Light Signals on Moving Bodies as Measured by Transported Rods and Clocks

Herbert E. Ives
Bell Telephone Laboratories, New York, N. Y.
(Received April 30, 1937)

1. Introduction

In an earlier paper[1] I have pointed out that the null result of the Michelson-Morley and Kennedy-Thorndyke experiments may be explained by the assumptions:

(1) Moving measuring rods are altered in length when in uniform motion in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n+1} : 1$$

in the direction of motion;

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n} : 1$$

at right angles to the direction of motion, and (2) Clocks are altered in frequency when in uniform motion in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{1-n} : 1,$$

where v is the velocity of the rods or clocks through the luminiferous ether, and c is the velocity of light.

No experiments, such as the Michelson-Morley experiment, using to and fro transits of light signals, are competent to establish the value of n in the above expression.[2] In the present paper I discuss the problem of light signals on moving bodies, taking these alterations of length and frequency as the basic physical phenomena, and assuming the value $n=0$ in the above expressions. This corresponds to a contraction of length in the direction of motion, of value $(1-v^2/c^2)^{\frac{1}{2}} : 1$; no alteration in length at right angles to the direction of motion; and a reduction of clock frequency in the ratio $(1-v^2/c^2)^{\frac{1}{2}} : 1$. This choice of value of n is that made by Fitzgerald, Lorentz, and Larmor, based on speculations and experiments apart from the Michelson-Morley experiment.[3] Neither the contraction of dimensions, nor the alteration of clock rate, has been subjected to direct experimental test.

2. The Frame of Reference and Tools of Measurement

In this discussion we shall assume light signals to be moving in a fixed ether, which we identify as the seat of the pattern of radiant energy received from the fixed stars. This pattern is substantially constant in character, suffering as a whole periodic variations with respect to the earth, shown by the phenomenon of stellar aberration. The argument for the necessity and reality of this frame of reference will be amplified at the close of the paper.

Since we are to consider the effects of variations in the properties of rods and clocks due to their motion through the ether, we shall compare these with rods and clocks which we shall postulate as *unaffected by transport*. Whether such rods and clocks can be realized in practice is broadly not important for carrying out a general argument. Using such rods and clocks, we define the *velocity of light* as the ratio of the distance between two points fixed in the ether, between which a light signal travels, to the time taken by transit. The velocity of light so defined we assume to be a constant, which we designate by c.

We note at once that the velocity of light with reference to bodies not fixed in the ether will differ from c, and that the value as measured on moving bodies with rods and clocks affected by motion may in general be expected also to be different from c. It is our problem to determine how different, in the case of the specific behavior of rods and clocks we have assumed.

[1] "Graphical Exposition of the Michelson-Morley Experiment," J. Op. Soc. Am. **27**, 177 (1937).

[2] This was recognized by Lorentz, who describes the corresponding factor in his expressions as "the origin of all our difficulties" (*Theory of Electrons*, p. 219), and chooses the value corresponding to our $n=0$ by consideration of the Kaufmann and Bucherer experiments. Birkhoff (*Relativity and Modern Physics*, p. 34) concludes that "the decision . . . must be made solely on the basis of facts established by means of physical experiments" and (p. 55) "It appears that no experimental data are available." He then discusses an experiment (the "transverse Doppler effect") which would be decisive.

[3] Larmor, *Aether and Matter*, 1900. The characterization of the frequency change as a prediction of the theory of relativity (1905) is chronologically unjustifiable.

3. The Measured Distances and Differences of Clock Readings Between Two Points on a Uniformly Moving Body

The most general method of measuring a length, e.g., the distance between points a and b on a body, is to place a "standard" measuring rod alongside the body, and read the positions of the two points a and b on divisions marked on this rod. In carrying out this comparison we ordinarily make the two readings at two different times, which would be without objection if the system were stationary. If the measuring rod is moving past the body, which is the general case, our observations must be made *simultaneously*. If our clocks at the two points of observation were unaffected by their separation this would present no complications. In our present problem, however, two clocks set alike at a differ when moved apart, and it becomes at once necessary to find what is the absolute time interval between measurements made "simultaneously" as indicated by our clocks.

Let the observing platform be moving with the velocity v. The velocity of light c, we shall assume to be obtainable to any desired degree of approximation, by a procedure which will be made clear in the sequel. Now assume our clock moved from a to b with the absolute velocity $v+W$. The true time [time in our framework with our unvarying clocks] T, which it takes to move between a and b is given by the relation

$$(v+W)T=L_s(1-v^2/c^2)^{\frac{1}{2}}+vT,$$

from which

$$T=L_s(1-v^2/c^2)^{\frac{1}{2}}/W \quad (1)$$

where L_s is the distance ab when the body is stationary, as measured by an unvarying measuring rod. Now the time as indicated by the clock on reaching b will be:

$$T_b=T\left[1-\frac{(v+W)^2}{c^2}\right]^{\frac{1}{2}} =\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W}\left[1-\frac{(v+W)^2}{c^2}\right]^{\frac{1}{2}}. \quad (2)$$

Meanwhile the clock at a will indicate, at the same instant,

$$T_a=T\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} \quad (3)$$

and there will be, after the clock, moved to b, has come to rest and resumed its rate of $\nu_0(1-v^2/c^2)^{\frac{1}{2}}$ (or a clock at b has been set at the indication of the moved clock as it passes) *a difference of setting* of the clock at b with respect to that at a

$$\Delta S=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{(v+W)^2}{c^2}\right)^{\frac{1}{2}}\right], \quad (4)$$

which is the interval between apparently simultaneous observations.

An observer at a, making an observation at a predetermined indicated time actually makes the observation earlier on his clock by this difference of settings, and this amount earlier, to be reduced to true time, is to be increased in the ratio $1/(1-v^2/c^2)^{\frac{1}{2}}$, since his clock runs slow in the ratio $(1-v^2/c^2)^{\frac{1}{2}} : 1$. The true time elapsed between the apparently simultaneous observations at a and b is therefore

$$\Delta T=L_s/W[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v+W)^2/c^2)^{\frac{1}{2}}]. \quad (5)$$

Let us now take a standard measuring rod, longer than the distance ab, which has been marked off in equal divisions, while stationary. Let the rod be moved past ab at an absolute velocity $v+Y$. The length of rod intercepted by ab is then

$$L_i=L_s(1-v^2/c^2)^{\frac{1}{2}}-Y\Delta T$$

or

$$L_i=L_s[(1-v^2/c^2)^{\frac{1}{2}}-(Y/W)\{(1-v^2/c^2)^{\frac{1}{2}} -(1-(v+W)^2/c^2)^{\frac{1}{2}}\}], \quad (6)$$

which since the moving rod is contracted in the ratio $(1-(v+Y)^2/c^2)^{\frac{1}{2}} : 1$, will be read as

$$L_i'=\frac{L_s\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\frac{Y}{W}\left\{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{(v+W)^2}{c^2}\right)^{\frac{1}{2}}\right\}\right]}{\left(1-\frac{(v+Y)^2}{c^2}\right)^{\frac{1}{2}}}. \quad (7)$$

We next express the absolute velocities Y and W of measuring rod and clock in terms of *observable* *quantities*, namely, their velocities as evaluated by divisions on the rod passing an observer on a clock per unit of time by the clock. In the case of the rod we imagine an observer at a, counting divisions against the "ticks" of his clock, whose rate is slow on the proportion $(1-v^2/c^2)^{\frac{1}{2}}:1$. The rod being contracted in the ratio $(1-(v+Y)^2/c^2)^{\frac{1}{2}}:1$, its *observed* velocity Y_0 is

$$Y_0=\frac{Y}{(1-v^2/c^2)^{\frac{1}{2}}(1-(v+Y)^2/c^2)^{\frac{1}{2}}}, \quad (8)$$

from which $$Y=\frac{Y_0(1-v^2/c^2)}{Y_0v/c^2+(1+Y_0^2/c^2)^{\frac{1}{2}}}. \quad (9)$$

For the moving clock, whose velocity relative to the body is W, we imagine an observer, on the clock, noting the time necessary for the light path ab, which he takes as his unit of length, to be traversed.[4] He thus observes his velocity increased over his true velocity, inversely as the length assumed by ab in motion, and inversely as his own clock rate, or

$$W_0=\frac{W}{(1-v^2/c^2)^{\frac{1}{2}}(1-(v+W)^2/c^2)^{\frac{1}{2}}}, \quad (10)$$

from which $$W=\frac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}. \quad (11)$$

We now have all the factors necessary to find the *observed* traversed distances and times of transit of light signals passing between the points a and b on a body moving with absolute velocity v.

4. Light Signals on a Single Moving Body

Consider first the apparent distance traversed by a light signal going from a to b, on the body moving with the uniform velocity v. This is the distance L_i' above. Substituting in it our values for W and Y in terms of the velocities as observed, W_0 and Y_0, we get

$$L_i'=L_s\frac{(1-v^2/c^2)^{\frac{1}{2}}-\dfrac{\dfrac{Y_0(1-v^2/c^2)}{Y_0v/c^2+(1+Y_0^2/c^2)^{\frac{1}{2}}}}{\dfrac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}}\left[(1-v^2/c^2)^{\frac{1}{2}}-\left\{1-\dfrac{\left(v+\dfrac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}\right)^2}{c^2}\right\}^{\frac{1}{2}}\right]}{\left[1+\left(\dfrac{v+\dfrac{Y_0(1-v^2/c^2)}{Y_0v/c^2+(1+Y_0^2/c^2)^{\frac{1}{2}}}}{c^2}\right)^2\right]^{\frac{1}{2}}}. \quad (12)$$

The solution of this equation[4] is

$$L_i'=L_s[(1+Y_0^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)(1-(1+W_0^2/c^2)^{\frac{1}{2}})]. \quad (13)$$

On inspecting this result we note at once the significant fact that it is *independent of* v; the read value of the distance traversed by the light signal is a function of the stationary value L_s, of the distance ab, of the velocity of light c, and the measured velocities of rods and clocks, obtained in the manner described. Further, on expanding $(1+W_0^2/c^2)^{\frac{1}{2}}$ we get

$$L_i'=L_s[(1+Y_0^2/c^2)^{\frac{1}{2}}+Y_0W_0/2c^2], \quad (14)$$

which for $Y_0=0$ gives

$$L_i'=L_s. \quad (15)$$

Consider next the observed time of transit of the light signal. The true time is

$$T_t=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{c-v}, \quad (16)$$

which will be read by any stationary clock on the

[4] If the observer is called upon to insure that the clock is moving with uniform velocity with respect to the body (which is moving uniformly by assumption) he must watch the passage of uniformly spaced divisions past the clock. These divisions can be those on our standard measuring rod whose uniform velocity with respect to the body (which might be zero) can be assured by independent observation, with the clock held stationary on the body. This procedure avoids any commitment as to the methods of length measurement on the moving body.

body as

$$T_t=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}(1-v^2/c^2)^{\frac{1}{2}}}{c-v}=(L_s/c)(1+v/c). \tag{17}$$

Now the clock at b is set back, due to its journey from a by the quantity given above (4). Substituting in this W_0, the observed value of W, we get

$$T_t'=\frac{L_s}{c}\left(1+\frac{v}{c}\right)-\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{\dfrac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{\left(v+\dfrac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}\right)^2}{c^2}\right)^{\frac{1}{2}}\right]. \tag{18}$$

The solution of this[5] is

$$T_t'=\frac{L_s}{c}\left[\frac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \tag{19}$$

Inspecting this we note once more that our value is *independent of* v as long as we keep the observed velocities of rods and clocks constant. (The absolute values to meet these conditions of course vary with every value of v.) Further, on expanding $(1+W_0^2/c^2)^{\frac{1}{2}}$, we find, if W_0 is small

$$T_t'=(L_s/c)(1-W_0/2c), \tag{20}$$

so that for infinitesimal velocities of the clock, the read time approaches L_s/c.

Next consider the *velocity of light* as measured by sending a light signal between two points on a moving body such as the earth. The observed or measured value will be

$$\frac{L_i'}{T_t'}=\frac{L_s}{L_s/c}\,\frac{\left[(1+Y_0^2/c^2)^{\frac{1}{2}}+Y_0/W_0(1-(1+W_0^2/c^2)^{\frac{1}{2}})\right]}{\left[\dfrac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]} \tag{21}$$

$$=c\mathfrak{L}/\mathcal{T}=c_m, \tag{22}$$

using $\mathfrak{L}$ and $\mathcal{T}$ for the measured quantities in brackets and c_m for the velocity of light measured with transported rods and clocks, *which is different from* c, our fundamental constant. Since $\mathfrak{L}$ and $\mathcal{T}$ are functions of c, it will be possible to express c entirely in time of measured velocities of clocks, rods and light. From the preceding work it follows that for $Y_0=0$, *the value* c *is approximated as the velocity of the rods and clocks is indefinitely reduced.*

A further observation on the absolute velocity of light, which we promised above would be shown to be a determinable quantity. From the foregoing discussion it follows that the value c can be obtained to any desired degree of approximation, by sending a light signal to a point on a moving body, whose distance is obtained by the use of measuring rods, and determining its time of arrival by a clock carried to that point, provided the rods and clocks used are moved at negligible velocities with respect to the moving body. The method used in all practical determinations of the velocity of light meets these requirements. The distance is measured by laying rods end to end, each rod being stationary on the body when the measurement is recorded; the time is measured by a clock stationary on the body, by virtue of the invariable procedure of sending the light signal to a mirror and back to the origin.

5. Light Signals on Several Moving Bodies

We now investigate the measured values of time and distance for a light signal traversing several moving bodies simultaneously. In Fig. 1, let B_0, B_1, B_2, etc. represent the several bodies in motion, B_0 being the luminiferous ether, B_1, B_2,

[5] See Appendix. I am indebted to Mr. T. C. Fry for the formal solutions.

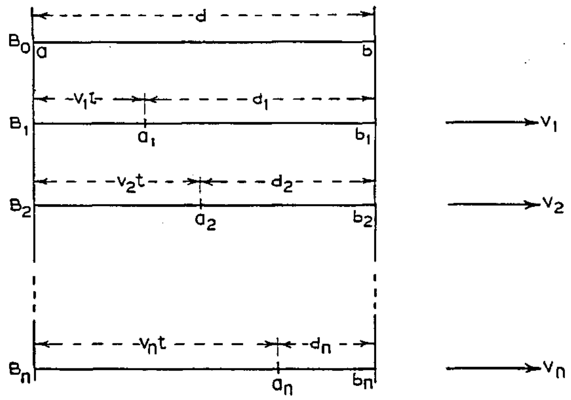

FIG. 1.

etc., being considered for purposes of visualization as moving bands all in immediate contact with each other, and with the light source positioned at a. Under these conditions, the light signal when emitted can be instanteously and simultaneously impressed, say upon a photosensitive surface, upon each moving band; and when received, as by a photoelectric cell at b, its reception may be instantaneously and simultaneously recorded on each moving band.

Let the distance traversed by the light signal in the ether (B_0) be designated as d, and let d_1, d_2, etc., be the distances between the initial and final traces of the light signals (b_1-a_1, b_2-a_2) on the several moving bands. Let the time of transit of the light signal from a to b be

$$t=d/c. \tag{23}$$

Then the exact physical statement of the event on each band is

$$\begin{aligned} d_1&=d(1-v_1/c),\\ d_2&=d(1-v_2/c),\\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ d_n&=d(1-v_n/c). \end{aligned} \tag{24}$$

Now the trace $d(1-v_1/c)$ on the band moving with the velocity v_1 would, if the band were stopped, taking the value of the contraction as $(1-v_1^2/c^2)^{\frac{1}{2}}$ in the direction of motion, assume the value

$$\frac{d(1-v_1/d)}{(1-v_1^2/c^2)^{\frac{1}{2}}}. \tag{25}$$

Now this is merely the L_s of our previous section. Therefore, to get the measured values, using our moved rods and clocks we have

$$\begin{aligned} d_1'&=\frac{d(1-v_1/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}[(1+Y_0^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)\{1-(1+W_0^2/c^2)^{\frac{1}{2}}\}],\\ d_2'&=\frac{d(1-v_2/c)}{(1-v_2^2/c^2)^{\frac{1}{2}}}[(1+Y_0^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)\{1-(1+W_0^2/c^2)^{\frac{1}{2}}\}],\\ d_n'&=\frac{d(1-v_n/c)}{(1-v_n^2/c^2)^{\frac{1}{2}}}[(1+Y_0^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)\{1-(1+W_0^2/c^2)^{\frac{1}{2}}\}]. \end{aligned} \tag{26}$$

These are different for each moving band.

Also from the previous section we have that the observed time of transit is $L_s/c\cdot\mathcal{T}$, so that we have

$$\begin{aligned} t_1'&=\frac{d}{c}\,\frac{(1-v_1/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}\left[\frac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]\\ &=\frac{t(1-v_1/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}\left[\frac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right],\\ t_2'&=\frac{t(1-v_2/c)}{(1-v_2^2/c^2)^{\frac{1}{2}}}\left[\frac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right],\\ t_n'&=\frac{t(1-v_n/c)}{(1-v_n^2/c^2)^{\frac{1}{2}}}\left[\frac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \end{aligned} \tag{27}$$

These values are different for each band.

The *observed velocity of light* on each band, provided we have adhered to the same observed manipulations of rods and clocks is

$$\frac{d_1'}{t_1'}=\frac{d_2'}{t_2'}=\frac{d_n'}{t_n'}=c\frac{[(1+Y_0^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)\{1-(1+W_0^2/c^2)^{\frac{1}{2}}\}]}{\left[\dfrac{1+W_0/c-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]}, \tag{28}$$

which for zero rod velocity and small clock velocity, approximates c.

An observation which is obvious is that our Eqs. (26), (27) for the case of infinitesimal velocities of rods and clocks are (for the direction of motion, which has alone been presented) the well-known Lorentz transformations,[6] for

$$d_1'=\frac{d(1-v_1/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}=\frac{d-v_1(d/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}=\frac{d-v_1t}{(1-v_1^2/c^2)^{\frac{1}{2}}}, \tag{29}$$

which is expressed by Lorentz as

$$x'=\frac{x-vt}{(1-v^2/c^2)^{\frac{1}{2}}} \tag{30}$$

and

$$t_1'=\frac{t(1-v_1/c)}{(1-v_1^2/c^2)^{\frac{1}{2}}}=\frac{t-(v_1/c)t}{(1-v_1^2/c^2)^{\frac{1}{2}}}=\frac{t-v_1d/c^2}{(1-v_1^2/c^2)^{\frac{1}{2}}}, \tag{31}$$

which is expressed by Lorentz as

$$t'=\frac{t-vx/c^2}{(1-v^2/c^2)^{\frac{1}{2}}}, \tag{32}$$

which he calls "local time." Since all the clocks (Fig. 1) are recording an event (arrival of the light signal) at *the same place, at the same time*, each giving a *different* value, the term "local time" is somewhat of a misnomer for the array of indications possible. "Clock readings" is a more accurately descriptive term.

We now develop an interesting consequence of the contractions we have assumed as the basis of our discussion, namely, that all the phenomena of measurement of light paths and times of transit may be expressed entirely in terms of the *measured* velocities of the bodies and of light.

Let us take any pair of read distances, such as d_1' and d_2' and solve for one in terms of the other.

$$\frac{d_2'}{d_1'}=\frac{d(1-v_1/c)}{d(1-v_2/c)}\left[\frac{1-v_2^2/c^2}{1-v_1^2/c^2}\right]^{\frac{1}{2}}, \tag{33}$$

from which

$$d_2'=d_1'\left[\frac{1-\dfrac{(v_2-v_1)}{c(1-v_1v_2/c^2)}}{\left\{1-\dfrac{(v_2-v_1)^2}{c^2(1-v_1v_2/c^2)}\right\}}\right], \tag{34}$$

with similar expressions for the measured times.

We note that this equation is similar in form to the expressions from which it is derived, the quantity $(v_2-v_1)/(1-v_1v_2/c^2)$ taking the place of v_1 and v_2. Now v_2-v_1 is an absolute difference of velocities, which suggests that we investigate the corresponding term for observed or measured differences of velocity.

Consider a point on a uniformly moving body of absolute velocity v_2, which is passing our observation platform moving at the velocity v_1, with the difference of velocity v_2-v_1. Now the absolute distance between two marks a, b, on our body is $L_s(1-v_1^2/c^2)^{\frac{1}{2}}$ so that the true time for the transit of the moving point from a to b is

$$L_s(1-v^2/c^2)^{\frac{1}{2}}/(v_2-v_1), \tag{35}$$

[6] In the procedure used to obtain the Lorentz transformations by starting with the postulate that the velocity of light is always c, is the concealed restriction that all clocks and rods are to be manipulated at infinitesimal speeds. The final results are subject to this same restriction, which when recognized leads to such dicta as "clocks must be excluded from time measurement," and that the times recorded by a clock are "incorrect." In view of the role played in both macroscopic and atomic physics (e.g., spectroscopy) by rods and clocks, no system which throttles them can be physically satisfactory.

which will be read by a clock on the body moving at velocity v_1, as

$$L_s(1-v^2/c^2)/(v_2-v_1). \tag{36}$$

Now from this is to be subtracted the change in clock setting due to the transport of the clock from a to b, giving

$$T'=\frac{L_s(1-v^2/c^2)}{v_2-v_1}-\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{(v+W)^2}{c^2}\right)^{\frac{1}{2}}\right]. \tag{37}$$

Substituting W_0 from (11) and solving we get

$$T'=\frac{L_s(1-v_1v_2/c^2)}{v_2-v_1}+\frac{L_s}{cW_0}\left[1-\left(1+\frac{W_0^2}{c^2}\right)^{\frac{1}{2}}\right], \tag{38}$$

from which, putting v_{2-1} for the observed velocity difference,

$$v_{2-1}=\frac{L_s}{T}=\frac{1}{\dfrac{1-v_1v_2/c^2}{v_2-v_1}+\dfrac{1}{cW_0}\left[1-\left(1+\dfrac{W_0^2}{c^2}\right)^{\frac{1}{2}}\right]}, \tag{39}$$

giving

$$\frac{v_2-v_1}{1-v_1v_2/c^2}=\frac{v_{2-1}}{1-v_{2-1}[1-(1+W_0^2/c^2)^{\frac{1}{2}}]}. \tag{40}$$

If now we substitute this value for $(v_2-v_1)/(1-v_1v_2/c^2)$ in the relation beween d_1' and d_2', and also substitute for c its value in terms of c_m we finally have the relation beween the measured values d_1' and d_2' *entirely in terms of the observed velocities of rods, clocks, and light.* Therefore, as long as we perform our measurements on the distances and times of transit of light signals, with rods and clocks which we move at the same measured velocities, we can give a complete description without reference to our absolute framework.

Returning to the expression

$$(v_2-v_1)/(1-v_1v_2/c^2),$$

note that for $W_0 \cong 0$ this becomes simply

$$\frac{v_2-v_1}{1-v_1v_2/c^2}=v_{2-1}. \tag{41}$$

Inserting this in the expression for d_2' in terms of d_1' we get

$$d_2'=\frac{d_1'(1-v_{2-1}/c)}{(1-(v_{2-1})^2/c^2)^{\frac{1}{2}}}. \tag{42}$$

Since c is also the measured value of the velocity of light for $W_0 \cong 0$, $Y_0=0$, we see that we have for this case an expression exactly similar in form to the original expressions (26)

$$d_1'=\frac{d(1-v/c)}{(1-v^2/c^2)^{\frac{1}{2}}}$$

in terms of absolute velocities and with rods and clocks moved infinitely slowly. It is probable that the expressions for d_2' in terms of d_1' for finite values of W_0 and Y_0 would take similar forms after the performance of the proper transformations.

6. Summary of Results

The postulates used in this discussion may be arranged and listed as follows:

A postulate covering the velocity of light and its measurement

I. The velocity of light in the luminiferous ether, when measured by light signals between two points at rest in the ether, by clocks and rods unaffected by transport, is a constant c.

A postulate covering the behavior of material rods and clocks

II. The lengths of material rods (in the direction of motion) and the frequencies of material clocks, are reduced by the factor $(1-v^2/c^2)^{\frac{1}{2}}$, where v is the velocity of the rod or clock, and c the velocity of light, measured as in I.

By the use of these postulates we *conclude* that

III. Measurements of light signals made on single moving bodies yield results which can be expressed entirely in terms of the measured velocities of rods, clocks, and light, and are identical whatever the velocities of the moving bodies, so long as the same measured velocities of rods and clocks are adhered to.

Using III in conjunction with I and II we find expressions for light signals traversing several moving bodies.[7] Principal conclusions are:

(1) Measured distances and times of transit of identical light signals are different for each moving body.

(2) The velocity of light measures the same on each body, the actual value being a function of the observed velocities of the measuring rods and clocks used, which approximates c as those velocities become small compared to the velocity of light.

(3) The expressions found approximate to the Lorentz transformations as the velocities of rods and clocks become small compared to the velocity of light.

7. Arguments for the Framework and Measuring Methods

In the treatment of light signals on moving bodies here presented we have assumed the existence of a luminiferous ether, and have used throughout the conception of true or absolute time, such as could be established if clocks could be moved from place to place through the ether without being affected by their transport. Both these are conceptions which it has become common to deny as necessary, in view of the possible elimination of all terms representing these quantities in the final equations which we have been able to derive, in which occur only quantities determined by measurements involving the moving bodies. I give now some considerations which I think, support these conceptions in spite of the common argument against their necessity.

In the case of the time factor the objection is commonly stated in terms of the idea of simultaneity at separated points. If this cannot be determined, it is argued that it has "no meaning." Now the distinction which must be made here is between *nonexistence* and *indeterminacy.* It is now well recognized in other departments of physics that phenomena which are altered by the process of measurement become as a consequence *indeterminate.* In the case of simultaneity we have seen that the transport of a clock to a distant point is such a function of the velocity that it prevents the velocity being known. This in turn prevents the correlation of the clock indications at the two points. The *existence* of true simultaneity is however easily established by the following observation:

Consider three clocks A, B and B', all having the same rates when together, and let B and B' be moved to a distant point on the common moving platform. Send a series of equidistant pulses from B to A, and let A be set to be synchronous with the pulses it receives. Then let the pulses sent from A be used to set B'. We will then have B and B' going at the same rate, but with a difference of phase. Let us now set up a number of other clocks at B, B' all of the same rate, but at phases between B and B'. Now by taking a sufficiency of these clocks, *one will be* beating simultaneously with A, irrespective of the fact that we cannot determine which it is, within the limits set by B and B', which give a measure of the indeterminacy.

Einstein offers as a substitute for this indeterminacy the proposal to *define* simultaneity as given by *half the* time taken for a to and fro light signal. This of course will agree with the simultaneity above discussed only if the system is stationary in the light transmitting medium. It corresponds to "local time" when clocks and rods are moved infinitely slowly. As already pointed out this involves us in the predicament that parallel light signals traversing several bodies are, when their arrival at a terminal point is recorded on each moving body, reported as arriving at a *different* time on each body, although constituting a *single event at a single place,*—simultaneity by the most rigid criterion.

[7] It is possible, by assuming I and III, to derive II, by imagining light signals, sent over a to and fro path, to take an invariant measured time, and by requiring that the measured distance and time at the far point shall be invariant whatever the velocities of the rods and clocks used in measurement. Both this line of attack (assumption of invariance), and that followed in the paper (assumption as to behavior of material rods and clocks) requires I, which is thus characterized as the fundamental assumption in the treatment of light signals.

The definition of simultaneity proposed by Einstein is violated immediately clocks moving at finite velocities on the bodies are used to establish "local" times. The time of arrival of a light signal at a distant point is then a function of the clock speed, given by (27) which is no longer half the interval recorded by a clock at the origin for a to and fro signal. These various differing measures of the time of arrival of a light signal at a distant point are simply manifestations of the dependence of our results on the characteristics of the tools of measurement used. They none of them give us the simultaneity above discussed, which could be found only with clocks and rods unaffected by motion, which have been here taken as the basis for discussing material clocks and rods which *are* affected.

We are saved from any *practical* difficulties in connection with distant synchronization by the extremely low speeds at which clocks and rods can be moved.

Turning next to the question of the ether, we admit at once its elusiveness by any measurements of the sort we have been considering (based on the value $n=0$ in the contraction expressions), for the result of the length and clock rate contractions is to make these measurements invariant with v, the velocity of the system with respect to the ether. Referring however to a preceding paper,[1] it was there stressed that an uncompensated difference in the character of the light signal for different velocities of the system remained, namely the *direction* of the pulse or wave front, which enters or leaves the interferometer at the *aberration angle.* This is a different characteristic of the light from the measurements of distances and times of transit to which we have directed attention here, and it is in this directional factor that we have the strongest evidence for the existence of the luminiferous ether.

This evidence we get from the *aberration of light from the stars.* The earth swims in a sea of radiant energy, resolvable into a pattern of wave fronts maintaining over long periods substantial constancy of relation in direction to each other. In the course of a year this pattern of radiant energy experiences, *as a whole*, a to and fro oscillation of direction. The interpretation of Bradley, that it is the earth which is moving in an orbit, through the substantially unchanging pattern of light waves, is as compelling today as ever. The alternative, that this entire energy pattern, together with the stellar universe with which it is associated (either immovably or with a steady state of drift, which alternative being indeterminate) oscillates back and forth yearly, is improbable in the ratio of the number of stars to the number of the observing platforms, namely one. The evidence is thus not metric, but by sheer weight of enormous probability. It is the same evidence that leads us to accept the Copernican as against the Ptolemaic system.[8]

It must not be forgotten in the discussion of this subject that the Michelson-Morley experiment, which is the original and crucial problem, only demands invariance of light signals with the velocity of the moving platform of measurement *on the premise that the earth is moving,*—there is no other motion involved in the experiment. If this is not agreed to then the null result proves nothing with regard to invariance, and the whole discussion is futile. Now the annual motion of the earth (which is the motion it is sought to detect in the Michelson-Morley experiment) we deduce from the energy pattern reaching us from outer space, in other words we observe that the earth moves with respect to this energy pattern. The Michelson-Morley experiment is performed on the assumption that a light signal initiated in the apparatus becomes a part of this energy pattern with respect to which the earth moves. The alternative would be that the signal partakes of the velocity of the source, which again would nullify the whole discussion. Calling upon the Michelson-Morley experiment as proof of the invariance of light signal phenomena therefore carries with it the acceptance of the luminiferous ether here assumed.

[8] The relativity treatments of stellar aberration all start with a "wave front" from a star, thus of necessity recognizing the pattern of radiant energy here considered as a manifestation of the ether. The common relativity treatment of aberration concerns itself with a broad section of a wave front, and shows that, due to the "local" time difference at points along the receiving surface the wave front will always be interpreted as being normal to the direction of propagation of the wave front section as a whole, with respect to the earth. This does not touch the question of the cause of the direction taken by the wave section or ray, and is incidentally not the case of a star image formed by a telescope. The second-order term in the "relativity" expression for aberration follows at once along the lines here followed, from the contraction of the measuring plane in the moving telescope.

8. Conclusion

In conclusion I wish to emphasize the three factors of this treatment which I believe to be most important. First, is the exact specification of the framework, and the characteristics of the tools of measurement and their use. Second, is the choice of the length and frequency contractions as the basic phenomena in the behavior of moving bodies. Third, is the admission of velocities of the measuring instruments other than infinitesimal, with the consequent recognition that the measured velocity of light is a function of the measuring conditions, and can have other values than the constant c.

Appendix

Solutions of equations involving W, W_0, Y and Y_0

(1) W and Y in terms of W_0 and Y_0.
Let $v/c=x$, $W/c=z$, $W_0/c=\alpha$.

(10) is then

$$\alpha=\frac{z}{(1-x^2)^{\frac{1}{2}}(1-(x+z)^2)^{\frac{1}{2}}},$$
$$z^2/\alpha^2=(1-x^2)(1-[x+z]^2)$$
$$=(1-x^2)(1-x^2-2xz-z^2),$$
$$z^2[1+\alpha^2(1-x^2)]+2zx\alpha^2(1-x^2)-\alpha^2(1-x^2)^2=0,$$
$$z=\frac{-x\alpha^2(1-x^2)\pm\{x^2\alpha^4(1-x^2)^2+\alpha^2(1-x^2)^2[1+\alpha^2(1-x^2)]\}^{\frac{1}{2}}}{1+\alpha^2(1-x^2)}$$
$$=\frac{\alpha(1-x^2)[-x\alpha\pm(1+\alpha^2)^{\frac{1}{2}}]}{((1+\alpha)^{\frac{1}{2}}-x\alpha)((1+\alpha)^{\frac{1}{2}}+x\alpha)},$$

taking the positive sign

$$=\frac{\alpha(1-x^2)}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}$$

or

$$\frac{W}{c}=\frac{W_0/c(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}.$$

The same method of solution gives a similar expression for Y in terms of Y_0.

(2) Solution of

$$L_i'=\frac{L_s\{(1-v^2/c^2)^{\frac{1}{2}}-(Y/W)[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v+W)^2/c^2)^{\frac{1}{2}}]\}}{(1-(v+Y)^2/c^2)^{\frac{1}{2}}},$$

adding the symbols y for Y/c and β for Y_0/c we have

$$L_i'=\frac{L_s[(1-x^2)^{\frac{1}{2}}-(y/z)\{(1-x^2)^{\frac{1}{2}}-(1-(x+z)^2)^{\frac{1}{2}}\}]}{(1-(x+y)^2)^{\frac{1}{2}}}.$$

Now

$$x+z=x+\frac{\alpha(1-x^2)}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}=\frac{\alpha+x(1+\alpha^2)^{\frac{1}{2}}}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}},$$

from which

$$(1-(x+z)^2)^{\frac{1}{2}}=\frac{(1-x^2)^{\frac{1}{2}}}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}$$

and

$$(1-(x+y)^2)^{\frac{1}{2}}=\frac{(1-x^2)^{\frac{1}{2}}}{x\beta+(1+\beta^2)^{\frac{1}{2}}}$$

so that

$$L_i'=\frac{L_s[(1-x^2)^{\frac{1}{2}}-y/z\{(1-x^2)^{\frac{1}{2}}-(1-x^2)^{\frac{1}{2}}/(x\alpha+(1+\alpha^2)^{\frac{1}{2}})\}]}{(1-x^2)^{\frac{1}{2}}/(x\beta+)1+\beta^2)^{\frac{1}{2}})}$$

$$=L_s\left\{1-\frac{\dfrac{\beta(1-x^2)}{x\beta+(1+\beta^2)^{\frac{1}{2}}}}{\dfrac{\alpha(1-x^2)}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}}\left[1-\frac{1}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}\right]\right\}(x\beta+(1+\beta^2)^{\frac{1}{2}})$$

$$=L_s[(1+\beta^2)^{\frac{1}{2}}+(\beta/\alpha)(1-(1+\alpha^2)^{\frac{1}{2}})]$$

or

$$L_i'=L_s[(1+Y_C^2/c^2)^{\frac{1}{2}}+(Y_0/W_0)\{1-(1+W_0^2/c^2)^{\frac{1}{2}}\}].$$

(3) Solution of

$$T_i'=L_s/c(1+v/c)-L/W(1-v^2/c^2)^{\frac{1}{2}}[(1-v^2/c^2)^{\frac{1}{2}}-(1-(x+W)^2/c^2)^{\frac{1}{2}}].$$

Making the substitutions indicated in (1) and (2)

$$T_i'=\frac{L_s}{c}(1+x)-\frac{L_s/c}{z}(1-x^2)^{\frac{1}{2}}[(1-x^2)^{\frac{1}{2}}-(1-(x+z)^2)^{\frac{1}{2}}]$$

$$=\frac{L_s}{c}(1+x)-\frac{L_s}{c}\frac{(1-x^2)-(1-x^2)^{\frac{1}{2}}(1-(x+z)^2)^{\frac{1}{2}}}{z},$$

putting $(1-(x+z)^2)^{\frac{1}{2}}=\dfrac{(1-x^2)^{\frac{1}{2}}}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}$ as above found, and substituting for z its value in terms of α

$$T_i'=\frac{L_s}{c}(1+x)-L_s(1-x^2)\left[\frac{1-\dfrac{1}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}}{\dfrac{\alpha(1-x^2)}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}}\right]$$

$$=\frac{L_s}{c}\left[\frac{\alpha+1-(1+\alpha^2)^{\frac{1}{2}}}{\alpha}\right]$$

$$=\frac{L_s}{c}\left[\frac{W_0/c+1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right].$$