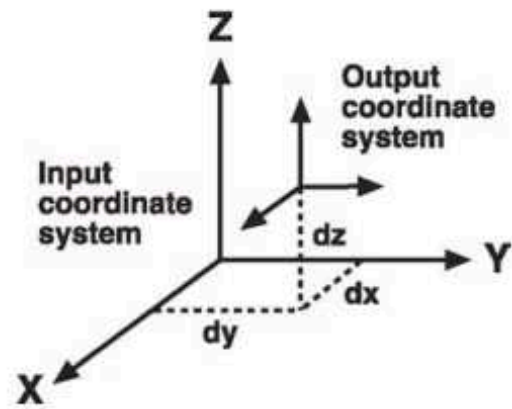
PLANAR (AZIMUTHAL) GRATICULE



ALL MAPS ARE THE PROJECTING THE GRATICULE

Three-parameter methods

The simplest datum transformation method is a geocentric, or three-parameter, transformation. The geocentric transformation models the differences between two datums in the X,Y,Z coordinate system. One datum is defined with its center at 0,0,0. The center of the other datum is defined at some distance ($\Delta X, \Delta Y, \Delta Z$) in meters away.



Usually the transformation parameters are defined as going 'from' a local datum 'to' WGS 1984 or another geocentric datum.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$

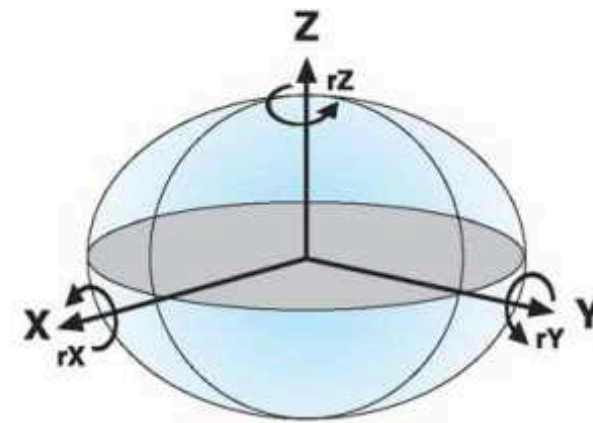
The three parameters are linear shifts and are always in meters.

Seven-parameter methods

A more complex and accurate datum transformation is possible by adding four more parameters to a geocentric transformation. The seven parameters are three linear shifts ($\Delta X, \Delta Y, \Delta Z$), three angular rotations around each axis (r_x, r_y, r_z), and scale factor(s).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+s) \cdot \begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$

The rotation values are given in decimal seconds, while the scale factor is in parts per million (ppm). The rotation values are defined in two different ways. It's possible to define the rotation angles as positive either clockwise or counterclockwise as you look toward the origin of the X,Y,Z systems.



The Coordinate Frame (or Bursa-Wolf) definition of the rotation values.

The equation in the previous column is how the United States and Australia define the equations and is called the Coordinate Frame Rotation transformation. The rotations are positive counterclockwise. Europe uses a different convention called the Position Vector transformation. Both methods are sometimes referred to as the Bursa-Wolf method. In the Projection Engine, the Coordinate Frame and Bursa-Wolf methods are the same. Both Coordinate Frame and Position Vector methods are supported, and it is easy to convert transformation values from one method to the other simply by changing the signs of the three rotation values. For example, the parameters to convert from the WGS 1972 datum to the WGS 1984 datum with the Coordinate Frame method are (in the order, $\Delta X, \Delta Y, \Delta Z, r_x, r_y, r_z, s$):

(0.0, 0.0, 4.5, 0.0, 0.0, -0.554, 0.227)

To use the same parameters with the Position Vector method, change the sign of the rotation so the new parameters are:

(0.0, 0.0, 4.5, 0.0, 0.0, +0.554, 0.227)

Unless explicitly stated, it's impossible to tell from the parameters alone which convention is being used. If you use the wrong method, your results can return inaccurate coordinates. The only way to determine how the parameters are defined is by checking a control point whose coordinates are known in the two systems.

Molodensky method

The Molodensky method converts directly between two geographic coordinate systems without actually converting to an X,Y,Z system. The Molodensky method requires three shifts ($\Delta X, \Delta Y, \Delta Z$) and the differences between the semimajor axes (Δa) and the flattenings (Δf) of the two spheroids. The Projection Engine automatically calculates the spheroid differences according to the datums involved.

$$(M+h)\Delta\varphi = -\sin\varphi\cos\lambda\Delta X - \sin\varphi\sin\lambda\Delta Y + \cos\varphi\Delta Z + \frac{e^2\sin\varphi\cos\varphi}{(1-e^2\sin^2\varphi)^{1/2}}\Delta a + \sin\varphi\cos\varphi\left(M\frac{a}{b} + N\frac{b}{a}\right)\Delta f$$

$$(N+h)\cos\varphi\Delta\lambda = -\sin\lambda\Delta X + \cos\lambda\Delta Y$$

$$\Delta h = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z - (1-e^2\sin^2\varphi)^{1/2}\Delta a + \frac{a(1-f)}{(1-e^2\sin^2\varphi)^{1/2}}\sin^2\varphi\Delta f$$

- h ellipsoid height (meters)
- φ latitude
- λ longitude
- a semimajor axis of the spheroid (meters)
- b semiminor axis of the spheroid (meters)
- f flattening of the spheroid
- e eccentricity of the spheroid

M and N are the meridional and prime vertical radii of curvature, respectively, at a given latitude. The equations for M and N are:

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}$$

$$N = \frac{a}{(1-e^2\sin^2\varphi)^{1/2}}$$

You solve for $\Delta\lambda$ and $\Delta\varphi$. The amounts are added automatically by the Projection Engine.

Abridged Molodensky method

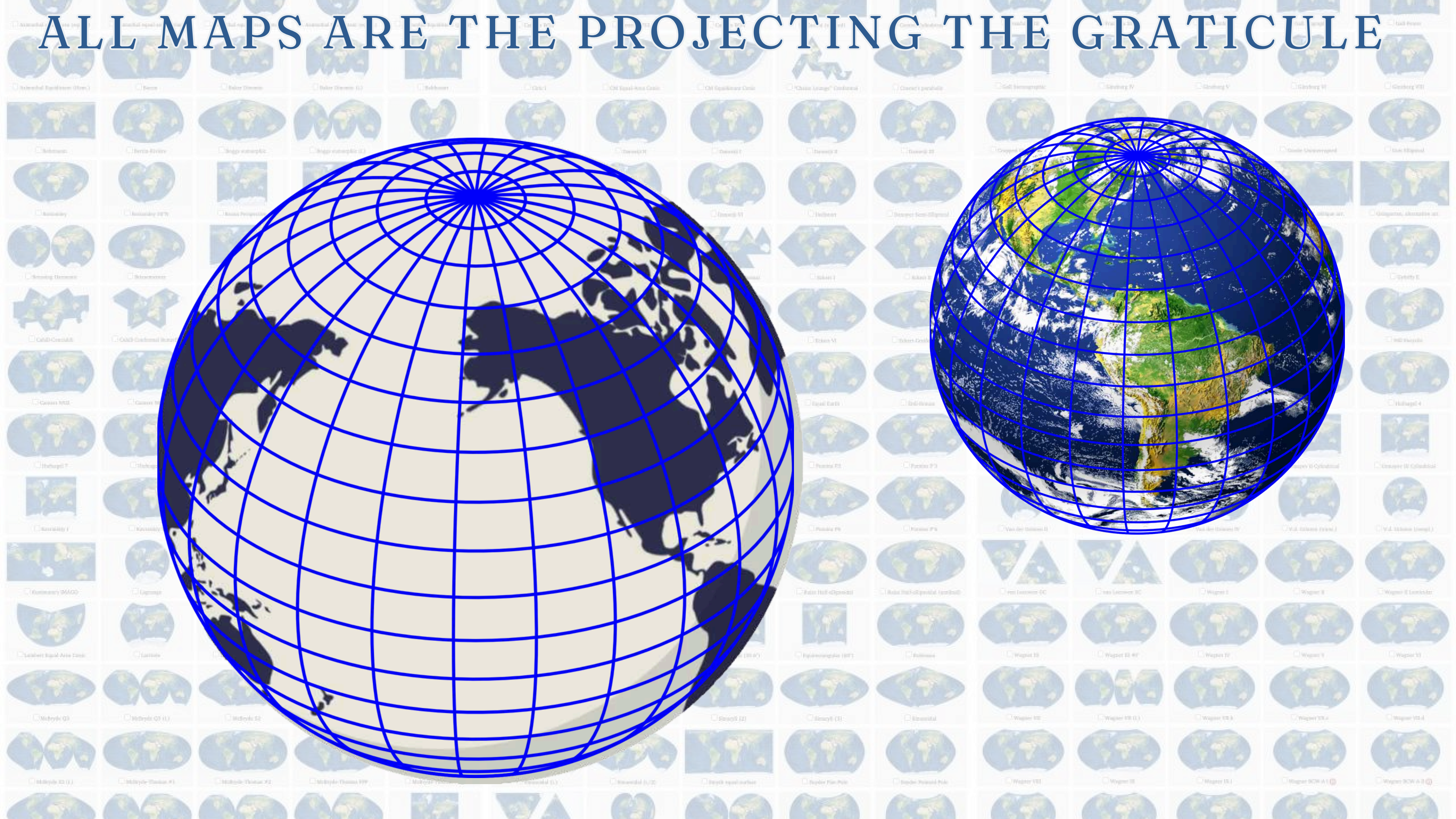
The Abridged Molodensky method is a simplified version of the Molodensky method. The equations are:

$$M\Delta\varphi = -\sin\varphi\cos\lambda\Delta X - \sin\varphi\sin\lambda\Delta Y + \cos\varphi\Delta Z + (a\Delta f + f\Delta a) \cdot 2\sin\varphi\cos\varphi$$

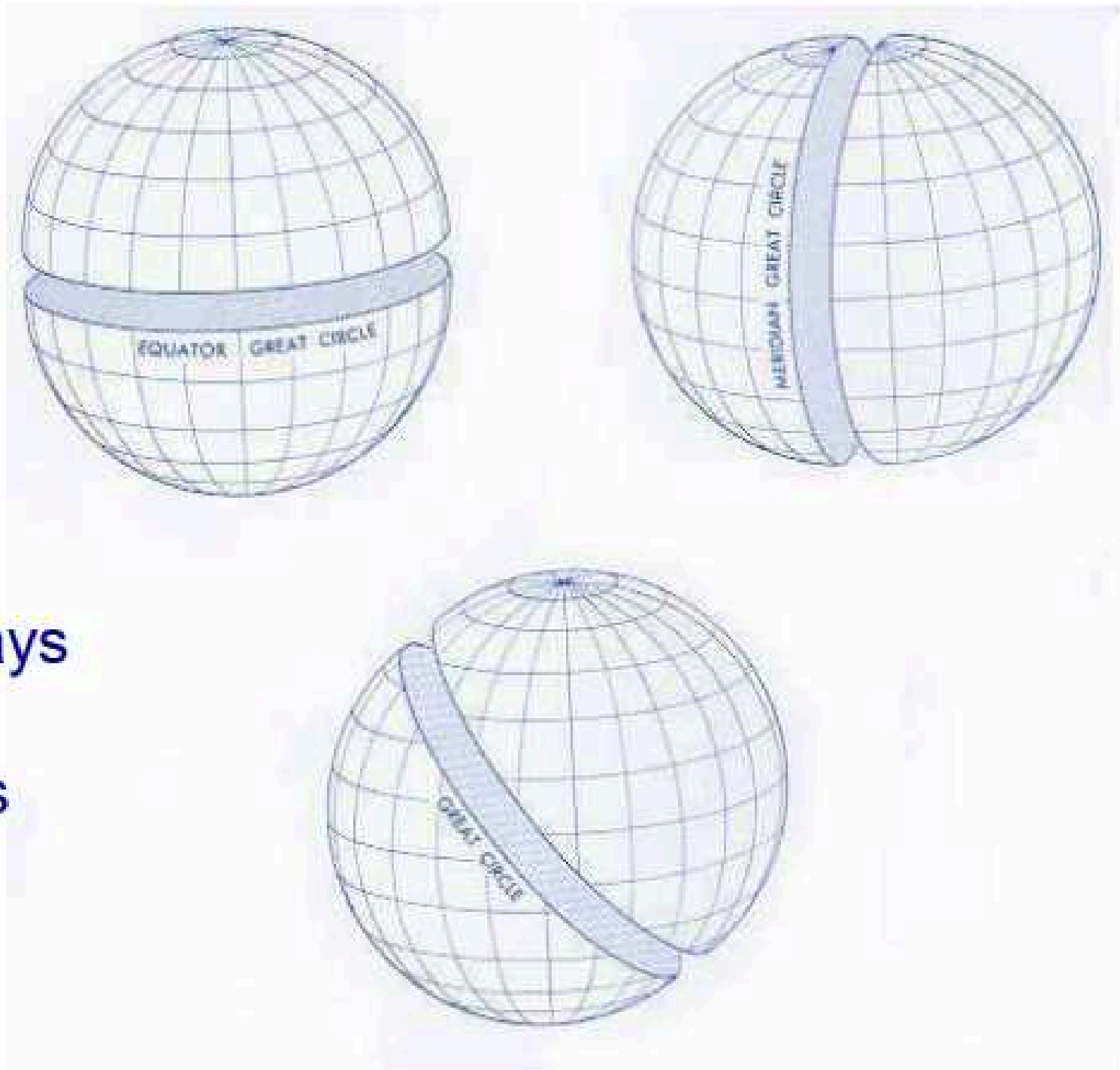
$$N\cos\varphi\Delta\lambda = -\sin\lambda\Delta X + \cos\lambda\Delta Y$$

$$\Delta h = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z + (a\Delta f + f\Delta a)\sin^2\varphi - \Delta a$$

ALL MAPS ARE THE PROJECTING THE GRATICULE



Properties of the Graticule



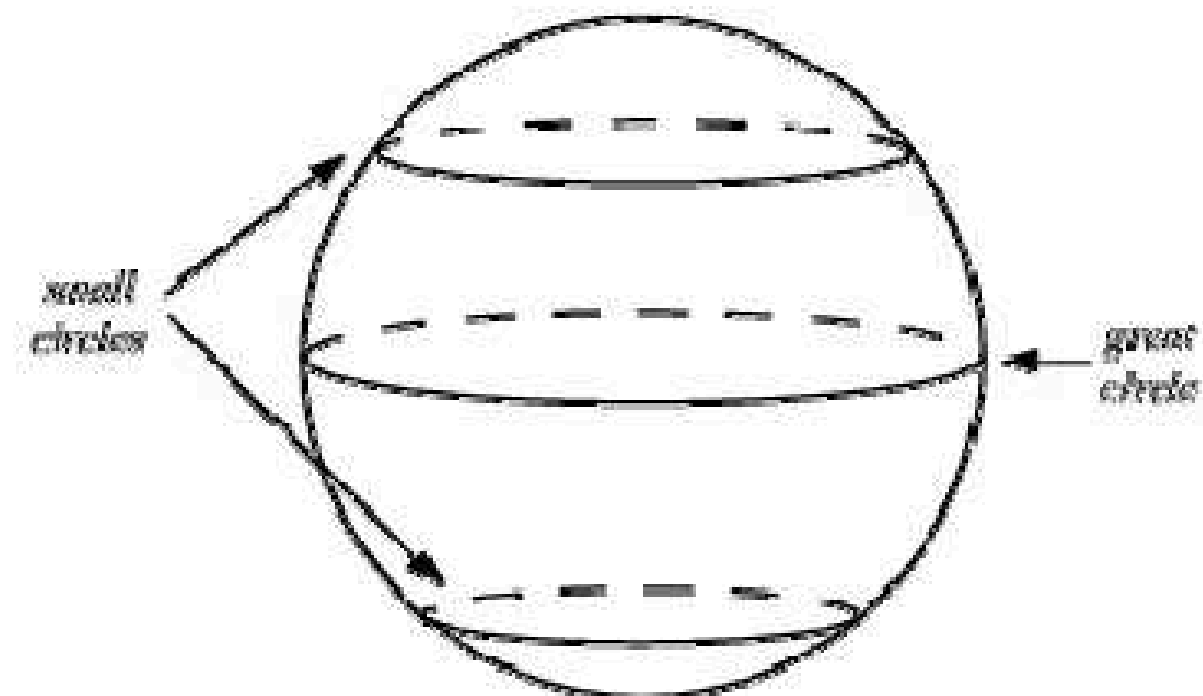
Different ways
of creating
great circles

See <http://www.csulb.edu/~rodrigue/geog140/lectures/geographicgrid.html>



Properties of the Graticule

- ✦ **Great Circle** - circle created by a plane that intersects 2 points and the center of the Earth (i.e. meridians or the equator)
- ✦ **Small Circle** - any circle created by an intersecting plane that doesn't cut through the center of the Earth (any parallel other than the equator)



Greater and smaller circles

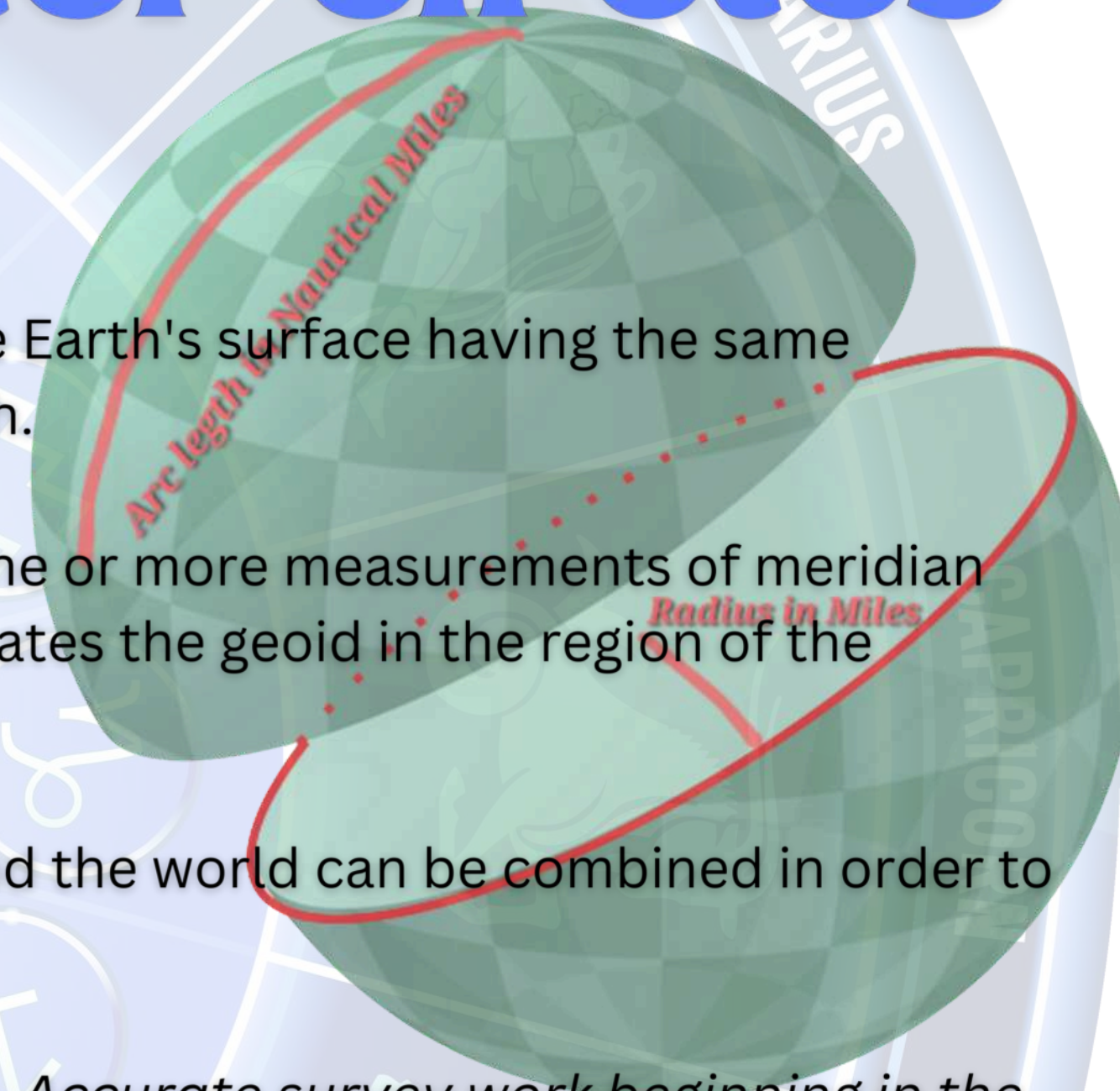
In geodesy and navigation, a meridian arc is the curve between two points on the Earth's surface having the same longitude. The term may refer either to a segment of the meridian, or to its length.

The purpose of measuring meridian arcs is to determine a figure of the Earth. One or more measurements of meridian arcs can be used to infer the shape of the reference ellipsoid that best approximates the geoid in the region of the measurement

Measurements of meridian arcs at several latitudes along many meridians around the world can be combined in order to approximate a geocentric ellipsoid intended to fit the entire world.

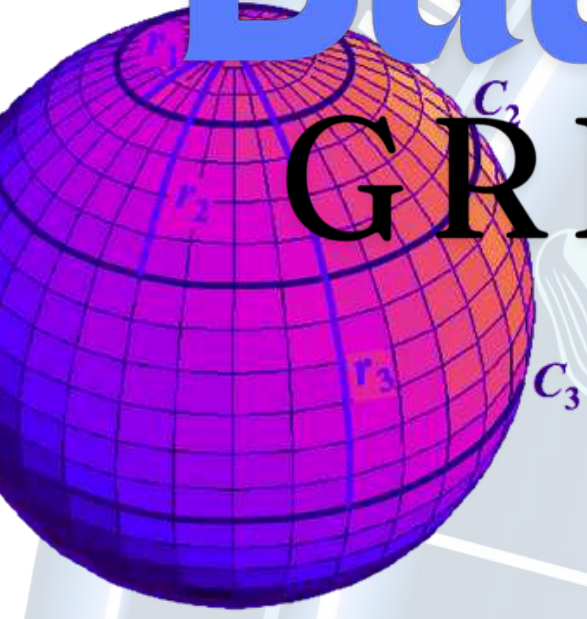
The earliest determinations of the size of a spherical Earth required a single arc. Accurate survey work beginning in the 19th century required several arc measurements in the region the survey was to be conducted, leading to a proliferation of reference ellipsoids around the world.

The latest determinations use **astro-geodetic measurements** and the methods of satellite geodesy to determine reference ellipsoids, especially the geocentric ellipsoids now used for global coordinate systems such as WGS 84 (see numerical expressions).



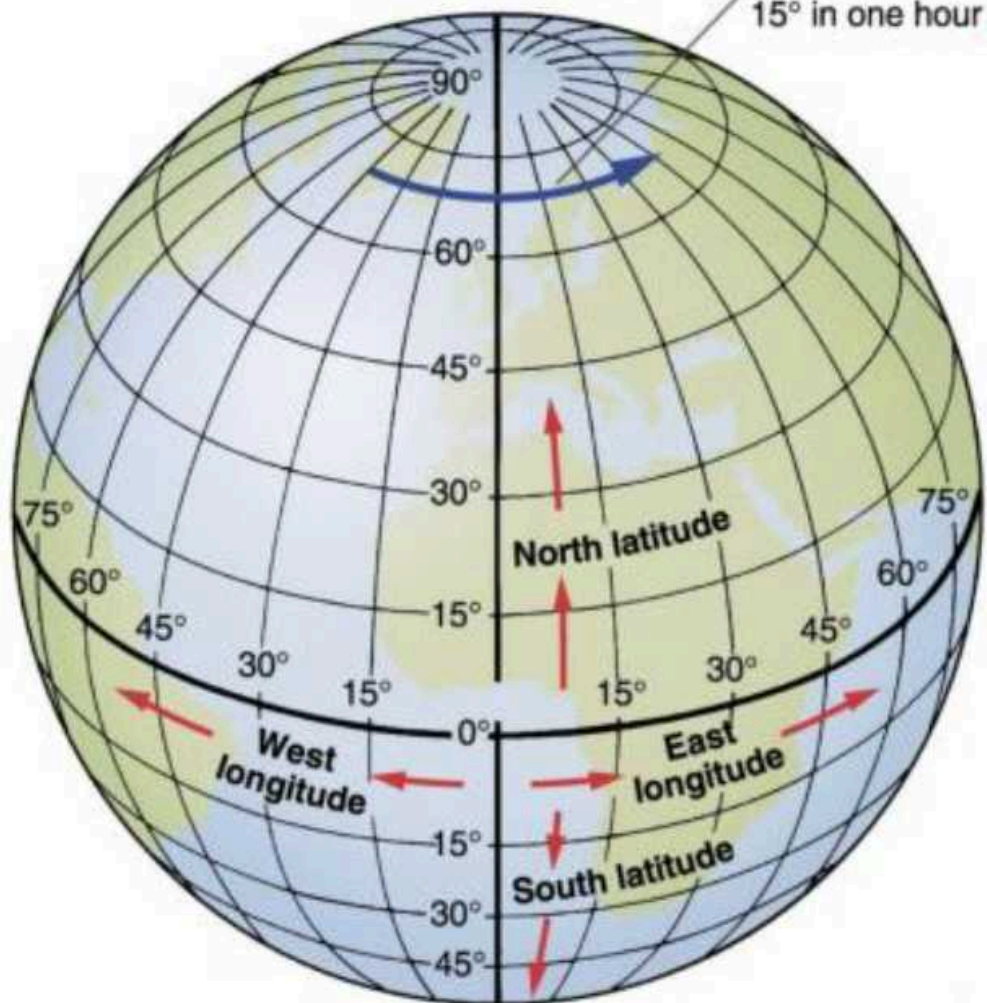
But what is a Meridian Arc?

GREATER AND SMALLER CIRCLES



Great circles- page 11

Earth turns 15° in one hour



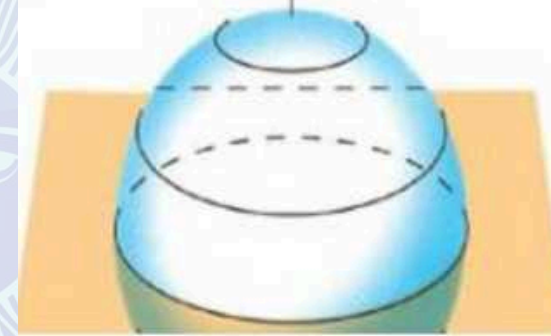
An imaginary circle through the center of Earth:

1. It divides Earth into equal halves= hemispheres
2. It is a circumference of Earth – 1 degree of latitude= 69 mi or 111km (40,000km/360 deg.=111km; 25,000mi/360 deg=69 mi)
3. It marks the shortest routes between locations on Earth (with a string)
4. Circle of illumination

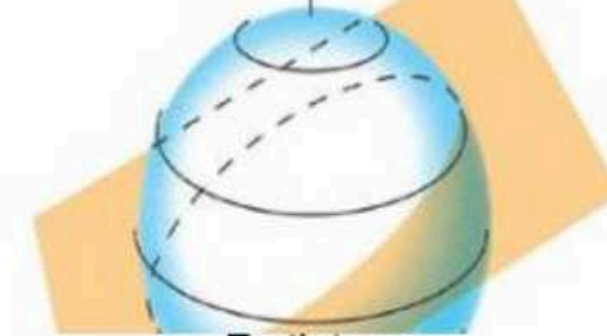
What other circles (latitudes) are important?

Download

p.11 Great Circle

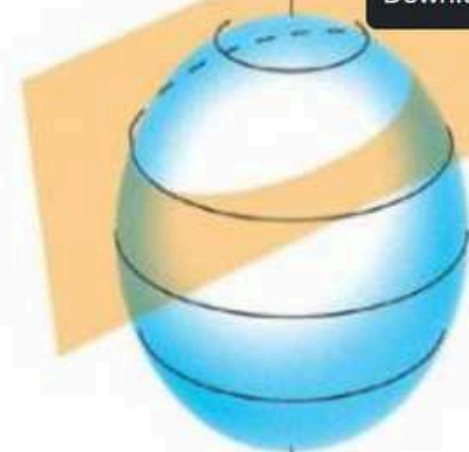


Great Circle



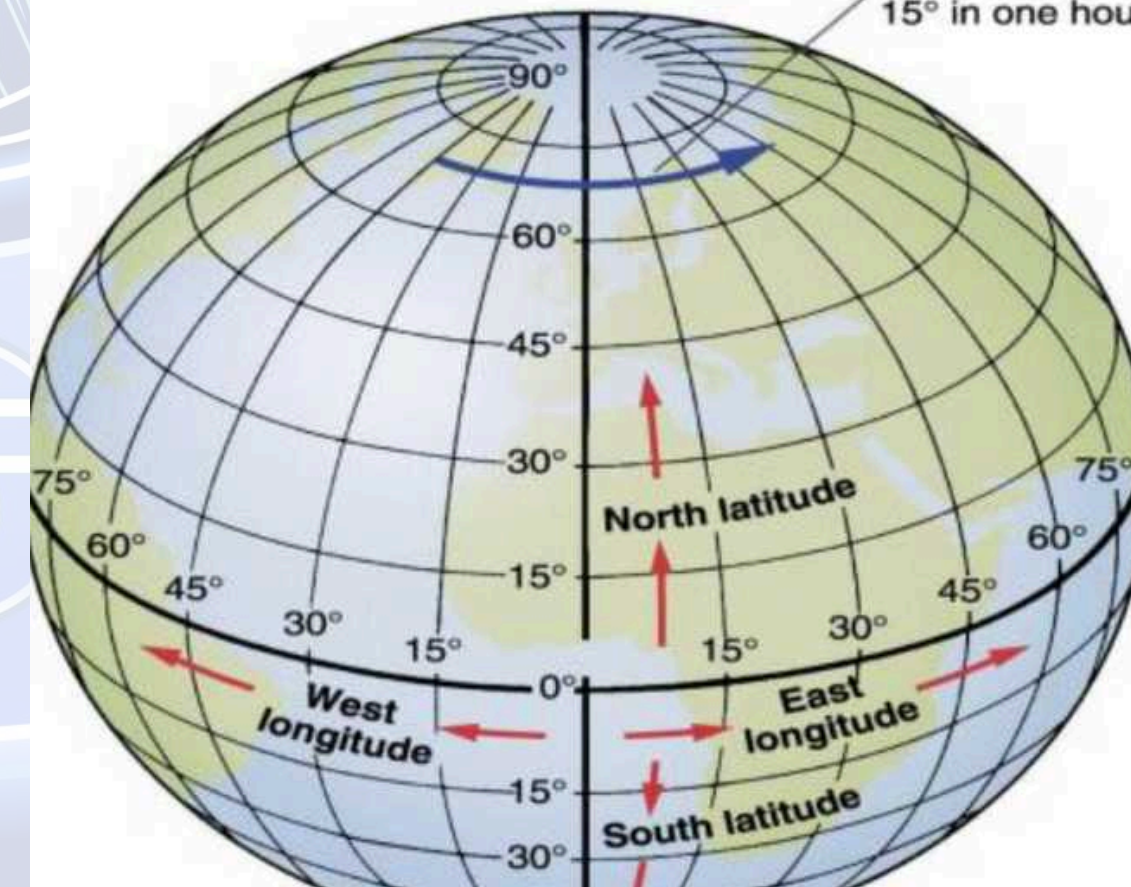
Earth turns 15° in one hour

small circle



Download

(c)



Only one latitude line= **parallel** is a Great Circle. Which one is it?

The Equator

All longitude lines= **meridians** are half Great Circles.

But what is a Meridian Arc ?

GREAT CIRCLES DEFINITION:

A GREAT CIRCLE IS ANY CIRCLE THAT DIVIDES A SPHERE INTO TWO EQUAL HEMISPHERES AND IS THE LARGEST POSSIBLE CIRCLE THAT CAN BE DRAWN ON A SPHERICAL SURFACE.

EVERY GREAT CIRCLE IS THE INTERSECTION OF THE SPHERE WITH A PLANE THAT PASSES THROUGH THE CENTER OF THE SPHERE.

EXAMPLES ON EARTH:

THE EQUATOR IS A NATURAL EXAMPLE OF A GREAT CIRCLE BECAUSE IT DIVIDES THE EARTH INTO THE NORTHERN AND SOUTHERN HEMISPHERES.

MERIDIANS OF LONGITUDE ARE ALSO EXAMPLES OF GREAT CIRCLES AS EACH PASSES THROUGH THE NORTH AND SOUTH POLES, SPLITTING THE EARTH INTO EASTERN AND WESTERN HEMISPHERES. EACH MERIDIAN AND ITS ANTIMERIDIAN (THE LINE OF LONGITUDE 180 DEGREES ON THE OPPOSITE SIDE OF THE GLOBE) TOGETHER FORM A GREAT CIRCLE.

MERIDIANS
DEFINITION:

A MERIDIAN IS A LINE OF LONGITUDE, RUNNING FROM THE NORTH POLE TO THE SOUTH POLE. BY DEFINITION, IT CONNECTS ALL POINTS WITH THE SAME LONGITUDE.

UNLIKE PARALLELS OF LATITUDE (EXCEPT FOR THE EQUATOR), ALL MERIDIANS ARE HALVES OF GREAT CIRCLES.

SIGNIFICANCE OF GREAT CIRCLES IN NAVIGATION

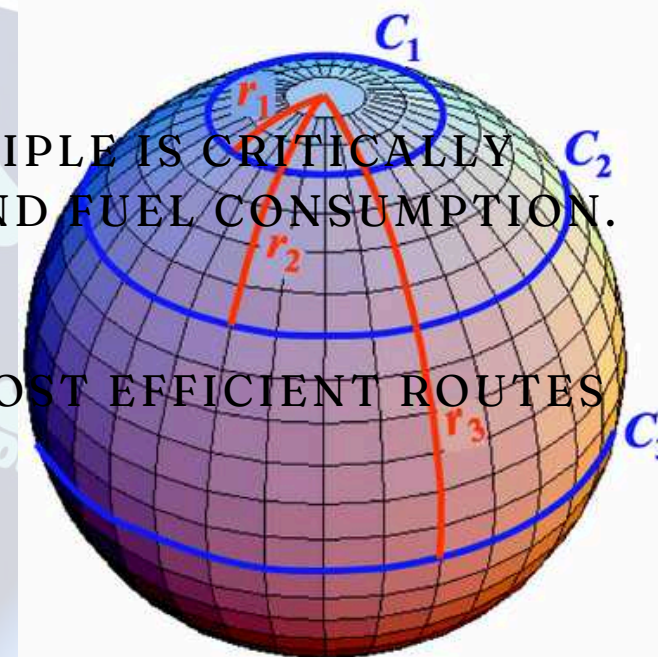
SHORTEST ROUTE:

TRAVEL ALONG A GREAT CIRCLE IS THE SHORTEST DISTANCE BETWEEN TWO POINTS ON A SPHERE. THIS PRINCIPLE IS CRITICALLY IMPORTANT IN AIR AND SEA NAVIGATION, WHERE FOLLOWING A GREAT CIRCLE ROUTE MINIMIZES TRAVEL TIME AND FUEL CONSUMPTION.

NAVIGATION AND GPS:

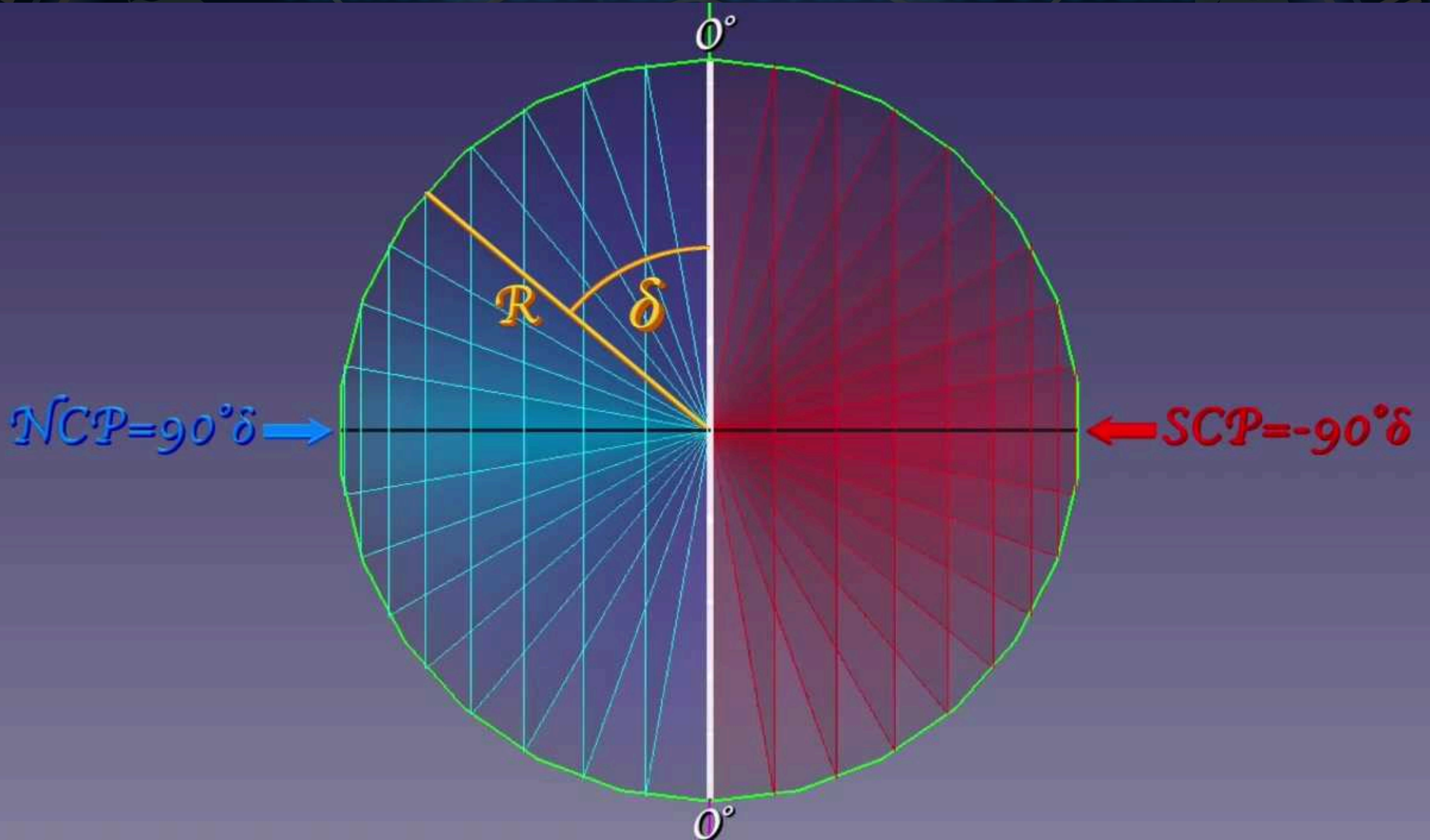
MODERN NAVIGATION SYSTEMS, INCLUDING GPS, UTILIZE THE CONCEPT OF GREAT CIRCLES TO CALCULATE THE MOST EFFICIENT ROUTES ACROSS THE EARTH'S CURVED SURFACE.

VISUALIZING GREAT MERIDIANS AND GREAT CIRCLES



Where is the spherical trig?

TRANSFER BETWEEN COORDINATE SYSTEMS



Three Families of Projections

- There are **three major families** of projections, each tends to introduce **certain kinds of distortions**, or conversely each has certain **properties** that it used to **preserve** (i.e. spatial characteristics that it does not distort):

- **Three families:**

1. Cylindrical projections
2. Conical projections
3. Planar projections

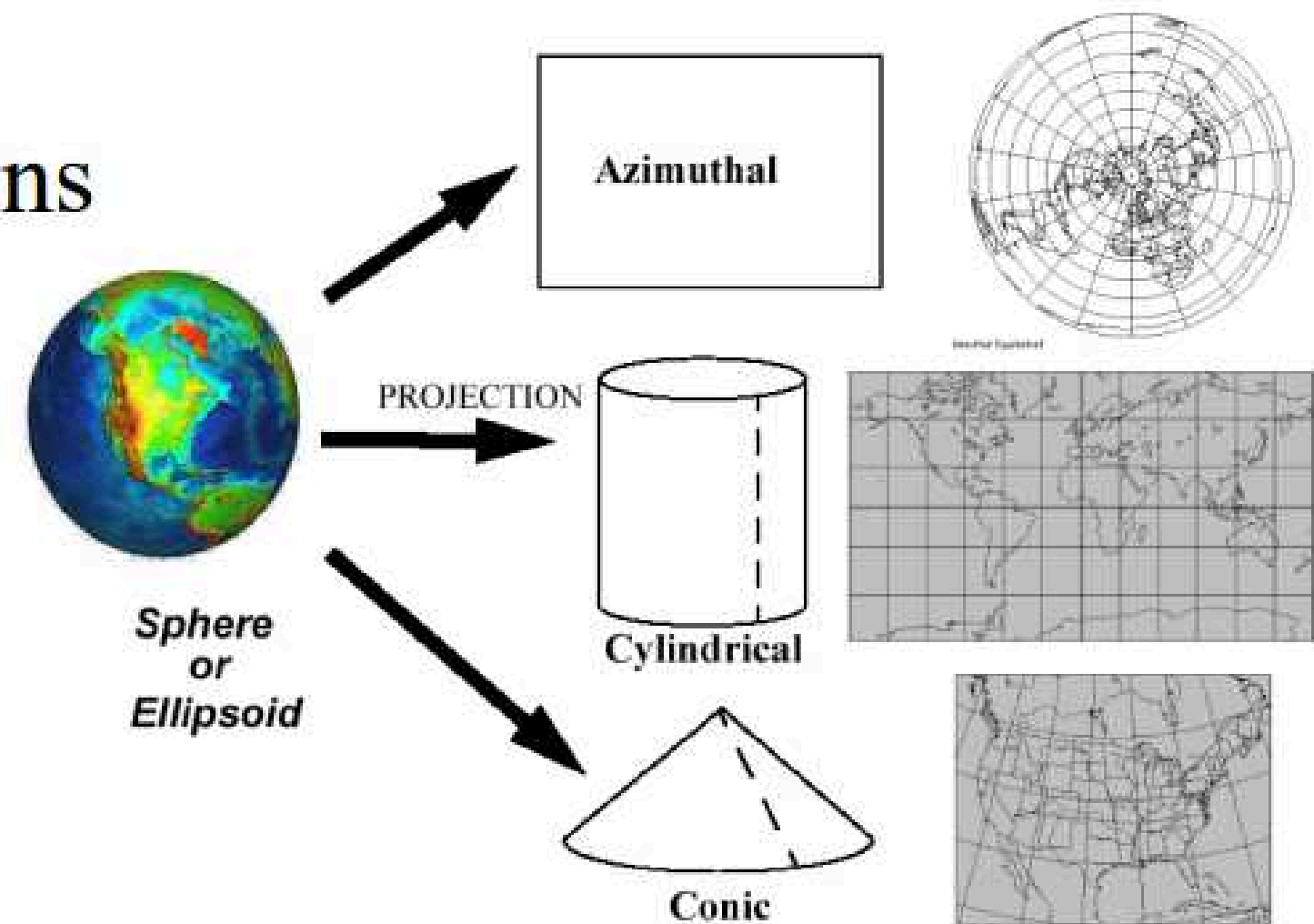
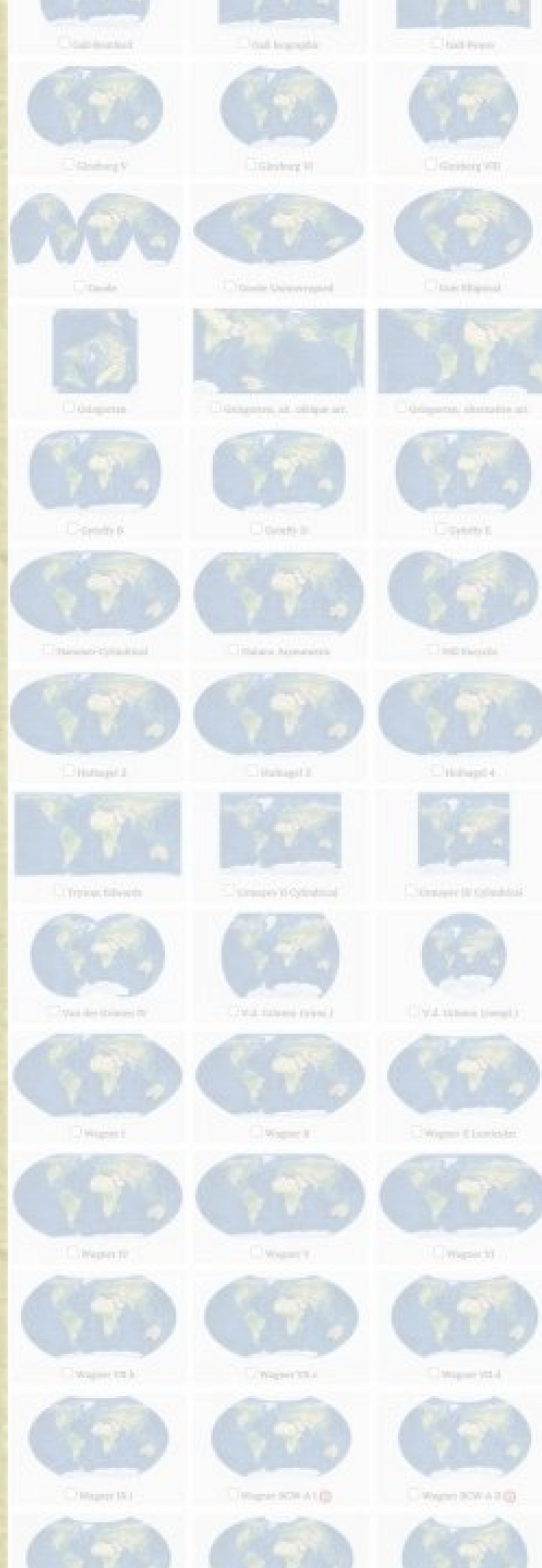
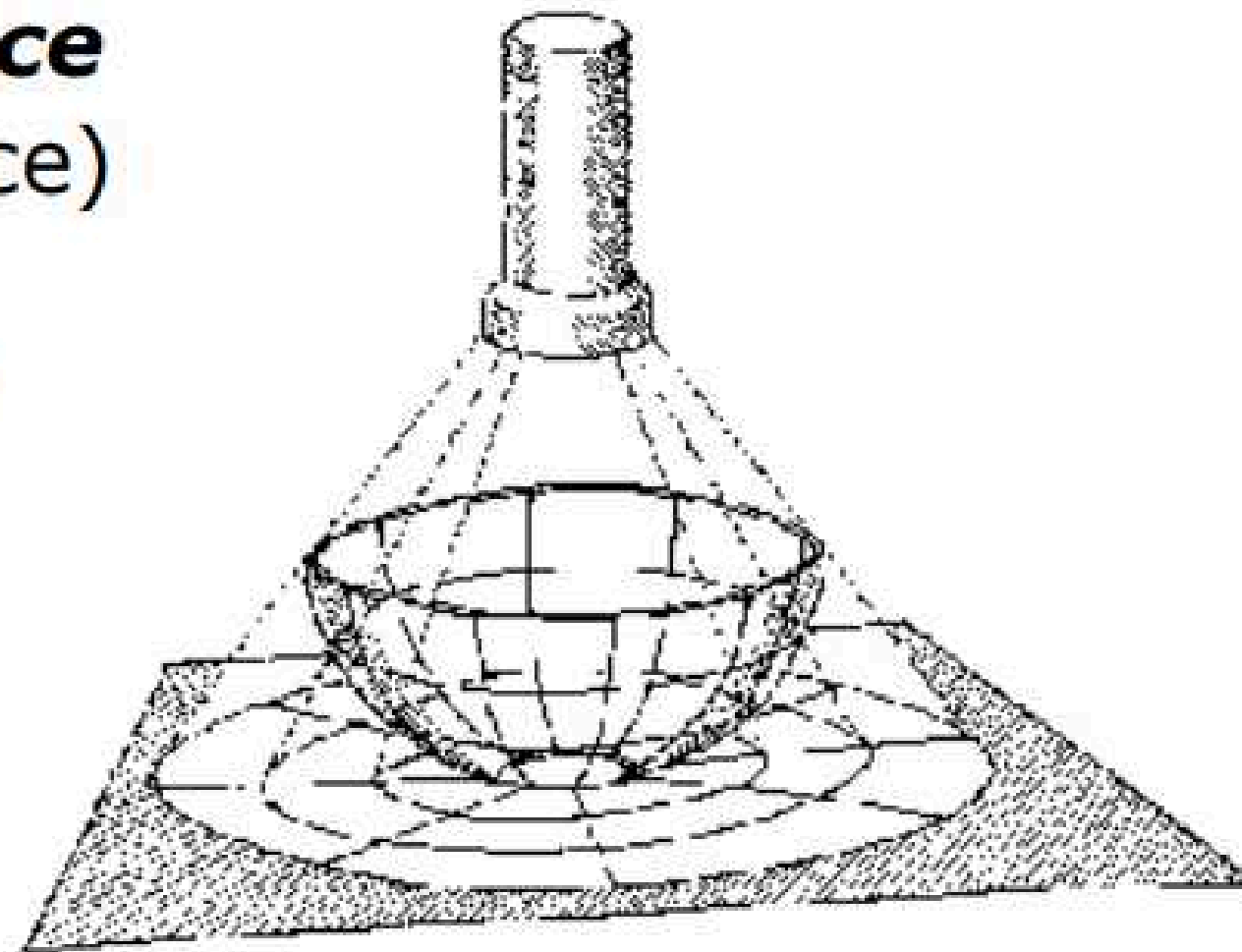


Figure 2.7 The earth can be projected in many ways, but basically onto three shapes that can be unrolled into a flat map: a flat plane, a cylinder, and a cone.

Producing Projections

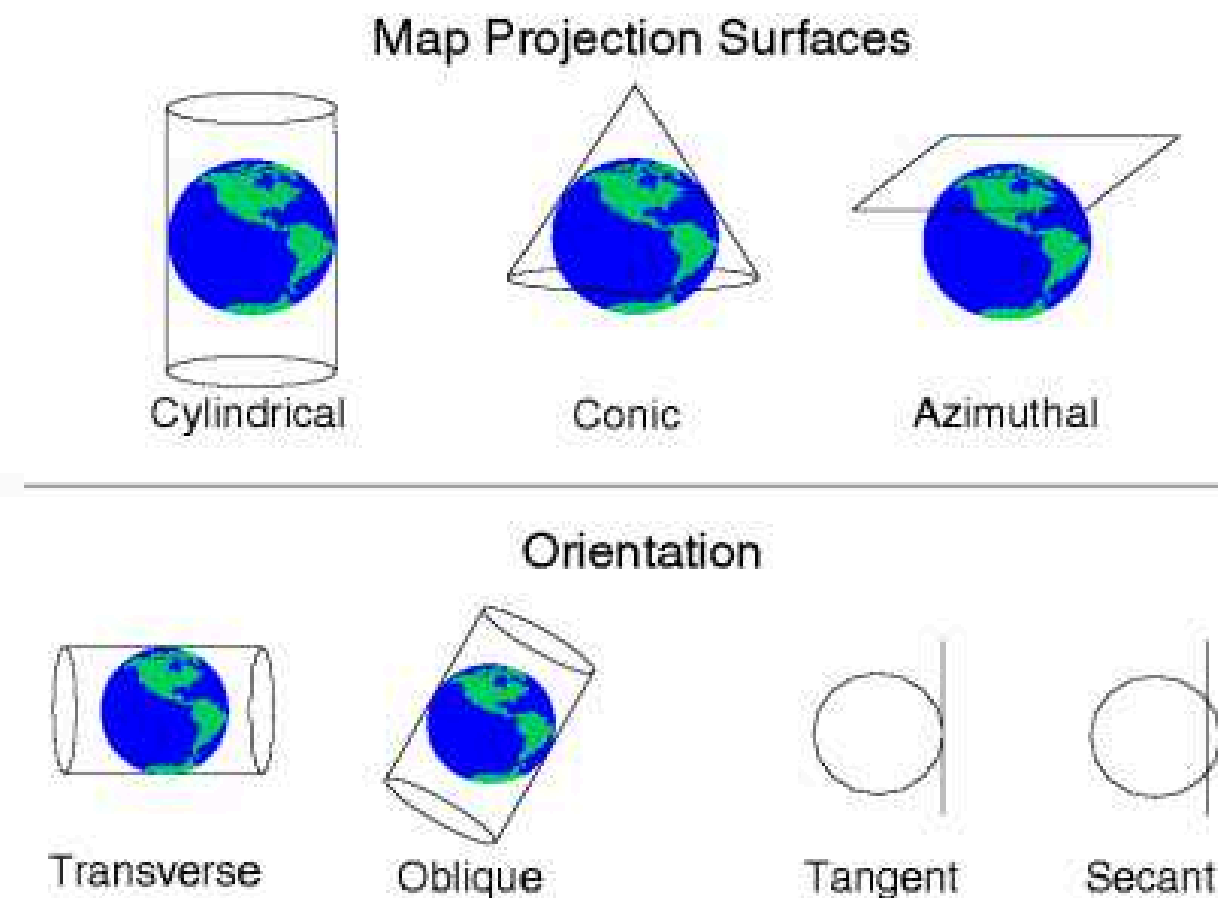
✦ Projections may be physically produced by “shining a light” from the center of the earth onto a *developable surface* that is being “wrapped around” or “tangent to” the reference globe

✦ A ***developable surface*** (aka flattenable surface) is a geometric shape that may be flattened without stretching its surface - **cones**, **cylinders**, and **planes**

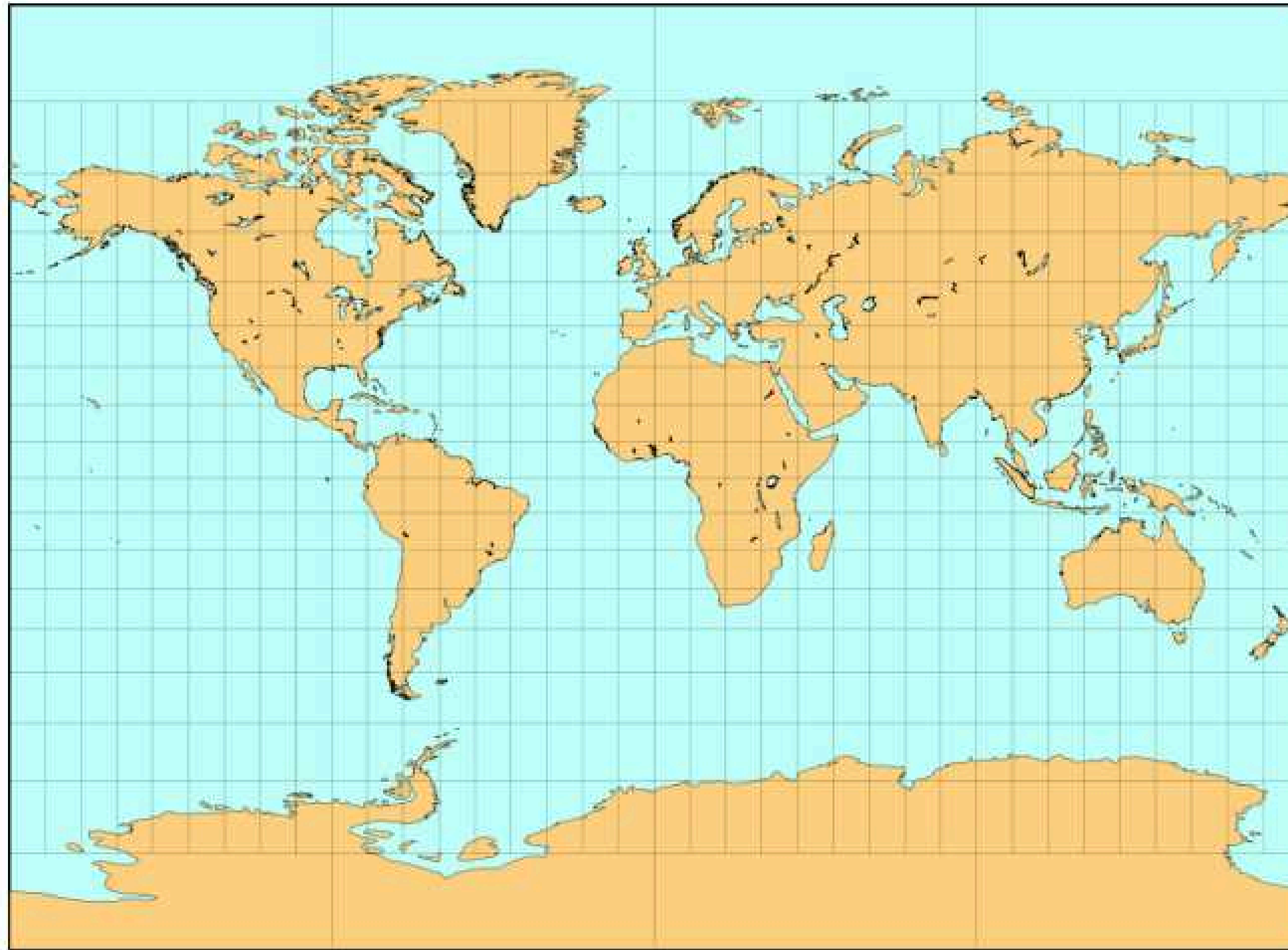


Producing Projections

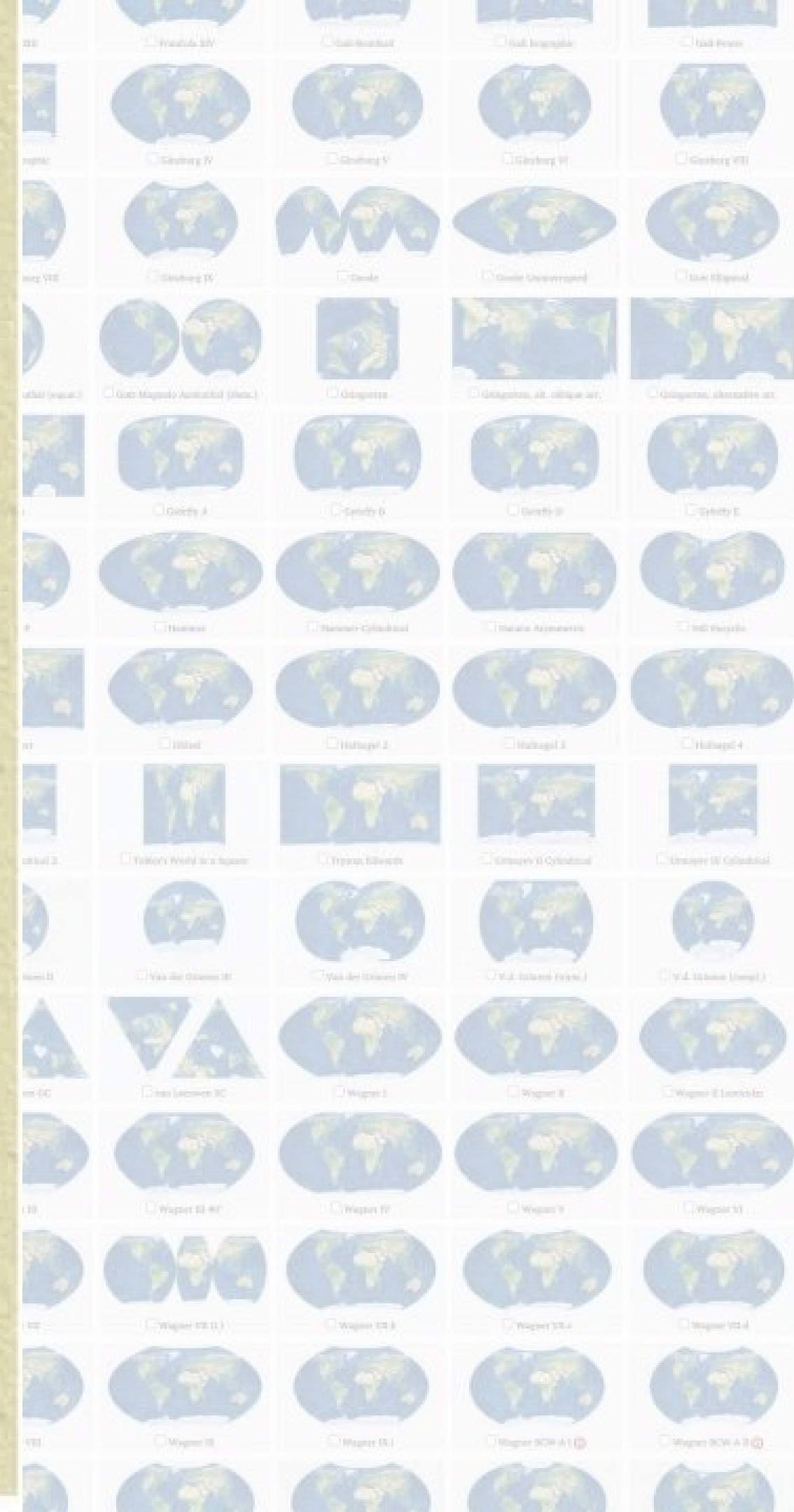
- ✦ First step is to create one or more points of contact between the developable surface and the reference globe - these are called points of tangency
- ✦ A **tangent projection** is one where the developable surface touches in one location



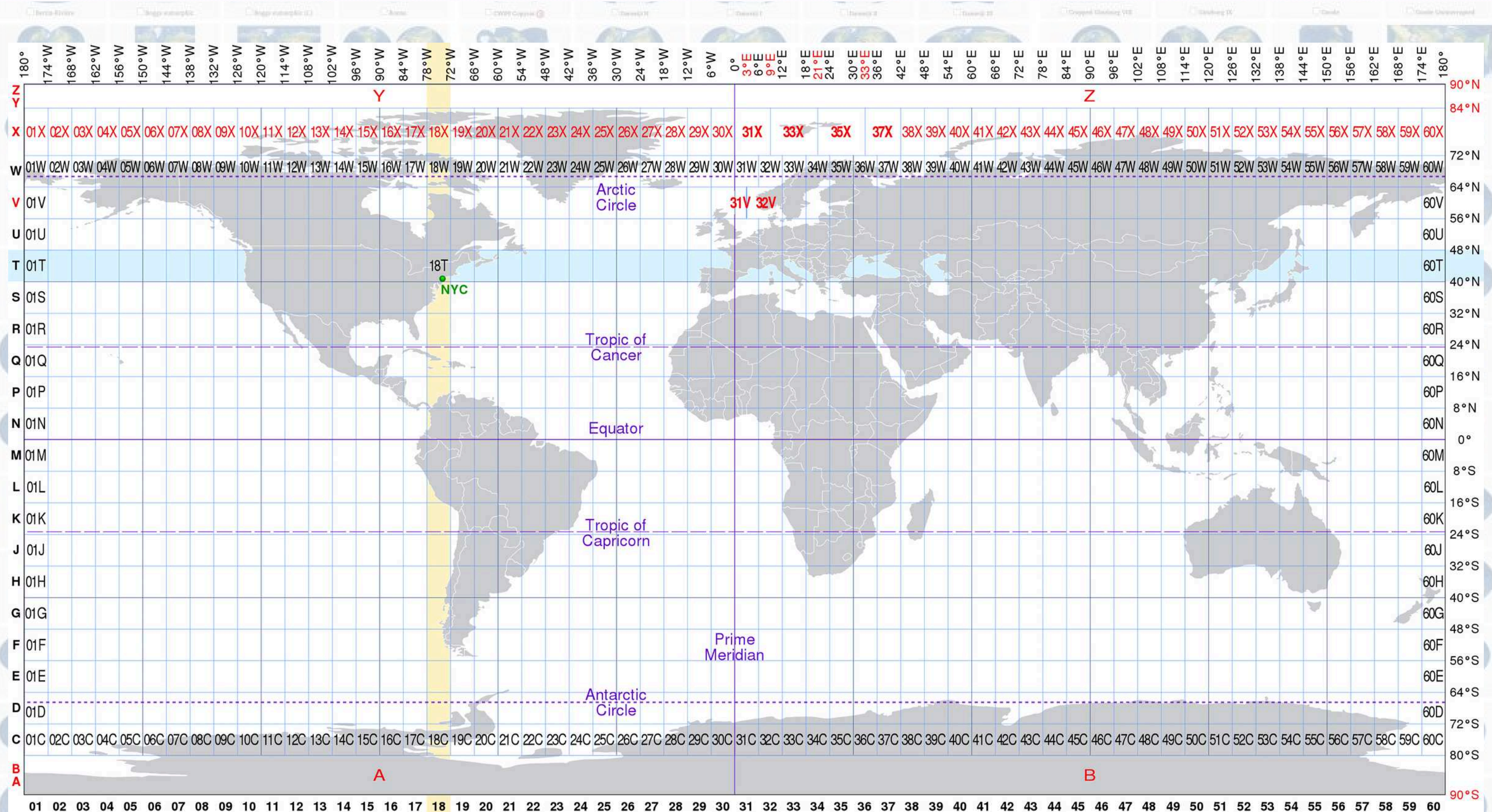
Cylindrical Projection - Example



Miller's Cylindrical Projection

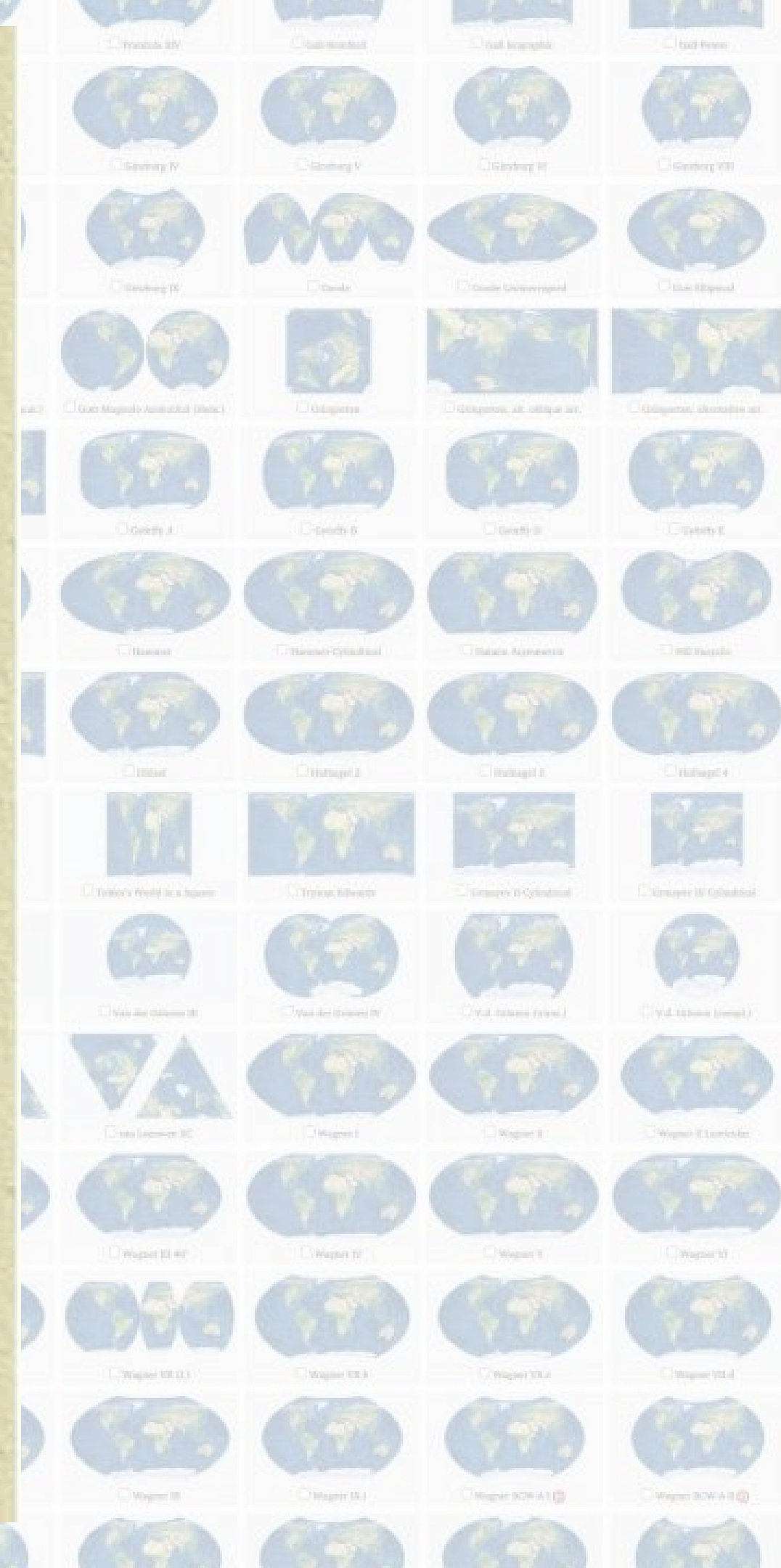
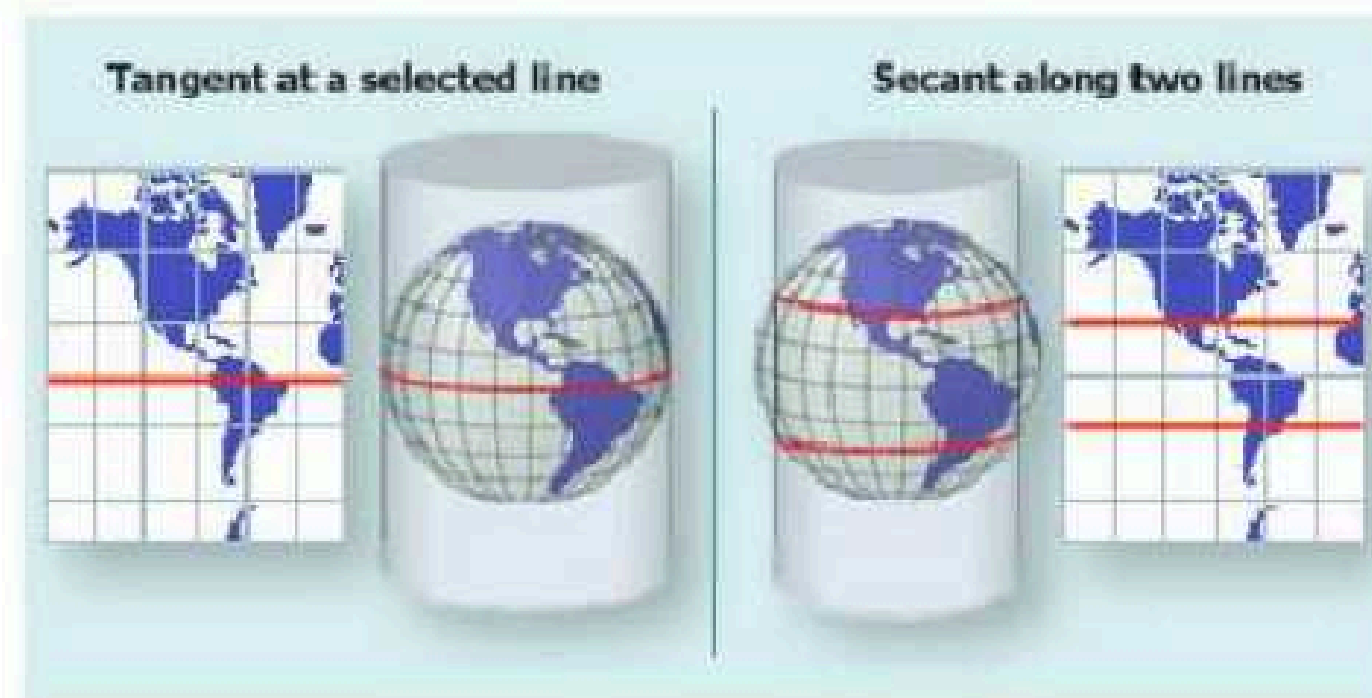


Cylindrical Projection - Example



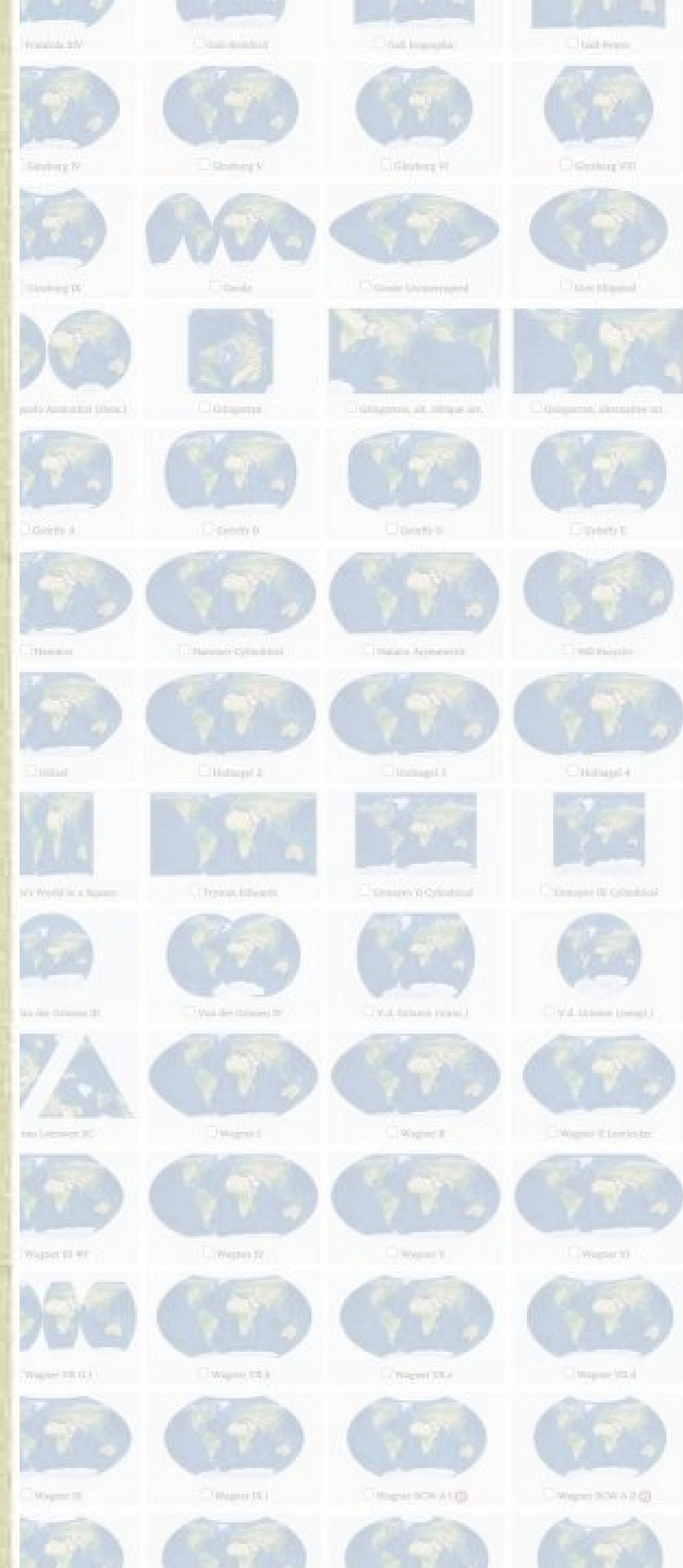
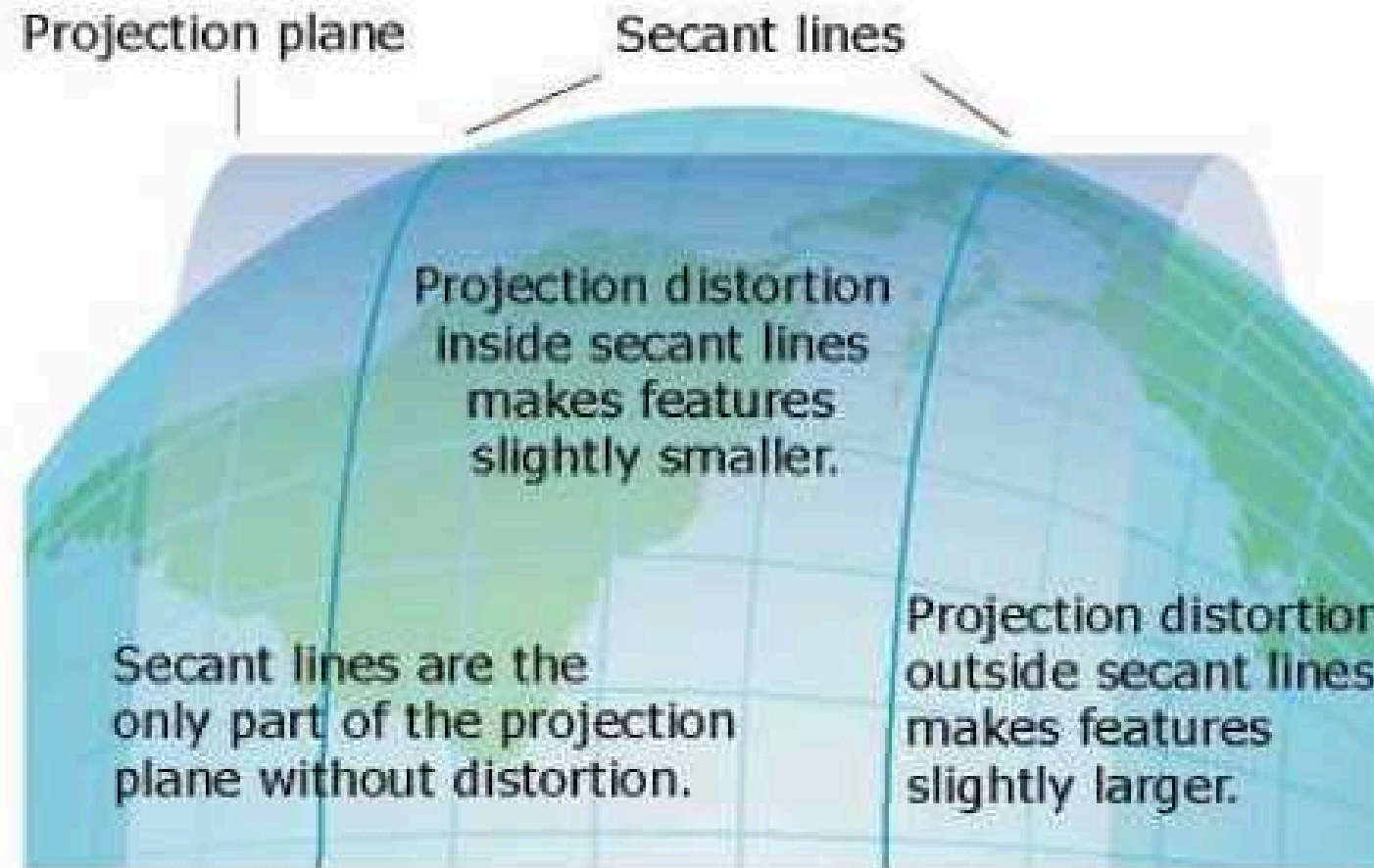
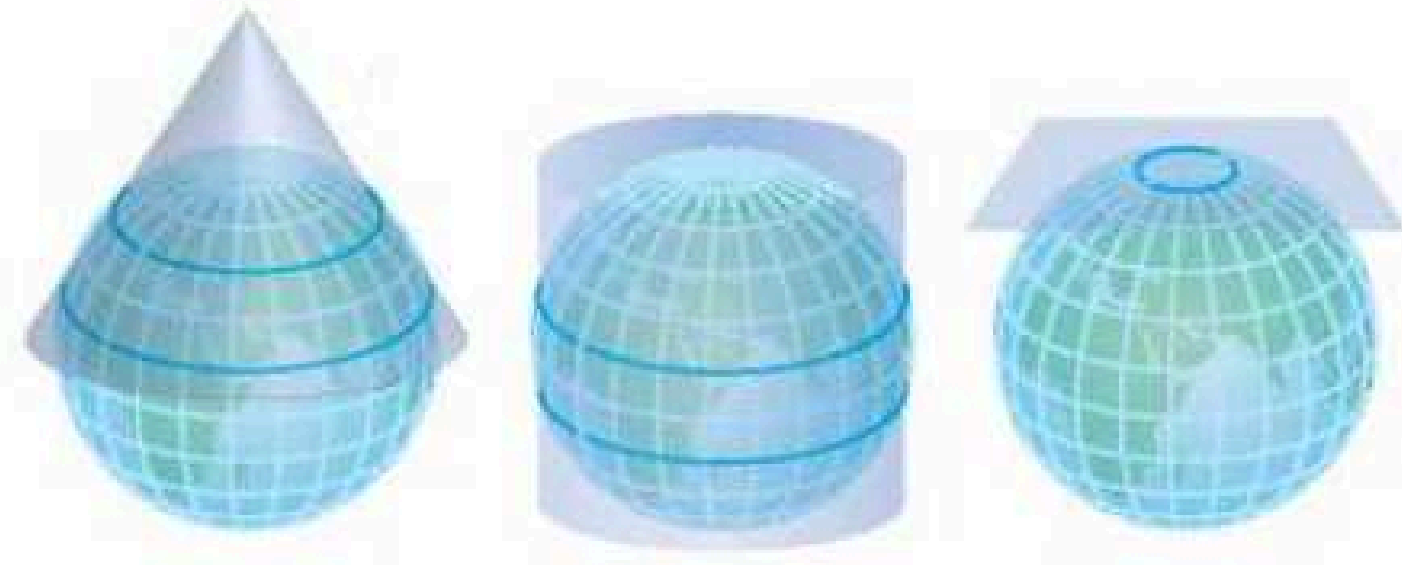
Producing Projections

- ✦ Where the dev. surface touches the globe is important - **this is a line of true scale**, often referred to as a *standard line*, such as a line of standard latitude (**standard parallel**) or standard longitude (**central meridian**)
- ✦ **Distortion is zero** along standard lines or points; distortion increases away from these standard locations

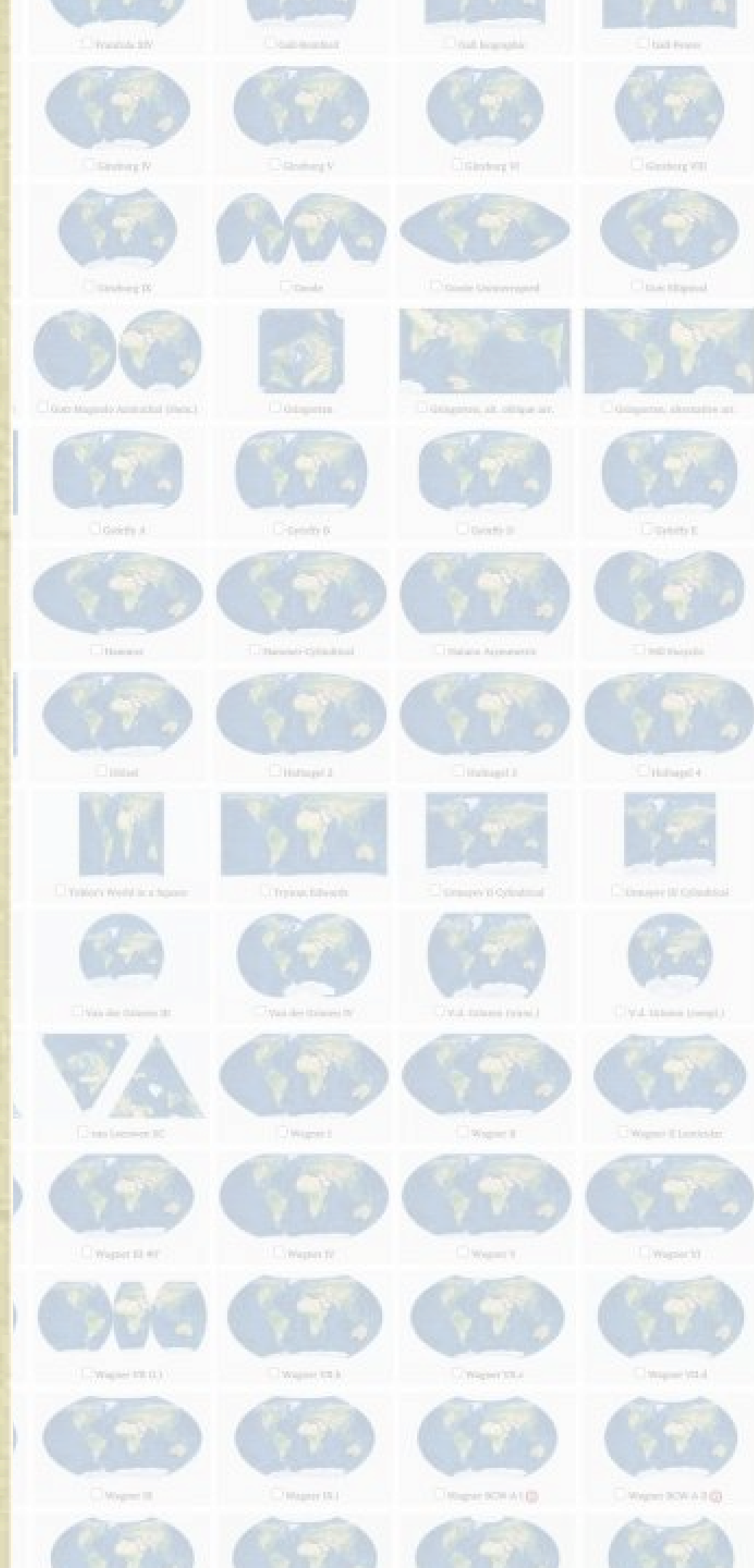
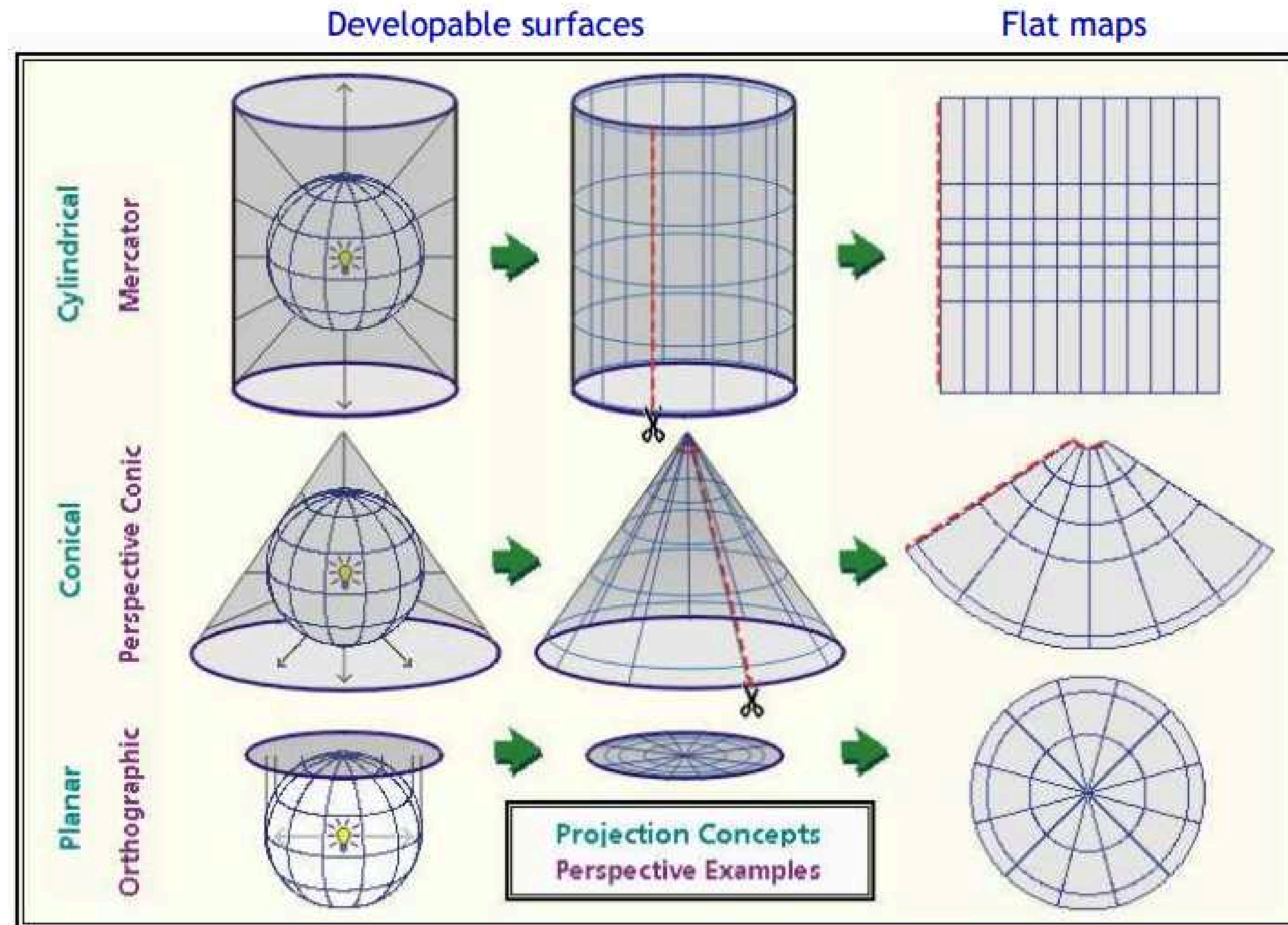


Producing Projections

✦ In a **secant projection** the developable surface intersects the globe in two places

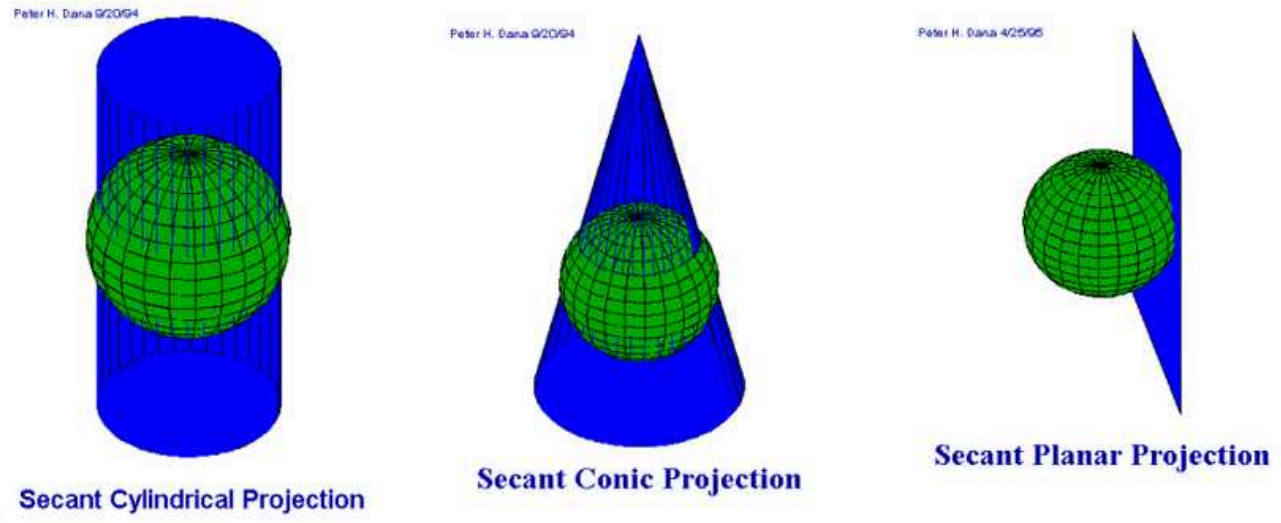
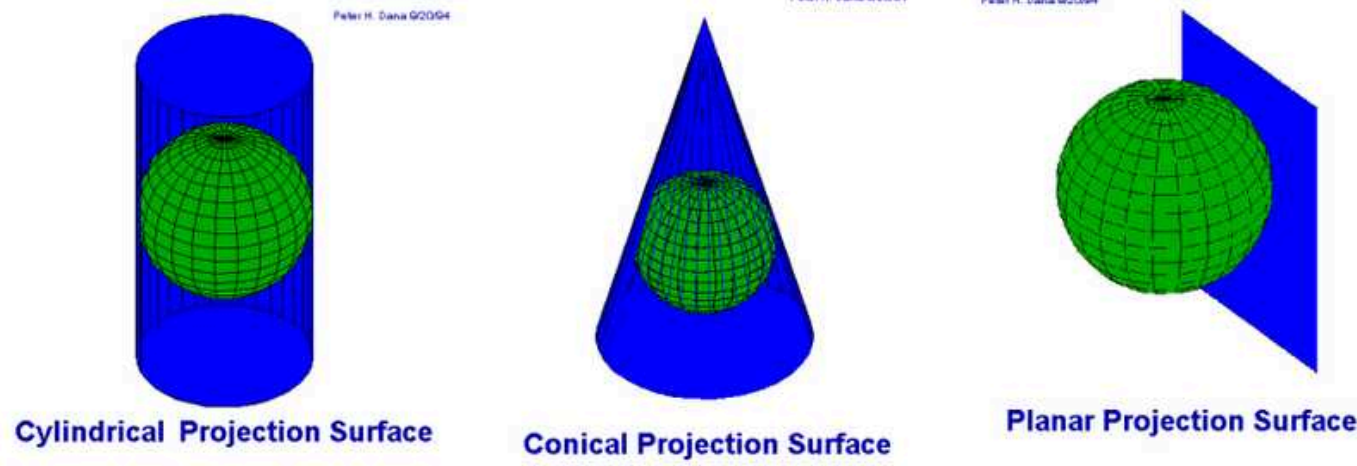


Producing Projections



Tangent Projections

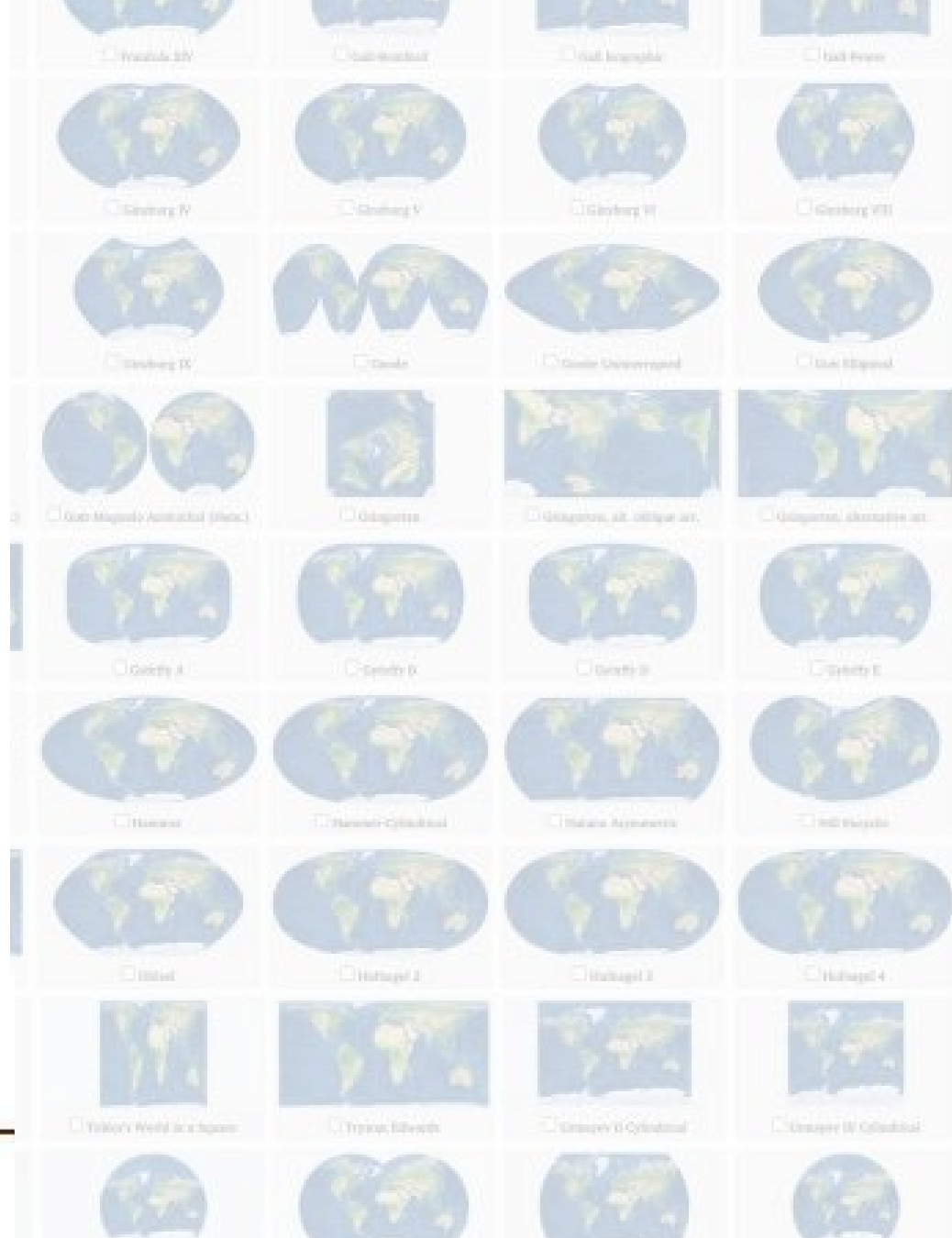
Secant Projections



•Tangent projections have a **single standard point** (in the case of planar projection surfaces) or a **standard line** (for conical and cylindrical projection surfaces) of contact between the developable surface and globe

•Secant projections have a **single standard line** (in the case of planar projection surfaces) or **multiple standard lines** (for conical and cylindrical projection surfaces) of contact between the developable surface and the globe

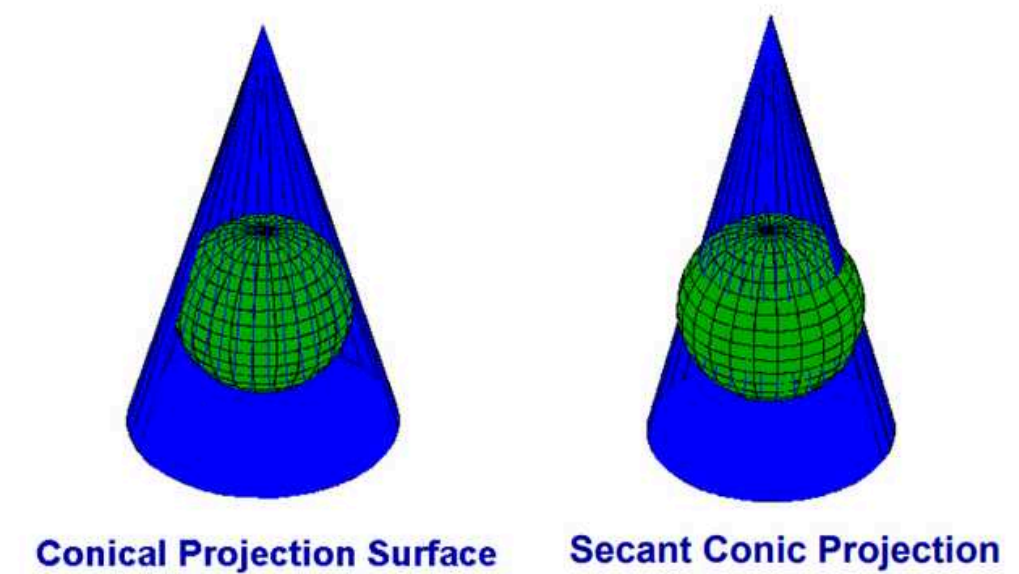
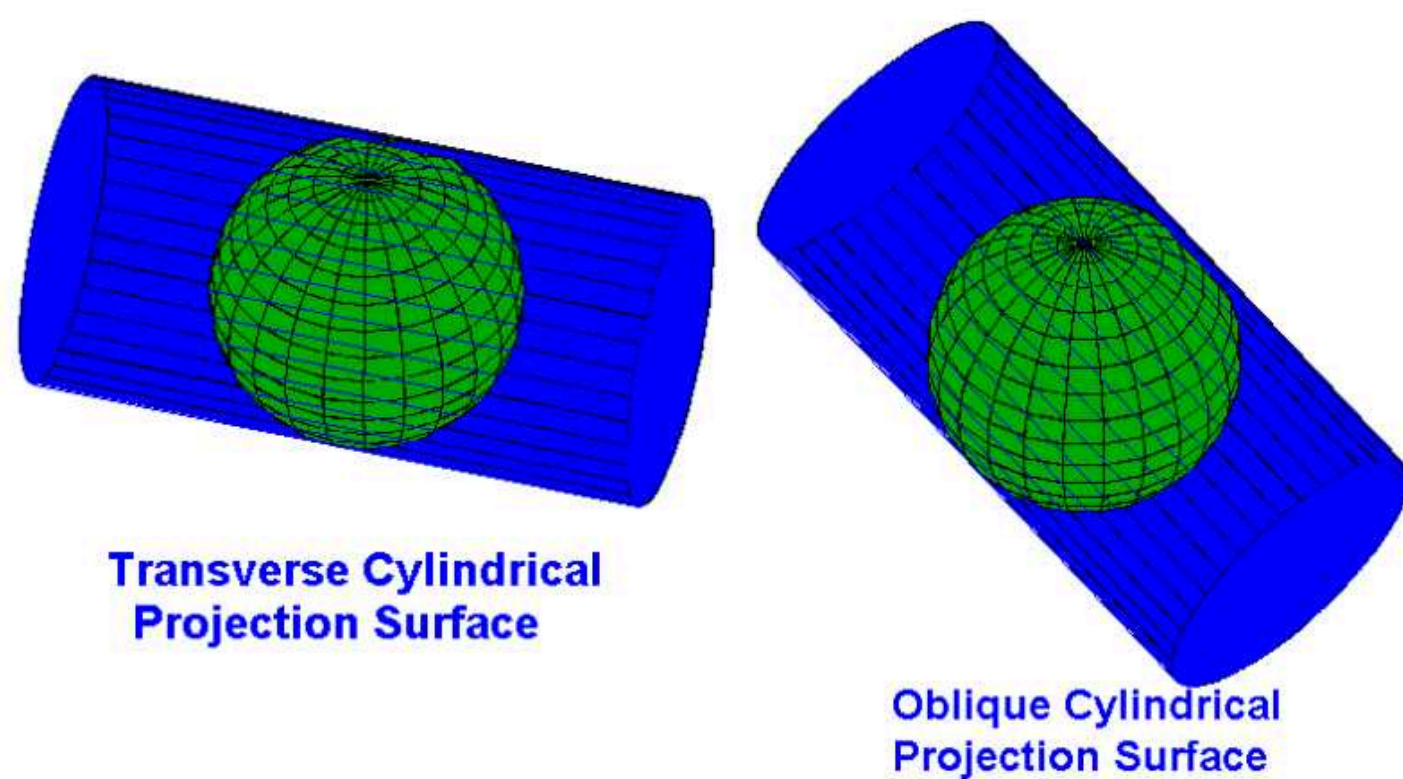
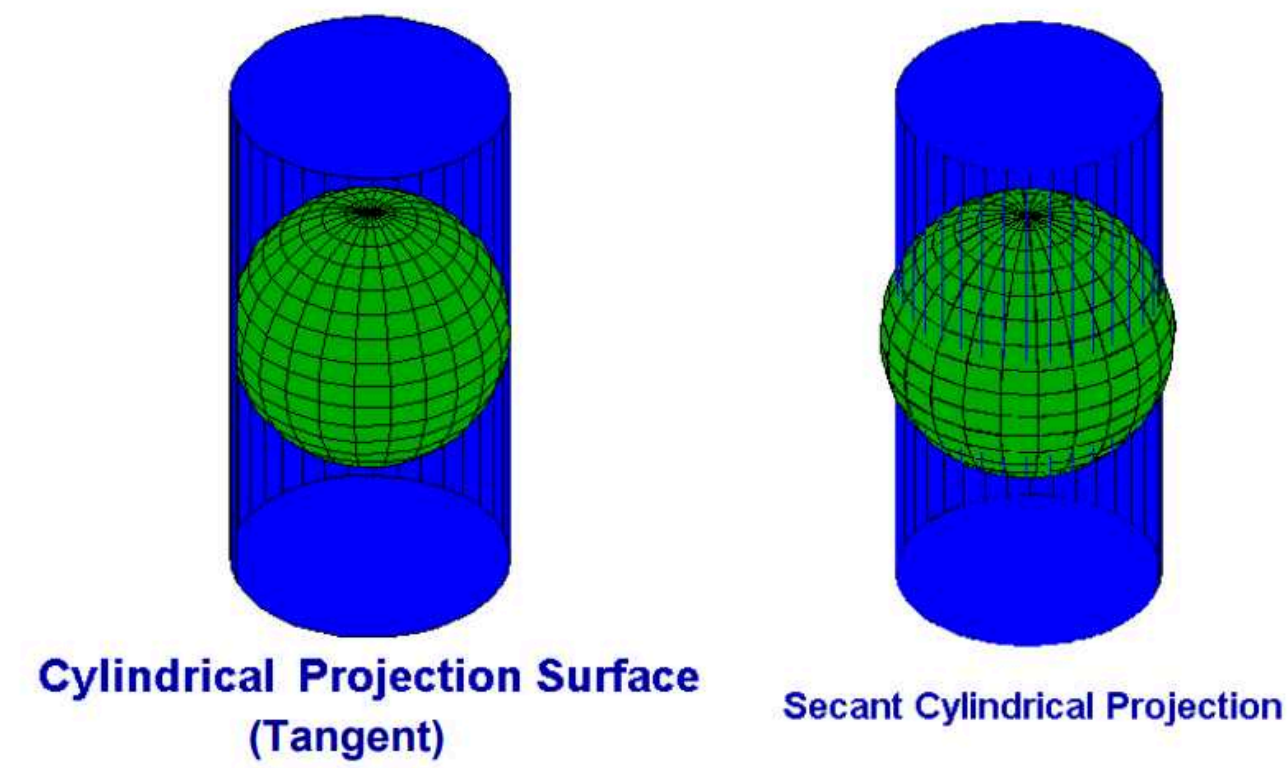
David Tenenbaum - EOS 281 - UMB Fall 2010

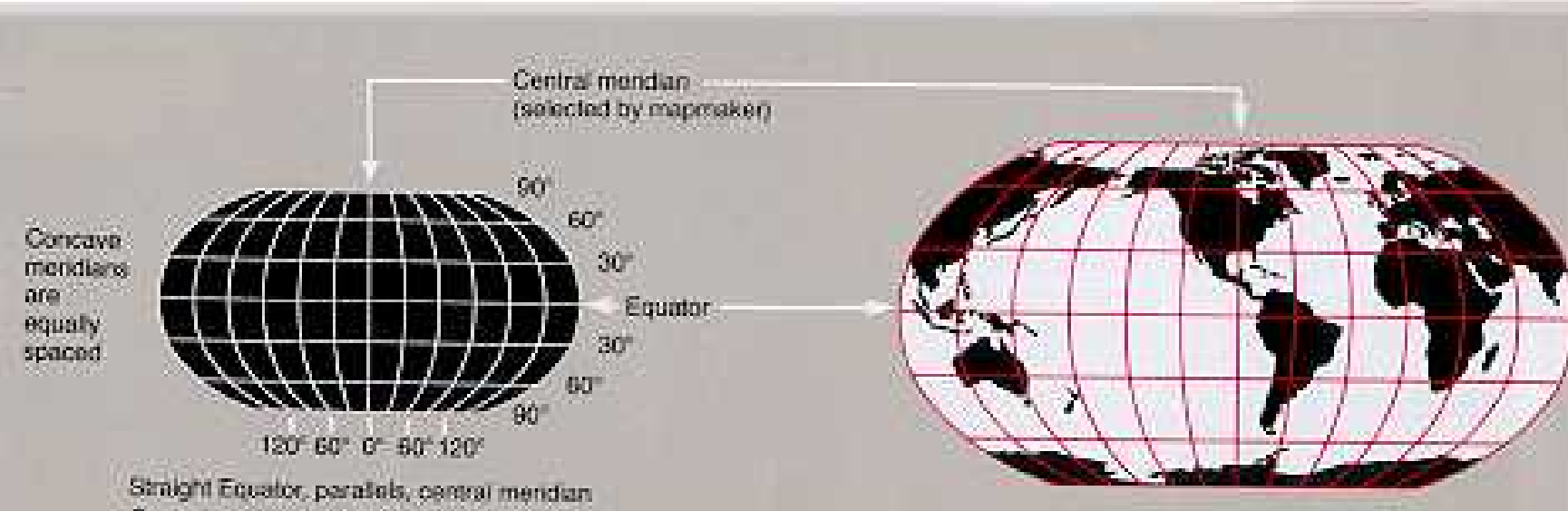
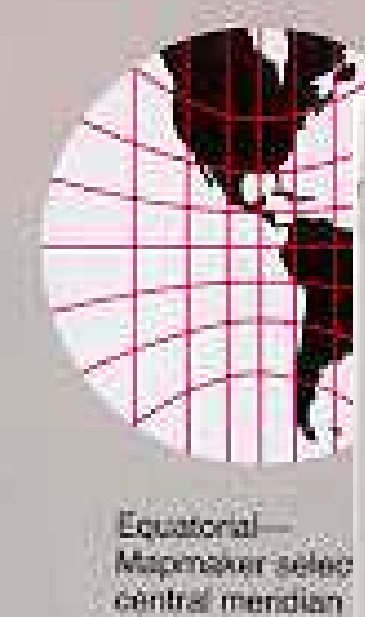
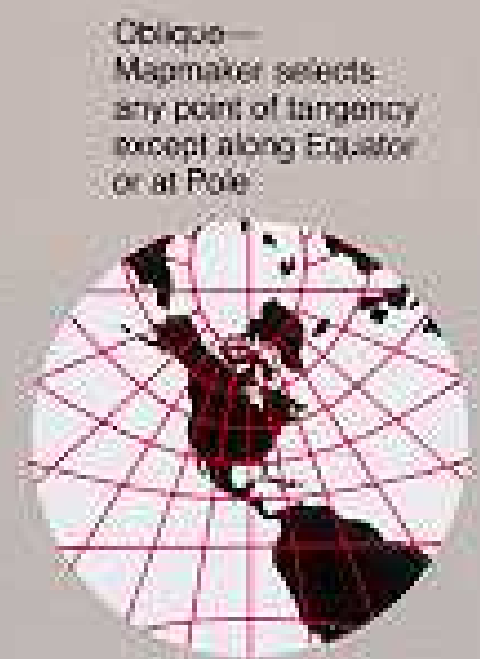
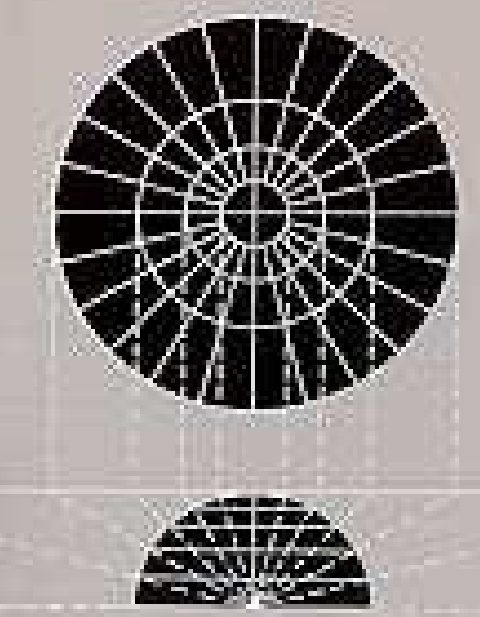
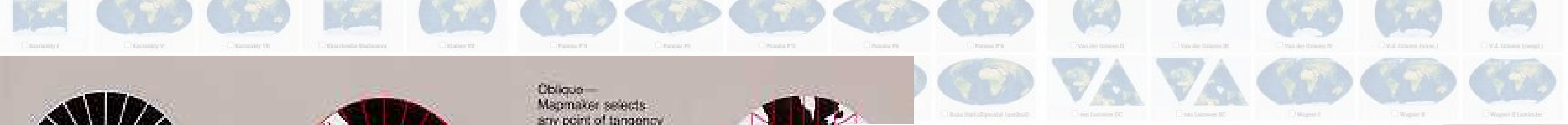
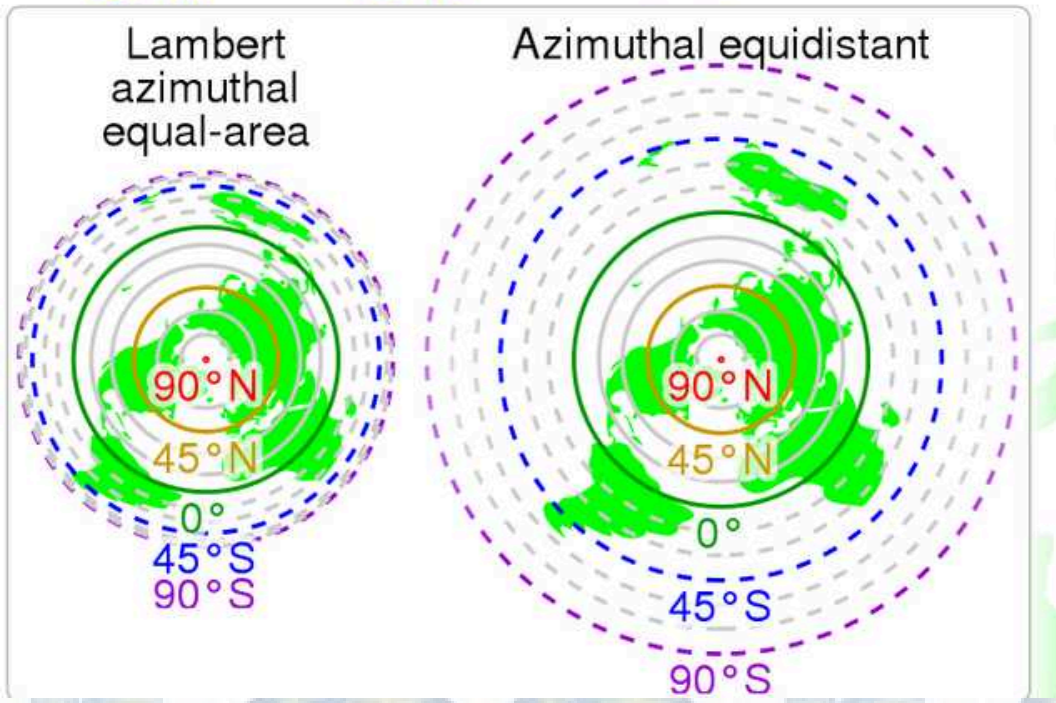
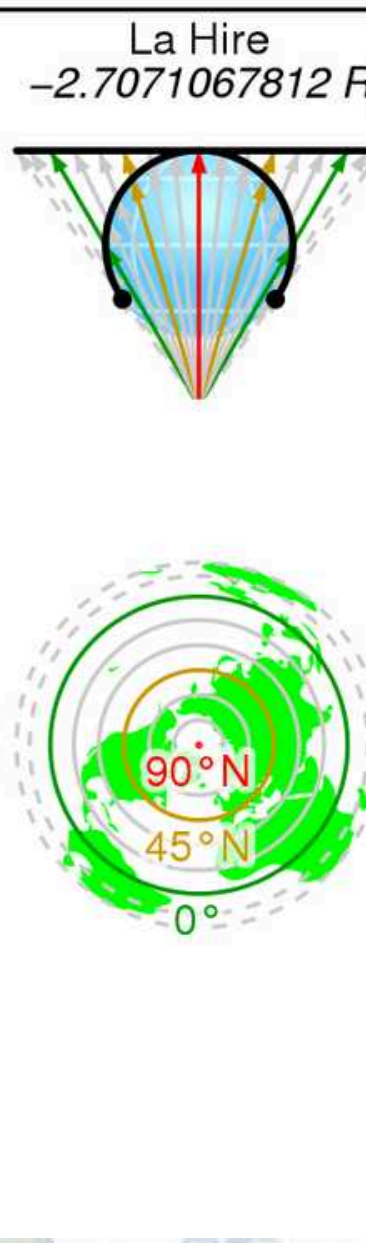
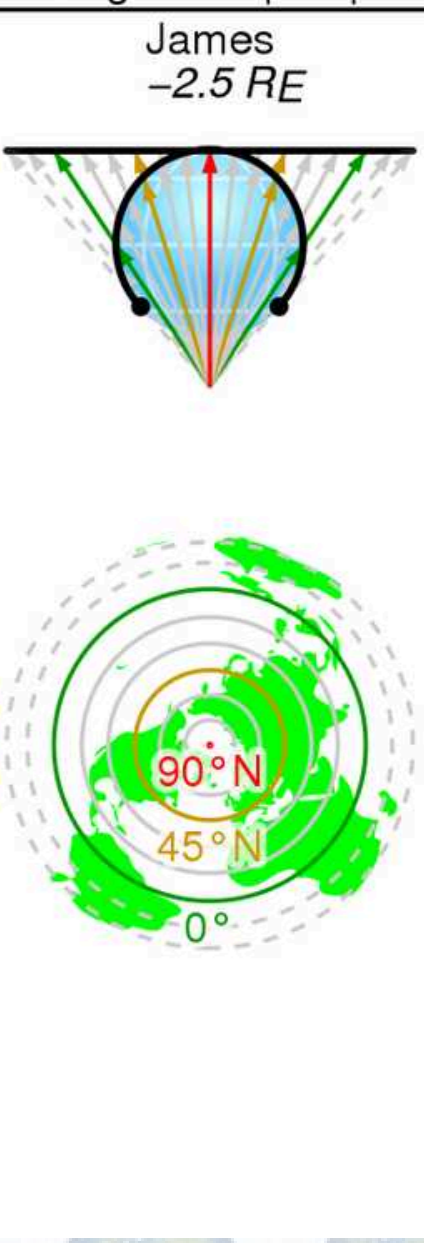
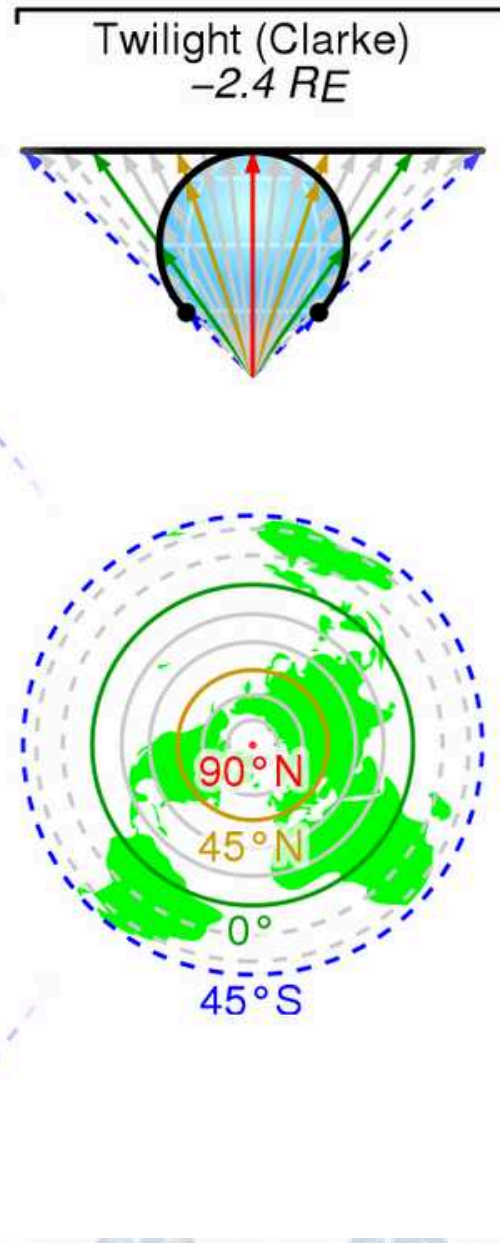
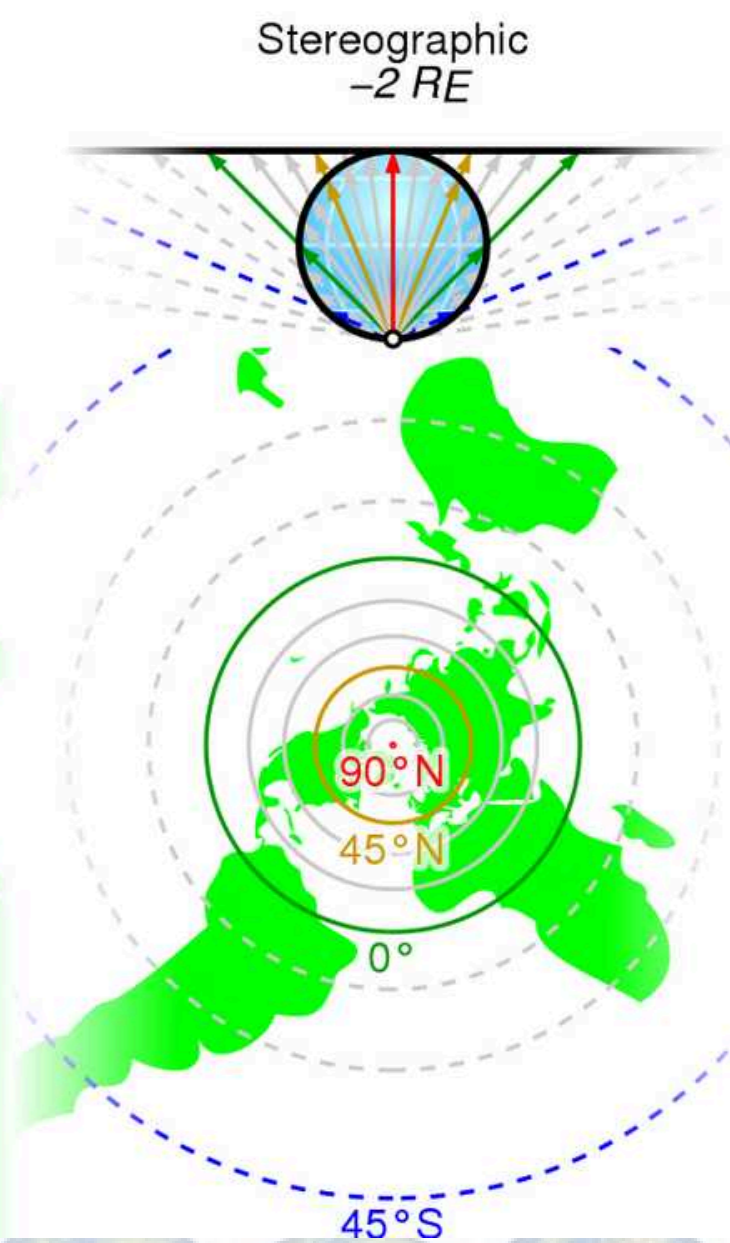
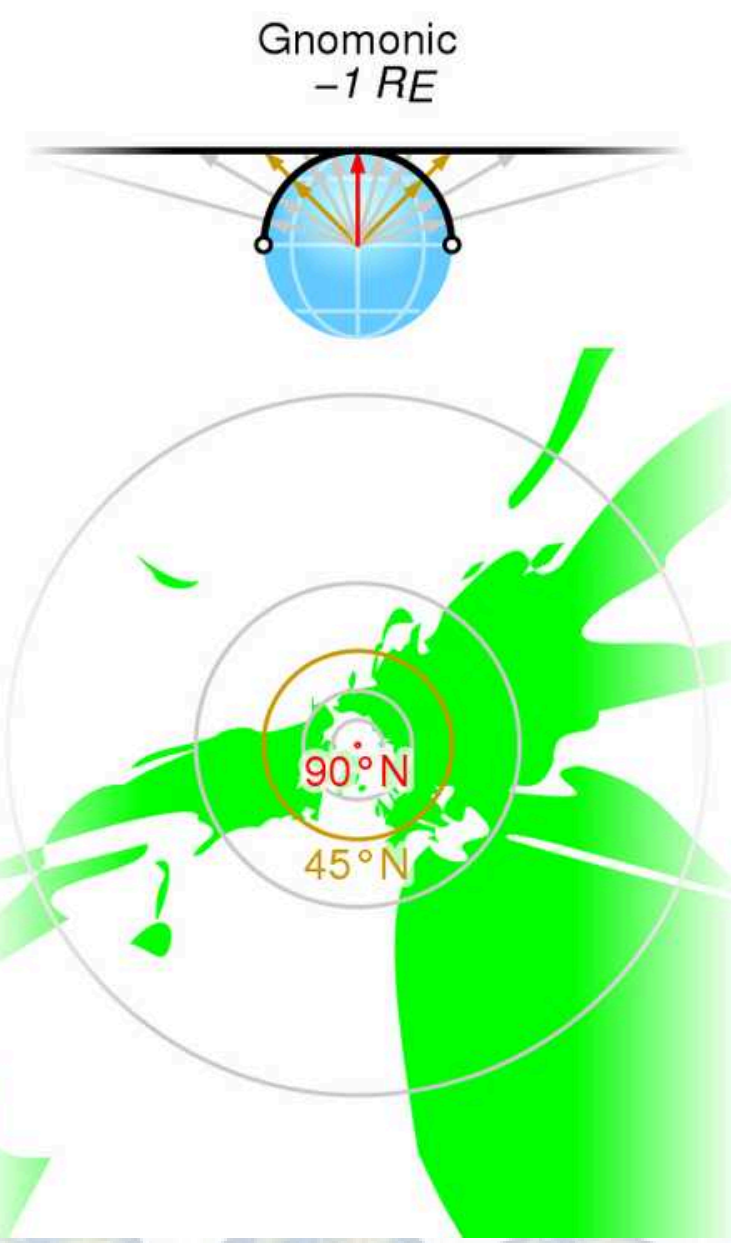
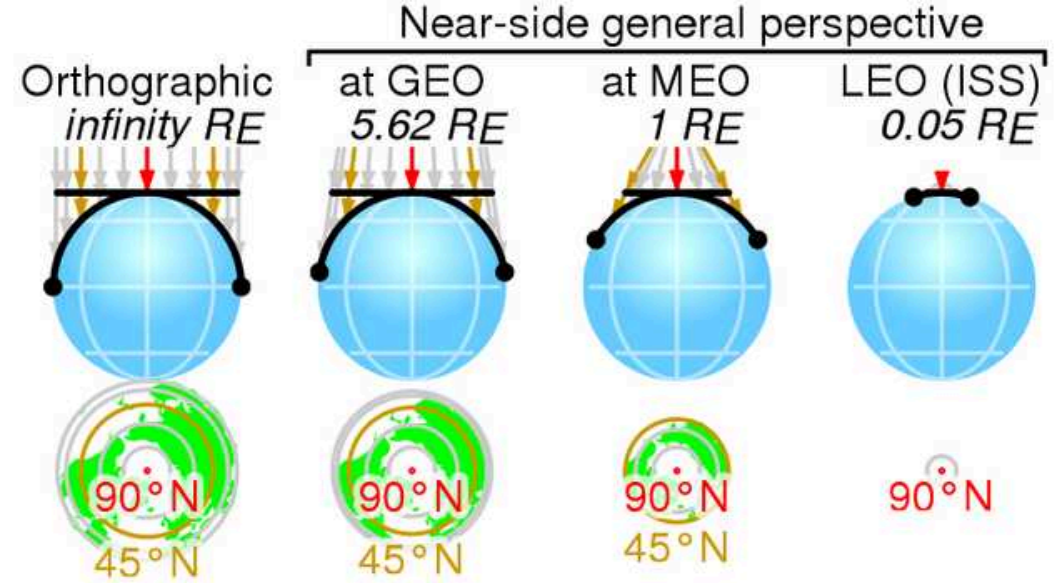


Cylindrical Projection – Conceptual View

Cylindrical Projection – Conceptual View

Conic Projection – Conceptual View





Three Families of Projections

There are **three major families** of projections, each tends to introduce **certain kinds of distortions**, or conversely each has certain **properties** that it used to **preserve** (i.e. spatial characteristics that it does not distort):

Three families:

1. Cylindrical projections
2. Conical projections
3. Planar projections

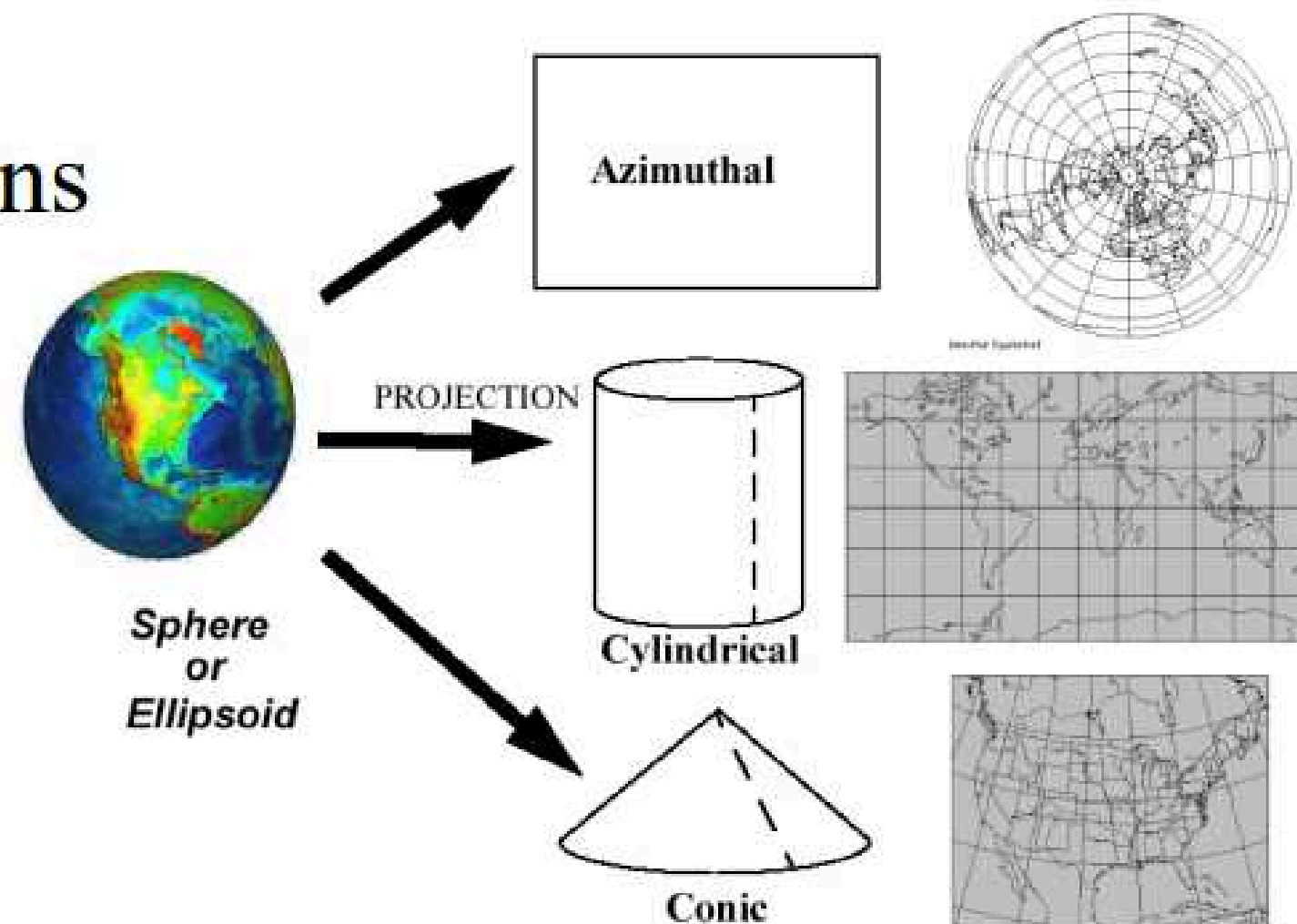


Figure 2.7 The earth can be projected in many ways, but basically onto three shapes that can be unrolled into a flat map: a flat plane, a cylinder, and a cone.

1. CYLINDRICAL PROJECTIONS: THESE PROJECTIONS INVOLVE WRAPPING A CYLINDER AROUND THE EARTH AND PROJECTING ITS FEATURES ONTO THE CYLINDRICAL SURFACE. EXAMPLES ARE THE MERCATOR, TRANSVERSE MERCATOR, AND MILLER CYLINDRICAL PROJECTIONS.

CONIC PROJECTIONS: FOR THESE PROJECTIONS, A CONE IS PLACED OVER THE EARTH, AND ITS FEATURES ARE PROJECTED ONTO THE CONICAL SURFACE. COMMON EXAMPLES ARE THE LAMBERT CONFORMAL CONIC AND ALBERS EQUAL-AREA CONIC PROJECTIONS.

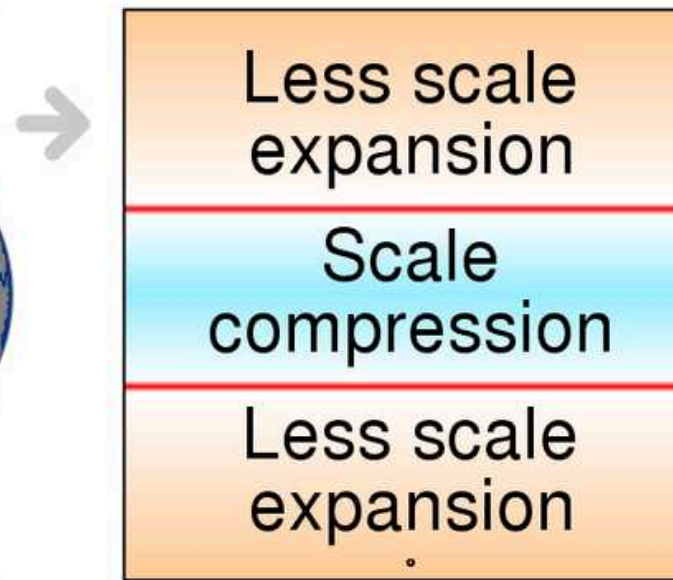
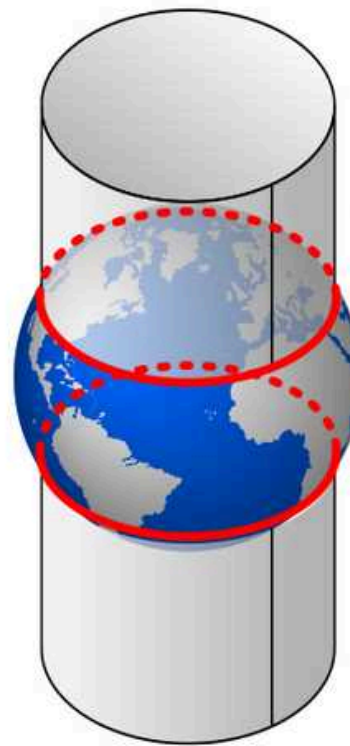
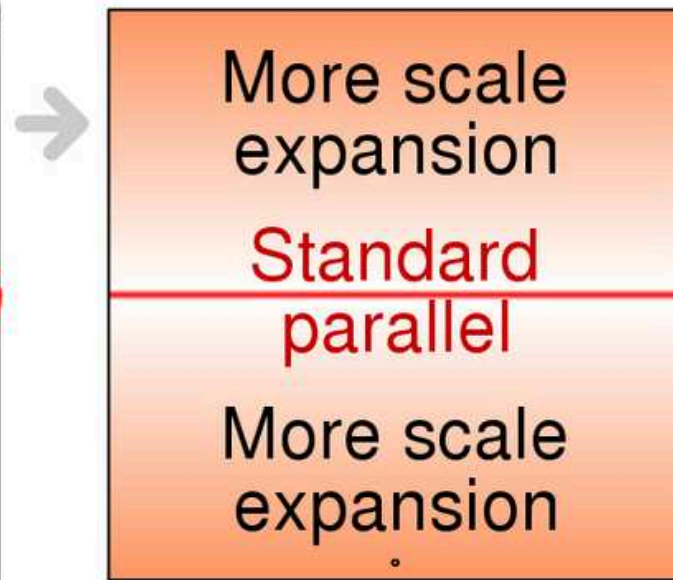
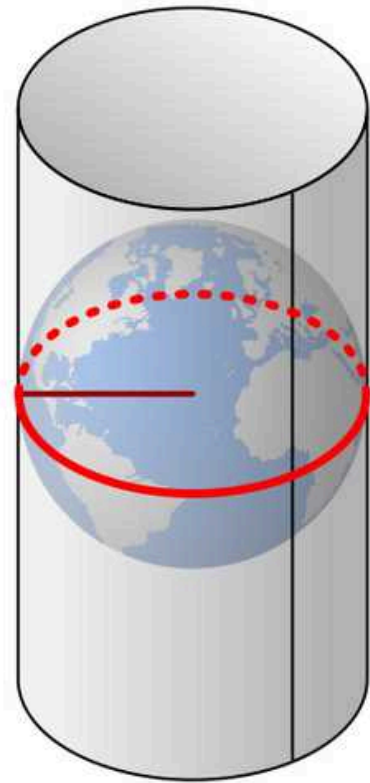
AZIMUTHAL PROJECTIONS: ALSO REFERRED TO AS PLANAR OR ZENITHAL PROJECTIONS, THESE USE A FLAT PLANE THAT TOUCHES THE EARTH AT A SINGLE POINT, PROJECTING THE EARTH'S FEATURES ONTO THE PLANE. AZIMUTHAL EQUIDISTANT, STEREOGRAPHIC, AND ORTHOGRAPHIC PROJECTIONS ARE EXAMPLES.

PSEUDOCYLINDRICAL PROJECTIONS: THESE PROJECTIONS RESEMBLE CYLINDRICAL PROJECTIONS BUT EMPLOY CURVED LINES INSTEAD OF STRAIGHT LINES FOR MERIDIANS AND PARALLELS. THE SINUSOIDAL, MOLLWEIDE, AND GOODE HOMOLOXINE PROJECTIONS ARE POPULAR EXAMPLES.

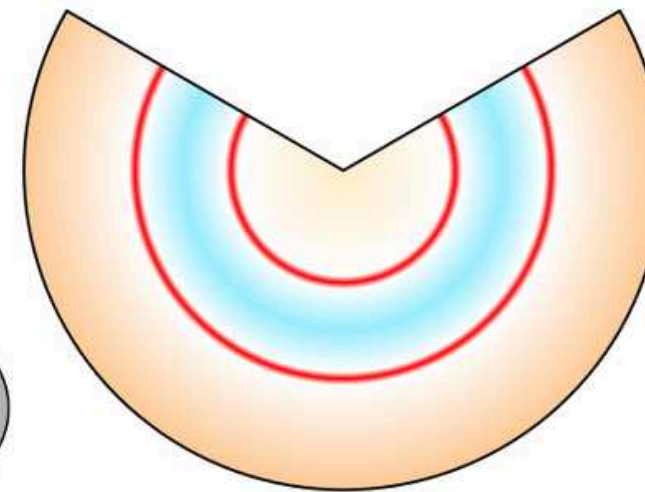
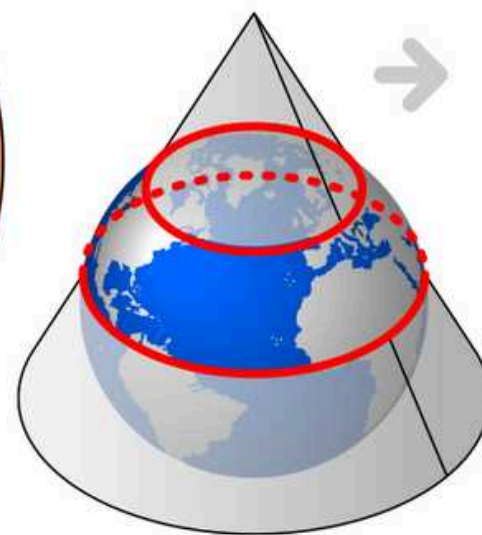
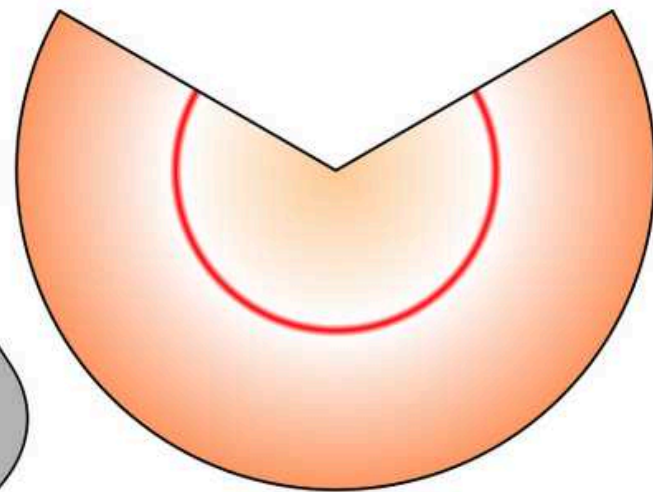
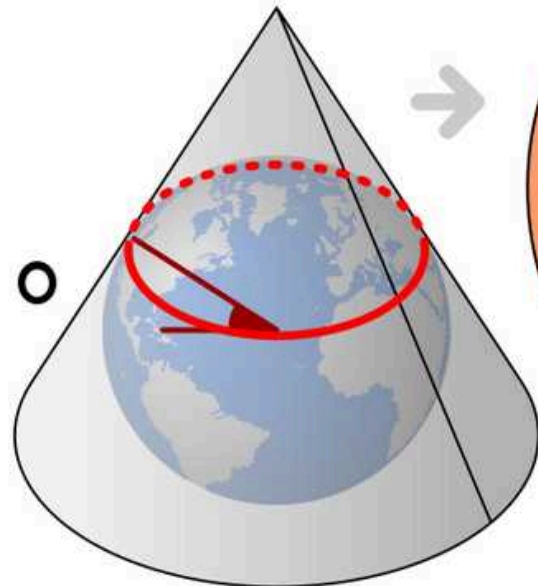
Tangent

Secant

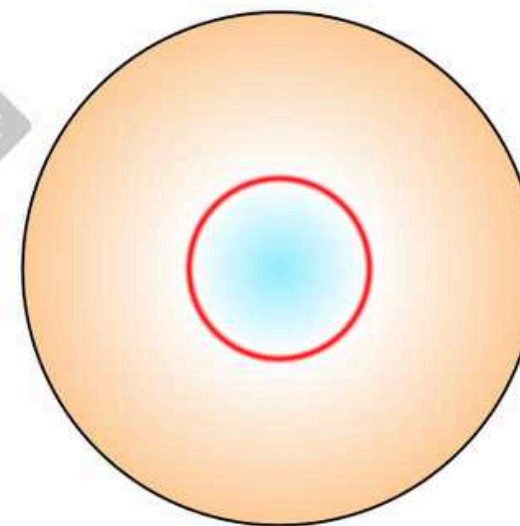
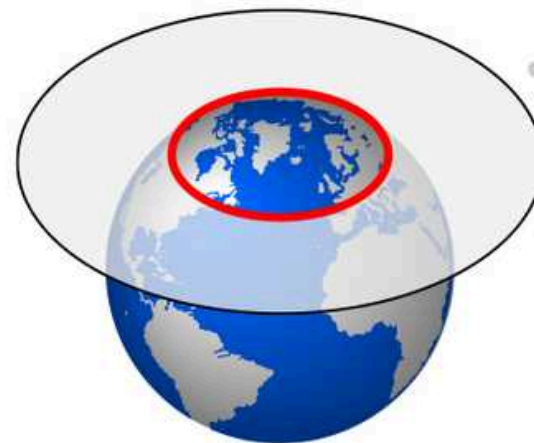
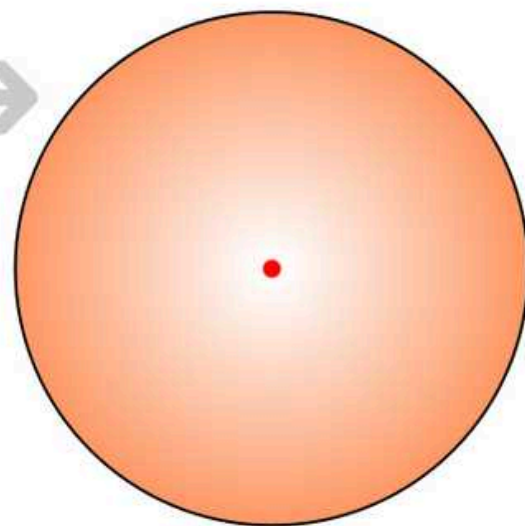
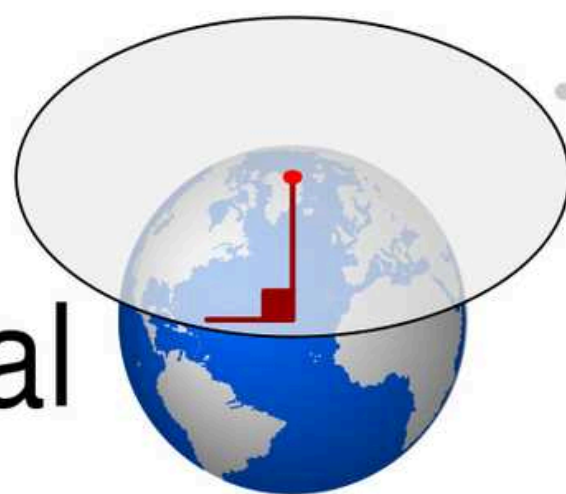
Cylindrical
 $\phi = 0^\circ$



Conic
 $0^\circ < \phi < 90^\circ$



Azimuthal
 $\phi = 90^\circ$



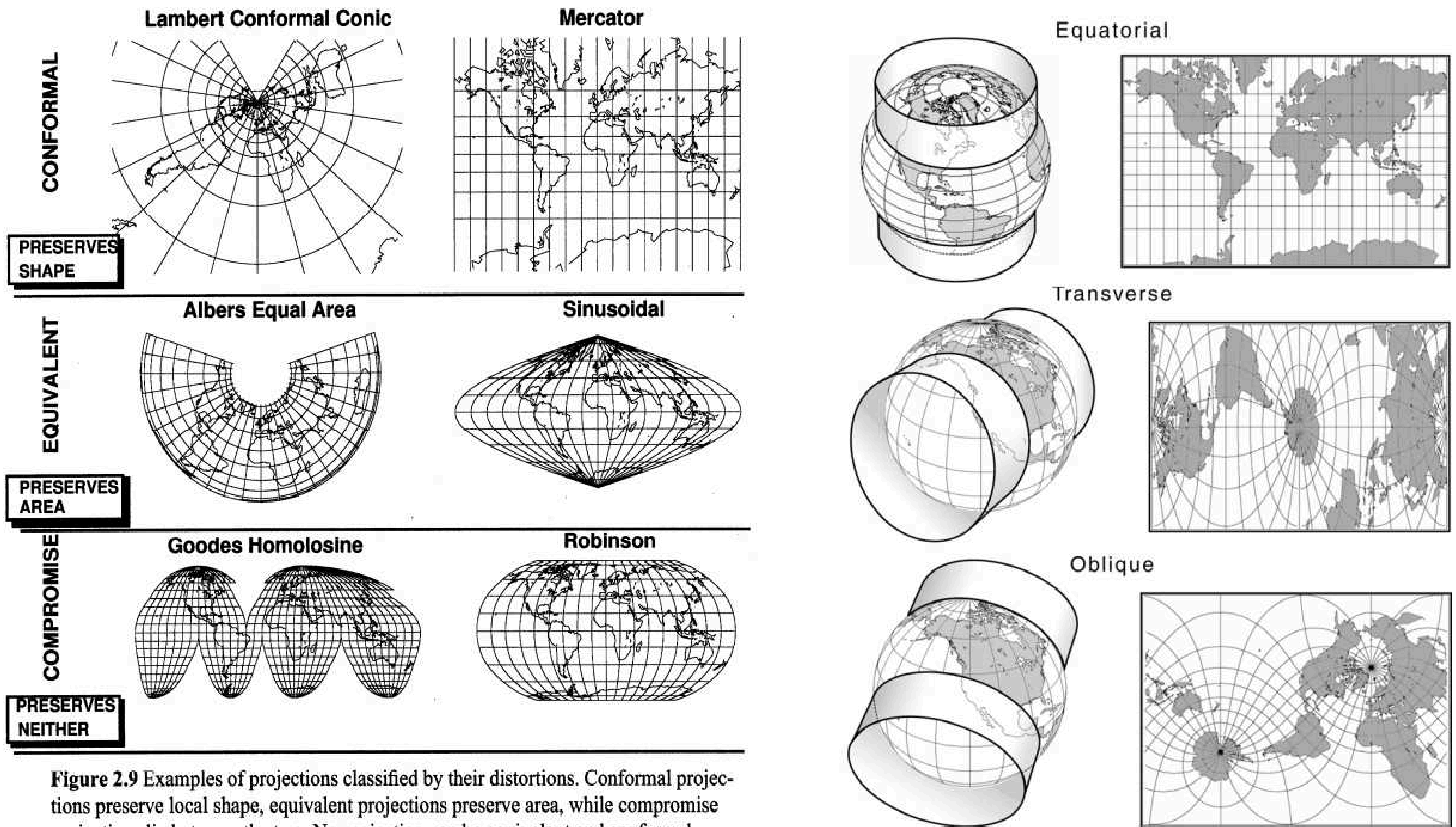
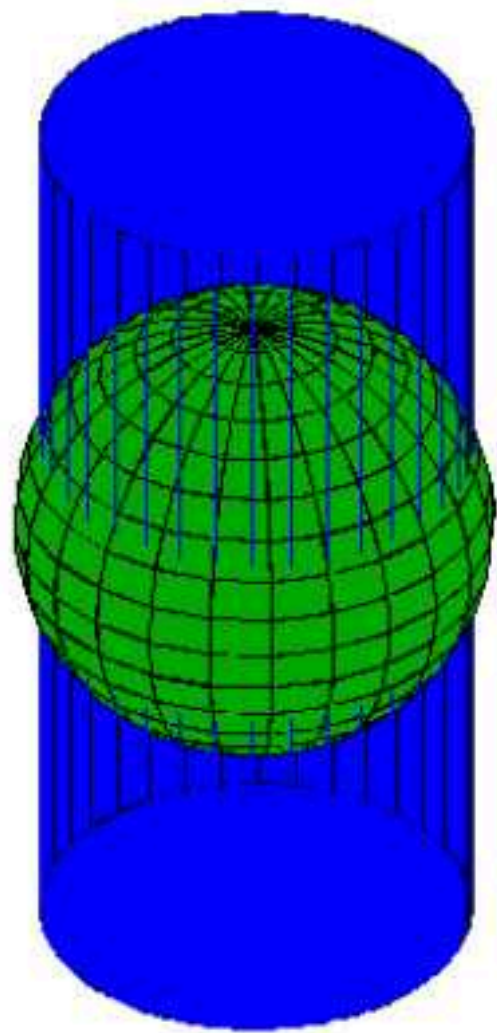
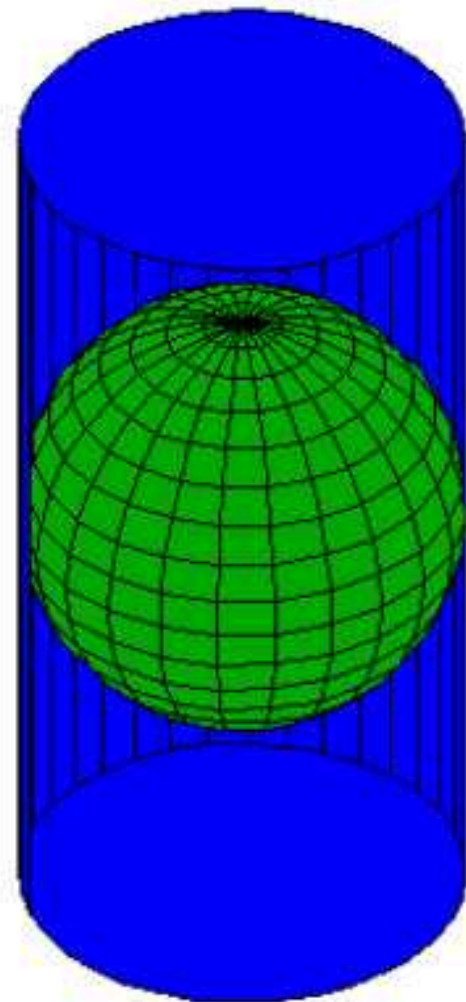


Figure 2.9 Examples of projections classified by their distortions. Conformal projections preserve local shape, equivalent projections preserve area, while compromise projections lie between the two. No projection can be equivalent and conformal.

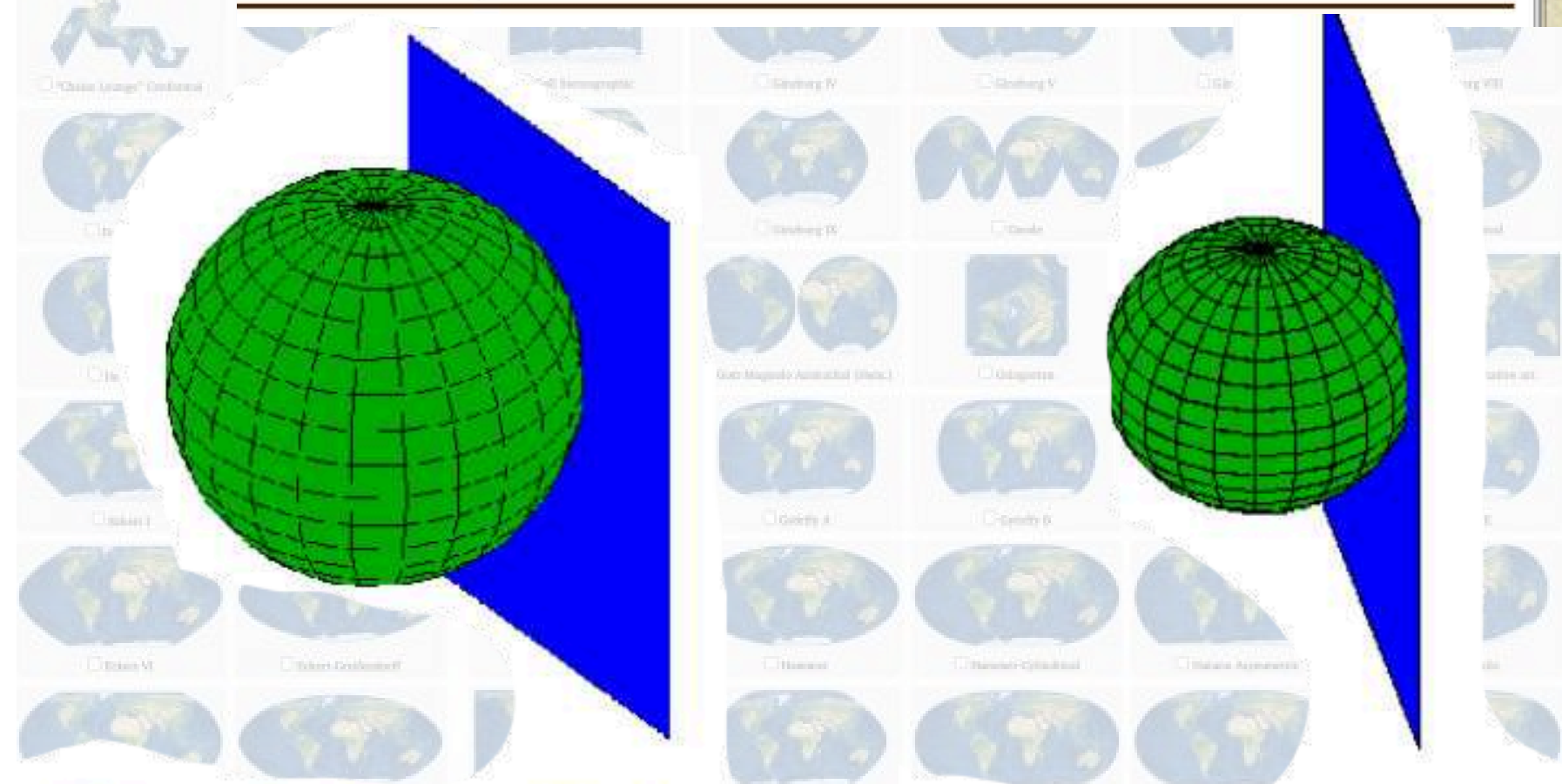


Secant Cylindrical Projection



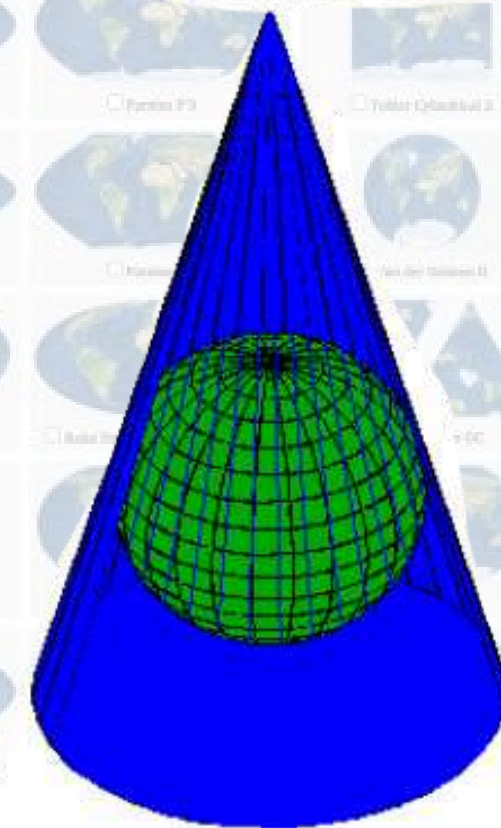
Cylindrical Projection Surface
(Tangent)

Planar (Azimuthal) Projection – Conceptual View

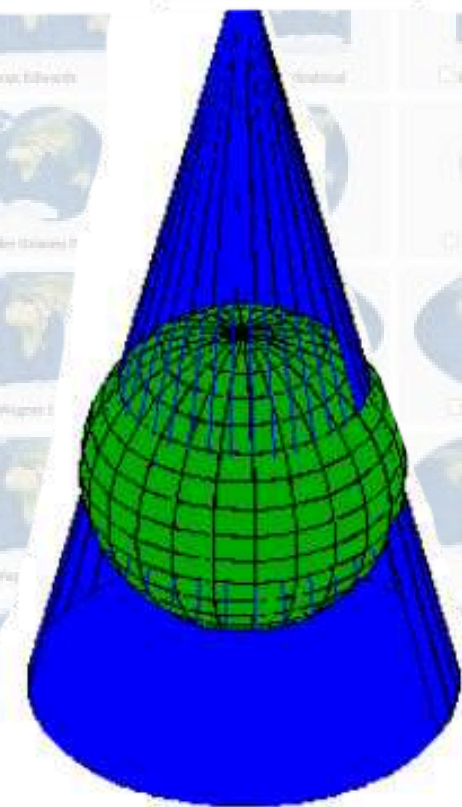


Planar Projection Surface

Secant Planar Projection



Conical Projection Surface



Secant Conic Projection



Distortion With Projections

✠ Shape (Conformality)

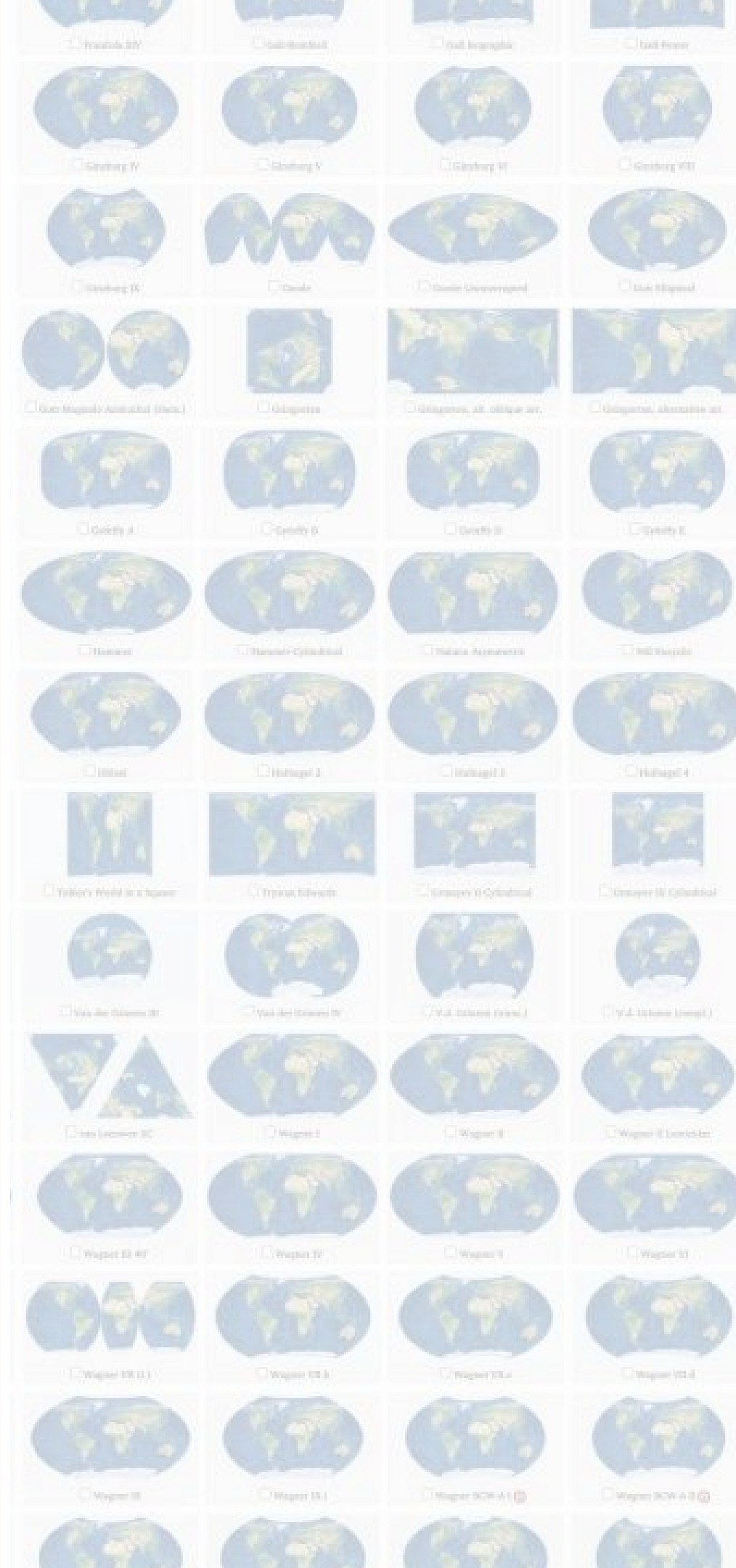
✠ Distance

✠ Direction

✠ Area

Scale also can be distorted, or differ, throughout a single map

Different map projections preserve some of these properties or attempt to reduce the distortion of some or all of these properties, but NO map projection preserves all these properties.



1. EQUAL-AREA (EQUIVALENT) PROJECTIONS: THESE PROJECTIONS PRESERVE THE CORRECT PROPORTIONS OF AREAS, SUCH AS IN THE ALBERS EQUAL-AREA CONIC AND MOLLWEIDE PROJECTIONS.

2. CONFORMAL (ORTHOMORPHIC) PROJECTIONS: THESE PROJECTIONS MAINTAIN LOCAL ANGLES AND SHAPES, AS SEEN IN THE MERCATOR AND LAMBERT CONFORMAL CONIC PROJECTIONS.

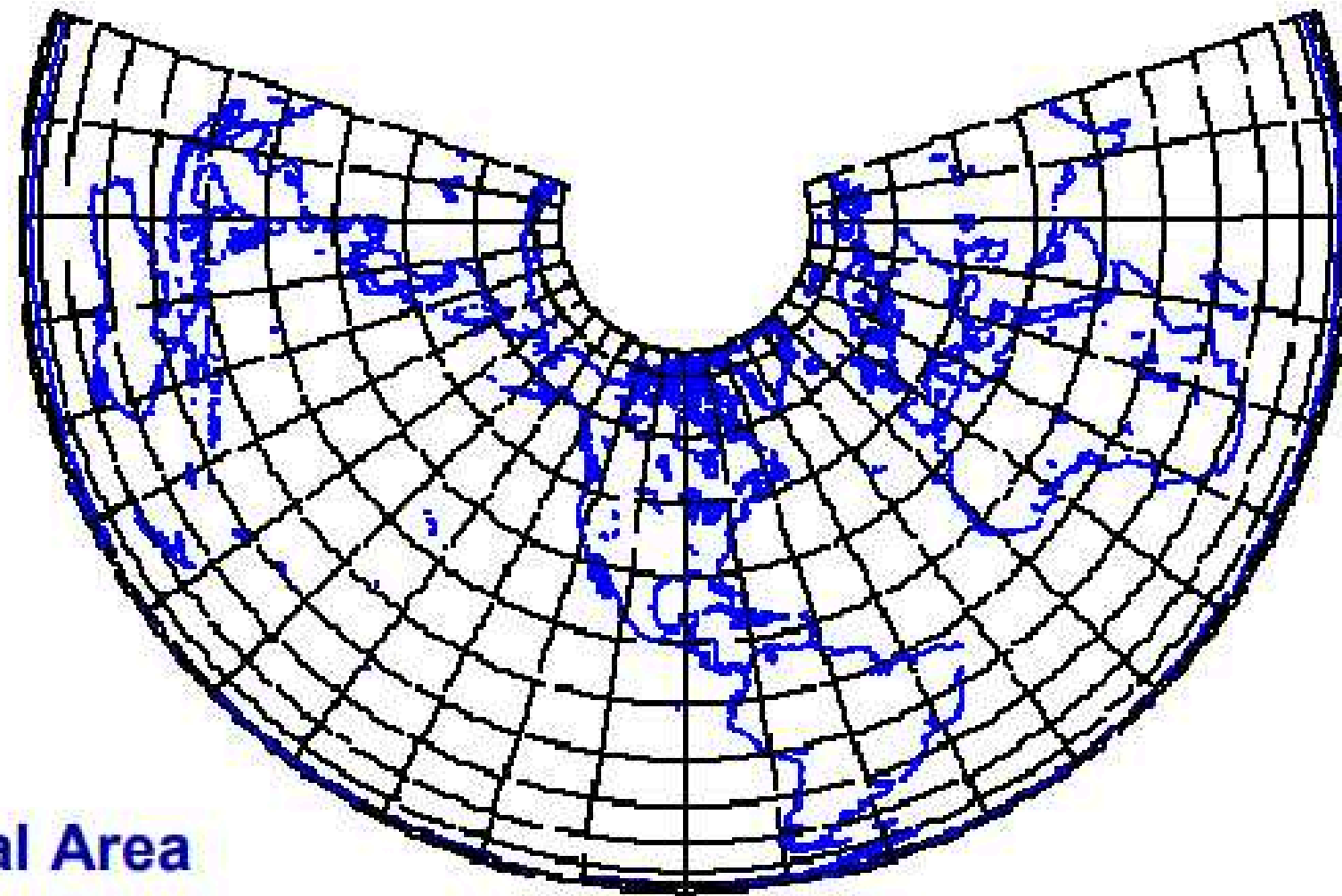
3. EQUIDISTANT PROJECTIONS: THESE PROJECTIONS RETAIN TRUE DISTANCES FROM ONE OR TWO POINTS TO ALL OTHER POINTS, AS IN THE AZIMUTHAL EQUIDISTANT PROJECTION.

4. AZIMUTHAL PROJECTIONS: THESE PROJECTIONS PRESERVE DIRECTIONS FROM A CENTRAL POINT, INCLUDING SOME CONFORMAL, ORTHOMORPHIC, AND AZIMUTHAL PROJECTIONS.

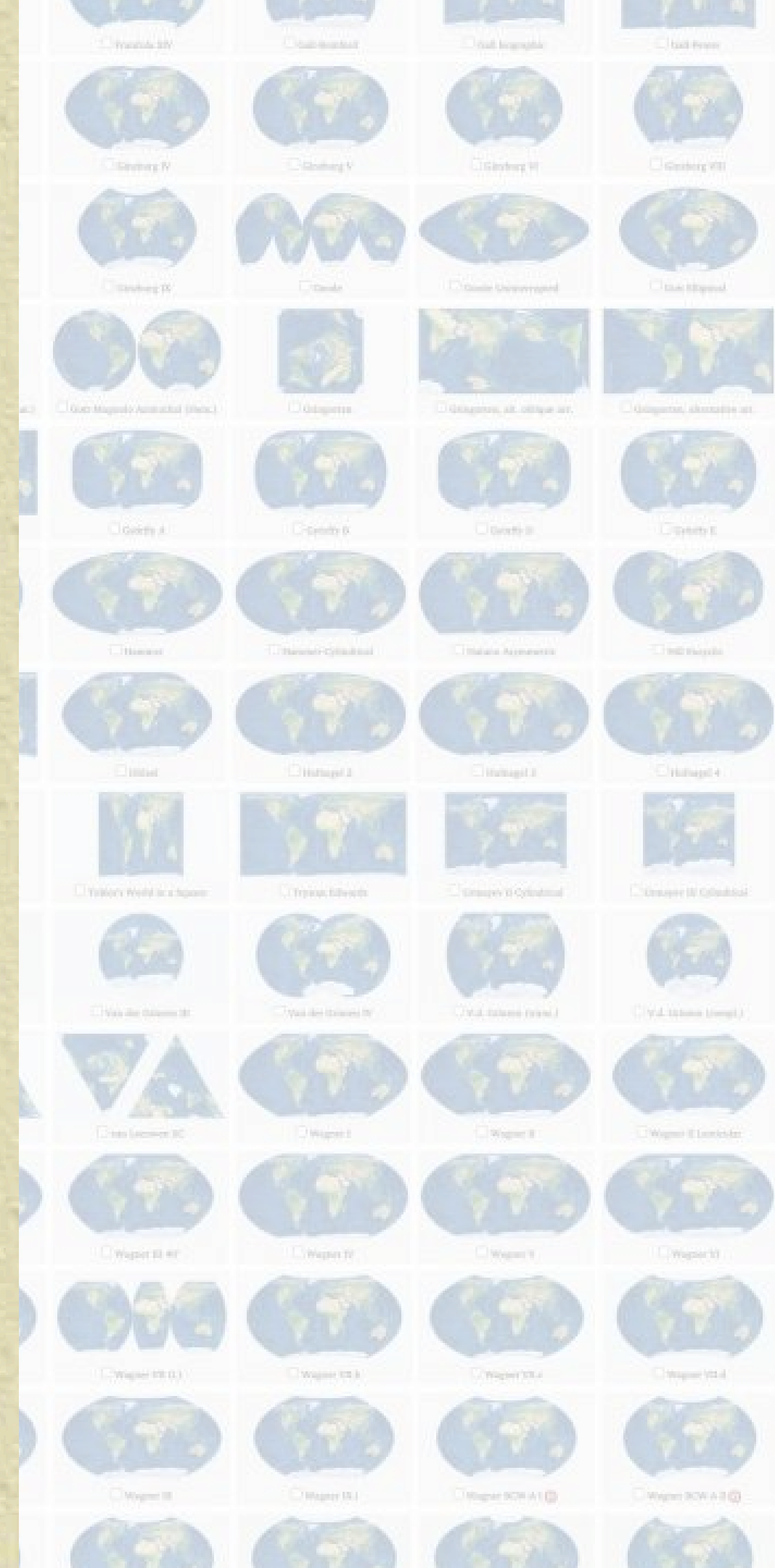
5. COMPROMISE PROJECTIONS: THESE PROJECTIONS ATTEMPT TO BALANCE VARIOUS DISTORTIONS INHERENT IN MAP PROJECTIONS, SUCH AS THE ROBINSON AND WINKEL TRIPEL PROJECTIONS.

Equal-area Projections

- ✦ **Equal-area** projections preserve the **relative area** of displayed features
 - Every part on the map, as well as the whole, has the same area as the corresponding part on the Earth, at the same reduced scale
 - Shape, angles, scale may be distorted
 - Graticule lines may not meet at 90-degree angles

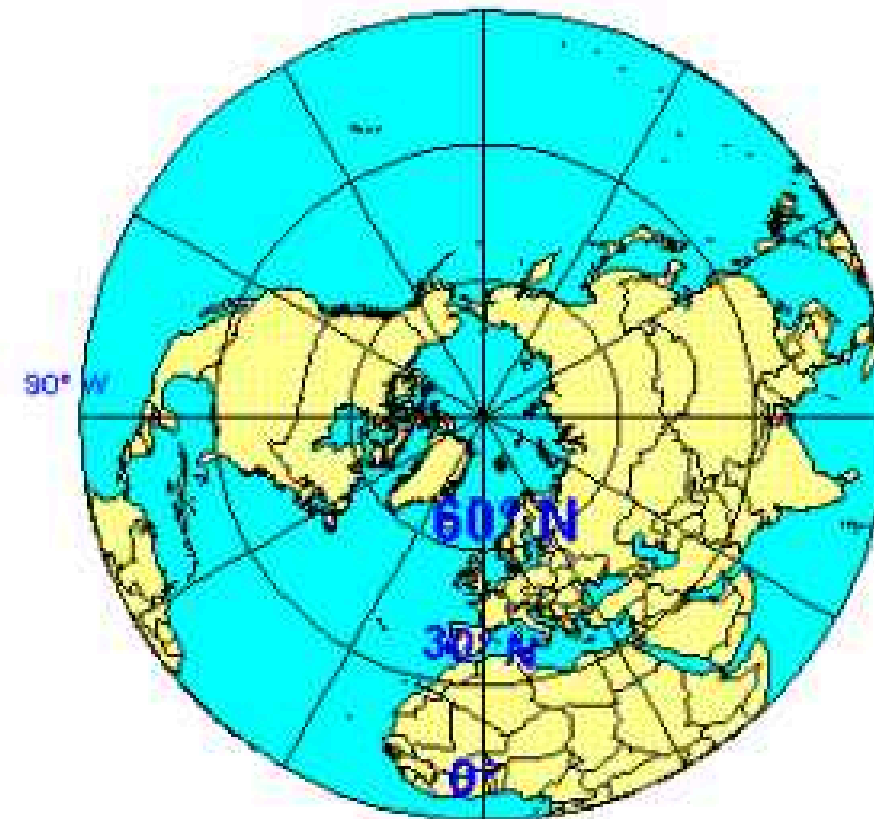


Albers Equal Area

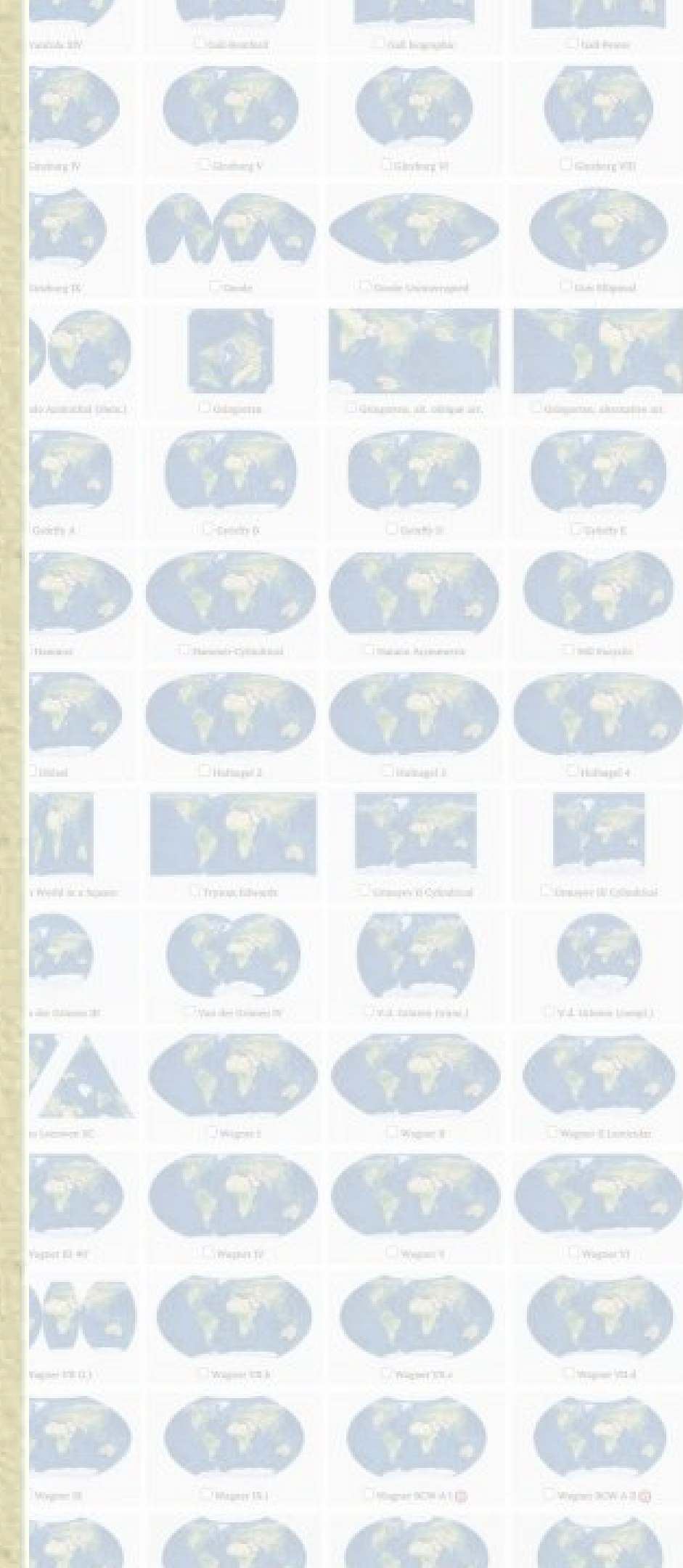


Equidistant Projections

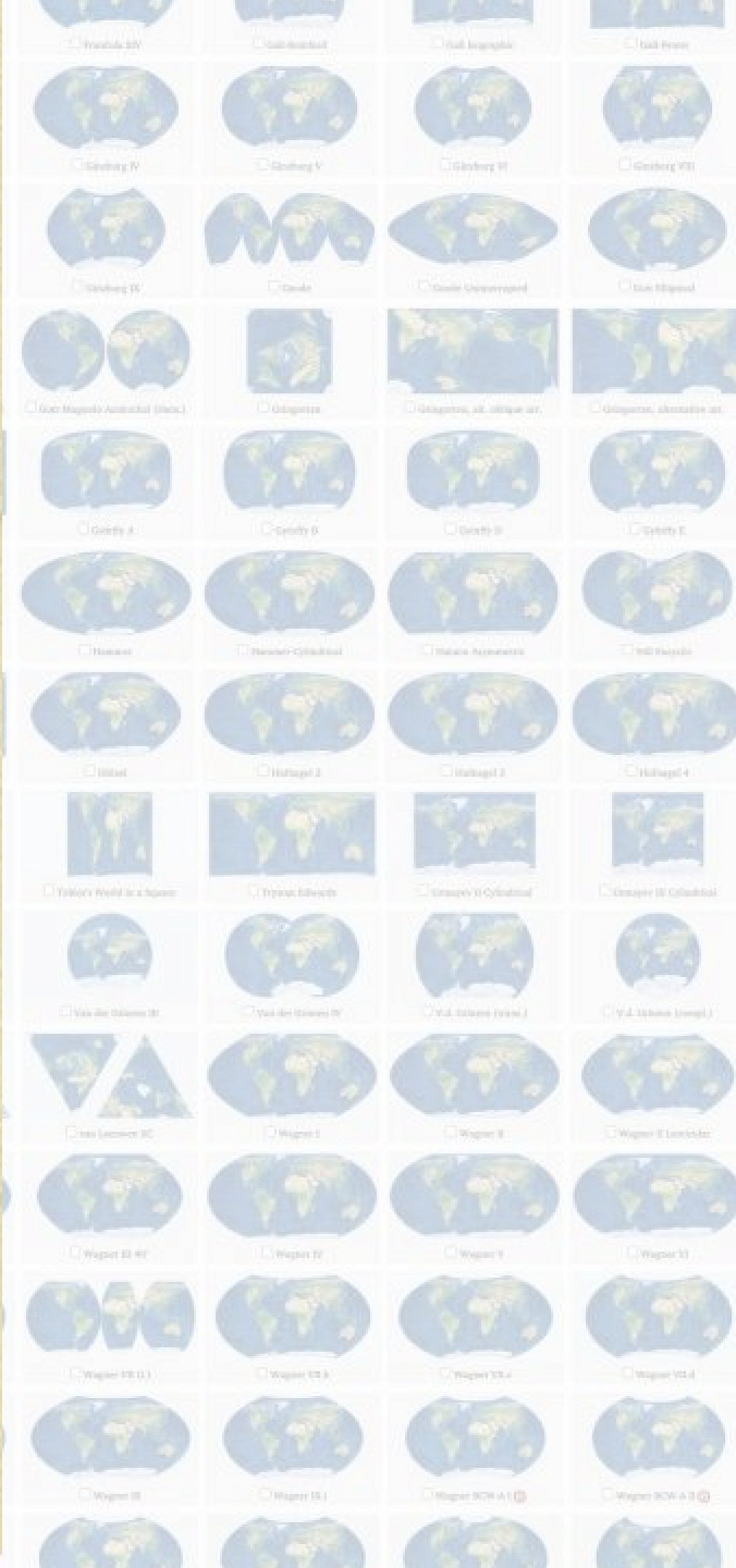
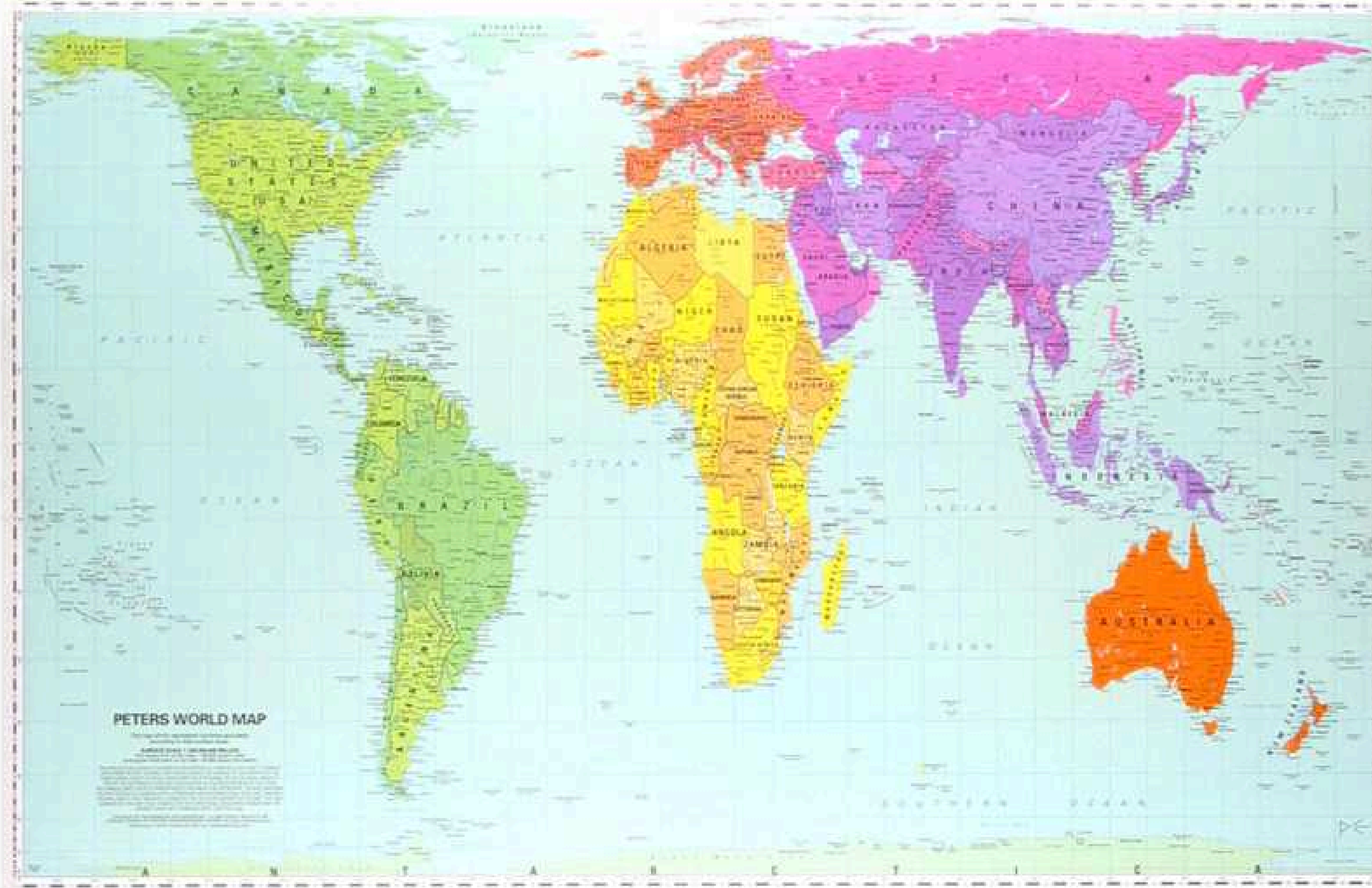
- ✦ **Equidistant** projections preserve the **distances** between certain points
 - Scale is maintained along certain lines on map in relation to its reference globe; the distances along these lines are *true*
 - True distances only from the center of the projection or along special lines
 - No projection is equidistant to and from all points on a map



Azimuthal Equidistant



Equal-area Projections

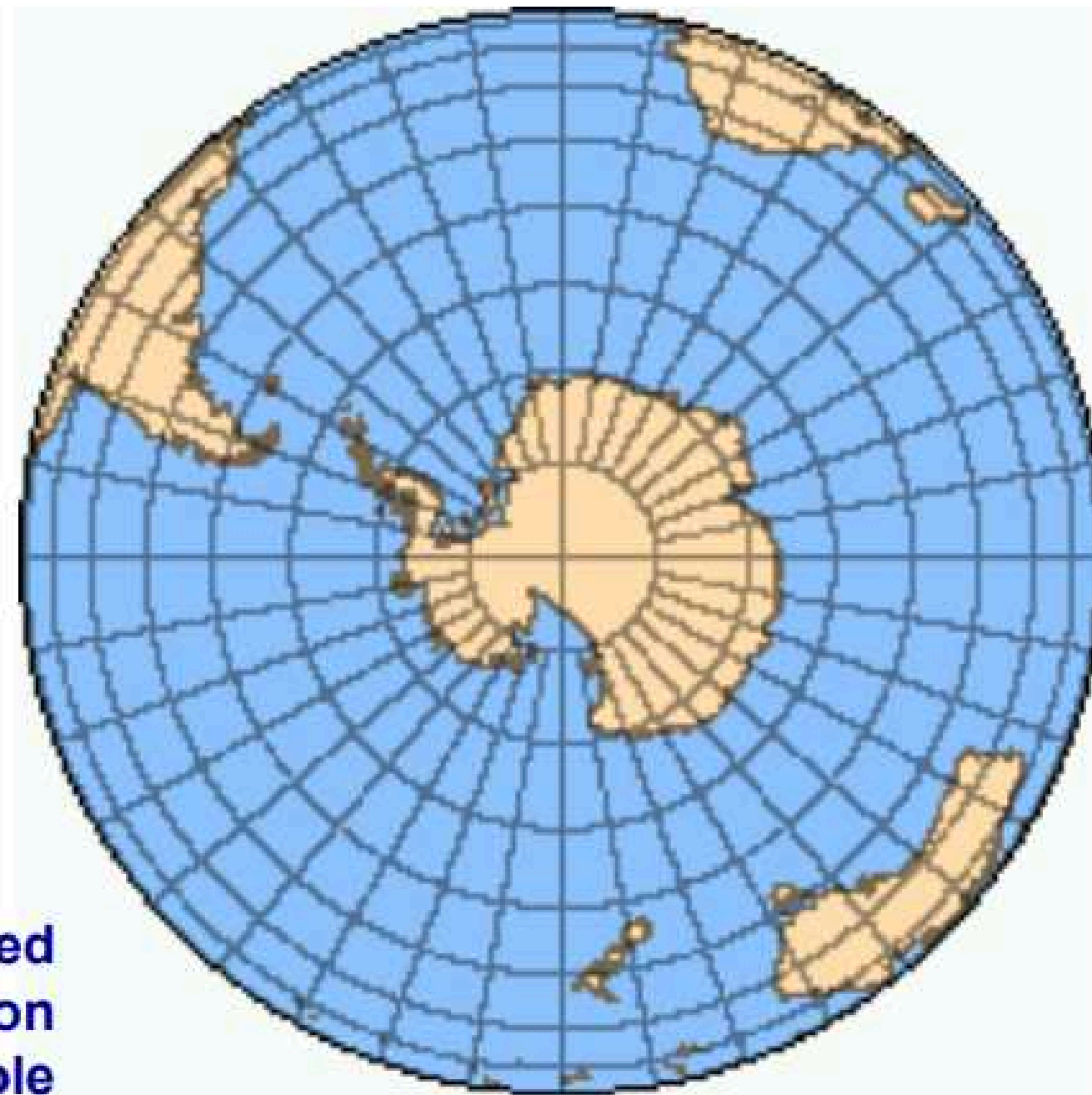
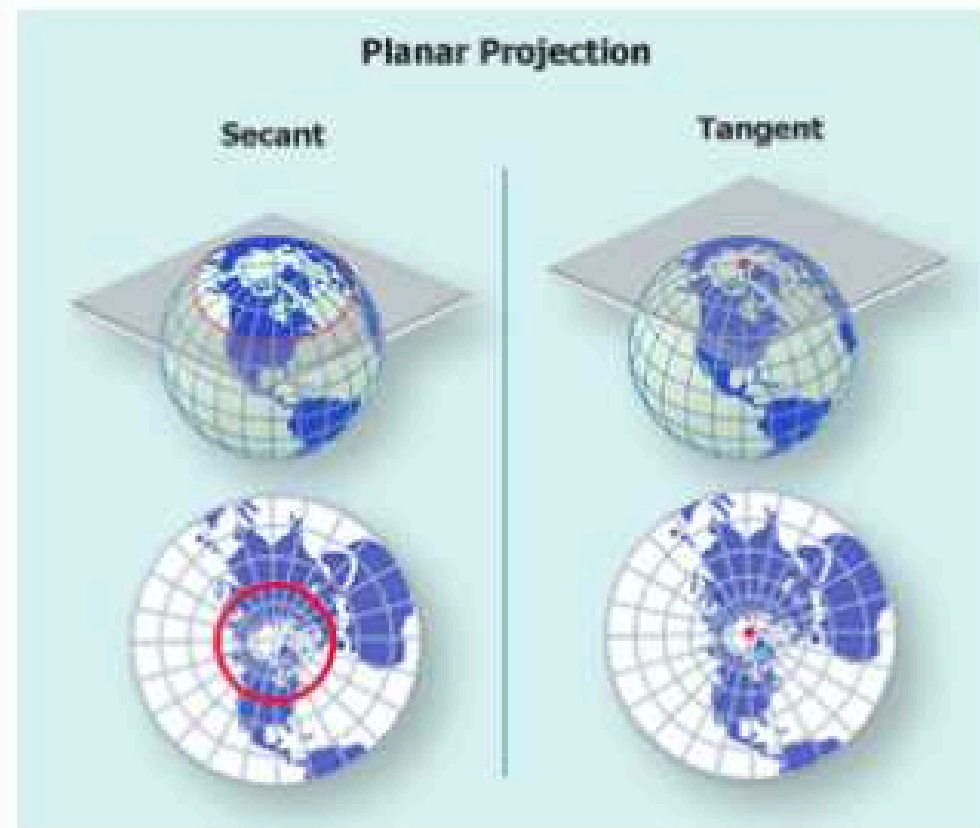


True-direction Projections

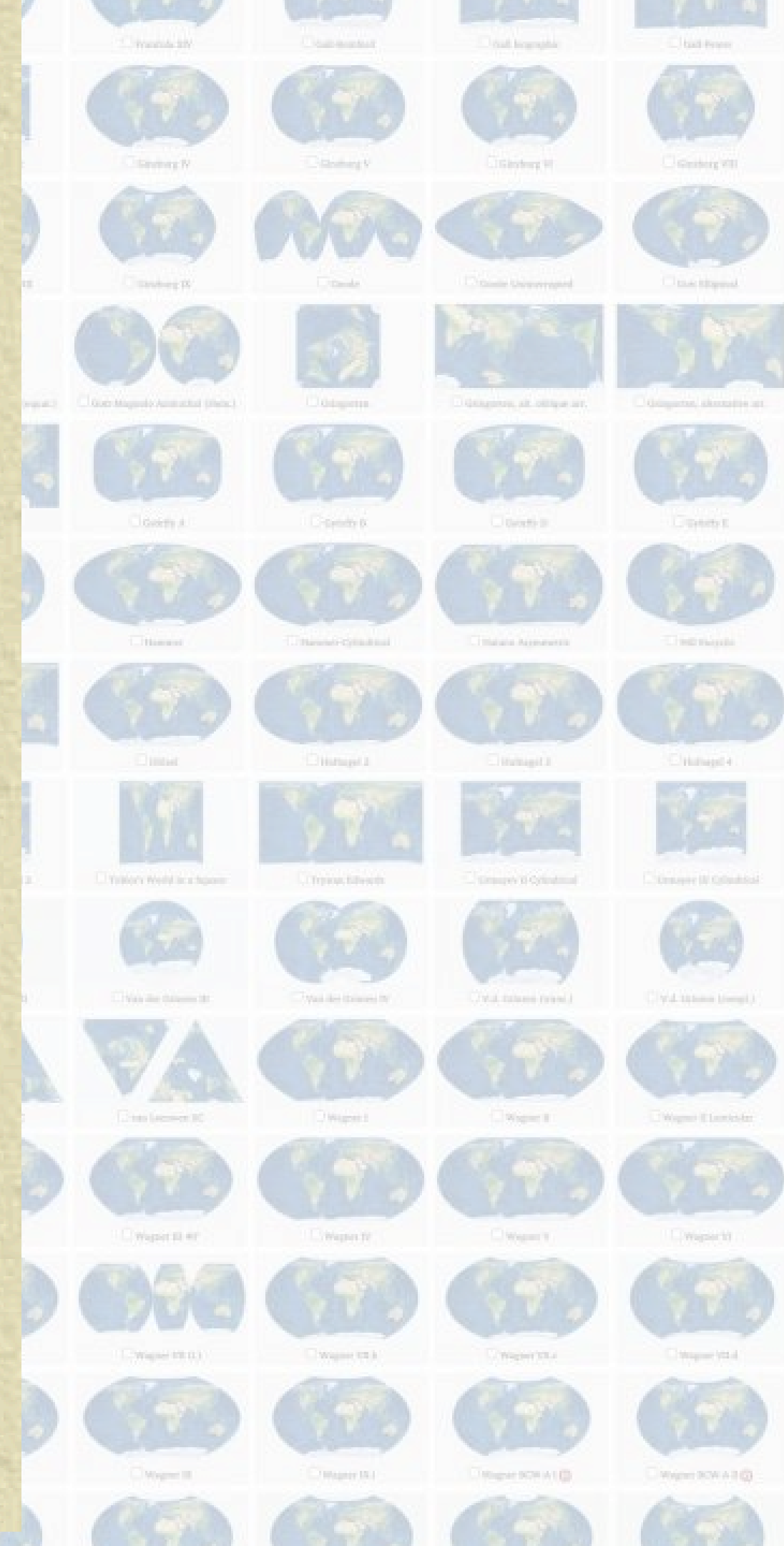
- ☀ **True-direction** or azimuthal projections preserve the direction of specified points on a great circle
 - The shortest route between two points on a curved surface such as the earth is along the spherical equivalent of a straight line on a flat surface - **called the great circle**
 - **Great circle arcs are *rectified*, or shown as straight lines**
 - Azimuths (angles from a point on a line to another point) are portrayed correctly in all directions

See <http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj.html> for a good list and examples of different projections

Planar (Azimuthal) Projection - Example



Example of a map produced with a planar projection centered on the South Pole

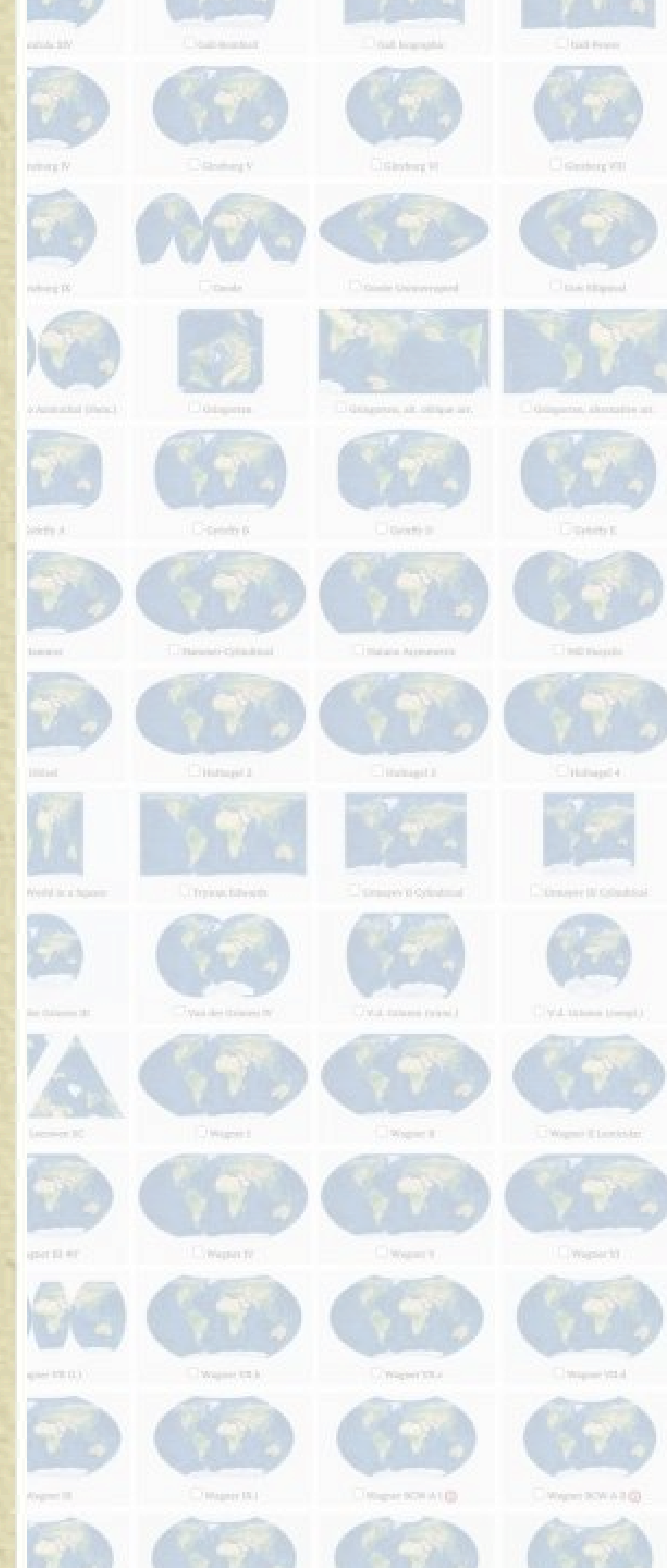
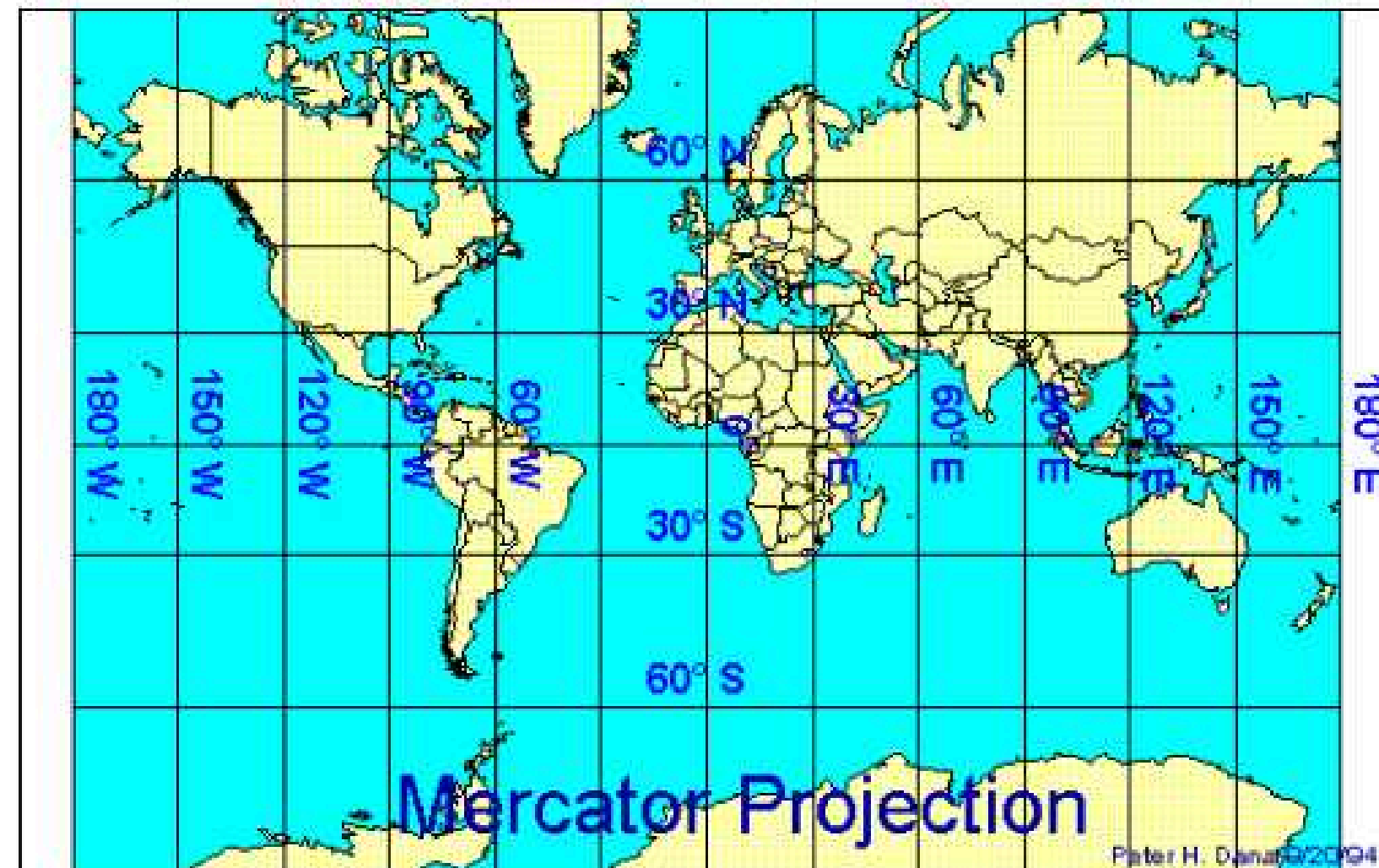


Conformal Projections

- ✦ **Conformal** projections preserve local **shape**
 - At every point the scale is the same in every direction
 - Graticule lines intersect at 90-degree angles
 - All angles between intersections of arcs are maintained
 - Shapes of very small areas and angles with very short sides are preserved.
 - Size of many areas are distorted

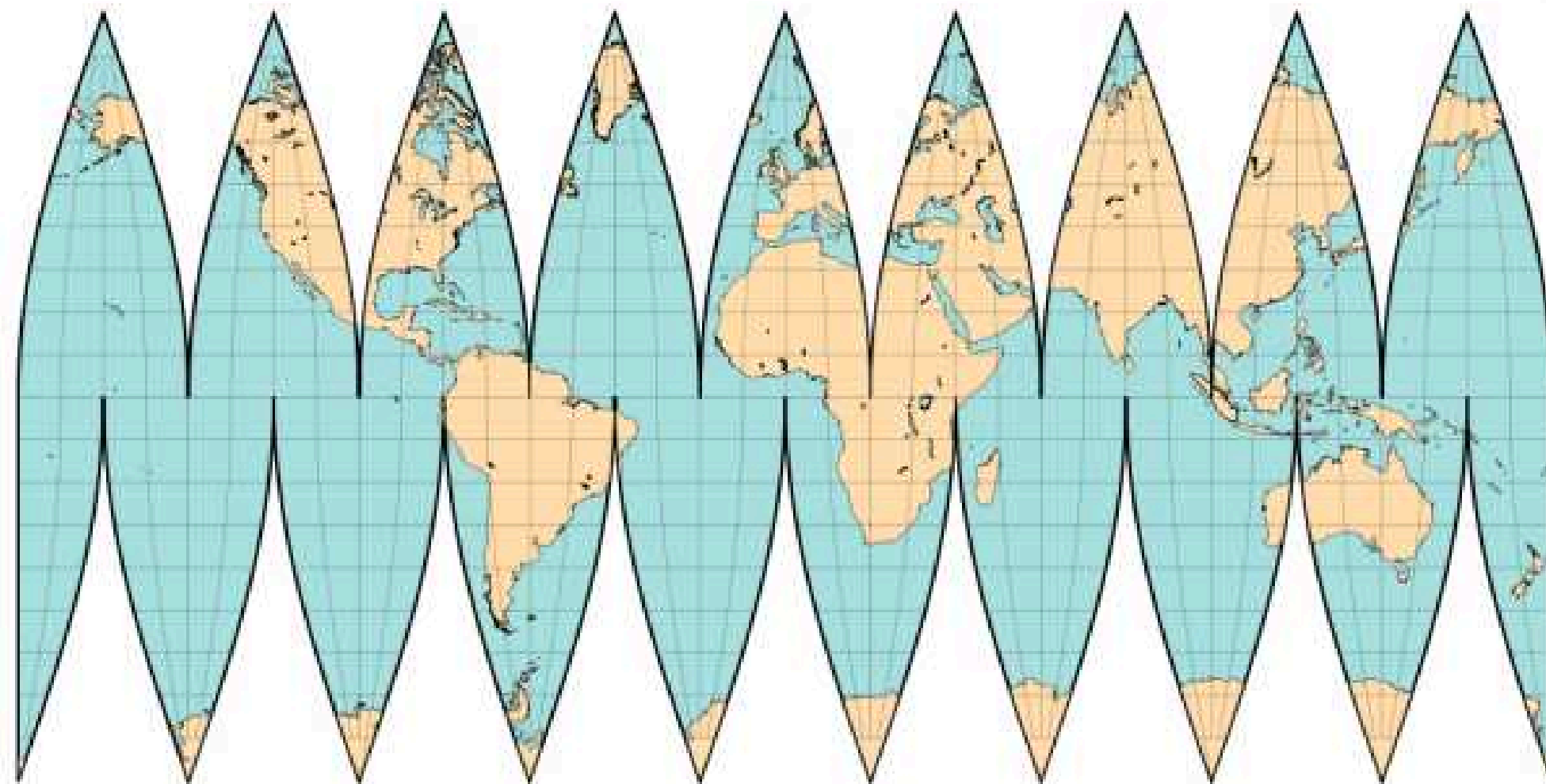


Stereographic
North Polar Aspect

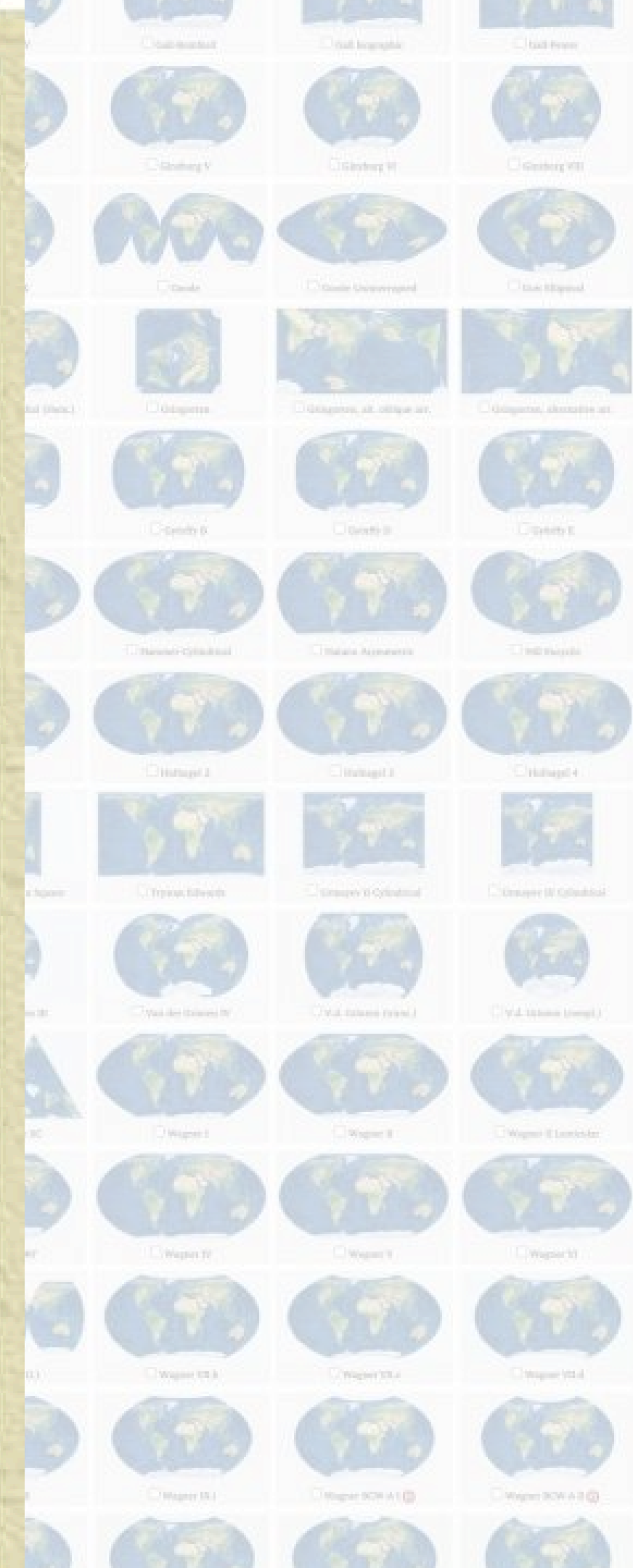


Interrupted Projections

- ✦ A compromise projection, "cutting" the earth's surface along arbitrarily chosen lines and projecting each section separately, which results in less stretching.



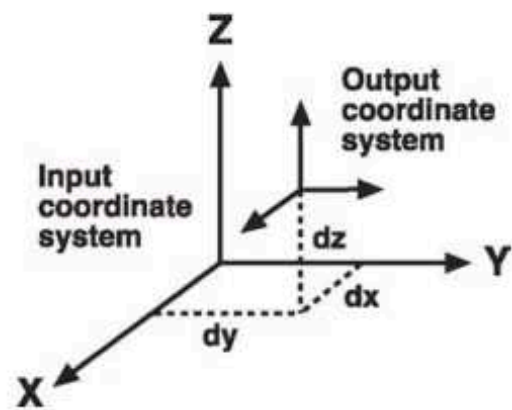
**Interrupted
sinusoidal
map**



ALL MAPS ARE THE PROJECTING THE GRATICULE

Three-parameter methods

The simplest datum transformation method is a geocentric, or three-parameter, transformation. The geocentric transformation models the differences between two datums in the X,Y,Z coordinate system. One datum is defined with its center at 0,0,0. The center of the other datum is defined at some distance ($\Delta X, \Delta Y, \Delta Z$) in meters away.



Usually the transformation parameters are defined as going 'from' a local datum 'to' WGS 1984 or another geocentric datum.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$

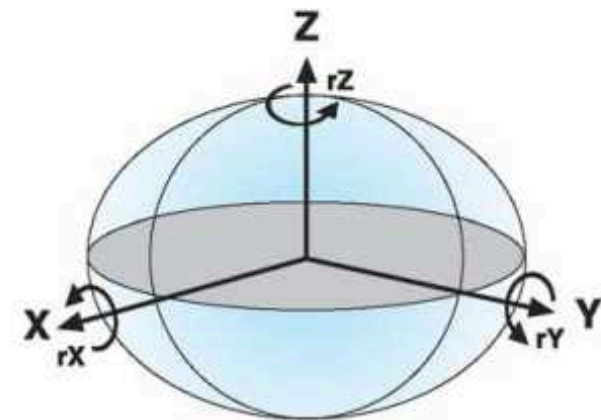
The three parameters are linear shifts and are always in meters.

Seven-parameter methods

A more complex and accurate datum transformation is possible by adding four more parameters to a geocentric transformation. The seven parameters are three linear shifts ($\Delta X, \Delta Y, \Delta Z$), three angular rotations around each axis (r_x, r_y, r_z), and scale factor(s).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+s) \cdot \begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$

The rotation values are given in decimal seconds, while the scale factor is in parts per million (ppm). The rotation values are defined in two different ways. It's possible to define the rotation angles as positive either clockwise or counterclockwise as you look toward the origin of the X,Y,Z systems.



The Coordinate Frame (or Bursa-Wolf) definition of the rotation values.

The equation in the previous column is how the United States and Australia define the equations and is called the Coordinate Frame Rotation transformation. The rotations are positive counterclockwise. Europe uses a different convention called the Position Vector transformation. Both methods are sometimes referred to as the Bursa-Wolf method. In the Projection Engine, the Coordinate Frame and Bursa-Wolf methods are the same. Both Coordinate Frame and Position Vector methods are supported, and it is easy to convert transformation values from one method to the other simply by changing the signs of the three rotation values. For example, the parameters to convert from the WGS 1972 datum to the WGS 1984 datum with the Coordinate Frame method are (in the order, $\Delta X, \Delta Y, \Delta Z, r_x, r_y, r_z, s$):

(0.0, 0.0, 4.5, 0.0, 0.0, -0.554, 0.227)

To use the same parameters with the Position Vector method, change the sign of the rotation so the new parameters are:

(0.0, 0.0, 4.5, 0.0, 0.0, +0.554, 0.227)

Unless explicitly stated, it's impossible to tell from the parameters alone which convention is being used. If you use the wrong method, your results can return inaccurate coordinates. The only way to determine how the parameters are defined is by checking a control point whose coordinates are known in the two systems.

Molodensky method

The Molodensky method converts directly between two geographic coordinate systems without actually converting to an X,Y,Z system. The Molodensky method requires three shifts ($\Delta X, \Delta Y, \Delta Z$) and the differences between the semimajor axes (Δa) and the flattenings (Δf) of the two spheroids. The Projection Engine automatically calculates the spheroid differences according to the datums involved.

$$(M+h)\Delta\varphi = -\sin\varphi\cos\lambda\Delta X - \sin\varphi\sin\lambda\Delta Y + \cos\varphi\Delta Z + \frac{e^2\sin\varphi\cos\varphi}{(1-e^2\sin^2\varphi)^{1/2}}\Delta a + \sin\varphi\cos\varphi\left(M\frac{a}{b} + N\frac{b}{a}\right)\Delta f$$

$$(N+h)\cos\varphi\Delta\lambda = -\sin\lambda\Delta X + \cos\lambda\Delta Y$$

$$\Delta h = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z - (1-e^2\sin^2\varphi)^{1/2}\Delta a + \frac{a(1-f)}{(1-e^2\sin^2\varphi)^{1/2}}\sin^2\varphi\Delta f$$

- h ellipsoid height (meters)
- φ latitude
- λ longitude
- a semimajor axis of the spheroid (meters)
- b semiminor axis of the spheroid (meters)
- f flattening of the spheroid
- e eccentricity of the spheroid

M and N are the meridional and prime vertical radii of curvature, respectively, at a given latitude. The equations for M and N are:

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}$$

$$N = \frac{a}{(1-e^2\sin^2\varphi)^{1/2}}$$

You solve for $\Delta\lambda$ and $\Delta\varphi$. The amounts are added automatically by the Projection Engine.

Abridged Molodensky method

The Abridged Molodensky method is a simplified version of the Molodensky method. The equations are:

$$M\Delta\varphi = -\sin\varphi\cos\lambda\Delta X - \sin\varphi\sin\lambda\Delta Y + \cos\varphi\Delta Z + (a\Delta f + f\Delta a) \cdot 2\sin\varphi\cos\varphi$$

$$N\cos\varphi\Delta\lambda = -\sin\lambda\Delta X + \cos\lambda\Delta Y$$

$$\Delta h = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z + (a\Delta f + f\Delta a)\sin^2\varphi - \Delta a$$

LATITUDE AND LONGITUDE ARE INDEED BASED ON OBSERVATIONS OF THE SKY (CELESTIAL NAVIGATION) AND ARE FUNDAMENTALLY TIED TO A SPHERICAL MODEL OF THE EARTH.

THE ENTIRE SYSTEM OF LATITUDE AND LONGITUDE IS BASED ON THE PREMISE THAT THE EARTH IS SPHERICAL. THIS IS EVIDENT IN SEVERAL WAYS:

GREAT CIRCLES: BOTH LATITUDE AND LONGITUDE LINES ARE BASED ON THE CONCEPT OF GREAT CIRCLES THAT DIVIDE THE GLOBE INTO EQUAL HALVES. THE EQUATOR AND ALL MERIDIANS ARE GREAT CIRCLES.

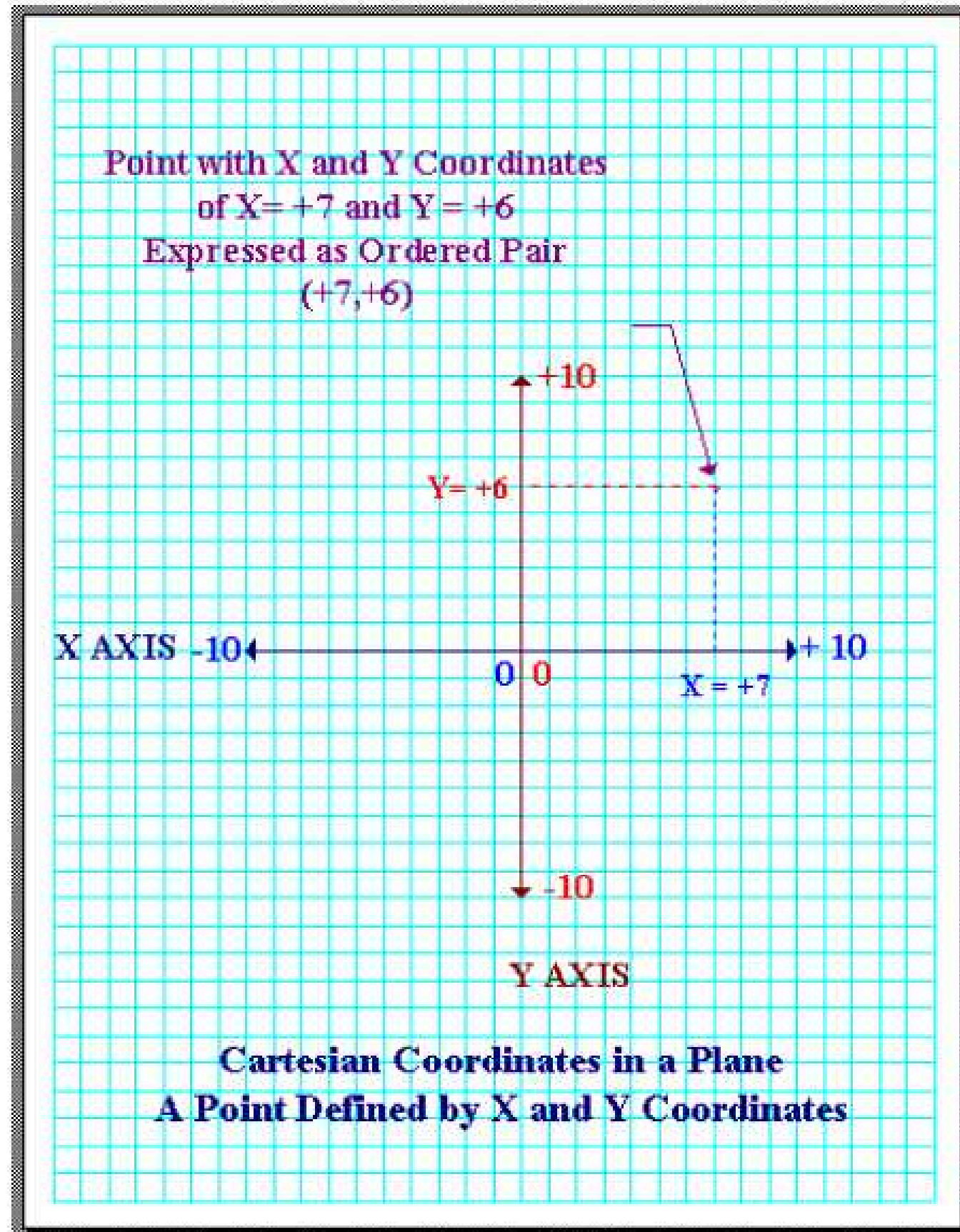
SPHERICAL TRIGONOMETRY: [ASTRONOMICAL TRIANGLE]THE CALCULATIONS FOR DISTANCES AND ANGLES BETWEEN DIFFERENT POINTS ON THE EARTH'S SURFACE USE SPHERICAL TRIGONOMETRY, ASSUMING THE EARTH IS A SPHERE.

NAVIGATION AND MAPPING: ALL NAVIGATIONAL AND MAPPING SYSTEMS THAT USE LATITUDE AND LONGITUDE TAKE INTO ACCOUNT THE EARTH'S INFLICTED CURVATUREFROM THE GEOGRAPHIC COORDINATE SYSTEM IT IS PROJECTING

GREAT CIRCLES VS. CIRCLES: A GREAT CIRCLE IS THE LARGEST CIRCLE THAT CAN BE DRAWN ON A SPHERE'S SURFACE, DIVIDING IT INTO TWO EQUAL HALVES. ON A SPHERE LIKE THE EARTH, THE SHORTEST DISTANCE BETWEEN TWO POINTS LIES ALONG THE ARC OF A GREAT CIRCLE. IN NAVIGATION, USING THE CONCEPT OF GREAT CIRCLES ACCOUNTS FOR THE EARTH'S CURVATURE, PROVIDING THE MOST EFFICIENT AND SHORTEST PATH BETWEEN TWO POINTS.

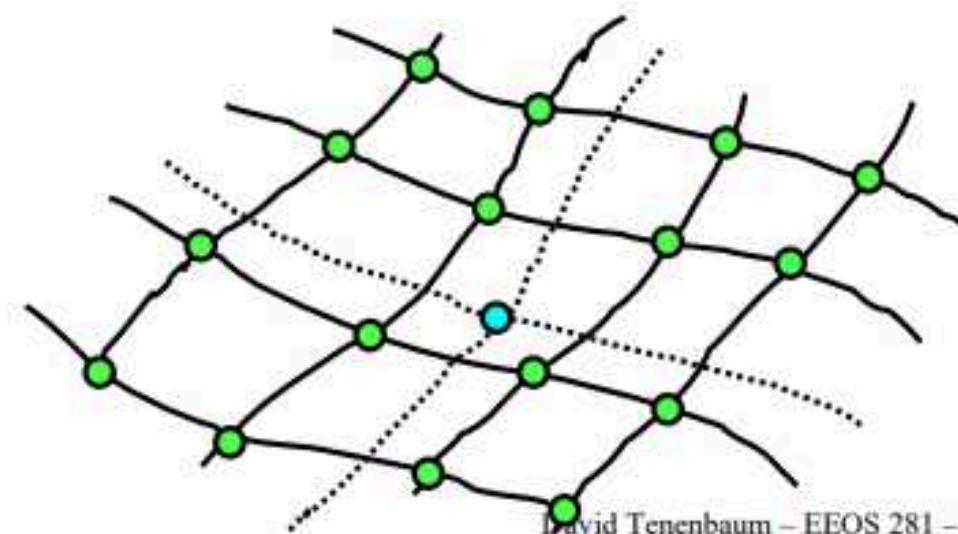
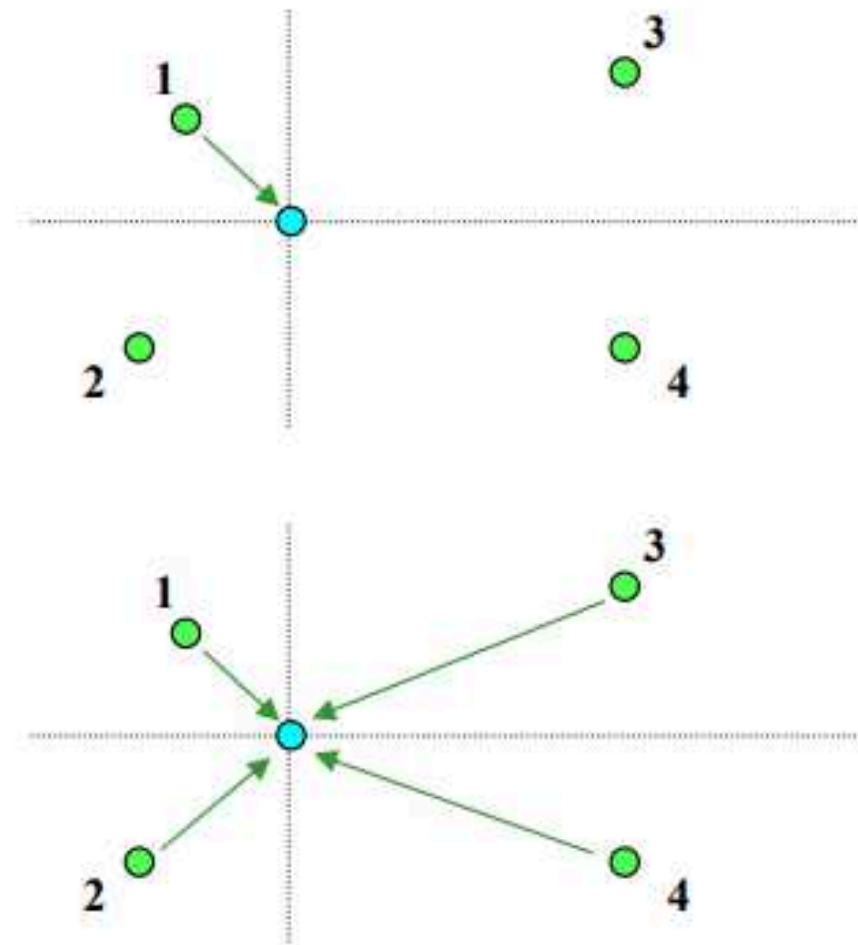
Planar Coordinate Systems

- Once we start working with **projected** spatial information, using latitude and longitude becomes **less convenient**
- We can instead use a **planar coordinate system** that has x and y axes, an arbitrary origin (a Cartesian plane), and some convenient units (e.g. ft. or m.)
- When applied in a geographic context:
 - **Eastings** are x values
 - **Northings** are y values



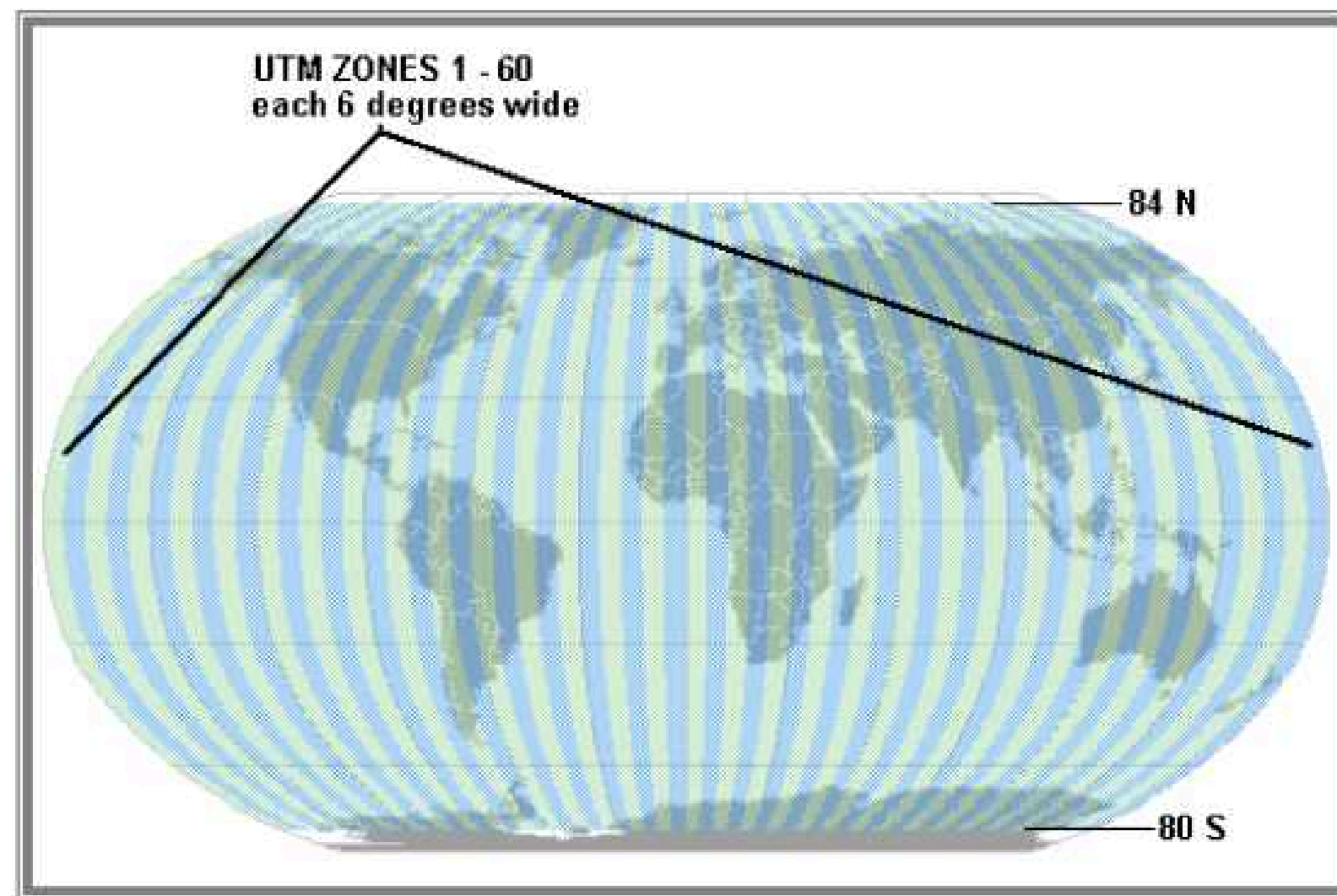
Geometric Correction

- Three Types of Resampling
 - **Nearest Neighbor** - assign the new BV from the closest input pixel. This method does not change any values.
 - **Bilinear Interpolation** - distance-weighted average of the BVs from the 4 closest input pixels
 - **Cubic Convolution** - fits a polynomial equation to interpolate a “surface” based on the nearest 16 input pixels; new BV taken from surface



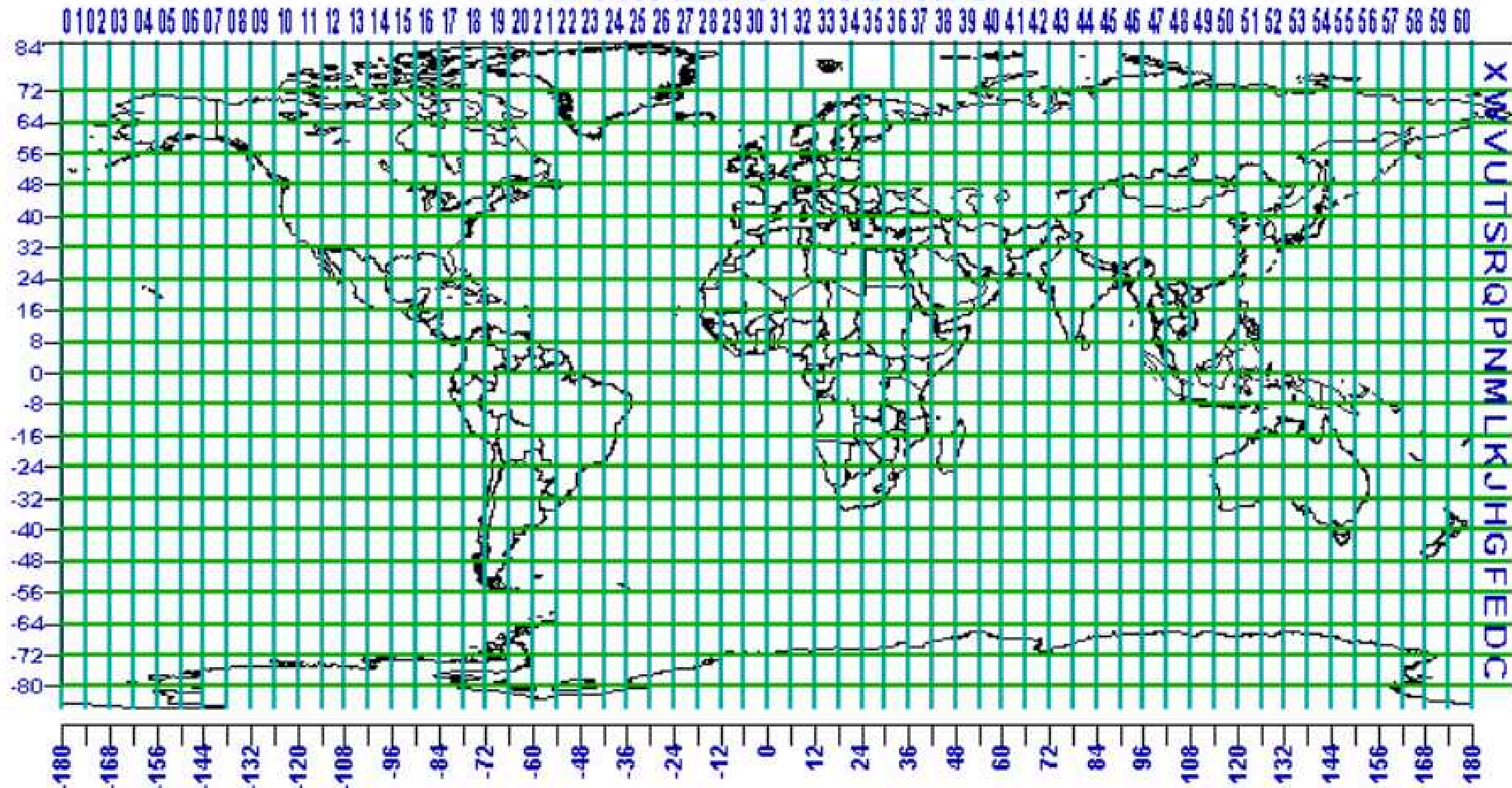
Universal Transverse Mercator

- In order to **minimize** the **distortion** associated with the projection, the UTM coordinate system uses a **separate Transverse Mercator projection** for every **6 degrees** of longitude → the world is divided into **60 zones**, each 6 degrees of longitude in width, each with its own UTM projection:



Universal Transverse Mercator

UTM Zone Numbers



UTM Zone Designators

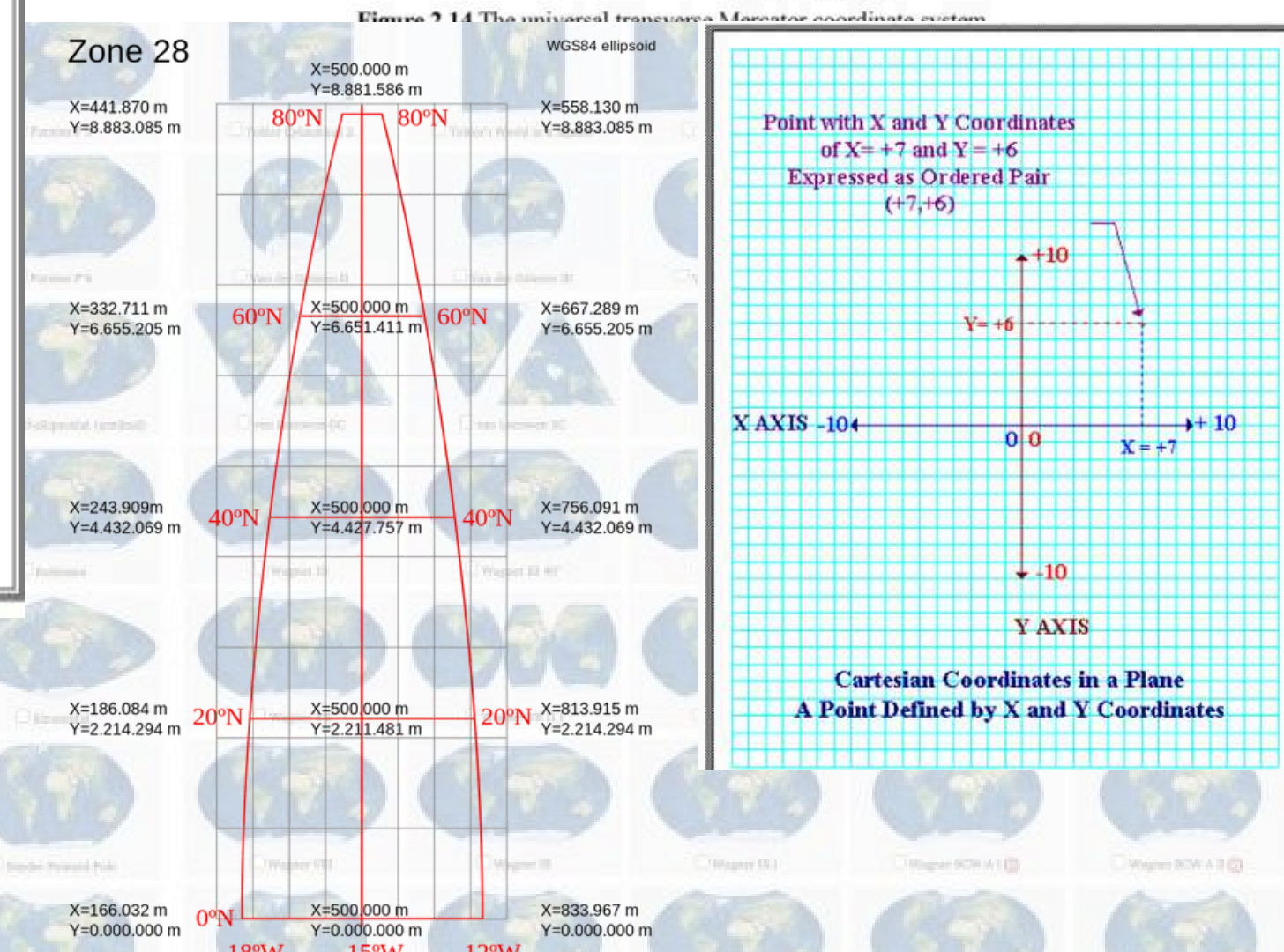
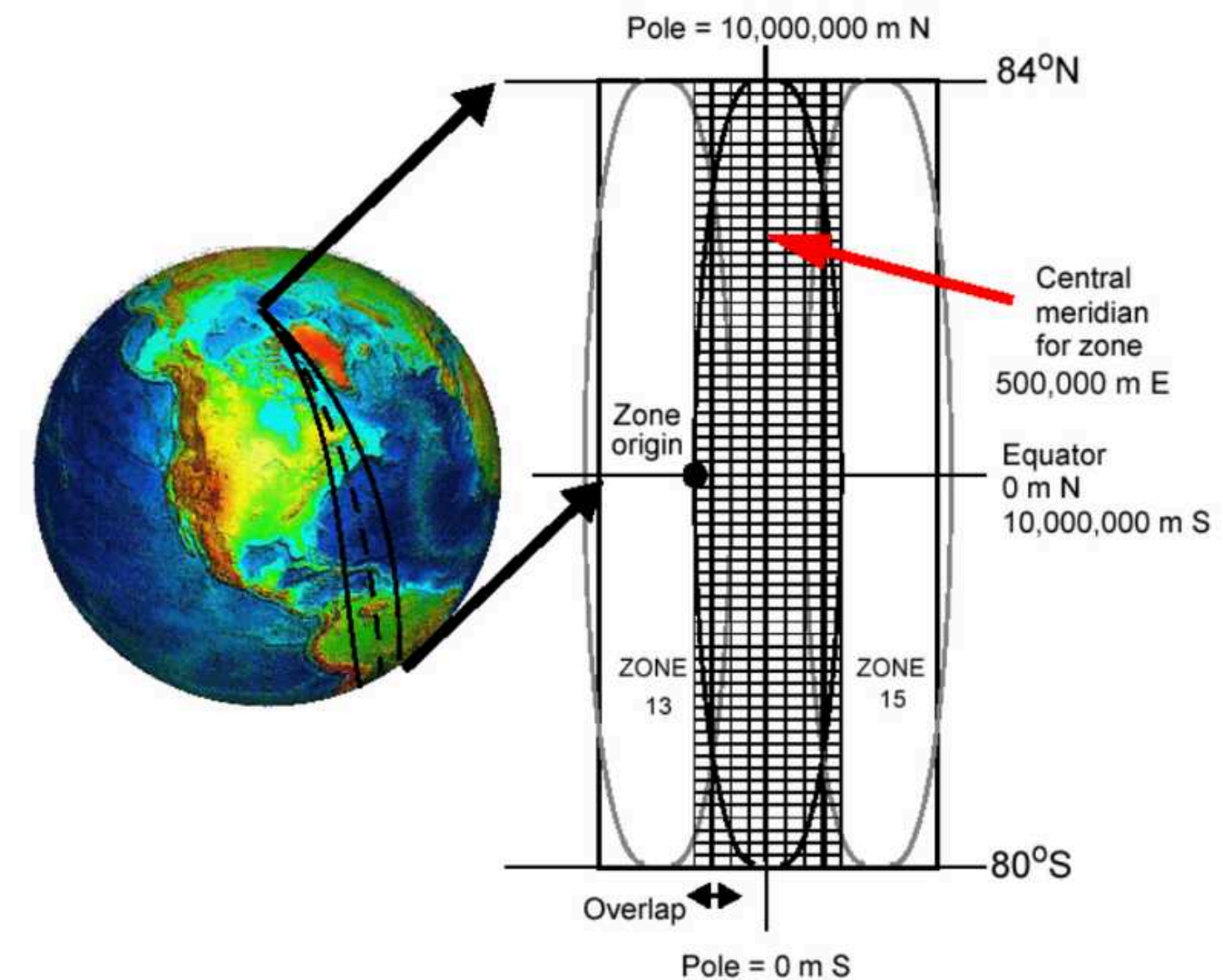
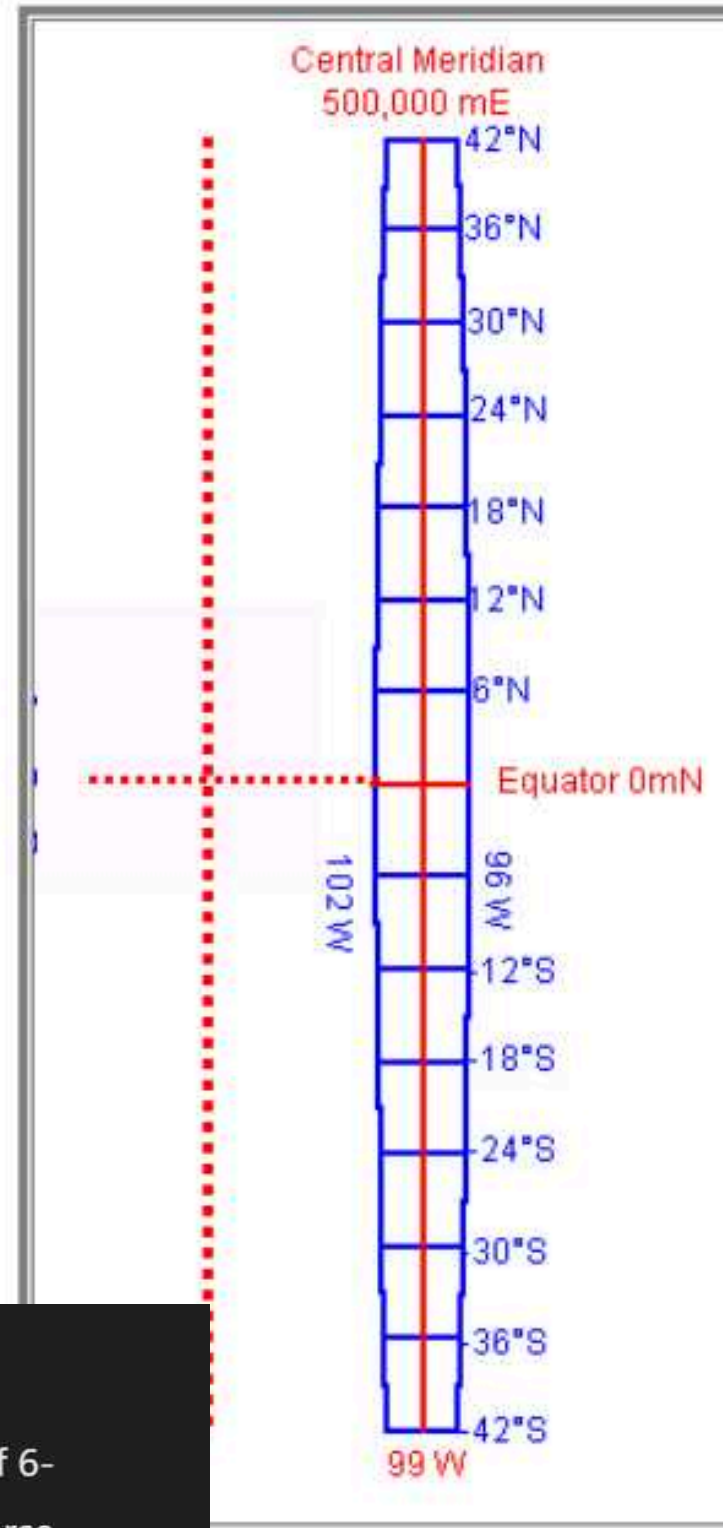
Universal Transverse Mercator (UTM) System

Peter H. Dana 9/7/94



Universal Transverse Mercator

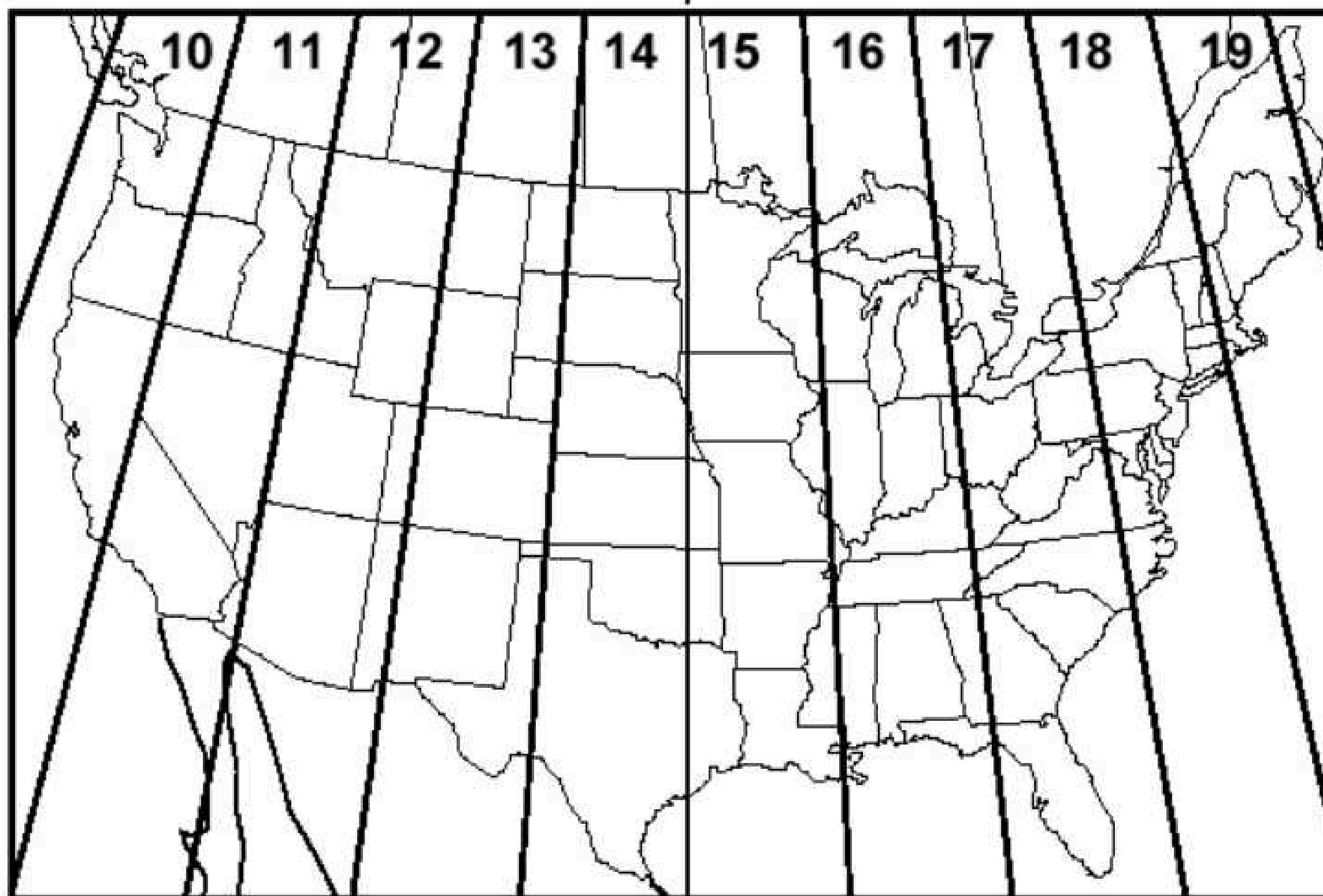
- The **central meridian**, which runs down the middle of the zone, is used to define the position of the origin
- **Distance units** in UTM are defined to be in **meters**, and distance from the origin is measured as an **Easting** (in the x-direction) and a **Northing** (in the y-direction)
- The x-origin is west of the zone (a false easting), and is placed such that the central meridian has an Easting of **500,000 meters**



4. Universal Transverse Mercator (UTM)

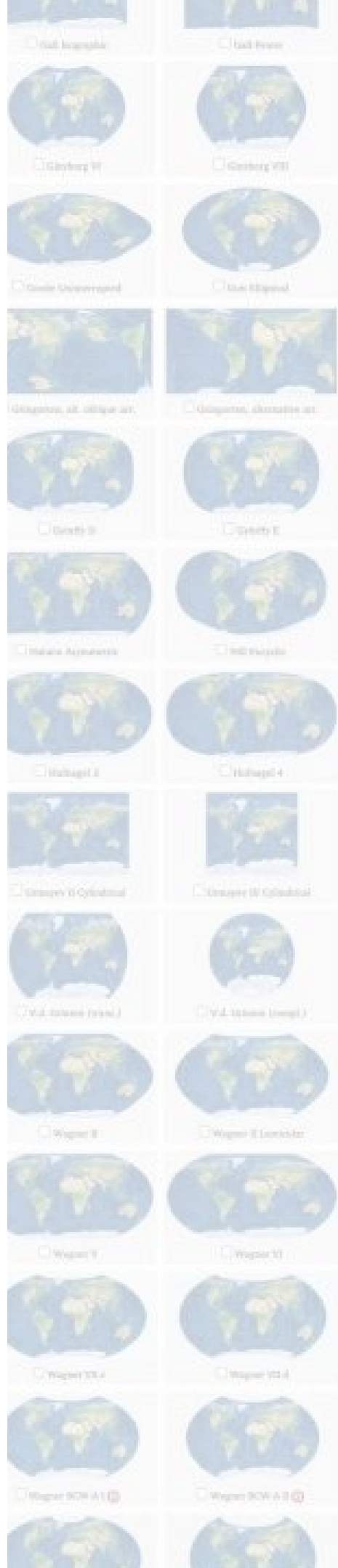
- **Overview:** UTM is a global map projection system that divides the world into a series of 6-degree longitudinal zones, each with its own central meridian. Each zone uses a transverse Mercator projection.
- **Curvature Handling:** The system reduces distortion within each zone by using a cylindrical projection around a meridian (north-south orientation), unlike the standard Mercator which projects around the equator. This localized approach greatly mitigates the error introduced by Earth's curvature in each zone, making it highly accurate for regional applications.

UTM Zones in the Lower 48



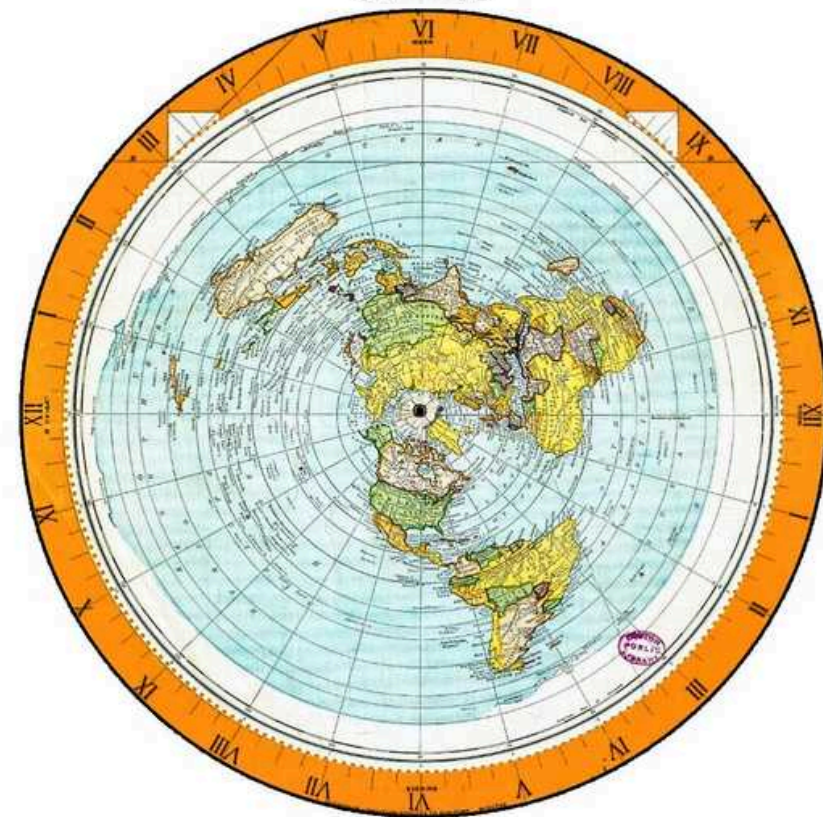
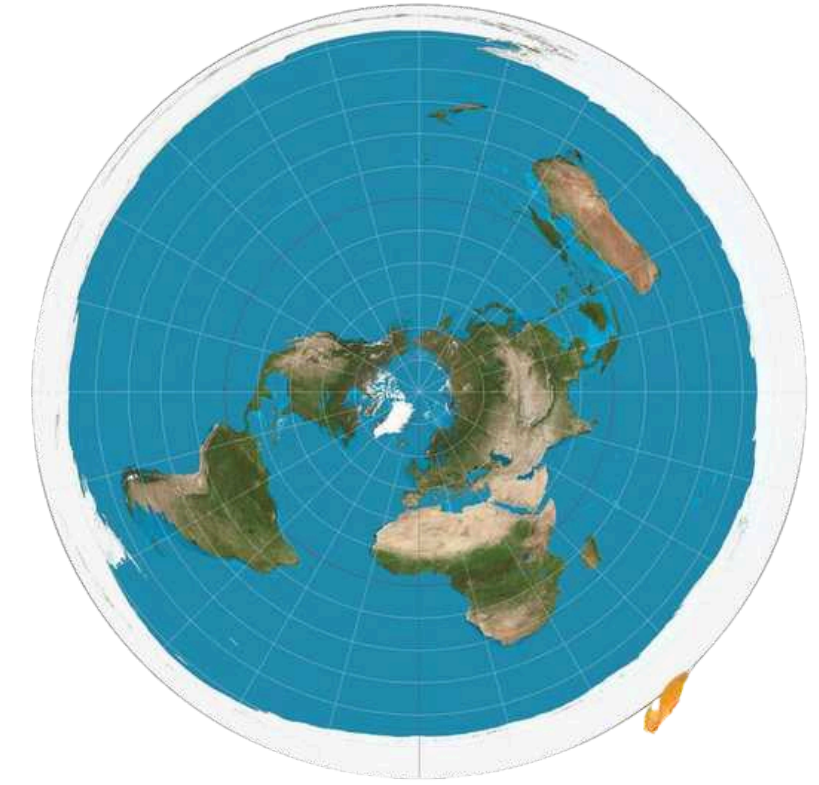
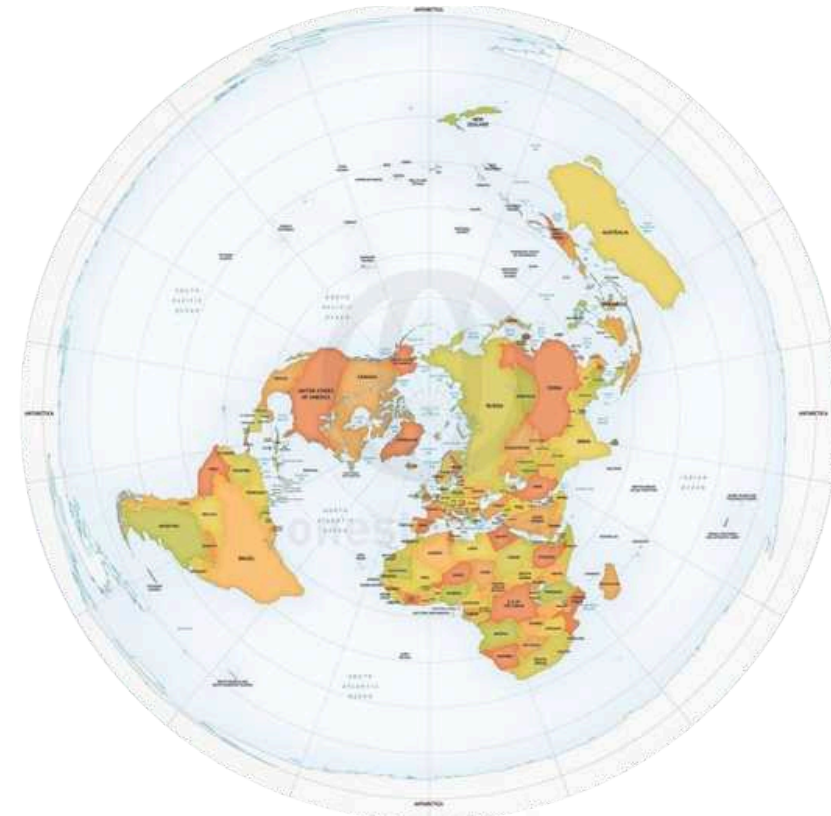
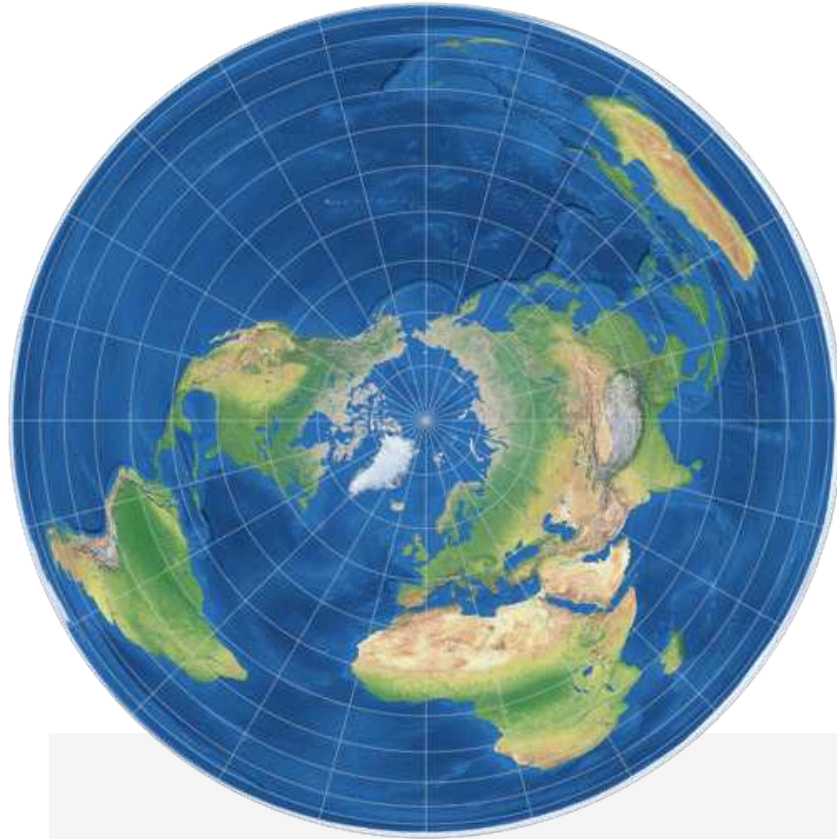
State Plane Coordinate Systems

- Each state in the U.S. has its own **planar coordinate system(s)** known as State Plane Coordinate Systems (SPCS)
 - Depending on the size of the state, its coordinate system **may be divided into multiple zones** (e.g. Alaska has 8 zones)
- These may **make use of three different projections, depending on the shape** of the state:
 - Lambert Conformal Conic
 - Transverse Mercator
 - Oblique Mercator



The A€ Map

AZIMUTHAL EQUIDSITANT PROJECTIONS



True-direction Projections

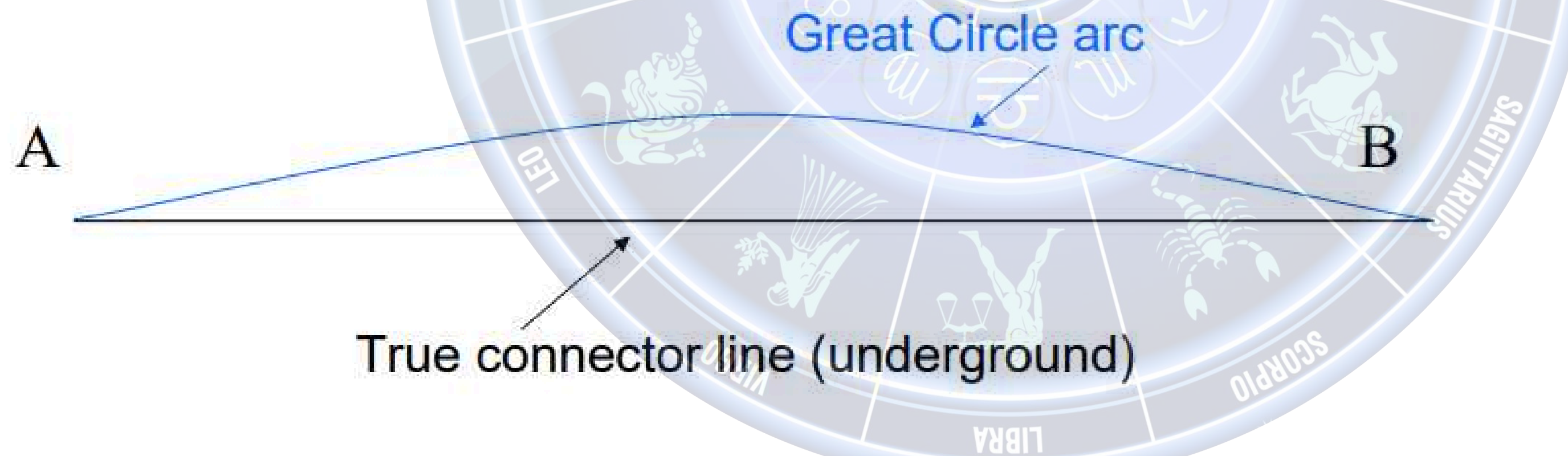
- ✦ **True-direction** or azimuthal projections preserve the direction of specified points on a great circle
 - The shortest route between two points on a curved surface such as the earth is along the spherical equivalent of a straight line on a flat surface - **called the great circle**
 - **Great circle arcs are *rectified*, or shown as straight lines**
 - Azimuths (angles from a point on a line to another point) are portrayed correctly in all directions

See <http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj.html> for a good list and examples of different projections



Measuring Distance

✦ The shortest distance between 2 points on a curved surface is the great circle arc above the true "connector line"



=ACOS(COS(RADIANS(90-C6)) *COS(RADIANS(90-C5)) +SIN(RADIANS(90-C6)) *SIN(RADIANS(90-C5)) *COS(RADIANS(D6-D5))) *3959

Place	Latitude	Longitude
Point 1	-13.928661	-155.078604
Point 2	15.000000	-155.078604
Miles		1,999
		1998.90061

Haversine Formula

$$S = 2r \sin^{-1} \sqrt{\sin^2 \left(\frac{\varphi_2 - \varphi_1}{2} \right) + \cos \varphi_1 \cos \varphi_2 \sin^2 \left(\frac{\gamma_2 - \gamma_1}{2} \right)}$$

Angle of Haversine Formula

$$\theta = 2 \sin^{-1} \sqrt{\sin^2 \left(\frac{\varphi_2 - \varphi_1}{2} \right) + \cos \varphi_1 \cos \varphi_2 \sin^2 \left(\frac{\gamma_2 - \gamma_1}{2} \right)}$$

Let's introduce you to the parameters of the Haversine Formula.
 φ_1 = Latitude of the first place
 φ_2 = Latitude of the second place
 γ_1^{**} = Longitude of the first place
 γ_2^{**} = Longitude of the second place

Now, I'll be showing you how to apply this formula in Excel step by step.

Steps:
 • First, make a cell to store the distance value and type the following formula in cell C8.
 =26400ASIN(SQRT((SIN(RADIANS((C6-C5)/2)))^2+COS(RADIANS(C5))COS(RADIANS(C6))
 (SIN(RADIANS((D6-D5)/2)))^2))

The formula uses ASIN, RADIANS, SQRT, SIN, and COS functions. It's pretty simple if you just look at the Haversine Formula. We measure the distance in kilometers, so we put the radius of the earth in kilometers which is 6400 km. ASIN refers to the inverse Sine or the ArcSine. If we compare the parameter angles of the Haversine Formula with our Excel formula, we get,

φ_1 = Latitude of Ohio (C5)
 φ_2 = Latitude of Alaska (C6)
 γ_1^{**} = Longitude of Ohio (D5)
 γ_2^{**} = Longitude of Alaska (D6)

• After that, press the ENTER button to see the distance between Ohio and Alaska in Kilometers.
 • Thereafter, if you want to measure the distance in miles, use the following formula in cell C8.

=2*3959*ASIN(SQRT((SIN(RADIANS((C6-C5)/2)))^2+COS(RADIANS(C5))*COS(RADIANS(C6))*
 (SIN(RADIANS((D6-D5)/2)))^2))

Convert the values into radians form and **COS** provides the cosine of the values, the cosines for latitude are multiplied then. The cosine value for the longitude difference between two locations.

As the diversion of longitudes from 90 in radians form and multiplied the sine values

The spherical law of cosines is used to calculate the distance between two points on the surface of a sphere, using their latitudes and longitudes. Here's how it's applied:

Formula:
 $d = r \cdot \arccos(\sin(\text{lat1}) \cdot \sin(\text{lat2}) + \cos(\text{lat1}) \cdot \cos(\text{lat2}) \cdot \cos(\text{lon2} - \text{lon1}))$

- Where:**
- d is the distance between the two points on the sphere's surface,
 - lat1 and lat2 are the latitudes of the two points in radians,
 - lon1 and lon2 are the longitudes of the two points in radians,
 - r is the radius of the Earth, approximately 6371 km.

Example:
 Calculating the distance between Madrid and Hong Kong using their coordinates:

- Madrid: 40.50°N, 3.67°W
- Hong Kong: 22.28°N, 114.17°E

AZIMUTHAL EQUIDISTANT Projection

Classifications

Azimuthal

Equidistant

Nonperspective

Graticule

Polar aspect (fig. 52B):

Meridians: Equally spaced straight lines intersecting at the central pole. Angles between them are the true angles.

Parallels: Equally spaced circles, centered at the pole, which is a point. The entire Earth can be shown, but the opposite pole is a bounding circle having a radius twice that of the Equator.

Symmetry: About any meridian

Equatorial aspect (fig. 52C):

Meridians: Central meridian is a straight line. Meridian 90° away is a circle. Other meridians are complex curves, equally spaced along the Equator and intersecting at each pole.

Parallels: Equator is a straight line. Other parallels are complex curves concave toward the nearest pole and equally spaced along the central meridian and the meridian 90° from the central meridian.

Symmetry: About the central meridian or the Equator

Oblique aspect (fig. 52D):

Meridians: Central meridian is a straight line. Other meridians are complex curves intersecting at each pole.

Parallels: Complex curves equally spaced along the central meridian

Symmetry: About the central meridian

Range

Entire Earth

Scale

True along any straight line radiating from the center of projection. Increases in a direction perpendicular to the radius as the distance from the center increases.

Distortion

Figure 52A shows distortion for the polar aspect. Other aspects have identical distortion at the same distance from the projection center. Only the center is free from distortion. Distortion is moderate for one hemisphere but becomes extreme for a map of the entire Earth.

Special features

The distance between any two points on a straight line passing through the center of projection is shown at true scale; this feature is especially useful if one point is the center.

Compromise in distortion between Stereographic (conformal) and Lambert Azimuthal Equal-Area projections

Usage

Commonly used in the polar aspect for maps of polar regions, the Northern and Southern Hemispheres, and the "aviation-age" Earth. The oblique aspect is frequently used for world maps centered on important cities and occasionally for maps of continents. The ellipsoidal form is used for topographic mapping of Micronesia and Guam.

Origin

Possibly developed in the polar aspect by Egyptians for star charts. Several users during the 16th century.

Other names

Postel (in France and Russia, for Guillaume

Postel, who was considered an originator, although he first used it in 1581)

Zenithal Equidistant

Similar projections

Two-Point **Azimuthal** (p. 144) shows correct azimuths (but not distances) from either of two points to any other point.

Two-Point Equidistant (p. 146) shows correct distances (but not azimuths) from either of two points to any other point.

Chamberlin Trimetric (p. 170) shows approximately true distances from three chosen points to any other points (cannot be exact). The three points are placed near the edges of the region being mapped to reduce overall distortion.

Airy (p. 140) and Breusing (p. 143) azimuthal projections have spacings very similar to those of the Azimuthal Equidistant if the extent is less than one hemisphere.

Berghaus Star projection (p. 156) uses the Polar Azimuthal Equidistant projection for the Northern Hemisphere.

"Tetrahedral" projection (p. 114) combines the Polar Azimuthal Equidistant projection with an interrupted Werner projection.

On the face of the map proper, and within another circle (still toward the center) is laid out the continents, principal islands, rivers and cities of the world; their latitudes and longitudes corresponding to the latitudes and longitudes of all other first class geographical globe maps or charts of the world.

On the face of the map are circular lines from the center or north pole to ninety degrees south representing the latitudes of the earth, both north and south of the equator. These circular lines are indicated by the letter, G'.

In operating with this map I employ two indicating arms G and H, pivoted together by means of a pin, a, having a flange, l, (see Fig. 3), the two arms G and H being put on said pin above the flange, b, then a light spring washer H, in the top of the arm H and the head of the pin riveted so that the two arms are held together by friction and can be turned on each other back and forth.

In the center of the map is an eyelet, c, and into the opening (through the eyelet, c), is put the lower end, d, of the pin, a, so that these indicating arms have two movements, a movement one on the other and one or both together around the center of the map, and may be detached at pleasure from the map if so desired, and are also made easily removable by simply lifting the pin, d, out of the eyelet c.

On these indicating arms are numerals, J, indicating degrees of latitude corresponding to the degrees of latitude as represented and marked on the map at B at thirty degrees west of Greenwich. By bringing either of the indicator arms to any given point, the latitude and longitude of the said point may at once be determined without future computation.

In order to ascertain the time of day or night, in any part of the world, corresponding to your own meridian time; first: place the lower indicating arm, G', into the center socket 0 or receptacle, letting the graduate edge of the arm be in line on your own meridian time, for instance if it be New York, which is the fifteenth meridian: Now you wish London's corresponding time? Place the arm H, on 5 the meridian of

In order to ascertain the time of day or night, in any part of the world, corresponding to your own meridian time; first: place the lower indicating arm, G', into the center socket 0 or receptacle, letting the graduate edge of the arm be in line on your own meridian time, for instance if it be New York, which is the fifteenth meridian: Now you wish London's corresponding time? Place the arm H, on 5 the meridian of Greenwich, which is London, and marked, F, at the same time holding the arm G in its place. You have now got the absolute corresponding difference of time between New York and London, which is five [00 hours in round numbers. Next look at your own pocket time or clock, and if it be just eleven o'clock; move the arm G to eleven and the arm H, will still retain its relative position to arm G, (as the two arms are held to each other by friction) and indicate six p. m. or the corresponding fractional parts of an hour be it more or less. Thus the time stands all ready computed to any child who is able to read the time of day from the face of an ordinary clock. Again, in order to give the child the most simple lesson first I would get the difference of the time between the two places as above mentioned, then placing the arm G at twelve, of course the arm H will stand at five p. m. for London, and there is no computation or counting for the child to make; he thus reads the hour and fractional part thereof from the dial of the map. The utility of such a computing map will be obvious, not only to the school child but for an adult or official person. The map is not so extorted as to lose the relative latitude and longitude of any places on the land or sea, but retains all latitudes and longitudes of places agreeing with other recognized authors; and as the proper relations of continents and countries all stand in their relative position to each other, they are thus impressed upon the mind of the student. The extorsion of the map from that of a globe consists, mainly in the straightening out of the meridian lines allowing each to retain their original value from Greenwich, the equator to the two poles.

LONGITUDE AND TIME CALCULATOR.

.....

FIRST.—Let it be borne in mind that the time dial or disk contains twenty-four hours, or the twenty-four hourly meridians.

SECOND.—The dial corresponds to the ordinary watch or clock dial, and between the Roman numerals representing time of day and time of night, there are sixty divisions or minutes to the hour marked on the periphery of the circle.

THIRD.—Longitude and time are reckoned from **Greenwich Meridian** marked “Noon XII,” or the cycle of the twenty-four hours.

FOURTH.—Let the student remember that there are 15° to the hour, and every fourth division on the dial or fourth minute sun time represents 1° of longitude; therefore, if four minutes of time represent a degree, one minute of time represents $15'$ arc or longitude.

FFTH.—In order to procure the **corresponding** time of day or night in any part of the world, first find the meridian your time piece is regulated to, next place one of the index arms on that meridian, the other on the place which you desire the corresponding time; you now have the difference of time between the two places in question; next, with your finger on your own meridian pointer, carry it to the time of day or night that your own watch indicates, and the pointer will stand at the corresponding time of day at the place desired. Thus it will be seen that as soon as a place can be located on the map, so soon the latitude, longitude and time of day or night can be ascertained.

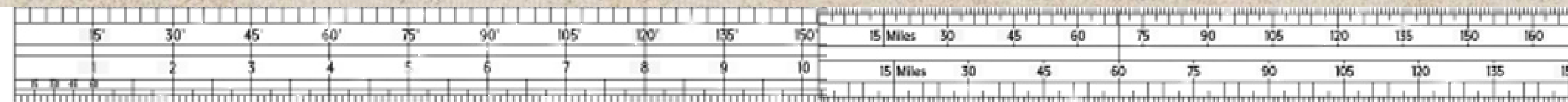
ALTITUDE OF THE SUN.

SIXTH.—The sun is always seen at an angle of 45° arc of the heavens; just 45° either latitude or longitude from his **daily path**. On the 21st and 22d of March it is vertical to the inhabitants of the Equator, and at an angle of 45° arc to the inhabitants of 45° north and 45° south, simultaneously at 12 o'clock noon. We will next notice the diagram at the top of the map, but first, will bear in mind: there is no method known by which an angle of 45° can be produced from the **four quarters** or cardinal points of a sphere or circle, without placing the sun in just the relative distance from the **earth** or **circle** that the above geometrical diagram places it; therefore,

SEVENTH.—If we take the two arms on the map and turn them, the two simultaneously, one to "A IX," the other to "A III," we will have the inhabitants at 45° north and 45° south, also the people on the Equator (O—See under "Morn." or "90") beholding each their sun, thousands of miles apart, on **parallel lines** which never converge, and never meet the required conditions, namely—all must see it at "E," and not at "S" for the south, "E" for the Equator and "N" for the north.

EIGHTH.—Now if we take the line "B O B" and its intersections at "A A" and view from the point of intersection, the conditions required will be met, and under no others can it be. The demonstration has reference to either considerations: the earth a globe or a plane—take your choice.

AUTHOR.

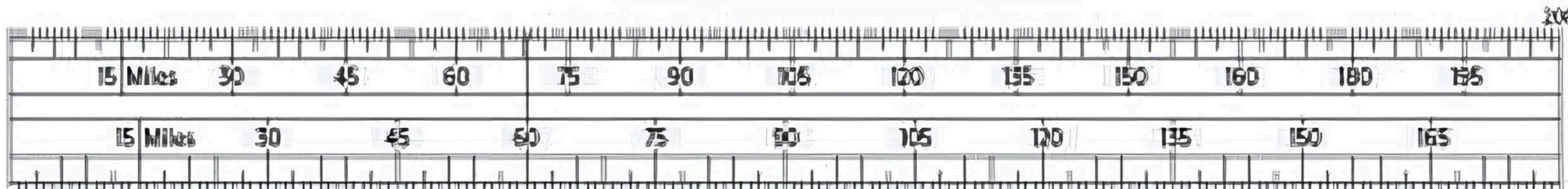


Minutes and Degrees of Longitude Corresponding to Minutes of Sun Time

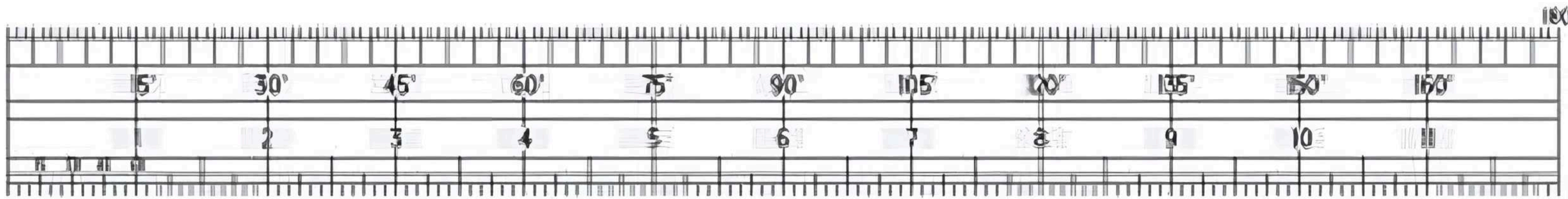
Miles Corresponding to Nautical Miles; 60 Miles to the Degree

DESCRIPTIVE KEY

New Standard Map of the World



Miles Corresponding to Nautical Miles; 60 Miles to the Degree



Minutes and Degrees of Longitude Corresponding to Minutes of Sun Time



Does the Azimuthal Equidistant polar projection display the same distances as all the other projections of the geographic coordinate system?

YES

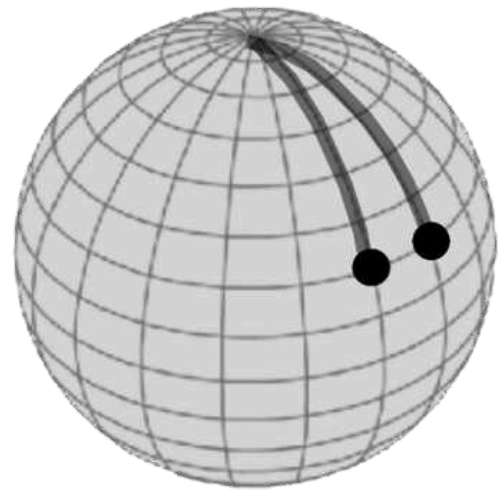
The Azimuthal Equidistant projection preserves true distances from the central point to any other point on the map, and the great-circle distance formulas using latitude and longitude apply universally across all map projections, including the Azimuthal Equidistant projection.

Therefore, it displays the same accurate distances as other projections of the geographic coordinate system.

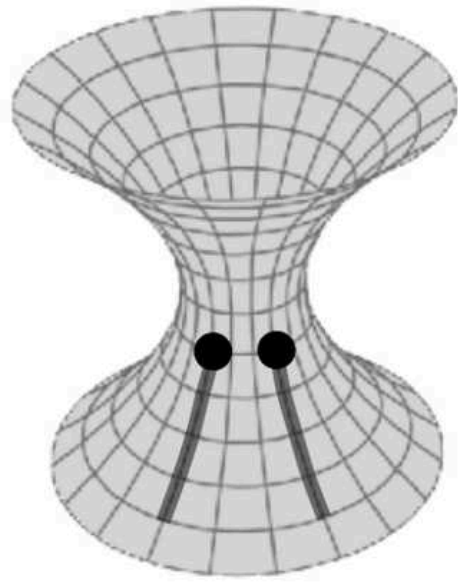
The Haversine formula and the spherical law of cosines are standard methods used to calculate great-circle distances between two points on the Earth's surface using their latitude and longitude. These formulas provide accurate distance calculations.

THE CURVATURE

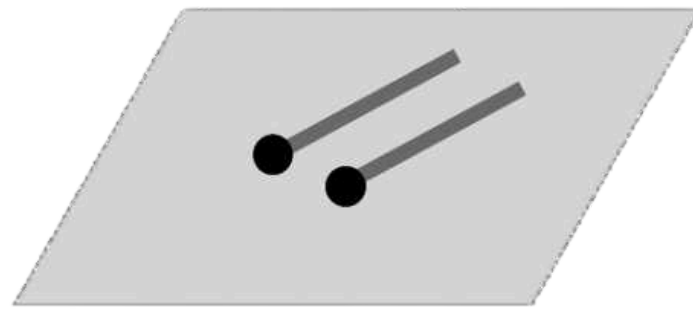
IF IT AINT THE EARTH, WHAT IS CURVED?



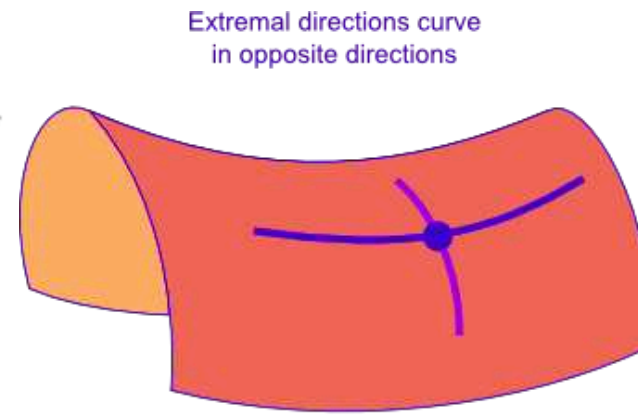
Spherical (>0)



Hyperbolic (<0)

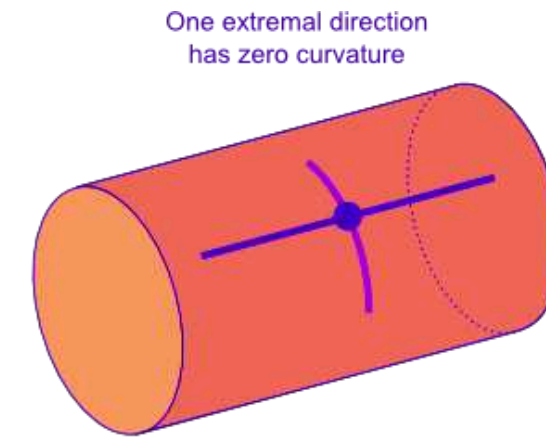


Euclidean ($=0$)



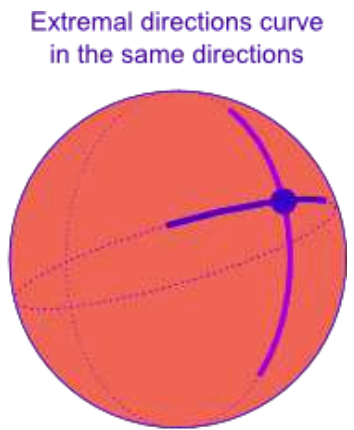
Extremal directions curve in opposite directions

Negative Curvature



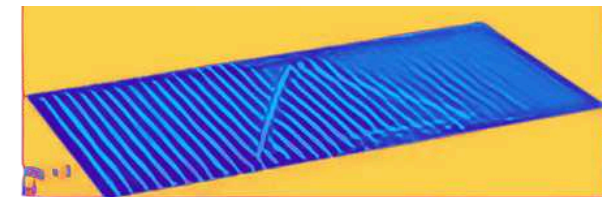
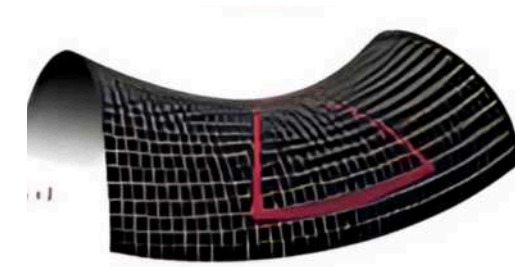
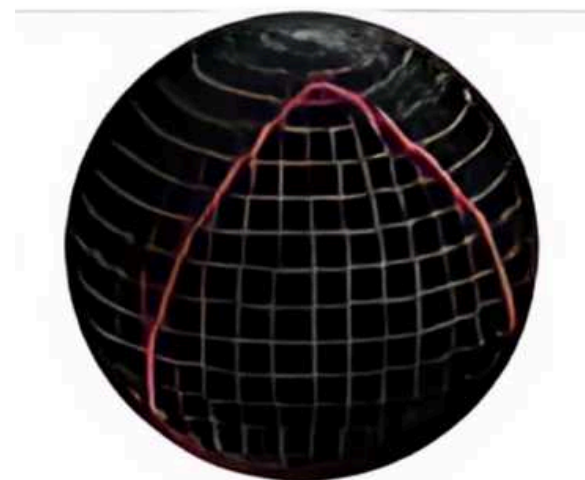
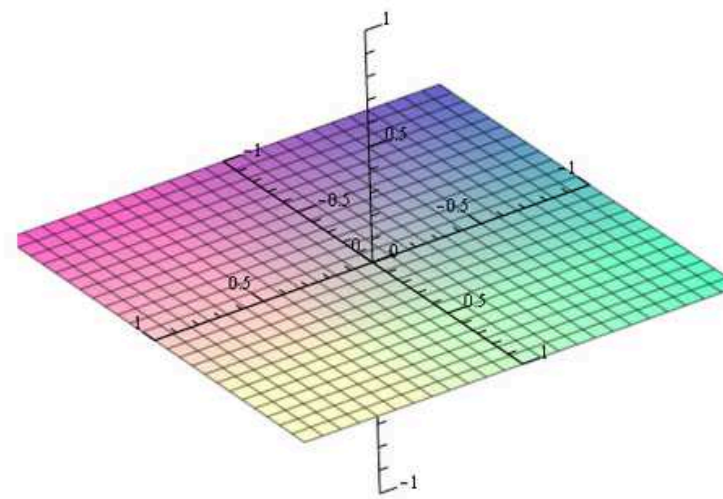
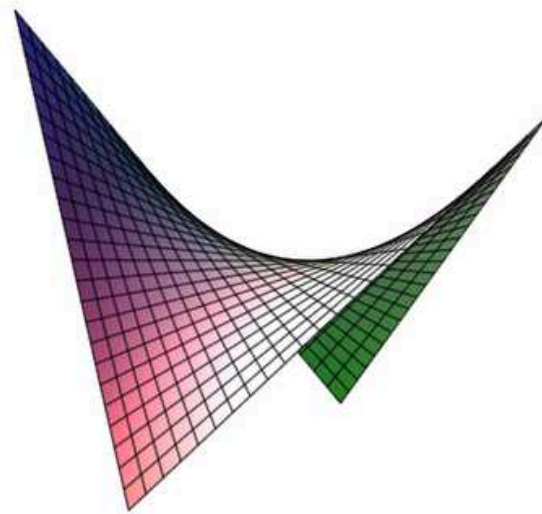
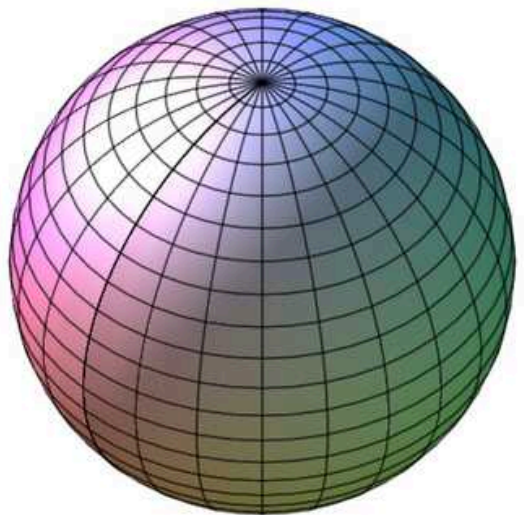
One extremal direction has zero curvature

Zero Curvature



Extremal directions curve in the same directions

Positive Curvature



COORDINATE SYSTEMS

BASED ON ANGLES TO THE STARS

Horizontal Coordinate System (**Altitude-Azimuth**):

- **Coordinates:** Altitude, Azimuth

Equatorial Coordinate System (**Right Ascension - Declination**):

- **Coordinates:** Right Ascension, Declination

Geographic Coordinate System

- **longitude/latitude**

Geocentric Coordinate Systems

Cartesian coordinates (X, Y, Z) or spherical coordinates (radius, latitude, longitude).

- **X:** Distance from the center of the Earth to the point in the plane of the equator, along the prime meridian.
- **Y:** Distance from the center of the Earth to the point in the plane of the equator, 90 degrees east of the prime meridian.
- **Z:** Distance from the center of the Earth to the point along the axis of rotation (positive towards the North Pole).

Ecliptic Coordinate System:

- **Coordinates:** Ecliptic Longitude, Ecliptic Latitude

Galactic Coordinate System:

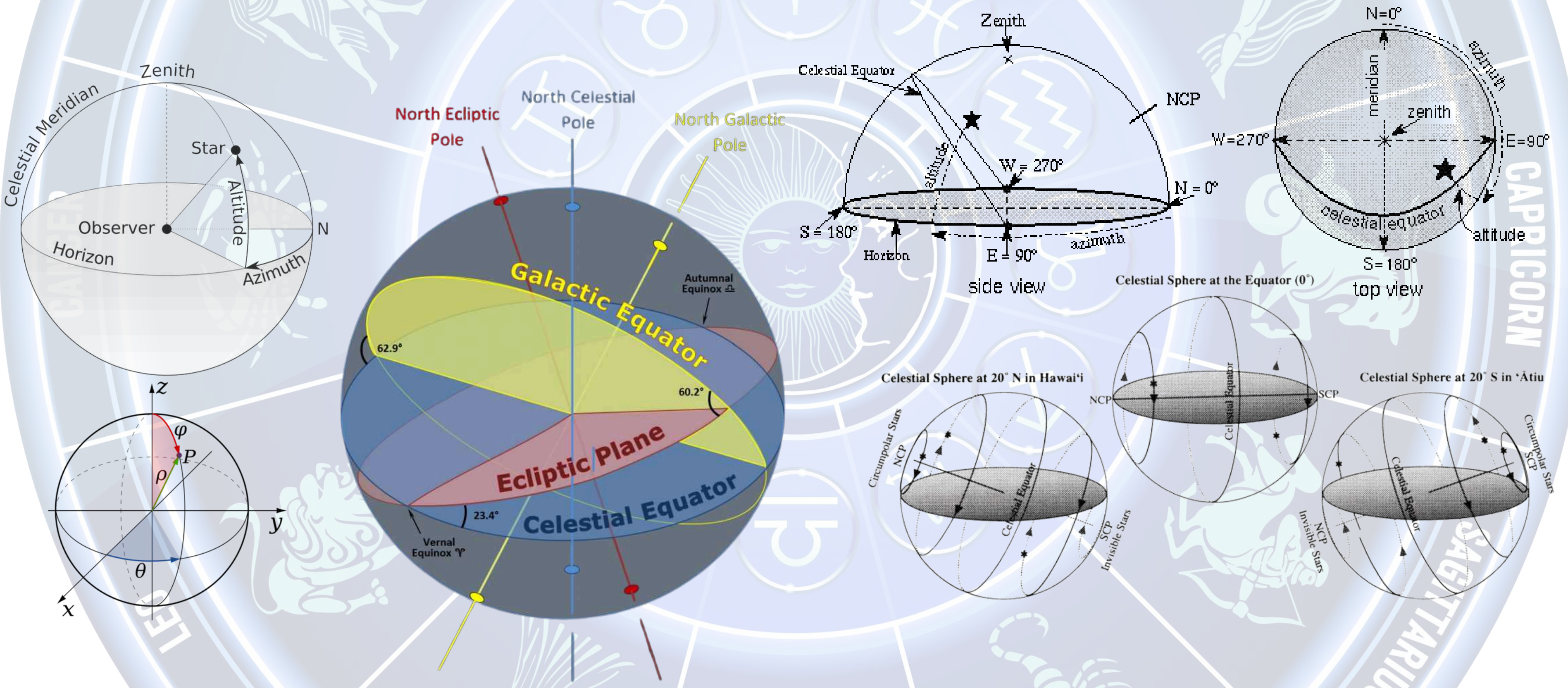
- **Coordinates:** Galactic Longitude, Galactic Latitude

Supergalactic Coordinate System:

- **Coordinates:** Supergalactic Longitude, Supergalactic Latitude

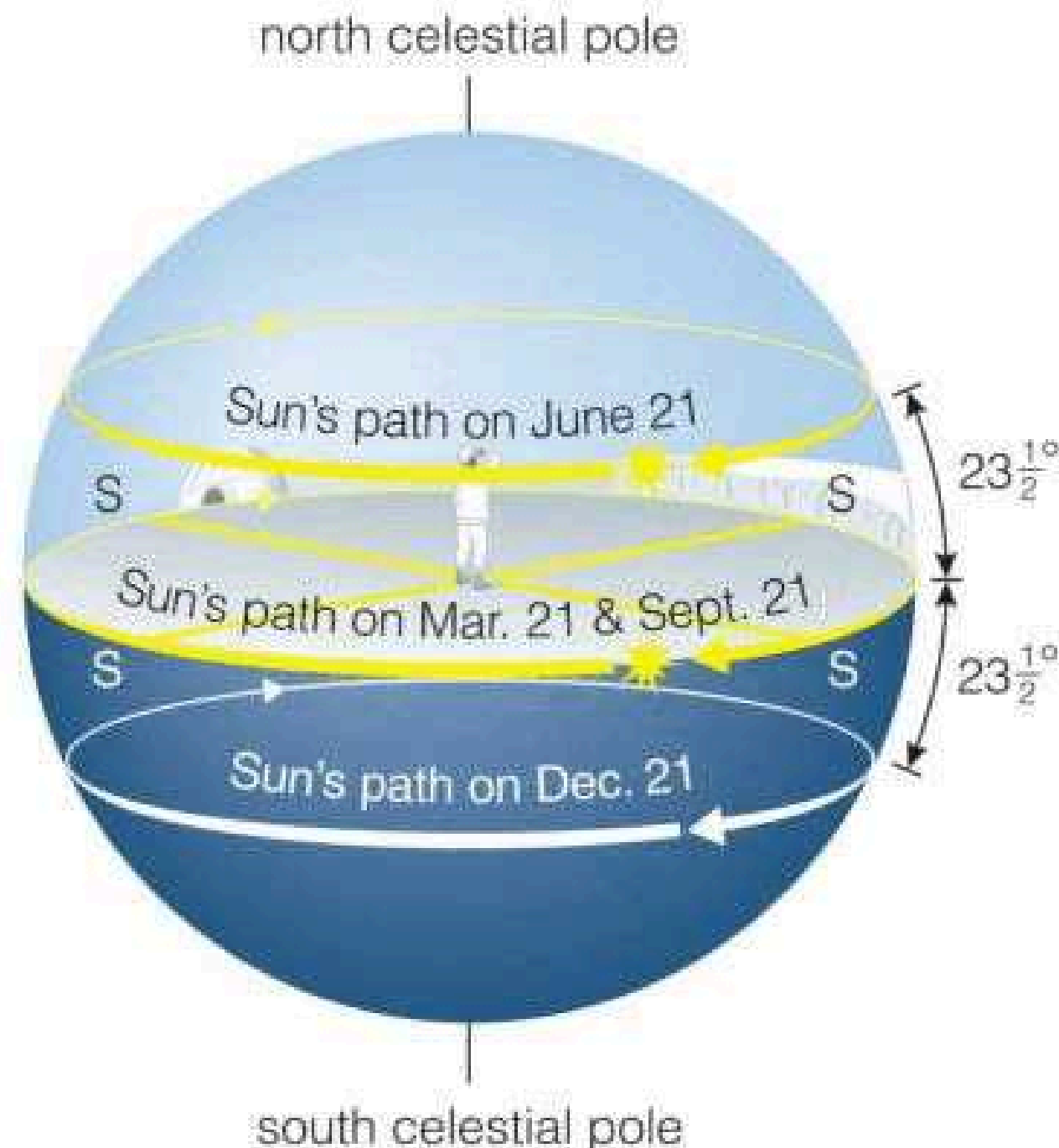
Coordinate Systems

BASED ON ANGLES TO THE STARS



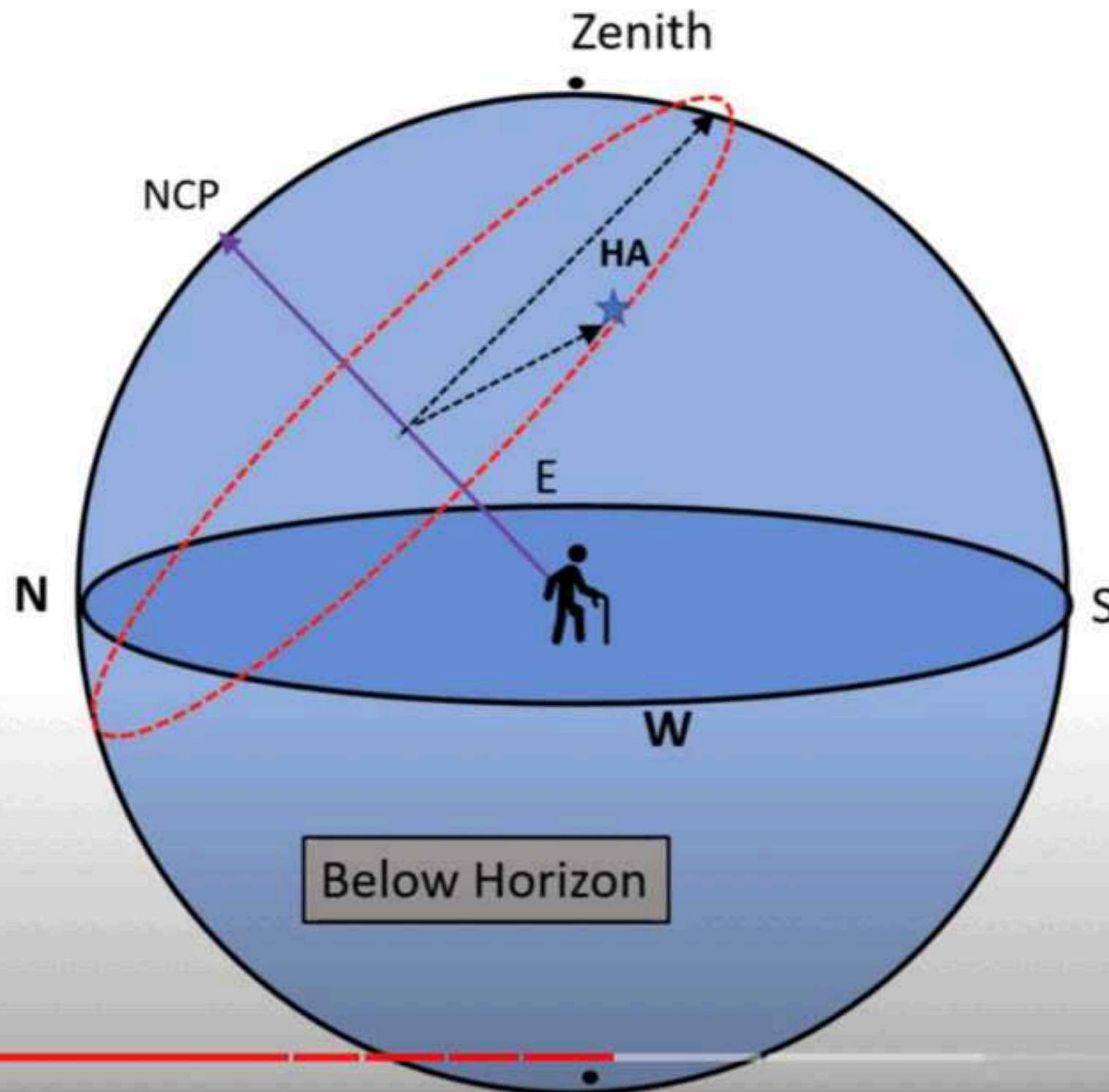
CELESTIAL COORDINATES

Sun's Path at North Pole



- Sun remains above horizon from spring equinox to fall equinox
- Altitude barely changes during a day

HOW IS THE CELESTIAL SPHERE REPRESENTED?



Hour angle (HA) of an object

Defined as:

The lapsed sidereal time since the object reached its highest elevation as crossed the meridian.

Can also be expressed in degrees by multiplying the value by 15 (24 hrs = 360°). E.g.

HA of 2 hrs 20 mins

HA (in degrees) is $2^{20}/_{60} \times 15 = 35^\circ$

When it crosses the meridian the HA of an object is 0 hrs

As the Earth rotates, the HA of an object *increases*



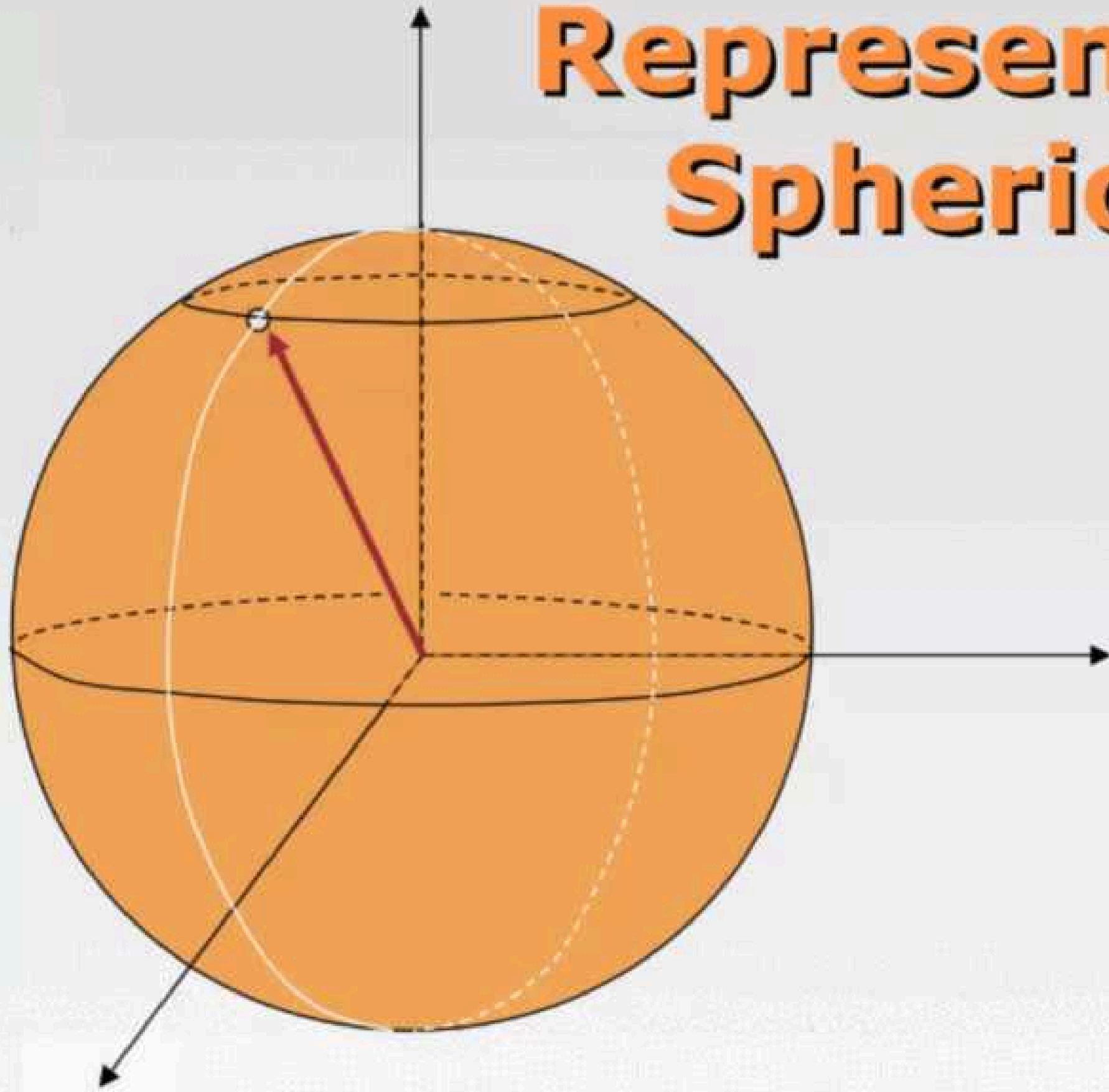
HOW IS THE CELESTIAL SPHERE REPRESENTED?

GLOBE E

.....



Representing 3D points in Spherical Coordinates



We use a method similar to the method used to measure *latitude* and *longitude* on the surface of the Earth.

Next, we draw a horizontal circle on the sphere that passes through the point.

COORDINATE SYSTEMS

ALTITUDE AZIMUTH / HORIZON

Alt-Azimuth Coordinate System

1. Horizontal Coordinate System (Altitude and Azimuth)

- **Based on Celestial Observations:** Yes.
- **Details:** Used primarily in astronomy, this system measures the altitude of celestial bodies above the horizon and their azimuth, which is the angular distance measured along the horizon from the north.

The Altitude-Azimuth coordinate system is the most familiar to the general public. The origin of this coordinate system is the observer and it is rarely shifted to any other point. The fundamental plane of the system contains the observer and the horizon. While the horizon is an intuitively obvious concept, a rigorous definition is needed as the apparent horizon is rarely coincident with the location of the true horizon. To define it, one must first define the zenith. This is the point directly over the observer's head, but is more carefully defined as the extension of the local gravity vector outward through the celestial sphere. This point is known as the astronomical zenith. Except for the oblateness of the earth, this zenith is usually close to the extension of the local radius vector from the center of the earth through the observer to the celestial sphere. The presence of large masses nearby (such as a mountain) could cause the local gravity vector to depart even further from the local radius vector. The horizon is then that line on the celestial sphere which is everywhere 90° from the zenith. The altitude of an object is the angular distance of an object above or below the horizon measured along a great circle passing through the object and the zenith. The azimuthal angle of this coordinate system is then just the azimuth of the object. The only problem here arises from the location of the zero point. Many older books on astronomy will tell you that the azimuth is measured westward from the south point of the horizon. However,

Altitude-Azimuth coordinate system

Based on what an observer sees in the sky.

Zenith = point directly above the observer (90°)

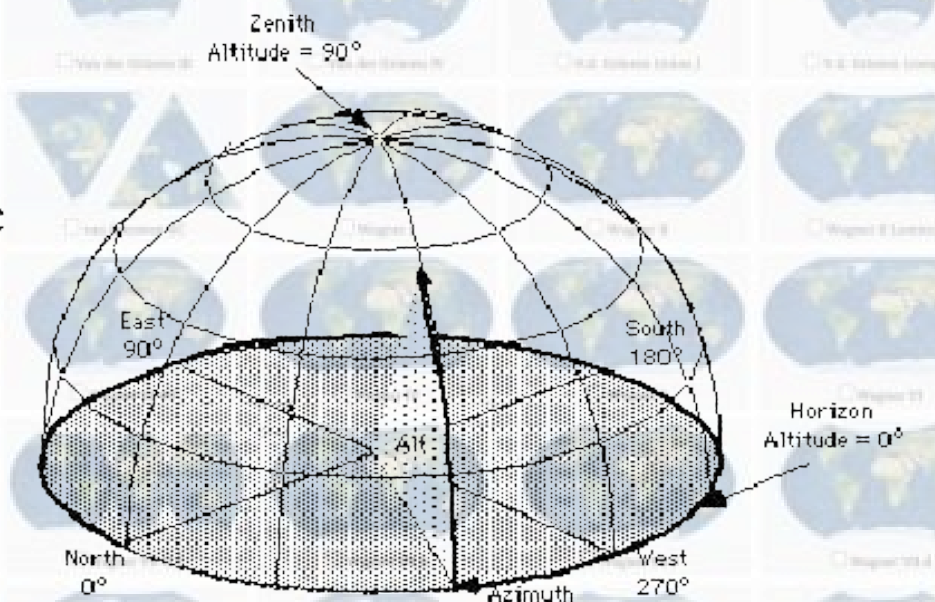
Nadir = point directly below the observer (-90°) – can't be seen

Horizon = plane (0°)

Altitude = angle above the horizon to an object (star, sun, etc)
(range = 0° to 90°)

Azimuth = angle from true north (clockwise) to the perpendicular arc from star to horizon
(range = 0° to 360°)

Note: lines of azimuth converge at zenith

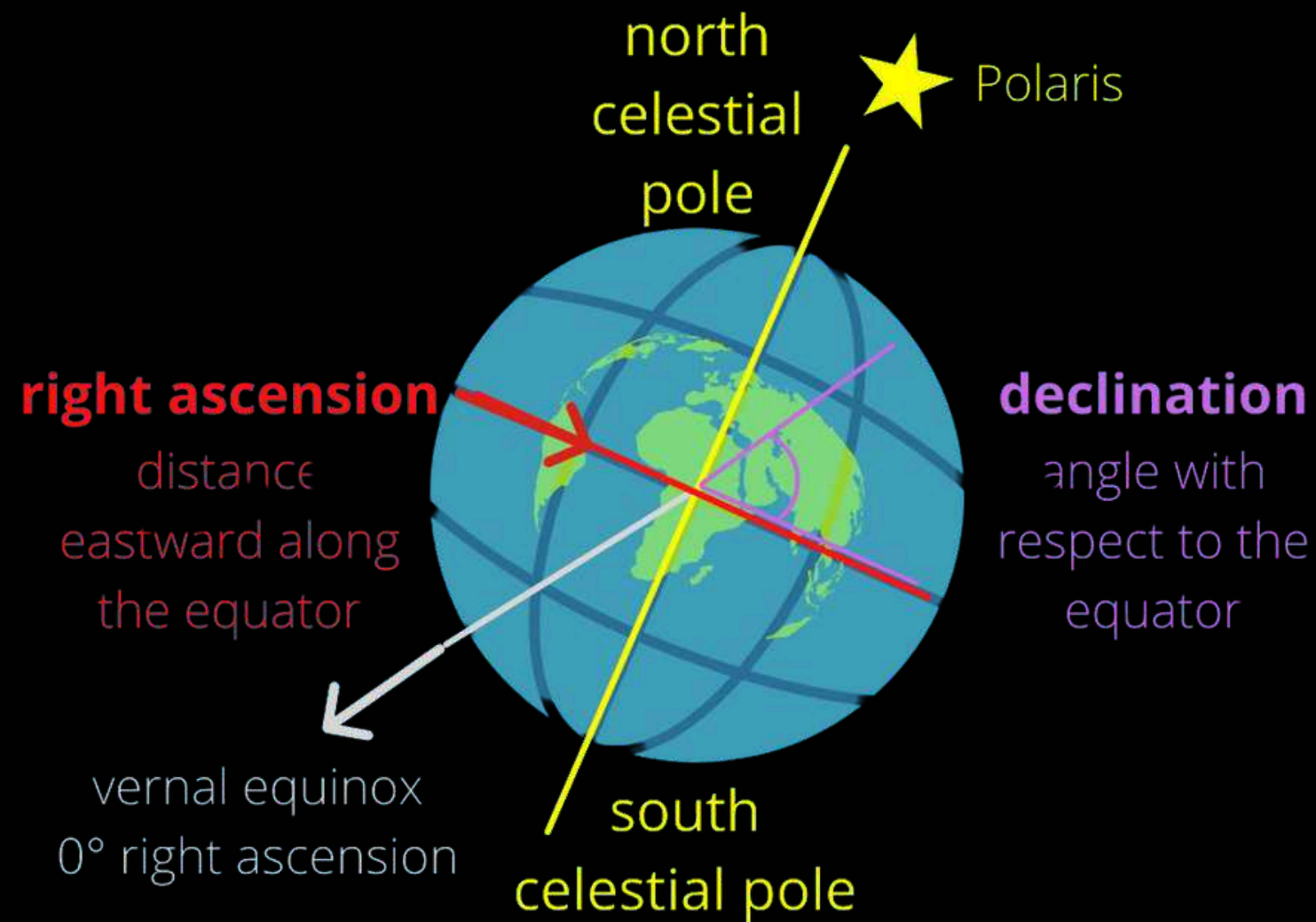


COORDINATE SYSTEMS

RIGHT ASCENSION AND DECLINATION

Right Ascension and Declination

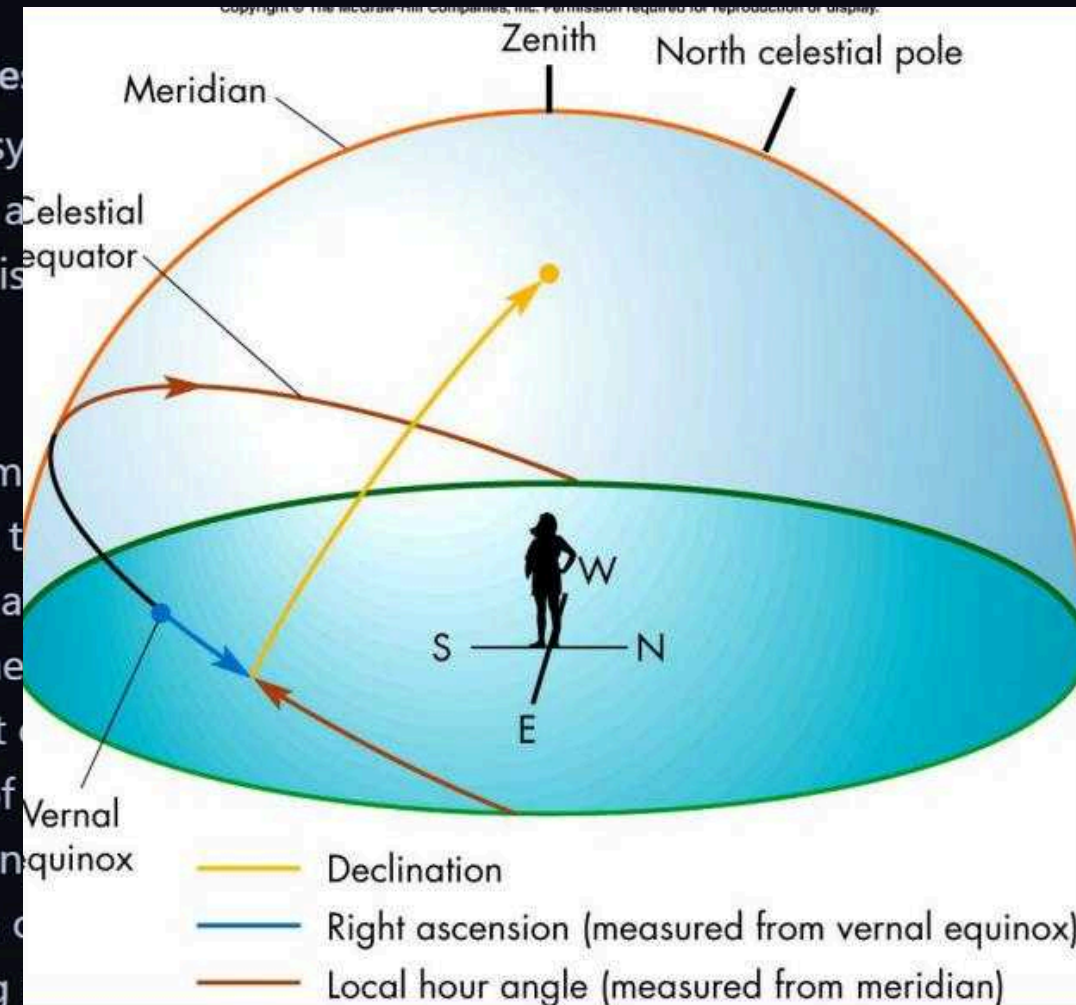
Right ascension and declination are the celestial equivalents of longitude and latitude, respectively.



The Right Ascension - Declination Coordinate System

1. Equatorial Coordinate System (Right Ascension and Declination)

- Based on Celestial
- Details: This system is used for celestial bodies. Right ascension is analogous to longitude. Declination is analogous to latitude. It is based on the celestial equator.



This coordinate system is based on the celestial equator. Right ascension is measured eastward along the celestial equator from the vernal equinox. Declination is measured north or south from the celestial equator. The origin of the system is the vernal equinox.

Right Ascension of rising or ascending stars increases with time. There is a tendency for some to face south and think that the angle should increase to their right as if they were looking at a map. This is exactly the reverse of the true situation and the notion so confused air force navigators during the Second World War that the complementary angle, known as the sidereal hour angle, was invented. This angular coordinate is just 24 hours minus the Right Ascension.

COORDINATE SYSTEMS

GEOCENTRIC COORDINATE SYSTEM

Cartesian coordinates (X, Y, Z) or spherical coordinates (radius, latitude, longitude).

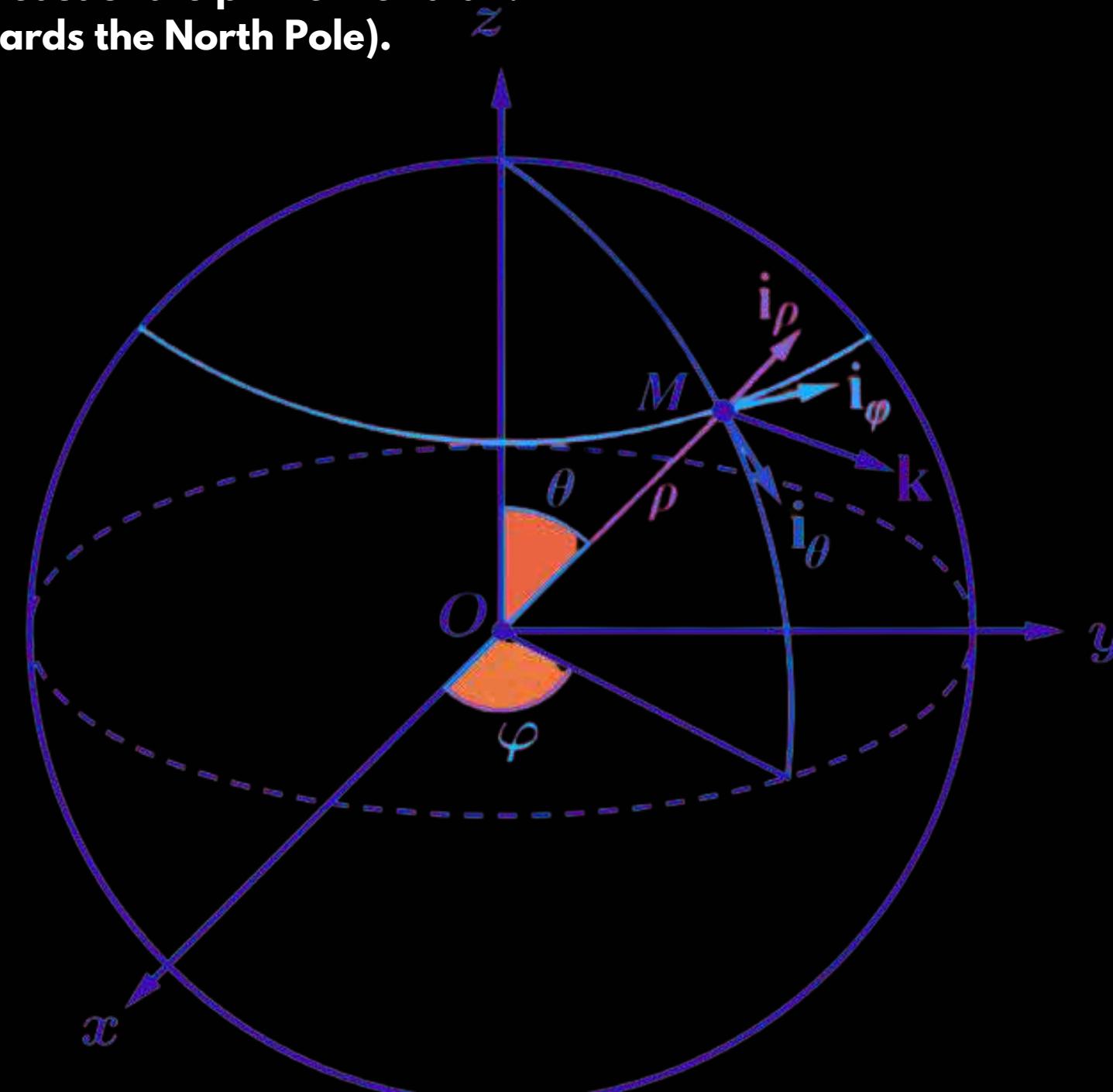
- **X:** Distance from the center of the Earth to the point in the plane of the equator, along the prime meridian.
- **Y:** Distance from the center of the Earth to the point in the plane of the equator, 90 degrees east of the prime meridian.
- **Z:** Distance from the center of the Earth to the point along the axis of rotation (positive towards the North Pole).

Spherical Coordinates

- **Radius (r):** Distance from the Earth's center to the point.
- **Latitude (φ):** Angle between the point and the equatorial plane.
- **Longitude (λ):** Angle from the prime meridian to the point's projection onto the equatorial plane.

c. The Geocentric Coordinate System

Consider the oblate spheroid that best fits the actual figure of the earth. Now consider a radius vector from the center to an arbitrary point on the surface of that spheroid. In general, that radius vector will not be normal to the surface of the oblate spheroid (except at the poles and the equator) so that it will define a different local vertical. This in turn can be used to define a different latitude from either the astronomical or geodetic latitude. For the earth, the maximum difference between the geocentric and geodetic latitudes occurs at about 45° latitude and amounts to about (11' 33"). While this may not seem like much, it amounts to about eleven and a half nautical miles (13.3 miles or 21.4 km.) on the surface of the earth. Thus, if you really want to know where you are you must be careful which coordinate system you are using. Again the geocentric longitude is defined in the same manner as the geodetic longitude, namely it is the angle between the local meridian and the meridian at Greenwich.



COORDINATE SYSTEMS

GEOGRAPHIC (LONGITUDE/LATITUDE)

The concepts of longitude and latitude are fundamentally based on measurements derived from observing celestial bodies and are designed for a **spherical model of the Earth**.

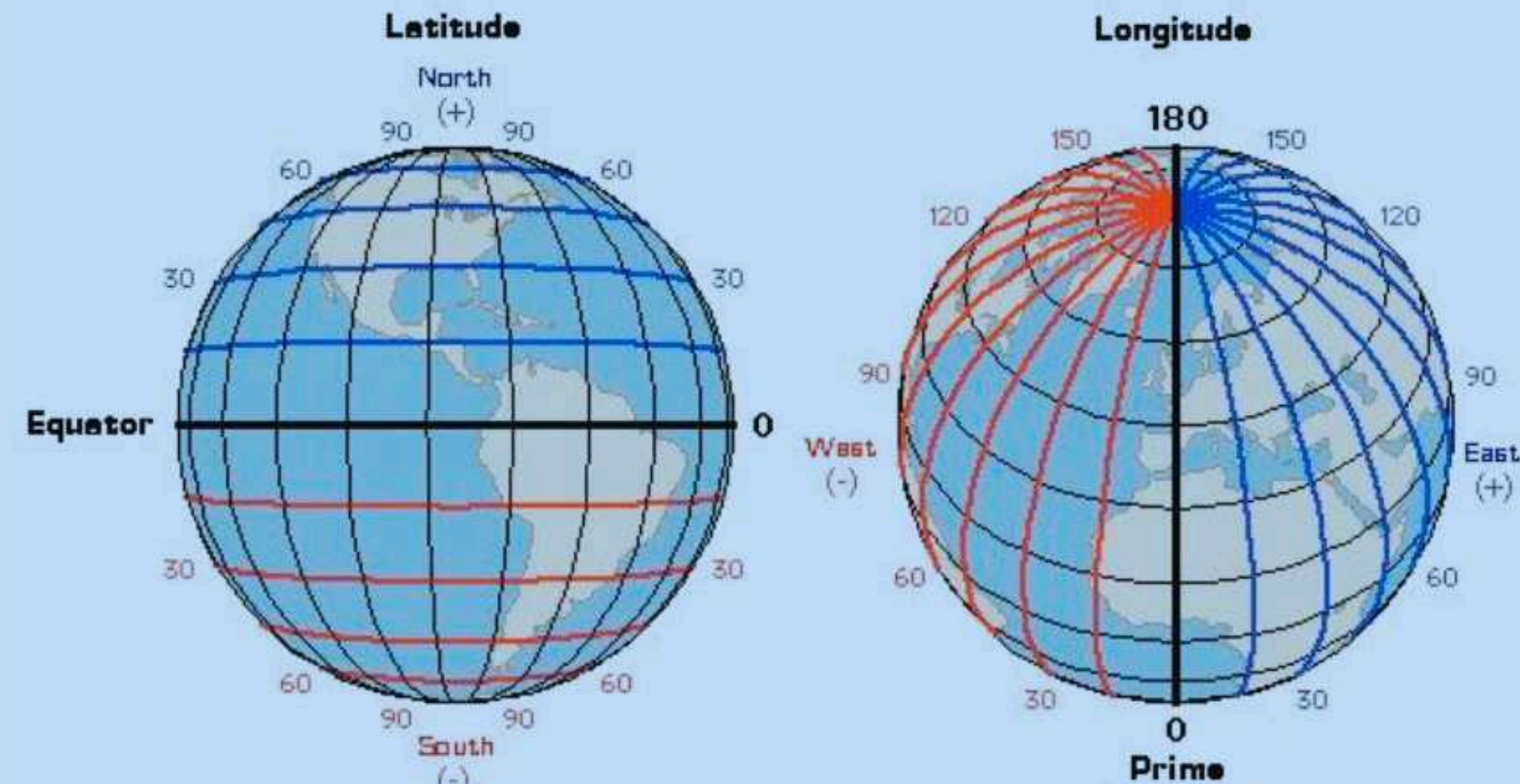
These geographic coordinates form a grid system used to pinpoint locations on the globe (**GRATICULE**)

Geographic Coordinate System (Latitude and Longitude)

Longitude and latitude form a coordinate system used to pinpoint locations on the Earth's surface.

- **Latitude:** Measures north-south position between the poles and the equator. The equator represents 0 degrees latitude, while the poles are at 90 degrees north and south. Latitude lines, or parallels, run parallel to the equator.
- **Longitude:** Measures east-west position and is expressed in degrees east or west from the Prime Meridian, which passes through Greenwich, England. Longitude lines, or meridians, converge at the poles and are widest at the equator.

Lat/Lon Invariance: Both flat and spherical Earth models use latitude and longitude for navigation and mapping. The azimuthal transformation maintains these coordinates invariant, meaning they do not change between models. This is crucial for preserving distances calculated by common formulas like the Haversine, which calculates distances between two points on a sphere based on their latitudes and longitudes.



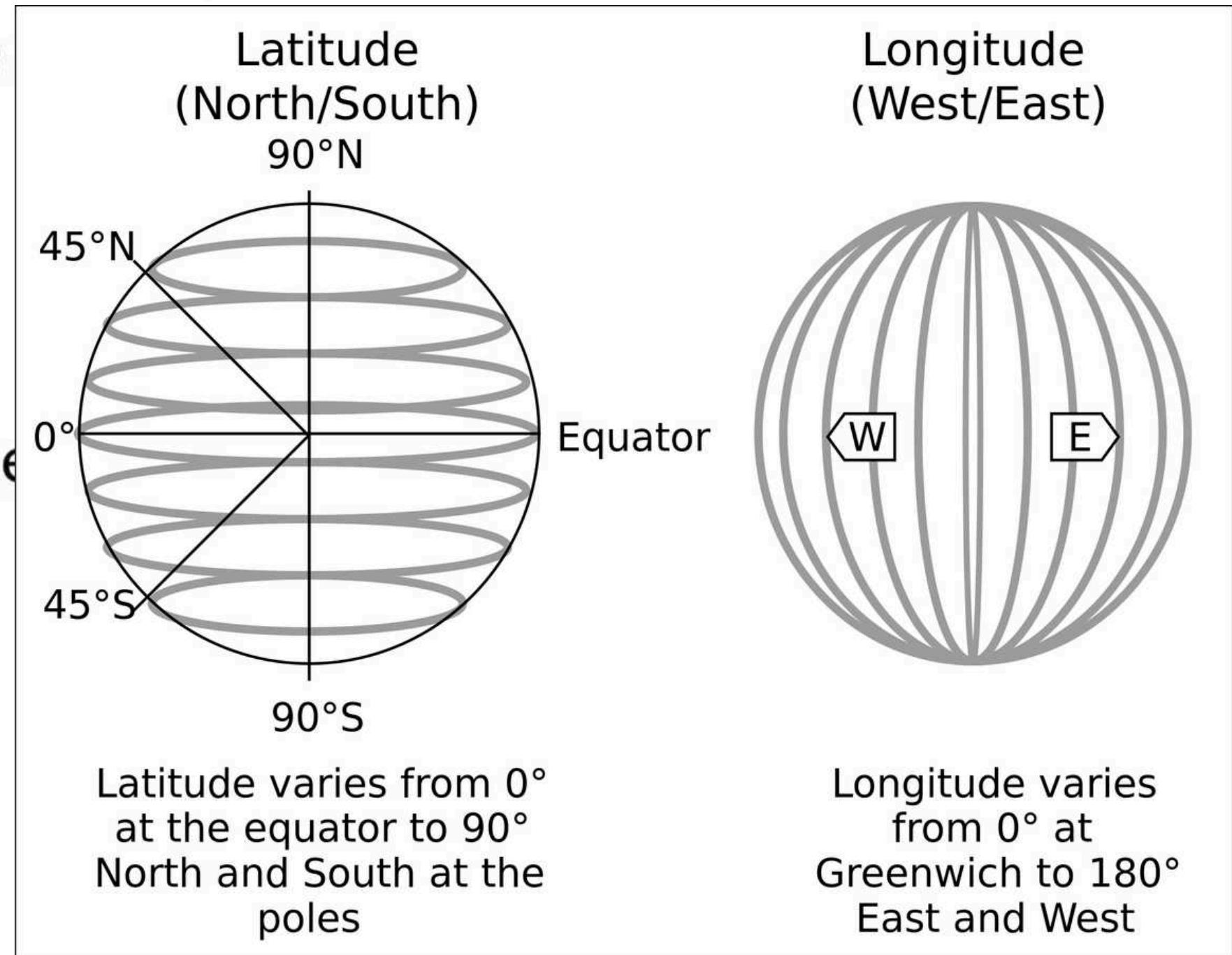
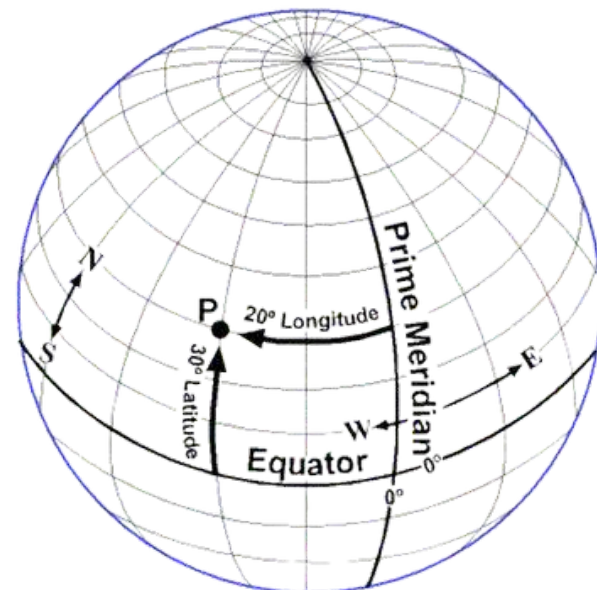
COORDINATE SYSTEMS

GEOGRAPHIC (LONGITUDE/LATITUDE)

- ✦ **Latitude** (y) - north-south distance from the equator - the "origin". Also called parallels.
- ✦ **Longitude** (x) - east-west angular distance from a prime meridian - the "origin". Also called meridians.
- ✦ Not a map projection - a set of spherical coordinates used to reference positions on the curved surface of the Earth for use in map projections.
- ✦ Basis for projected coordinate systems

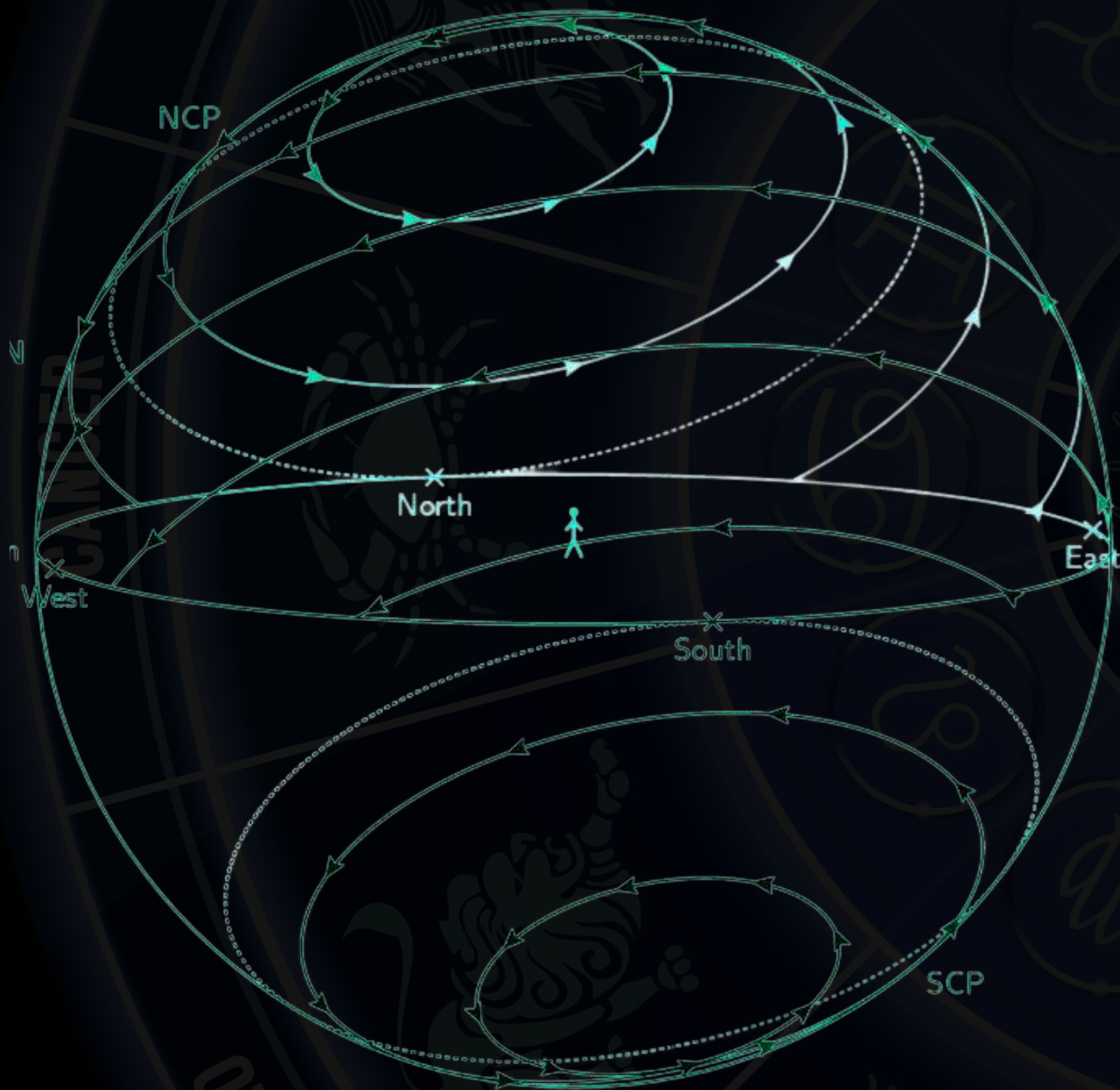
The Geographic Coordinate System

Viewing latitude and longitude angles from a 3D perspective:



Longitude and Latitude are CURVED?

WITHOUT A DOUBT!



The entire system of latitude and longitude is based on sphericity.

- **Great Circles:** Both latitude and longitude lines are based on the concept of great circles that divide the globe into equal halves. The equator and all meridians are great circles.
- **Spherical Trigonometry:** The calculations for distances and angles between different points on the Earth's surface use spherical trigonometry, assuming the Earth is a sphere.
- **Navigation and Mapping:** All navigational and mapping systems that use latitude and longitude take into account the Earth's curvature. For example, flight routes and maritime courses plotted using these coordinates reflect adjustments for the globe's shape.
- **Great Circles vs. Circles:** A great circle is the largest circle that can be drawn on a sphere's surface, dividing it into two equal halves. On a sphere like the Earth, the shortest distance between two points lies along the arc of a great circle. In navigation, using the concept of great circles accounts for the Earth's curvature, providing the most efficient and shortest path between two points. In contrast, a circle (in the context of a two-dimensional plane) does not account for the varying distances and directions encountered on a three-dimensional spherical surface due to curvature.

Coordinate Systems

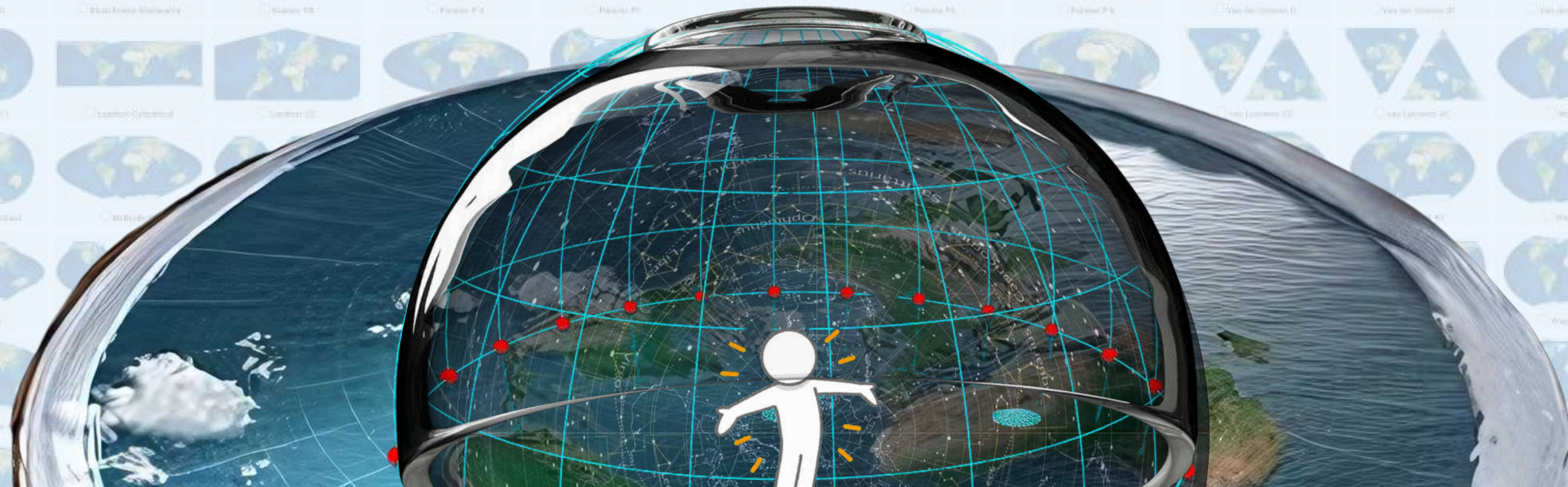
BASED ON ANGLES TO THE STARS

- Any rotating sphere has two poles at each end of the axis of rotation, and an equator which bisects the sphere in a plane that is perpendicular to the axis of rotation. However, In reality,
 - In reality, we are only seeing a half sphere, or a hemisphere of stars at any one time. The tricky part is that they, and everyone else, assume that when the celestial objects are out of sight, they are actually beneath our feet, executing a perfect circle before returning the following day.
 - Never the less, they have denoted both celestial poles and the celestial equator, imagining this sphere to continue beneath your feet. The great part is that although the objects are actually across the earth, the math for a sphere depending on angles taken from the bottom hemisphere actually work perfectly.
 - That is to say, if you use math to predict when you will see objects return, they could be represented equally well in either coordinate system.
- ***The celestial sphere conceptualizes imaginary lines inscribed on the celestial sphere through the use of coordinate systems. These lines rotate with the celestial sphere, and therefore do not depend on the observer's location, time of observation, or horizon, but are linked to the axis rotation of the sky.**

COORDINATE SYSTEMS

BASED ON ANGLES TO THE STARS

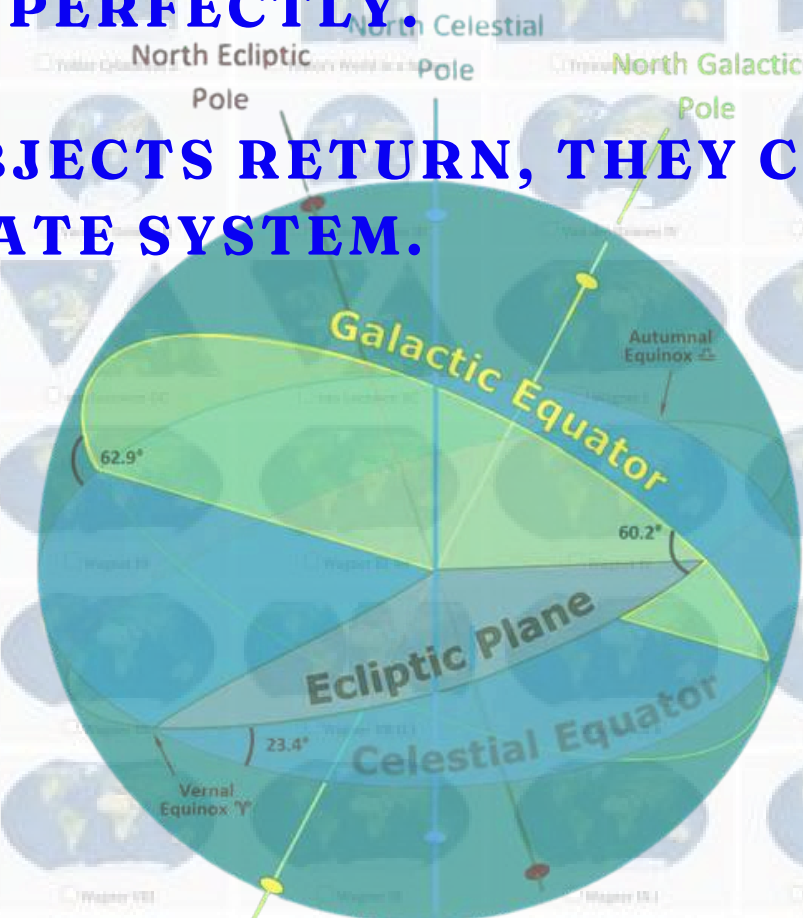
- **THE NIGHT SKY LOOKS LIKE AN UPSIDE DOWN BUT BOWL, AS IT TURNS AROUND DURING THE NIGHT AND THIS MAKES IT IS EASY TO THINK OF IT AS A GIANT SPHERE.**



COORDINATE SYSTEMS

BASED ON ANGLES TO THE STARS

- **IN REALITY, WE ARE ONLY SEEING A HALF SPHERE, OR A HEMISPHERE OF STARS AT ANY ONE TIME. THE TRICKY PART IS THAT THEY, AND EVERYONE ELSE, ASSUME THAT WHEN THE CELESTIAL OBJECTS ARE OUT OF SIGHT, THEY ARE ACTUALLY BENEATH OUR FEET, EXECUTING A PERFECT CIRCLE BEFORE RETURNING THE FOLLOWING DAY.**
- **NEVER THE LESS, THEY HAVE DENOTED BOTH CELESTIAL POLES AND THE CELESTIAL EQUATOR, IMAGINING THIS SPHERE TO CONTINUE BENEATH YOUR FEAT. THE GREAT PART IS THAT ALTHOUGH THE OBJECTS ARE ACTUALLY ACROSS THE EARTH, THE MATH FOR A SPHERE DEPENDING ON ANGLES TAKEN FROM THE BOTTOM HEMISPHERE ACTUALLY WORK PERFECTLY.**
- **THAT IS TO SAY, IF YOU USE MATH TO PREDICT WHEN YOU WILL SEE OBJECTS RETURN, THEY COULD BE REPRESENTED EQUALLY WELL IN EITHER COORDINATE SYSTEM.**



Here's how these systems incorporate curvature

BASED ON ANGLES TO THE STARS

- **Celestial Sphere:** All of these systems conceptualize the sky as a spherical surface with the observer at its center. This spherical model is critical as it mirrors the true nature of the sky as observed from Earth, which appears dome-like due to the Earth's curvature and the vast distances of celestial objects.
- **Observer-Centric Modeling:** In systems like the Horizontal Coordinate System, the observer's horizon and zenith define the fundamental plane and the highest point directly overhead, respectively. This setup naturally forms a sphere segment from the observer's perspective, reinforcing the idea of curvature as it relates to the observer's immediate environment.
- **Spherical Geometry:** The use of spherical geometry in these coordinate systems is crucial for managing angular measurements and relationships between objects in the sky. Spherical trigonometry, which is employed to calculate positions and convert between coordinate systems, depends on the principles of spherical geometry—confirming that the sky's curvature is not merely perceived but geometrically integral to the systems.
- **Reference Planes and Great Circles:** Each coordinate system uses specific reference planes (such as the celestial equator or ecliptic plane) and measures angles along these planes. These planes, intersecting the celestial sphere, create great circles that are the shortest paths between points on the sphere, emphasizing the inherent spherical nature of the sky.
- **Equatorial and Ecliptic Systems:** These systems further underscore curvature by aligning their primary coordinates with Earth's rotation axis and orbit around the Sun, respectively. The Right Ascension and Declination in the Equatorial system, or Ecliptic Longitude and Latitude in the Ecliptic system, are measured in terms of angles on the celestial sphere, consistent with spherical coordinates.

This is you.

Shane St. Pierre

ASTORIA



The Curvature of the Celestial Sphere

BASED ON ANGLES TO THE STARS

Coordinate systems based on angles to the stars, such as the Horizontal, Equatorial, Ecliptic, and Galactic coordinate systems, inherently assume and account for the curvature of the celestial sphere.

This curvature is essential for accurately mapping the positions and movements of celestial objects as seen from an observer's vantage point on Earth.

This is the only thing in our world that IS CURVED! The celestial sphere (which represents all tangent points from any observer and a fundamental plane through the middle) inherently uses spherical trigonometry and a host of other factors that require us to treat it as a curved sphere

The Curvature of the Celestial Sphere

BASED ON ANGLES TO THE STARS

Coordinate Systems and the Celestial Sphere: Takes into account the apparent curvature of the sky as seen from the observer's vantage point.

Coordinate systems based on angles to the stars account for the curvature of the celestial sphere by treating the sky as a spherical surface surrounding the observer.

The altitude measures how high an object is in the sky, directly accounting for the observer's horizon.

- The azimuth provides the horizontal direction, with the horizon forming the fundamental plane

THE CURVATURE OF THE CELESTIAL SPHERE

BASED ON ANGLES TO THE STARS

Each system defines a fundamental plane (horizon, celestial equator, ecliptic, or galactic plane) that helps in positioning objects on the curved celestial sphere.

Angles are measured relative to the fundamental plane and other reference points (e.g., vernal equinox for right ascension, north point for azimuth).

Calculations on the celestial sphere use **spherical trigonometry** to account for the curvature. This allows for accurate transformations between different coordinate systems.

Transformation Equations: Equations transform coordinates between different systems (e.g., from horizontal to equatorial coordinates).

Coordinate Systems

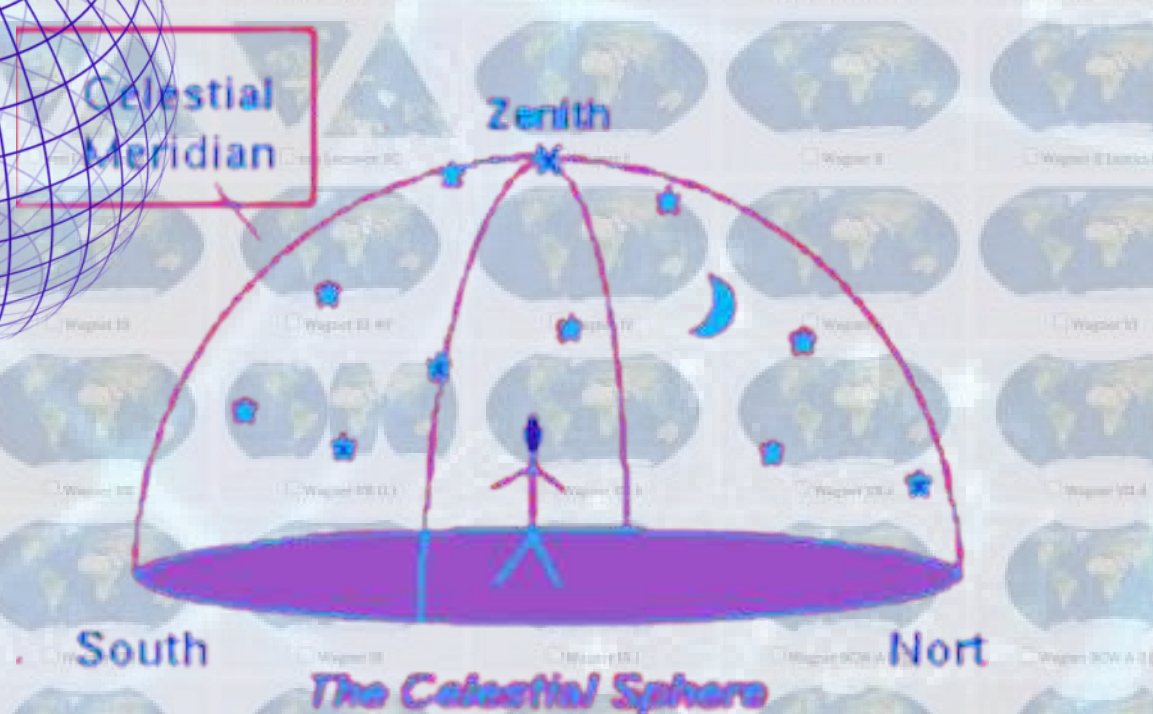
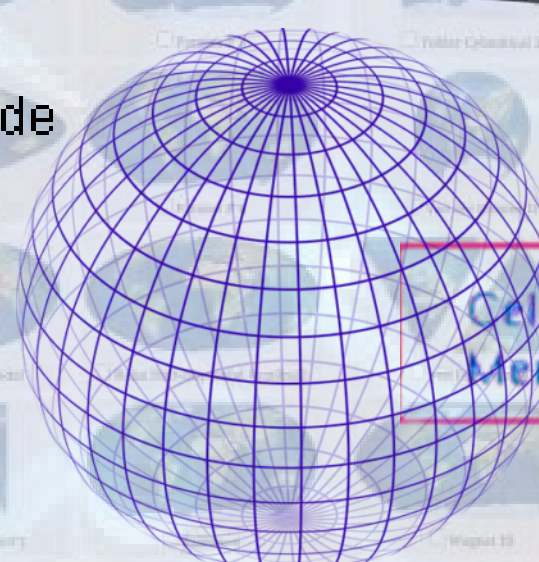
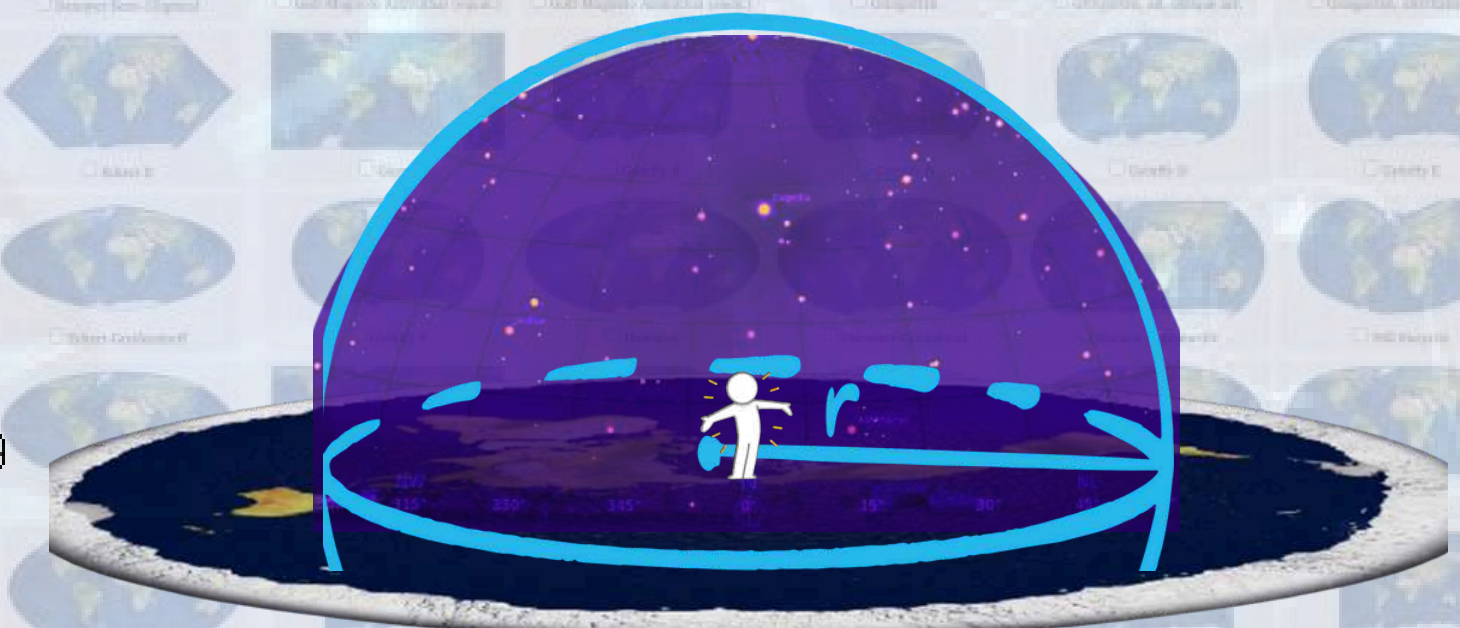
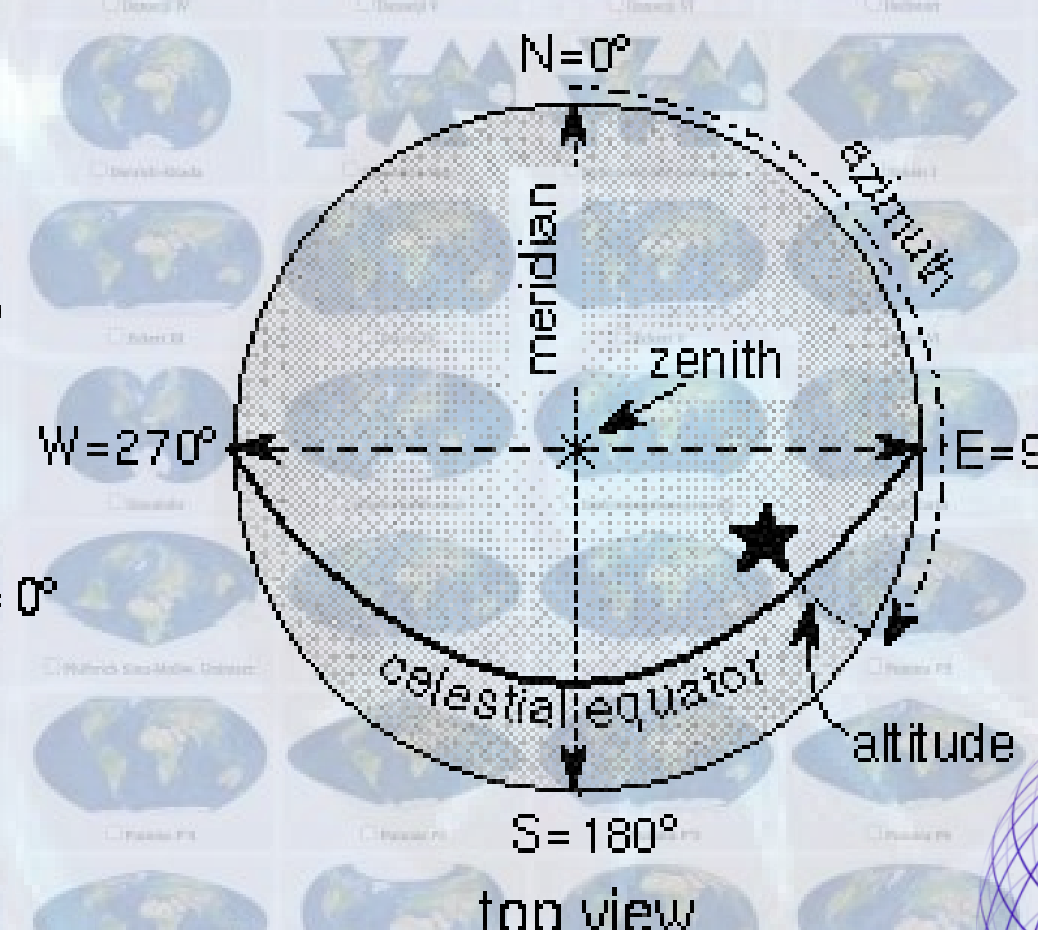
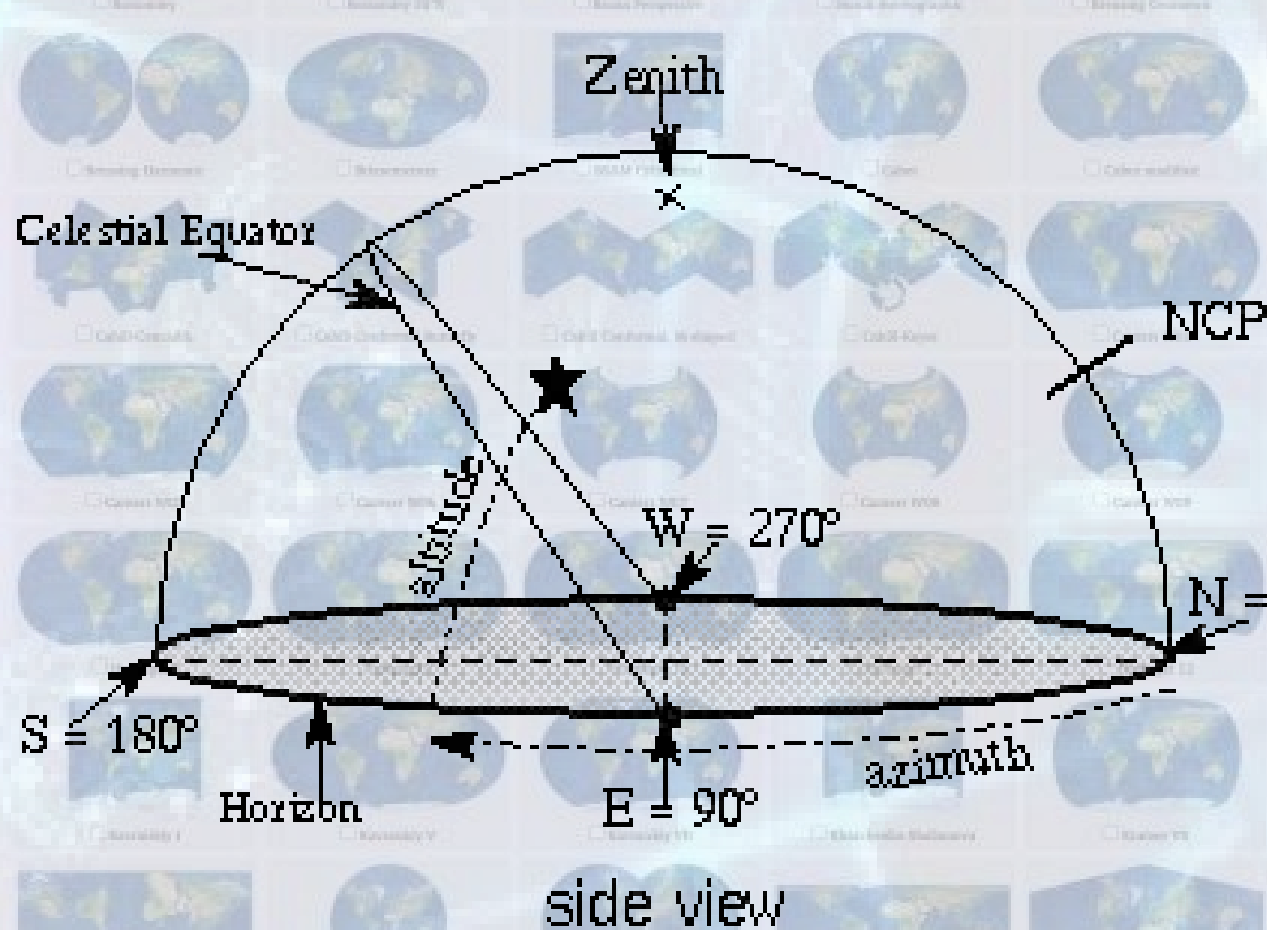
BASED ON ANGLES TO THE STARS

When you perform calculations such as distances using azimuthal transformations, whether the Earth is considered flat or spherical in model, the **distances like those measured in nautical miles remain consistent.**

This is because the formulas that **calculate distances based on angles to the stars** (like the Haversine formula) remain valid regardless of the underlying shape assumption of the Earth.

This means that for navigation and mapping, using either a **flat or spherical** model does not impact the practical outcomes when using properly adjusted coordinate transformations. Measurements like **nautical and statute miles** retain their values and utility **PRECISELY** because **their definition ties back to the coordinate system** (latitude and longitude), **which remains invariant between transformations.**

THIS IS MUCH SIMPLER THAN THEY ARE MAKING IT.....



A star's position in the altitude-azimuth coordinate system. The azimuth=120° and the altitude=50°. The azimuth is measured in degrees clockwise along the horizon from due North. The azimuths for the compass directions are shown in the figure. The altitude is measured in degrees above the horizon. The star's altitude and azimuth changes throughout the night and depends on the observer's position (here at the intersection of the N-S line and E-W line). The star's position does not depend on the location of the NCP or Celestial Equator in this system.

Geographical to Celestial

1 FOR 1

FOR EVERY POINT ON THE EARTH

(SPECIFIED BY ϕ AND λ),

THERE IS A UNIQUE POINT IN THE SKY

(SPECIFIED BY Δ AND A)

CONVERSELY, EVERY POINT IN THE SKY

UNIQUELY CORRESPONDS TO A POINT ON THE EARTH'S

SURFACE

(AT A GIVEN MOMENT, DEPENDING ON THE OBSERVER'S

LONGITUDE, LATITUDE, AND THE SIDEREAL TIME)

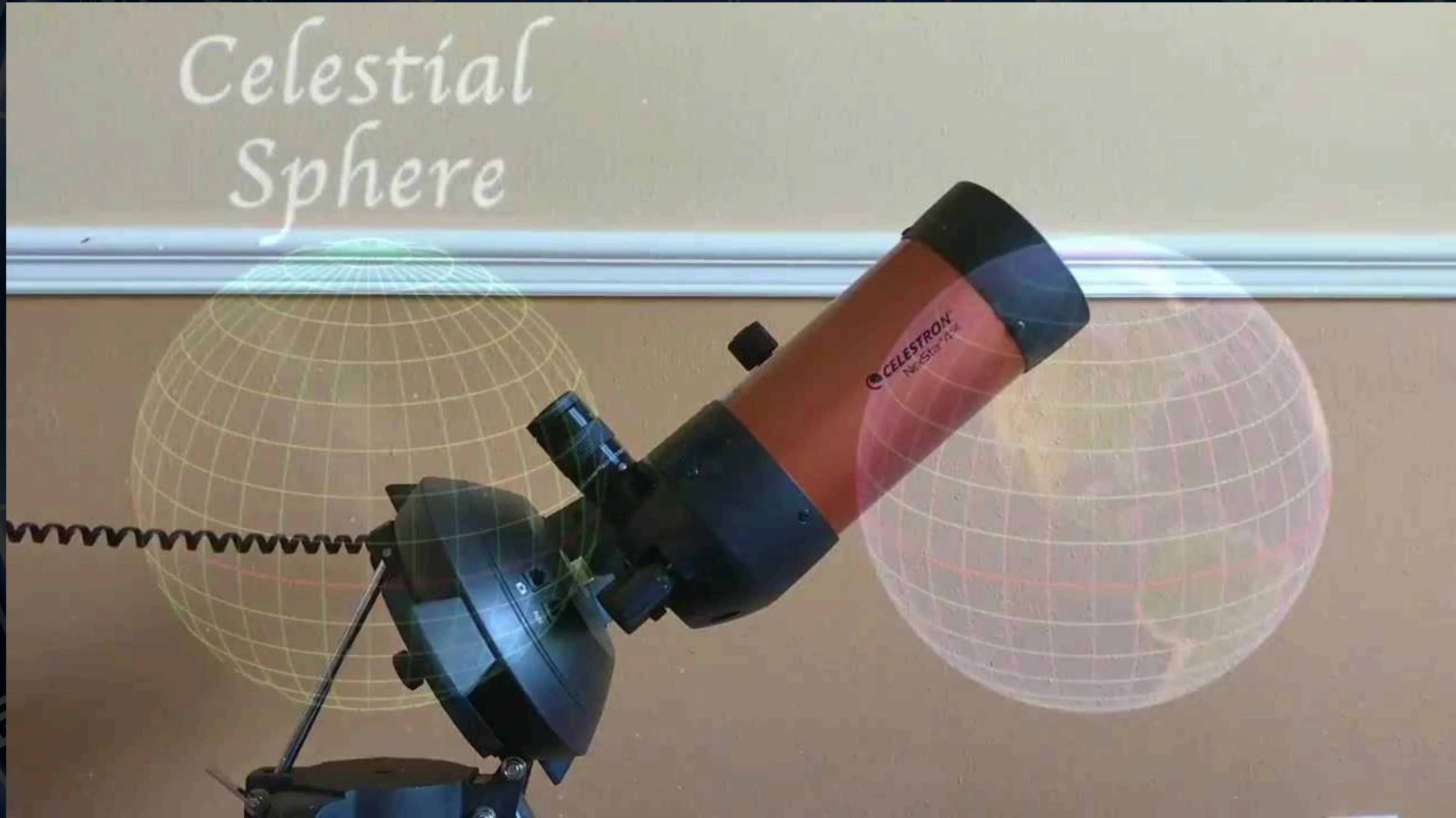
Geographical to Celestial

1 FOR 1

- BOTH SYSTEMS ARE BASED ON GREAT CIRCLES: LATITUDE AND DECLINATION ARE MEASURED FROM THE EQUATORIAL PLANES (EARTH'S EQUATOR AND CELESTIAL EQUATOR), WHILE LONGITUDE AND RIGHT ASCENSION ARE MEASURED FROM PRIME MERIDIANS (GREENWICH FOR EARTH AND THE VERNAL EQUINOX FOR THE SKY).
- EACH USES A FUNDAMENTAL CIRCLE (EQUATOR) AND A STARTING POINT FOR MEASURING EAST-WEST COORDINATES (GREENWICH MERIDIAN FOR LONGITUDE, VERNAL EQUINOX FOR RIGHT ASCENSION).
- TIME IS INTRINSICALLY LINKED TO THESE COORDINATE SYSTEMS. EARTH'S ROTATION, WHICH DEFINES THE MEASUREMENT OF A DAY, AFFECTS BOTH LONGITUDE (THROUGH TIME ZONES) AND RIGHT ASCENSION (THROUGH THE SIDEREAL DAY, ABOUT FOUR MINUTES SHORTER THAN THE SOLAR DAY).
- THE PRACTICE OF CELESTIAL NAVIGATION INVOLVES MEASURING THE ANGLES BETWEEN CELESTIAL BODIES AND THE HORIZON, AND COMPARING THESE OBSERVATIONS WITH TABLES BASED ON RIGHT ASCENSION AND DECLINATION. THIS DATA CAN THEN BE TRANSLATED INTO TERRESTRIAL LATITUDE AND LONGITUDE FOR NAVIGATION.
- LATITUDE TO DECLINATION:
 - THE CELESTIAL SPHERE USES DECLINATION
 - (Δ) INSTEAD OF LATITUDE, BUT BOTH MEASURE THE ANGULAR DISTANCE NORTH OR SOUTH OF THE EQUATOR. THUS, EARTH'S LATITUDE IS DIRECTLY PROJECTED ONTO DECLINATION: $\delta = \text{LATITUDE}$
- LONGITUDE TO RIGHT ASCENSION:
 - RIGHT ASCENSION
 - (α) IS ANALOGOUS TO LONGITUDE, ALTHOUGH IT IS MEASURED IN TIME UNITS (HOURS, MINUTES, AND SECONDS) ON THE CELESTIAL SPHERE.
 - LONGITUDE IS CONVERTED TO RIGHT ASCENSION THROUGH A SCALING FACTOR (1 HOUR EQUALS 15 DEGREES).
- THE RELATIONSHIP ALSO INVOLVES THE EARTH'S ROTATION AND THE POSITION OF THE VERNAL EQUINOX: $\alpha = \text{GST} + \text{LONGITUDE}$
 - (WHERE GST IS GREENWICH SIDEREAL TIME, ADJUSTED FOR THE EARTH'S ROTATION TO ALIGN WITH THE VERNAL EQUINOX).
 - THE EARTH'S ROTATIONAL AXIS PROJECTS OUTWARD TO DEFINE THE NORTH AND SOUTH CELESTIAL POLES ON THE CELESTIAL SPHERE. IF YOU WERE STANDING AT THE EARTH'S NORTH POLE, FOR INSTANCE, THE CELESTIAL NORTH POLE WOULD BE DIRECTLY OVERHEAD.
 - THE EARTH'S EQUATOR PROJECTS DIRECTLY OUTWARDS TO FORM THE CELESTIAL EQUATOR. THIS IS THE FUNDAMENTAL PLANE OF THE CELESTIAL COORDINATE SYSTEM, ANALOGOUS TO THE EQUATOR ON EARTH.

Geographical to Celestial

1 FOR 1

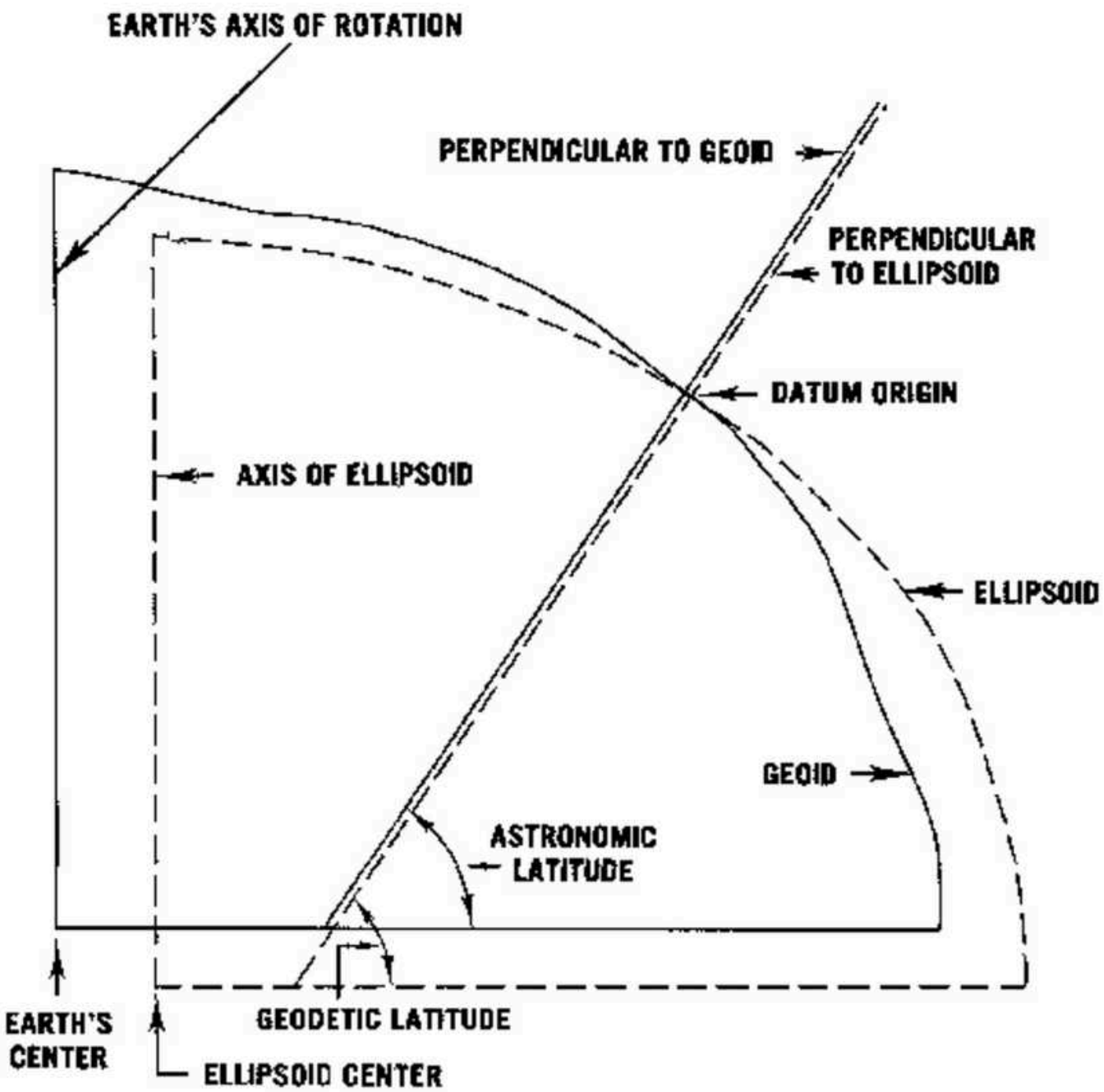


ASTROGEODESIC USES

IT GETS A LITTLE CRAZY

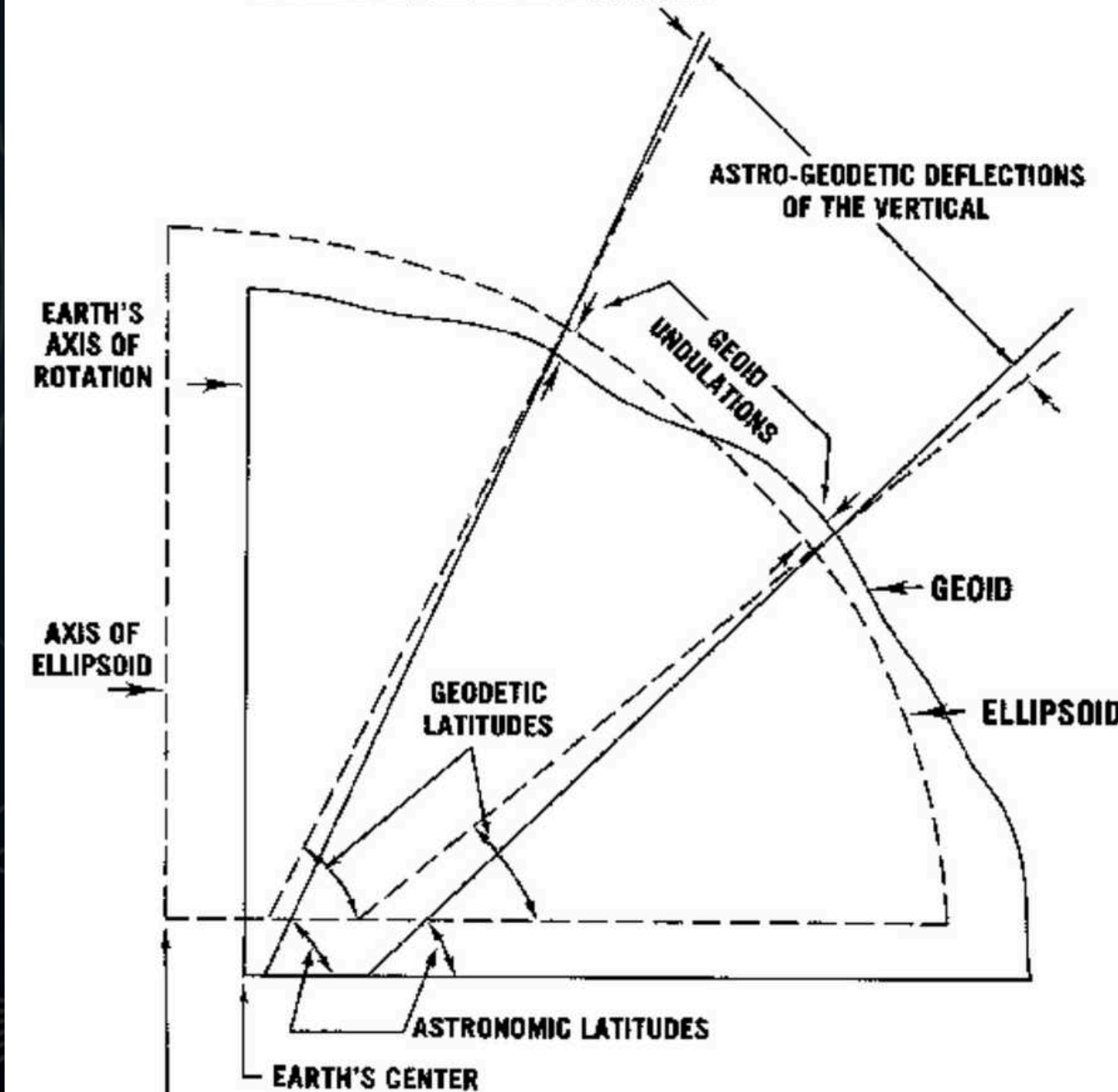
SINGLE ASTRONOMICAL STATION DATUM ORIENTATION

(PERPENDICULAR TO ELLIPSOID MADE COINCIDENT WITH PERPENDICULAR TO GEOID AT DATUM ORIGIN)



ASTRO-GEODETTIC DATUM ORIENTATION

GEOID AND ELLIPSOID ARE ORIENTED SO THAT THE SUM OF THE SQUARES OF SEVERAL DEFLECTIONS OF THE VERTICAL SELECTED THROUGHOUT THE GEODETTIC NETWORK IS MADE AS SMALL AS POSSIBLE

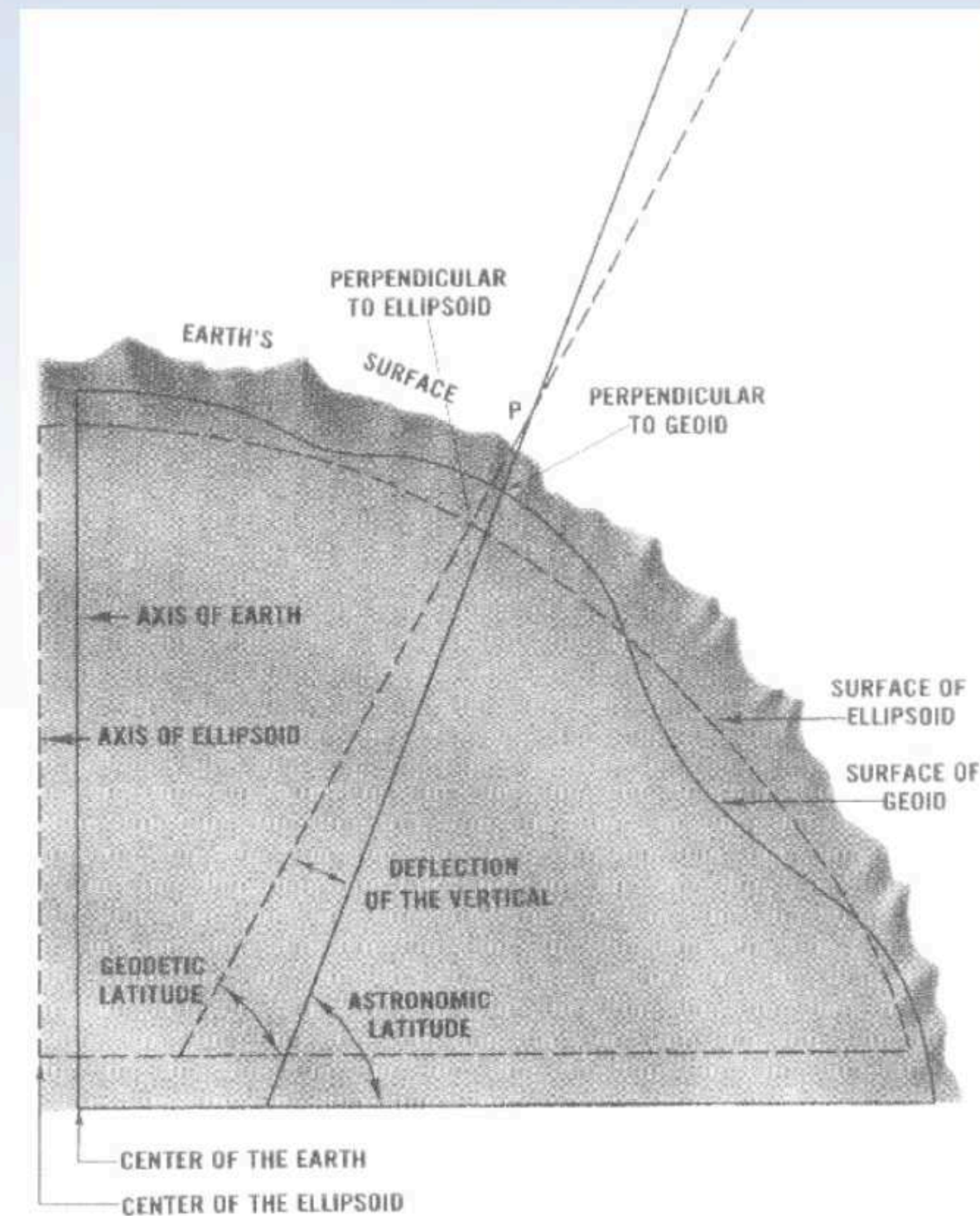


ASTROGEODESIC USES

IT GETS A LITTLE CRAZY

Coordinates

- Astronomic or “natural” coordinates
 - Astronomic latitude and longitude: Which way is up?
 - Orthometric height: How far have I traveled up or down from the geoid?
- Geodetic coordinates
 - Geodetic or ellipsoidal latitude, longitude, and height: Where am I in three-dimensional space?
- The deflection of the vertical gives the difference between astronomic and geodetic latitude and longitudes
- The difference between orthometric and ellipsoidal heights is the geoid undulation



1984, Geodesy for the Layman

ASTROGEODESIC USES

IT GETS A LITTLE CRAZY

The deflection of the vertical (θ) is the angular difference between the direction of the gravity vector (g), or plumbline at a point, and the corresponding ellipsoidal normal through the same point for a particular ellipsoid (Figure 1). Since the plumblines are orthogonal to the level surfaces by definition, the deflection of the vertical also gives a measure of the gradient of the level surfaces (including the geoid) with respect to a particular ellipsoid. Accordingly, the deflection of the vertical is classified as absolute when it refers to a geocentric ellipsoid and relative when it refers to a local ellipsoid. Depending on the choice of ellipsoid, the deflection of the vertical can reach 20" in lowland regions and up to 70" in regions of rugged terrain (Bomford, 1980). In Australia, the largest measured deflection of the vertical with respect to the ANS is around 30" (Fryer, 1971).

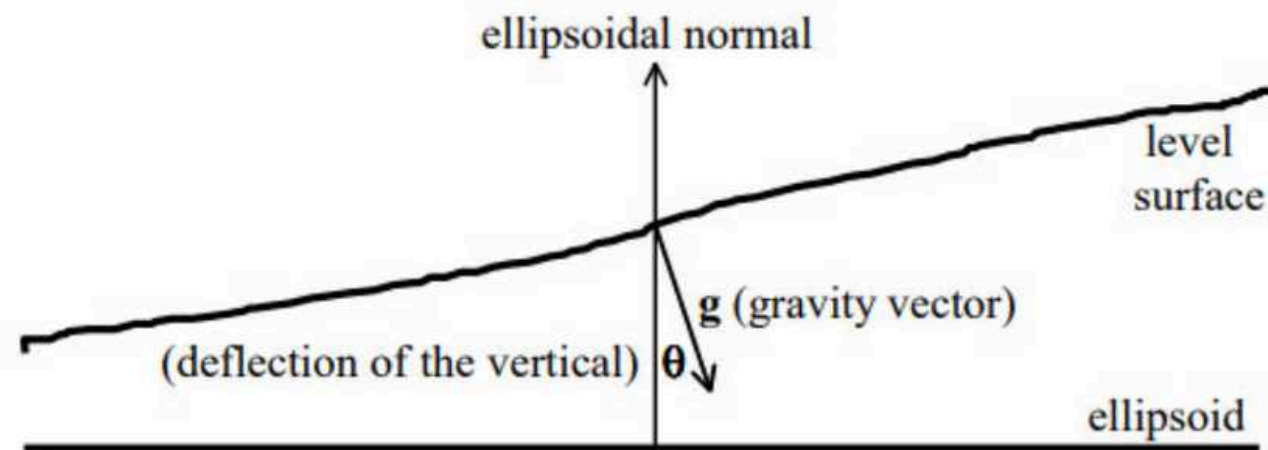


Figure 1. The deflection of the vertical (θ)

The deflection of the vertical, which is a vector quantity, is usually decomposed into two mutually perpendicular components: a north-south or meridional component (ξ), which is reckoned positive northward, and an east-west or prime vertical component (η), which is reckoned positive eastward. In other words, the deflection components are positive if the direction of the gravity vector points further south and further west than the corresponding ellipsoidal normal (Vanicek and Krakiwsky, 1986), or the level surface is rising to the south or west, respectively, with respect to the ellipsoid

Techniques used for the determination of the deflections of the vertical and the geoid separation are reviewed. These may be basically described as astro-geodetic, gravimetric, dynamic and geometric satellite, and combination methods.

Vertical deflection

[Article](#) [Talk](#)

From Wikipedia, the free encyclopedia

For the CRT rastering method, see [Cathode ray tube](#).

The **vertical deflection (VD)** or **deflection of the vertical (DoV)**, also known as **deflection of the plumb line** and **astro-geodetic deflection**, is a measure of how far the gravity direction at a given point of interest is rotated by local mass anomalies such as nearby mountains. They are widely used in geodesy, for surveying networks and for geophysical purposes.

The vertical deflection are the angular components between the true zenith–nadir curve (plumb line) tangent line and the normal vector to the surface of the reference ellipsoid (chosen to approximate the Earth's sea-level surface). VDs are caused by mountains and by underground geological irregularities and can amount to angles of 10" in flat areas or 20–50" in mountainous terrain).^[*citation needed*]

ASTROGEODESIC USES

IT GETS A LITTLE CRAZY

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Vertical Deflection at the Earth's Surface

The deflection of the vertical at the surface of the Earth (θ_s) is defined by Helmert (Torge, 1991) as the angular difference between the direction of the gravity vector and the ellipsoidal normal through the same point at the Earth's surface. This can also be an absolute or relative quantity. The deflection of the vertical at the surface of the Earth is of more practical use than the deflection of the vertical at the geoid, because survey measurements are made at the Earth's surface and are thus affected by the deflection of the vertical at this point.

The deflection of the vertical at the Earth's surface can be computed simply by comparing astronomical and geodetic coordinates at the same point on the Earth's surface. The corresponding deflection of the vertical in the prime vertical is the difference between astronomical latitude (Φ) and the geodetic latitude (ϕ) of the same point. Likewise, the deflection of the vertical in the meridian is the difference, scaled for meridional convergence, between astronomical longitude (Λ) and the geodetic longitude (λ) of the same point. These are given, respectively, by

$$\xi_s = \Phi - \phi \tag{5}$$

$$\eta_s = (\Lambda - \lambda) \cos \phi \tag{6}$$

where the subscript s is used to distinguish these components of the deflection of the vertical at the surface of the Earth, and it is assumed that the minor axis of the ellipsoid is parallel to the mean spin axis of the Earth's rotation (Bomford, 1980).

Probably the most important implication of the relations in equations (5) and (6) is to choose the relative deflection of the vertical to be as small as possible through an

ASTROGEODESIC USES

IT GETS A LITTLE CRAZY

measurements that he used to confirm his results. Gauß pointed out that some of the measurements Gerling was citing were geodetic and others astronomic longitude determinations and that he, Gauß, did not expect them to show the same results. He mentioned his former latitude measurements in northern Germany (letter no. 296, Schäfer 1927). It was this remark of Gauß's, that opened Gerling's eyes: *he had performed the first measurement of the deflection of the vertical in longitude, and the deviation of the astronomic and geodetic measurements was a new and very valuable result.* His measurements initiated new quality of measurements. He replied to Gauß (letter no. 297, Schäfer 1927): "If I must accuse myself here of gross error and lack of thoroughness in applying your § [symbol refers to a section of Gauß' article from 1828], then perhaps I can find comfort in that there are probably "not five persons existing in Europe" who have taken heed of the § in this sense. I therefore feel

In terrestrial surveying, the deflection of the vertical has three primary

1. transformation of astronomical coordinates to geodetic coordinates;
2. conversion of astronomic azimuth to geodetic azimuth; and
3. reduction of vertical and horizontal angles to the spheroid.

Transformation of Coordinates

The deflections of the vertical provide the transformation between astronomical (natural) coordinates (Φ, Λ) , observed with respect to the gravity vector, and the desired geodetic coordinates (ϕ, λ) on the ellipsoid. Rearranging equations (5) and (6), and adhering to the same approximations, gives the coordinate transformation as

$$\phi = \Phi - \xi_s \quad (8)$$

$$\lambda = \Lambda - (\eta_s \sec \phi) \quad (9)$$

where the deflections of the vertical refer to the surface of the Earth, since this is the point at which the astronomic coordinates are usually measured. If the deflections of the vertical at the geoid are used in equations (8) and (9), the limitation imposed by the curvature of the plumbline should be acknowledged.

1. Define the Geographic and Celestial Coordinate Systems:

- **Geographic Coordinates:** Latitude (φ) and Longitude (λ)
 - Latitude measures north-south position with respect to the equator.
 - Longitude measures east-west position from the Prime Meridian.
- **Celestial Coordinates:**
 - Right Ascension (α) and Declination (δ)
 - Right Ascension is equivalent to celestial longitude, measuring eastward along the celestial equator.
 - Declination is equivalent to celestial latitude, measuring northward or southward from the celestial equator.
 -

1. Relation via Projection:

- The celestial coordinate system can be thought of as a projection of the Earth's surface onto the celestial sphere, with the north and south poles extending to the north and south celestial poles, and the equator to the celestial equator.

Transform Geographic Coordinates to Celestial Coordinates:

- Assume a standard projection where:
 - The celestial equator aligns with the Earth's equator.
 - The **Prime Meridian aligns with a fixed point in the sky, such as the vernal equinox** (where the sun **crosses the celestial equator at the March equinox**), which serves as the zero point for right ascension.
- This setup implies that:
 - **Declination (δ) is directly equal to the latitude (φ).**
 - **Right Ascension (α) can be derived from the longitude (λ)** by considering the Earth's rotation and the current sidereal time.

The Transformation Explained

Coordinate Systems

- **Geographic Coordinates:** These include latitude and longitude.
- **Celestial Coordinates:** These are typically right ascension (α) and declination (δ).

Projection of Geographic Coordinates onto the Celestial Sphere**:

- **Latitude to Declination:** The celestial sphere uses declination (δ) instead of latitude, but both measure the angular distance north or south of the equator. Thus, Earth's latitude is directly projected onto declination:
 - $\delta = \text{Latitude}$
- **Longitude to Right Ascension:** Right ascension (α) is analogous to longitude, although it is measured in time units (hours, minutes, and seconds) on the celestial sphere.
- Longitude is converted to right ascension through a scaling factor
 - (1 hour equals 15 degrees).
 - The relationship also involves the Earth's rotation and the position of the vernal equinox:
 - $\alpha = \text{GST} + \text{Longitude}$
 - (where GST is Greenwich Sidereal Time, adjusted for the Earth's rotation to align with the vernal equinox).

- Sidereal Time at Greenwich (θ_0) measures the right ascension directly overhead at the Greenwich meridian. This time varies with Earth's rotation, accounting for the difference between solar time and sidereal time.
- To find the right ascension corresponding to a particular longitude:
 - $\alpha = \theta$
 - $0 + \lambda$

(where λ is adjusted for the time of day and year, considering that Earth completes a full rotation relative to distant stars approximately four minutes earlier than relative to the sun each day).

5. Proving One-to-One Correspondence:

- For every point on the Earth
 - (specified by φ and λ),
 - there is a unique point in the sky
 - (specified by δ and α)
 - when considering the sidereal time.
- Conversely, every point in the sky uniquely corresponds to a point on the Earth's surface at a given moment, depending on the observer's longitude, latitude, and the sidereal time.

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Convert Earth Longitude to Right Ascension:

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- The celestial equator aligns with the Earth's equator.

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- This setup implies that:
 - **Declination (δ) is directly equal to the latitude (φ).**
 - **Right Ascension (α) can be derived from the longitude (λ)** by considering the Earth's rotation and the current sidereal time.

- **North and South Poles to Celestial Poles:** The Earth's rotational axis projects outward to define the north and south celestial poles on the celestial sphere. If you were standing at the Earth's **North Pole**, for instance, **the celestial north pole** would be directly overhead.

4. Equatorial Projection:

- **Equator to Celestial Equator:** **The Earth's equator projects** directly outwards to form the **celestial equator**. This is the fundamental plane of the celestial coordinate system, analogous to the equator on Earth.

Visualizing the Transformation

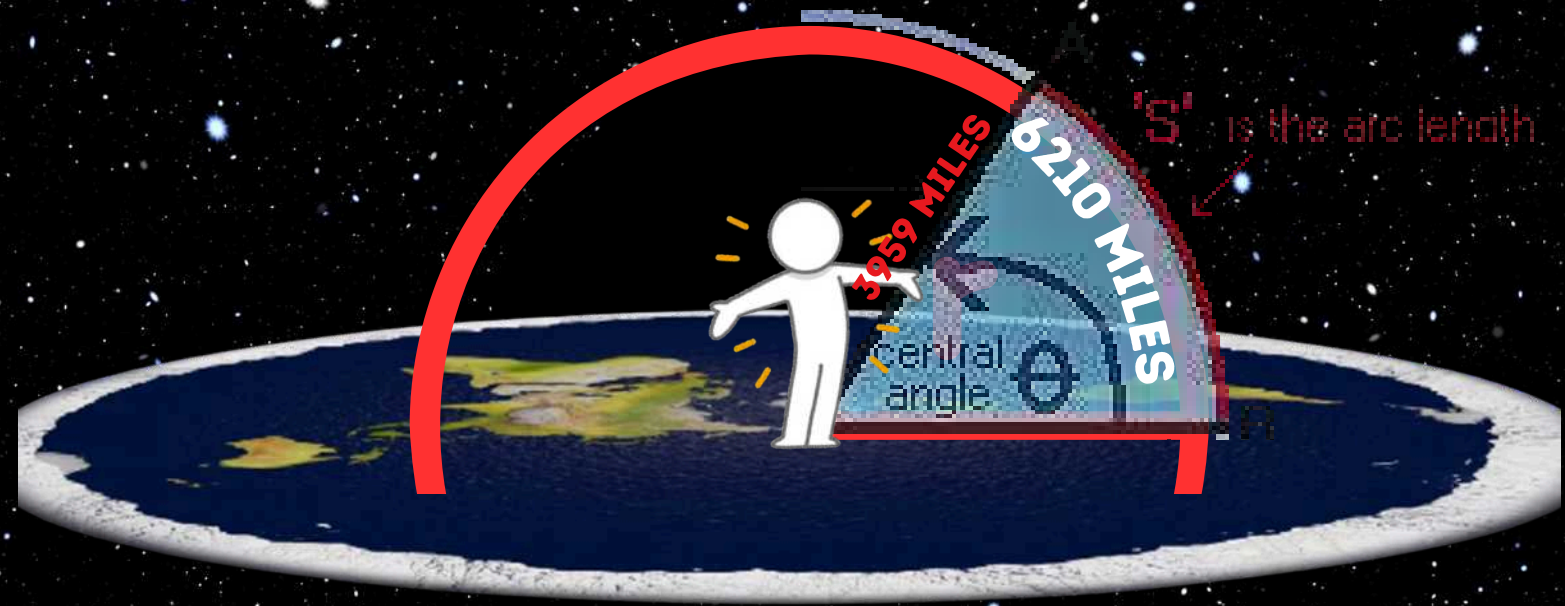
Imagine the Earth surrounded by a giant, transparent sphere (the celestial sphere). If you extended **lines from the center of the Earth outward** through every point on the Earth's surface, these lines would **intersect the celestial sphere**. The points where they intersect would define the celestial equivalents of the Earth's geographical points:

- The line passing through any location on the Earth's equator **would intersect the celestial sphere at the celestial equator**.
- The lines from the **geographic poles** would meet at the **celestial poles**.

The Implications of Coordinate Transformation Equivalence

1. Equivalence in Different Models:

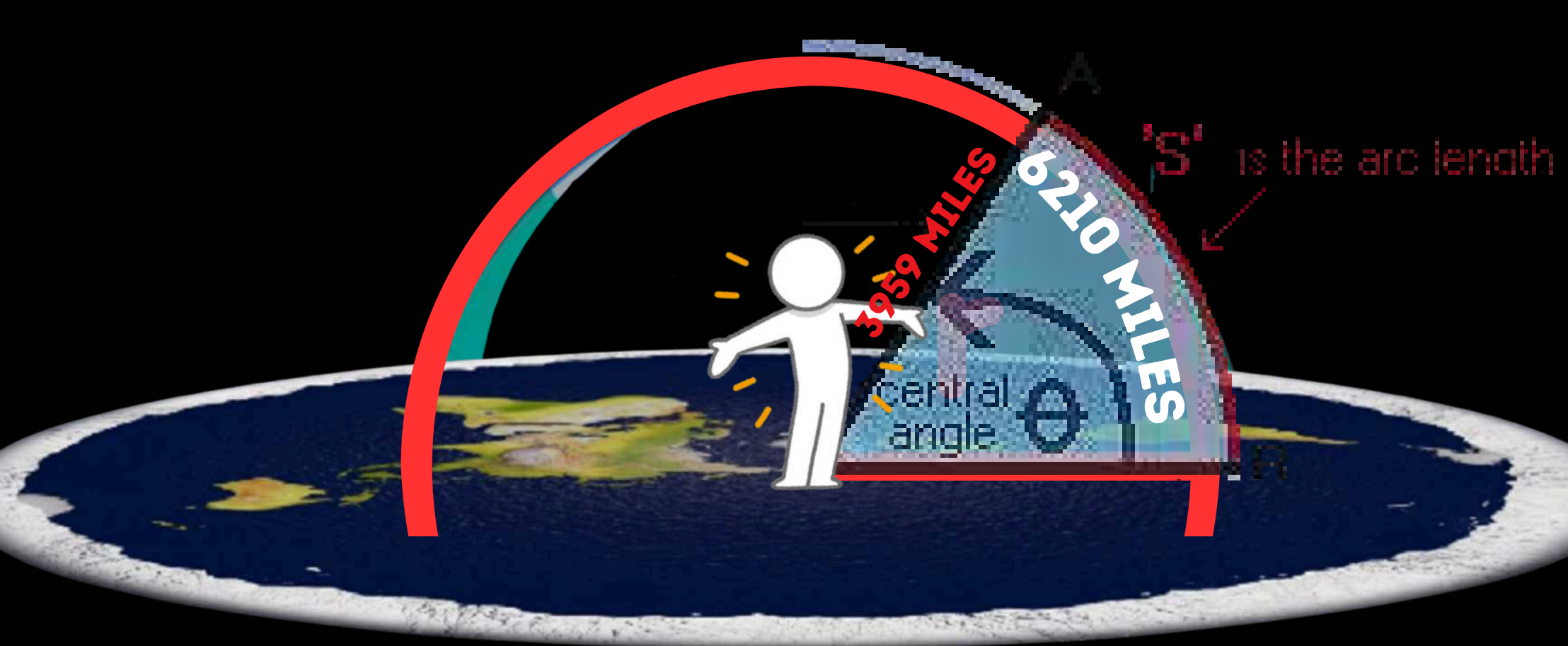
- When you perform calculations such as **distances using azimuthal transformations**, whether the Earth is **considered flat or spherical** in model, the distances like those **measured in nautical miles remain consistent**. This is because the **formulas that calculate distances based on angles** (like the Haversine formula) **remain valid** regardless of the underlying shape assumption of the Earth.



THE SPHERICAL LIMIT OF OUR VIEW

WHICH THEY USED TO FAKE THE GLOBE

Hemisphere and Sphere



Corporate needs you to find the difference between this picture and this picture

[half of] 22,859 mi

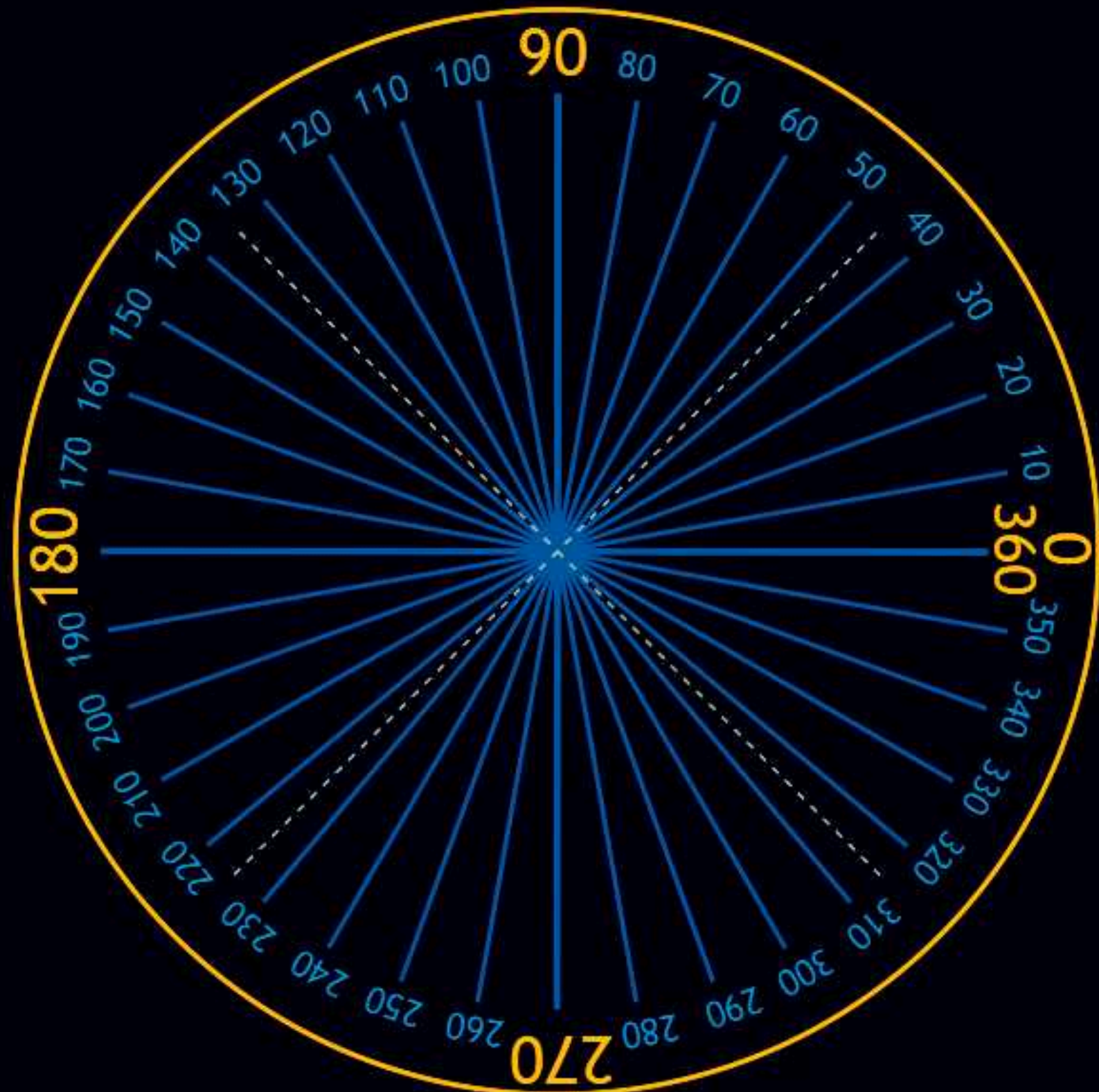




They're the same picture

The Circumference

THE ONLY THING EVER MEASURED IN REALITY

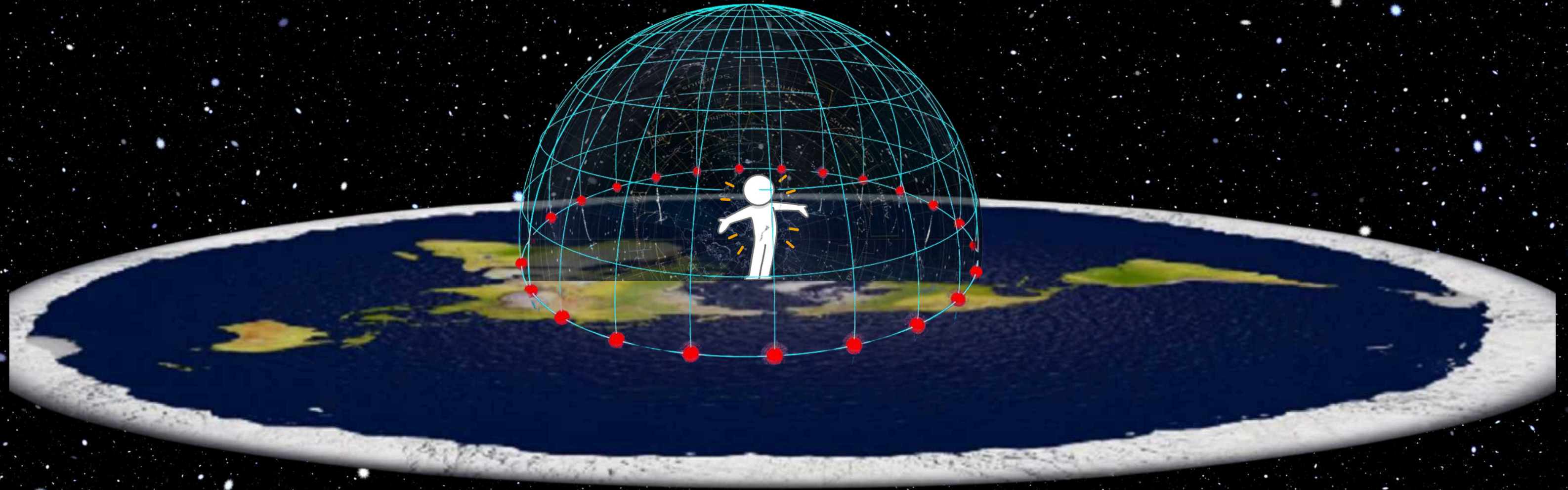


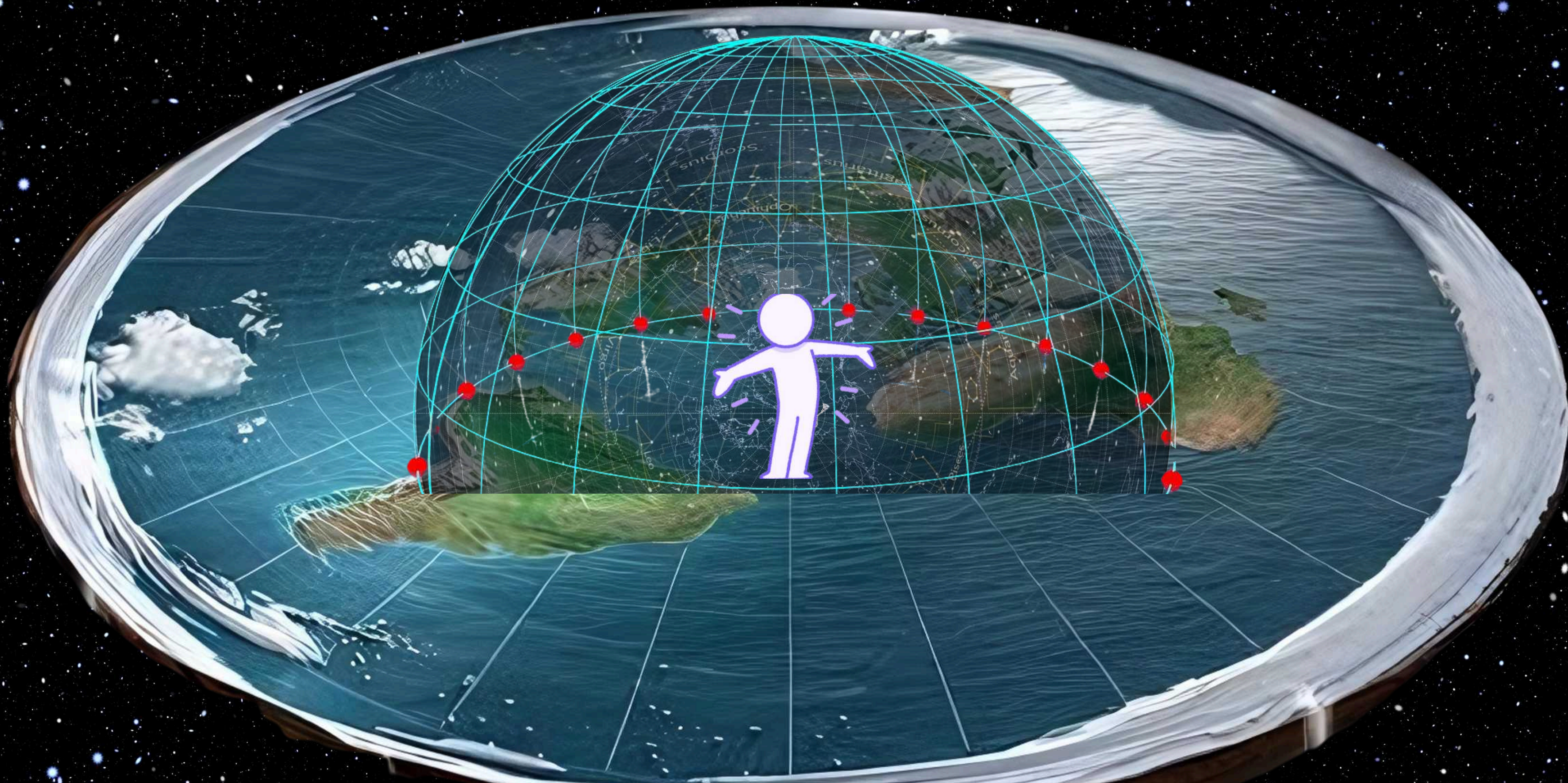
The Full Circle

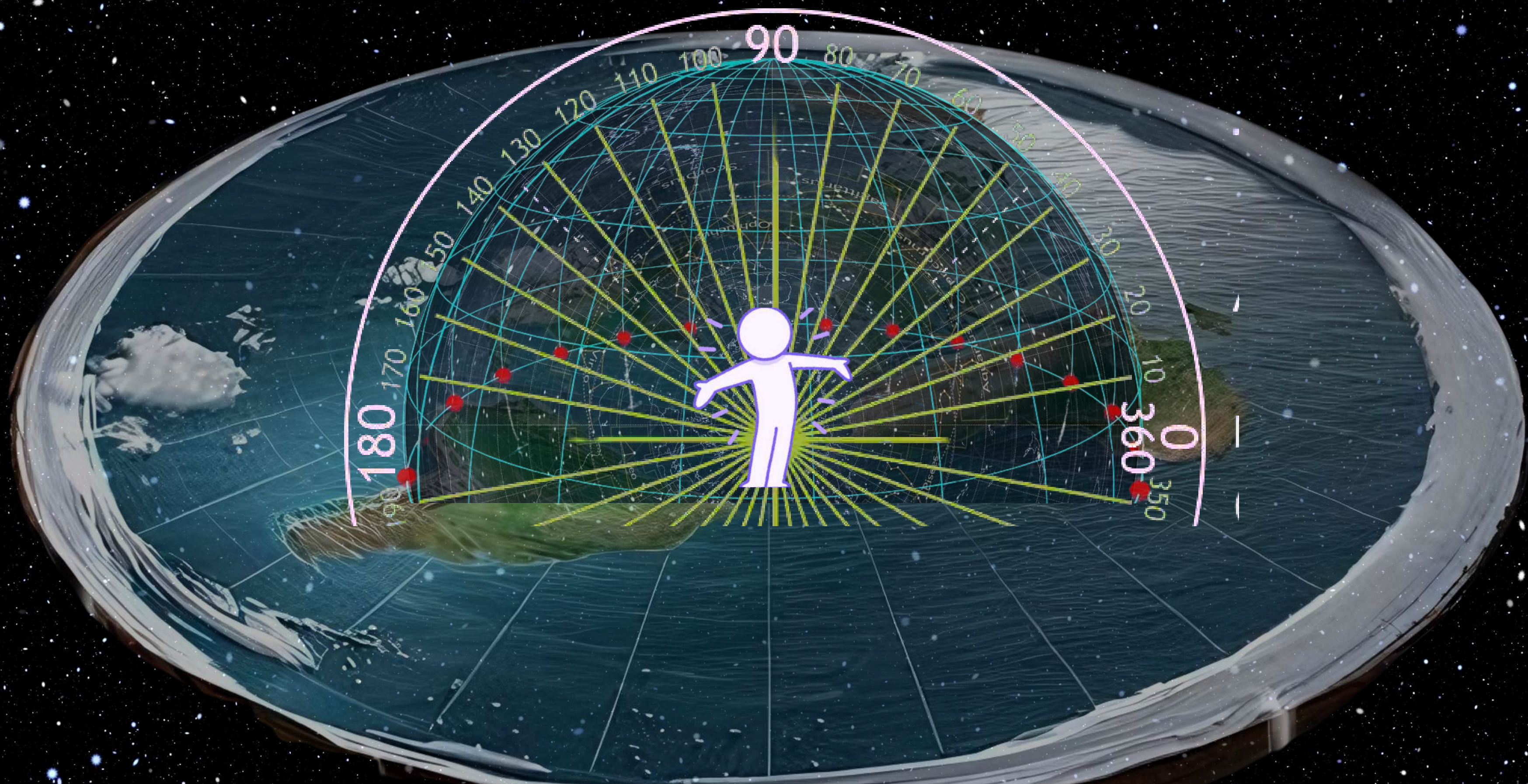
A Full Circle is 360°

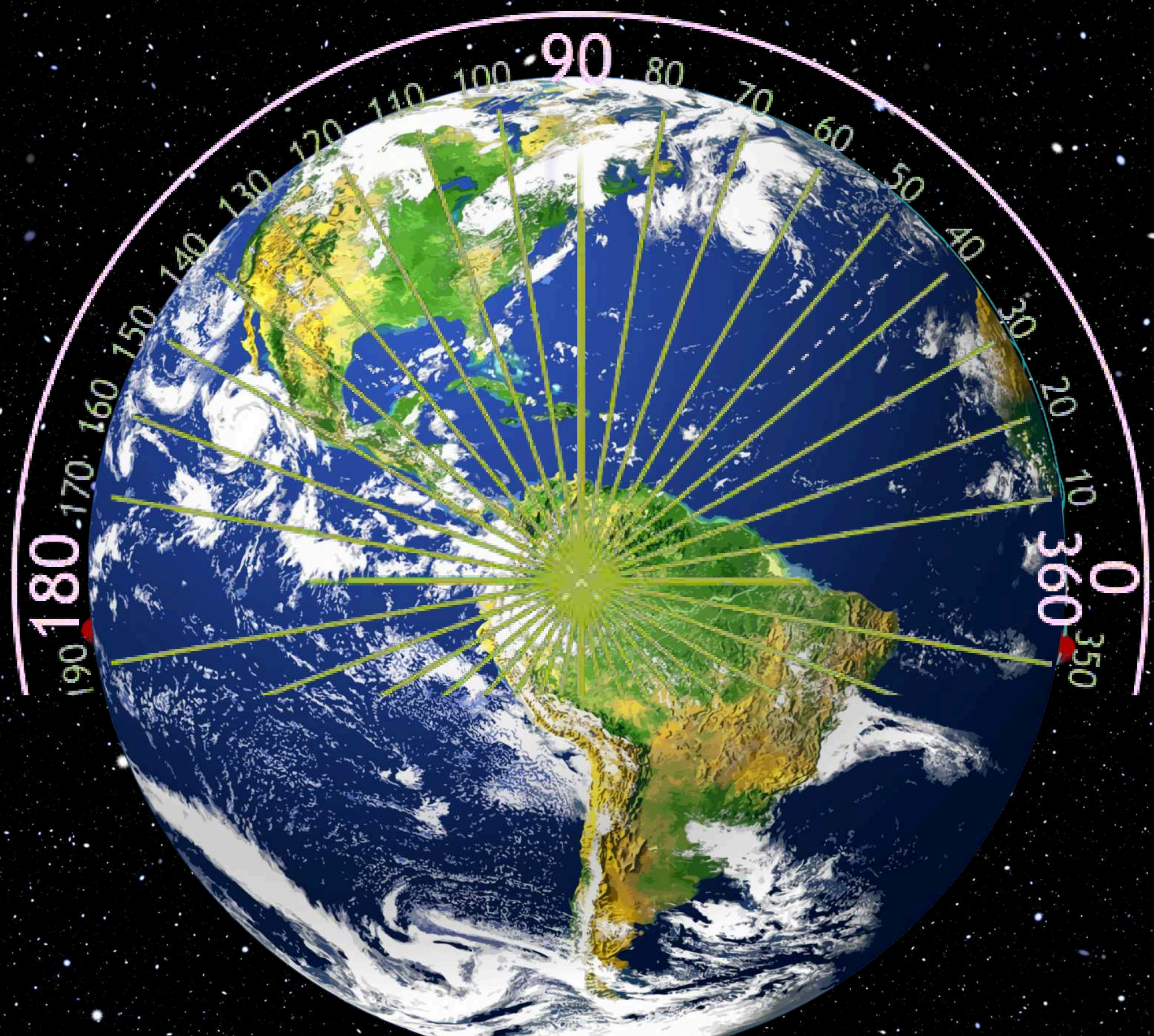
Half a circle is 180°
(called a Straight Angle)

Quarter of a circle is 90°
(called a Right Angle)









Coordinate Systems

NOT BASED ON ANGLES TO THE STARS

The three best-known coordinate systems: the Cartesian, the circular cylindrical, and the spherical.

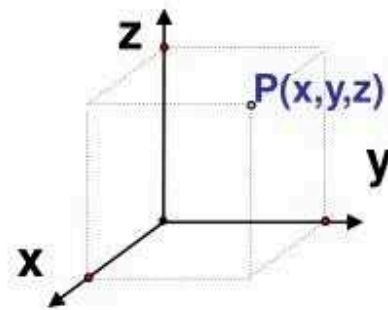
Orthogonal Coordinate Systems:

1. Cartesian Coordinates

Or

Rectangular Coordinates

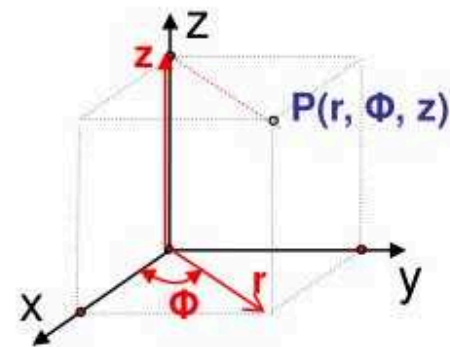
$P(x, y, z)$



2. Cylindrical Coordinates

$P(r, \Phi, z)$

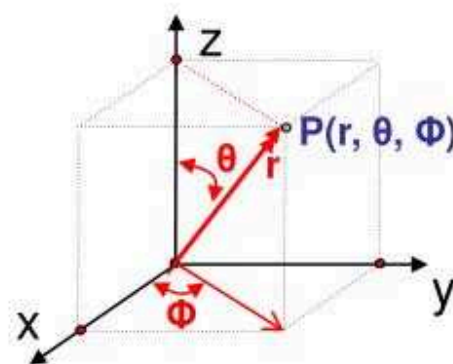
$$\begin{aligned} X &= r \cos \Phi, \\ Y &= r \sin \Phi, \\ Z &= z \end{aligned}$$



3. Spherical Coordinates

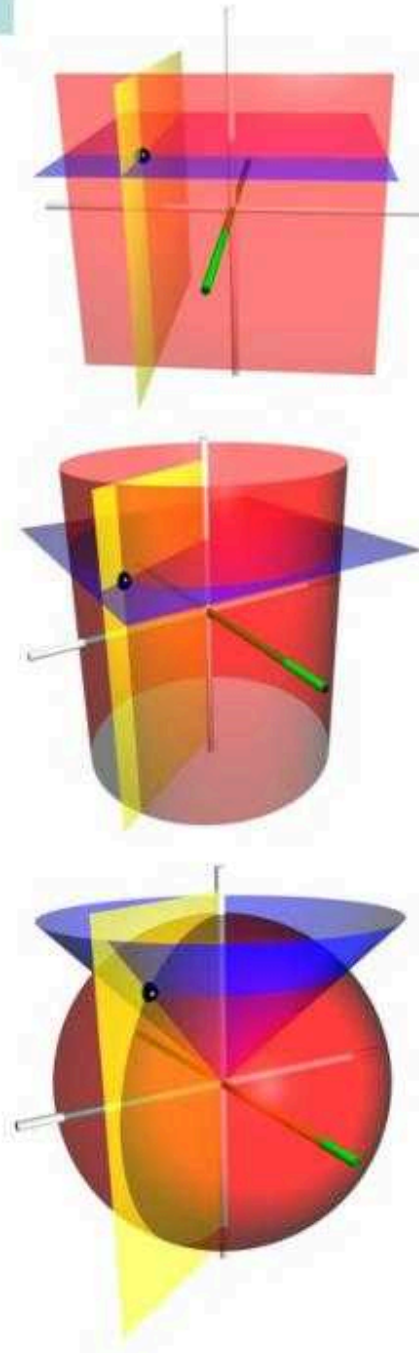
$P(r, \theta, \Phi)$

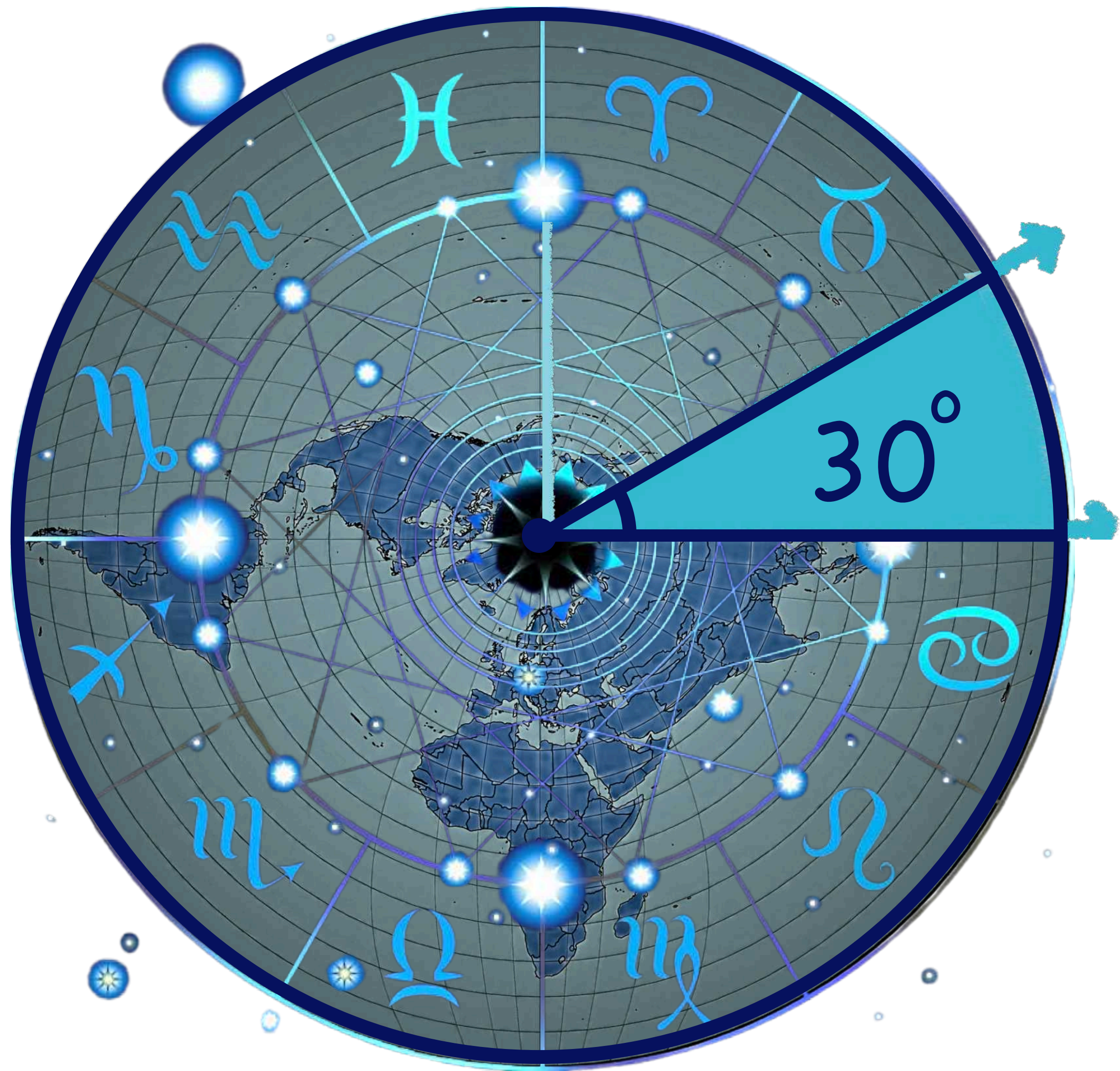
$$\begin{aligned} X &= r \sin \theta \cos \Phi, \\ Y &= r \sin \theta \sin \Phi, \\ Z &= r \cos \theta \end{aligned}$$



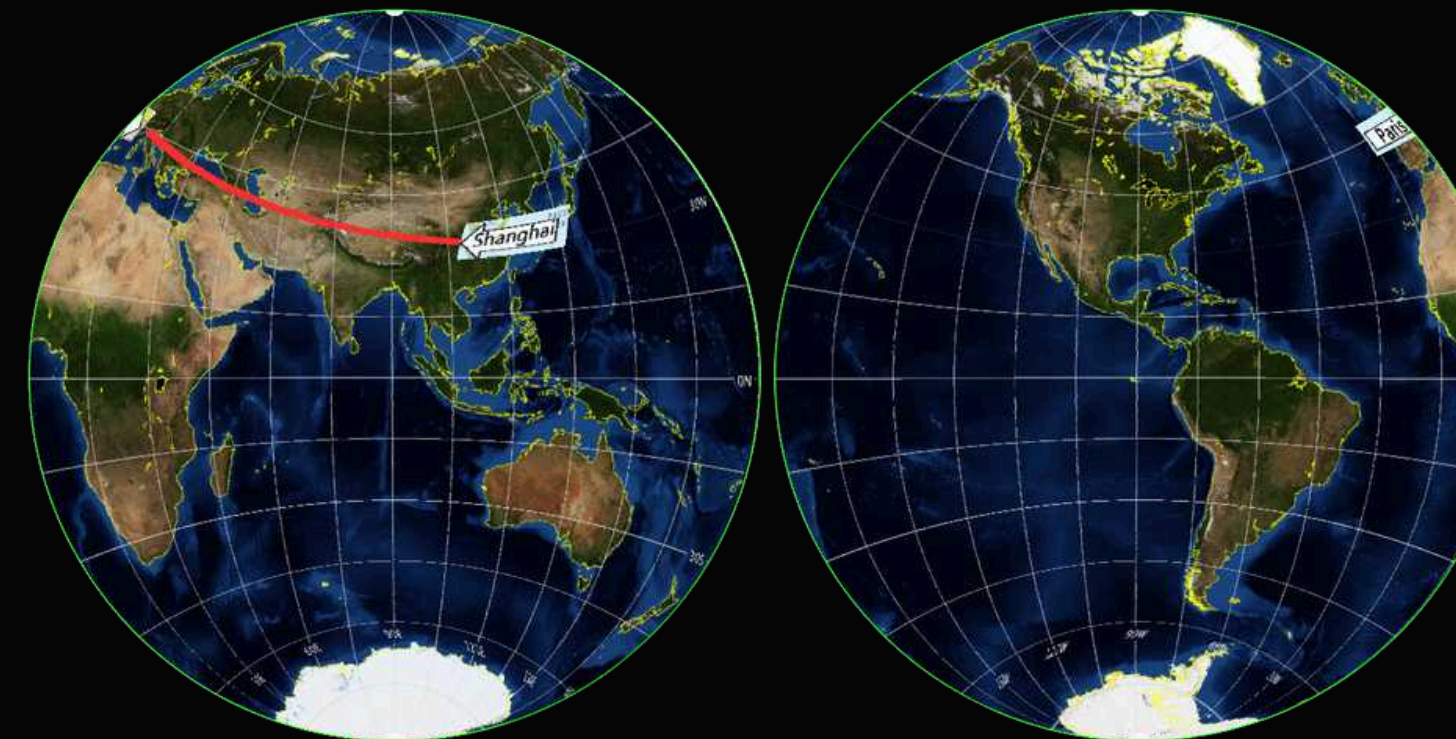
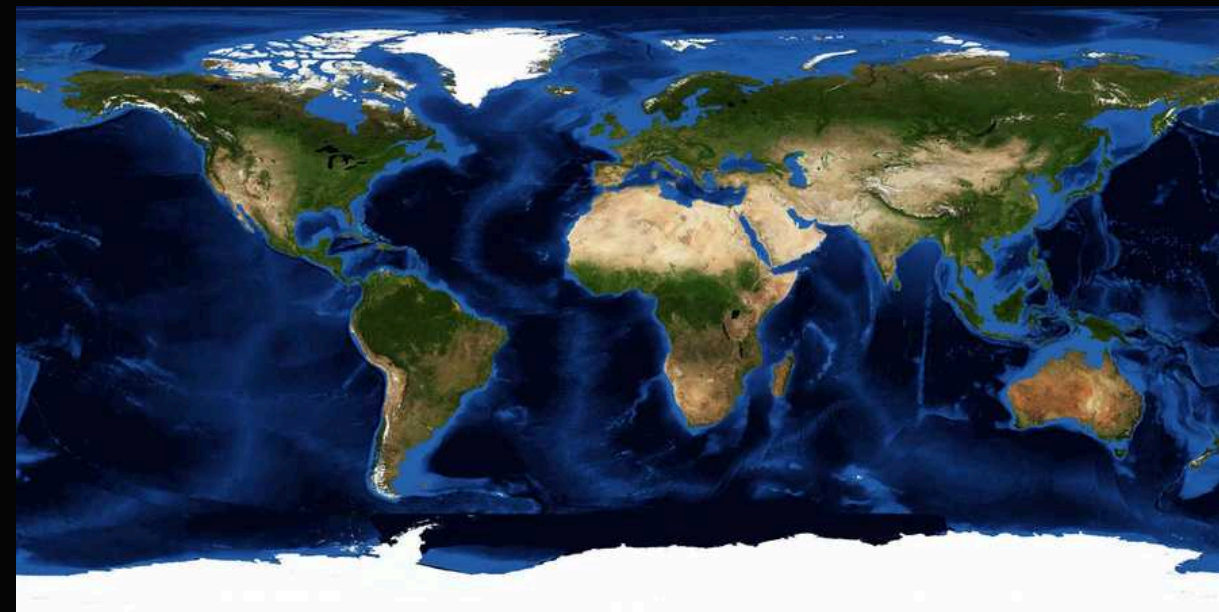
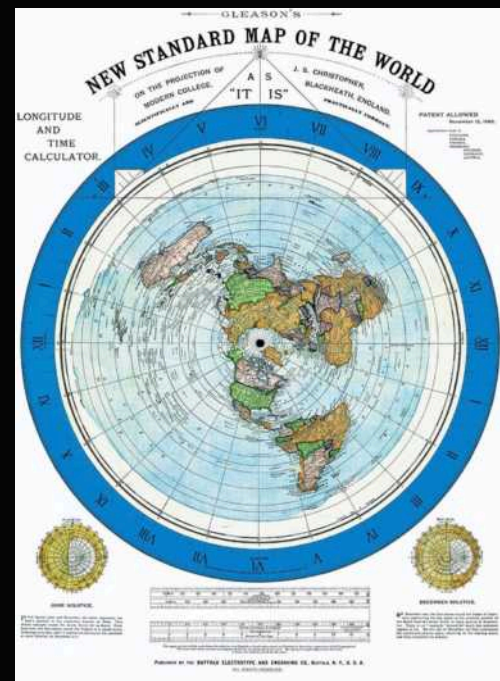
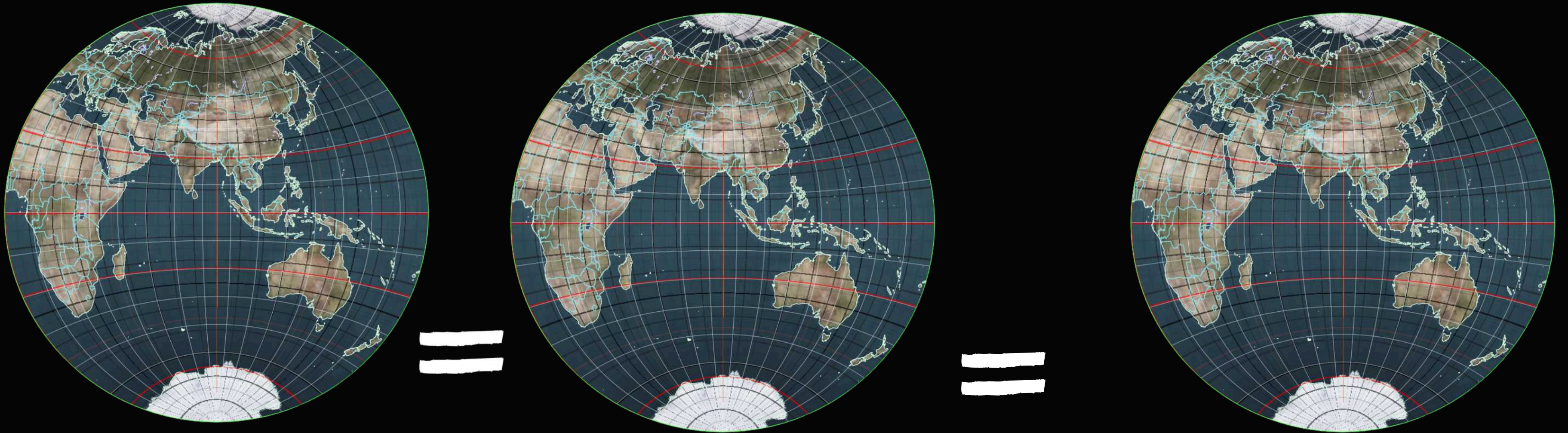
Examples of orthogonal coordinate systems:

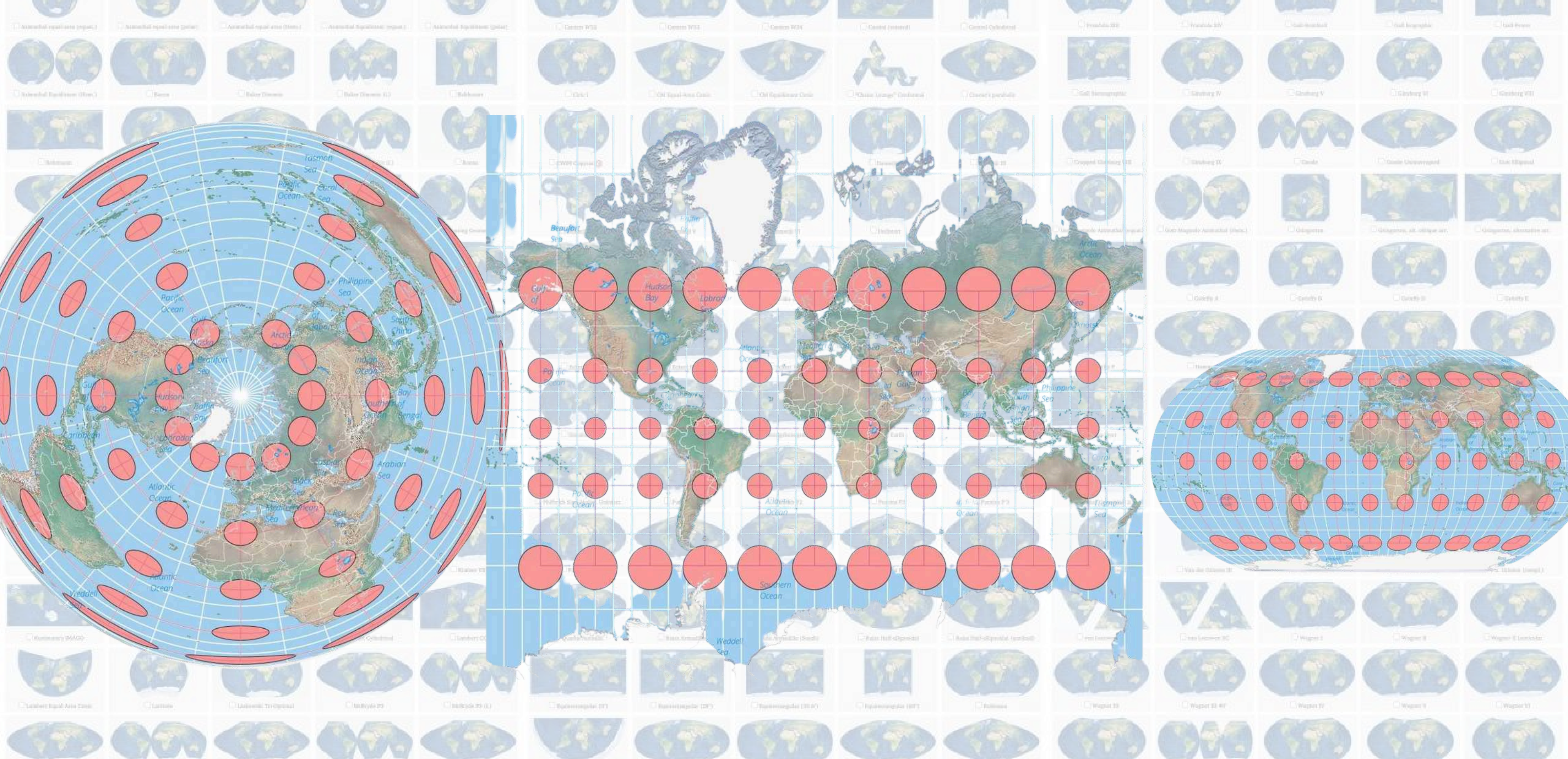
the Cartesian (or rectangular),
the circular cylindrical,
the spherical,
the cylindrical,
the conical,
the spheroidal,
and the ellipsoidal.





QUIT COMPARING GLOBE DATA TO GLOBE DATA





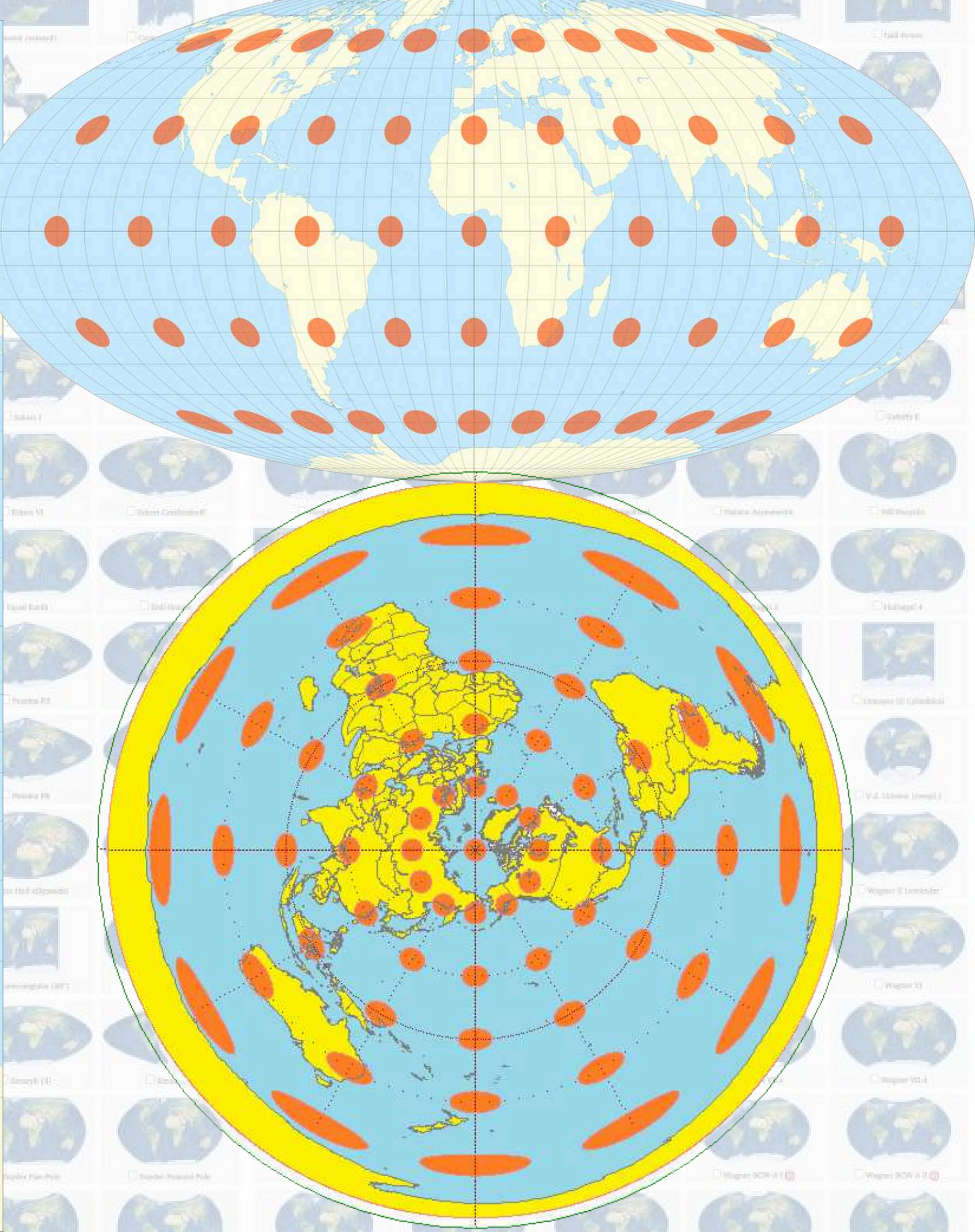
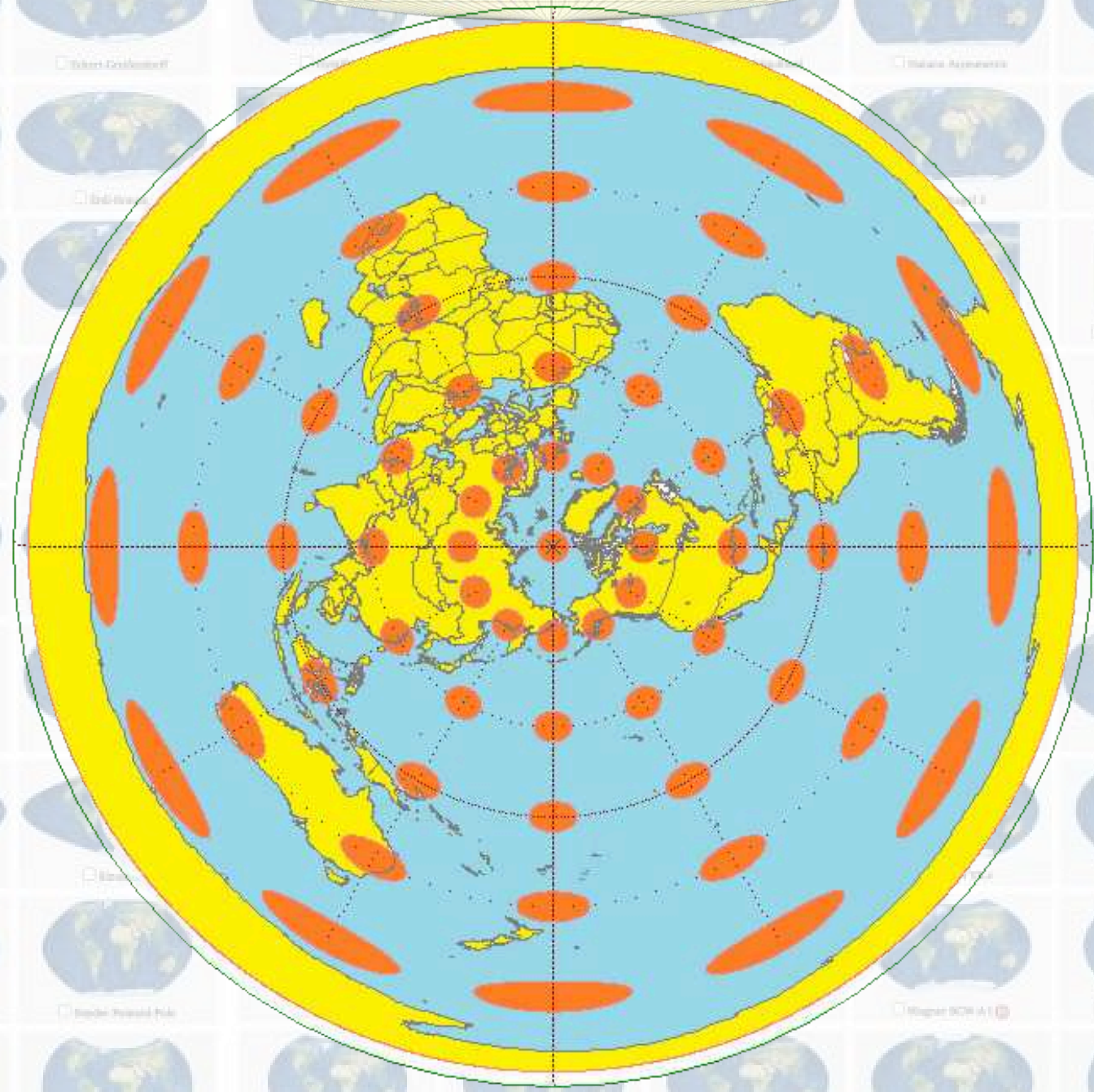
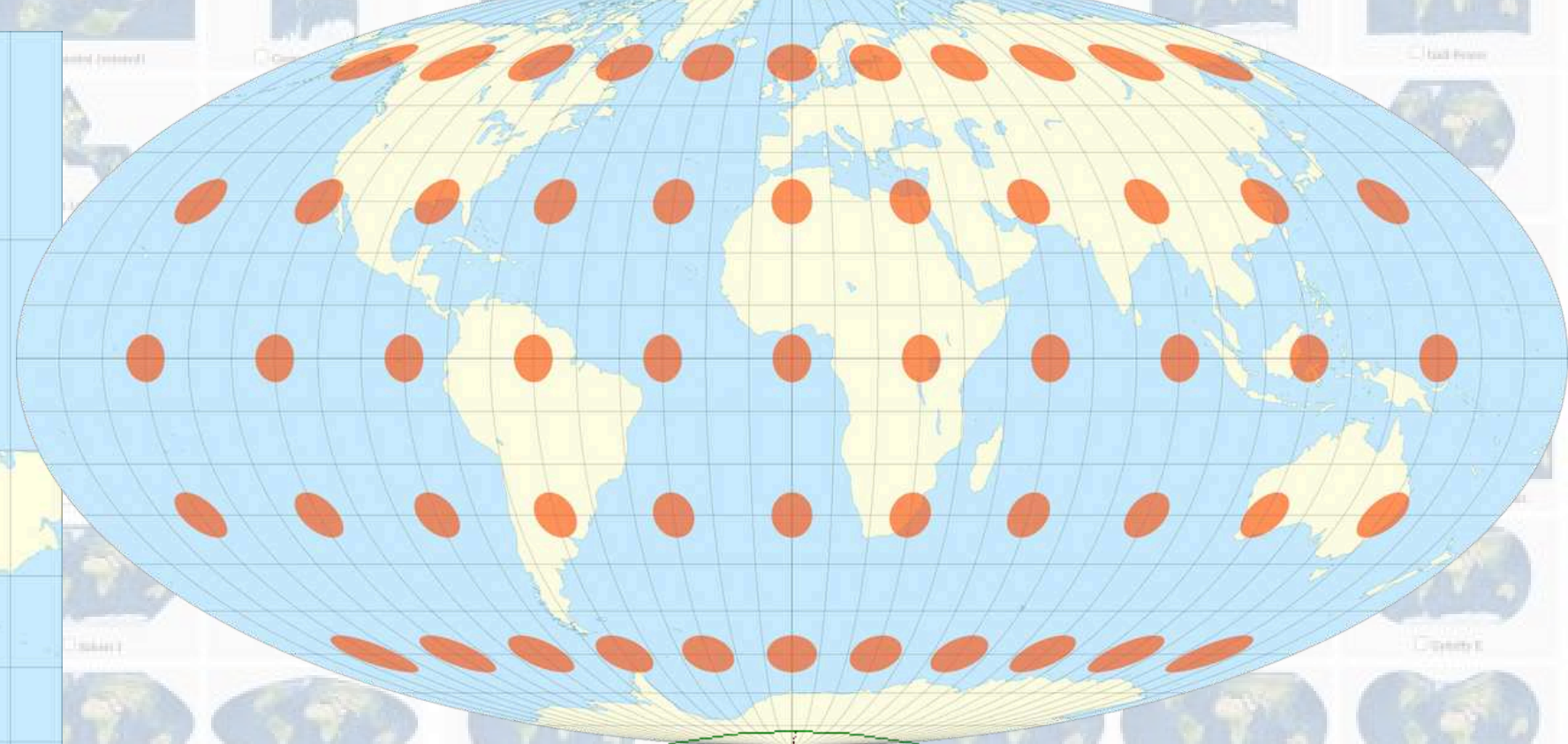
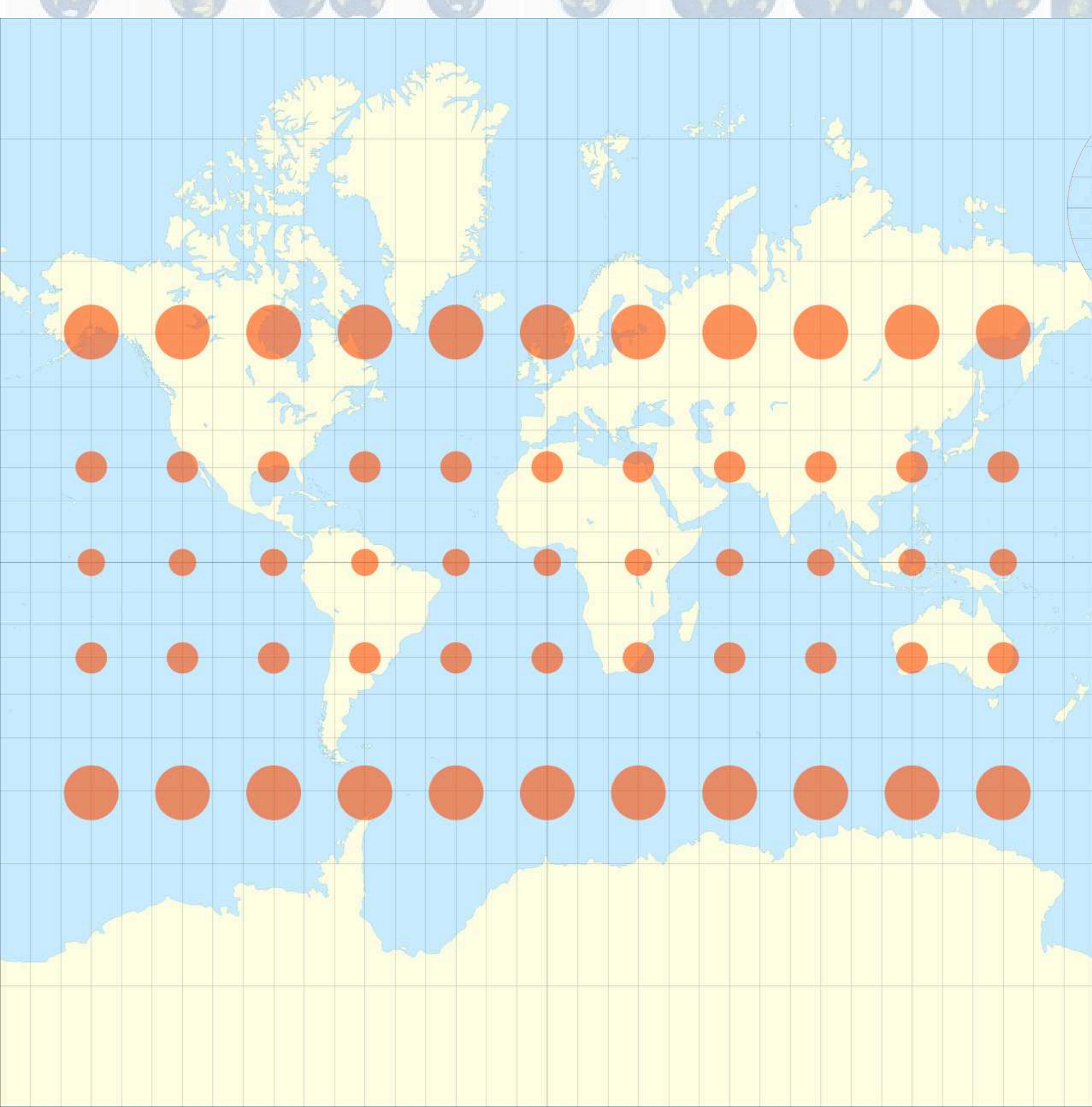
TISSOT'S INDICATRIX WAS CREATED BY A FRENCH MATHEMATICIAN NAMED NICOLAS AUGUSTE TISSOT BETWEEN 1859-1871. HE SHOWED HOW THE GEOMETRY OF PUTTING AN OBJECT LIKE A GLOBE ONTO A MAP CREATES AN ELLIPSE THAT HAS AXES INDICATING TWO DIRECTIONS ALONG A SCALE OF MAXIMAL AND MINIMAL POINTS ON A MAP.

TISSOT FOUND A WAY TO INDICATE HOW MUCH A [MAP'S POINTS WERE DISTORTED USING HIS SCALE]([HTTPS://WWW.GEOGRAPHYREALM.COM/CHANGING-MAP-SCALE-PANTOGRAPH/](https://www.geographyrealm.com/changing-map-scale-pantograph/)). DISTORTION VARIES ACROSS A MAP, WHICH MAKES THE SCALE IMPORTANT FOR KNOWING WHAT IS THE MOST DISTORTED AND WHAT IS ONLY SLIGHTLY DISTORTED.

THE BEST WAY TO VISUALIZE TISSOT'S INDICATRIX IS BY OVERLAYING CIRCLES ON TO A MAP. WHEN TISSOT'S INDICATRIX IS APPLIED, THE CIRCLES ARE ALTERED IN SIZE AND/OR SHAPE BASED ON HOW MUCH DISTORTION APPLIES TO THAT PART OF THE MAP

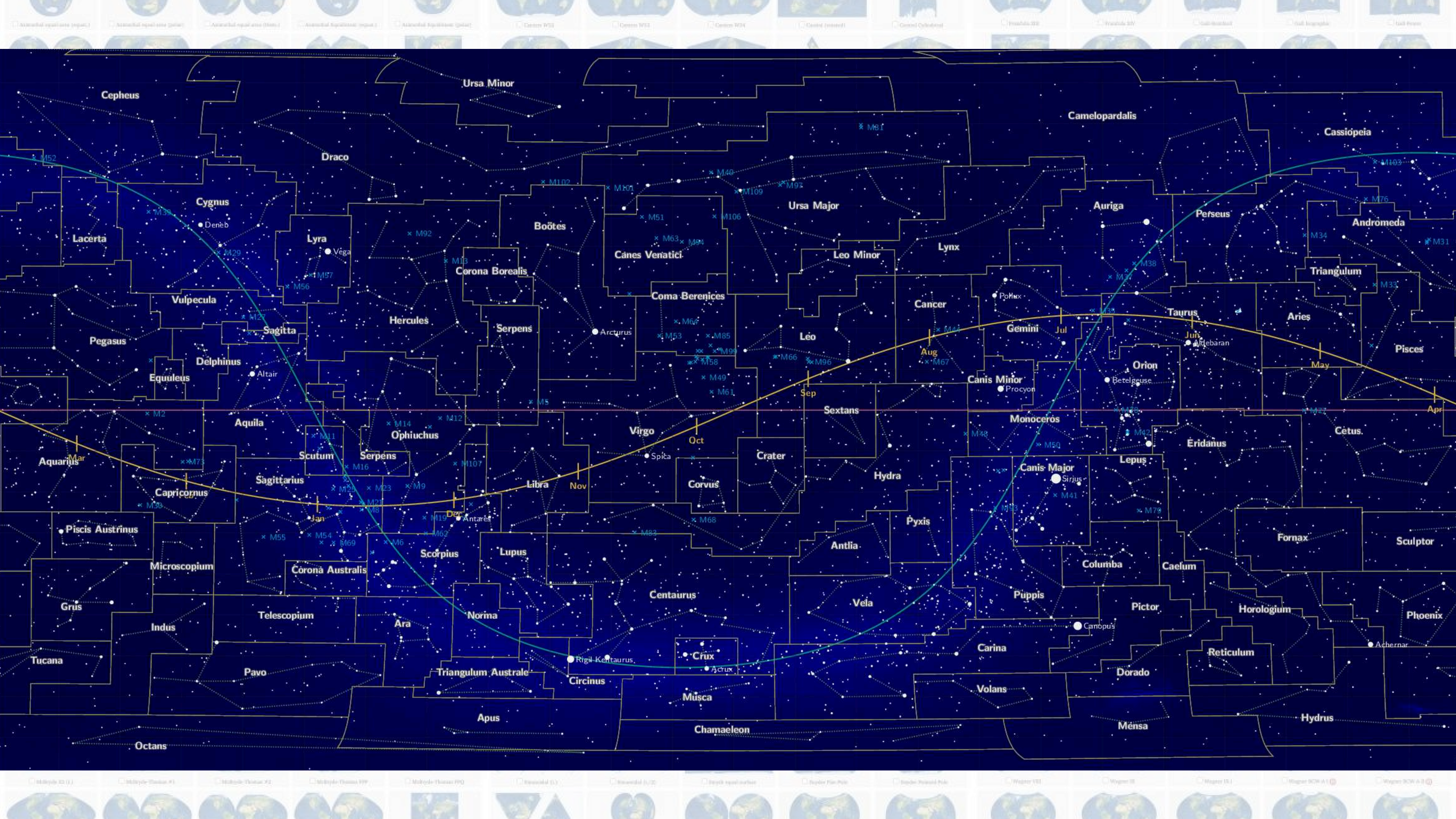
THE INDICATRIX NOT ONLY SHOWS WHERE THE MAP'S DISTORTIONS ARE, BUT HOW MUCH THEY ARE DISTORTED USING A SCALE OF MAGNITUDE.

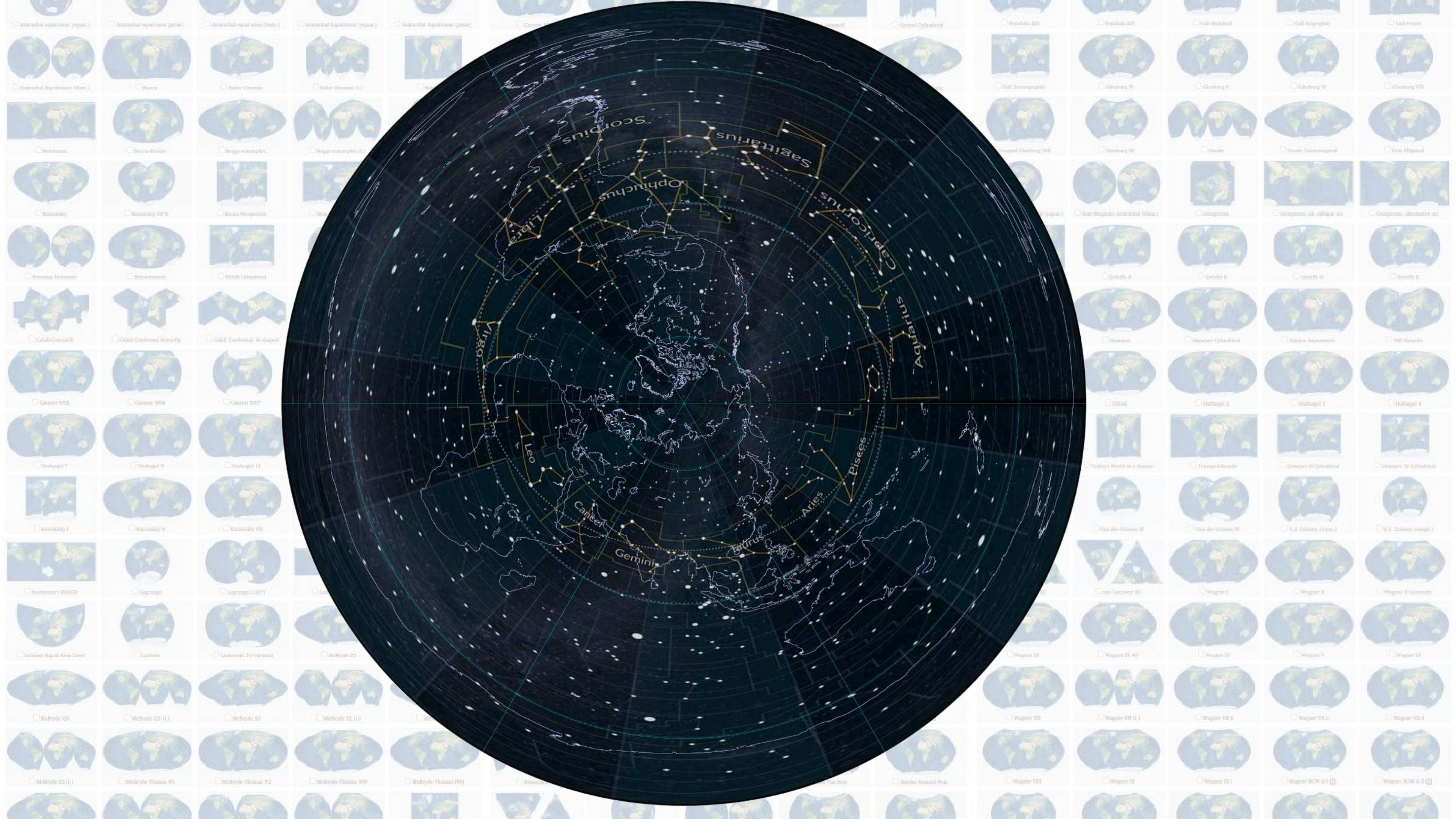
[HTTPS://EN.WIKIPEDIA.ORG/WIKI/TISSOT'S_INDICATRIX](https://en.wikipedia.org/wiki/Tissot's_Indicatrix)



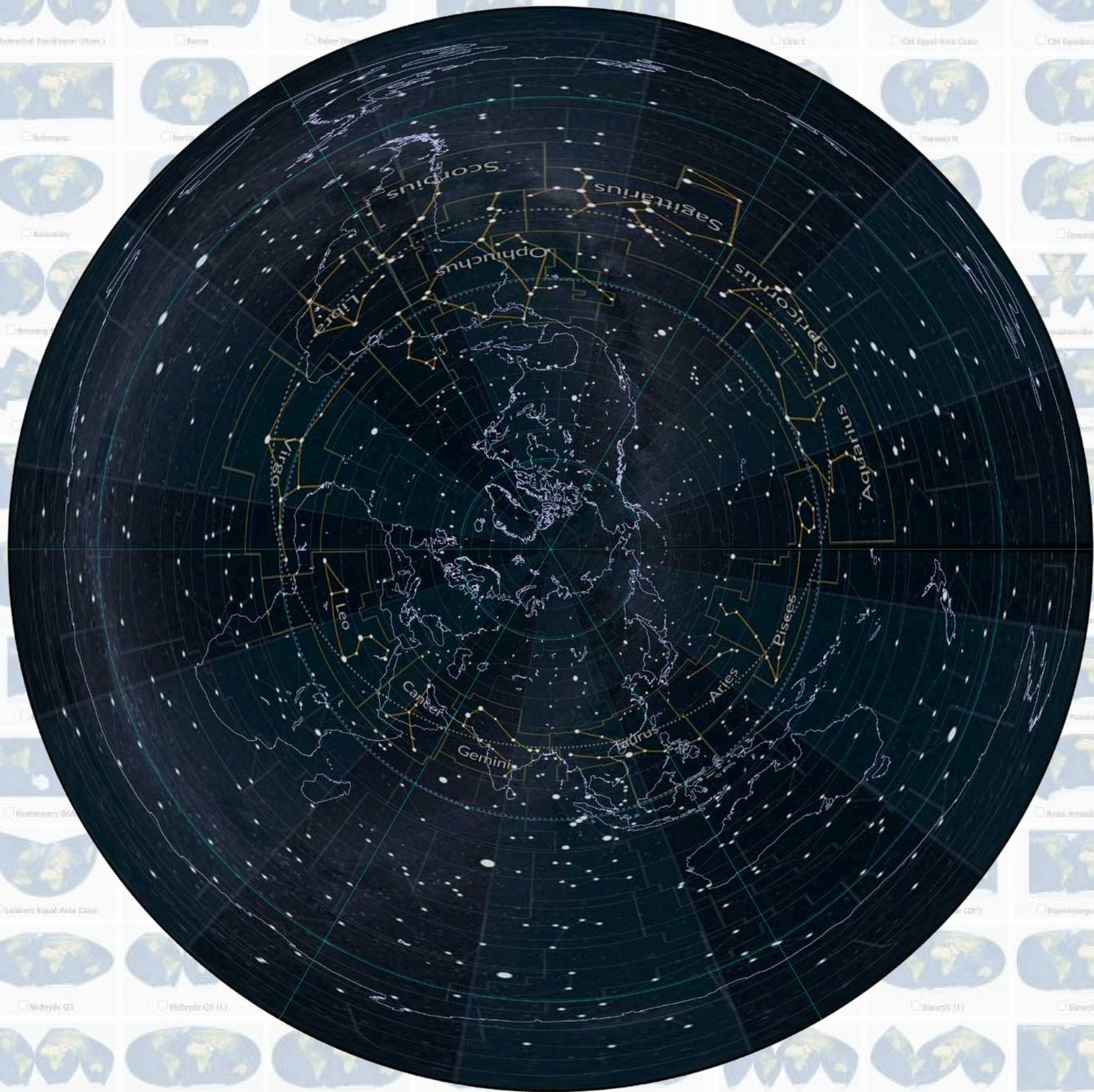


Cosmography











Thank
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