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# Earth Rotation Observed by Very Long Baseline Interferometry and Ring Laser

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*Abstract*—We present a systematic and unified treatment of Earth rotation from the geodetic, astronomical, and geophysical point of view. A precise terminology of precession-nutation and polar motion is of great importance for understanding the interacting physical phenomena leading to rotational irregularities of the Earth. In total, four poles of the equatorial plane are defined for the description of Earth rotation. The various components of the Earth Orientation Parameters, i.e., precession-nutation, polar motion, and universal time (or length of day) are summarized. This paper shows how very long baseline interferometry and ring laser differ in terms of Earth rotation and presents the current state-of-the-art measurement accuracy that can be achieved.

**Key words:** Earth rotation, very long baseline interferometry, ring laser, reference frame.

**Nomenclature**

| | |
|---|---|
| CIP | Celestial Intermediate Pole |
| e-VLBI | Electronic VLBI |
| ENSO | El Niño Southern Oscillation |
| EOP | Earth Orientation Parameters |
| FCN | Free Core Nutation |
| FICN | Free Inner Core Nutation |
| GCRF | Geocentric Celestial Reference Frame |
| GPS | Global Positioning System |
| IAG | International Association of Geodesy |
| IAU | International Astronomical Union |
| IERS | International Earth rotation and Reference system Service |
| IRP | Instantaneous Rotation Pole |
| IRV | Instantaneous Rotation Vector |
| ITRF | International Terrestrial Reference Frame |
| LOD | Length of day |
| MJD | Modified Julian Date |
| MJO | Madden-Julian Oscillation |

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| | |
|---|---|
| NDFW | Nearly Diurnal Free Wobble |
| PM | Polar motion |
| PN | Precession-nutation |
| SG | Superconducting gravimeter |
| UT1 | Universal Time |
| UTC | Universal Time Coordinated |
| VLBI | Very Long Baseline Interferometry |

## *1. Introduction*

For the study of Earth rotation, being both a dynamical and kinematic problem, a conventional geocentric terrestrial reference frame (e.g. the International Terrestrial Reference Frame ITRF2005) is adopted, which moves in space, in order to appropriately describe the instantaneous position of a material point on the Earth. It is essential to describe the motion of the axes of the ITRF, to which we refer individual observatories on the surface of the Earth with respect to space. The other set of axes with the directions fixed in space and defined by stable positions of quasars, represents a conventional Geocentric Celestial Reference Frame (GCRF). The GCRF is not an inertial frame, a fact that should be accounted for when considering the dynamics of Earth rotation. The GCRF allows us to describe the moving axes of the ITRF. This idea was adopted by Euler: The ITRF axes represent a material system, whose motion can be monitored or predicted by three angles about the GCRF axes.

The change of the instantaneous axis of rotation of the Earth relative to the Earth-centered ITRF can be interpreted as an equilibrium condition for the vanishing of the resultant of five torques: Those of the Euler forces, the centrifugal forces, the de Coriolis forces, the forces due to acceleration of particles, and the external forces. The first four torques arise due to effects, which occur in the interior of the Earth and in its surficial layers (including the atmosphere) (THOMSON and TAIT, 1912). The fifth torque is caused by the gravitational attraction of the celestial bodies to a nonspherical and tilted heterogeneous Earth and is responsible, among others, for precession-nutation.

Four poles are required for a complete description of the equatorial Earth rotation: The pole of a GCRF, the pole of an ITRF, the celestial intermediate pole (CIP), and the instantaneous rotation pole (IRP). As an approximation, the CIP represents precession-nutation (PN) arising due to the external torques (e.g., lunisolar) applied to system Earth with respect to a GCRF. Currently, no observing technique is able to estimate pure, unbiased PN from lunisolar-planetary torques. In fact, all techniques are obliged to resort to an additional convention and define the frequencies for PN of the CIP. We distinguish between two types of polar motion (PM), i.e., PM of the CIP and PM of the IRP. Similarly, we distinguish between two types of PN: PN of the CIP and PN of the IRP. We note that the motion of the IRP is unique once both conventional poles of reference, i.e., the GCRF and the ITRF are adopted.

The role of the instantaneous rotation vector in astrometric observations seems to have first appeared in a foreword by JEFFREYS (1963), whose idea was further elaborated by the fundamental work of ATKINSON (1973). Their conclusion was that "the instantaneous pole of rotation ... does not enter directly into any observations at all". However, of course, it enters indirectly and we should understand the details if we want to relate Very Long Baseline Interferometry (VLBI) and ring laser data, which is the main motivation of this work. Any difference in the PM signature of the IRP obtained from either VLBI or ring laser (for frequencies to which both techniques are sensitive) is an indication of an intertechnique bias. The main benefit is that there is a huge amplification of amplitudes in the subdiurnal band when moving from the CIP to the IRP. This means that the ring laser manifests larger amplitudes with respect to those observed from space geodetic techniques.

Since the advent of VLBI, it became apparent that there is real geophysical polar motion at the diurnal retrograde frequencies that can be interpreted as PN of the CIP. This led the scientific community to adopt a definition which separates the effects by frequency (see Fig. 1). An alternate suggestion would be to separate the effects by cause. The free core nutation (FCN) is a phenomenon that manifests itself in the PN of the CIP and therefore enters indirectly into the PM of the IRP. The latter is however presently undetectable due to its smallness ($< 1\ \mu$as). The FCN is of geophysical origin and involves no external torques of celestial bodies. Another example is the retrograde diurnal contribution of ocean tides, which is of geophysical origin, however included in current nutation models of the CIP, mainly because the ocean tide contributions cannot be separated from the VLBI observations of PN of the CIP.

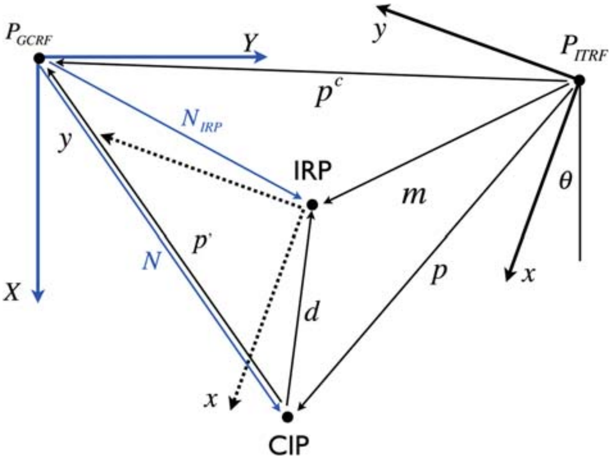

Figure 1
Poles of reference in the equatorial plane with respect to an ITRF and a GCRF.

## 2. *Strengths of VLBI and Ring laser*

Earth rotation encloses PN, variations in universal time (UT1) or length of day (LOD), and PM. Since almost three decades, VLBI has proven its ability to separate these five geodetic parameters at a daily resolution. But even more crucial is the possibility to estimate accurate (unbiased) subdaily UT1 and variations in the PM of the CIP from VLBI. The latter are only estimable if modeled PN of the CIP is fixed to the best available *a priori* values.

Presently, VLBI data processing still introduces a time delay of a few days between observation and parameter availability due to transport of data storage media and data preprocessing. Yet, one improvement is being investigated through electronic VLBI (e-VLBI). Recently, e-VLBI has been successfully tested for UT1 variations with a time delay of less than 1 hour (HAAS *et al.*, 2008).

A major advance in Earth rotation science is the signature in the PM of the IRP in time series of ring laser measurements at diurnal periods (SCHREIBER *et al.*, 2004). But let us recall some facts from another type of instrument: Since more than a decade, PM signatures of the CIP are usually subtracted from gravity observations. LOYER *et al.* (1999) performed the correct reduction for PM of the IRP (LOYER *et al.*, 1999). To date, efforts to estimate PM of the IRP from such gravity observations have not been successful, as one common problem to superconducting gravimeters (SG) or ring lasers is the nonlinear unpredictable instrumental drift. Absolute gravimeters (AG) are able to redress the instrumental drift observed in SG observations. In this paper, PM of the IRP from gravimeters will not be pursued any longer, but must remain an alternative for the future. Regarding the ring laser technology, no other *absolute* instrument is able to redress the instrumental drift yet. A combination of VLBI and ring laser data would benefit from the accumulated advantages and encompass a better understanding of the Earth rotation spectrum from hours to decades (RAUTENBERG *et al.*, 1997). The strength of VLBI definitely resides in the absence of instrumental drifts, while the strength of the ring laser technology is its high resolution and its real-time data acquisition capability. Ring laser observables could fill up the subdaily gap for PN and PM of the CIP as well as for LOD variations. VLBI observations would provide for the drift of those parameters.

One of the most critical parts for a successful combination of both techniques resides in the consistent reduction of the observables, closely related to kinematical aspects. The commonly used International Earth rotation and Reference system Service (IERS) Conventions 2003 applicable to VLBI reductions are not rigorously correct when applied to ring laser observables. Transformations to the adequate poles of reference are imperative, leading inevitably to changes in amplitudes of the spectral motions.

Both techniques, VLBI and ring lasers, are in some way sensitive to the instantaneous Earth rotation vector. The main difference with respect to VLBI is that the ring laser needs no physical observations outside the Earth. In fact, VLBI and ring laser are two totally different approaches to measuring Earth rotation, i.e., the former is geometric while the latter is dynamical. Besides, VLBI requires an interpolation of PN values if subdaily values for the motion of the IRP need to be computed. Additionally, numerical differentiation of empirical data is needed for the VLBI technique (as nutation and polar motion rates of the CIP are required), and this is always a critical step, which amplifies the noise in the high-frequency domain. In VLBI, any defect of the pole of the GCRF will alias into the PM of the IRP. The ring laser observations include a mixture of retrograde and prograde components of diurnal PM of the IRP, while the VLBI observations indirectly distinguish between them. Hence, if a comparison is made at the level of the Sagnac signal, the prograde diurnal signal in the VLBI-derived PM series of the IRP needs to be added, i.e., the prograde component of diurnal PM of the IRP, which is excited by ocean tides and by the influence of tidal gravitation upon the triaxial structure of the Earth. Care must be taken, as most common models related to Earth rotation in the IERS Conventions 2003 describe PM of the CIP.

In this paper we exclude the signature of rotational motions induced by strong earthquakes. Such signatures have been discussed in IGEL *et al.* (2005).

## 3. *Definitions*

First, some basic definitions need to be recalled for the further development, where Figure 1 serves as a schematic description of the equatorial Earth rotation. Let us denote by

$$\vec{\omega} = (\omega_1, \omega_2, \omega_3)^T = \omega_0 (m_1, m_2, 1 + m_3)^T \tag{1}$$

the instantaneous rotation vector (IRV) with respect to an ITRF, where $\omega_0$ is the mean angular speed of the sidereal rotation of the Earth as given in the IERS numerical standards by the International Association of Geodesy (IAG) in 1999, i.e., $7.2921150(1) \cdot 10^{-5}$ [rad/s]. The CIP is defined in the resolution B1.7 of the International Astronomical Union (IAU) General Assembly 2000 so that its periodic celestial motion contains only terms with periods longer than two days; all other motions are interpreted as polar motion (CAPITAINE *et al.*, 2002). As shown in Figure 2, nutation is the retrograde motion of the CIP with frequencies between 1 cycle in 48 hours and 1 cycle in 16 hours (sidereal) with respect to an ITRF.

### 3.1. *Equatorial Motion*

The PM of the IRP is denoted by

$$m = m_1 + i m_2, \tag{2}$$

where $i = \sqrt{-1}$.

The terrestrial motion of the pole of the GCRF is composed of two parts and will be denoted by $p^c$

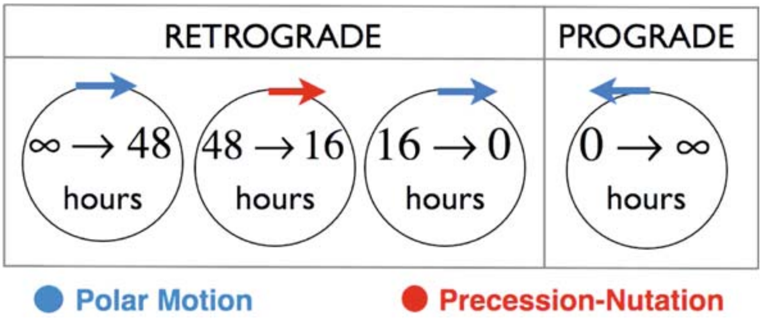

Figure 2
Definition of precession-nutation vs. polar motion of the CIP as described in the IERS Conventions 2003. Periods are given in sidereal hours with respect to an ITRF.

$$p^c = p + p' = p^c_x - ip^c_y, \tag{3}$$

where $p = p_x - ip_y$ represents PM of the CIP, and $p' = p'_x - ip'_y$ stands for the negative PN of the CIP. All phenomena listed in Table 1, except the Oppolzer terms (Frede and Dehant, 1999), pertain to the PM of the CIP, i.e., to the quantity $p$.

In the following, the description is a first-order theory with several limitations, i.e., some relationships are valid for small quantities, e.g., $\delta X$ and $\delta Y$, which are the celestial pole offsets of the CIP from an *a priori* precession-nutation model, but do not hold for the complete precession-nutation angles $X$ and $Y$. However, extended formulas can be used as presented in Brzezinski and Capitaine (1993) to account for larger angles: The Earth rotation vector can always be obtained more rigorously from a numerical derivation applied to the transformation matrix connecting an ITRF to a GCRF (Bolotin *et al.*, 1997).

Considering these limitations, the PN of the CIP is given by

$$N = \delta\psi \sin \epsilon_0 + i\delta\epsilon \approx \delta X + i\delta Y, \tag{4}$$

where, $\delta\psi$, $\delta\varepsilon$ are the corresponding PN angles and $\varepsilon_0$ the mean obliquity at epoch J2000.0.

The set $\{p_x, p_y, \delta X, \delta Y\}$ or $\{p_x, p_y, \delta\psi, \delta\varepsilon\}$ includes four of the five geodetic Earth Orientation Parameters (EOP), which are routinely determined by the space geodetic techniques and provided to the users through the IERS.

The quantities $p$ and $m$ are small, so that to first order, i.e., $10^{-6}$ considering also the long periodic PM, simple linear relationships relating PM of the IRP and CIP hold (Brzezinski and Capitaine, 1993). This means that we assume uniform rotation about the CIP axis, equivalent to the one for the IRP axis. In the following, $\theta = \omega_3 t + \theta_0 \approx \omega_0 t + \theta_0$ denotes the sidereal rotation angle and $\theta_0$ is a constant phase, depending on the initial condition.

Table 1

*Description of contributions to PM of the CIP. The symbol X denotes detectability, while O denotes that it is presently undetectable. P/R denotes prograde (+)/retrograde ( − ) motion. Note that the Oppolzer terms only appear in the PM of the IRP. Information taken from* Gross *(2007) and IERS* Conventions *2003 (2004)*

| Phenomenon/Mechanism | P/R | Period/Direction | Amplitudes | VLBI |
|---|---|---|---|---|
| **Linear trend** | | | | |
| glacial isostatic adjustment | N/A | ~ 79° West | ~ 3.5 mas/year | X |
| **Decadal variations** | | | | |
| core-mantle interactions | +/− | ~ 20–30 years | ~ 30 mas | X |
| **Chandler wobble** | | | | |
| atmosphere-ocean-hydrology | + | ~ 433 days | ~ 44–280 mas | X |
| **Annual wobble** | | | | |
| atmosphere-ocean-hydrology | +/− | ~ 365 days | ~ 65–145 mas | X |
| **Other seasonal wobbles** | | | | |
| atmosphere-ocean-hydrology | +/− | ~ 182 days | > 3 mas | X |
| | +/− | ~ 120 days | > 3 mas | X |
| **Nonseasonal wobbles** | | | | |
| atmosphere-ocean | +/− | ~ 1–4 years | > 1 mas | X |
| | +/− | ~ 1–4 months | > 1 mas | X |
| **Gravitational tidal effects** | | | | |
| ocean tides | +/− | long periodic | ~ 0.080 mas | X |
| | + | ~ 1 day | < 0.526 mas | X |
| | + | ~ 0.5 days | < 0.152 mas | X |
| | − | ~ 0.5 days | < 0.549 mas | X |
| tidal gravitation | +/− | long periodic | ~ 0.030 mas | X |
| | + | ~ 1 day | < 0.046 mas | X |
| **Thermal tides** | | | | |
| Satmosphere-ocean | + | ~ 1 day | ~ 0.010 mas | O |
| | +/− | ~ 0.5 days | ~ 0.010 mas | O |
| **Oppolzer terms** | − | ~ 1 day | ~ 28 mas | X |

The PN of the CIP is given by Brzezinski and Capitaine, (1993) or Moritz and Műller (1988)

$$p' = -Ne^{-i\theta}. \tag{5}$$

The PM of the IRP reads (Brzezinski and Capitaine, 1993)

$$m = p^c - i\frac{\dot{p}^c}{\omega_0}. \tag{6}$$

The so-called Oppolzer terms (see Brzezinski, 1986; Chao, 1985; Frede and Dehant, 1999; Moritz and Műller, 1988) of the Earth are caused by the external torques of the celestial bodies. To first order, these Oppolzer terms visible in the motion of the IRP are obtained as the relative terrestrial motion of the IRP with respect to the CIP. This motion was modeled by Brzezinski (1986) for an Earth having a liquid core.

In the past, PM of the IRP when viewed from a GCRF was called “sway” as stated in Capitaine (1986) or Chao (1985). This terminology was confined to variations entirely due to phenomena on Earth, whereas pure PN depends on the gravitational forces of celestial bodies. Therefore, PN of the IRP is contaminated by geophysical sway. The designation sway has been abandoned in recent years. Besides, the part of the PM of the IRP due to the external lunisolar-planetary effect has been called "diurnal nutation".

Furthermore, given PM ($p$) and PN of the CIP ($N$), PM of the IRP can be summed up as (Brzezinski and Capitaine, 1993; Eubanks, 1993; Gross, 1992, 2007)

$$m = \left[p - i\frac{\dot{p}}{\omega_0}\right] + \left[i\frac{\dot{N}}{\omega_0}e^{-i\theta}\right] = p + \frac{i}{\omega_0}\left(\dot{N}e^{-i\theta} - \dot{p}\right). \tag{7}$$

The PM of the IRP given by equation (7) can be split into its equatorial components $m_1$ and $m_2$

$$m_1 = \frac{1}{\omega_0}\left[\omega_0 p_x - \dot{p}_y + \delta\dot{X}\sin\theta - \delta\dot{Y}\cos\theta\right], \tag{8}$$

$$m_2 = \frac{-1}{\omega_0}\left[\omega_0 p_y + \dot{p}_x - \delta\dot{X}\cos\theta - \delta\dot{Y}\sin\theta\right], \tag{9}$$

where $\delta\dot{X} = \delta\dot{\psi}\sin\epsilon_0$ and $\delta\dot{Y} = \delta\dot{\epsilon}$.

The PN of the IRP is

$$N_{IRP} = N - \frac{i}{\omega_0}\left(\dot{N} - \dot{p}e^{i\theta}\right). \tag{10}$$

The difference between the PM of the IRP and the CIP is denoted by $d = m - p$ in Figure 1 and is a function of the time derivatives of PM and PN of the CIP. The advantage of the IRP is that it is independent of the reduction models and parameterization.

Observations of one ring laser are sensitive to the signature of the PM of the IRP, which arises from both geophysical and astronomical phenomena.

PM of the IRP derived from VLBI observations, as well as its signature obtained by one ring laser can be compared to theoretical models of forced diurnal polar motion, as proposed by Brzezinski (1986).

Table 1 describes the contribution to polar motion of the CIP as determined by the VLBI technique. Presently, the Wettzell G ring laser (Schreiber *et al.*, 2009) is only sensitive to the Oppolzer terms, which do not appear in the PM of the CIP; however, they do appear in the PM of the IRP.

### *3.2. Axial Motion*

The total axial dimensionless perturbation term $m_3$ describes the deviation in angular speed of the IRV with respect to the nominal quantity $\omega_0$. It can be linked to a variation of LOD or universal time UT1 by

$$m_3 = \frac{-\delta LOD}{T_0} = \frac{\partial(UT1 - UTC)}{\partial t}, \tag{11}$$

where $T_0$ is the nominal length of the sidereal day and $UTC$ (Universal Time Coordinated) is derived from worldwide atomic clocks.

The total axial perturbation $m_3$ can be decomposed into two parts. The first part describes the effect of the external torques to the Earth's system on the rotational speed, while the second one expresses the internal torques on the Earth's system. For a rotationally symmetric Earth, the axial component of the external torque vanishes, and therefore the first part is zero. As shown by BRZEZINSKI and CAPITAINE (2002), the effect of lunisolar perturbations on a triaxial Earth on UT1 reaches a maximum of only 5.2 μs in the semidiurnal band. The largest part of the axial perturbation $m_3$ is of geophysical origin.

Table 2 describes the detectability to UT1 variations by VLBI. The Wettzell G ring laser is at present insensitive to LOD variations.

### *3.3. Motions in the Frequency Domain*

In the frequency domain, the PM of the CIP when transformed to the IRP, with $p = p(\sigma)\ e^{i\sigma t}$ and $\sigma$ being the terrestrial angular frequency of the signal under consideration, reads

$$r = p(\sigma)\left(\frac{\sigma}{\omega_0}\right)e^{i\sigma t}. \tag{12}$$

Table 3 shows the maximum amplitude of $r$ for specific periods. For other long-period motions of the CIP (not shown in this table), the change of amplitudes in $r$ is small, less than 10 $\mu$as, and arises due to tidal gravitation or ocean tide contributions.

In the frequency domain, the PN of the CIP when transformed to the PM of the IRP, with $N = N(\sigma')\ e^{i\cdot\sigma'\cdot t}$ and $\sigma'$ being the celestial angular frequency of the signal, reads

$$m' = -N(\sigma')\left(\frac{\sigma'}{\omega_0}\right)e^{i[\sigma' t - \theta]}. \tag{13}$$

The Nearly Diurnal Free Wobble (NDFW) (MATHEWS and SHAPIRO, 1992), as it is often called by the geophysical community (ZŰRN, 1997), is excited as the rotation axes of mantle and fluid outer core are misaligned. The NDFW is resonant to the tidal forcing for diurnal tides, and the reaction of the rotating Earth is a damped wobble of the IRP around the pole of the axis of the greatest moment of inertia and a nutation in space of the IRP around the pole of the total angular momentum axis. The NDFW is a phenomenon which can be detected from subdaily polar motion estimates. With respect to a GCRF, this motion appears as part of the PN of the CIP, and has a period of about 430 days. The terminology conventionally adopted for this eigenmode is FCN (MATHEWS and SHAPIRO, 1992). The effect of the FCN is presently not detectable in the PM of the IRP because of

Table 2

*Description of contributions to UT1 variations. The symbol X denotes detectability, while O denotes that it is presently undetectable. MJO stands for Madden-Julian Oscillation and ENSO for El Niño Southern Oscillation. Information taken from* Gross *(2007)*

| Phenomenon | Period | Amplitude | VLBI |
|---|---|---|---|
| **Tidal effects** | | | |
| zonal solid Earth tides | ~9.1 days | ~2.205 mas | X |
| | fortnightly | ~18.975 mas | X |
| | monthly | ~18.885 mas | X |
| | semiannual | ~73.892 mas | X |
| | annual | ~24.706 mas | X |
| | ~18.6 years | ~2534.688 mas | X |
| ocean tides | ~1 day | < 1.108 mas | X |
| | ~0.5 days | < 0.560 mas | X |
| | fortnightly | ~1.723 mas | X |
| | monthly | ~1.725 mas | X |
| tidal friction | secular | ~544 as/century$^2$ | O |
| **Non-tidal effects** | | | |
| atmosphere | | | |
| winds | annual | ~1.095 mas | X |
| | semiannual | ~0.185 mas | X |
| | terannual | ~0.015 mas | X |
| MJO | ~30–60 days | | X |
| pressure | annual | ~0.110 mas | X |
| ocean | | | |
| winds-pressure | annual | ~0.055 mas | X |
| atmosphere-ocean ENSO | ~5 months | | X |
| mantle-core | decadal | ~1917.562 mas | X |
| coupling mantle | | | |
| inner core | ~6 years | ~11.834 mas | X |
| **Triaxiality** | ~0.5 days | ~0.049 mas | X |

Table 3

*Maximum difference in r as derived from space geodetic techniques. P/R denotes prograde (+)/retrograde ( − ) motion. Periods are given with respect to an ITRF*

| Phenomenon | P/R | Period | Amplitude | *r* |
|---|---|---|---|---|
| **Eigenmodes** | | | | |
| Chandler wobble | + | ~433 days | < 280 mas | < 647 µas |
| NDFW | − | ~1 day | ~200 µas | ~200 µas |
| Inner Core Wobble | − | ~2400–2500 days | < 3 mas | < 2 µas |
| **Nontidal atmospheric and oceanic forcing** | | | | |
| Annual wobble | +/− | 365 days | < 100 mas | < 274 µas |
| **Tidal effects** | | | | |
| ocean tides | + | ~24 hours | < 526 µas | < 526 µas |
| | + | ~12 hours | < 152 µas | < 304 µas |
| | − | ~12 hours | < 549 µas | < 1098 µas |
| tidal gravitation | + | ~24 hours | < 46 µas | < 46 µas |
| atmospheric diurnal tide | + | ~24 hours | ~10 µas | ~10 µas |

its smallness (less than 1 $\mu$as, see Table 4). The amplitudes of the FCN and NDFW are identical, as can be proven by equation (5), however their impacts onto the quantities $\dot{m}$ or $r$ are quite different (see Tables 3 and 4).

The prograde Free Inner Core Nutation (FICN) is assumed to range between 500 and 1500 days with respect to a GCRF, according to different references (see GUO and NING, 2002; MATHEWS and SHAPIRO, 1992; and MATHEWS *et al.*, 2002), but thus far has not been detected from nutation observations.

Equations (12) and (13) are especially useful to evaluate the changes in amplitudes of polar motion and precession-nutation for the transformation from the CIP to the IRP or *vice versa*. Both equations are used to compile Tables 3 and 4.

## *4. Results*

### *4.1. Very Long Baseline Interferometry (VLBI)*

The VLBI technique records signals at two or more sites of observation (see e.g., SOVERS *et al.*, 1998). These signals are then cross-correlated to produce the interference pattern. Currently, the group delay $\tau$ is used in VLBI analysis. The group delay is the time derivative of the phase with respect to the angular radio frequency and yields information about the EOP, besides positional information of stations and quasars. All noise and nongeometric effects should either be removed or estimated within the parameter estimation process.

*4.1.1 Precession-nutation of the CIP.* VLBI allows session-wise the estimation of precession-nutation corrections with respect to an *a priori* model. Gaps of a few days are common between successive VLBI sessions. However, the IERS applies an interpolating, filtering and smoothing scheme, in combination with other space geodetic techniques and encompassing all EOP, in order to produce *a posteriori* continuous daily values. Since the adoption of the MHB2000 precession-nutation model in the year 2003 (MATHEWS *et al.*, 2002), official daily corrections provided by the IERS to this model do not exceed 2 cm ($\sim$0.66 mas) when projected to Earth's surface (see Fig. 3). At present, an apparent drift

Table 4

*Maximum value of $\dot{m}$ as derived from space geodetic techniques. P/R denotes prograde (+) retrograde (−) motion. Periods are given with respect to a GCRF*

| Phenomenon | P/R | Period | Amplitude | $\dot{m}$ |
|---|---|---|---|---|
| **Precession-nutation of CIP** | +/− | all | | < 29 mas |
| **Eigenmodes** | | | | |
| Free Core Nutation (FCN) | − | ~430 days | ~200$\mu$as | < 1 $\mu$as |
| Free Inner Core Nutation (FICN) | + | (23) ~1025 days | ? | ? $\mu$as |

of PN of $-1.4$ mm/year ($\sim -46$ $\mu$as/year) can be depicted from the corrections in the $Y$ component. With time, the latter could become of long-periodic nature.

Furthermore, we performed a spectral analysis of the IERS EOP 05 C04 precession-nutation residuals with respect to MHB2000 for 1751 days since January 1, 2003. The largest quasi-circular space-referred retrograde motion, called FCN, has a mean period of about 438 days and an amplitude of approximately 0.2 mas ($\sim$6 mm). Other studies have shown that the amplitude of the FCN is variable ranging from 0.1 to 0.3 mas ($\sim$3 to 9 mm). Its nominal period is usually considered as 430.2082 solar days (see MATHEWS and SHAPIRO, 1992 and MATHEWS *et al*., 2002). The semiannual and monthly signals in the residuals of precession-nutation mainly arise from the motion of the $Y$ component (see Fig. 4 and Table 5).

*Hourly polar motion of the CIP and universal time corrections.* VLBI sessions from 2003 to 2007.5 were processed for hourly polar motion values of the CIP. All known effects, with available models (see list of Table 1 marked by a cross), were removed. In total there are 12710 epochs. However, outliers at 331 epochs were removed because their deviation, i.e., residuals, exceeded 2 mas ($\sim$6 cm). The standard deviation, using equal weights, of the residuals in the components $p_x$ and $p_y$ is about 1 cm ($\sim$0.3 mas), see Figure 5.

As regards hourly universal time values, we decided to remove 239 epochs because their deviation, i.e., residuals, exceeded 0.15 ms ($\sim$6.75 cm) after reduction for the effect of ocean tides on UT1. The standard deviation, using equal weights, of the residuals in the UT1 component is again about 1.2 cm ($\sim$27 $\mu$s), see Figure 6. However,

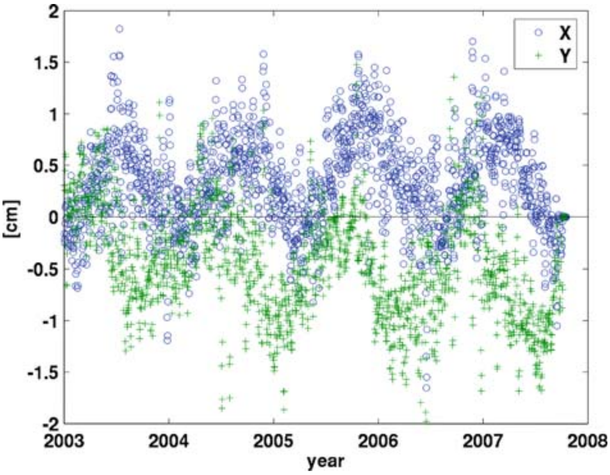

Figure 3
Correction of observed precession-nutation from the IERS with respect to MHB2000. Units: 1 cm corresponds to $\sim$0.3 mas.

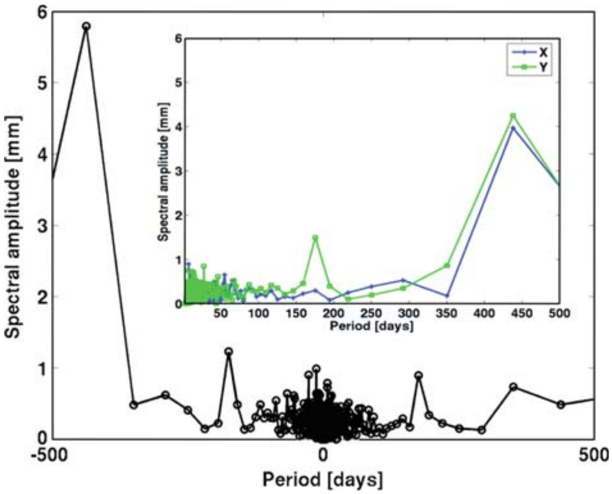

Figure 4
Spectral analysis of IERS EOP 05 C04 precession-nutation residuals with respect to MHB2000 for 1751 days since January 1, 2003. Units: 1 mm corresponds to ~30 $\mu$as.

Table 5

*Signals found in the residuals of the MHB2000 precession-nutation from the IERS EOP 05 C04 time series, with respect to a GCRF*

| Phenomenon | P/R | Period [days] | Amp [mm] |
|---|---|---|---|
| FCN | – | ~438 | 5.8 |
| semiannual | – | ~175 | 1.2 |
| fortnightly | – | ~13.69 | 1.0 |
| monthly | – | ~27.38 | 0.9 |
| semiannual | + | ~175 | 0.9 |

the standard deviation of unity weight, by using weights obtained from the formal errors of UT1 estimates, is reduced to 1 mm (~2$\mu$s).

### 4.2. *Wettzell G Ring Laser*

A detailed description of the Wettzell G ring laser is given in Schreiber *et al.* (2009). The basic relation between the relative change in the Sagnac frequency (see Post, 1967 and Anderson *et al.*, 1994), corrected for latitudinal tilt variations due to local effects, and the perturbation vector of the IRV, i.e., $m_1$, $m_2$, and $m_3$, is given by Mendes Cerveira *et al.* (2009)

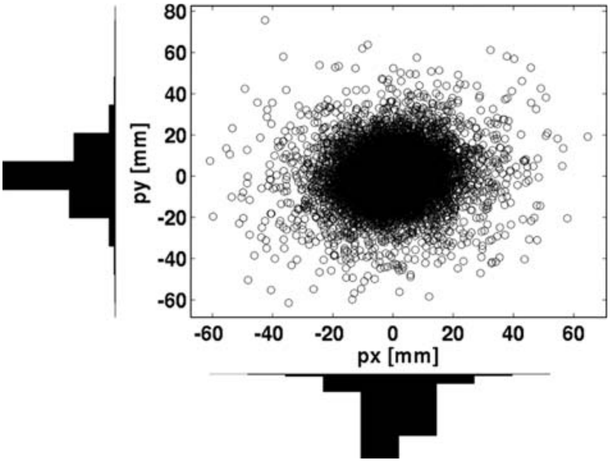

Figure 5
2-D histogram of residuals in hourly polar motion observed by VLBI. Units: 1 mm corresponds to $\sim 30\ \mu$as.

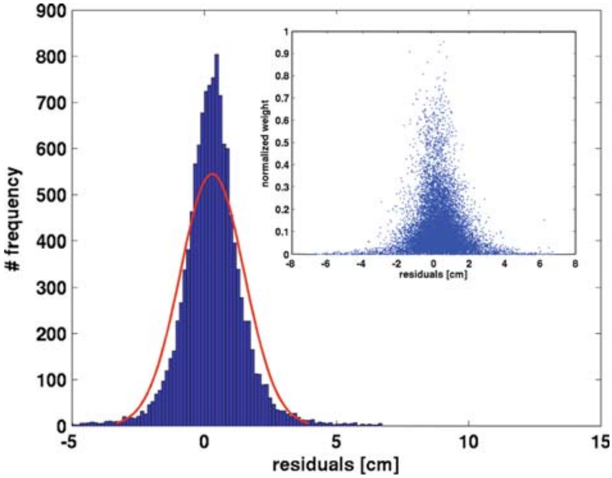

Figure 6
Histogram of residuals in hourly universal time. Units: 1 cm corresponds to $\sim 20\ \mu$s.

$$\Delta S_{RLG} \approx \cot \phi_0[-m_1 \cos \lambda_0 + m_2 \sin \lambda_0] + m_3, \tag{14}$$

where $\phi_0$ and $\lambda_0$ are the nominal geographic latitude and longitude of the ring laser position, respectively. The paper (Mendes Cerveira *et al.*, 2009) focuses on the partial derivatives from VLBI and ring laser data in terms of Earth rotation parameters.

Latitudinal tilt corrections, including deformation and attraction, due to the effect of solid Earth tides on station displacements, can be computed with (see MENDES CERVEIRA *et al.*, 2009)

$$\Delta S_{tilt} \approx -\frac{1-h_2+k_2}{l_2}\cot\phi_0\Delta\phi \approx -8.1163\cot\phi_0\Delta\phi, \tag{15}$$

where $h_2$, $k_2$, and $l_2$ are the nominal degree-2 Love and Shida numbers and $\Delta\phi$ is the geocentric latitudinal deflection. However, a change in the coefficient of equation (15) is required for the ring laser correction $\Delta S_{TR}$, because the ring laser is only sensitive to the geometric effect of tidal deformation, i.e.,

$$\Delta S_{TR} \approx -\frac{h_2}{l_2}\cot\phi_0\Delta\phi \approx -7.1759\cot\phi_0\Delta\phi. \tag{16}$$

Combining equations (8), (9), (11), and (14) suggests that similar to the Global Positioning System (GPS) (ROTHACHER *et al.*, 1999), the ring laser is, in principle, sensitive to the PM of the CIP, to its rate of change, to the rate of change of the PN of the CIP, and finally to LOD variations. Thus, a single relative Sagnac frequency at one site for one epoch is confronted to a set of seven unknown parameters $\{p_x, \dot{p}_x, p_y, \dot{p}_y, \delta\dot{X}, \delta\dot{Y}, \delta LOD\}$.

Equation (14) is complete in the sense that we assume a very high stability of the instrument in terms of the area of the beam circuit and its perimeter. In fact, the sensitivity of a ring laser depends on the area and perimeter of the beam circuit. A compromise is necessary, because on the one hand the sensitivity should be as high as possible, but on the other hand we would like to have a small instrument. One problem arising in small ring lasers is however the backscatter coupling, which is difficult to correct for. Besides, we suppose that all tilt-related signals (see CHAO, 1991; RAUTENBERG *et al.*, 1997; SCHREIBER *et al.*, 2003) have been removed from the ring laser data (MENDES CERVEIRA *et al.*, 2009).

*4.2.1. Data analysis.* The Wettzell G ring laser data was investigated for 144 days since the Modified Julian Date (MJD) 54000, i.e., September 22, 2006.

First, the Oppolzer motion has been removed by the model of BRZEZINSKI (1986) for an Earth having a liquid core. Then, the latitudinal displacement due to the solid Earth tides was computed for the Wettzell station following the IERS Conventions 2003. Subsequently, this displacement was transformed to a latitudinal tilt correction by applying equation (16). This refers to version V1 of Table 6. A second version V2 used the tiltmeter data, as described in SCHREIBER *et al.* (2009), to correct the ring laser data for locally induced tilt variations.

Figure 7 shows the residual spectral amplitude as obtained from the remaining ring laser signal, corrected for the Oppolzer motion and the latitudinal solid-Earth tides displacement. It shows a clear spectral peak for the O1 tide (4 cm) and for the S2 tide (2 cm). A difference of O1 should be detectable by VLBI in the PN of the CIP with a period of 13.66 days. Additional ring lasers could resolve this contradiction.

## 5. *Discussion and Conclusions*

Currently, the model of forced PM of the IRP developed by Brzezinski (1986) is aligned to the IAU1976 precession and the IAU1980 nutation models and is used to remove the retrograde diurnal PM signature of the IRP from the relative Sagnac frequency variation. In this respect, a change from the IAU1980 to the IAU2000 precession-nutation model only affects the retrograde diurnal PM of the IRP by less than 50 $\mu$as. This quantity is more than one order of magnitude smaller than that which is currently detectable by the Wettzell G ring laser.

To date, only one signal related to Earth rotation has been extracted from the Wettzell G ring laser, i.e., the signature of the retrograde diurnal PM of the IRP, and thus far no signatures of LOD variations have been detected. A second signal, which is not in direct relation to Earth rotation, i.e., the periodic latitudinal displacement of the ring laser produced by the solid-Earth tides, is also unambiguously present in the ring laser data. When projected to Earth's surface, the retrograde diurnal PM signature of the IRP attains a maximum amplitude of about 85 cm for an Earth model consisting of an elastic mantle and a liquid core (Brzezinski, 1986). The tilt signal visible in the Wettzell G ring laser

Table 6

*Tidal analysis of the residual ring laser signal. SNR denotes signal-to-noise ratio. V1 uses the model of the solid Earth tides, while V2 uses the tiltmeter data for the reduction. The model of Brzezinski (1986) has been applied in both cases, i.e., for V1 and V2*

| Tide | Period [hours] | Amp [mm] | $\sigma$Amp [mm] | Pha [deg] | $\sigma$Pha [deg] | SNR |
|---|---|---|---|---|---|---|
| **O1** | 25.8193 | | | | | |
| V1 | | 40.3 | 13.5 | 254.3 | 21.2 | 7.2 |
| V2 | | 48.3 | 16.9 | 256.9 | 16.3 | 8.2 |
| **M2** | 12.4206 | | | | | |
| V1 | | 10.9 | 7.0 | 78.1 | 45.6 | 2.4 |
| V2 | | 6.8 | 6.8 | 140.6 | 68.2 | 1.0 |
| **L2** | 12.1916 | | | | | |
| V1 | | 7.3 | 7.0 | 166.1 | 68.0 | 1.1 |
| V2 | | 8.4 | 7.3 | 167.3 | 57.9 | 1.3 |
| **S2** | 12.0000 | | | | | |
| V1 | | 19.5 | 7.7 | 33.3 | 21.5 | 6.4 |
| V2 | | 18.7 | 8.2 | 14.9 | 28.3 | 5.2 |
| **SK3** | 7.9927 | | | | | |
| V1 | | 7.1 | 6.6 | 190.9 | 53.6 | 1.2 |
| V2 | | 6.6 | 7.1 | 183.4 | 67.3 | 0.9 |
| **S4** | 6.0000 | | | | | |
| V1 | | 4.6 | 4.3 | 357.2 | 60.7 | 1.1 |
| V2 | | 4.1 | 4.8 | 352.9 | 71.9 | 0.7 |

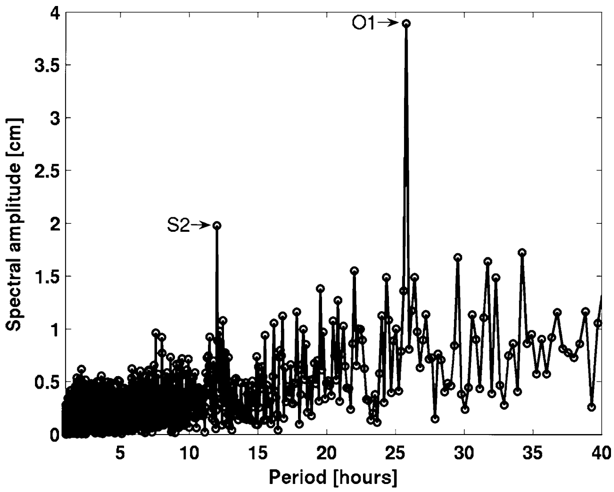

Figure 7
Spectral amplitude of the residuals from the Wettzell G ring laser after reduction of the effect of solid-Earth tides and Oppolzer terms.

data, induced by a latitudinal station displacement, reaches an amplitude of about 50 cm at the latitude of Wettzell.

The word "instantaneous" in association with Earth rotation calls for a temporal resolution of shorter than 2 to 3 hours. At present, this is the highest reasonable frequency achievable from both techniques for sensing Earth rotation variations. With the upcoming VLBI2010 system (see BEHREND *et al.*, 2008; and WRESNIK *et al.*, 2008), this limit will surely be reduced, leading to a resolution close to the period of Earth's free oscillations excited by strong earthquakes.

Finally, we emphasize that for geophysical interpretation we do not need the motion of the IRP. However, the exact relationship between the motion of the CIP and the IRP is required if ring laser data is to be combined with the VLBI technique. The ring laser data contains information pertinent to certain components of the state vector including the Earth-rotation components and their time derivatives. Therefore, the Kalman-filter procedure is perfectly suited to improve or update predicted unknowns by such observations. This filter corresponds to a sequential adjustment in the static case. In the future, a well-designed Kalman filter will be the perfect tool for combining VLBI and ring laser data in terms of EOP.

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