The Determination of Einstein's Light-Deflection in the Gravitational Field of the Sun

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SUMMARY

The problem of the observational determination of the light-deflection in the gravitational field of the Sun, as predicted by EINSTEIN's General Theory of Relativity, is outlined. All available results obtained at eclipse expeditions until now, as far as these have been successful, and their critical discussions are briefly summarized. Each set of observations is represented diagrammatically by the particular star field, and the published measures are shown for each star separately. The relevant details of each attempt are tabulated. An extensive bibliography covers most of the essential work on the problem.

1. HISTORICAL INTRODUCTION

ACCORDING to classical theory, in empty space light should be propagated in a straight line. At the beginning of the 19th century (when there was still some dispute as to whether light should be considered as corpuscular or purely as a wave-motion), SOLDNER (1801) investigated the behaviour of a light-ray in a gravitational field of the classical Newtonian type, assuming the corpuscular theory. Unfortunately, his formula contains the erroneous factor 2. Correcting for this, and using modern constants, it can be shown that light coming from a star, and just grazing the limb of the Sun before reaching an observer on the Earth, should be deviated by an angle of $0\rlap{.}''87$.

In 1908, and again in 1911, before he had developed his General Theory of Relativity, and probably without knowing anything of SOLDNER's earlier work, EINSTEIN investigated a possible deflection of light in a gravitational field (EINSTEIN, 1908, 1911). Starting from the conception of the equivalence of a uniform gravitational field and an accelerated system of reference, he arrived at the conclusion that radiation energy (light, etc.) must have inertia or mass, and that this mass must be subject to gravitational forces. From the principle of equivalence he derived directly a formula for the deviation of light (as it should appear to a terrestrial observer looking at a star, the light of which had passed through a Newtonian gravitational field near the Sun). He found the expression

$$\alpha = \frac{2kM}{c^2 r}, \qquad \ldots . (1)$$

where the angle of deflection is α (Fig. 1); k is the constant of gravitation, M the mass of the Sun, r the distance at which the light ray passes the centre of gravity, and c the speed of light in vacuum. For the limb of the Sun he found the value $\alpha = 0\rlap{.}''87$. This, as is to be expected, is the same value as that found by SOLDNER, although derived in a far more general way.

A first, somewhat tentative attempt to prove the existence of such an effect by an examination of older plates, taken for other purposes, gave no conclusive results

(FREUNDLICH, 1913). Special observations planned by the Cordoba Observatory at the eclipse of 1912, probably the first expedition aiming at the determination of the light-deflection, failed because of bad weather (PERRINE, 1923). A number of further suggestions were made to check the expected light-deflection by special observations during total solar eclipses (FREUNDLICH, 1913; CURTIS, 1913; FREUNDLICH, 1914). Even a proposal of making day-time observations for this purpose was thoroughly discussed (LINDEMANN, 1916a, 1916b). Later on, also the possibility of making suitable observations of the major planets was considered (TRUMPLER, 1929a, 1929b). Another attempt for observing the light-deflection in 1914 was prevented because of the outbreak of war; (FREUNDLICH, 1930). Shortly afterwards EINSTEIN (1916) published his famous General Theory of Relativity, in which he used his new law of gravitation, differing from NEWTON's classical law by small terms; it nevertheless become appreciable for the light-deflection close to the Sun. His formula reads

$$\alpha = \frac{4kM}{c^2 r} \qquad \ldots . (2)$$

and with the modern values

$$\begin{aligned} k &= 6{\cdot}67 \,.\, 10^{-8}\ \mathrm{cm^3 g^{-1} sec^{-2}} \\ M &= \text{mass of the Sun} = 1{\cdot}991 \,.\, 10^{33}\mathrm{g} \\ c &= 2{\cdot}998 \,.\, 10^{10}\ \mathrm{cm\ sec^{-1}} \\ r &= \text{radius of the Sun} = 6{\cdot}956 \,.\, 10^{10}\ \mathrm{cm}, \end{aligned}$$

we obtain for α (at the Sun's limb) the usually quoted maximum value $L = 1''.75$. Another calculation of this value was given by M. v. LAUE (1920). A confirmation of the formula (2) would obviously provide a crucial test for the whole concept of EINSTEIN's General Theory. As the formula cannot be verified by any experiment in a terrestrial laboratory, its confirmation by adequate astronomical observations is of fundamental importance.

Such work has so far been possible only during the brief and rare opportunities offered by total solar eclipses, because only then can we observe some stars sufficiently near to the Sun's limb.

At the 1918 eclipse a Lick Observatory expedition succeeded in obtaining photographs showing up to 50 stars around the Sun. But since the special technique required for these very difficult observations and their reduction was not yet fully developed, no conclusive results were obtained (CAMPBELL, 1919). Since then, a large number of further attempts were made by different observers. Owing to the great technical difficulties and the scarcity of solar eclipses, only very few of these have been even moderately successful, and more accurate observations, especially within one solar radius beyond the limb, are still urgently needed.

Actual results from observations of the light deflection in the gravitational field of the Sun have so far been published only for six eclipses. This is not surprising, for two reasons. As can be seen from what follows, these observations must be considered still as amongst the most difficult which can be attempted at an eclipse. Furthermore, from 1916 when EINSTEIN first published his formula (2) up to 1958 only 24 total solar eclipses have taken place, giving altogether a total observing time of not more than about 90 minutes. (The longest possible duration of a total solar eclipse is about $7\frac{1}{2}$ minutes (LEWIS, 1931) and such an occasion occurs very seldom.) A considerable number of these 24 eclipses could not be used, because either the star field surrounding the Sun was unsuitable, or the duration of the eclipse was too short; bad weather at the moment of the eclipse, and uncertain political circumstances have prevented any observations at quite a number of other eclipses.

During earlier years there was much discussion and controversy as to whether the predicted Einstein effect may be influenced, or masked, or simulated by other physical factors in the near surroundings of the Sun; for instance, by diffraction in the

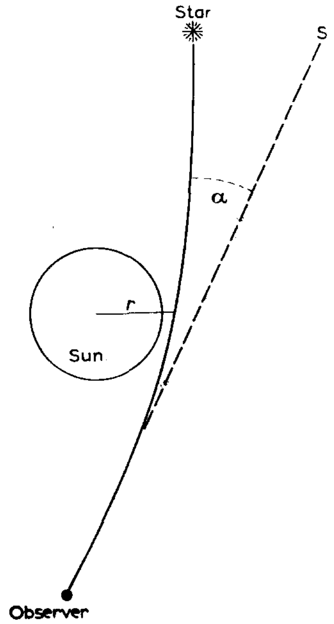


FIG. 1. The light-deflection as seen by a terrestrial observer. The General Theory of Relativity predicts a shift of a star image by the angle α, away from the Sun, according to equation (2). If the light ray grazes the Sun's limb, α reaches its maximum 1".74; this value is usually quoted as the "Einstein Effect", denoted in this paper by L.

Sun's atmosphere, or even by effects in the Earth's atmosphere (JONCKHEERE, 1918; LINDEMANN, 1918; EDDINGTON, 1918a, 1918b, 1918c; ANDERSON, 1919, 1920a, 1920b, 1922, 1924; DINES, RICHARDSON and ANDERSON, 1918; JEFFREYS, 1919; Royal Astronomical Society discussion, London, 1919a, 1919b; EDDINGTON and CROMMELIN, 1919; NEWALL, 1919, 1920; BAUER and PETERS, 1920; FERRIER, 1922a, 1922b; EMDEN, 1920, 1922; v. GLEICH, 1928; CAMPBELL-TRUMPLER, 1923a, 1928). But it has been shown convincingly that in the light of our present knowledge such effects must be expected to be quite negligible. This holds also for the so-called Courvoisier Effect (COURVOISIER, 1920, 1932). Modern values for the refractive index in the solar atmosphere have been given, for instance, by PROISY (1949).

Other objections have been based on possible distortion of the photographic emulsion (SILBERSTEIN, 1920; SLOCUM, 1921; ROSS, 1920; see also GOLLNOW and HAGEMANN, 1956), and photometric effects (BOTTLINGER, 1920a, 1920b; WOLF, 1920). More serious are troubles possibly introduced in some circumstances by the optical system (e.g. STROHMEYER, 1921). More recently also a possible deviation of photons in the gravitational field of the Sun has been discussed (WHEELON, 1952; PAPAPETROU, 1953).

Reports and discussions on the light-deflection have been frequently published ever since this topic was first raised by EINSTEIN. While the more specialized and important papers will be quoted individually in their proper context below, references to a number of more general papers are also given here. (EDDINGTON, 1918a, 1918b, 1918c; BOTTLINGER, 1920a, 1920b, FORSYTH, 1921a, 1921b; HEPPERGER, 1922; HOPMANN, 1928; TRUMPLER, 1929c; KOPFF, 1932; STOYKO, 1932; DANJON, 1932a, 1932b; HERMANN, 1936; DYSON and WOOLLEY, 1937; TIKHOV, 1937; BUCERIUS, 1938; OVENDEN, 1952; FREUNDLICH, 1952, 1953, 1955; TRUMPLER, 1956; MATTIG, 1956; MIKHAILOV, 1956, 1957, 1959. Some essentially historical remarks can be found in TRUMPLER, 1923; PERRINE, 1923; POOR, 1927; HALLUIN, 1942).

2. GENERAL TREATMENT OF THE PROBLEM

Before giving a report of the actual observations and the results so far available, we would like to outline the general conditions which should be fulfilled to ensure successful observations and to point out the many difficulties the observers have to cope with.

The light deflection α, as predicted by Einstein's formula (2), can be represented by a hyperbola of the kind shown in Fig. 2. To decide whether the curve is a hyperbola

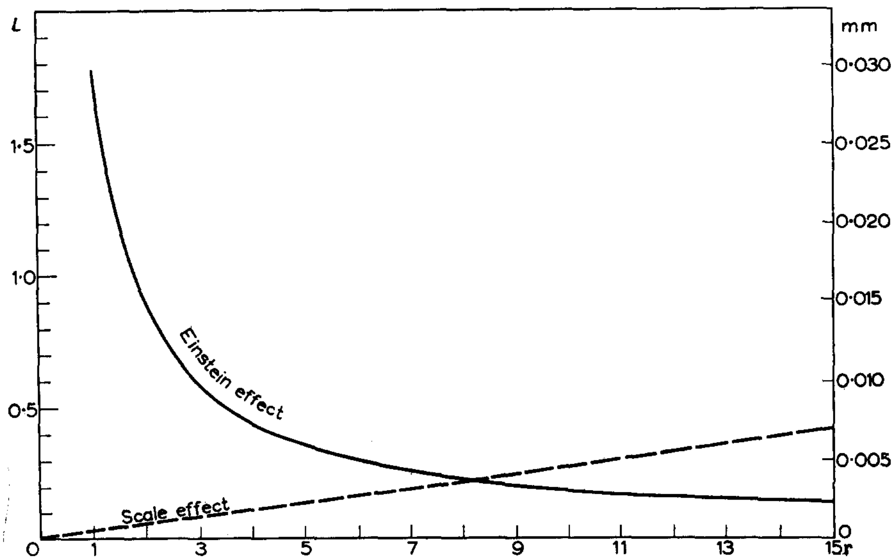


FIG. 2. This graph, a hyperbola, shows the behaviour of the predicted light-deflection, plotted as a function of the distance r from the centre of the Sun. The broken straight line indicates the "Scale Effect" (see p. 53), produced by an alteration of 0·1 mm in the focal setting of a "Normal Astrograph" (f = 343 cm).

Abscissae: distances from the centre of the Sun, expressed in units of the solar radius.
Left-hand ordinate: light-deflections in seconds of arc, as predicted by EINSTEIN (1916). See eq. (2).
Right-hand ordinate: the same, but expressed in millimetres on the photophraphic plate, assuming the focal length of the telescope to be 343 cm.

and, if so, to determine its form satisfactorily, is possible only if sufficient observations are available for $r < 2$. That is, enough stars must be available near the Sun's limb at the moment of totality, and their images on the photographic plate must be measurable with high precision. This condition raises the primary difficulty that the Sun passes rather few suitable stars in its yearly movement along the ecliptic, and that obviously such near and favourable approaches will coincide very seldom with the precise moment of a total eclipse. Furthermore, the Sun's corona becomes rapidly brighter when one approaches the Sun's limb, and therefore reduces seriously the contrast between the star images and their background on the photographic plate. This obliteration by the corona is the reason why at all eclipses which permitted successful observations of the light-deflection, the most interesting stars near the Sun's limb have been very difficult to measure, or were actually lost altogether. For the 1929 eclipse, which was observed through a sky of excellent transparency, some average values were determined of the limiting stellar magnitude which could be successfully observed near the Sun, as a function of their distance from the Sun's limb (FREUNDLICH, V. KLÜBER, and V. BRUNN, 1933a; V. KLÜBER, 1932b). A graph showing this relation is reproduced here in Fig. 3, but it must

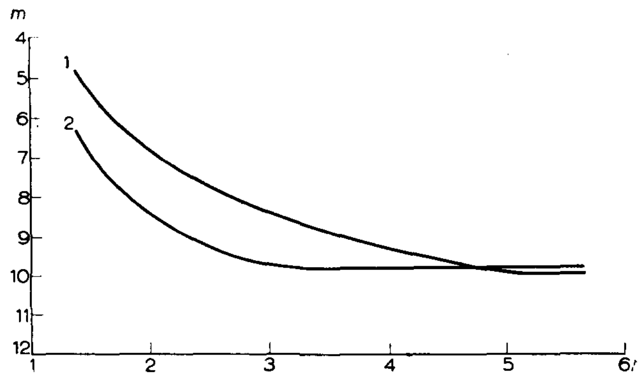


FIG. 3. This diagram indicates in a general manner the faintest stellar magnitude m which may on the average, be recorded on an eclipse photograph, taken with 60 seconds exposure time under favourable conditions, plotted as a function of the distance r of the star from the Sun's centre. (1) Telescope of 20 cm aperture and focal length $f = 343$ cm; (2) telescope of 20 cm aperture and $= 850$ cm. *Abscissae*: distances from the centre of the Sun in solar radaii. *Ordinates*: faintest stellar magnitude to be expected. (V. KLÜBER, 1932b).

be emphasized that only approximate values can be given in this way. The contrast between corona and star image depends, for instance, upon the colour temperature of the stars, upon the actual intensity of the corona at the position occupied by the star, and especially also upon the photographic emulsion and the processing (to which therefore much consideration has to be given). Taking into account that the spectral distribution of the corona light is identical with that of the Sun, it may perhaps be possible to reach somewhat fainter stars by using a suitable red filter and a red-sensitive emulsion. The intensity of the corona, furthermore, depends upon the cycle of solar activity, and quite appreciably upon the position angle with respect to the Sun's axis. At $r=2$, differences in the surface brightness of the corona of the order of more than one magnitude may occur (VAN DE HULST, 1950). Also larger f-ratios than those given in our example in Fig. 3 may help in reaching fainter starts. It follows from what we have just said that it will probably not be possible ever to measure the full maximum value of 1″.75 of the deflection itself, which in other problems of astrometry would be a fairly large and easily measurable amount. The nearest stars so far observed were, with a single exception, at distances from the Sun's centre of about $r \geqq 2{\cdot}0$, corresponding to a deflection of the order of only 1″.0 (see below, Table 1, column 13). The most interesting part ($r < 2$) for the hyperbola in Fig. 2 is therefore hardly covered by observations at all.

Total solar eclipses are visible only from within a very narrow belt (100–200 kilometres) of a length of many thousands of kilometres, which the shadow of the Moon draws more or less at random over the surface of the Earth. Besides the trouble introduced by the light of the corona, the main and principal technical difficulty for observing the light-deflection is therefore, that it requires the absolute measurement of a very small quantity during a particular short time-interval under the usually quite difficult conditions of a temporary field-station in some more or less remote part of the world.

The technique of reducing plates for the determination of the light deflection is related to that used for many years in the photographic method for stellar parallaxes and proper motions (ZURHELLEN, 1904; KÖNIG, 1933). The principle is shown in a simple sketch in Fig. 4. The Sun with surrounding stars is indicated in (a) which

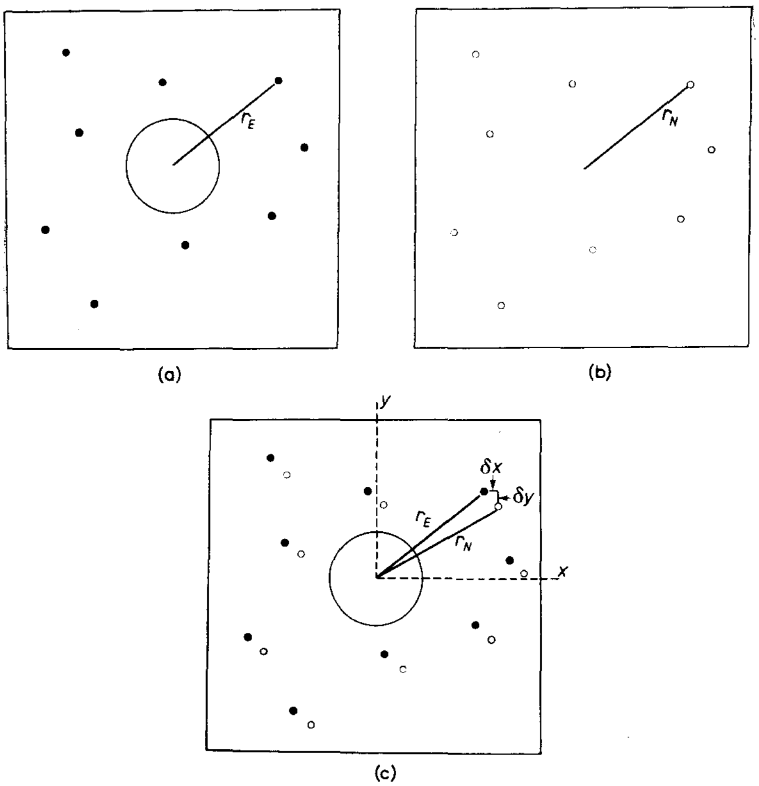


FIG. 4.

The notations are explained on p. 53.
(a) = "eclipse plate" showing the Sun and surrounding stars;
(b) = the corresponding "night plate" taken of the same star-field when visible at night, i.e. several months after (or before) the eclipse;
(c) = both plates combined, as they appear in the measuring machine, for the purpose of determining relative rectangular co-ordinate differences.

is usually called the eclipse-plate. The positions of the stars with the high accuracy needed here, cannot be obtained simply from a star catalogue. Therefore, the same star-field has to be photographed before the eclipse, or more often after, at a time when this field is visible in the night sky. This is done with the same instrument, and so far as possible when the field has the same position in the sky as during the eclipse. This produces the so-called night-plate (b) and, for convenience in later measurements, this is usually taken through the reverse side of the plate. For then the eclipse-plate and the night-plate can be adjusted emulsion to emulsion in the measuring-machine in such a way that very near to each star of the eclipse-plate the corresponding image from the night-plate will be visible (c). It is then easily possible to measure the relative position of each of such star-pairs in rectangular co-ordinates δx and δy as indicated in Fig. 4. Their values will depend upon the accidental way in which the two plates are clamped together in the measuring-machine (involving a lateral shift and a rotation); furthermore, on the possible change in the scale of the instrument (mainly due to the difference in the setting of the focus, which may have occurred between the exposure of the eclipse-plate and the exposure of the night-plate), on some other plate parameters, and on the light-deflection itself. Minor corrections may be necessary for differential atmospheric refraction, for differential aberration, for proper motion and parallax; but these can generally be applied in the usual way without much difficulty. If sufficient stars are available (at least 6), the various required parameters can be determined by a straightforward analytical method, and, finally, the Einstein effect L can be derived.

It will be seen from what follows that the main trouble in the reduction arises from the quite unavoidable fact that for instrumental reasons there will generally be a more or less conspicuous difference in the scale-value between the eclipse-plate and the night-plate. This can be easily visualized from our Fig. 2. Supposing the Sun is near the centre of the plate of a Normal Astrograph of focal length $f = 343$ cm; then the Einstein effect (full line) will shift the star's image at the Sun's limb by about $1''.75$, or by about 0·031 mm on the plate. At $r = 8$ solar radii from the centre of the Sun, the Einstein effect will have dropped to only $0''.27$ or about 0·005 mm. But if the effective focal length of the instrument or its focal setting (i.e. the distance between the principal plane of the objective and the surface of the photographic plate) has changed between the eclipse and the night exposure (which are necessarily separated by a period of at least 4 months) by only 1/10 mm, it would cause a scale-correction which is indicated in our sketch by the broken line, i.e. a deviation increasing linearly with distance from the plate's centre. At about $r = 8$ this scale-correction is already of the same order as the Einstein effect at the same distance. Whilst the Einstein effect is decreasing hyperbolically with distance from the Sun, the scale-correction is increasing linearly (Fig. 2). It is obvious that the determination of the Einstein effect will be more and more difficult with increasing distance of the available stars from the Sun, not only because it becomes much smaller, but mainly because the unavoidable scale error becomes more and more effective.

In this just outlined relative measurement of an eclipse-plate against the night-plate in rectangular co-ordinates each star i at a distance r_i from the Sun's centre will contribute one value δx_i and one value δy_i (Fig. 4). The analytical procedure to extract from these measurements the desired Einstein effect, and to separate it from the other instrumental parameters, proceeds usually by forming for each star two equations of the form:

$$\begin{aligned}\delta x_i = {} & A_x + B_x y_i + C_x x_i^2 + D_x x_i y_i + S_x x_i + L_x \frac{x_i}{r_i^2} + \\ & + E_x x_i^3 + F_x x_i^2 y_i + G_x x_i y_i^2 + H_x y_i^2 + J_x y_i^3\end{aligned} \quad \ldots . (3)$$

$$\begin{aligned}\delta y_i = {} & A_y + B_y x_i + C_y x_i y_i + D_y y_i^2 + S_y y_i + L_y \frac{y_i}{r_i^2} + \\ & + E_y y_i^3 + F_y y_i^2 x_i + G_y y_i x_i^2 + H_y x_i^2 + J_y x_i^3,\end{aligned} \quad \ldots . (4)$$

where we should have

$$B_x = -B_y, \quad C_x = C_y, \quad D_x = D_y, \quad E_x = E_y, \quad F_x = F_y, \quad G_x = G_y,$$
$$H_x = H_y, \quad J_x = J_y, \quad L_x = L_y.$$

To most of the coefficients a geometric significance can be given as follows:

A = arbitrary relative translatorial shift and
B = arbitrary relative rotational shift,
of the two plates during the measurements
C and D = inclination of the plate against the optical axis
S = Scale-value
L = light-deflection
E and F = scale-value, inclination, optical distortion
G = scale-value and inclination
H and J = bending of plate.

In most cases only the first six parameters of these equations have been used and these have often been found sufficient for solving the problem. For n stars we have $2n$ equations of this kind, and these are solved in the usual way by the method of least squares. The number of stars must obviously be at least equal to the number of unknown parameters used in the equations, i.e. usually at least six.

As we have already seen from purely geometric considerations there is a very unpleasant connection between the scale-value S and the Einstein effect L. This can be demonstrated, of course, even better by a detailed analytical investigation as it was carried out by several authors (e.g. FREUNDLICH and v. BRUNN, 1933; FREUNDLICH and GLEISSBERG, 1935; FREUNDLICH and LEDERMANN, 1944). Let us assume that we require a standard deviation σ_L for L of the order of $\pm\ 0''.1$; the weight W_L for L will then be $1/\sigma_L = 100$. From measurements of k plates the standard deviation σ_0 of δx_i and δy_i becomes

$$\sigma_0 = \pm \frac{0''.3}{\sqrt{k}},$$

or the weight

$$W_0 = \frac{k}{0{\cdot}09},$$

and therefore the corresponding relative weight

$$\frac{W_L}{W_0} \geqq \frac{9}{k}.$$

From the theory of least squares for the solution of normal equations *including* the parameters L and S, and for a fairly symmetrical star distribution of n stars, we get

$$\frac{W_L}{W_0} = n\left(\frac{1}{h} - \frac{1}{a}\right) \qquad \text{with}$$

$$\frac{1}{h} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{r_i^2} \qquad \text{and}$$

$$a = \frac{1}{n}\sum_{i=1}^{n} r_i^2,$$

where r_i is the distance of the star i from the Sun's centre. The expression in the round bracket will always be a small quantity. If however the scale-value of S is *not* included in the equations, but is known independently by some suitably arranged observations, we would get the much more favourable expression

$$\frac{W_L}{W_0} = \frac{n}{h}.$$

The sensitive dependence of L on the scale-value S, and therefore the accuracy required for the determination of the scale value (or of the equivalent focal-lengths f), can be seen from the following expression, which can be derived in an elementary manner from the normal equations

$$\delta L = -h\delta S = -\bar{r}^2\delta S = -\frac{\bar{r}^2 \,.\, df}{f},$$

$\bar{r}$ being the mean distance of the stars from the centre of the Sun. If we want δL to be of the order of 0″1, i.e. 10^{-4} of the solar radius, and accept $r = 5$, we get

$$\frac{df}{f} = 4 \,.\, 10^{-6}$$

which means that the scale (in other words the effective focal-length) must be known to better than $\pm\ 10^{-5}$ (FREUNDLICH and v. BRUNN, 1933). This is a very strict experimental requirement in view of the conditions under which eclipse observations are made, for it means that the change of the focal setting for a Normal Astrograph (of $f = 343$ cm) between the exposure of the eclipse-plate and the night-plate should be known to less than $\pm\,0{\cdot}03$ mm. Hence one of the main tasks of the observer is to determine the scale-value of his plates as accurately as possible. This, of course, has been realized right from the beginning and special observational techniques have been developed to achieve this end. If no special arrangements for the determination of the scale-value are made, then the equations (3) and (4), containing the parameter S as one of the unknowns, must be applied straight away. As we have seen, the weight of the resulting L may be appreciably lowered by the coupling of S and L in the equations. To meet this difficulty, as early as 1919 observers had arranged to obtain photographs of an independent field of stars (the "check-field") at a sufficient distance from the Sun, with the same instrument, either during the eclipse itself or in the night before or after the eclipse; this procedure should provide some kind of check of the scale-value. A specially promising method was tried several times: during the eclipse itself the telescope was pointed first to the Sun, and then to a check-field so far away that the Einstein effect could be considered as negligible. The plates then carried two star fields superimposed upon each other, the intention being that the instrumental parameters of the equations and especially the scale-value would be obtained from the check-field, quite independent of the Einstein effect. A disadvantage of this procedure is that the movement of the telescope, even through a fairly small angle, may itself cause a small alteration of the focal setting which, as we remember, ought to be kept constant to about $\pm 0{\cdot}01$ mm (FREUNDLICH, v. KLÜBER and v. BRUNN, 1933). Probably it would be worthwhile to try the same procedure with a horizontal camera, merely swinging the coelostat mirror through a small angle during the eclipse (FREUNDLICH, v. KLÜBER, and v. BRUNN, 1931a).

In another method a plane-parallel plate (or a semi-transparent mirror, or even a real quartz-mirror which is smaller than the objective), tilted with respect to the optical axis of the telescope, has been used in front of the main objective (MIKHAILOV, 1949; VAN BIESBROECK, 1949; POPOV and FJODOROV, 1954). The star-field surrounding the Sun is photographed through this plate, while at the same time a check-field at some distance from the Sun is reflected by the plate onto the same photographic plate. Again two star fields superimposed upon each other are obtained, in this case with strictly simultaneous exposures. This very attractive method has been criticized because the two star-fields are not imaged by strictly the same optical arrangement, the one being transmitted and the other one reflected by the semi-transparent plate. Any small distortion of this plate may affect the reflected beam, and thus the scale-value which is supplied by the check-field more than the transmitted beam (FREUNDLICH, 1950). In principle, therefore, the scale-value taken from the check-field might again be not quite reliable. But if the dimensions of the reflecting mirror are kept small enough, so that each point of the check-field is imaged over the same whole mirror-surface, then the mirror will obviously not alter the scale-value of the check-field with respect to the eclipse-field by a significant amount; this method seems, therefore, to be rather promising. Its main disadvantage is that at the best it necessarily causes a loss of about half the light in each field.

Another very elaborate observing-method was developed and used at several eclipses by the Potsdam observers (FREUNDLICH, 1930; FREUNDLICH, v. KLÜBER and v. BRUNN, 1931a, 1931b; v. KLÜBER, 1926, 1929a, 1929b, 1931; MATTIG, 1956). They worked with a large horizontal double-camera, and the scale-value was again determined by an independent star-field during the eclipse itself. Furthermore, with the help of a large collimator, photographic reseaux were printed photographically on all plates to determine scale changes between the plates.

The scale value depends essentially on the distance between the objective and the emulsion of the photographic plate: SCHÜRER's (1954) attempt to control this distance by three invar wires was frustrated by poor weather.

Before considering in more detail the few expeditions so far successful in obtaining values for the Einstein effect, we should just mention briefly some other important technical requirements. The extremely high accuracy needed in controlling the effective focal setting obviously necessitates also very high accuracy in the definition of the position of the photographic plate, that is, of the plate holder on the telescope itself (KÖNIG and v. KLÜBER, 1941). Such techniques are known from parallax work.

Besides an independent determination of the scale value, as just explained, first-class guiding is absolutely essential; many observations in the past have failed in this respect. The scale-value of a Normal Astrograph ($f = 343$ cm) is $1'' = 0{\cdot}017$ mm, and star positions on the plates should be measured to about $\pm 0{\cdot}002$ mm, an accuracy not attainable unless the star images are symmetrical and of the best quality. Experience has shown that to guide rather large telescopes or coelostats under field conditions at an eclipse needs a great deal of consideration and special precautions (v. KLÜBER, 1932a).

Furthermore, high quality lenses and, if coelostats are used, quartz mirrors, are essential, despite the fact that usually a strictly differential method of reduction will be applied.

From general considerations a star-field with as symmetrical a distribution around the Sun as possible is desirable. As this depends entirely upon the accidental circumstances of the eclipse, this condition unfortunately is often not sufficiently fulfilled and then, if reductions are not carried out very carefully, systematic errors may result.

Because of their importance and the general interest of the results, nearly every published determination of the light deflection has led to a considerable amount of discussion. It is rather confusing to find that, in addition to the results published by the authors themselves, later discussions by others have often produced rather different figures from the same observational material. The chief reason for this is found in the fact already mentioned, that the final value for the light-deflection L is always quoted as extrapolated to the limb of the Sun, while the actual observations so far cover only ranges beyond $r > 2$, where the effect because of its hyperbolic decrease is very much smaller. A very small alteration in the much disputed scale-value or in the grouping or weighting of the stars, etc., can easily cause a rather large alteration of the result when extrapolated right up to the Sun's limb.

In Table 1 (column 16) we quote the numerical results in the same way as they were given by the authors themselves. Corrections proposed later by various critics will be mentioned below. There are, furthermore, two papers giving a more general discussion of the available measurements, one by DANJON (1932b, 1932c), and another by MIKHAILOV (1956, 1959). As these are both very interesting in demonstrating the nature of the relation between the observations and the final result, they are quoted below as well. In order to find possible corrections of the light-deflection and of the scale-value, after having eliminated the other parameters by a first approximation, DANJON proceeded in the following way: We denote by r_i the distance of the star from the centre of the Sun in units of the solar radius; by Δr_i the radial shift found for this star in the first approximation; by L the value of

Table 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	Observatory (site)	Eclipse	Focal length f (cm)	Aperture ϕ (cm)	ϕ/f	Instrument (lens)	Field of plate	Number of plates	Exposure (sec)	Limiting stellar magnitude	Number of stars	$r_{\min}$ (solar radii from centre)	$r_{\max}$ (solar radii from centre)	Check-field	L	m.s.e.	Reference
1	Greenwich (Brazil)	1919 May 29	570	10	1:57	Coelostat (Double)	2°4x2°0	7	28	6(ph)	7	2	6	no	1″98	0″16	DYSON-EDDINGTON-DAVIDSON, 1920
			343	20	1:17	Coelostat (Double)	2·7×2·7	16	5–10	6(ph)	11	2	6	no	0·93	—	
2	Greenwich (Principe)	1919 May 29	343	20	1:17	Coelostat (Double)	2·7×2·7	2	2–20	6(ph)	5	2	6	no	1·61	0·40	
3	Adelaide-Greenwich (Australia)	1922 Sept. 21	160	7·5	1:21	Astrograph (Quadruple)	7×8	2	20–30	8·3	11–14	2	10	yes	1·77	0·40	DODWELL-DAVIDSON, 1924
4	Victoria (Australia)	1922 Sept. 21	330	15	1:22	Astrograph (Quadruple)	—	2	45	9·0	18	2	10	not used	1·75 1·42 2·16	—	CHANT-YOUNG, 1924.
5	Lick I (Australia)	1922 Sept. 21	450	12	1:37	Doubl. Astrograph (Double)	5×5	4	120–125	10·5(ph)	62–85	2·1	14·5	yes	1·72	0·15	CAMPBELL-TRUMPLER, 1923a.
6	Lick II (Australia)	1922 Sept. 21	150	10	1:15	Astrograph (Quadruple)	15×15	6	60–102	10·4(ph)	145	2·1	42	yes	1·82	0·20	CAMPBELL-TRUMPLER, 1928
7	Potsdam I (Sumatra)	1929 May 9	850	20	1:42	Coelostat (Double)	3×3	4	40–90	8·9	17–18	1·5	7·5	yes	2·24	0·10	FREUNDLICH-v. KLÜBER-v. BRUNN, 1931a.
8	Potsdam II (Sumatra)	1929 May 9	343	20	1:17	Astrograph (Triplet)	7·5×7·5	3	14–56	9·5	84–135	4	15	yes	—	—	FREUNDLICH-v. KLÜBER-v. BRUNN, 1933.
9	Sternberg (U.S.S.R.)	1936 June 19	600	15	1:40	Astrograph (Double)	3·5×3·5	2	25–35	9·6	16–29	2	7·2	not used	2·73	0·31	MIKHAILOV, 1949.
10	Sendai (Japan)	1936 June 19	500	20	1:25	Coelostat (Double)	2·9×2·9	2	80	8·6 (vis.)	8	4	7	no	2·13 1·28	1·15 2·67	MATUKUMA, 1940a.
11	Yerkes I (Brazil)	1947 May 20	609	15	1:40	Astrograph (Triplet)	4×4	1	185	10·2	51	3·3	10·2	not used	2·01	0·27	VAN BIESBROECK, 1949.
12	Yerkes II (Sudan)	1952 Feb. 25	609	15	1:40	Astrograph (Triplet)	4×4	2	60–90	8·6	9–11	2·1	8·6	yes	1·70	0·10	VAN BIESBROECK, 1953

the light-deflection as predicted by EINSTEIN at the limb of the Sun; by k a possible correction factor to L; and by S a possible correction to the scale-value, as found in the first approximation. We then have:

$$\Delta r_i = \frac{L}{r_i} + S_i r_i, \qquad \text{or}$$

$$\frac{\Delta r_i}{r_i} = \frac{1{\cdot}75\,.\,k_i}{r_i^2} + S_i.$$

Inserting

$$x_i = \frac{1{\cdot}75}{r_i^2} \quad \text{and} \quad y_i = \frac{\Delta r_i}{r_i},$$

this can be written as the equation of a straight line (as shown as example in Fig. 5).

$$y_i = k_i x_i + S_i.$$

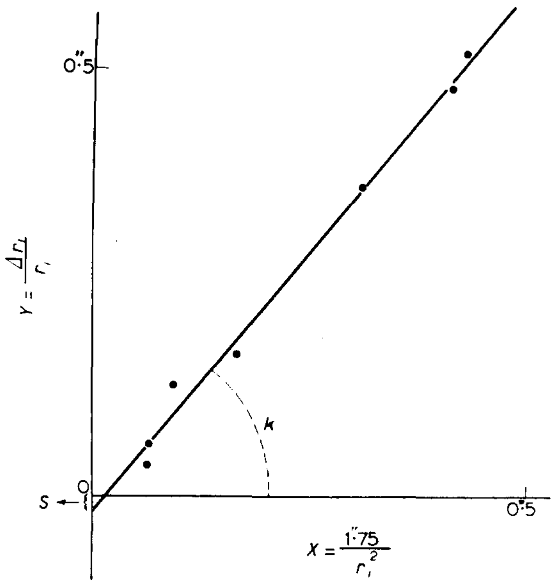


FIG. 5. Explanation of DANJON's method (pp. 58-59) for the final reduction of the measurements. The quantity $\tan k$ is the correction-factor to be applied to the predicted light-deflection L; S represents the scale-correction (DANJON, 1932c). The above figure illustrates the reduction of the Greenwich expedition of 1919, giving $\tan k = 1''18$, and therefore $L = 1{\cdot}74 \times 1''18 = 2{\cdot}06$; the scale-correction is $S = -0''014$.

Its slope gives k, the correction to the light-deflection; and its intercept on the ordinate axis gives the correction to the scale-value. Both corrections can thus be visualized conveniently. This procedure again has been questioned because of the distribution of weights between the two unknowns (MIKHAILOV, 1956).

MIKHAILOV (1956), on the other hand, has taken most of the observations so far published and applied a final least-squares solution, using the equation of condition

$$\Delta r_i = \frac{L_i}{r_i} + S_i r_i$$

which gives final correcting values for L and for the scale parameter S. His results are quoted below.

3. Survey of the Eclipse Results

After these more general considerations we give next a short account of the few expeditions so far successful in obtaining values for the Einstein effect. (See also Table 1.)

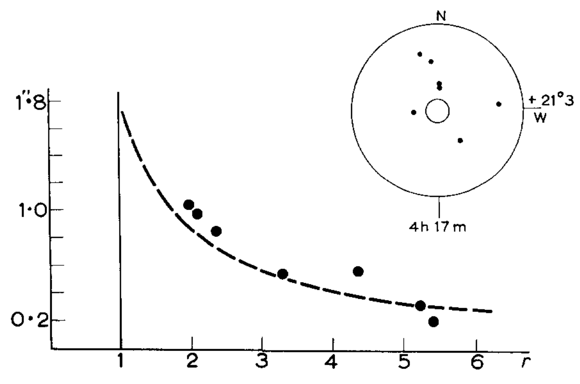
Fig. 6 represents all star-fields which have so far contributed to the determination of the light-deflection, up to a distance of about $8r$; they contain only those stars which have really been measured and used for the reductions. The same figure shows for each of these determinations the radial shift for each star (expressed in seconds of arc, as ordinates), plotted against its distance from the Sun's centre in units of the solar radius (i.e. 15 minutes of arc), as abscissae; only the values given by the authors themselves have been used. Different weights, as sometimes adopted by the authors, are not indicated. EINSTEIN's predicted deflection is shown in each

FIG. 6. These 9 combined diagrams show the actually measured light-deflections for each star, as far as available, using only the data given by the authors themselves, without having regard to individual weights or group-means. Some small amendments, mainly due to scale correction, may have to be applied to the one or the other of these observational sets. The broken hyperbola represents the Einstein Effect as it should be expected from theory.

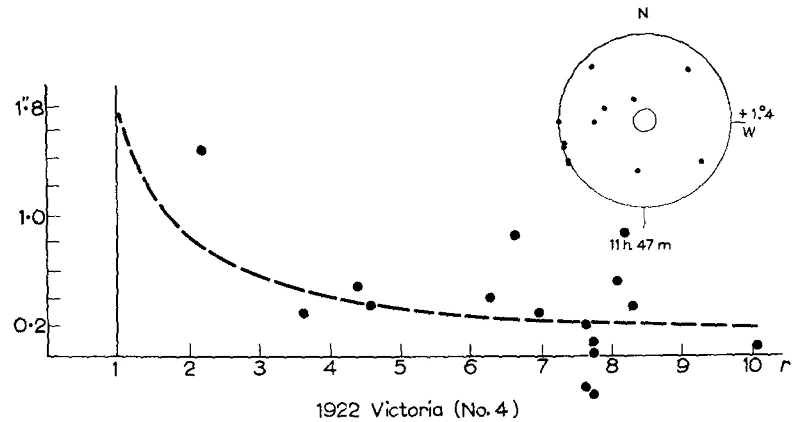
Abscissae: distances from the centre of the Sun in solar radii.

Ordinates: measured light-deflections in seconds of arc.

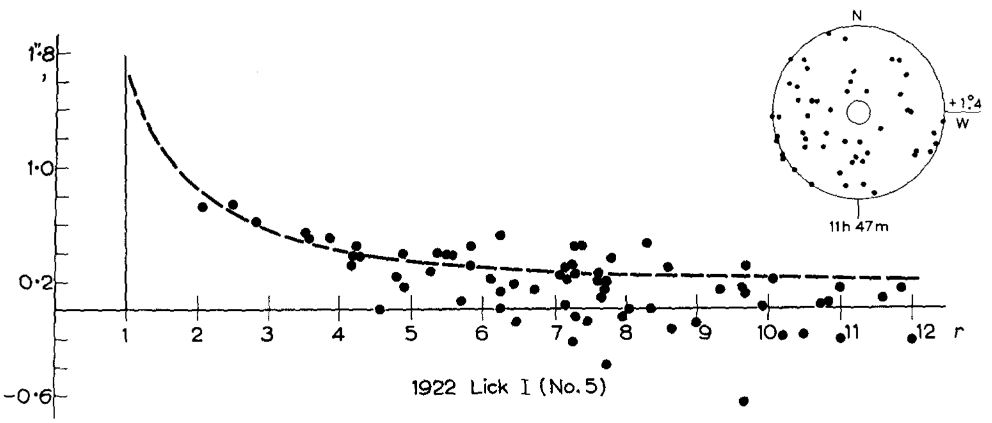
Inserted into the top right corners are the corresponding star fields, to a distance of about 8 solar radii from the Sun's centre. Only actually measured stars are plotted, without regard to the weight given by the observers. Co-ordinates are indicated, giving the positions of the Sun's centre for 1855·0 (B.D.-charts).



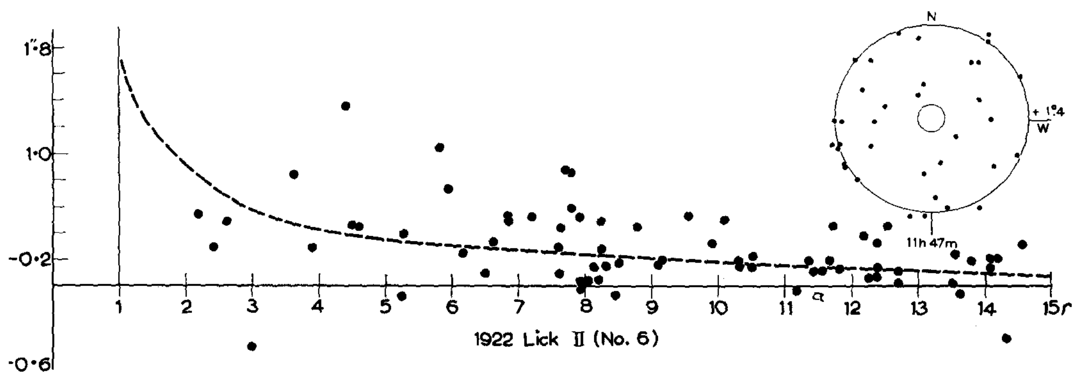
1919 Greenwich, (see Table 1, Nos. 1 and 2)



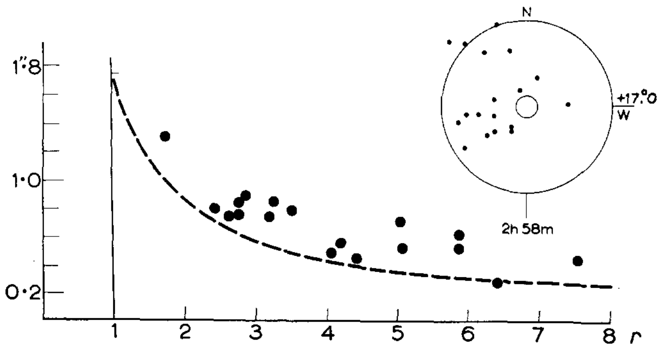
1922 Victoria (No. 4)



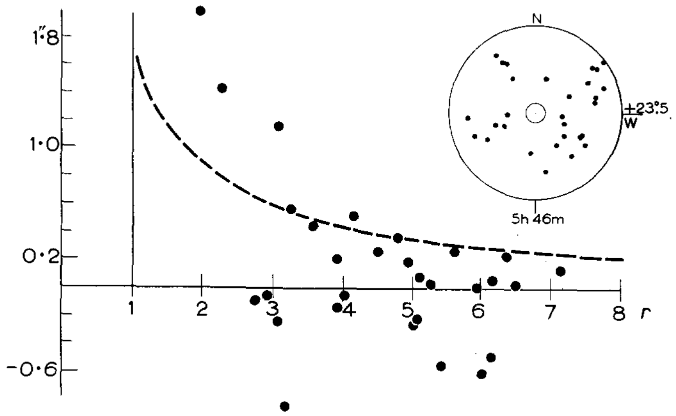
1922 Lick I (No. 5)



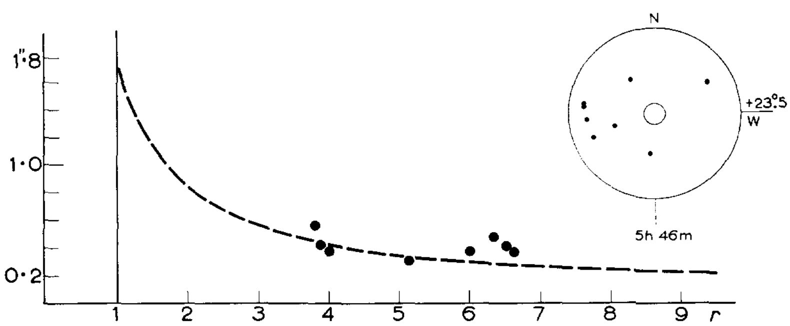
1922 Lick II (No. 6)



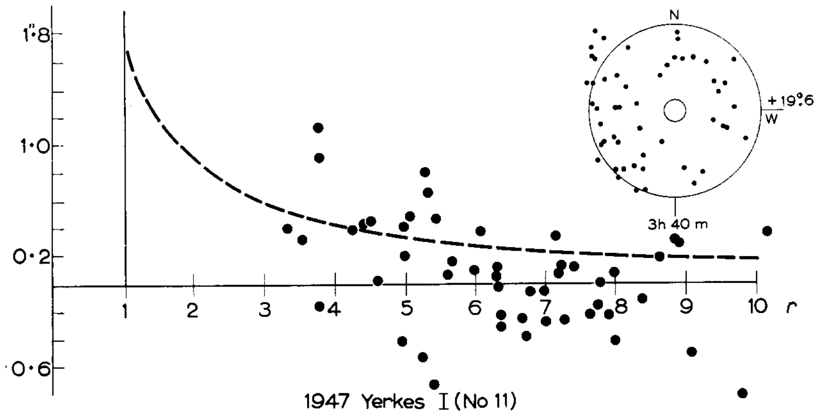
1929 Potsdam I.(No 7)



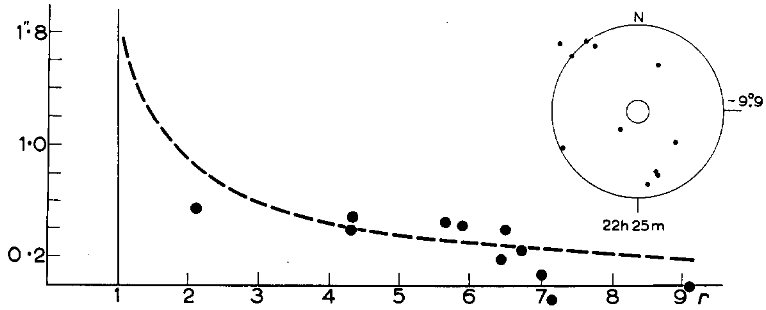
1936 Sternberg (No 9)



1936 Sendai (No 10)



1947 Yerkes I (No 11)



1952 Yerkes II (No. 12)

case by the dashed curves. These sketches emphasize the fact that the most important part of the hyperbola is scarcely covered by stars at all. In fact, most observations could be represented simply by straight lines (MIKHAILOV, 1956), and it is quite obvious that the actual deflection-law cannot be determined by the observations available at present. It is also necessary to keep in mind that in nearly every case represented in this figure, some small correction or other has been proposed by the authors or by various critics for reasons briefly mentioned above, mostly because of a correction in the scale-value, or because of different weighting or grouping of the stars or of the plates. In the following paragraphs the bold numbers are identical with those given in the first column of Table 1 and in the diagrams, (Fig. 6, etc.).

(**1**), (**2**) The first success in measuring the Einstein effect (EDDINGTON, 1919; Report joint Eclipse-Meeting, 1919; CROMMELIN, 1919a, 1919b; DAVIDSON and CROMMELIN, 1919; FREUNDLICH, 1920) was obtained by two British teams observing on 1919 May 29 from *Sobral* (Brazil) and from the Isle de *Principe* (Gulf of Guinea), respec-

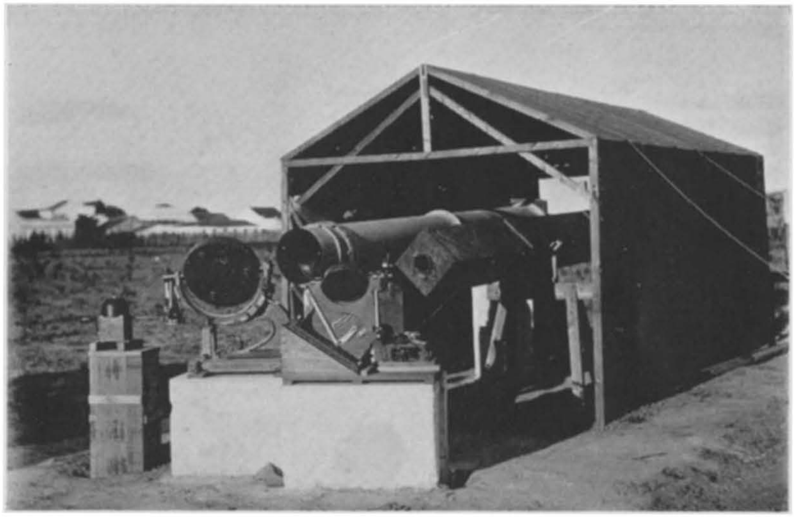


FIG. 7. The instrument used by the Greenwich expedition in 1919 at Sobral (Brazil). The two coelostats are feeding two horizontal telescopes: $f = 343$ cm and aperture 20 cm, on the left; $f = 570$ cm and 10 cm aperture, on the right; DYSON, EDDINGTON and DAVIDSON, 1920. (Photo. C. R. DAVIDSON.)

tively; (DYSON, EDDINGTON, DAVIDSON, 1920). A large number of plates were obtained with an astrograph and with horizontal telescopes fed by coelostats, but only a few of them were found suitable for reduction. No independent determination of the scale-value had been carried out, but for the observations on Principe photographs of a check-field were obtained. The result for the Sobral station, considered as the more accurate, is $L = 1''98 \pm 0''16$ m.e., and that for Principe is $L = 1''61 \pm 0''40$. The star-field for this eclipse is reproduced in Fig. 6, together with a graph giving the results for the individual stars. Fig. 7 shows the two horizontal telescopes used at Sobral ($f =$ 570 and 343 cm) with their coelostats. Some discussion arose later from the fact that corrections for aberration and refraction had been incorporated in the least squares solution in such a way as actually to lower the resulting weight, while all such corrections would better have been taken into account independently.

Objections were also raised because there were still some small tangential components left in the vectors for the radial shifts of the stars (RUSSELL, 1920), and because of suspected disturbances from refraction (BAUER, PETERS, 1920; ANDERSON, 1919–1924; DINES, RICHARDSON and ANDERSON, 1918; JEFFREYS, 1919; Royal Astronomical Society discussion, London, 1919a, 1919b; NEWALL, 1919; EDDINGTON and CROMMELIN, 1919), or because optical disturbances (STROHMEYER, 1921), or photographic ones (SLOCUM, 1921) were suspected. DYSON and WOOLLEY (1937) re-discussed these findings later, taking into account second-order terms in refraction and aberration, but no substantial alterations resulted. A re-discussion of the Sobral-values by HOPMANN (1923b), on the other hand, gave $L = 2''.16$.

Further re-discussions of the Sobral data by DANJON (1932) gave $L = 2''.06$, while MIKHAILOV (1956) obtained $L = 1''.95 \pm 0{\cdot}088$ (or, with a scale-correction, $L = 2''.07 \pm 0{\cdot}085$). DANJON's result is represented in Fig. 5.

(3) While on 1922 September 21 the weather was very unfavourable for expeditions working from Christmas Island (Indonesia), (FREUNDLICH, 1923a, HOPMANN, 1923a), a success was achieved at this eclipse by a combined team of Australian and British astronomers. The observations were made from *Condillo Downs, Australia* (DODWELL and DAVIDSON, 1924). This time a high-quality four-lens astrograph of the rather short focal length of 160 cm was used. A check-field was photographed during the eclipse and plate-parameters were taken from this check-field for the reduction of the star-field surrounding the Sun. The reduction of two plates gave $L = 2''.36$ and $L = 1''.18$, from which the observers deduced the mean value $L = 1''.77 \pm 0''.4$. The star field is very similar to that given in Fig. 6 for the Victoria team (No. 4), but the authors give no final values for the radial-shift of individual stars.

(4) At the same 1922 eclipse a Canadian team from the Dominion Astrophysical Observatory at Victoria also observed the light-deflection from *Wallal in Australia* (CHANT, 1923; YOUNG, 1923; CHANT and YOUNG, 1924). Two plates were obtained with a quadruple astrograph; reductions were made by using the linear expression of the equations only, and there was no independent determination of the plate parameters. The results were found to depend rather critically on the selection of stars used for the reduction. The authors gave the solution $L = 1''.75$ from all stars; but excluding various somewhat doubtful star-images, the resulting values for L became $1''.42$ or $2''.16$, with mean errors of about $\pm 0''.40$.

(5), (6) The same eclipse favoured by the fine weather at *Wallal, Australia*, as well as by a very good star-field, was also successfully observed by a Lick expedition at Wallal (CAMPBELL and TRUMPLER, 1923a, 1923b; CAMPBELL, 1923; FREUNDLICH, 1923b). These observations produced one of the best determinations of the light-deflection so far. Two instruments were used, a special double camera of $f = 450$ cm (Lick I), shown in Fig. 8; and a wide-angle quadruple-astrograph of $f = 150$ cm, (Lick II), (CAMPBELL and TRUMPLER, 1928). Each of the four plates of Lick I carried, superimposed on the eclipse-field, a check-field taken with the same instrument on the nights before and after the eclipse. Furthermore, other check-plates from the same star-field were secured some months later. In the reductions the second-order terms were taken from the check-field and then, as a first step, the linear plate-parameters were determined from a large number of stars at $r > 2°$ (without taking into account the Einstein effect which at such distances was considered negligible for this approximation). A last solution, and a small correction for the scale-value eventually gave as a mean for the four plates $L = 1''.72 + 0''.15$. Some small, remaining systematic deviations appeared indicated in the check-field; if a correction were applied for these, it would lead to small differences only, mainly for the stars more

FIG. 8. Double astrograph, parallactically mounted, (f = 450 cm, aperture 12 cm), as used by the Lick expedition (Lick I) in 1922 at Wallal, Australia, by CAMPBELL and TRUMPLER. (Photo: Lick Observatory.)

distant from the Sun; but the value for L at the Sun's limb would then go up to $L = 2''.05$. The results given by the authors were later disputed by the Potsdam observers (FREUNDLICH, v. KLÜBER and v. BRUNN, 1931a, 1931b, 1932b; TRUMPLER, 1932a, 1932b, 1932c), who, after some alterations in the method of reduction, derived from the same observations a value of $L = 2''.21$.

The application of DANJON's methods (1932c) to the observations of Lick I gave a value $L = 2''.00$ (or, with a small correction to the scale-value, $L = 2''.05$). MIKHAILOV's (1956) re-discussion gave $L = 1''.83 + 0''.20$, a figure very nearly identical with the one given by the authors themselves. Here again it is very significant that a straight line would also fit the observations quite well, demonstrating that even this comparatively good material, with a good star-field and the largest number of stars used so far, is not sufficient to decide purely empirically the form of the deflection-law.

A report on the results of Lick I deals also with possible trouble from the COURVOISIER effect (HOPMANN, 1923b; TRUMPLER, 1924), whilst criticisms were raised because of too large a scatter of the individual values for the different stars (ESCLANGON, 1924a, 1924b, 1924c), or because there were still small tangential vectors left in the resulting radial shifts (PORTER, 1929; POOR, 1930; COMAS SOLÁ, 1928; TRÜMPLER, 1929c.) Even a quite different explanation of the deflection was proposed (GLEICH, 1931).

The short-focus instrument of Lick II (CAMPBELL and TRÜMPLER, 1928; TRÜMPLER, 1928), covering a field of $15° \times 15°$, gave plates which showed up to more than 500 stars with a limiting magnitude of about $10^{m}5$; check-fields were taken in a similar manner as for Lick I. In the fairly elaborate reduction all third-order terms were included, and the scale values were first determined from the outer stars of the field. A second solution eventually gave a final and small scale-correction, and the Einstein effect became $L = 1''82 \pm 0''20$. A later reduction by the Potsdam observers deduced a larger value, $2''07$, from the same material.

The Lick II observations, too, have also been re-discussed; DANJON (1932c) used the more important stars only and was able to show how the presence and the weight given to a single star can influence the whole result. His value is $L = 2''07$ for the Lick II observations.

The radial shifts from the 15 best stars from both the Lick I and Lick II observations, are shown in Fig. 9 which gives a good idea as to how well on the whole the measured shifts agree with the expected light-deflection.

(7), (8) A further successful determination of the Einstein effect was made at the eclipse of 1929 May 9 in *Takengon* (Sumatra), after most careful preparation with elaborate instrumental equipment, by an expedition of the Astrophysical Observatory of Potsdam (FREUNDLICH, v. KLÜBER and v. BRUNN, 1931a, 1931b; FREUNDLICH, 1930; v. KLÜBER, 1929a, 1929b, 1931; WATTENBERG, 1932). The large instrument of Potsdam I was a double horizontal camera of $f = 850$ cm, so far the longest focal length used for this problem (Fig. 10). It was fed by a coelostat with a special drive of high precision, constructed by ZEISS in Jena (v. KLÜBER, 1932a), giving first-class automatic guiding, electromagnetically controlled by a chronometer. During the eclipse one of the cameras took exposures of the eclipse field, whilst the other one simultaneously photographed a suitable check-field, both using the same coelostat. Immediately before and again after the eclipse a scale reseau from a large thermally insulated collimator was photographed on all plates, using the coelostat again at the same reflection angles as during the eclipse. The same procedure was repeated about six months later with the same star field, then in the night sky. The reduction procedure demanded a very large amount of measurement and calculation, and was designed to give an independent absolute determination of the scale-value. For the collimator and the reseau the latter could be determined from the check-field and from this the scale-value of the eclipse-field could be found independently. That this whole cycle of observations could work very satisfactorily

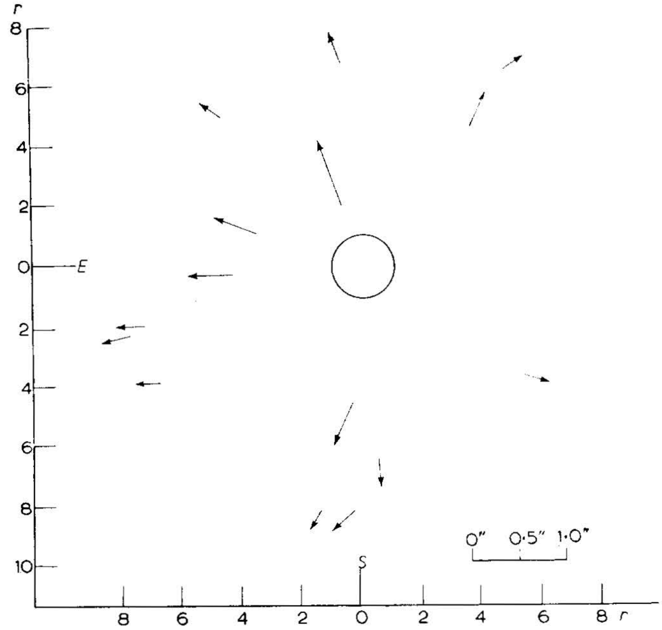


FIG. 9. Vector diagram of the radial shifts, derived from means of the 15 best stars by the Lick I and Lick II observations in 1922. It presents a very good indication of the existence of the light-deflection (CAMPBELL and TRUMPLER, 1928)

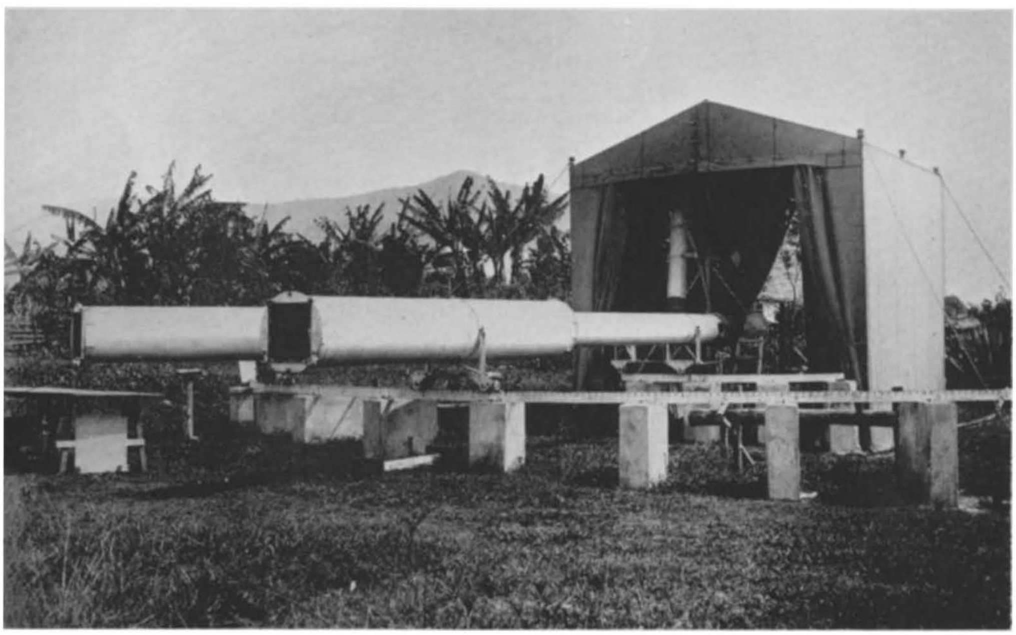


FIG. 10. Large double horizonta camera (f = 850 cm, aperture 20 cm) as used by the Potsdam expedition in 1929 to Takengon, Sumatra (Potsdam I). The coelostat and the collimator for printing the reseau are visible under the tent (FREUNDLICH, v. KLÜBER, v. BRUNN, 1931a). (Photo: v. KLÜBER.)

was demonstrated by a separate investigation, in which a star-field at night was treated by the same method as that employed at the eclipse itself (v. BRUNN and v. KLÜBER, 1937).

Favoured by very good weather, this instrument yielded four good plates; the outcome of the whole elaborate reduction was $L = 2''.24 \pm 0''.10$, showing a remarkably low m.e. The mean error of one plate was found to be $\pm 0''.30$. The star field at this eclipse and the radial shifts given by the different stars are shown in Fig. 6, while Fig. 11 represents the radial shift of all measured stars from the mean of all four plates.

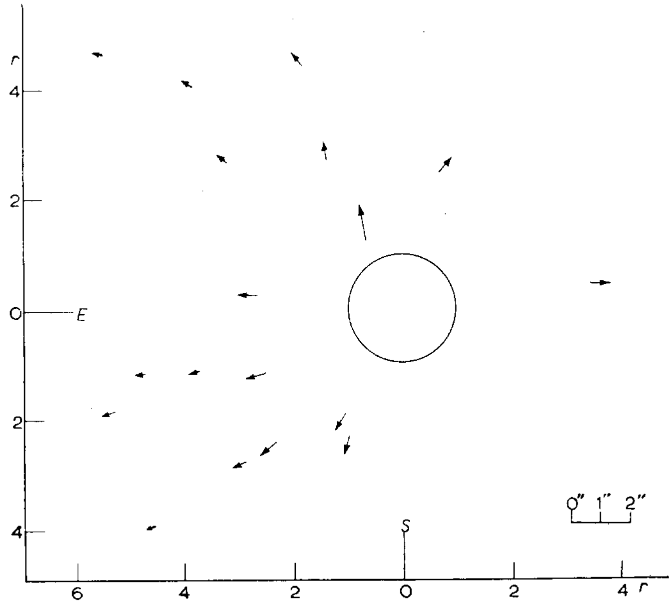


FIG. 11. Vector diagram indicating the observed light deflection for each star of the Potsdam I observations; mean of all four plates. The radial shift as expected by Einstein's theory is very clearly indicated (FREUNDLICH, V. KLÜBER, V. BRUNN, 1931a).

This paper again caused some controversy, partly because of the unfortunate asymmetry of the star field (LUDENDORFF, 1932; FREUNDLICH, V. KLÜBER and V. BRUNN, 1932b), and partly because small systematic residual errors were suspected for various reasons (TRUMPLER, 1932a, 1932b; FREUNDLICH, V. KLÜBER and V. BRUNN, 1932a, 1932b). LUDENDORFF deduced from the same material a value of $L = 1''.90 \pm 0''.15$, while TRUMPLER found that $L = 1''.75 \pm 0''.19$ satisfied the observations with good internal consistency. Another discussion by JACKSON (1931) produced the value $L = 1''.98 \pm 0''.20$. DANJON (1932c) found from his graphical method that he could represent the measures very well with $L = 2''.06$, while MIKHAILOV (1956), assuming a slight correction of the scale-value, gave $L = 1''.96 \pm 0''.11$.

Two further attempts, with the same instrument, at the eclipses of 1954 and 1955 failed because of bad weather (v. KLÜBER, 1955, 1956; MATTIG, 1956; NICHOLSON, 1956).

At the above mentioned eclipse of 1929 the Potsdam team used, in addition, a specially constructed strong astrograph, parallactically mounted, with a Zeiss triplet lens ($f = 343$ cm, $\phi = 20$ cm) of high quality (Potsdam II); see Fig. 12

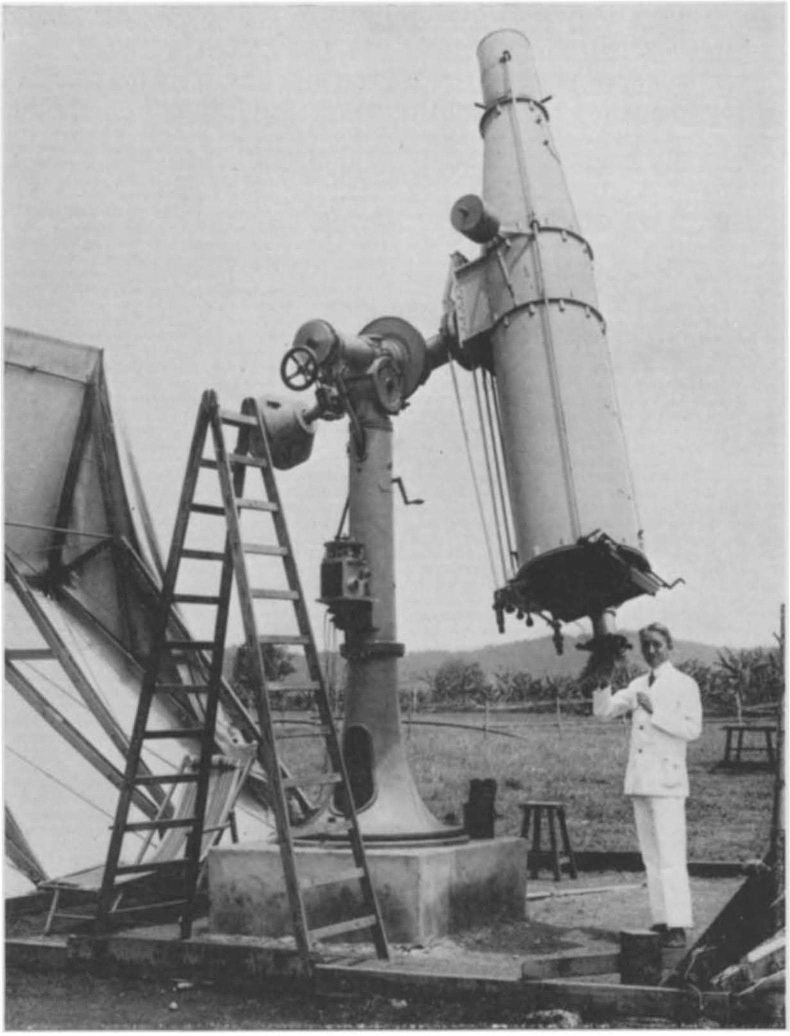
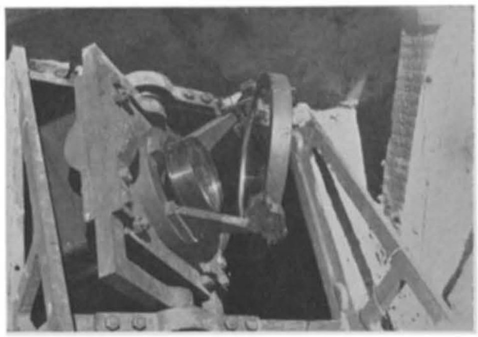


FIG. 12. Large parallactically mounted Zeiss astrograph ($f = 343$ cm, 20 cm aperture) with electrically controlled automatic drive, covering the large field of $7^\circ\!.5 \times 7^\circ\!.5$, as used in 1929 by the Potsdam observers (Potsdam II). During the eclipse itself a check star-field was photographed on each of the three plates taken of the Sun's surrounding, by pointing the astrograph alternately at a star-field distant from the Sun (FREUNDLICH, v. KLÜBER, v. BRUNN, 1933). (Photo: v. KLÜBER.)

(FREUNDLICH, v. KLÜBER and v. BRUNN, 1933). After each exposure of the star-field surrounding the Sun, the instrument was pointed during the eclipse itself onto a field at some distance from the Sun, so that each of the three plates obtained carried two star-fields superimposed on each other. Each of the plates, covering a field of $7^\circ\!.5 \times 7^\circ\!.5$, contained about 100 stars surrounding the Sun. This method seemed ideal for the determination of the plate parameters, and, especially, of the scale-value. But even when all third-order terms were included in the reduction, and in spite of the fact that the star images were excellent, the resulting vectors for the different stars scattered widely. This unsatisfactory result was probably due to a very small relative mechanical deformation of the instrument, caused by moving the whole telescope during the eclipse from the one star-field to the other. Recalling what has been said above, namely, that the effective focal setting must be kept constant for both exposures to within a few hundredths of a millimeter, this failure is perhaps not surprising. The experiment with this instrument certainly demonstrates that this apparently promising method is in practice of rather doubtful value.

The extensive discussion of the Potsdam investigations by the observers themselves and by other authors has brought out very clearly, in an actual practical case, how critically the results depend upon the scale-value, how very important it is to obtain in some way or other an independent value for the scale-correction, and also how small the mechanical and optical tolerances of the instrument must be in order to secure good results (FREUNDLICH and v. BRUNN, 1933; KÖNIG, 1957).

(a)



(b)

FIG. 13

(a) A. A. MIKHAILOV's instrument, showing the specially designed plate-carrier and its drive, as used in 1936 and on later expeditions.

(b) MIKHAILOV's plane-parallel plate in front of the objective, for photographing simultaneously on the same plate the eclipse-field and a check-field.

(Photos: A. A. MIKHAILOV, Pulkovo Observatory.)

(9) At the eclipse of 1936 June 19, a Russian expedition from the Sternberg Observatory (MIKHAILOV, 1940a, 1940b, 1942, 1949) to *Kubishev* (Siberia), succeeded in obtaining further observations of the Einstein effect; (Fig. 13). This work included the first attempt to photograph the check-field and the eclipse-field simultaneously, by using a plane parallel plate in front of the objective of the telescope, as outlined above. Unfortunately, in taking the night photographs the check-field was missed, so that the reductions had to be carried out in the usual straightforward manner by determining all the plate parameters from the eclipse field itself, using all second-order terms. Four different reductions of the two best plates gave $L = 2''.73 \pm 0''.31$, which is quite appreciably more than to be expected from EINSTEIN's theory. A later reduction (MIKHAILOV, 1956) gave $L = 2''.70 \pm 0''.40$. Another similar attempt by the Poltawa Observatory, in 1954, using a quartz mirror instead of the plane-parallel plate, failed because of bad weather (POPOV and FJODOROV, 1954).

(**10**) At the same eclipse of 1936 the Japanese Imperial University at Sendai made observations (SOTOME and HOSHIDA, 1936; MATUKUMA, 1940a, 1940b) at *Kosimizu* (Japan). A double-camera of $f = 500$ cm combined with a coelostat was used. Only one plate was obtained, and this showed not more than eight measurable stars. In the process of reduction this plate was combined with two different comparison night-plates; no special attempt was made to determine the scale-value separately; the measurements were reduced in the usual manner, including second-order terms, as well as a term for the scale-value, and another one for the light-deflection. The two plate-combinations gave $L = 2''.13 \pm 1''.15$ and $L = 1''.28 \pm 2''.67$, respectively, indicating excessively large mean errors. The diagram in Fig. 6 represents the mean of the two plates.

(**11**) Another attempt to use a plane-parallel plate in front of the objective of a long-focus astrograph ($f = 609$ cm), Fig. 14, was made by VAN BIESBROECK at the eclipse of 1947 May 20 from a station near *Bocajuva* (Brazil) (VAN BIESBROECK, 1949). A half-silvered plate was used, giving an equal loss of intensity in both the eclipse- and the check-field of about one magnitude. Only one photograph was obtained, on which the star images of the check-field appeared far worse than the images of the simultaneously exposed eclipse-field. In the opinion of the observer, this was probably caused by temperature distortion of the plane-parallel plate, and if so, this would be an interesting illustration of the objection mentioned above that the optical paths from the eclipse-field and the check-field are different. This leads to some doubt as to whether a scale-value determined from the check-field can safely be used for the reduction of the eclipse-field (FREUNDLICH, 1950; VAN BIESBROECK, 1950). In this case the check-field was not used. A simple graphical reduction like the one suggested by DANJON (1932c) was used by the author and gave $L = 2''.01 \pm 0''.27$. MIKHAILOV (1956) raised some objections to this simplified reduction, and in a re-reduction he found $L = 2''.20 \pm 0''.35$.

(**12**) At the eclipse of 1952 February 25, observing at *Khartoum* (Sudan), VAN BIESBROECK successfully obtained two more plates employing the same method as 1947, with special precautions to protect the plane-parallel plate from any kind of optical distortion. In a short paper (VAN BIESBROECK, 1953) the author derives from these two plates the value $L = 1''.70 \pm 0''.10$; the mean error is remarkably low. Unfortunately all stars, except one at $r = 2{\cdot}1$, are at a distance from the Sun $> 4{\cdot}3$, so that the most vital part of the expected hyperbola is not covered by stars at all. The figures given in this publication have also been re-reduced by MIKHAILOV (1956), who then obtained $L = 1''.43 \pm 0''.16$.

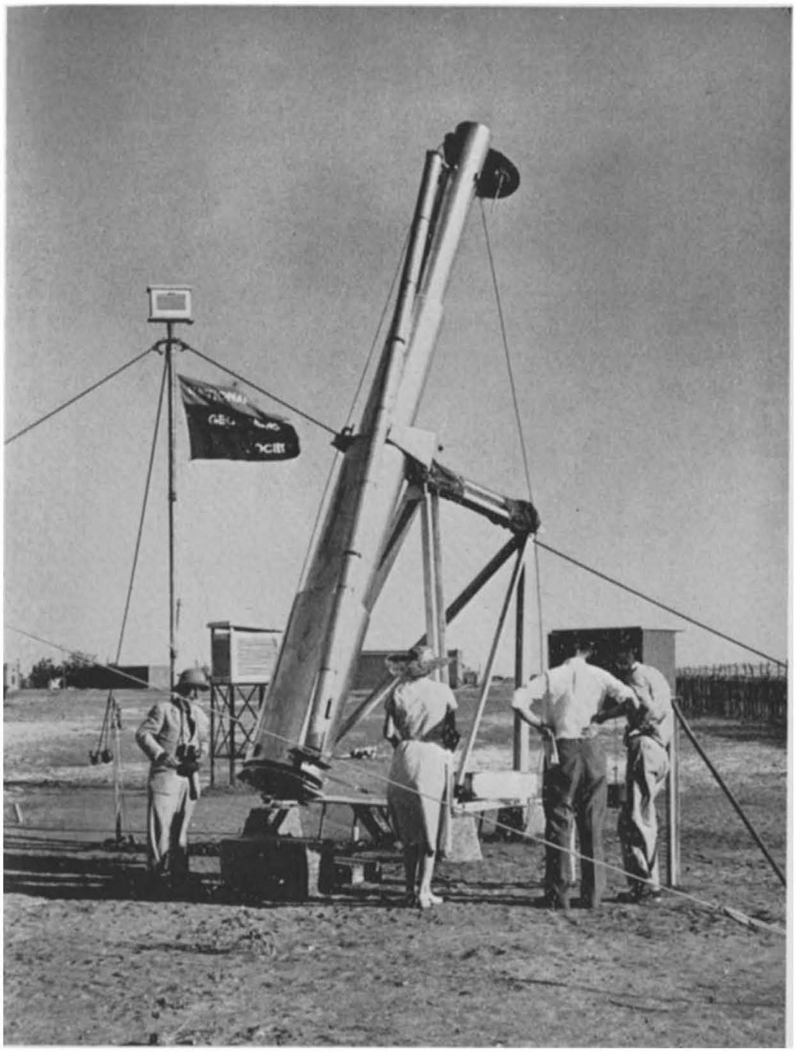


FIG. 14. Telescope (f = 609 cm, aperture 15 cm) with special mounting and drive, as used for the Yerkes II observations in 1952. A check-field was superimposed on the eclipse field during the eclipse itself, using a semi-transparent mirror in front of the objective; (VAN BIESBROECK, 1953). (Photo: v. KLÜBER.)

4. CONCLUSIONS

Summing up the results of the foregoing investigations, which comprise all observations available so far (1959), the following points can be made:

(a) Without exception, all observations indicate clearly that a light-deflection effect of the kind expected quite obviously exists in the neighbourhood of the Sun. But the observations are not sufficient to show decisively whether the deflection really follows the hyperbolic law predicted by the General Theory of Relativity, mainly because so far it has not been possible to obtain a satisfactory number of star-images sufficiently near to the Sun. As things are at present, most observations could be represented quite well even by straight lines (MIKHAILOV, 1956).

(b) If the existing observations are extrapolated to the limb of the Sun, according to the supposed hyperbolic law, then there is an indication that the resulting constant L appears to be somewhat larger than the value expected by the General Theory of Relativity. Whether this behaviour is real, or only introduced by some systematic observational error, cannot at present be decided with certainty; (PAPAPETROU, 1953). WHEELON (1952) recently pointed out that if one were to assume photons with a non-vanishing rest-mass, then the effect of the non-zero mass would increase the light-deflection in accordance with the values for the rest-mass and the light-frequency. But if larger effects were produced in this way, there should also exist certain laboratory phenomena which are not known to occur.

(c) The fairly large discrepancies between the results of various authors, even when based on the same observational material, is not so surprising if one recalls that the value for L is in fact an extrapolation which is very sensitive to small alterations in the position of the actually observed points, if these lie rather far out on the supposed hyperbola. Furthermore, the quantity sought is unfortunately strongly coupled to the scale-value itself, which is very difficult to determine. A good impression of all available results may be obtained from the set of graphs in Fig. 6 and from Table 1.

To determine the relativistic light-deflection as accurately as possible is without any doubt a most important experiment because of its relation to fundamental consequences of the Theory of Relativity. As observations of this kind are so difficult and can be carried out only at total solar eclipses, progress in future will probably be rather slow. From the experience gained so far, the following points seem to be important for further investigations:

(I) First of all, every possible effort should be made to obtain somehow or other an independent determination of the scale-value. This is probably an absolutely essential condition for achieving any real progress. If any other plate-parameter could be determined independently as well, it would certainly increase the weight of the result considerably.

(II) The usual minimum number of stars needed in the eclipse field is six, but actually as many stars as possible are required; and if no stars are available for $r < 2$, the actual law of deflection (supposed to be hyperbolic) can probably not be determined by the observations alone. Furthermore, a symmetrical distribution of the stars around the Sun is most desirable. As the limit for measurable star images will be probably round about the tenth photographic magnitude, all these conditions place a rather severe restriction on the star fields suitable for further observations.

Certainly not every future total eclipse will fulfil these conditions. Possibly more star images could be obtained through the corona, by working in the red part of the spectrum only and/or by making use of the radial polarization of the corona light.

(III) First-class guiding is another essential condition, and we must emphasize most strongly how difficult this is to obtain at the temporary field-station of an eclipse expedition. Many observers in the past have underestimated this difficulty. The focal length of the instrument should be larger than 300 cm, if possible at least 600 cm; a suitable aperture ratio would be about 1 : 40. A larger value may cause some loss of star images because of the larger apparent surface brightness of the corona. The whole instrument must be constructed so rigidly that under working conditions the scale-value can be maintained to something of the order of 10^{-6}. This in itself is a very severe condition.

(IV) High-quality optics are desirable, but it must be kept in mind that optics which are too complicated may, under field conditions, possibly not fulfil some of the requirements just mentioned. Mirrors, of course, should be made of fused quartz.

(V) It is in principle desirable to obtain several plates at an eclipse, since this increases the weight of the results appreciably.

Finally, we should like to state quite clearly that further observations of the light-deflection are only justified, if real progress is to be expected as a result of fulfilling as nearly as possible the stringent conditions summarized above. Even so, such observations remain among the most difficult of all those which can be attempted at a total solar eclipse.

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