ULTRA-SHORT-WAVE REFRACTION AND DIFFRACTION*

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SUMMARY

The work of G. N. Watson on the propagation of electric waves over a spherical earth has been extended by the author to take account of the finite resistivity of the earth, the effect of which is of great importance in ultra-short-wave transmission.

The work is in such a form that the field intensities above the earth can be computed numerically. The effect of refraction in the earth's atmosphere can also be taken into account. The results for a range of wavelength between 2 and 10 m and for heights up to 4 000 m and distances up to 400 km, are published in the paper in a set of curves, the general properties of which are discussed.

The effect of atmospheric refraction is considered, and a comparison between observation and theory is made, in which good agreement is obtained, on the average, when neglecting refraction. Major changes may, however, be produced occasionally by refraction.

PROPAGATION CURVES FOR ULTRA-SHORT WAVES

It is probable that in the near future ultra-short waves, i.e. those below 10 m, will be extensively exploited for television and for short-distance point-to-point and aircraft communications. It is therefore essential to know the ranges and physical transmission properties of these waves. The curves reproduced in this paper are intended to give this information in graphical form.

The curves show the field intensity for all distances up to 400 km and at all heights up to 4 000 metres, produced by a vertical doublet—a radiator so short as to give a cosine diagram—situated on the earth's surface and radiating 1 kW. Also, by the reciprocal theorem, the rôles of receiver and sender can be reversed, so that the curves give the field produced by a raised transmitter at a receiver on the surface of the earth, within the ranges of height and distance already specified. Finally, they can be made to give the field at any given height h_1, from a transmitter at any other height h_2, measured above the spherical earth surface.

As in the case of the longer-wave broadcast transmission, the field depends to a great extent on the earth's conductivity (σ) and permittivity (κ), but whereas, in the long-wave case, oversea transmission can be regarded as equivalent to transmission over a perfect conductor, at the ultra-short wavelengths the behaviour over sea departs very markedly from that for transmission over a perfect conductor. Curves are therefore calculated for the following earth constants (in e.m.u.): $\sigma = 10^{-13}$, $\kappa = 5$, for overland transmission; $\sigma = 10^{-11}$, $\kappa = 80$, for oversea transmission. Although the curves are self-sufficient, a few words of explanation are required to indicate the assumptions made and the limitations involved.

Ultra-short-wave transmission has generally been treated by the methods of geometrical optics. Thus, in Fig. 1, a transmitter T at a height h_1 is supposed to have a visual range TR, where R is so chosen that the plane TR is tangential to the earth's surface at R.

The points on the surface of the earth beyond R are in the shadow of the " bulge " of the earth, and, according to geometrical optics, nothing would be received at such distances.

As a concession to the wave theory, it is generally admitted that some energy leaks into the shadow region, but the amount of spread is generally left to guesswork. Some such concession is necessary, because otherwise, according to the geometrical ray theory, a transmitter on the surface of the earth should have practically no range at all. It is clear that the problem is not one of geometrical optics but one that requires the full wave theory for its solution.

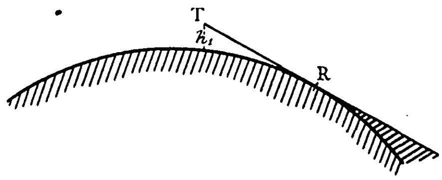

Fig. 1

This theory has been worked out by many eminent mathematicians, including Poincaré, Nicholson, Love, and MacDonald. A certain amount of disagreement was cleared up by the work of G. N. Watson, and it is generally admitted that his mathematical results are beyond reproach.

The following well-known diffraction formula, giving numerical results, has been derived by Van der Pol from Watson's analytical results:—

$$\epsilon = \frac{0 \cdot 5386 h I e^{-23 \cdot 9 \theta / \lambda}}{\lambda^{\frac{1}{3}} \sin^{\frac{1}{2}} \theta}$$

where ϵ = field intensity, in millivolts per metre; h = " effective height," in km; I = current (r.m.s.), in amps.; λ = wavelength, in km; and θ = angular distance between transmitter and receiver. In the derivation of this formula it was assumed that, in effect, the conductivity of the earth was so high that it could be considered a perfect conductor; again, the field at a height above the earth's surface was not considered. Thus the formula is not adequate to describe the behaviour of ultra-short-wave transmission where the

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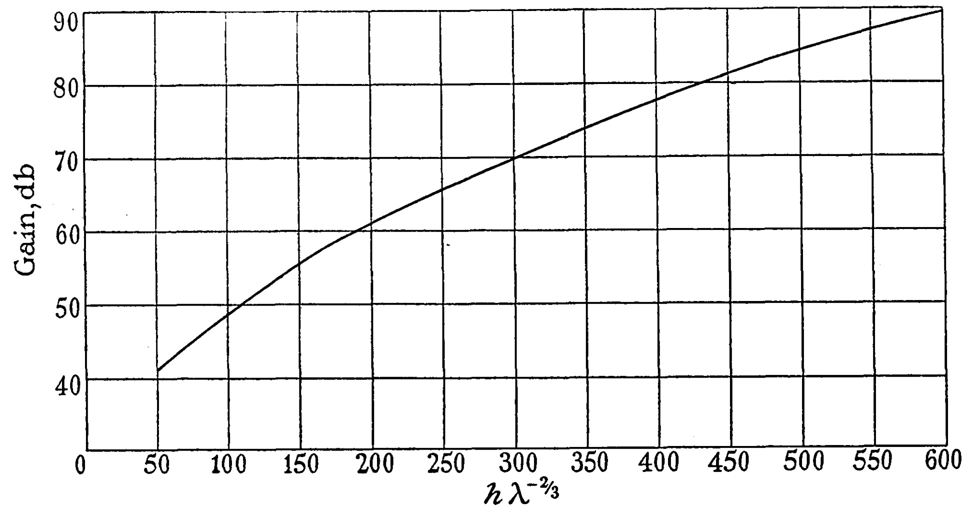

Fig. 2.—Height/gain curves (h and λ in metres).

The following are the figures of decibels to be subtracted:—
Land. 2 m, 0 db; 4 m, 2·2 db; 6 m, 3·7 db; 8 m, 4·9 db; 10 m, 5·9 db.
Sea. 2 m, 13·6 db; 4 m, 18·0 db; 6 m, 20·8 db; 8 m, 22·9 db; 10 m, 24·4 db.

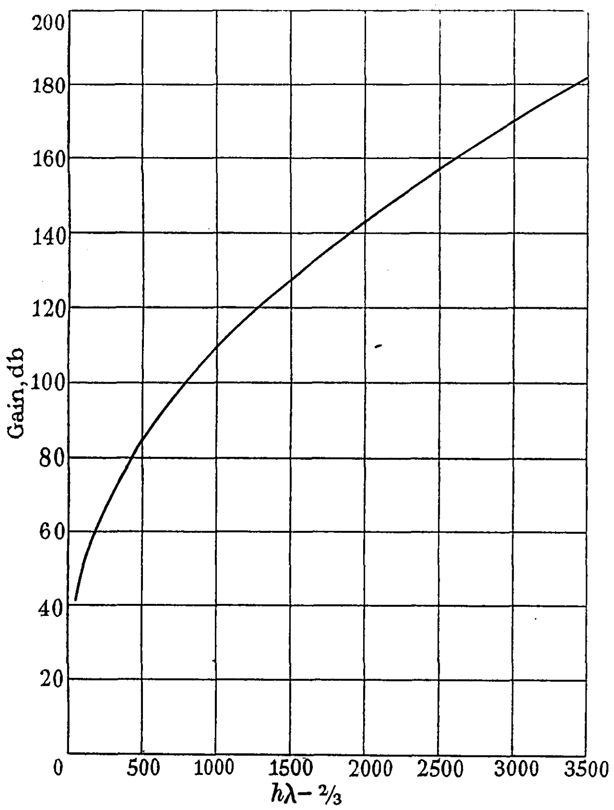

Fig. 3.—Height/gain curves (h and λ in metres).

The following are the figures of decibels to be subtracted:—
Land. 2 m, 0 db; 4 m, 2·2 db; 6 m, 3·7 db; 8 m, 4·9 db; 10 m, 5·9 db.
Sea. 2 m, 13·6 db; 4 m, 18·0 db; 6 m, 20·8 db; 8 m, 22·9 db; 10 m, 24·4 db.

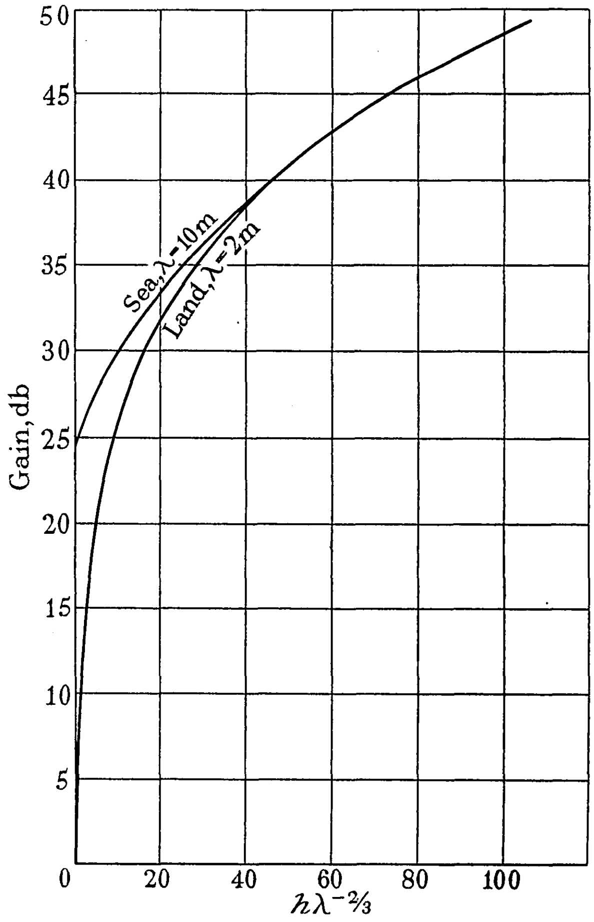

Fig. 4.—Height/gain curves (h and λ in metres).

The following are the figures of decibels to be subtracted:—
Land. 2 m, 0 db; 4 m, 2·2 db; 6 m, 3·7 db; 8 m, 4·9 db; 10 m, 5·9 db.
Sea. 2 m, 13·6 db; 4 m, 18·0 db; 6 m, 20·8 db; 8 m, 22·9 db; 10 m, 24·4db.

effect of the height of the receiving and transmitting aerial requires to be known and where the effect of the earth's constants must be included. A short-cut method enabled the present author* to determine the modification of the exponential factor in the above equation due to the finite conductivity of the earth. A development of this method, by which certain characteristic numbers involved in Watson's solutions can be recalculated for the case of finite earth conductivity, has enabled the complete solution to be obtained at various heights and distances. The results are in such a form that the effect of atmospheric refraction can be included. The curves here presented are derived from these mathematical results, but the effect of refraction is neglected. They are based, therefore, on the assumption that the earth is of uniform conductivity σ and permittivity κ, and that the permittivity of the regions outside the earth is constant and equal to unity. The general solution is obtained as the sum of a number of terms of which the first is predominant at sufficiently large distances and for sufficiently small heights. The curves (Figs. 6–25) are calculated in the regions where the first term is predominant. In each figure, the broken line gives the inverse distance field for propagation over a plane perfectly-conducting earth. The estimated uncertainty in the curves outside these regions is not greater than 2 or 3 db.

CHARACTERISTICS OF CURVES

The most obvious feature of the curves is that in the region where the first term of the diffraction formula is predominant, the curves for various heights are all parallel. This implies that the gain in decibels resulting from an increase in height of receiver (or transmitter) above the earth's surface, is independent of the distance from the transmitter. Again, on ultra-short wavelengths the slope of the curves (loss, in db per km) is independent of the earth constants.

Finally, a close scrutiny shows another general property possessed by the curves, a property which is also, of course, implicit in the mathematical analysis. It is this: above a certain height h_0, which is only a slowly varying function of the wavelength, the gain in field strength with height is, to a high degree of approximation, independent of the earth constants. It is consequently only a function of h and λ in the form $f(h\lambda^{-2/3})$. Experimental measurement of the height/gain relation, as well as measurements of the slope of the field-intensity/distance curves, afford, therefore, a very useful check on the theory, since in this comparison between observation and theory no doubtful earth-constants are involved. A single master curve, in which the gain in decibels above h_0 is plotted as a function of $h\lambda^{-2/3}$, can therefore be constructed. This auxiliary curve is shown in Figs. 2 and 3. It gives the part of the curve for the height/gain relation for heights above h_0. The value of h_0 is such that $h_0\lambda^{-2/3}$ is practically equal to 50.

An example will make the use of this curve clearer. Thus, if we wish to know the ratio of the field intensity at a height h to that at h_0, we first calculate $h\lambda^{-2/3}$ (h and λ are expressed in metres). Suppose h is 800 m and λ is 8 m; then $h\lambda^{-2/3} = 200$. The value on the decibel scale corresponding to this is 61 db. The value at h_0 is 41 db. Therefore the ratio of the signal intensity at $h = 800$ m, $\lambda = 8$ m, to that at h_0 is (61 − 41) db, i.e. 20 db, or 10 to 1.

Below h_0, however, the height/gain relation is very sensitive to the earth constants. Thus, in raising the receiver or transmitter from the ground to a height h_0, there is a very much greater gain over land than over sea. This is shown in Figs. 4 and 5, where the single curve

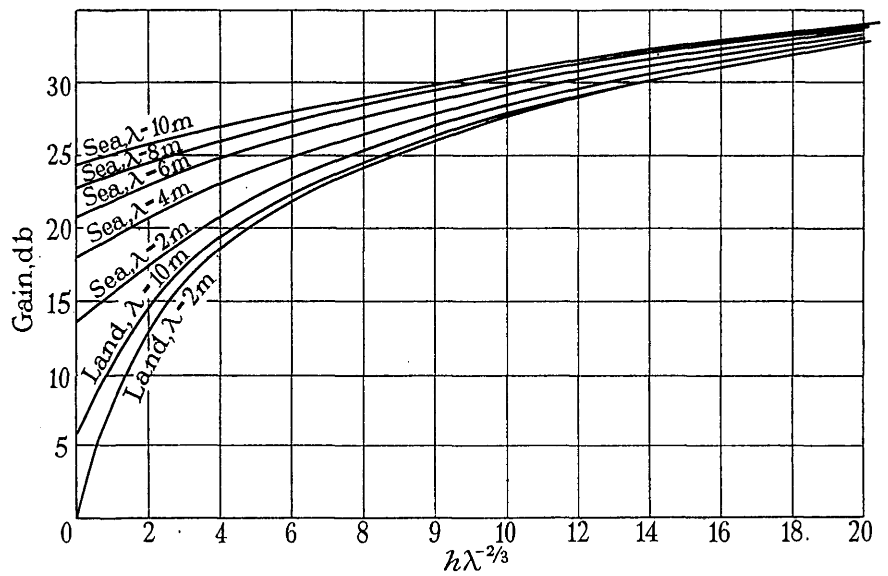

Fig. 5.—Height/gain curves (h and λ in metres).

The following are the figures of decibels to be subtracted:—
Land. 2 m, 0 db; 4 m, 2·2 db; 6 m, 3·7 db; 8 m, 4·9 db; 10 m, 5·9 db.
Sea. 2 m, 13·6 db; 4 m, 18·0 db; 6 m, 20·8 db; 8 m, 22·9 db; 10 m, 24·4 db.

* *Proceedings of the Royal Society*, A, 1932, vol. 136, p. 499.

giving the relation between $h\lambda^{-\frac{2}{3}}$ and decibels gain, which is appropriate for $h\lambda^{-\frac{2}{3}} > 50$, fans out into a intensity above the ground intensity by using the figures appended to the curves. Thus, if we obtain directly

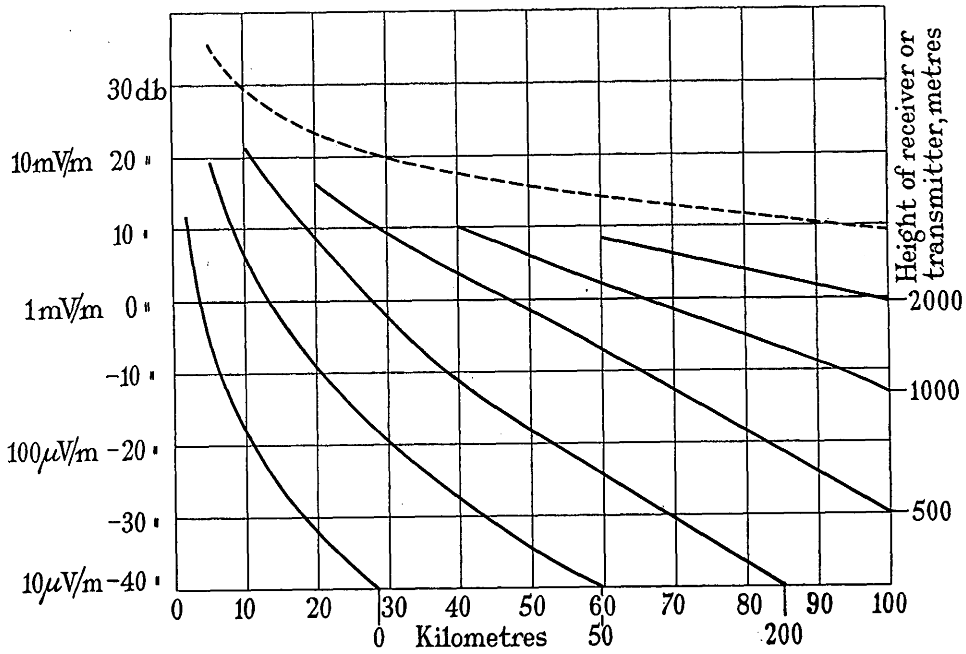

Fig. 6.—Results obtained over sea; $\lambda = 2$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

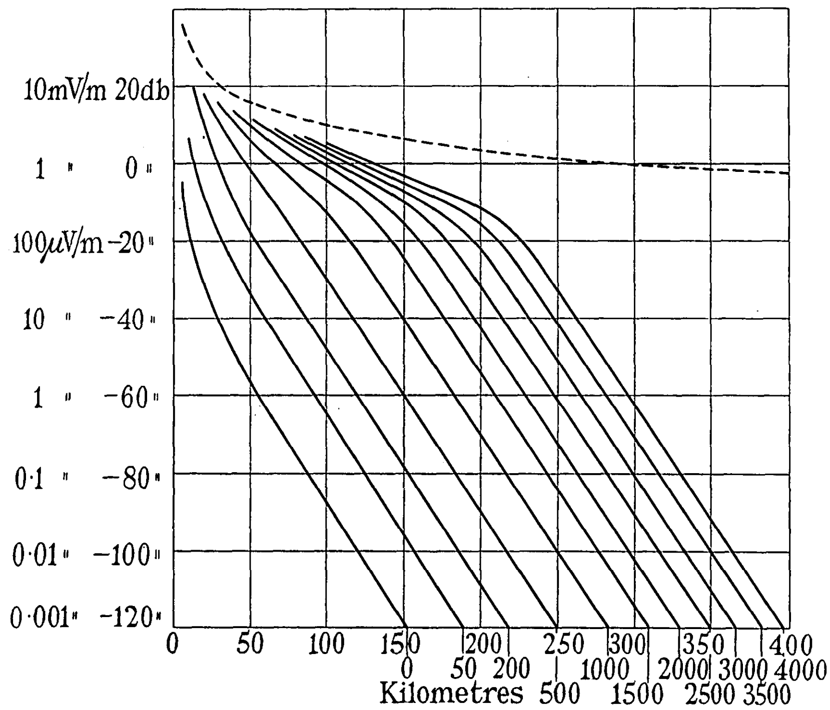

Fig. 7.—Results obtained over sea; $\lambda = 2$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.
Figures against curves indicate height of receiver or transmitter in metres.

sheaf of curves for different values of the earth constants and wavelengths in the region where $h\lambda^{-\frac{2}{3}}$ lies between 0 and 50. Figs. 2–5 can be used to obtain the gain from the curves the decibels corresponding to a given height h, then we have only to subtract the figure appropriate to the nature of the transmission (e.g. for

8 m over land, 4·9 db) to obtain the gain (on a wavelength of 8 m) due to raising the receiver to a height h above the land.

curves (Figs. 6 to 25), and then the gain on raising the receiver to a height h_2 is given by the curves of Figs. 2, 3, 4, and 5. The resultant field is thus found.

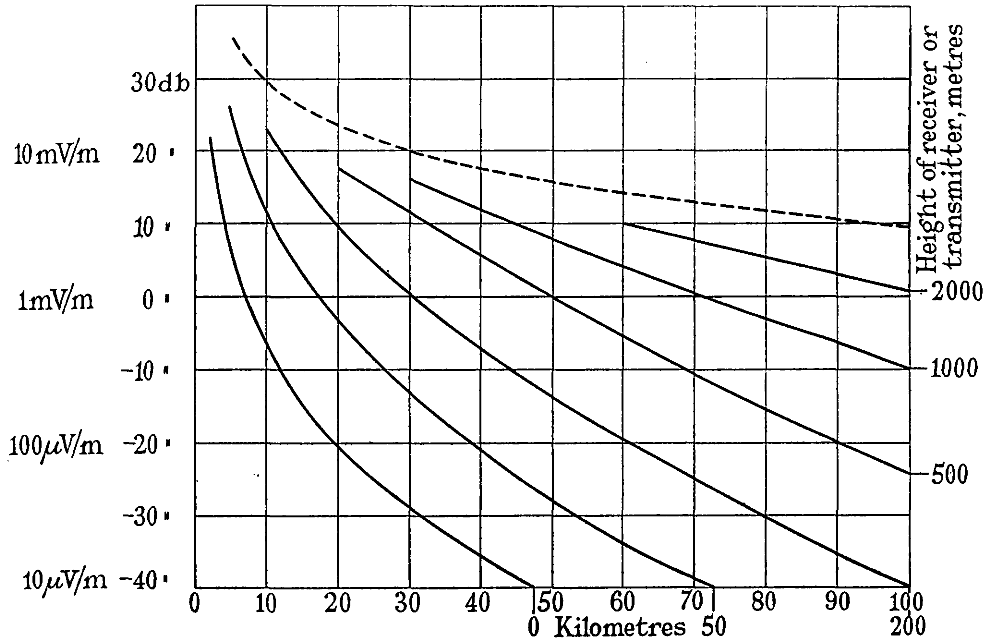

Fig. 8.—Results obtained over sea; $\lambda = 4$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

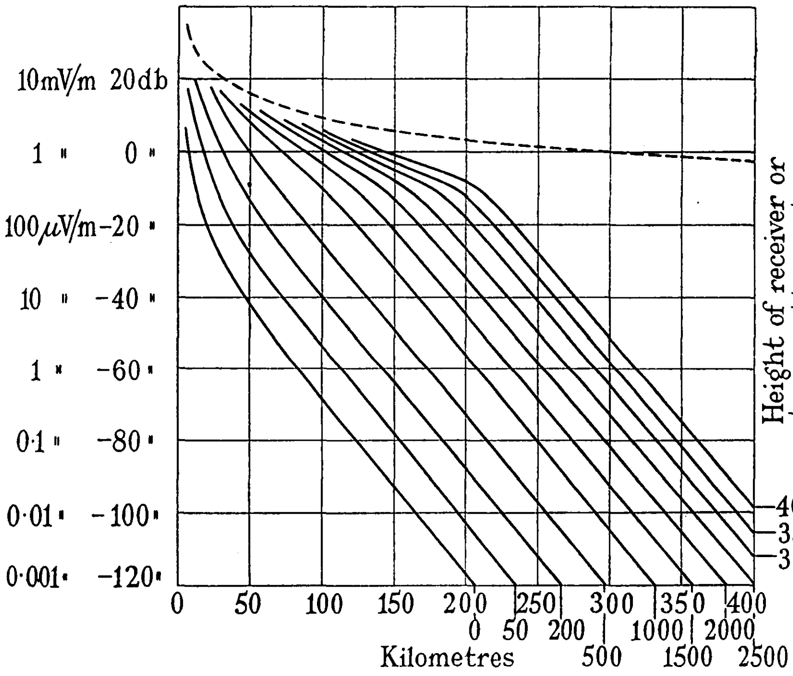

Fig. 9.—Results obtained over sea; $\lambda = 4$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

We can use these curves to give the field at a height h_2 at the receiver due to a transmitter at height h_1. Thus the field on the ground at the required distance from a transmitter at height h_1 is first determined from the

The characteristics of the usual working theory, in which diffraction effects are only introduced beyond the geometric " visual " range, are illustrated diagrammatically in Fig. 26. The field (curve A) maintains its inverse distance characteristics up to the visual range, after which it drops suddenly in the shadow region. The true curve for heights above 200 metres illustrates these charac-

distance, transmission behaves approximately as the rough working theory suggests. On the other hand, for the lower heights, up to 200 metres or so (curve B), such

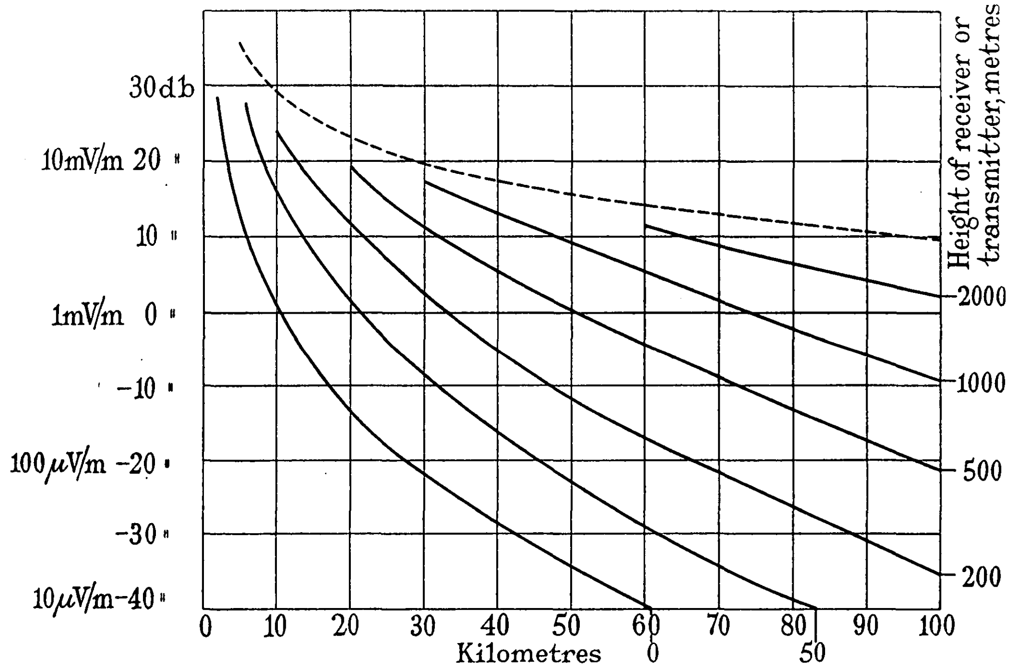

Fig. 10.—Results obtained over sea; $\lambda = 6$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

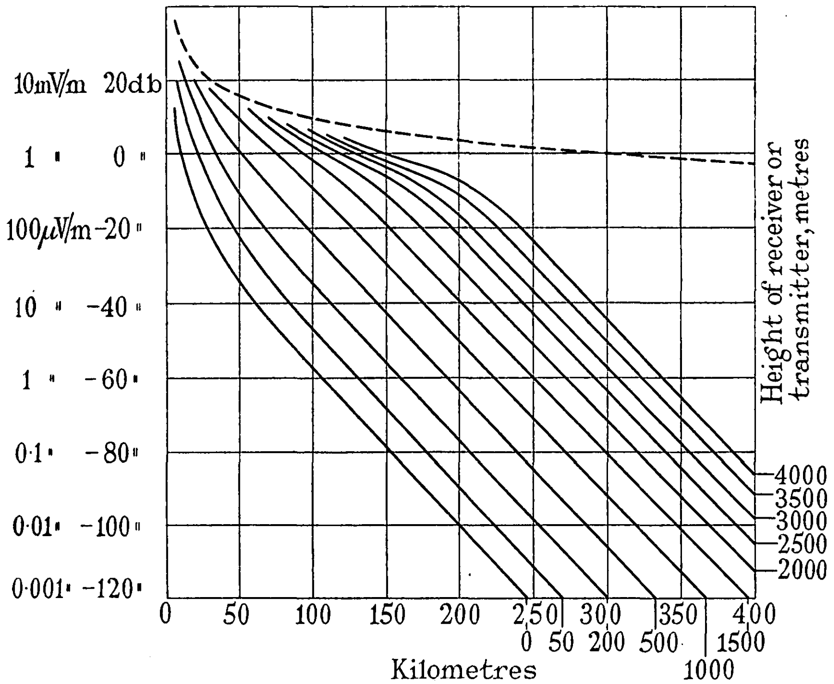

Fig. 11.—Results obtained over sea; $\lambda = 6$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

teristics, though the point of inflection P at the visual range is very much rounded off, and the actual field at the visual range is some 10 to 30 db below the unobstructed (inverse distance) value. Thus at sufficient height and

as might be used in television practice, there is absolutely no indication on the curve of any inflection such as occurs at the visual range in the other cases.

To express the results in terms of visual range, in the expectation of a sudden drop of field strength beyond this, is, if not false, at least meaningless. Physically, the meaning of the results is fairly clear. The earth has taken such a toll of energy in the case of relatively low transmitters, even within visual range, that the extra shadow effect beyond this range is entirely obscured.

This characteristic, namely the absence of marked quasi-optical visual-range effect, is associated with the following fact noted in the mathematical treatment. It is found that the results can be expressed in a quasi-optical manner with respect to a fictitious radius (R_0) of the earth which is greater than the actual radius (R). The difference between R_0 and R is of the order of

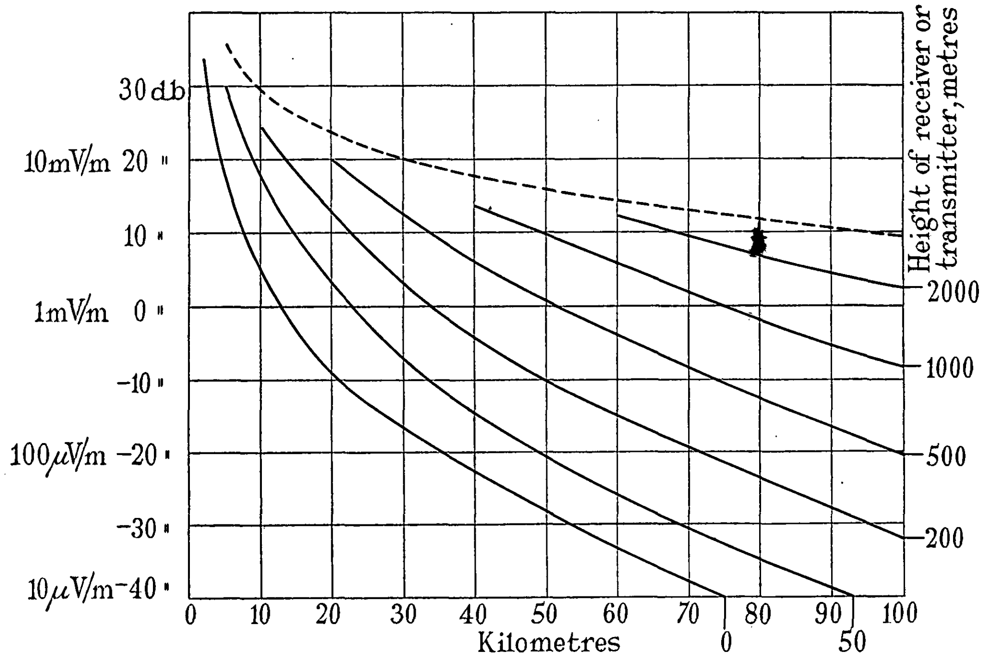

Fig. 12.—Results obtained over sea; $\lambda = 8$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

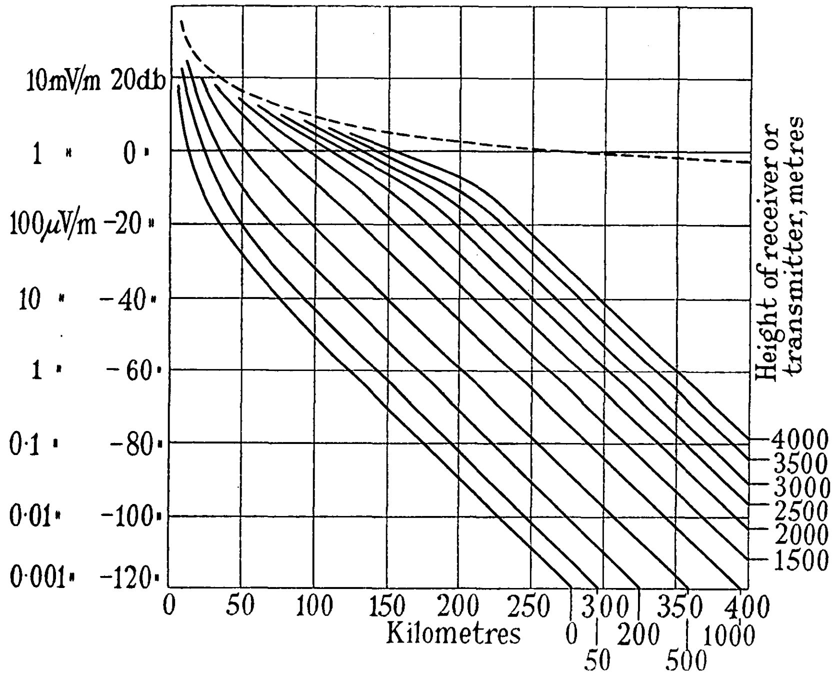

Fig. 13.—Results obtained over sea; $\lambda = 8$ m, $\kappa = 80$, $\sigma = 10^{-11}$, 1 kW radiated.

200 metres, for a wavelength of 10 metres, and is a slowly varying function of the wavelength as well as of the earth's constants. The height, $R_0 - R$, is the appear. These results are illustrated in Fig. 28(*a*), from which it is obvious that, if the height of the transmitter is less than ($R_0 - R$), no meaning can be attributed to visual

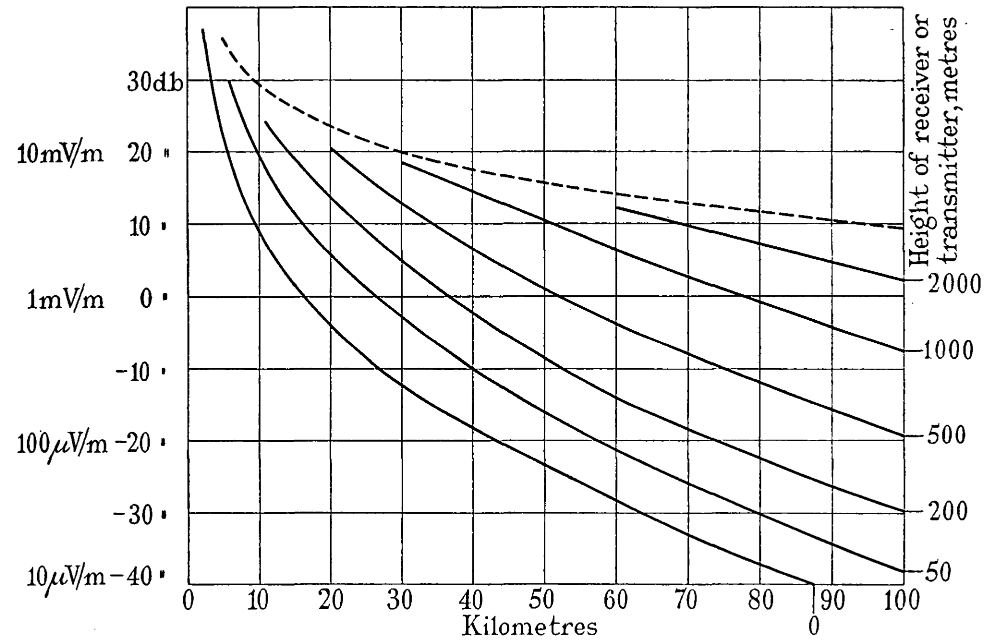

Fig. 14.—Results obtained over sea; λ = 10 m, κ = 80, $\sigma = 10^{-11}$, 1 kW radiated.

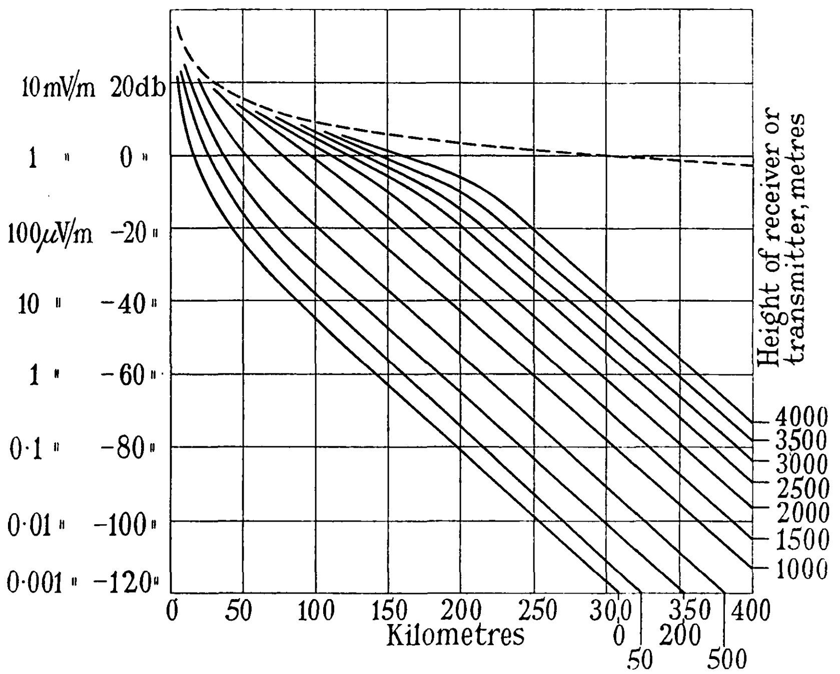

Fig. 15.—Results obtained over sea; λ = 10 m, κ = 80, $\sigma = 10^{-11}$, 1 kW radiated.

height h_0 referred to in the previous paragraphs, and is shown in Fig. 27 as a function of the wavelength. It is only when the transmitter (or receiver) is well above this radius that the quasi-optical visual-range effects begin to range in the altered sense described above. This leads to the following practical rule: If unobstructed transmission is required between points T and B, the line TB must not approach the earth nearer than ($R_0 - R$.)

For heights less than about 500 metres, and distances less than 50 km but greater than about 3 km, the field at R is calculated as the resultant of the direct ray TR and the reflected ray TER [Fig. 28(*b*)]. The

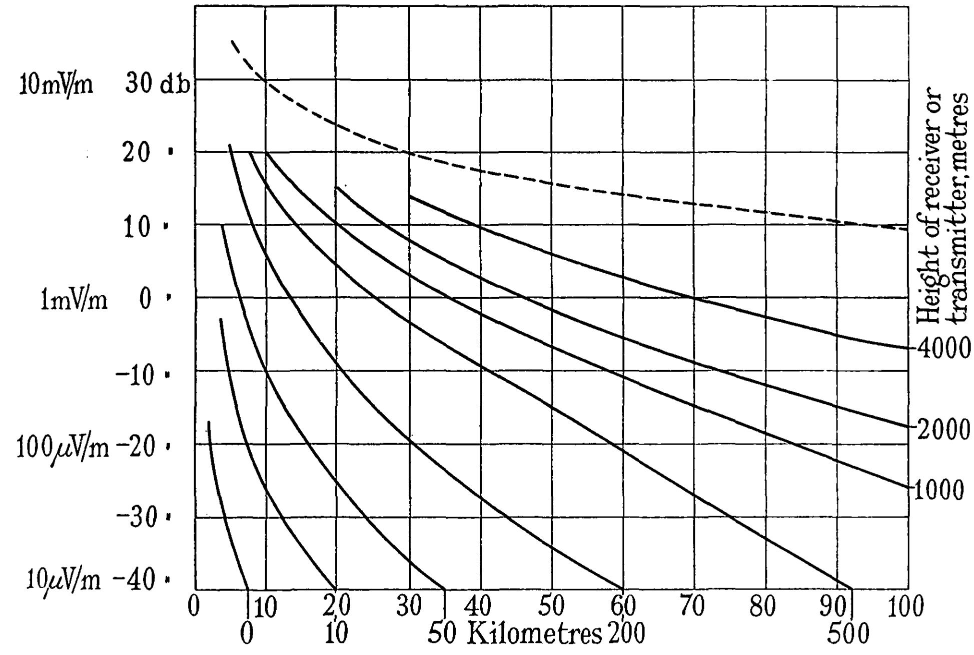

Fig. 16.—Results obtained over land; $\lambda = 2$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

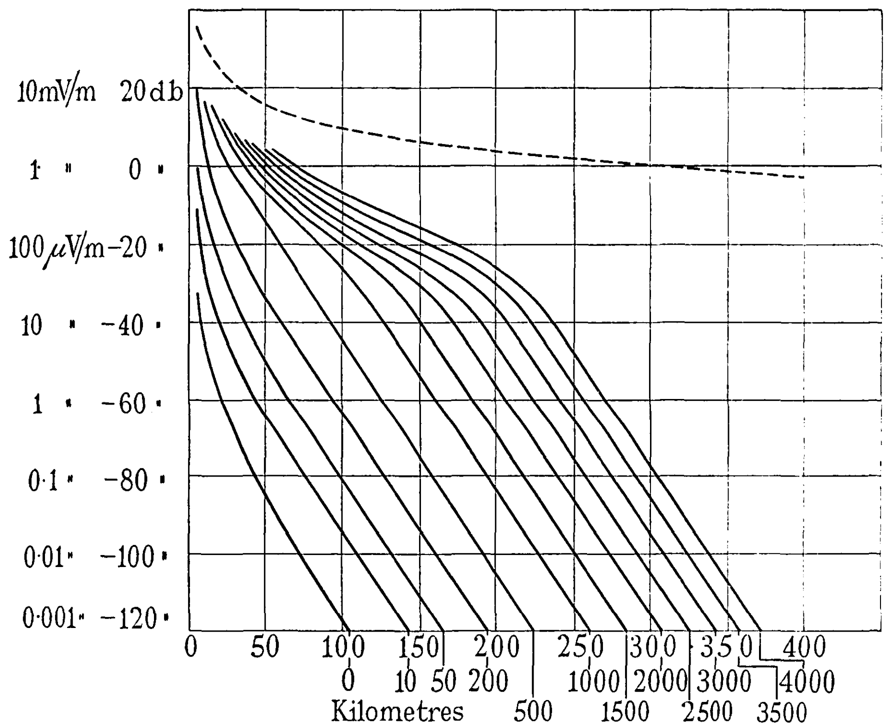

Fig. 17.—Results obtained over land; $\lambda = 2$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.
Figures against curves indicate height of receiver or transmitter in metres.

intensity is proportional to the inverse square of the distance. This follows from Sommerfeld's theory, or equally well from the approximate theory by which the larger-scale curves for distances up to 100 km are calculated in this manner, and the transition to the diffraction curves is reasonably well defined (see Fig. 29). These results have been recently checked experimentally,* but as long ago as January, 1925, the same law was determined experimentally in a 10-metre transmission in a range C.C.I.R. Documents, Lisbon (vol. 1, Propositions, p. 1294), in which the measured attenuation is shown to agree closely with the calculated value. The theoretical gain

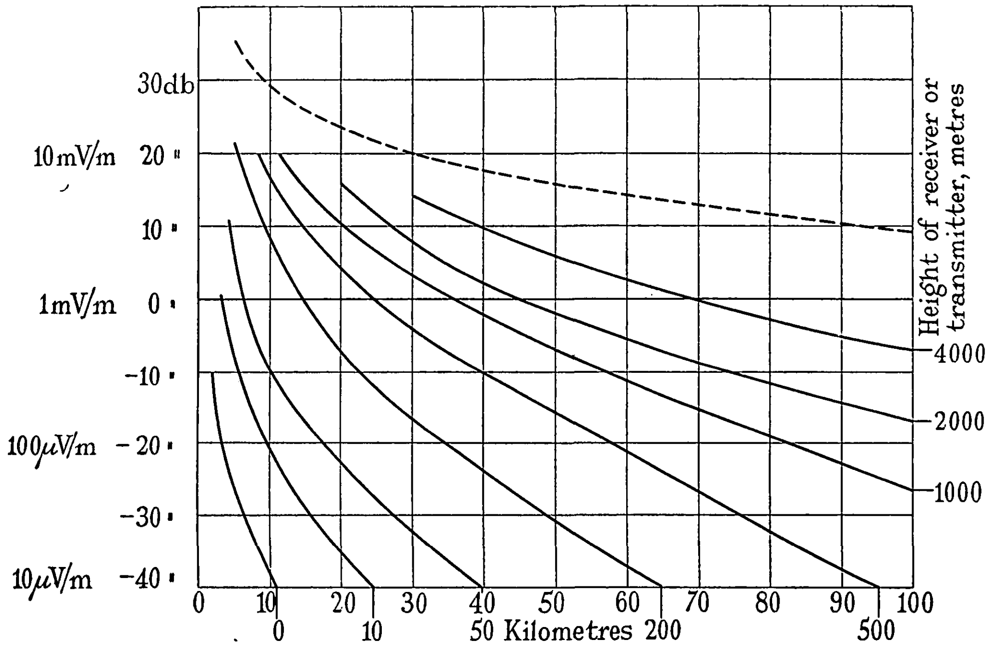

Fig. 18.—Results obtained over land; $\lambda = 4$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

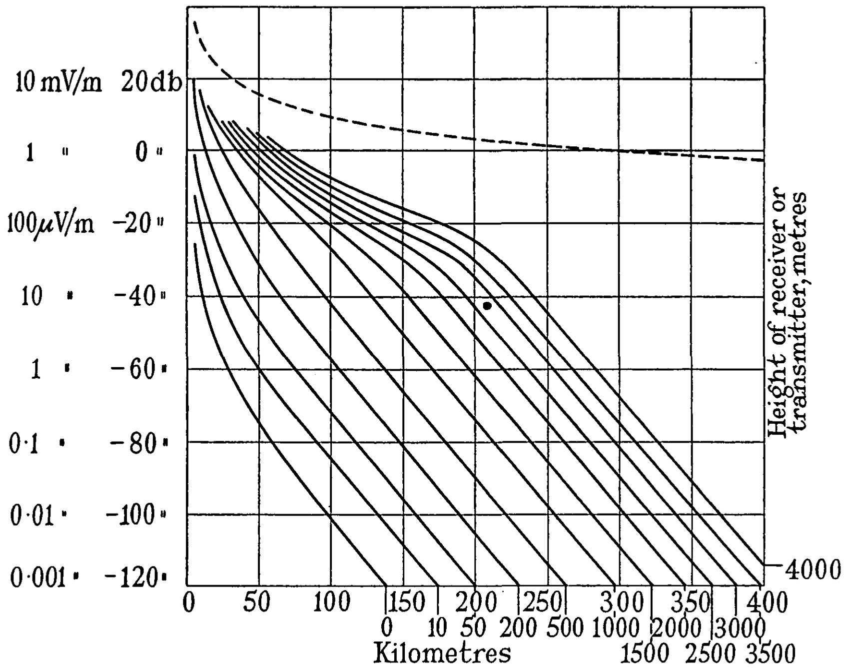

Fig. 19.—Results obtained over land; $\lambda = 4$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

between 5 and 25 miles and correctly explained as a consequence of Sommerfeld's theory. Other experimental confirmations of these diffraction curves are given in in decibels as a function of the height has also been checked against the experimental autogyro results obtained by Trevor and Carter and by Jones, and good agreement found. The above experimental checks constitute a considerable body of evidence that, although

* B. Trevor and P. S. Carter: *Proceedings of the Institute of Radio Engineers*, 1933, vol. 21, p. 387; also C. R. Burrows, L. E. Hunt, and A. Decino: *Bell System Technical Journal*, 1935, vol. 14, p. 253.

fading at extreme distances may imply a small degree of refraction, the major part in ultra-short-wave transmission is played by diffraction.

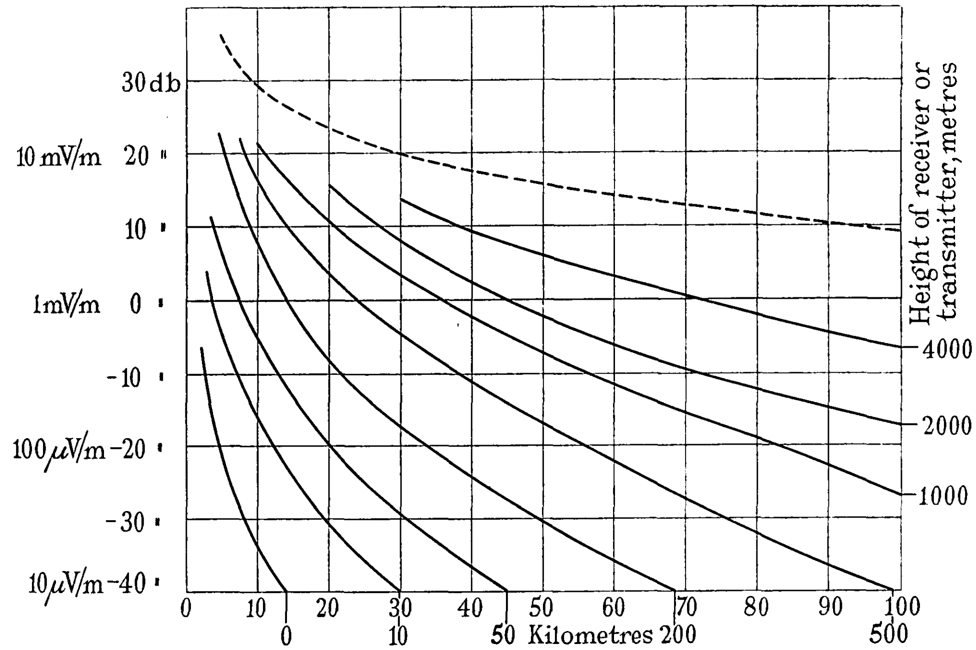

Fig. 20.—Results obtained over land; $\lambda = 6$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

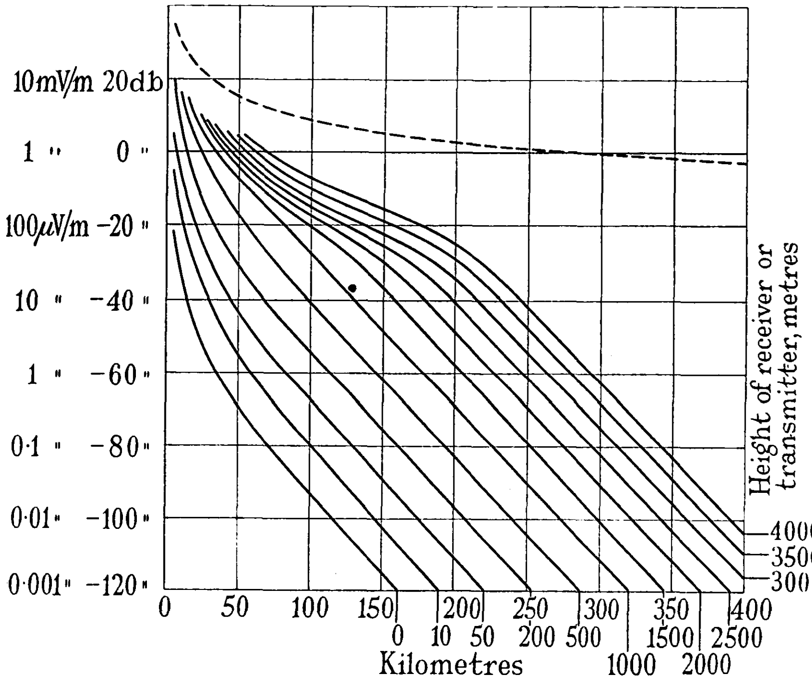

Fig. 21.—Results obtained over land; $\lambda = 6$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

REFRACTION

Although, as stated above, the major factor controlling ultra-short-wave propagation is diffraction, there is not wanting evidence of variation of signal intensity at extreme distances, which the author considers can only be caused by variation in the gradient of the refractive index of the air near the surface of the ground. The alternative hypothesis that the variations are due to reflections from the ionosphere seems very doubtful, for, although there is evidence of reflections at normal incidence of waves well above the critical frequency, the reflections occur in sporadic bursts and are very unlike the slow fadings observed in ultra-short-wave propagation. On the other hand the extreme ranges, recently recorded, of police-car signals on 9 metres from America, and television signals from Germany, are evidence of reflection from the ionosphere at glancing incidence.

These reflections are likely to occur at the upper limit of the ultra-short-wave band. With the approach of sun-spot maximum conditions, waves of this kind are likely to be propagated over long distances and so may interfere with local reception on these waves, but regular long-distance working with such waves will be impossible on account of unreliability. Waves below 10 metres will usually be too close to the limit to give reliable working.

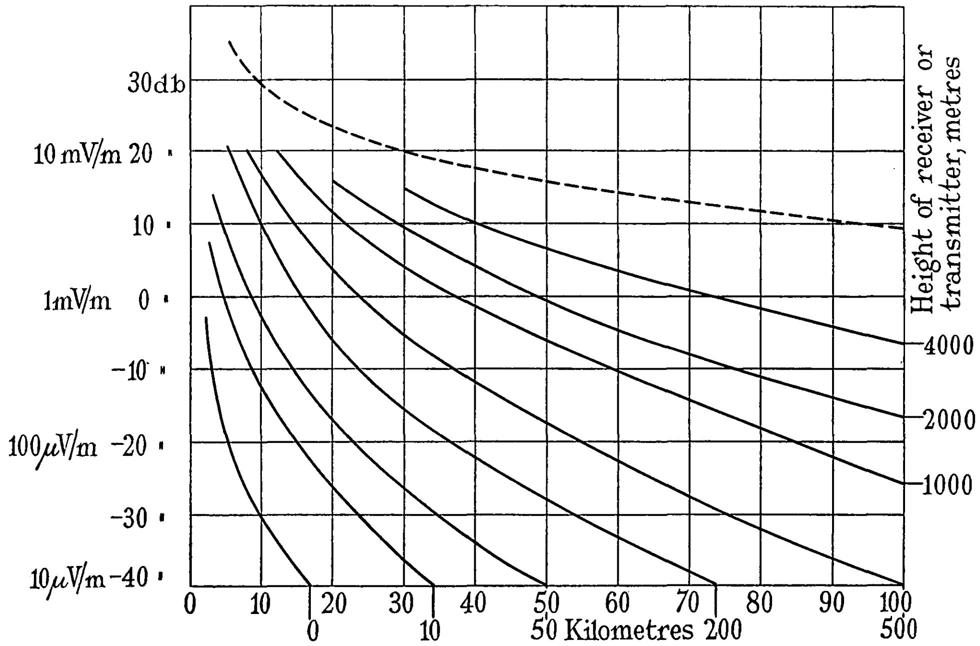

Fig. 22.—Results obtained over land; $\lambda = 8$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

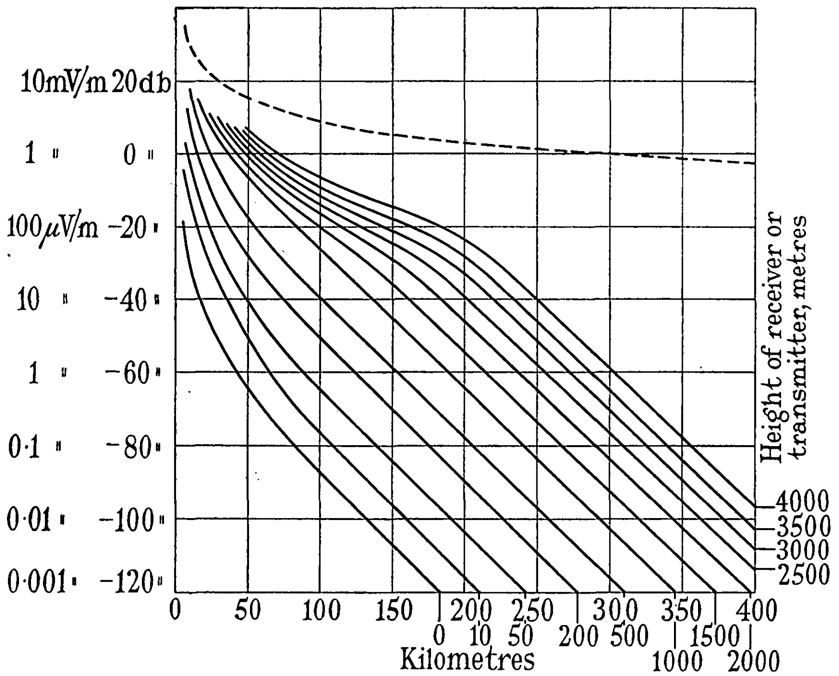

Fig. 23.—Results obtained over land; $\lambda = 8$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

This ultra-short-wave fading is of extreme importance from the practical point of view, for in general it sets a limit to the range of a service long before the average

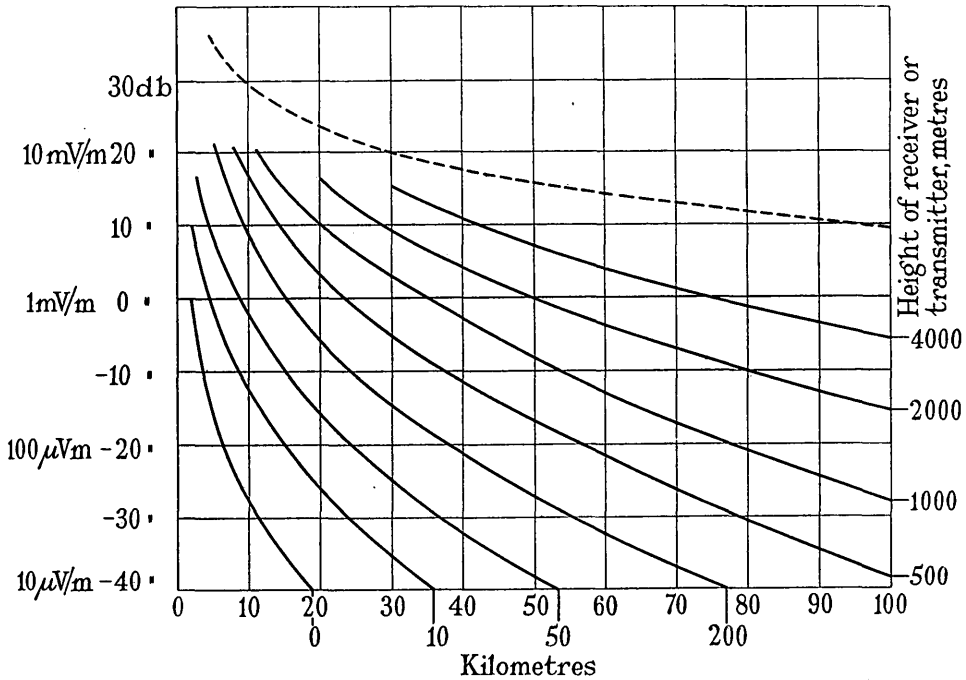

Fig. 24.—Results obtained over land; $\lambda = 10$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

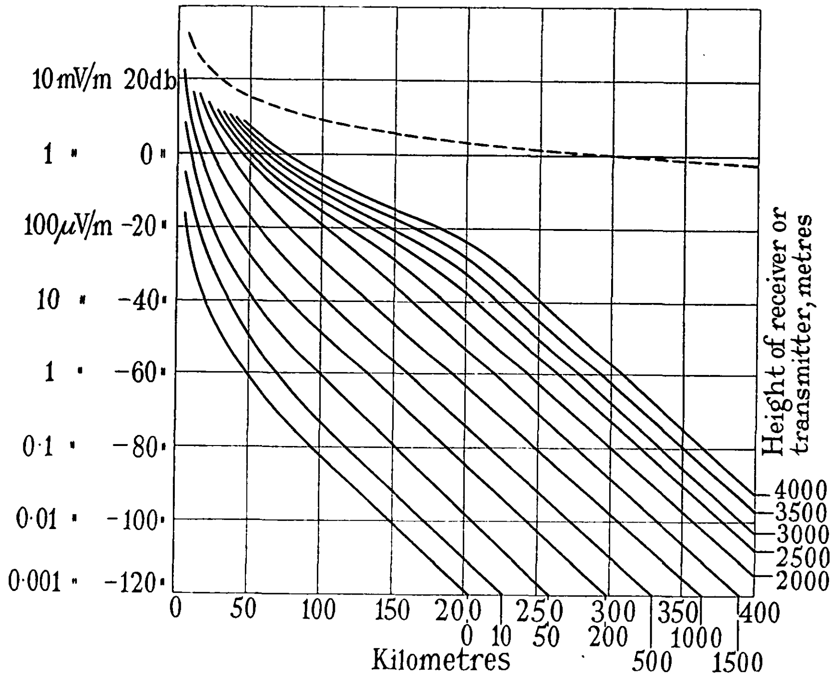

Fig. 25.—Results obtained over land; $\lambda = 10$ m, $\kappa = 5$, $\sigma = 10^{-13}$, 1 kW radiated.

signals become too weak. Thus the study of ultra-short-wave propagation would be incomplete without an analysis of the effect of refraction. This is in the main not an optical ray problem, for diffraction lies outside the scope of such methods. If ray methods are used, they require justification by a more complete wave analysis. Fortunately the wave method, by which the diffraction curves above were calculated, allows an easy extension to the case where the atmosphere above the earth has a vertical gradient of refractive index.

The field intensity beyond the visual distance, where fading is in evidence, can be expressed in the form

$$Ae\,\frac{-(2\pi)^{\frac{1}{3}}\beta_n d}{R^{\frac{2}{3}}\lambda^{\frac{1}{3}}}$$

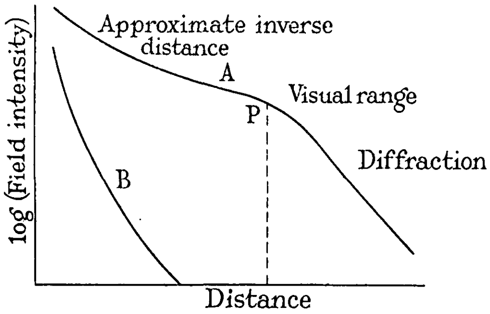

Fig. 26

where β_n is a constant determined in the diffraction analysis, d is the distance, and R_0 is the earth's radius. The field intensity is mainly controlled by the exponential e^{-kd}. A is the amplitude factor, similar to the factor $\frac{0\cdot5386hI}{\lambda^{\frac{7}{6}}\,\sin^{\frac{1}{2}}\theta}$ in Van der Pol's diffraction formula, and modified by a factor γ (< 1) which takes account of the earth constant. It is only a very slowly varying function of distance.

Assuming a practically uniform gradient in the atmosphere, which would be a natural consequence of the upward decrease in molecular density in the atmosphere, it is found that the effect of this gradient is to modify the exponential coefficient by substituting an equivalent earth's radius R_0' for R_0, where

$$R_0' = R_0\frac{R_1}{R_1 - R_0}$$

The distance R_1, which depends on the gradient of refractive index (μ) being $-\frac{1}{\mu}\frac{\delta\mu}{\delta h}$, is readily interpreted as the radius of curvature of the ray caused by the vertical gradient of the refractive index.

The physical meaning of the modification to the diffraction formula is now apparent. The circumstances are illustrated in Fig. 30(a), where TS is the earth's surface and TR is the tangent ray, curved downwards by the effect of the gradient of refractive index in the atmosphere above the earth's surface. If we now make a mathematical transformation such that any radius r becomes $r[R_1/(R_1 - r)]$, the radius of curvature of TR, where $r = R_1$, becomes infinite, and that of the earth $R_0R_1/(R_1 - R_0)$. In the transformed space the ray TR is a straight line T_1R_1 [Fig. 30(b)], and the earth's radius is increased from R_0 to $R_0R_1/(R_1 - R_0)$.

The problem of propagation in the transformed space is now one in which there is no effective gradient of refractive index outside the earth (T_1R_1 being a straight line), and the solution is the same as that previously given, with R_0' in place of R_0. Shelleng, Burrows, and Ferrell have employed this method for determining the effect of refraction, using an argument similar to the above.

It can now be seen that this argument is justified from the wave point of view. It should be noted, however, that the coefficient A should also be changed, but this change is small so long as R' is considerably greater than R_0, as is usually the case. The change in A cannot be allowed for by putting R_0' in the place of R_0.

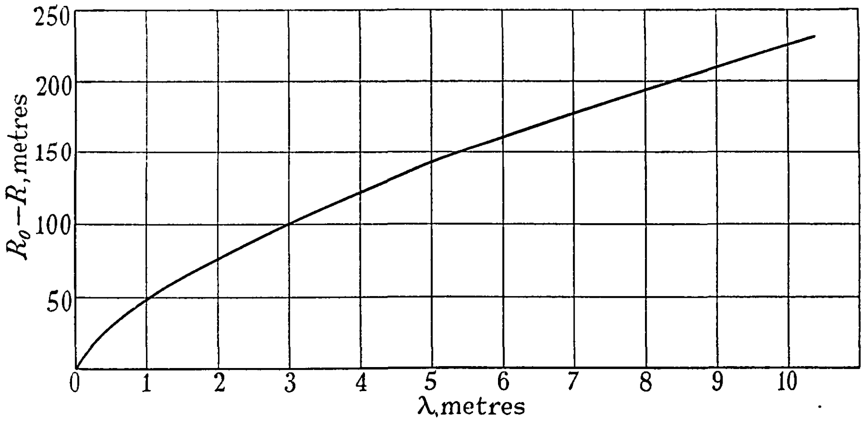

Fig. 27.—Values of R_0—R.

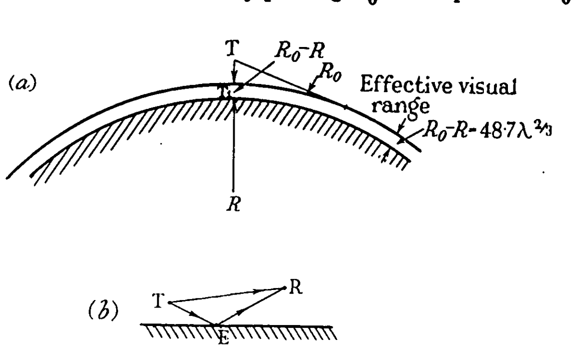

Fig. 28

In (a), T_1 has no effective visual range. λ is expressed in metres.

One point is significant in the analysis: it appears that it is mainly, if not entirely, the gradient of the refractive index in the first few hundred metres which

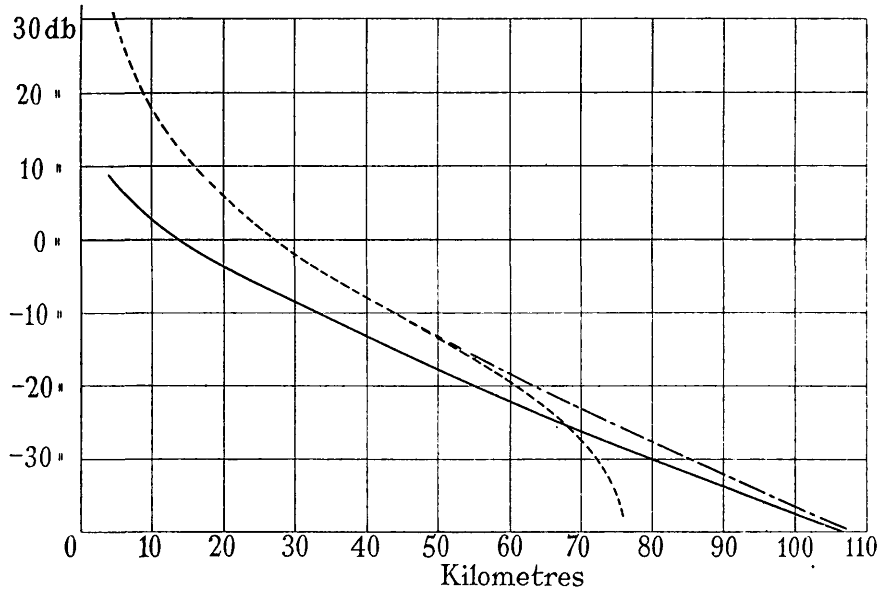

Fig. 29.—Continuity of reciprocal and diffraction values: results obtained over land, $\lambda = 10$ m, $h = 500$ m.

— — — — Reciprocal theorem.
———— Diffraction theory.

counts, at least as far as transmission from one point on the ground to another on the ground is concerned.

The magnitude of the refraction effects depends on the gradient of the refractive index of the atmosphere. The refractive index of air is well known, and is proportional to the density. Since the density decreases upwards in the atmosphere there is a vertical gradient of μ which can be computed when the pressure gradient

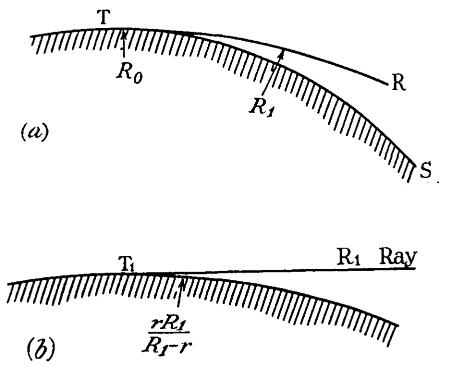

Fig. 30

(a) Actual space.
(b) Transformed space.

is known. Water vapour has a high refractive index, and the effect of a small percentage is very pronounced.* The relative refractive indices of air and water vapour are given in the following table:—

Temperature	μ (air)	μ (water vapour)
45° C.	1·000574	1·01033
63° C.	1·000554	1·00965
83° C.	1·000534	1·00898

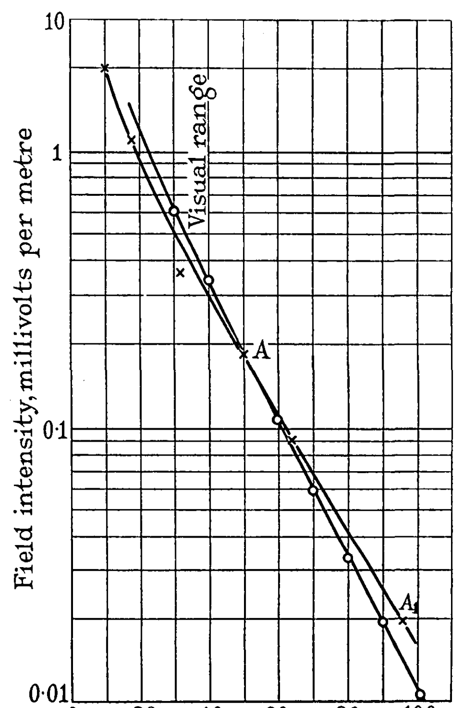

Fig. 31.—Ultra-short-wave diffraction. Results obtained by Jones at 61 megacycles per sec., in transmission from Empire State Building, New York.

The diffraction curve is adjusted to fit at A.
x x Observed field intensity.
o o Calculated relative field intensity.

The difference of μ from 1 in water vapour is nearly 20 times as great as that for air, so that the effect of even a small percentage of water vapour is very considerable.

* The relevant data can be found in a paper by C. R. ENGLUND, A. B. CRAWFORD, and W. W. MUMFORD: "Further Results of a Study of Ultra-Short-Wave Transmission Phenomena," *Bell System Technical Journal*, 1935, vol. 14, p. 383. Further data may be obtained from Humphrey's "Physics of the Air," and from an article on "The Thermodynamics of the Atmosphere" in the "Dictionary of Applied Physics" (vol. 3, p. 44). The diagram on p. 61 of the latter is of particular interest, as it shows the large increase of saturated water-vapour density in summer as compared with winter.

Englund, Crawford, and Mumford have given the following formula for calculating R_1:—

$$R_1 = \frac{M \times 10^6}{31\,185} \times \frac{1}{\frac{\delta}{\delta h}\left\{\beta\left[211 + \alpha\left(\frac{10\,159}{\tau} - 0\cdot293\right)\right]\right\}}$$

where M is the molecular weight of the gas involved, α the percentage of water vapour, τ the absolute temperature, and h the height. A typical numerical example has been worked out for average conditions on the surface of the earth, with 1·37 per cent water vapour, giving $R' = 23\,000$ km. In the absence of the 1·37 per cent of water vapour, R' would be 39 000 km. In the former case the increase in range at which a given field intensity occurs is

$$\left(\frac{R'_0}{R_0}\right)^{\frac{3}{4}} = \left(\frac{R_1}{R_1 - R_0}\right)^{\frac{3}{4}} = \left(\frac{23\,000}{23\,000 - 6\,360}\right)^{\frac{3}{4}} = 1\cdot274$$

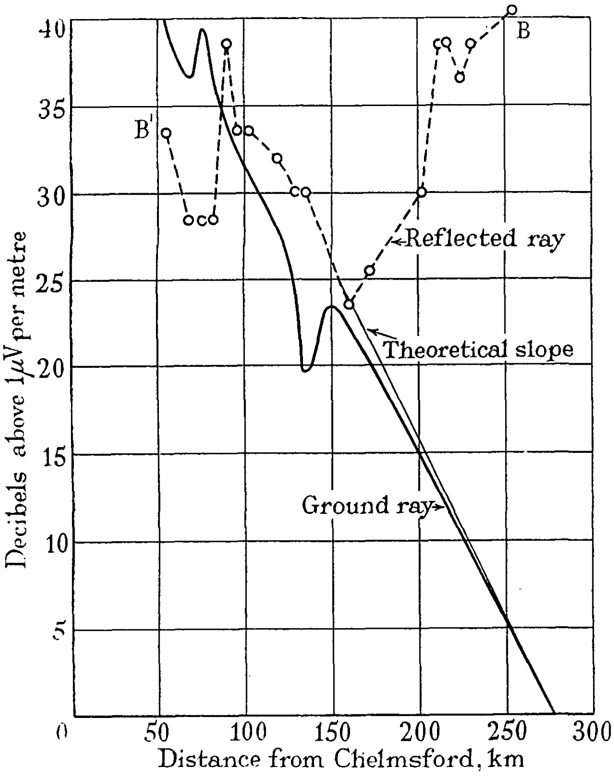

Fig. 32.—Aeroplane test. Results obtained on wavelength of 53·4 m, 1st October, 1935.

i.e a 27·4 per cent increase. In the latter case the increase is $[39\,000/(39\,000 - 6\,360)]^{\frac{3}{4}}$, i.e. 14·2 per cent.

These considerations, taking into account the considerable increase in water-vapour density in the summer, give a very plausible explanation of the now well-established result that the signal-intensity values well beyond the visual distance are much greater in summer than in winter. The effect of refraction is to increase the range, especially when, for instance, there are temperature inversions at the surface of the earth.

The varying refractive index of the atmosphere is probably the cause of the fading observed at extreme distances, which limits the reliable range. A given percentage variation in the gradient will produce a much greater percentage effect where the total attenuation is large than where it is small. Serious fading will therefore set in at ranges where the total attenuation is large. It must be left to observation to determine how large, but from the limited results obtained by Englund, Crawford, and Mumford it is suggested that

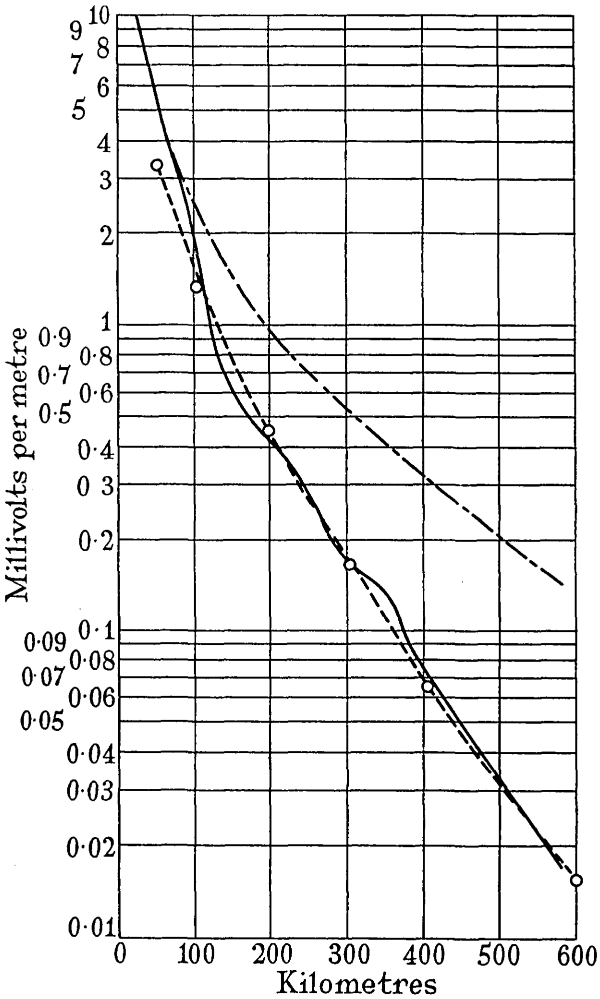

Fig. 33.—Aeroplane test. Results obtained on wavelength of 950 m.

— — — — Diffraction curve.
———— Observed values (Fassbender, Eisner, and Kurlbaum).
— - — - Sommerfeld curve.

an attenuation of about 30 db below the "free-space field" or inverse distance is reasonable.

EXPERIMENTAL CHECKS OF DIFFRACTION FORMULA FOR VARIOUS WAVELENGTHS

It is very desirable to check, experimentally, the theoretical conclusions discussed above, if only for the purpose of finding out whether there are important factors in ultra-short-wave transmission, e.g. lower-atmosphere refractions, other than those taken into account in the main analysis.

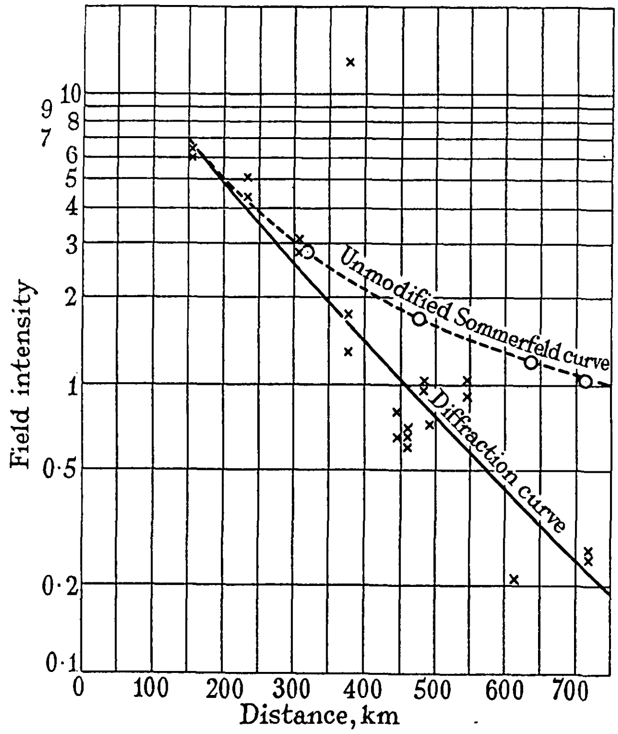

Fig. 34.—Daylight field strength of Daventry 5XX. Results obtained at midday, July, 1931; $\lambda = 1\,550$ m; normal power (25 kW).

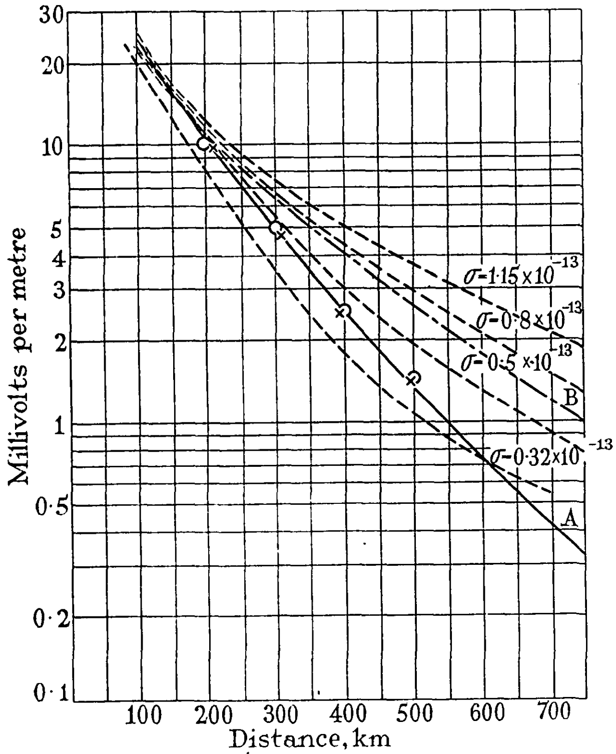

Fig. 35.—Results obtained on Warsaw; $\lambda = 1\,400$ m, 100 kW radiated.

A { × Observed results.
  { ○ Modified diffraction formula, $\sigma = 1{\cdot}15 \times 10^{-13}$.
B – ——— – Original Watson diffraction formula.
– – – – – – Sommerfeld formula.

In choosing the quantities to be compared with experimental observations it is desirable to leave out those which depend on the absolute amount of power radiated by the transmitter, since, in general, there is considerable difficulty in determining this quantity accurately. Relative, but not absolute, intensities can be checked fairly well. For the purposes of comparison use is made of the slope of the curve relating the logarithm of the field intensity to the distance, to compare with the calculated slope. Experimental data can be obtained from American results.*

An example is shown here (Fig. 31) in which AA_1 is the average curve of a large amount of data obtained by Jones in experimental transmissions from the Empire State Building, New York, on a frequency of 61 Mc. The observed rate of attenuation (or slope) is some 14 per cent less than the calculated; this difference may

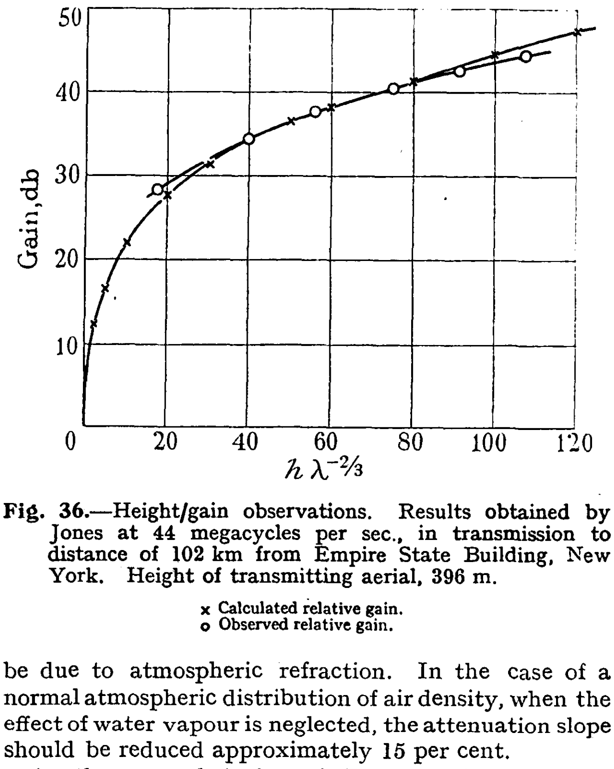

Fig. 36.—Height/gain observations. Results obtained by Jones at 44 megacycles per sec., in transmission to distance of 102 km from Empire State Building, New York. Height of transmitting aerial, 396 m.

× Calculated relative gain.
○ Observed relative gain.

be due to atmospheric refraction. In the case of a normal atmospheric distribution of air density, when the effect of water vapour is neglected, the attenuation slope should be reduced approximately 15 per cent.

Another example is shown in Fig. 32, which represents the ground-ray intensity from a 53·4-metre transmitter in an aeroplane flight from Croydon to Cologne. Beyond 150 km, the visual range, the slope of the log (field)/distance curve is constant and agrees closely with the slope calculated according to the diffraction theory. The measurement of the ground ray at these distances, where the reflected ray was generally predominant, was made possible by means of the impulse technique, in which the ground ray was separated from the reflected ray by an interval of about 2 milliseconds.

To illustrate the scope of the diffraction theory the following examples on medium waves are given.

Fig. 33 gives an example for 950 metres, and consists of field measurements of medium-wave transmissions

* *Proceedings of the Institute of Radio Engineers*, 1933, vol. 21, pp. 349–464.

from an aeroplane. The experimental curve is compared with (*a*) the diffraction curve ($\sigma = 7 \cdot 5 \times 10^{-14}$), and (*b*) the Sommerfeld curve ($\sigma = 7 \cdot 1 \times 10^{-14}$). The agreement with the former is very good.

better agreement with the results than the Sommerfeld curve. The scattered observations are probably caused by the hilly country in which the measurements were taken, i.e. North England and Scotland. Finally, the

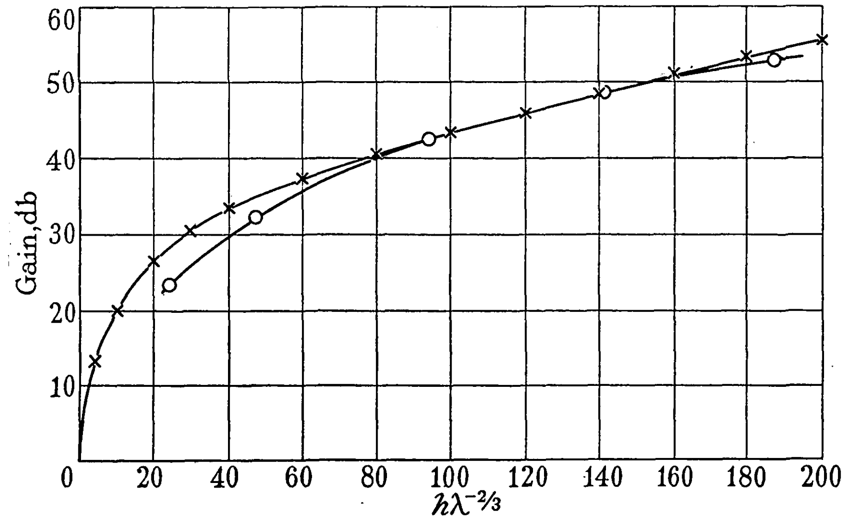

Fig. 37.—Height/gain observations. Results obtained by Trevor and Carter at 34 megacycles per sec., in transmission to distance of 59·6 km from Rocky Point. Height of transmitting aerial, 39 m.

x Calculated relative gain.
o Observed relative gain.

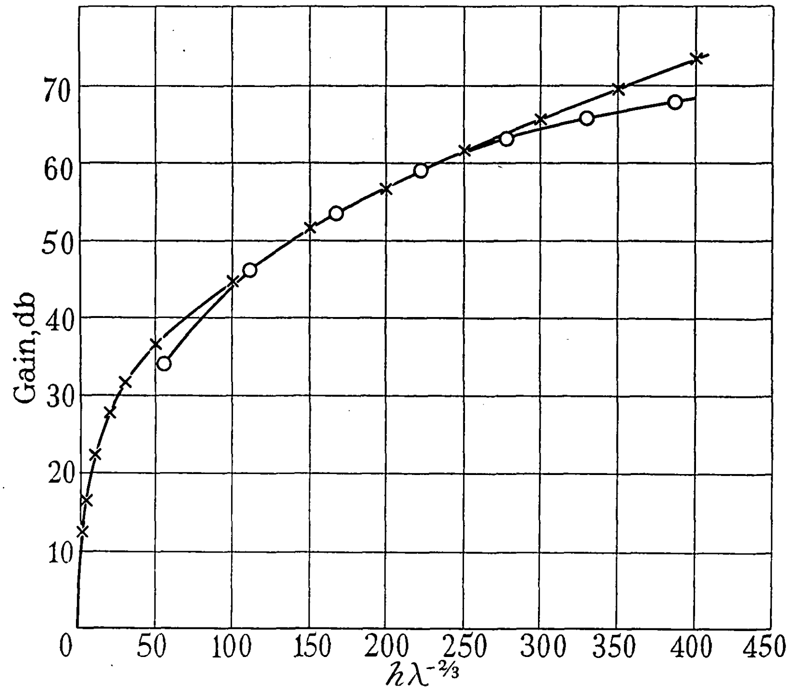

Fig. 38.—Height/gain observations. Results obtained by Trevor and Carter at 44 megacycles per sec., in transmission to distance of 185 km from Empire State Building, New York.

x Calculated relative gain.
o Observed relative gain.

In Figs. 34 and 35, measured values of Daventry and Warsaw are compared with the diffraction values. In the case of Daventry the points fall rather irregularly, but there is no doubt that the diffraction curve is in

Warsaw results (Fig. 35) show very good agreement with the diffraction values. The country is flat, and the observed curve is a smooth one agreeing closely with the calculated curve for $\sigma = 1 \cdot 15 \times 10^{-13}$.

With regard to the examples shown in Figs. 31 and 32, it should be emphasized that the calculated results do not involve any arbitrarily-assumed earth conductivity or permittivity. The calculated attenuation depends only on the wavelength (and, of course, the earth's radius) so long as the wavelength is so short that

$$\sigma^{\frac{1}{2}}\lambda^{\frac{5}{6}} < 10^{-7}$$

Even with sea water this condition is satisfied, and in every case of overland transmission the quantity $\sigma^{\frac{1}{2}}\lambda^{\frac{5}{6}}$ should be well below the limiting value. The reason for this is that in the diffraction attenuation factor $e^{-kd}/\lambda^{\frac{1}{3}}$ the parameter k approaches a definite limit k_1 when the frequency is high enough, and k_2 when the frequency is low enough.

Perhaps the best quantity to compare with experiment is the gain in signal strength with height above the earth in regions beyond the visual range. This gain of signal intensity with height has been measured by many observers in America, using either aeroplanes or autogyros. Examples of the results are given in Figs. 36, 37, and 38, which show that the calculated gains agree with the theoretical gains within a few decibels over quite a considerable range of height. The gains are plotted as a function of $h\lambda^{\frac{2}{3}}$, since the theory shows that they should be a function of this and this only. Thus it is again possible to check the theory without invoking the help of any arbitrarily-estimated earth constants. Although in the main agreement of observation and theory is very fair, individual cases of wide divergence have been recorded. Englund, Crawford, and Mumford have given an example where there was practically no gain with height well beyond the visual range. Again, they have found occasions where the field intensity just beyond the visual range has varied by as much as 20 db from one day to another. Such effects are no doubt due to refraction, which may on occasion produce profound variations, but if the present author's reading of the average results is correct such occasions are rather the exception than the rule. An estimate of the effect of atmospheric refraction can be obtained by comparing the observed results with the calculated ones.