Qeios

The Sagnac-Wang interferometers and absolute vs. relative simultaneity

Facultad de Ciencias Exactas y Naturales,Universidad Nacional de

Asunción,Ruta Mcal. J. F. Estigarribia, Km 11 Campus de la UNA, San

Lorenzo-Paraguay

Justo Pastor Lambare*

Abstract

Gianfranco Spavieri, George T. Gillies & Espen Gaarder Haug (The Sagnac effect and the role of simultaneity in relativity theory, Journal of Modern Optics (2021)), claim that the Sagnac effect reveals the theory of relativity is incorrect and inconsistent. We prove that when standard relativity is appropriately interpreted, it is observationally correct and logically sound.

Keywords: Sagnac effect, relativity theory, synchronization, absolute simultaneity, relative simultaneity.

1 Introduction

The so-called absolute Lorentz transformations (LTA) have been hailed in some quarters as a valid alternative to the usual Lorentz transformations (LT) [1–5]. Considering only one spatial dimension the LT is,

$$\left.\begin{array}{lcl} t' & = & \gamma(t - vx/c^2) \\ x' & = & \gamma(x - vt) \end{array}\right\} (LT) \tag{1}$$

*email: jupalam@gmail.com

https://doi.org/10.32388/6C4WDR

while the LTA,

$$\left.\begin{array}{rcl} t' & = & t/\gamma \\ x' & = & \gamma(x - vt) \end{array}\right\} (LTA) \tag{2}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$.

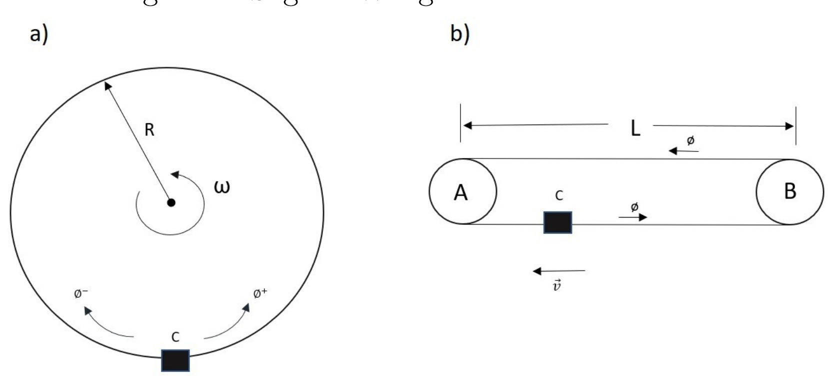
The alleged virtue of (2) would be that it allows us to keep the absolute character of simultaneity. Without discussing their empirical adequacy, the most evident problem with the LTA is that it lacks theoretical and conceptual support. While the LT can be derived from clear physical principles, as was done for the first time by A. Einstein in 1905, the LTA is an ad hoc alteration of the LT with the only purpose of making simultaneity absolute.

We shall refrain from referring to the LTA from here. We aim to prove that relativity theory is free from inconsistencies the Sagnac effect purportedly reveals.

In Ref. [4], the authors analyze two versions of the Sagnac experiment. The ring interferometer (Fig.1 a) and the Wang, Zheng, and Yao [6] linear version (Fig.1 b). In both instances, we have a light source and a detector in the same position inside a moving system. The experiments purportedly reveal two problems: 1) for the comoving observer, the speed of light is different from its universal value c, and 2) time is "discontinuous" and a clock cannot be synchronized with itself. The first issue would constitute empirical evidence against relativity, while the second would reveal its inconsistency.

In section 2, we examine the Wang's modified Sagnac experiment, and in section 3, we analyze the ring interferometer that purportedly suffers from similar difficulties.

Figure 1: Sagnac-Wang Interferometers



2 The Wang linear interferometer

Spavieri et al. analyze two examples of the Wang modified Sagnac experiment. Since the problems that those examples presumably reveal are very similar, we only reexamine here the first example given in Ref. [4].

Following a similar approch, all calculations are simplified to first order. This means that we take $\gamma = 1$ in all relativistic formulas, and we can neglect time dilation and Lorentz contraction effects. Also, the time elapsed at the turning points A and B is assumed to be null, i.e., at those points, both the clock and the photon undergo infinite acceleration.

Spavieri et al. note that even with these simplifications within this order of approximation, we cannot neglect the term vx/c^2 in the time transformation (1) responsible for the lack of absolute simultaneity.

The objective is to evaluate the elapsed time T between the emission and posterior reception of the photon ϕ, as registered by the clock C (Fig. 1 b). Since the total distance covered by the photon is $2L$ and the relative velocity is $c + v$,

$$T = \frac{2L}{c+v} = \frac{2L}{c(1+\beta)} \tag{3}$$

We shall prove that an observer comoving with the clock C finds the same value (3) applying relativity theory. Indeed, since (3) is the Newtonian result and we disregard second-order effects, both results are expected to coincide. This is most obvious by calculating

$$\tau = \int \frac{dt}{\gamma} \tag{4}$$

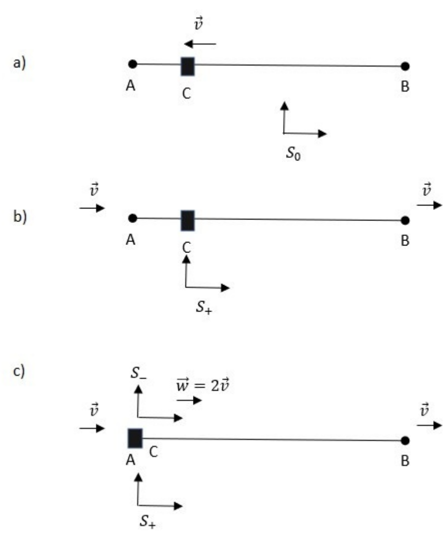
over the clock trajectory since to first order $\gamma = 1$. However, Spavieri et al. explicitly analyze the role of relative simultaneity to highlight the alleged inconsistent nature of relativity theory.

Thus, we shall use a similar approach to Spavieri et al. We start with the observer in the inertial frame S_+ where the clock C is at rest. The observer in S_+, sees the device moving at speed v from left to right (Fig 2 b). We assume that point A reaches the clock position at the same time photon ϕ reaches point B. If the speed of light in S_+ is c, the time registered by the clock at this point is,

$$T_{out} = \frac{L}{c} \tag{5}$$

As seen from the inertial frame S_0 where the interferometer is stationary (points A and B stationary, Fig 2 a), the clock C undergoes a sudden acceleration and then continues with its inertial motion in the opposite direction in the frame S_-. From the point of view of an observer in S_+, the clock passed to the the frame S_- with relative speed $w = 2v$ (Fig. 2 c).
We want to evaluate the time measured by the clock, now in the frame S_-,

Figure 2: Frames S_0, S_+, S_-



when the photon reaches it. We reset the clock reading to zero and use (1) to first order. The event corresponding to the photon at B when the clock is at A has spacetime coordinates $(0, L)$ in S_+. The same event in S_- has spacetime coordinates $(-2vL/c^2, L)$.

Thus, according to the time in S_-, the photon was at B when $(t_-)_1 = -2vL/c^2$. It is irrelevant that the clock was not in S_- at $t_- = (t_-)_1$. Once it joints S_-, its time is assumed synchronized with the time of that frame and the photon has already left point B when $t_- = 0$.

Let $t_2 = T_{ret}$ be the clock reading when the photon arrives at C. If the distance is covered with speed c, we have that $(t_-)_2 - (t_-)_1 = T_{ret} - (t_-)_1 =$

L/c,

$$T_{ret} = \frac{L}{c} + (t_-)_1 \tag{6}$$

$$T_{ret} = \frac{L}{c} - \frac{2vL}{c^2} \tag{7}$$

The total elapsed time is,

$$T = T_{out} + T_{ret} \tag{8}$$

$$T = \frac{L}{c} + \frac{L}{c} - \frac{2vL}{c^2} \tag{9}$$

$$T = \frac{2L}{c}(1-\beta) \tag{10}$$

Since to first order $1-\beta = 1/(1+\beta)$, from (10) we have,

$$T = \frac{2L}{c(1+\beta)} \tag{11}$$

Thus, the Newtonian result (3) coincides with the relativist one (11).

Now, the question is, where do Spavieri et al. see the inconsistency? According to them, since in frame S_-, the photon was at B in the past of the event "$E \equiv$ Clock turning around point A in frame S_- at $t_- = 0$" we have a missing part of the section $\overline{AB} = L$ in that frame. The missing part would correspond to the distance the photon traveled during the past of the event E. The idea seems to be that it is impossible or absurd that the photon traveled that distance in the past of E. Therefore, it had to jump from B to its position corresponding to $t_- = 0$.

However, that is true only if we assume that the photon was not indeed at B in the past of E (in frame S_-), which merely means that we are rejecting the relative character of simultaneity between frames in relative motion.

In other words, Spavieri et al.'s reasoning reduces to proving that relative simultaneity leads to inconsistencies by assuming that it is inconsistent.

Their reasoning can be considered a more elaborate case of Einstein's train-embankment thought experiment [7]. Einstein conceived his example to show that if we accept the existence of an invariant universal speed c, we must jettison the absolute character of the simultaneity of distant events. Similarly, the authors of [4] proved that an observer moving with the clock cannot measure the speed c unless the event E is not simultaneous with the event “photon at B” in S_-. Thus, they present another proof that an invariant universal speed and absolute simultaneity of distant events cannot stand together.

From a purely logical stance, it is possible to deny relativity. But we have to reject its postulates from the start. However, it is pointless trying to prove its internal inconsistency, i.e., once we accept its premises, they do not lead to contradictions or, at least, no one so far was able to find one.

3 The ring interferometer

We shall prove that the correct application of relativistic formulas gives an invariant speed of light equal to the universal constant c for observers within the rotating platform and that the puzzling time gap is a natural consequence of the relativity of simultaneity and has nothing to do with the alleged impossibility of synchronizing a clock with itself or with time being discontinuous.

Doing this correctly requires the use of the formalism introduced in general relativity. This does not mean that we need gravitation theory to describe noninertial systems. The last point is a widespread misconception, as Pepino and Mabile [8] recently pointed out. To avoid possible misunderstandings, we clarify that the following formalism is based on the validity of the LT and the application of special relativity through the principle of locality [9].

The need of the general metric formalism is necessary because when we pass to a noninertial system we need general nonlinear transformations $y^\mu = y^\mu(x^0, x^1, x^2, x^3)$. The Minkowski metric $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$ becomes,

$$g_{\mu\nu} = \frac{\partial x^\alpha}{\partial y^\mu}\frac{\partial x^\beta}{\partial y^\nu}\eta_{\alpha\beta} \tag{12}$$

The metric tensor components $g_{\mu\nu}$ become functions of the spacetime coordinates. We assume the Einstein summation convention. Greek letters indicate spacetime indices varying from 0 to 3, while Latin indices are space indices ranging from 1 to 3. The interval ds in a noninertial frame can be expressed as,

$$\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu && (13)\\
&= g_{00}(dx^0)^2 + 2g_{0i} dx^0 dx^i + g_{ik} dx^i dx^k && (14)\\
&= g_{00}\left[(dx^0)^2 + 2\frac{g_{0i} dx^0 dx^i}{g_{00}}\right] + g_{ik} dx^i dx^k && (15)\\
&= g_{00}\left[(dx^0)^2 + 2\frac{g_{0i} dx^0 dx^i}{g_{00}} \pm \frac{g_{0i}}{g_{00}}\frac{g_{0k}}{g_{00}} dx^i dx^k\right] + g_{ik} dx^i dx^k && (16)\\
&= g_{00}(\underbrace{dx^0 - g_i dx^i}_{\delta x^0})^2 + \underbrace{\left(g_{ik} - \frac{g_{0i} g_{0k}}{g_{00}}\right)}_{-\gamma_{ik}} dx^i dx^k && (17)
\end{aligned}$$

where $g_i = -g_{0i}/g_{00}$. The quantities δx^0 and γ_{ik} has relevant physical meanings. The 3-tensor γ_{ik} determines the geometry of space with respect to the noninertial frame, while the quantity δx^0 is the "synchronized" time interval (see, Refs. [10–12]).

As is well known by relativists, in a noninertial frame the geometry of space can be non-Euclidean notwithstanding that we are dealing with flat spacetime without gravitation [11].

Furthermore, synchronization of clocks as in inertial systems where all clocks can be set to show the same reading simultaneously is in general impossible inside a noninertial system.

The proper elapsed time indicated by a physical clock in the same point of space within the noninertial frame is,

$$d\tau = \frac{\sqrt{g_{00}}}{c} dx^0 \tag{18}$$

However, owed to the lack of synchronization of clocks in different points of space, the elapsed time as mark by two different neighboring clocks is

$$\delta\tau = \frac{\sqrt{g_{00}}}{c} \delta x^0 \neq d\tau \tag{19}$$

Velocities should be calculated with (19), not with (18) (see, Ref. [13]). In particular, the speed of a given particle is,

$$v = \frac{dl}{\delta\tau} \tag{20}$$

not the usual formula $dl/d\tau$ used in inertial systems with globally synchronized clocks.

3.1 The speed of light in a rotating platform

Using cylindrical coordinates in an inertial frame with origin at the center of the platform (Fig. 1 a),

$$ds^2 = c^2 dt'^2 - dr'^2 - r'^2 d\varphi'^2 - dz'^2 \tag{21}$$

We perform a transformation to a frame rotating with the platform,

$$t = t', \qquad r = r', \qquad \varphi = \varphi' - \omega t, \qquad z = z' \tag{22}$$

In the rotating frame the interval is,

$$ds^2 = \left(1 - \beta^2\right) c^2 dt^2 - dr^2 - r^2 d\varphi^2 - dz^2 - 2\beta r c\, dt\, d\varphi \tag{23}$$

where $\beta = \omega r / c$. The values of g_i and γ_{ik} are,

$$\begin{aligned} (g_i) &= (0, \frac{\beta r}{1-\beta^2}, 0) & (24)\\ dl^2 &= \gamma_{ik} dx^i dx^k & (25)\\ &= dr^2 + \frac{r^2}{1-\beta^2} d\varphi^2 + dz^2 & (26) \end{aligned}$$

The null geodesics giving the light paths on the rim of the platform is obtained by setting $ds = 0$ in (23),

$$\begin{aligned} R\frac{d\varphi^+}{dt^+} &= c(1-\beta) & (27)\\ R\frac{d\varphi^-}{dt^-} &= -c(1+\beta) & (28) \end{aligned}$$

Applying the correct formula for the speed, according to (17), (19), (20), (24), (26), and (27),

$$\begin{aligned} \frac{dl^+}{\delta\tau^+} &= \frac{\gamma c(1-\beta)}{\sqrt{1-\beta^2}(1 - \frac{\beta R}{c(1-\beta^2)}\frac{d\varphi^+}{dt^+})} & (29)\\ &= \frac{\gamma^2 c(1-\beta)}{1 - \frac{\beta}{c(1-\beta^2)} R\frac{d\varphi^+}{dt^+}} & (30)\\ &= \frac{\gamma^2 c(1-\beta)}{1 - \frac{\beta}{1+\beta}} & (31)\\ &= \gamma^2 c(1-\beta^2) & (32)\\ &= c & (33) \end{aligned}$$

Similarly, we can prove that $dl^-/\delta\tau^- = c$.

Thus when we correctly apply the relativistic formulas with the appropriate formalism, we find that observers in the rotating platform measure with their physical instruments, clocks and rulers, the correct invariant value of the speed of light.

As a further consistency test, we can calculate the total times T^+ and T^- that take for both light rays to complete a closed path around the platform according to observers inside it. That time must be calculated by summing the actual (corrected) time increments along their paths,

$$T^+ = \oint \delta\tau^+ \tag{34}$$

$$T^+ = \int_0^{2\pi} \sqrt{1-\beta^2}\left(dt^+ - \frac{\beta R}{c(1-\beta^2)}d\varphi^+\right) \tag{35}$$

$$T^+ = \int_0^{2\pi} \sqrt{1-\beta^2}\left(\frac{R}{c(1-\beta)}d\varphi^+ - \frac{\beta R}{c(1-\beta^2)}d\varphi^+)\right) \tag{36}$$

$$T^+ = \gamma\frac{2\pi R}{c} \tag{37}$$

According to (26), total length L^+ of the closed path is

$$L^+ = \oint dl \tag{38}$$

$$= \gamma 2\pi R \tag{39}$$

The mean speed along the closed path is again $L^+/T^+ = c$. Similarly,

$$T^- = \oint \delta\tau^- \tag{40}$$

$$T^- = \int_0^{-2\pi} \sqrt{1-\beta^2}\left(dt^- - \frac{\beta R}{c(1-\beta^2)}d\varphi^-\right) \tag{41}$$

$$T^- = \int_0^{-2\pi} \sqrt{1-\beta^2}\left(-\frac{R}{c(1+\beta)}d\varphi^- - \frac{\beta R}{c(1-\beta^2)}d\varphi^-)\right) \tag{42}$$

$$T^- = \gamma\frac{2\pi R}{c} \tag{43}$$

Since $L^- = L^+$ we also have $L^-/T^- = c$ for the clockwise beam.

Thus the mean speeds, when correctly evaluated, also coincide with their local value c. One could still think that we have a paradox here because as is well known the clockwise and counterclockwise beams do not arrive at C simultaneously. But that is also easily explained because the desynchronizing effect is different in both directions.

3.2 Clock synchronization in a rotating platform

This is perhaps the only point on which we partially agree with Spavieri et al. They claim that relativity implies an inconsistency arising from the synchronization of distant clocks. Next, we explain that the alleged inconsistency is only a consequence of the inappropriate definition of simultaneity that sometimes is given.

The simultaneity of distant events and synchronization of distant clocks cannot be consistently defined in arbitrary reference frames. Specifically, the case that concerns us here is that of stationary fields.

A gravitational field is called constant if the components $g_{\mu\nu}$ are independent of the time coordinate x^0. Besides, when the cross terms $g_{0i} = g_{0i} = 0$, the field is called static; otherwise they are called stationary. The same classification applies to the metric field $g_{\mu\nu}$ when we are in flat spacetime without gravitation.

In a rotating platform we have, according to (24), $g_{0\varphi} \neq 0$, so we must be careful when defining simultaneity of distant events.

We can always define simultaneity for infinitesimally near points. The difference of coordinate time Δx^0 that correspond to simultaneous infinitesimally near points in space is given by δx^0 in (17). The fact that $\Delta x^0 \neq 0$ for simultaneous events simply means that clocks cannot be assumed to be synchronized.

What about simultaneity of distant events? Here is where the confusion arises because some authors define simultaneity of distant events by integration of the infinitesimal concept,

$$(x^0)_B = (x^0)_A + \int_{C_{AB}} g_i dx^i \tag{44}$$

According to (44) we have that $(x^0)_A$ and $(x^0)_B$ would be time labels for simultaneous events at A and B. Since in general we have that

$$\oint g_i dx^i \neq 0 \tag{45}$$

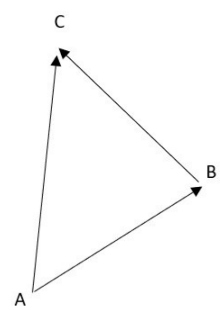
Spavieri and other authors [4, 5, 14] have correctly pointed out such definition leads to the obvious inconsistency that a given event would not be simultaneous with itself! Therefore relativity is inconsistent.

As we mentioned at the beginning of this section, the problem does not resides in the theory but in the incorrect definition of simultaneity according to (44). Indeed any consistent definition of simultaneity requires a property that (44) lacks, namely, transitivity. Really, we have that in general (Fig. 3),

$$\int_{C_{AC}} g_i dx^i \;\neq\; \int_{C_{AB}} g_i dx^i + \int_{C_{BC}} g_i dx^i \tag{46}$$

(46) means that if A is simultaneous with B, and B simultaneous with C,

Figure 3: Paths of simultaneity



A is no longer simultaneous with C.

There is one case when (44) can lead to a consistent definition of distant simultaneity. It is when the integration is path independent. In those cases, we have that,

$$\vec{g} \;=\; -\nabla f \tag{47}$$

However, in such cases it can be proved that a change in the time coordinate $x'^{\,0} = x^0 + f(x^1, x^2, x^3)$, leaving invariant the space coordinates, leads to $\vec{g} = 0$, so we are indeed in a static field [13].

In the case that concerns us, i.e., the rotating platform, (47) is not verified because,

$$\frac{\partial g_\varphi}{\partial r} \neq \frac{\partial g_r}{\partial \varphi} \tag{48}$$

(48) simply means the concept of distant simultaneity does not make sense for observers within the rotating platform.

4 Conclusions

We have proved that the alleged inconsistencies and empirical evidence that the Sagnac effect presumably reveals against relativity theory are based on incorrect applications of the theory and the use of erroneous definitions.

Regarding the modified Sagnac experiment of Wang et al., the explication is no more puzzling than relative simultaneity itself. So, it cannot be considered inconsistency proof unless we declare relative simultaneity inconsistent. This type of circular reasoning is typical of arguments trying to find internal inconsistencies in relativity theory.

As long as the relative nature of distant simultaneity does not lead to observable or logical contradictions, its absolute character shall remain a forsaken relic of our past metaphysical prejudices.

References

[1] Reza Mansouri and Roman U. Sexl. A Test Theory of Special Relativity: I. Simultaneity and Clock Synchronization. *General Relativity and Gravitation*, 8(7):497–513, 1977.

[2] F. Selleri. Noninvariant one-way speed of light and locally equivalent reference frames. *Foundations of Physics Letters*, 10(1):73–83, 1997.

[3] Chunghyoung Lee. Simultaneity in cylindrical spacetime. *American Journal of Physics*, 88(2):131–136, 02 2020.

[4] Gianfranco Spavieri, George T. Gillies, and Espen Gaarder Haug. The Sagnac effect and the role of simultaneity in relativity theory. *Journal of Modern Optics*, 68(4):202–216, 2021.

[5] Edward T. Kipreos and Riju S. Balachandran. Assessment of the relativistic rotational transformations. *Modern Physics Letters A*, 36(16):2150113, 2021.

[6] Ruyong Wang, Yi Zheng, and Aiping Yao. Generalized Sagnac Effect. *Phys. Rev. Lett.*, 93:143901, Sep 2004.

[7] A. Einstein. *Relativity: The Special and General Theory - Scholar's Choice Edition*. Creative Media Partners, LLC, 2015.

[8] Ron A. Pepino and Risley W. Mabile. A Misconception Regarding the Einstein Equivalence Principle and a Possible Cure Using the Twin Paradox. *The Physics Teacher*, 61(2):118–121, 02 2023.

[9] Bahram Mashhoon. The hypothesis of locality in relativistic physics. *Physics Letters A*, 145(4):147–153, 1990.

[10] L.D. Landau and E.M. Lifshitz. *Course of Theoretical Physics, Volume 1, Mechanics*. ELSEVIER, Oxford, UK, third edition, 1976.

[11] Anatolii A Logunov and Yu V Chugreev. Special theory of relativity and the Sagnac effect. *Soviet Physics Uspekhi*, 31(9):861, sep 1988.

[12] Justo Pastor Lambare. On the Sagnac Effect and the Consistency of Relativity Theory. *Qeios*, 2024.

[13] L.D. Landau and E.M. Lifshitz. *Course of Theoretical Physics, Volume 2, The Classical Theory of Fields*. ELSEVIER, Oxford, UK, fourth edition, 1975.

[14] Robert D. Klauber. Comments regarding recent articles on relativistically rotating frames. *American Journal of Physics*, 67(2):158–159, 02 1999.