ROBERT HANSEN

This Curving World: Hyperbolic Linear Perspective

> ... sag ich, dass alle, auch die gerädesten Linien, so nit directe contra pupillam stracks vor dem Aug stehen, oder durch sein Ax gehn, nothwendig umb etwas gebogen erscheinen. Das glaubt gleichwol kein Mahler, darumb mahlen sie die gerade Seitten eines Gebäws mit geraden Linien, wiewol es nach der wahren Perspectiffkunst eigentlich zu reden nit recht ist. ... Das Nüsslein beisset auf Ihr Künstler!
>
> Wilhelm Schickhardt, 1624

> ... the more the eye approaches the subject [a sphere] the more one believes to see and the less one does see.
>
> . . . .
>
> ... our author's perspective images are straight reproductions of the natural visual process, which is determined by angles and distances and thus results in an image projected on the *interior of a sphere* rather than on a plane surface.
>
> Codex Huygens (c. 1580)

> In point of fact (modern central perspective) is a mathematically exact abstraction substituted for a physiological image which is wholly deceptive. We see not with one fixed eye, but with two constantly moving eyes; the image which we receive on the retina is a spheroid world projected on a concave plane. Thus, in a perspective drawing, straight lines are presented as straight, but in our visual image they are actually curved. Through the use of the printed page and mathematical perspective we are now accustomed to discount mentally this image; but for the ancients this curving world was an accepted phenomenon.
>
> A. G. M. Little

I

BY ATTENDING strictly to the *appearance* of straight lines in my environment, and by testing my observations with the help of a generation of students, I believe that I have reason to claim that the system of linear perspective described in this paper offers a significant refinement of previous diagrams of curvilinear perspective and a correction

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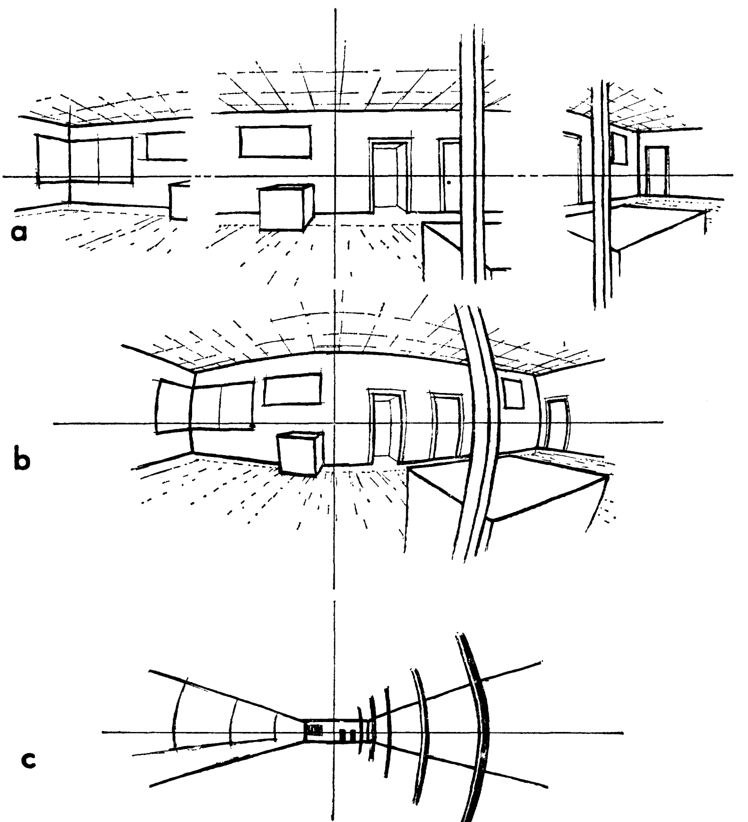
of generally accepted perspective "laws." It may also suggest a new application of the views of Leonardo da Vinci as interpreted by art historians, but there is no intention here to question those interpretations. I do hope to challenge certain traditional ideas about vision, particularly the assumption that classical linear perspective represents the way the world appears.

We see curves wherever we look at straight lines. Or is it more accurate to say that we are looking at curves wherever we believe we see straight lines? We tend to deny the empirical validity of these curves and feel that we must explain them away and correct them. It is my intention first to discuss the general nature of our perception of straight lines, briefly review the history of linear perspective in recent centuries, and then to present a system of five-point hyperbolic perspective that will come closer to representing "this curving world" than do our inherited rules; closer, I believe, than any system heretofore proposed.

It is true that in many situations, when I am at a "comfortable distance" from the focus of my attention, this curvature is negligible. But if I am quite close to a large cube or rectangular solid, or, as is the case with most of us the greater part of every day, enclosed within the walls of a box, the lines of that box swell and bend on all sides—especially if I enlarge my focus and give attention to the whole of my field of vision. The curves are most subtle directly in front of my eyes, but they are emphatic at the periphery of vision.

A rectangle's curvature becomes even more apparent when I move, or when the rectangle moves past me. Indeed, a street or a corridor veritably ripples as I walk along it, its apparently largest section accompanying me precisely as I move. Standing on a railway platform watching a passing train



presents the simplest experience of the swelling of the nearest section; but every vehicle, pedestrian, or animal moving past us on a straight street exhibits the same curved passage.

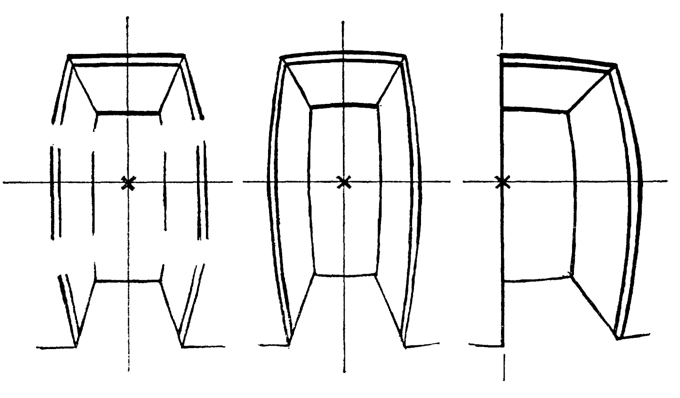
In your own room, you can simulate the moving rectangle by moving your eyes or your head. It is most readily done by approaching to within a couple of feet of a long wall, preferably one containing a number of windows, doors, shelves, or other perpendiculars, and while looking straight ahead, wag the head vigorously from right to left. While moving the head, pay special attention to the floor and ceiling lines, but keep the head level. Thus are connected in one continuous panorama the view to the right, the view straight ahead, and the view to the left, all of them familiar to us separately, those at right and left offering radical foreshortening and inescapable diagonal convergence of the parallel horizontal edges of walls and windows, etc.

To see the verticals behave in the same way, stand in a doorway, with your toes nearly touching the threshold, and nod the head up and down, keeping the door frame and other verticals visible out of the corners of the eyes. In both examples the center segment, the portion of the line nearest to your eye, will appear to contain the major curvature. This head movement near the wall should serve primarily as an introduction to curvature perception. After some practice, you should be able to see the curving wall lines and door lines from a comfortable distance, and without moving. It will always be easier at close range, and by attending to peripheral areas of the field of vision.

There are exceptions, lines that do not appear to curve. Some are "orthogonals," to be discussed later, the lines moving toward the horizon directly in front of me. There are also the lines that pass through the center of vision, intersecting that line projected straight forward from the point between the eyes—the line "stracks vor dem Aug" in the Schickhardt quotation.

Thus, every line that coincides with the horizon, that is, every horizontal line at eye level, will appear absolutely straight. One may see any straight line without curvature by tilting or veering the head until the line passes through that imaginary perpendicular line. Turn to look directly at the door frame edge, and it resumes its straight verticality. Look directly up at the line formed by the meeting of ceiling and wall, and it becomes in a real sense your horizon, precisely straight. Look down at the floor line, and it in turn becomes the visibly straight "horizon," all other horizontals curving above and below it. From a sufficient height the horizon itself is seen to curve. From the astronaut's point of view, our familiar straight horizon has of course become a circle. And a wheel or a jig-saw shape when seen edge-on presents a straight line, which in turn may be *seen* as a straight line only if it intersects that line projected straight forward from the point between the eyes, the center of vision.

It has been suggested that physiological factors may either account for or argue against the "reality" of these curves: bin-



ocular vision, a spherical retinal surface, and the mind's interpretation of the retinal information.

Monocular vision of course presents a simpler case. For some individuals, closing one eye, if it were not somewhat uncomfortable, might facilitate these experiments. Also, the horizontal axis shared by the two eyes favors a clear measuring of horizontal lines, and somewhat confuses the observation of verticals. However, binocular and stereoptical effects do not alter the phenomenon of parallel lines appearing to converge simultaneously in opposite directions. Convergence follows naturally from the more basic universal observation that every object appears smaller as it recedes from the eye.

Even in the case of a flat retina, or of a compound eye, an object just larger than the eye must necessarily appear larger than the entire field of vision if the object is so close that it covers the eye. Only in a world in which receding objects and distances appear to retain their size on the retina could straight lines conceivably retain their straight appearance in all circumstances. As long as the height of a wall appears to diminish both to right and to left, straight lines must appear to curve. For this reason, I do not believe that the spherical cornea and the concave retina are at all relevant to the question of whether we actually see straight lines as curves.

Some friends who are unable (or unwilling) to perceive these curves object that my mind has interpreted as curves the raw rectilinear data of the retina, in order to satisfy my prior reasoning. I can only say that nearly everyone who has undertaken the head-wagging just described has quickly come to acknowledge the curving appearance of demonstrably straight lines. Furthermore, it seems to me that precisely the opposite occurs: the lines that appear to curve, with the exceptions noted, are the raw, uninterpreted sensory data; we have been persuaded by centuries of drawings, paintings, and photographs (by lenses selected to eliminate curvature) that our brain must reject what does not appear straight. Anyway, the curvature in the center of our usual view is always slight and can be easily missed unless a special effort is made to attend to very long lines and not just to small segments near the center of vision.

I will discuss a widely read refutation of the curvature thesis that appeared in E. H. Gombrich's *Art and Illusion*.[1] Briefly, he claims that the eye sees the curvature only by turning the head, thus requiring the artist to construct "a compromise that does not represent one aspect but many.... What we call 'appearance' is always composed of such a succession of aspects ... which allows us to estimate distance and size; it is obvious that this ... can be imitated by the movie camera but not by the painter with his easel." But is it always necessary to separate into distinct "aspects" the single sensation I experience when I move my head? By simply revolving the eyes without moving the head, I can easily perceive the curves in a wall directly before me; and with a little practice, these curves can be seen as a single static pattern without moving as much as an eye-muscle, even at some distance from the wall. By recording a sufficiently wide view a painter or a still camera can then easily imitate this discrete visual image.

Gombrich disposes of other claims with disdain verging on contempt. "It may well be ... that a taut string held very close to our eyes ... 'looks like a curved string.' With strings held very close to our eye, judgment becomes uncertain and we may make mistakes. But to say that all straight lines in our field of vision look curved seems to me a much more doubtful statement. It would imply that all straight strings look like curved strings, and that is manifestly not the case."[2] I must say that straight lines appeared manifestly straight to me until I examined my vision closely. However, in attacking the claim that *all* straight strings look curved, he is misrepresenting the curvilinear proposition. He ignores exceptions that are generally acknowledged: the essentially straight appearance of all orthogonals, and the lines which pass "straight before the eyes" no matter at what distance from us. The "string held very close to our eyes" is particularly unlikely to appear curved unless it is placed at the periphery of the visual field.

It is not enough, as Gombrich and others suggest, to allow the natural foreshortening of the rectangular picture itself to supply whatever foreshortening that might exist in the subject matter parallel to the picture plane. The variable viewing distance separating the spectator from the picture must alter the curvilinear foreshortening at every step he takes. It is true of both one-point perspective and of curvilinear systems that there is only one position at which the spectator can experience a near-replica of the artist's view of the subject. For a small conventional picture of large objects such as architecture, that situation in which the straight lines might supply "natural" curvature places the spectator uncomfortably close to the picture surface, forcing him to turn and tilt his head in order to see the entire rectangle.

At another point Gombrich says, "one cannot insist enough that the art of perspective aims at a correct equation: it wants the image to appear like the object and the object like the image. . . . It does not claim to show how things appear to us, for it is hard to see what such a claim should mean." But can "to appear like the object" have any other meaning than *to appear as the object appears?* Must the image, in order to satisfy Gombrich, represent all measurements in the same scale and not depict distant columns smaller than near columns? I believe that traditional perspective, in distinguishing sizes of objects as they recede from us, does indeed claim to describe appearance. It is appearance at any rate, and only appearance, that I have attempted to measure in this paper.[3]

Finally, while discussing Leonardo's three columns, Gombrich acknowledges, "all this is no doubt a little confusing; if it is a consolation to the reader, let me state my conviction that many writers on perspective have also become confused at this point, not excluding myself, of course." As John White writes, "the straight lines of common architectural usage . . . are indeed all that is seen by the average modern man."

II

The artist has for four hundred years learned the principles of diminishing size and foreshortening with vanishing points, a system that was discovered and "perfected" in the fifteenth and sixteenth centuries. Moreover, whereas Oriental and medieval devices for depicting distance and volume were obvious metaphors (or were they mere conventions to the Chinese or Gothic painter?), Renaissance and Baroque systems have been accepted, at least today, as imitative of nature, to judge from the nonchalance with which painters, architects, and illustrators have portrayed "one-point" and "two-point" cubes, which, I submit, are not to be observed "in nature." (As figures 4, 5, 6, and 7 indicate, one-point becomes five-point and two-point becomes four-point in curvilinear systems.) From Giotto through Masaccio to Claude Lorraine and Turner, the science of *aerial perspective* advanced with man's growing worship of the material universe. The work of Newton led to the optical *color* experiments of Monet and Seurat. But *linear perspective* today is assumed to operate (that is, we assume that our eyes see) according to fifteenth-century rules, the formula of Leon Battista Alberti.

The following review of the history of curvilinear perspective is taken largely from John White's *The Birth and Rebirth of Pictorial Space.*[4] Quotations and pages cited are from this work.

"It is in Alberti's *Della Pittura,* which he wrote in 1435, that a theory of perspective first attains formal being outside the individual work of art. Theoretical dissertation replaces practical demonstration" (p. 121). The practical demonstrations include fourteenth- and fifteenth-century paintings by Giotto, Lorenzetti, Maso di Banco, and others, which utilize in a groping, experimental fashion an oblique foreshortening depicting architecture in what we would characterize as two-point perspective, and to two panels that Brunelleschi evidently painted expressly to demonstrate geometric perspective. These "painted manifestos" of Brunelleschi have not survived but contemporary descriptions of them indicate that one panel depicted the Palazzo della Signoria from the diagonally opposite corner of the Piazza in what may have been oblique two-point perspective, in which the north and west walls of the Palazzo were seen in their entirety (p. 118).[5]

Alberti's theory incorporated some of Brunelleschi's space but was restricted to a single vanishing point. Moreover, Alberti's codified perspective dictated that "(a) there is no distortion of straight lines, and (b) there is no distortion, or foreshortening, of objects or distances parallel to the picture plane" (p. 123). According to White, "Alberti shows no sign of any awareness ... of the points at which these achievements are made possible only by the acceptance of a geometrical convention which runs counter to the artist's visual experience.... The contrast between the mathematical and the empirical must not, however, be taken too far. It is not a question of the replacement of a method which is all fidelity to experience by another which is all convention. It is a substitution of one aspect of truth and one convention, for a different convention and another truth" (p. 125).

Within a few years, Alberti's single central vanishing point had conquered. His rule dictated that all horizontal and vertical lines parallel to the picture plane must be theoretically and practically parallel and perpendicular in the picture. Only those "orthogonal" lines, receding toward or radiating from the single central point remained diagonal and converging. This ingenious system, developed by architects, had simplified the artist's craft. Oblique two-point perspective virtually disappeared from painting and relief sculpture. Alberti's powerful treatise had apparently become dogma by 1500, and remained inviolate for more than a century. When seventeenth-century northern artists freed the depiction of architecture from this strict frontality, they returned to the more natural visual experiments of the fourteenth century, when several painters often implied the curved concave plane upon which we see the real world projected. Giotto, Maso di Banco, and, later, Uccello had expressed in numerous paintings some of the diagonal elements of "this curving world." And in an unattributed fresco in the lower church at Assisi, a complex building is seen in the center in horizontality, but on both flanks in two-point obliquity.

It is, however, only in the work of Jean Fouquet (1420?–1481?) that these adjacent sections are allowed to fuse in a continuous curve, in both ceiling and floor of the *Annunciation of the Death of the Virgin* (Museé Condé, Chantilly) and in the *Arrival of the Emperor at St. Denis* (Bibliothèque National, Paris) where pavement tiles behave in the visual way, curving up toward a horizon on both left and right, upon which, however, a procession is seen marching from left to right in an unyielding flat horizontality. And in another Fouquet manuscript illustration, *The Building of the Temple at Jerusalem* (Bibliothèque Nationale, Paris), as well as in Donatello's relief rondo, *The Assumption of St. John* (S. Lorenzo, Florence), verticals converge noticeably toward a zenith vanishing point.[6] It is the extreme rarity of vertical convergence that makes these last examples astounding. To our prejudiced eye, their curves and diagonals may look crude and unsure, but they must be seen as the application of a sensitive eye, not yet intimidated by formula. It is in Fouquet's manuscript painting that the sloping upper sections of the temple imply a curve as they rise from the almost vertical ground-level section of piers and columnar sculpture of the Gothic façade. Even the *trompe l'oeil* tours de force of the eighteenth century fall short of the vision of this fifteenth-century Frenchman, who expressed the curvature he saw in architectural lines in spite of the inconvenient adjustments necessary on a two-dimensional surface.

There was one other notable exception to the rule. Although Leonardo da Vinci did not publish a systematic theory of "synthetic" perspective, and did not follow these principles in his own painting, we know something of his thoughts from his own notes and from Cellini's description of

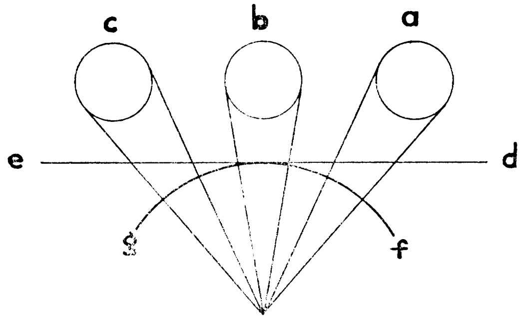


FIG. 3. Adapted from Leonardo's Manuscript A, 1492, as reproduced in White.

a manuscript book by Leonardo, a treatise on perspective that has not survived (pp. 207 ff.). In Manuscript A (1492), he depicts the intersection of the cone of vision by a surface concave to the eye (g–f) where flanking columns appear smaller than the nearer, central column, rather than equal or larger, as in the conventional plane intersection (e–d).

Commenting on another section of the same manuscript, White states, "There is no escape from the conclusion that, in his definition of simple perspective, Leonardo is visualizing a concave spherical surface, the three-dimensional counterpart of the arcs centered on the eye that have already appeared amongst his diagrams. No other surface can be 'equally distant from the eye in every part.' It is the first step towards the theory of synthetic perspective" (p. 211). But the die had been cast. One-point perspective was law, and the development of synthetic perspective was never consummated. Leonardo himself appears to have counseled against depicting such close views that curvature would be difficult to avoid. He recognized the practical advantages of reasonably distant views and of the one-point system.[7]

Other similar observations have been made from time to time. A didactic drawing manual dating from c. 1580, influenced by and possibly copied from Leonardo's work, had applied these principles to drawings of statuary and the human figure. Wilhelm Schickhardt, an obscure Tübingen linguist, mathematician, and dilettante etcher, published in 1624 a pamphlet in which observations on the paths of meteors are combined with general comments on linear perspective which include the taunt addressed to artists that is quoted at the beginning of this paper. Scholars, mostly concerned with architecture or with Leonardo, not with the problem of representational drawing, have debated these optical effects since the early nineteenth century, when curves were discovered in entablatures and stylobates of Greek temples.[8]

But artists were not affected. Except for occasional hints of simultaneous vanishing points at right and left in separate details of interiors by Dutch painters in the seventeenth century (see paintings by Steen, Jordaens, and Brouwer), there exist almost no indications that artists since Jean Fouquet have ever found fault with straight line formulas.[9] Two-point perspective became commonplace by the nineteenth century, and melodramatic use has been made of three-point perspective, more often perhaps in twentieth-century illustration than in painting. But Fouquet's curves have not been seen again. Even Piranesi's theatrical *Carceri* retain strictly parallel verticals. Taking into account his extremely wide-angle architectural views, it is difficult to avoid the tug of simultaneous foreshortening from zenith and nadir in Piranesi's drawings.

Ever since Fouquet and Donatello, verticals have remained conveniently sacrosanct, absolutely parallel. And there seems to have been no questioning of the propriety of the Albertian rectilinear system. Worse, it has been assumed in too many academies, by too many artists, as well as by art historians and laymen, that the inherited system represents not only propriety and practical metaphor, but essential fidelity to ordinary "natural" vision. Although common false assumptions about vision were successfully challenged in the area of color by Impressionism and more recently by Op, I know of no proposal since Leonardo either to revise Albertian rules or to consider Leonardo's diagrams critically. Numerous scholars, including John White, to whom I am indebted for much of the foregoing historical survey, have analyzed and expanded the

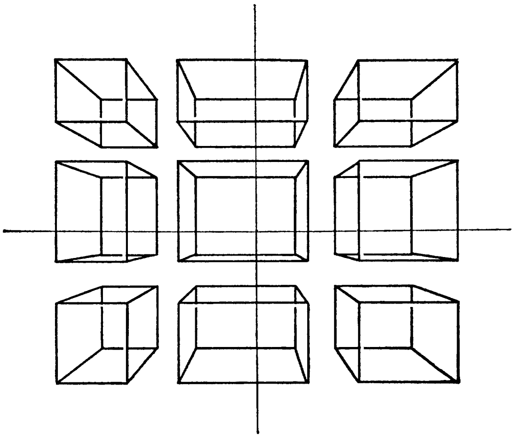


FIG. 4. 15th Century 1-point space (Albertian "artificial" perspective).

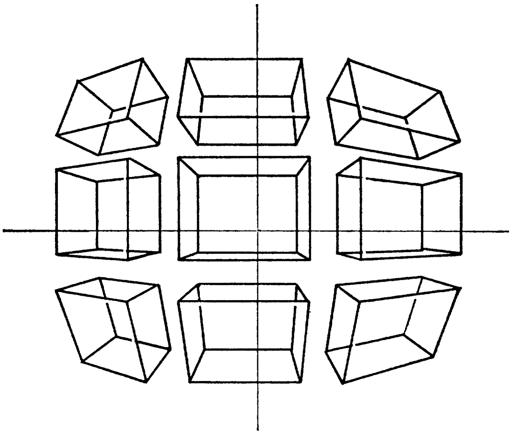


FIG. 5. 19th Century 1-point, 2-point, and 3-point space. Whenever three planes of an opaque cube are visible, three points are operative; two planes, two points; one plane, one point.

implications of Leonardo's theories. In doing so however, they have retained the same arcs and intersecting straight lines implied by Leonardo, and which have appeared in the diagrams of Schickhardt, Guido Hauck, and others interested in the problem.[10]

III

I now suggest one fundamental change in these diagrams of synthetic perspective, a change which will for the first time, I believe, produce a perspective convention that is in virtual agreement with the artist's visual experience:

> All straight lines, except those that pass through a line projected straight ahead from a point between our eyes, appear curved, *not as arcs, but as hyperbolas.*

The hyperbolic curve achieves these advantages: first, it is the way the world appears to me and, I believe, to all of us; second, it is more effective than the arc diagram as a synthesis of the separate right and left, up and down, foreshortening of straight lines. Perhaps most important, the hyperbola permits all straight lines to share the same visual form. It reconciles a seeming contradiction in synthetic perspective involving the orthogonal lines. All curvilinear diagrams to date suggest two kinds of lines: those lines receding directly in front of the eye toward the central vanishing point are depicted as straight lines, while all other receding lines, to right, left, up, and down, are arcs.[11] Why? The briefest answer is that an arc viewed edge-on, from a 90° angle, appears straight, just as a right-angle profile of a wheel is a straight line. But a more empirical analysis may be in order:

Your view of the walls of your room is one of lines that curve appreciably only where they are nearest you, and *approach straightness as they recede in any direction;* that is, toward any one of five vanishing points. One of these is, like Alberti's, in the center as you face a wall. The others are to be sensed at your extreme left, at your extreme right, directly above your head, and directly below your feet, all of course infinitely distant in space. Note that all points are located not somewhere diagonally in front of you, but, except for the center one, are on a plane with your forehead, so to speak, and at the infinitely distant circle where that plane converges with all other planes parallel to it.

There is of course a sixth point, directly behind you. This sixth point is not an unfleshed abstraction. All you need do to activate it is to turn your head. It is then immediately integrated into the panoramic six-point system within which all perpen-

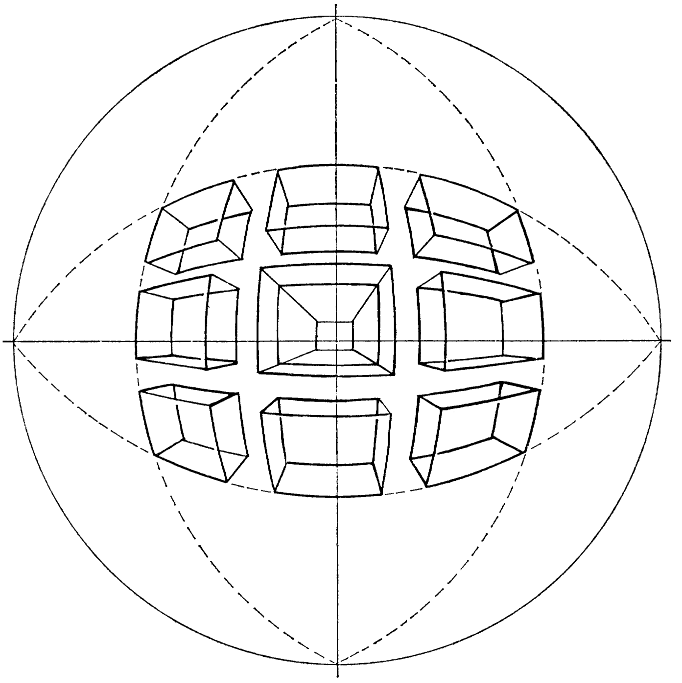


FIG. 6. Circular 5-point space, derived from Leonardo's writings. All lines are simple arcs except the orthogonals which are straight. This arrangement resembles the compressed view of architecture observed in convex mirrors and in extreme wide-angle photographs.

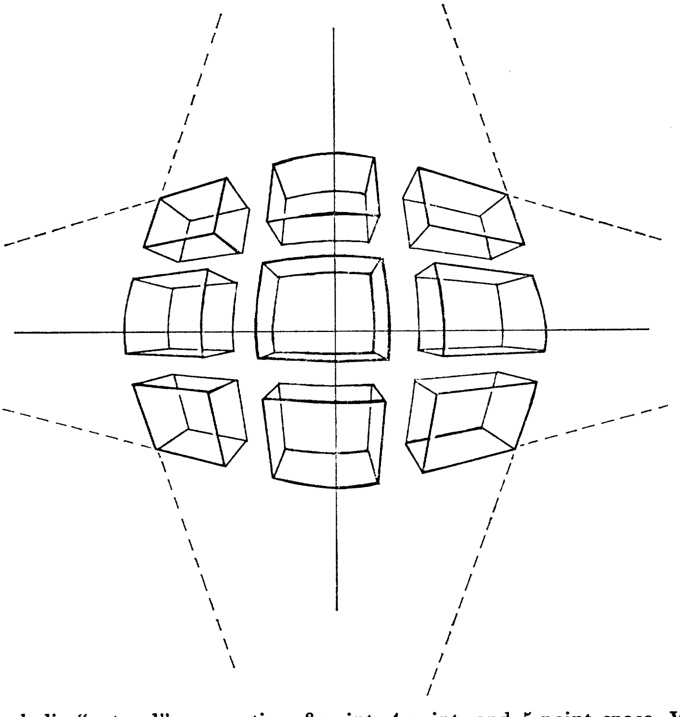


FIG. 7. Hyperbolic "natural" perspective: 3-point, 4-point, and 5-point space. When three planes of a cube are visible, three points are operative; two planes, four points; one plane, five points.

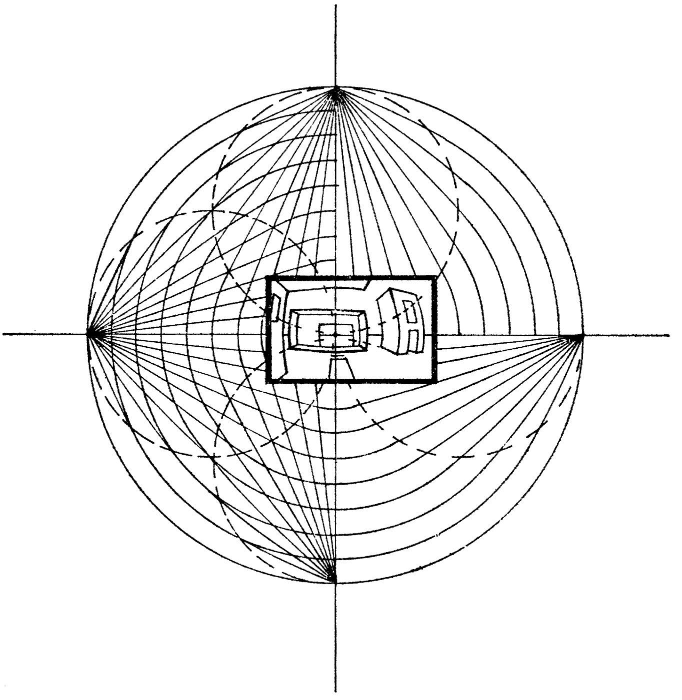


FIG. 8. Two points are illustrated: 1. The framed picture represents a relatively wide-angle view. Most of our attention in normal viewing is limited to a much smaller area, often no larger than the smallest rectangular "window" in the center, where the degree of curvature is negligible. 2. The portion of the hyperbola that appears to curve is very small in the lines near the center of vision, and increases at the periphery of vision. The smaller circles enclose the nearly straight segments of the hyperbolas.

dicular planes are comprehended. As the side walls and the ceiling and floor approach you, defined by apparently straight orthogonals, they increase in size, are at their widest and highest nearest your eye, but passing behind you, over your head and beneath you, they decrease in size as they approach that sixth point. In addition, all lines that define oblique planes curve sooner or later at the point nearest your eye, their extensions curving sometimes over or under your shoulders toward a vanishing point behind you on that infinite sphere where all planes meet. This spherical surface of which your eye is the center contains all of the vanishing points of all possible rectangular planes and straight lines. Of course, you have only to turn your head 90° up, down, right, or left, in order to make the erstwhile orthogonals into lines parallel to your new picture plane, which, answering now to a new pair of polar opposite vanishing points, exhibits the same curvature nearest your eye that you saw in the previous direction. As you turn, the curving horizontals that had been oriented left-right gradually become straight orthogonals. Your eye, then, in the center of that sphere, is on a plane with every orthogonal and therefore sees the arc edge-on as a straight line.

Now the question remains: are the curves really arcs?

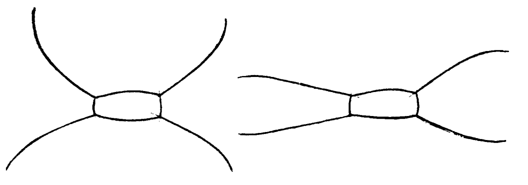
The answer, I believe, is: probably. Without moving the head, we of course cannot see accurately the entire area between opposite vanishing points. But my guess, based on the curves that I can see clearly in front of me, is that these curves do continue as arcs, and that convex mirrors and wide-angle lenses give us a fair picture of the circular nature of all straight lines as they would exist if we could see the entire 180° field.

However, do we *see* arcs? If in fact we must turn our head in order to see the extremities of each line, and if as the head turns the curving line gradually straightens, *is it not this graduation from curve to straight line that characterizes our experience?* I am not prepared to prove that we see mathematically defined hyperbolas, but I am convinced that every straight line appears to us *like* a hyperbola, a curving ligament connecting two straight orthogonal asymptotes. The task of reducing these hyperbolic lines to mathematically exact coordinates I must leave to others. Inasmuch as these irrational curves depend more on freehand skill than on compass and straightedge, quantification, if it is possible at all, may prove to serve no practical function beyond satisfying the scholar's urge to know. Variations in application resulting from subjective interpretations are not likely to exceed the variations observable when conventional straight line systems are used.

If a drawing, painting, or photograph is to reflect our dynamic visual experience, I propose that the hyperbolic system (figures 1 and 7) is more faithful and more effective than any other. It is as logically consistent as other systems and at the same time surpasses them in the empirical test—it looks like this curving world, whether we move through it panoramically, or view it at rest, in separate limited views.

Thus, even in a static view, all verticals are precisely perpendicular to the horizon only at the horizon, and immediately bend toward upper and lower vanishing points. They curve only as they approach and cross the horizon. Similarly, all horizontals parallel to the picture plane curve as they approach and pass beyond the line from zenith to nadir, which I call the zenith-line, and are perpendicular to the zenith-line only *at* the zenith-line. Except for areas near horizon and zenith-line, these gentle curves, moving to the visual periphery (beyond the frame of the picture) as we turn to follow them, lose all appreciable curvature and appear to lead *straight* to one of the four external vanishing points. The orthogonal lines, however, appearing perfectly straight as they emerge from the central internal vanishing point, may seem to curve only as they pass out of my field of vision over my ears or under my elbows, on their way to the sixth point behind me.

But I am here incapable of a decision: which way do the orthogonals appear to curve? For I must turn my head in order to see these lines pass near me. Now my decision must depend on favoring either a vertical movement of my head, making the

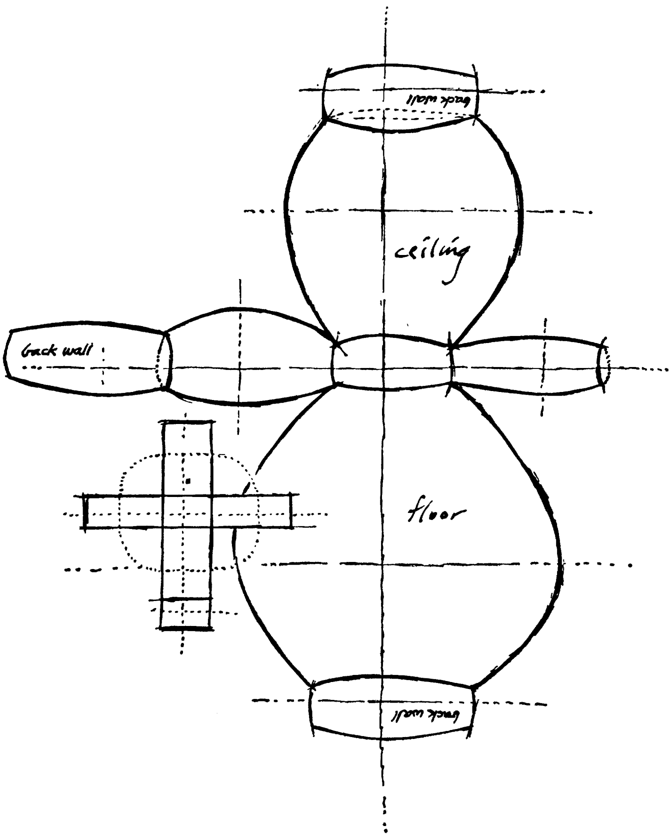


ceiling and floor convex and the walls concave; or a lateral movement which produces convex walls and concave ceiling and floor. I must remain neutral. The curvature of these orthogonal planes at any rate occurs outside the normal cone of vision.

It would appear sheer fantasy to attempt to symbolize this sixth vanishing point and the wall behind me in a two-dimensional drawing. Still, because this panorama is undeniably empirical, some graphic projection of this visual continuum may appear desirable. A globe enclosing my head is the only solution I can conceive.

The visible world that I carry with me can be metaphorically located at any moment on a concave spherical surface at all points equidistant from my eye, the picture plane implied by Leonardo. It is as if I wore at all times such a transparent sphere on my shoulders enclosing my head (actually two globes, one for each eye, requiring constant focusing), so that wherever I turned my eyes or my head, my world could be graphically plotted on it.

To transfer such a three-dimensional graphic diagram to a two-dimensional surface would require either an elastic paper to be wrapped first like a balloon over the globe, and then somehow stretched flat on a drawing board or canvas, a kind of Mercator's projection of any momentary view. Or else one or another kind of "equal-area" projection of the sort that cartographers



devise by cutting gores and flat patterns which simulate ovoid sections of the earth's surface, leaving gaps at the corners of a rectangular flat picture. It seems to me that such a procedure offers the artist a symbol closer to our visual perceptions than any other heretofore proposed. I believe that this hyperbolic system is both more logical and more faithful to raw sensory data than either conventional straight-line systems (whose most glaring lapse is the sanctity of all parallel verticals) or curvilinear systems employing arcs.

IV

Photography and twentieth-century painting have of course cast suspicion on the traditional attitude that the artist must be acquainted with his eyes' picture of objective shapes, proportion, and color. In surrealist painting, where an illusion of infinite physical distances might seem appropriate, Dali, Tanguy, and de Chirico manipulated academic formulas in creating their super-real fantasies, producing eloquent metaphors, but not real replicas in the same sense as Baroque landscape. Developing under the influence of the academy, even photography, which began to reveal "this curving world," has been carefully limited by lenses that maintain "true" rectilinearity. Of course a wide-angle, "fish-eye" lens is expected to produce this swollen effect, which is then considered an oddity and a distortion. Wide-angle views, and reflections seen in spoons or in convex mirrors (like that in Van Eyck's *Arnolfini Wedding Portrait*) do indeed distort our retinal image, but they depict only an exaggeration, a difference in degree, not in kind.

Most twentieth-century figurative painters have attempted no substantial replication of a three-dimensional world, paying more attention either to existential and intuitional visual patterns (notably figurative Diebenkorn) or to third-hand clichés (Pop and Common Object), most of them insistently flat. For a hundred years artists have shown little interest in imitating deep space, apparent exceptions like Hopper and Sheeler notwithstanding. The current Photo-realist movement shows signs of reviving artists' portrayal of "natural" vision, but the majority of its practitioners preserve the flatness of the photographic surface, a subtle but important distinction when the original subject matter implies space, modeling, and foreshortening. The subject matter of many examples of the new realism is really the photographic print, or the printed page that reproduces the photograph, and not the three-dimensional subject that the camera first recorded. Linear perspective in most cases must follow the parallax-corrected rectilinear pattern provided by the normal (approximately 55 mm.) lens.

Moreover, the visible world seems to many of us less objective than it used to be—less sharable—and more directly derived from each observer's retina—more private. Anyway, each artist's unique psychic and social associations make every realist painting an interpretation, not a copy. Only a selection of available small-scale sense data are noticed, and then translated into the language of the medium: brush strokes, glazes, spray-dots, inevitably conferring a fresh and personal meaning on the physical characteristics of the thing described.[12] For a sensing of the real third dimension, we must nowadays participate. While Op seldom referred to the "objective world," it often *required* us to perceive volume on a flat surface. Films, dance, happenings, light shows, sound sculpture, conceptual art, and political art gesture all seem to obviate the artist's need to examine visual space.

We continue to discover aspects that enlarge our perceptions or contradict our assumptions. The macroscopic and microscopic have had their analogies in nonobjective painting. The assumption that reality can be measured only when it or the observer is static has been cast in doubt by kinetic sculpture and by film. Motion pictures have increasingly focused our attention on the fleeting, the images given us when the camera pans quickly. We have begun to accept as real these vague, blurred patterns, which were also forcefully presented by certain action paintings, and less directly by Futurism.

Outside of Photo-realism and Op, there seems no good reason for us to consider at this time in art history any amendments to the laws of linear perspective. Who cares? What does it profit the artist or student of art if he becomes aware of a curving world, when his friends, patrons, and critics live in a perfectly adequate rectilinear one? Even filmmakers, photographers, illustrators, and painters of camera images may find a theory of appearance amusing but irrelevant to their craft. However, beyond verisimilitude, it is the subjective personal bias and the implied respect for primary information afforded us by our visual sensations that may be of particularly contemporary relevance. Moreover, the sense of a voluptuous experience conveyed by these swinging irrational curves seems to me to better express the vital kinetic world we are coming to acknowledge than do rectilinear and arc diagrams. In spite of the limitations of an admittedly metaphoric convention, the hyperbolic system may be considered "true" in more than one sense of the word.

The five-point hyperbolic system here proposed is then offered as a contribution to the science of perception, and only incidentally as a practical formula which artists might employ. It is presented first to convince skeptics that curvilinear perspective is natural and consistent with experience, and further that hyperbolic curves, not arcs of circles, most accurately represent the appearance of architecture in perspective.

For my part, if "often" implies exceptions to the general rule, I would maintain that the optical information that remains open to analytic introspection near the center of vision, even as we move, is sufficient to merit the effort to indentify and characterize those visual data. I do not believe that Gibson's writings justify the extreme judgment that "sensations are not entailed in perception at all," even when he demonstrates that many additional factors must be considered in order to understand our complex perceptual systems.

[1] E. H. Gombrich, *Art and Illusion* (London, New York, 1959), pp. 250–60.

[2] The same unthinking rejection occurs in William G. Lycan, "Gombrich, Wittgenstein, and The Duck-Rabbit" *JAAC* 30 (Winter 1971): 235.

[3] Psychologist James Gibson makes a useful distinction between the objective "visual world" and the subjective "visual field" and argues that other factors, such as the decision to attend to this or that aspect, are more basic to perception than are subjective sensory data. Several of his observations bear upon Gombrich's contentions.

From Gibson's *The Perception of the Visual World* (Boston, 1950):

> If human beings had a visual field whose width included the entire horizon—if they could see all the way around at the same time like a rabbit—the field during locomotion would appear to open up ahead and close in behind in a rather astonishing manner. Such characteristics of the visual field created a great deal of difficulty for the early students of perspective and for painters who wished to represent a large sector of the visual world on a picture-plane.
>
> Actually, of course, no rabbits and relatively few men have ever adopted the peculiar attitude of psychologists, artists, and geometers which enables them to see their visual field. They are, with good reason, perfectly content with the visual world as it is normally perceived, conforming to the rules of Euclidean geometry (p. 122).
>
> Ordinary visual perception is not delimited by an oval-shaped boundary, nor does it have a clear center and a vague fringe. These are the characteristics of that unusual kind of visual experience, the visual field, which we get when we fixate a point and take note of the experience, concentrating on how it feels to see.... Can we find an explanation in the facts of ocular movement for the absence of the above characteristics in the visual world—its lack of boundaries, its more nearly uniform clarity and its possession of what we might call a panoramic character?... Unquestionably the panoramic visual world depends on a temporal series of excitations and just as unquestionably the succession of the excitations is not represented in the final experience (pp. 155, 157).
>
> The visual world, it will be remembered, differs from the visual field in a number of ways. First, it has depth or distance, and it includes the experience of solid objects which lie behind one another. Second, it is Euclidean in the sense that neither the objects nor the spaces between them appear to change their dimensions in perception when the observer moves about. This is a general way of saying that they tend to remain constant. Third, it is stable and upright; things as seen have constant directions-from-here when the observer moves his eyes and the perceived ground remains horizontal when the observer tilts his head. Fourth, it is unbounded; our experience of the world does not have any visible margins or limits such as the visual field or a picture has. Finally, it has a characteristic to which we have scarcely referred but which, in a way, is the most important of all: it is composed of phenomenal things which have meaning (p. 164).

And from Gibson's *The Senses Considered as Perceptual Systems* (Boston, 1966); italics mine:

> A man, if he tries, can *almost* see the world as it would project on a glass plate in front of his face—the inverse of his retinal projection, or a so-called retinal "image." He can never quite do so, for there is always some compromise with natural perception. If it were easy to detect pure sensations, we could all be representational painters without training.... I would now maintain that the optical (not retinal) gradients and the other invariants that carry the information for perception are *often* not open to analytic introspection, and that perception is therefore, in principle, not reducible to sensations.... The problem of depth perception considered as the conversion of two-dimensional experience into three-dimensional experience seems to me quite insoluble. In this book, and implicitly in my earlier book (Gibson, 1950), the problem disappears. If sensations are not entailed in perception at all, why speculate about how they might be changed into perceptions? (pp. 237, 238)

[4] John White, *The Birth and Rebirth of Pictorial Space* (London, 1957), esp. chapt. 8, 13, 14, 15.

[5] For another interpretation, see Decio Giosefi, "Perspective," *Encyclopedia of World Art* (New York 1966), p. 204.

[6] The four examples cited are illustrated in White.

[7] Carlo Pedretti, in "Leonardo on Curvilinear Perspective," *Bibliotheque d'Humanisme et Renaissance, XXV* (Geneva, 1963), pp. 69–87, argues that Leonardo alluded only to the lateral foreshortening of the panel picture itself and never advocated depicting such foreshortening.

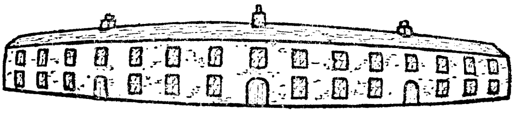
[8] Goodyear, *Greek Refinements* (New Haven, 1912), pp. 55–57, 75–76.

[9] As Patrick Heelan has recently pointed out in "Toward a New Analysis of the Pictorial Space of Vincent Van Gogh," *Art Bulletin* (Dec., 1972), Van Gogh appears to have used segments of hyperbolic curves in a few paintings, although inconsistently and unsystematically even within a given painting. Heelan is mistaken, I believe, when he implies, following Rudolf Luneburg, that hyperbolic curves especially characterize binocular vision and not monocular vision, which he says is rectangular. I believe that he may also be misapplying Luneburg's metric when he segregates zones of convex and concave curvature in natural vision.

[10] A rare example of applied curvilinear perspective is the drawing accompanying the following ex-

cerpt from Jan Gordon, *Modern French Painting* (New York, 1926):

> ... We are, of course, well aware that horizontal lines going away from us run into perspective, and we have accepted this fact of vision as pictorial possibility, although the Chinese have not. We must at the same time be aware that vertical lines are also subject to the same laws, and this the photograph shows clearly. [He refers, I take it, to views with the camera pointed up or down.] But this law of nature we are disinclined to adopt as an artistic fact. We are aware also that all lines sufficiently long are subjected to this perspective effect, there being in fact only two lines in nature which are visibly straight, the lines passing vertically and horizontally through the centre of vision. Thus, if we are facing a long building we get the following effect, the long lines curving in obedience to the laws of perspective:—



> Since these facts of perspective are ignored in art, and since our eyes are not trained to look for them, we ignore them in nature, and, indeed, in spite of their existence, they are difficult to perceive. It is difficult to realize that when we are sitting in a room the walls visually slope inward, or that when we face a piece of architecture such as Hampton Court it is usually barrel shaped in outline. The steps of St. Paul's are built with a slight horizontal curve to defeat this phenomenon.
>
> If we claim that the difference between the camera and our eye is that the camera has a fixed centre of vision, whereas we swing our vertical centre of sight as we turn round, we do not make nature any the more stable by our attempt to escape from error. As we swing about, each vertical line as we look at it becomes upright, the other lines going off into perspective, and so in fact all of them swing about in the most confusing way as we move our eyes—the Greeks, with the good taste that inspired their architecture, avoided all vertical straight lines in their best work (pp. 14, 15).

[11] Orthogonals appear straight, for instance, in White's 1957 book. But in an earlier article by him, a similar diagram indicated the orthogonals as curves. The question, which way must they curve? was avoided by the 1957 version. "Developments in Renaissance Perspective," *Journal of the Warburg and Courtauld Institutes,* no. 12 (London 1949), 59.

[12] In the light of Marcus Hester's "Are Paintings and Photographs Inherently Interpretative?" *JAAC* 32 (Winter 1972): 235.

Perhaps I am using a meaning of *interpretation* more normal in scholarship than in everyday usage, as in *the interpretation of data.* Certainly a scientist or scholar does not limit his critique of meaning to "persons and actions" but in addition examines abstract facts and patterns large and small. It is in this sense, I believe, that Goodman and others apply the word to aesthetics.

Relativity of vision is most forcefully demonstrated by simply closing each eye alternately, especially if one eye is in sun and the other in shade. When my two eyes disagree, I am forced to *interpret* the objective color and perspective of the scene before me as something close to the average of the two images, in order to depict it in two dimensions.

The new realism appears to avoid interpretation of data. But anyone who compares the actual paintings will recognize the distinct differences in style (texture, color, etc.) between such skillful Photorealists as McLain, Goings, Estes, and Eddy, for instance. The magazine reproductions of the paintings, on the other hand, three or four generations removed from the original subject, are extremely misleading in this respect. (See the special issue of *Art in America,* Nov.–Dec. 1972.)