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STUDIES IN RADAR CROSS SECTIONS XXXII - ON THE THEORY OF THE DIFFRACTION OF A PLANE WAVE BY A LARGE PERFECTLY CONDUCTING CIRCULAR CYLINDER

by

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PREFACE

This is the thirty-second in a series of reports growing out of the study of radar cross sections at The Radiation Laboratory of The University of Michigan. Titles of the reports already published or presently in process of publication are listed on the preceding pages.

When the study was first begun, the primary aim was to show that radar cross sections can be determined theoretically, the results being in good agreement with experiment. It is believed that by and large this aim has been achieved.

In continuing this study, the objective is to determine means for computing the radar cross section of objects in a variety of different environments. This has led to an extension of the investigation to include not only the standard boundary-value problems, but also such topics as the emission and propagation of electromagnetic and acoustic waves, and phenomena connected with ionized media.

Associated with the theoretical work is an experimental program which embraces (a) measurement of antennas and radar scatterers in order to verify data determined theoretically; (b) investigation of antenna behavior and cross section problems not amenable to theoretical solution; (c) problems associated with the design and development of microwave absorbers; and (d) low and high density ionization phenomena.

K. M. Siegel

SUMMARY

The problem is solved by a method somewhat different from those previously given by other authors. No new results are obtained, but observations are made on particular aspects not heretofore considered. The fundamental idea is explained and carried out by means of a Fourier integral representation. Useful integral expressions for the radiation part of the scattered field, and for the total scattering cross-section, are derived rather easily by accepted, though not entirely rigorous, Fourier techniques. Three devices are then proposed for overcoming the convergence difficulties which arise in the derivation of useful integral expressions for the field at a finite distance. Similarities and distinctions between the present method and those of other authors are detailed.

INTRODUCTION

So many papers have been written on the theory of the diffraction of monochromatic waves by circular cylinders and spheres in the difficult case when the radii of the obstacles are large compared to the wavelength that some justification should perhaps be offered for the appearance of another. Any justification must be implicit in the genealogy of the theory, and one line of descent, which also serves usefully as an introduction, is now described. Only the briefest possible account is given, and only a few key references are mentioned. A more detailed discussion of previous work of particular relevance is conveniently reserved for the end of the paper (§ 7), when comparison can be made with the present method.

At the beginning of this century the success of radio propagation over the earth stimulated great activity into the theory of diffraction by a large smooth sphere. The culmination of this effort was a famous paper by Watson (1918). Watson's work, however, applied explicitly only to the field well within the shadow region. Early attempts to cater also for the illuminated region (Bromwich (1920), White (1922)) were not entirely successful, but success was achieved by Van der Pol and Bremmer (1937) and by Fock (1945). More recently still, Imai (1954) and especially Franz and collaborators (Franz (1954), Franz and Galle (1955), Franz and Beckmann (1956, 1957)) have reconsidered the theory for the circular cylinder and sphere, and have persuaded it to yield a remarkable form of solution useful for calculating the field at any point in space.

The general mathematical technique in the papers just mentioned, including those of Imai and Franz, follows very closely that adopted by Watson (1918), in the sense that it starts from the classical series solution and rewrites this as a contour integral which is then put into a useful form by appropriate distortion of the contour of integration. The innovation consists essentially in treating the illuminated region by splitting the integrand into two parts, for one of which the contour of integration is closed around a set of poles to yield exponentially decaying terms analogous to those found by Watson, whilst for the other the contour is taken through a saddle-point and evaluated asymptotically by the method of steepest descents to give a term corresponding to the directly reflected ray of geometrical optics.

It is hardly to be expected that the form of the solution for the circular cylinder derived in the papers of Imai and Franz just referred to can be substantially improved for general purposes. It is physically illuminating, and numerically tractable even for quite modest values of the radius. However, the analysis is sufficiently complicated, and the technique sufficiently in the nature of a dodge, notwithstanding its long history, for the exploration of alternative approaches, and a closer examination of certain aspects of the problem, to be worthwhile. Such is the aim, on a limited front, of the papers by Jones (1956) and Wu (1956); and likewise of the present paper, in which a new method is given, having, perhaps, certain advantages.

In some respects the method is admittedly close to that described by Friedlander (1954) in a paper chiefly devoted to pulse diffraction by a circular cylinder, but it is felt that the reader will find here no unnecessary duplication of Friedlander's work. Attention ought also to be drawn to the fact that half a century ago Debye* (1908) gave a not dissimilar mathematical formulation; his development, however, took a different direction, and in any case was not pursued very far.

At this stage the clearest procedure seems to be, first to describe the new method as a self-contained theory, referring to previous work only to avoid duplication of specific calculations; and subsequently, with an explicit statement available, to compare it with other theories.

The incident field is taken to be a plane wave, and for the most part the discussion is presented in terms of the two-dimensional E-polarization problem, in which the electric vector is parallel to the axis of the cylinder. In § 2, the fundamental idea is stated and expressed mathematically by means of Fourier integral analysis. In § 3, the radiation field, including that in the forward direction which gives the total scattering cross-section, is considered; it is treated with some ease by accepted, if not entirely rigorous Fourier techniques. In § 4, attention is turned to the field at any finite distance from the axis of the cylinder; for this case there is some difficulty in securing the convergence of the integral representations, and three alternative devices for achieving

*I am indebted to Professor S. Silver for telling me of this reference.

convergence are proposed. In § 5, the behaviour of the current on the cylinder is related to that of the field in space. In § 6, the closely analogous case of H-polarization, in which the magnetic vector is parallel to the axis of the cylinder, is briefly discussed. Finally, in § 7, the present method is compared with that of other authors.

2

THE GENERAL NATURE OF THE SOLUTION

The problem discussed is that of the plane wave specified by*

$$E_z^i = e^{-ikr\cos\theta} \tag{1}$$

falling on the perfectly conducting cylinder $r = a$, where (r, θ, z) are cylindrical polar coordinates (see figure 1). The convention adopted is that the physical space is embraced by the range of values of θ between $-\pi$ and π.

Now the field is, of course, periodic in θ. If, however, it is made up by the superposition of fundamental solutions of the wave equation which are individually periodic in θ, the result is the classical series which is well known to become intractable as <u>a</u> increases much beyond a wavelength. The possibility must therefore be examined of expressing the solution in terms of functions which are not themselves periodic in θ.

*The suppressed time factor is $\exp(i\omega t)$

Moreover, it is evidently desirable that such non-periodic functions should have the following properties: they should individually be evaluable without undue difficulty; and the combination of only a few of them should approximate closely to the actual field in the physical space $-\pi < \theta \leqslant \pi$.

How may the suggested aim be achieved?

In the first place, it is easy to find non-periodic functions of θ which in combination give the actual periodic field. For, by Fourier integral analysis, the function

$$\int_{-\infty}^{\infty} p\,(\theta_1, \theta_2;\, \nu)\, e^{i\theta\nu}\, d\nu, \tag{2}$$

where

$$p(\theta_1, \theta_2;\, \nu) = \frac{1}{2\pi} \int_{\theta_1}^{\theta_2} e^{-ika\cos\phi}\, e^{-i\nu\phi}\, d\phi, \tag{3}$$

is equal to

$$\begin{cases} e^{-ika\cos\theta} & \text{for } \theta_1 < \theta < \theta_2, \\ \frac{1}{2}\, e^{-ika\cos\theta} & \text{for } \theta = \theta_1,\ \theta = \theta_2, \\ 0 & \text{for } \theta < \theta_1,\ \theta > \theta_2, \end{cases} \tag{4}$$

for arbitrary values of θ_1, θ_2 ($\theta_2 > \theta_1$). Likewise, the function

$$-\int_{-\infty}^{\infty} \frac{p\,(\theta_1, \theta_2;\, \nu)}{H_\nu^{(2)}(ka)}\, H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu \tag{5}$$

has the value minus (4) on $r = a$. It is thus an outgoing field which on the surface of the cylinder cancels the incident field for θ between θ_1 and θ_2, and is zero there for θ less than θ_1 or greater than θ_2. Clearly, then, an exact representation of the scattered electric vector component is given by the superposition of all functions (5) corresponding to non-overlapping ranges $\left[\theta_1, \theta_2\right]$ which together span the full range $-\infty$ to ∞.

The next step is to choose the ranges $\left[\theta_1, \theta_2\right]$ so that the non-periodic functions (5) have the two desirable properties previously mentioned. The choice depends somewhat on the position of the point at which the field is to be calculated, and is made precise in the particular cases considered in subsequent sections. These, however, can usefully be prefaced by one or two remarks which serve to indicate the trend of argument.

For $ka \gg 1$ it might be expected that the scattered field would be given to a good approximation by the expression (5) with the range $\left[\theta_1, \theta_2\right]$ roughly spanning that of physical space, namely $\left[-\pi, \pi\right]$; and that the further the range $\left[\theta_1, \theta_2\right]$ is from some such "primary" range, the smaller, at a given point in physical space, the expression (5) would become. The expectation is confirmed by the mathematics. For this shows that if the range $\left[\theta_1, \theta_2\right]$ is outside the primary range, then (5) essentially decays exponentially with increasing $|\theta|$; and moreover, from (3),

$$p\,(\theta_1 - 2n\pi,\ \theta_2 - 2n\pi;\ \nu) = e^{2i\pi n\nu}\ p\,(\theta_1,\ \theta_2;\ \nu), \quad (6)$$

for n = ± 1, ± 2, ± 3, ..., so that a decrease of $2n\pi$ in both θ_1 and θ_2 is equivalent in (5) to an increase in θ of $2n\pi$. Here is indicated the genesis of an interpretation in terms of rays traveling around the cylinder and being attenuated in the process.

3

THE RADIATION FIELD

3.1

THE CASE $\theta \neq 0$

In this section the radiation field, that is, the part of the scattered field of order $(kr)^{-\frac{1}{2}}$ for $kr \gg 1$, is considered for all directions other than that in which the incident wave is traveling.

For any fixed ν, as $kr \to \infty$,

$$H_\nu^{(2)}(kr) \sim \sqrt{\left(\frac{2}{\pi}\right)}\, e^{1/4\, i\pi}\, e^{\frac{1}{2} i\pi\nu}\, \frac{e^{-ikr}}{\sqrt{(kr)}}\,, \tag{7}$$

and (5) is consequently asymptotic to

$$-\sqrt{\left(\frac{2}{\pi}\right)}\, e^{1/4\, i\pi}\, \frac{e^{-ikr}}{\sqrt{(kr)}} \int_{-\infty}^{\infty} \frac{p\,(\theta_1,\, \theta_2;\, \nu)}{H_\nu^{(2)}(ka)}\, e^{i(\theta + \frac{1}{2}\pi)\nu}\, d\nu. \tag{8}$$

The last statement requires some justification, because ν runs to infinity in (5) and (8). This is easily provided. For since (5) converges, the integration limits may be taken to be finite, $\pm \nu_o$ say, without sensible error if ν_o is sufficiently large; the expression corresponding to (8) then likewise has limits $\pm \nu_o$ and is undoubtedly valid. But the integral (8) is even more rapidly convergent than (5); again without sensible error, therefore, the infinite limits may be restored.

Without loss of generality, save for the special case $\theta = 0$, it is assumed that $0 < \theta \leqslant \pi$.

Now, from (3), $p(\theta_1, \theta_2; \nu)$ is free of singularities throughout the entire finite part of the complex ν-plane, and its asymptotic behaviour as $|\nu| \to \infty$ is determined predominantly by the value of $\exp(-i\nu\phi)$ at the end points $\phi = \theta_1, \theta_2$. Hence, the only singularities in the complex ν-plane of the integrand in (8) are simple poles arising from the zeros of $H_\nu^{(2)}(ka)$; and, furthermore, the path of integration in (8) may be replaced by one enclosing the poles in the upper half-plane if $\theta_2 < -\pi/2$, and by one enclosing the poles in the lower half-plane if $\theta_1 > 3\pi/2$ (such paths, C_1 and C_2, are shown in figure 5).

For any range $\left[\theta_1, \theta_2\right]$ outside $\left[-\pi/2, 3\pi/2\right]$ the integral (8) can therefore be evaluated by the calculation of residues. Moreover, in view of the remark immediately following equation (6), the corresponding contribution to the scattered field is most simply expressed by taking successive ranges $\left[\theta_1, \theta_2\right]$ to be $\left[-\pi/2 \pm 2n\pi, 3\pi/2 \pm 2n\pi\right]$ for $n = 1, 2, 3, \ldots$

The main contribution to the scattered field is contained in the integral (8) with $\theta_1 = -\pi/2$, $\theta_2 = 3\pi/2$. In this case the appropriate method of evaluation is that of steepest descents. In order to apply this method, p $(-\pi/2, 3\pi/2; \nu)$ is expressed as the sum of terms whose individual behaviour can be represented, for the most part, by exponential functions with comparatively simple exponents. This is achieved by distorting the ϕ path of integration in (3) in the way indicated in figure 2, where the regions of convergence at infinity are shown shaded. The contribution to p $(-\pi/2, 3\pi/2; \nu)$ of the curved portions of the new ϕ path of integration is

$$\frac{1}{2} e^{-\frac{1}{2}i\pi\nu} \left[H_\nu^{(1)}(ka) + H_\nu^{(2)}(ka) \right] . \tag{9}$$

Furthermore, the contributions from the straight portions of the path are conveniently taken in conjunction with the functions p $(-\pi/2 \pm 2n\pi,$ $3\pi/2 \pm 2n\pi ; \nu)$, $n = 1, 2, 3, \ldots$. In effect, then, these functions can be replaced respectively by

$$\frac{1}{2} e^{\mp 2i\pi n\nu} e^{-\frac{1}{2}i\pi\nu} \left[H_\nu^{(1)}(ka) + H_\nu^{(2)}(ka) \right] . \tag{10}$$

The radiation part of the scattered field now appears in the form

$$E_z^s \sim \sqrt{\left(\frac{2}{\pi}\right)} e^{-\frac{1}{4}i\pi} P(\theta) \frac{e^{-ikr}}{\sqrt{(kr)}} , \tag{11}$$

where

$$P(\theta) = -\frac{i}{2}\int_{-\infty}^{\infty}\left[1+\frac{H_{\nu}^{(1)}(ka)}{H_{\nu}^{(2)}(ka)}\right]e^{i\theta\nu}d\nu$$

$$-\frac{i}{2}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\left[1+\frac{H_{\nu}^{(1)}(ka)}{H_{\nu}^{(2)}(ka)}\right](e^{-2i\pi n\nu}+e^{2i\pi n\nu})e^{i\theta\nu}d\nu. \tag{12}$$

It is evident that complete rigour has been sacrificed at this point, because the integrals in (12) are not strictly convergent. They are, however, of a type familiar in the application of Fourier analysis, and may be treated in the accepted manner without fear of error. The purely exponential parts of the integrands are therefore discarded, where, for the first integral, this involves the assumption $\theta \neq 0$. If also, for convenience, the sign of ν is reversed in half of the integrals in the sum, the result is

$$P(\theta) = -\frac{i}{2}\int_{-\infty}^{\infty}\frac{H_{\nu}^{(1)}(ka)}{H_{\nu}^{(2)}(ka)}e^{i\theta\nu}d\nu$$

$$-\frac{i}{2}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{H_{\nu}^{(1)}(ka)}{H_{\nu}^{(2)}(ka)}e^{-2i\pi n\nu}\left[e^{-i\theta\nu}\;e^{-i(2\pi-\theta)\nu}\right]d\nu. \tag{13}$$

The integral in the first term of (13) can be evaluated by the method of steepest descents. If the Debye asymptotic forms are used for the

Hankel functions, the saddle-point is seen to be located at ν=ka cos(θ/2), and the steepest descent path is somewhat as shown in figure 3, being asymptotic at infinity to the lines of zeros of $H_\nu^{(1)}(ka)$. The resulting contribution to P(θ) is

$$-\frac{\sqrt{\pi}}{2}\, e^{1/4\, i\pi} \sqrt{\left[ka \sin\left(\frac{1}{2}\,\theta\right)\right]}\; e^{2ika \sin\left(\frac{1}{2}\theta\right)} + 0\left[(ka)^{-\frac{1}{2}}\right], \qquad (14)$$

where some of the higher order terms are given explicitly by Imai (1954) and Franz and Galle (1955).

The integrals in the sum in (13) can be evaluated by closing the contour around the poles in the lower half-plane, these being at the zeros, $\nu = \nu_s$ say, s = 1, 2, 3, ..., of $H_\nu^{(2)}(ka)$; the new path of integration, C_2, is shown in figure 5. The resulting contribution to P(θ) is

$$-\pi \sum_{n=0}^{\infty} \sum_{s=1}^{\infty} \frac{H_{\nu_s}^{(1)}(ka)}{\left\{\frac{d}{d\nu}\left[H_\nu^{(2)}(ka)\right]\right\}_{\nu=\nu_s}}\; e^{-2i\pi n\, \nu_s}\left[e^{-i\theta\, \nu_s} + e^{-i(2\pi-\theta)\nu_s}\right]. \qquad (15)$$

The summation over n can, of course, be written in closed form; but the expression as it stands makes explicit the interpretation in terms of rays traveling around the cylinder. Detailed investigations of the nature of the terms in (15) have been given in several of the papers already referred to.

3.2

THE CASE $\theta = 0$

The analysis of the previous section is now supplemented by a consideration of the radiation field in the forward direction. This gives the total scattering cross-section, σ, through the relation

$$\sigma = -\frac{4}{k} \text{ Im } P(0). \tag{16}$$

The required modification of the analysis of § 3.1 is quite simple. In fact it is only necessary to preserve the obvious symmetry now present in the problem by taking the original primary range for $[\theta_1, \theta_2]$ as $[-3\pi/2, 3\pi/2]$ rather than $[-\pi/2, 3\pi/2]$. The ϕ path of integration in the integral (3) for $p(-3\pi/2, 3\pi/2; \nu)$ is then distorted as shown in figure 4, and the rest of the argument proceeds essentially as before, with the result

$$P(0) = -\frac{i}{2}\int_{-\infty}^{\infty}\left[1+\left(1+e^{2i\pi\nu}\right)\frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)}\right] d\nu$$

$$-\frac{i}{2}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\left\{\left[1+e^{2i\pi\nu}\frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)}\right]e^{2i\pi n\nu}+\left[1+\frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)}\right]e^{-2i\pi n\nu}\right\}dv. \tag{17}$$

By a procedure similar to that indicated in the remarks immediately following equation (12), the contribution to P(0) of the sum in (17) may be written

$$- i \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)} e^{-2i\pi n\nu} \, d\nu \; ; \tag{18}$$

and by closing the path of integration around the poles $\nu = \nu_s$, $s = 1, 2, 3, \ldots$, in the lower half-plane, (18) in turn is seen to be

$$- 2\pi \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} \frac{H_{\nu_s}^{(1)}(ka)}{\left\{ \frac{d}{d\nu} \left[H_\nu^{(2)}(ka) \right] \right\}_{\nu=\nu_s}} e^{-2i\pi n\nu_s}, \tag{19}$$

a result given by Franz and Beckmann (1957).

The first integral in (17) can be made strictly convergent by taking the path to infinity along radial directions falling within the first and third quadrants of the complex ν-plane. It is then in precisely the form derived by Franz and Beckmann (1957). In view of the asymptotic behaviour of $H_\nu^{(1)}(ka)/H_\nu^{(2)}(ka)$ as $|\nu| \to \infty$, the corresponding contribution to P(0) is appropriately written

$$-ika - i\int_{ka}^{\infty} \left[1 + \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)} \right] d\nu - i\int_{ka}^{\infty'} \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)} \, d\nu \, , \tag{20}$$

where the path of integration in the first integral is along the real axis, and in the second integral goes to infinity ultimately along the line of zeros of $H_{\nu}^{(1)}(ka)$ in the lower half-plane. For both integrals the main contribution comes from the initial part of the path of integration, and they can be evaluated asymptotically by using approximations to the Hankel functions valid when order and argument are nearly equal. Some details and results of the calculation are given by Franz and Beckmann (1957) and Wu (1956).

4

THE FIELD AT AN ARBITRARY DISTANCE FROM THE CYLINDER

In this section the problem is considered for unrestricted values of kr. If kr is not large enough for (5) to be validly approximated by (8) the analysis is complicated by the fact that the retention of the factor $H_{\nu}^{(2)}(kr)$ in the various integrands makes the convergence of the integrals a somewhat delicate matter. For any given range $[\theta_1, \theta_2]$ the integral (5), as it stands, is comfortably convergent; but it would become wildly divergent if $p(\theta_1, \theta_2; \nu)$ were replaced by the function (9), so that an argument precisely parallel to that of § 3.1 cannot be sustained. The final results, nevertheless, are very similar to those of § 3.1; the task is to derive them with adequate rigour. Three alternative procedures by which this may be achieved are offered for consideration.

I. Equations (11) and (13) give an exact expression for the radiation part of the scattered field for $0 < \theta \leq \pi$. In (13) the path of integration for the first integral can be taken as in figure 3, and for the integrals in the sum can be taken around the poles in the lower half-plane to yield the residue series (15).

Now consider the expression obtained from (11), (13) and (15) by writing

$$\tfrac{1}{2}\, e^{-\frac{1}{2} i\pi\nu}\, H_\nu^{(2)}(kr), \tag{21}$$

with the appropriate value of ν, in place of

$$\frac{e^{\frac{1}{4} i\pi}}{\sqrt{(2\pi)}}\;\frac{e^{-ikr}}{\sqrt{(kr)}}, \tag{22}$$

(22) being the asymptotic approximation to (21) as $kr \to \infty$. The expression is

$$-\tfrac{1}{2}\int \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)}\, H_\nu^{(2)}(kr)\, e^{i(\theta - \frac{1}{2}\pi)\nu}\, d\nu$$

$$+ i\pi \sum_{n=0}^{\infty}\sum_{s=1}^{\infty} \frac{e^{-\frac{1}{2} i\pi\nu_s}\, H_{\nu_s}^{(1)}(ka)\, H_{\nu_s}^{(2)}(kr)}{\left\{\frac{d}{d\nu}\left[H_\nu^{(2)}(ka)\right]\right\}_{\nu=\nu_s}}\, e^{-2i\pi n\nu_s}\left[e^{-i\theta\nu_s} + e^{-i(2\pi-\theta)\nu_s}\right], \tag{23}$$

where the integral is convergent with the path of integration as in figure 3, provided that $\pi/2 < \theta < 3\pi/2$.

It is observed that (23) is an outgoing solution of the wave equation, free of singularities for all values of $r > a$, whose asymptotic form for $kr \to \infty$ is precisely that established in §3.1 for E_z^s, the z-component of the actual scattered electric field.

Hence, for $\pi/2 < \theta \leqslant \pi$, E_z^s is given by (23), with the path of integration as in figure 3. It should be noted that the convergence of the integral is a delicate matter, in that the regions of convergence at infinity in the complex ν-plane are quite restricted, and that for $0 < \theta < \pi/2$ there is no common path of convergence for the corresponding integrals in (13) and (23).

II. Another argument is put forward for obtaining the result just established.

Referring back to the initial formulation of the solution in terms of the functions p $(\theta_1, \theta_2; \nu)$, if θ is restricted to the range $\pi/2$ to π the primary range $[\theta_1, \theta_2]$ is $[0, 3\pi/2]$. For any range $[\theta_1, \theta_2]$ outside $[0, 3\pi/2]$ the path of integration in (5) can be one of the paths C_1, C_2 which respectively enclose the sets of poles $\nu = -\nu_s$, $\nu = \nu_s$, s = 1, 2, 3, ..., as indicated in figure 5. If to the integral taken along C_1 is added

$$- \int_{C_1} \frac{p_1(\nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu, \qquad (24)$$

where

$$p_1(\nu) = \frac{1}{2\pi} \int_0^{\pi/2 + i\infty} e^{-ika\cos\phi}\, e^{-i\nu\phi}\, d\phi, \qquad (25)$$

and to the integral taken along C_2 is added

$$- \int_{C_2} \frac{p_2(\nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu \,, \tag{26}$$

where

$$p_2(\nu) = \frac{1}{2\pi} \int_{3\pi/2}^{3\pi/2 - i\infty} e^{-ika\cos\phi}\, e^{-i\nu\phi}\, d\phi, \tag{27}$$

then as far as the residue evaluation is concerned the functions $p\,(\theta_1, \theta_2; \nu)$ may in effect be replaced by Hankel functions in the manner of § 3.1, and the double summation part of (23) is obtained.

The rest of the solution, however, now appears in the form

$$\int_{C_1} \frac{p_1(\nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu + \int_{C_2} \frac{p_2(\nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu$$

$$- \int_{-\infty}^{\infty} \frac{p\,(0, 3\pi/2; \nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr) e^{i\theta\nu}\, d\nu, \tag{28}$$

and the aim is to show that (28) is the same as the first term in (23). This seems not entirely straightforward. It is true that

$$p\,(0, 3\pi/2; \nu) - p_1(\nu) - p_2(\nu) = \tfrac{1}{2}\, e^{-\frac{1}{2}i\pi\nu}\, H_\nu^{(1)}(ka), \tag{29}$$

but the integrals in (28) have no common path of convergence; indeed, none of the paths of integration in (28) may legitimately be deformed into a path, like that in figure 3, along which the integral in (23) converges.

Without attempting to go into details, some such device as the following is suggested for completing the proof. First, each path of integration in (28) is taken to lie as closely as possible to the path of figure 3. Next, each integrand is multiplied by a factor $\exp(\gamma \nu^2)$, where γ is a complex number whose argument, different, if need be, for each integrand, is such that the paths of integration in the individual integrals may be distorted into that of figure 3. Finally, the identification of (28) with the first term in (23) is completed by allowing each $|\gamma|$ to tend to zero.

III. In methods I and II an expression for the field at an arbitrary distance kr is obtained only for the case $\pi/2 < \theta \leq \pi$; that is, in effect, for the half-space on the illuminated side of the plane through the axis of the cylinder perpendicular to the direction of propagation of the incident wave. The other half-space includes the shadow region, where the total field can be very small. For its consideration, therefore, it seems desirable to discuss the total field rather than the scattered field alone; and this device in fact proves effective in removing convergence difficulties, as emphasized by Franz and Beckmann (1956).

The appropriate representation of the incident field is obtained merely by writing r for a in (2), (3), and (4). If θ is allowed to be in the range $0 < \theta \leqslant \pi$, the primary range for $[\theta_1, \theta_2]$ is $[-\pi/2, 3\pi/2]$, and the corresponding contribution to the total field is

$$\int_{-\infty}^{\infty} \left\{ p_r(-\pi/2, 3\pi/2; \nu) - \frac{p(-\pi/2, 3\pi/2; \nu)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr) \right\} e^{i\theta\nu} \, d\nu, \quad (30)$$

where

$$p_r(-\pi/2, 3\pi/2; \nu) = \frac{1}{2\pi} \int_{-\pi/2}^{3\pi/2} e^{-ikr\cos\phi} e^{-i\nu\phi} \, d\phi. \quad (31)$$

The analysis can now proceed in a manner closely parallel to that of § 3.1. The ϕ path of integration in p $(-\pi/2, 3\pi/2; \nu)$ and $p_r(-\pi/2, 3\pi/2; \nu)$ is distorted as in figure 2, and the contributions to (30) associated respectively with the curved and straight portions of the ϕ path can again be treated separately without giving divergent ν integrals. For the former, $p(-\pi/2, 3\pi/2; \nu)$ in (30) is replaced by (9), and $p_r(-\pi/2, 3\pi/2; \nu)$ by (9) with r written for a; it is therefore

$$\frac{1}{2} \int_{-\infty}^{\infty} \left\{ H_\nu^{(1)}(kr) - \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)} H_\nu^{(2)}(kr) \right\} e^{i(\theta - \frac{1}{2}\pi)\nu} \, d\nu. \quad (32)$$

The latter is taken in conjunction with the contributions arising from ranges $\left[\theta_1, \theta_2\right]$ outside the primary range, and together they yield the double summation of (23).

If $\pi/2 < \theta \leqslant \pi$, the path of integration in (32) can be distorted into that of figure 3. Comparison with the first term of (23) then shows agreement provided

$$\frac{1}{2}\int H_\nu^{(1)}(kr)\, e^{i(\theta - \frac{1}{2}\pi)\nu}\, d\nu = e^{-ikr\cos\theta} . \qquad (33)$$

Equation (33) does, in fact, hold for the range of θ, $\pi/2$ to $3\pi/2$, for which the integral converges. This is proved by Franz and Beckmann (1956); it can also be established by Fourier analysis in conjunction with the sort of device explained in the final paragraph of method II.

If $0 < \theta < \pi/2$, the path of integration in (32) can be closed around the line of poles in the lower half of the complex ν-plane. Thus (32) may be written

$$-\frac{1}{2}\int \frac{H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)}\, H_\nu^{(2)}(kr)\, e^{-i(\frac{1}{2}\pi - \theta)\nu}\, d\nu , \qquad (34)$$

taken over C_2 (see figure 5) or an equivalent path.

For points of observation well within the shadow region a residue evaluation is appropriate, and the resulting terms appear as natural additions to the series in (23).

For points of observation sufficiently far from the shadow region, (34) can be evaluated by the method of steepest descents. But, as pointed out by Franz and Beckmann (1956), the steepest descents path now traverses two saddle-points; one near $\nu = ka \cos(\theta/2)$ associated with the reflected wave, and one near $\nu = kr \sin\theta$ associated with the incident wave.

5

THE CURRENT ON THE CYLINDER

The surface current density on the cylinder at the point (a, ϕ) has only a z-component which is given by*

$$J_z(\phi) = -\, i\, Y \frac{\partial E_z}{\partial (kr)} \qquad (r = a,\ \theta = \phi), \tag{35}$$

where E_z is the z-component of the electric vector of the total field.

With the help of the expression for the Wronskian of the two Hankel functions, the contribution of the part (32) of E_z to the right hand side of (35) is easily seen to be

$$\frac{2Y}{\pi ka} \int_{-\infty}^{\infty} \frac{e^{-i(\frac{1}{2}\pi - \phi)\nu}}{H_\nu^{(2)}(ka)}\, d\nu\ , \tag{36}$$

and the remaining contribution can be written in the like form

$$\frac{2Y}{\pi ka} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-i(\frac{1}{2}\pi - \phi + 2n\pi)\nu}}{H_\nu^{(2)}(ka)}\, d\nu\ . \tag{37}$$

---

*M.k.s. units; Y is the admittance of free space.

Added together, (36) and (37) give the true current $J_z(\phi)$ which generates the scattered field E_z^s. A current distribution made up by the superposition of (36) and a finite number of terms of (37) would be a good approximation to $J_z(\phi)$, and the field generated by it would be a good approximation to E_z^s. However, it is instructive to see that the approximations to E_z^s obtained in this way are closer to the original forms given in this paper, which involve the functions $p(\theta_1, \theta_2; \nu)$, than to the final forms, such as (32), which involve the related Hankel functions.

Attention is confined, for simplicity, to the radiation field. That generated by the current distribution on the cylinder is given in terms of $P(\theta)$, defined in (11), where

$$P(\theta) = -\frac{ika}{4Y} \int_{-\pi}^{\pi} J_z(\phi)\, e^{ika\cos(\phi-\theta)}\, d\phi. \tag{38}$$

The current density $J_z(\phi)$ is, of course, periodic in ϕ. However, in view of the form of (36) and (37), it is conveniently represented as the superposition of two terms, each of which is given by (36), one with ϕ allowed to run from π to $-\infty$, the other with ϕ allowed to run from $-\pi$ to ∞. Consider, then, the contribution to $P(\theta)$ from a current density represented by (36) with ϕ running from π to $\pi - \phi_1$. It is

$$-\frac{i}{2\pi} \int_{\pi-\phi_1}^{\pi} \left\{ \int_{-\infty}^{\infty} \frac{e^{-i(\frac{1}{2}\pi-\phi)\nu}}{H_\nu^{(2)}(ka)}\, d\nu \right\} e^{ika\cos(\phi-\theta)}\, d\phi. \tag{39}$$

If the order of integration is reversed, and the new integration variable $\psi = \pi + \theta - \phi$ introduced in place of ϕ, (39) becomes

$$-i \int_{-\infty}^{\infty} \frac{p(\theta, \theta + \phi_1; \nu)}{H_{\nu}^{(2)}(ka)} e^{i(\theta + \frac{1}{2}\pi)\nu} d\nu . \qquad (40)$$

Similarly, the contribution to $P(\theta)$ from a current density represented by (36) with ϕ running from $-\pi$ to $-\pi + \phi_2$ is

$$-i \int_{-\infty}^{\infty} \frac{p(\theta - \phi_2, \theta; \nu)}{H_{\nu}^{(2)}(ka)} e^{i(\theta + \frac{1}{2}\pi)\nu} d\nu . \qquad (41)$$

The combination of (40) and (41) gives the approximation

$$P(\theta) = -i \int_{-\infty}^{\infty} \frac{p(\theta-\phi_2, \theta + \phi_1; \nu)}{H_{\nu}^{(2)}(ka)} e^{i(\theta + \frac{1}{2}\pi)\nu} d\nu . \qquad (42)$$

Evidently the primary range $\left[-\pi/2, 3\pi/2\right]$ associated with the function $p(\theta_1, \theta_2; \nu)$ in § 3.1 for $0 < \theta \leqslant \pi$ appears in (42) if $\phi_2 = \pi/2 + \theta$, $\phi_1 = 3\pi/2 - \theta$. The approximate forms of solution which are the point of departure for the present paper can therefore be recovered by integrating over the standard approximate forms of the current distribution, but the requisite range of integration depends, as might be expected, on the position of the point of observation.

6

THE CASE OF H-POLARIZATION

The foregoing analysis needs only slight modification for the case when the plane wave incident on the cylinder is specified by

$$H_z^i = e^{-ikr\cos\theta} . \qquad (43)$$

In fact, the contribution to H_z^s analogous to (5) is

$$-\int_{-\infty}^{\infty} \frac{q(\theta_1, \theta_2; \nu)}{H_\nu^{(2)\prime}(ka)} H_\nu^{(2)}(kr)\, e^{i\theta\nu}\, d\nu , \qquad (44)$$

where the dash denotes differentiation of the Hankel function with respect to its argument, and

$$q(\theta_1, \theta_2; \nu) = -\frac{i}{2\pi} \int_{\theta_1}^{\theta_2} \cos\phi\, e^{-ika\cos\phi}\, e^{-i\nu\phi}\, d\phi . \qquad (45)$$

For since $YE_\theta = i\,\partial H_z / \partial(kr)$, it is clear that on $r = a$ the part of E_θ^s corresponding to (44) cancels the incident field for $\theta_1 < \theta < \theta_2$, and is zero for $\theta < \theta_1$, $\theta > \theta_2$.

Furthermore, from (3) and (45),

$$q(\theta_1, \theta_2; \nu) = \frac{\partial}{\partial(ka)} \left[p(\theta_1, \theta_2; \nu)\right] . \qquad (46)$$

Evidently, then, the analysis for the case of E-polarization can be adapted to that of H-polarization merely by replacing $H_{\nu}^{(1)}(ka)$, $H_{\nu}^{(2)}(ka)$ throughout by $H_{\nu}^{(1)\prime}(ka)$, $H_{\nu}^{(2)\prime}(ka)$ respectively.

7

COMPARISON WITH OTHER METHODS

It is emphasized in the Introduction that the method of the present paper is only offered as an interesting alternative to several other ways of treating the problem. In this final section an attempt is made to indicate in what respects other discussions are similar to or different from that given here.

Because of its greater importance in physical applications the sphere has received a good deal more attention than the cylinder. However, the two problems have much in common, and in describing methodology in general terms there is no need to distinguish between them. It should, perhaps, be remarked that it is hoped to treat the sphere by a method analogous to that of the present paper with the help of an infinite Legendre integral transform.

The quantity of literature which could be regarded as relevant is enormous, so in what follows references are only given to those papers with which detailed comparison is made. Much more complete lists of references can be found in the reports by Sensiper (1953) and Weil, Barasch and Kaplan (1956).

Probably the most frequently adopted approach is that based essentially on the Watson technique, with a modification of the type described briefly in the Introduction. From a strictly mathematical point of view this cannot be adversely criticized. But, in contrast to the present method, it does seem to the writer to have the following disadvantages: a) it starts from an inappropriate form of solution; b) there is no obvious guide as to what precisely should be split off the integrand before the contour of integration is deformed around the $1/H_{\nu}^{(2)}(ka)$ (or corresponding) poles (this, in fact, accounts for some of the variations in the form of solution finally obtained); c) the relation of the transformed solution to the boundary conditions is obscured; d) the physical significance of the transformed solution is a matter of a posteriori interpretation.

Another not uncommonly adopted approach certainly aims at avoiding these disadvantages. In terms of the cylinder problem, with E-polarization, this consists in starting from the solution for the total field as a linear combination of all terms

$$H_{\nu_s}^{(2)}(kr)\ e^{i\nu_s\theta},$$

where ν_s, s = 1, 2, 3, ..., are zeros of $H_{\nu}^{(2)}(ka)$. Several discussions of the problem of radio propagation over the surface of the earth are on these lines, though approximations for the purpose of simplifying the mathematics and clarifying the physics are often introduced at the outset.

The method raises the question as to whether or not the ν_s form a complete set, and appears only comparatively recently to have been made rigorous in this respect. There is some similarity with the present treatment in that, in the course of the analysis, θ is thought of as a variable which takes all values from $-\infty$ to ∞. However, the form of solution adopted in this approach is still, of course, not applicable to all circumstances; there remains plenty of transforming to do (see, for example, Kear (1955) and Kodis (1957)), and results covering all positions of the point of observation do not seem to have been obtained in this way.*

A third approach aims explicitly at an approximate determination of the current density on the surface of the obstacle, with particular reference to the "penumbra" region. The advantage might be claimed that bodies of arbitrary shape can be treated, provided they are convex with large radii of curvature. But for the cylinder and the sphere more information is obtained more easily from an exact solution; and for bodies of less simple geometrical shape it is debatable whether the knowledge furnished by these exact solutions does not itself provide an equally adequate basis for an approximate treatment.

Two rather recent papers, mentioned in the Introduction, have points of similarity with the present one. The first, by Jones (1956), is limited to the derivation of the total scattering cross-section of a cylinder by an approximate treatment which gives the main correction to the geometrical optics formula. The idea is to concentrate on finding

* A general mathematical theory developed by Marcuvitz (1951) and Felsen (1957) which embraces both the methods just described should be noted.

a field which satisfies the boundary conditions at the surface of the body in the vicinity of the penumbra region. This is achieved by means of Fourier analysis, which is the feature common to Jones' and the present paper. Unfortunately, however, the resulting integrals are just as complicated as those arising in the exact treatment.

The second paper, by Wu (1956), has in common with the present paper the idea of building up the periodic solution by means of a superposition of non-periodic solutions. A recipe for doing this is given; but there is no explicit recognition that such a process is not unique, and in fact the resulting representation seems less useful than that given in §2.

Finally, as mentioned in the Introduction, a paper by Friedlander (1954) on diffraction of a pulse by a circular cylinder contains ideas close to those on which the present paper is based. In particular, an infinite Fourier transform with respect to θ is used. It is used, however, somewhat differently, being applied to the governing partial differential equation. This is thereby reduced to an ordinary differential equation with independent variable r (Bessel's equation, in fact), to which the appropriate physical solution must be found. There results the expression of the total field in the form (32) together with terms obtained by adding to θ all integral multiples of 2π.

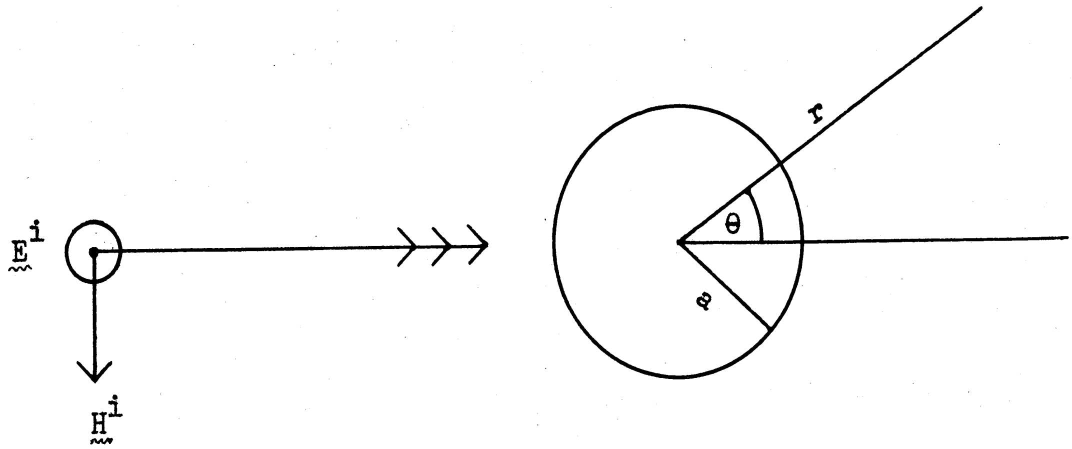

FIGURE 1. THE CONFIGURATION

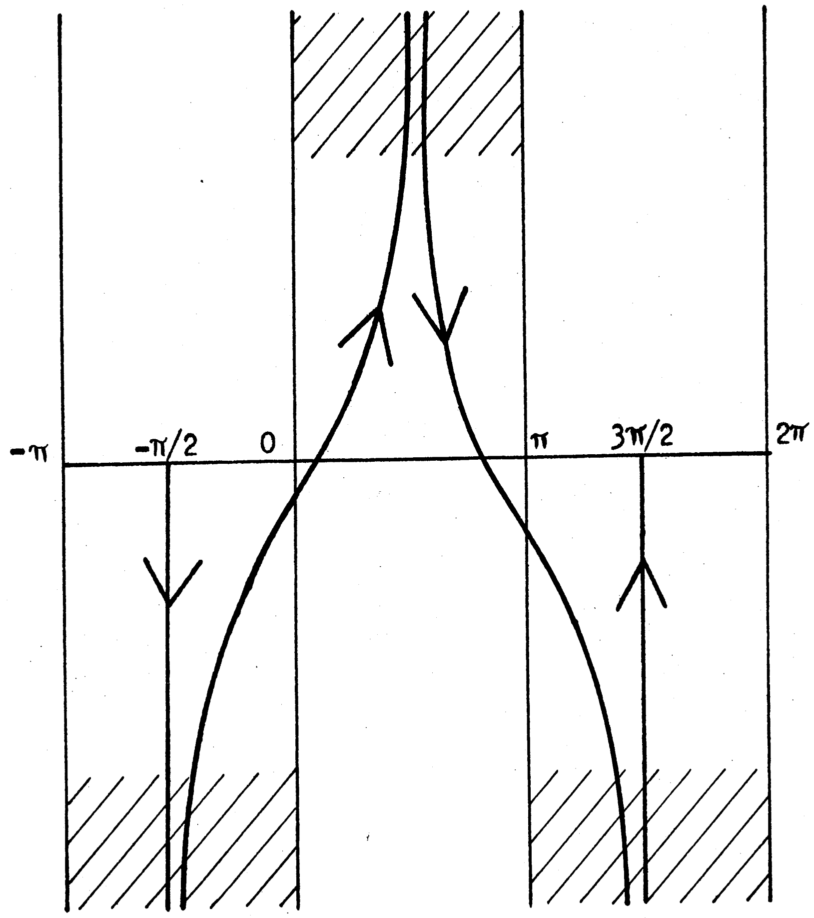

FIGURE 2. DISTORTION OF THE ϕ PATH OF INTEGRATION IN $p(-\pi/2, 3\pi/2;\nu)$ DEFINED BY (3)

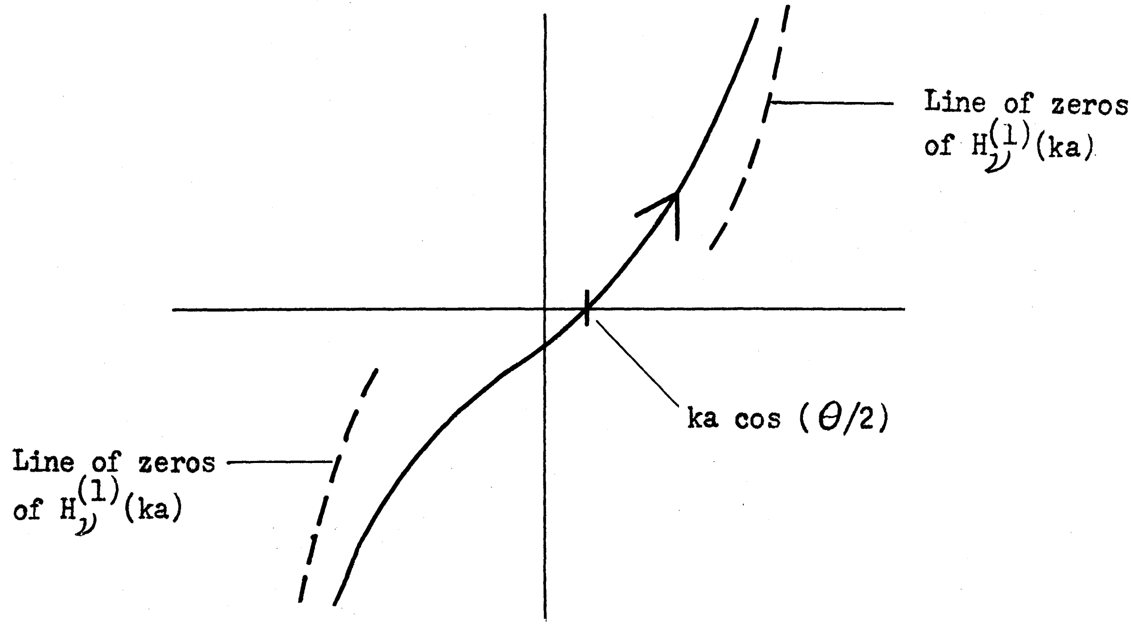

FIGURE 3. THE STEEPEST DESCENTS PATH IN THE COMPLEX ν-PLANE FOR THE FIRST INTEGRAL IN EQUATION (13)

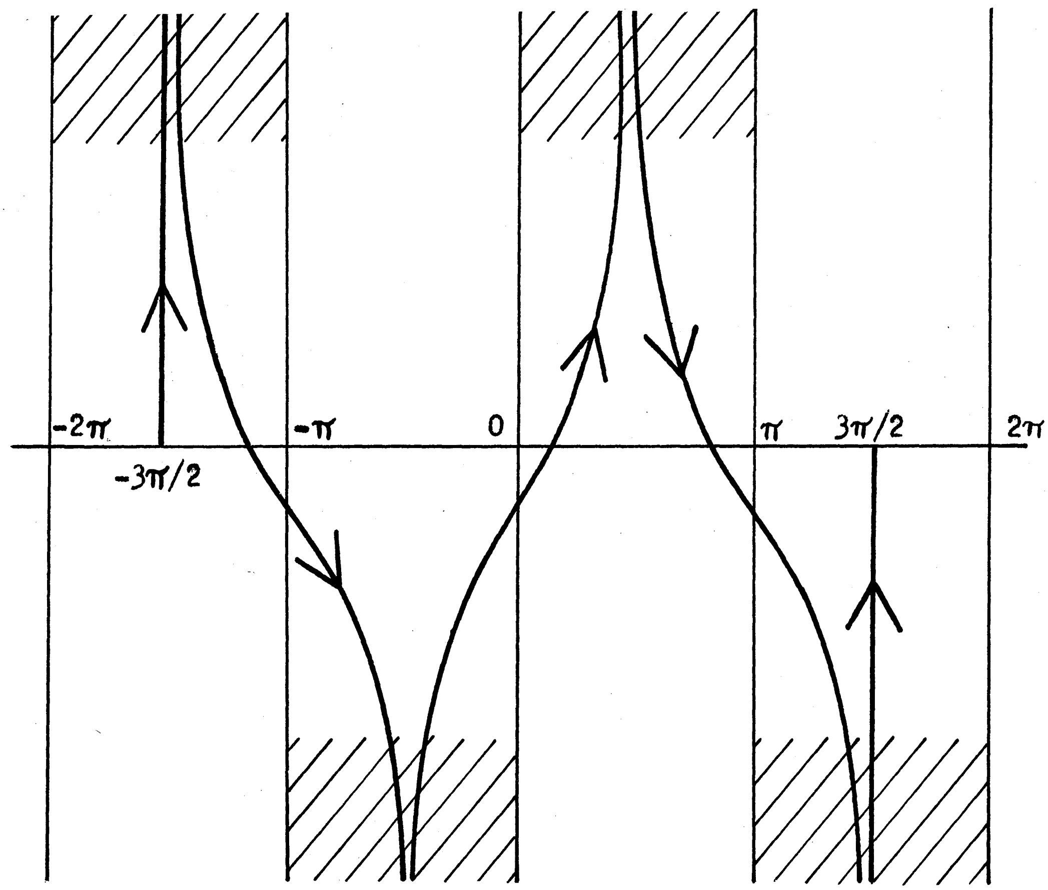

FIGURE 4. DISTORTION OF THE ϕ PATH OF INTEGRATION IN $p(-3\pi/2, 3\pi/2; \nu)$ DEFINED BY (3)

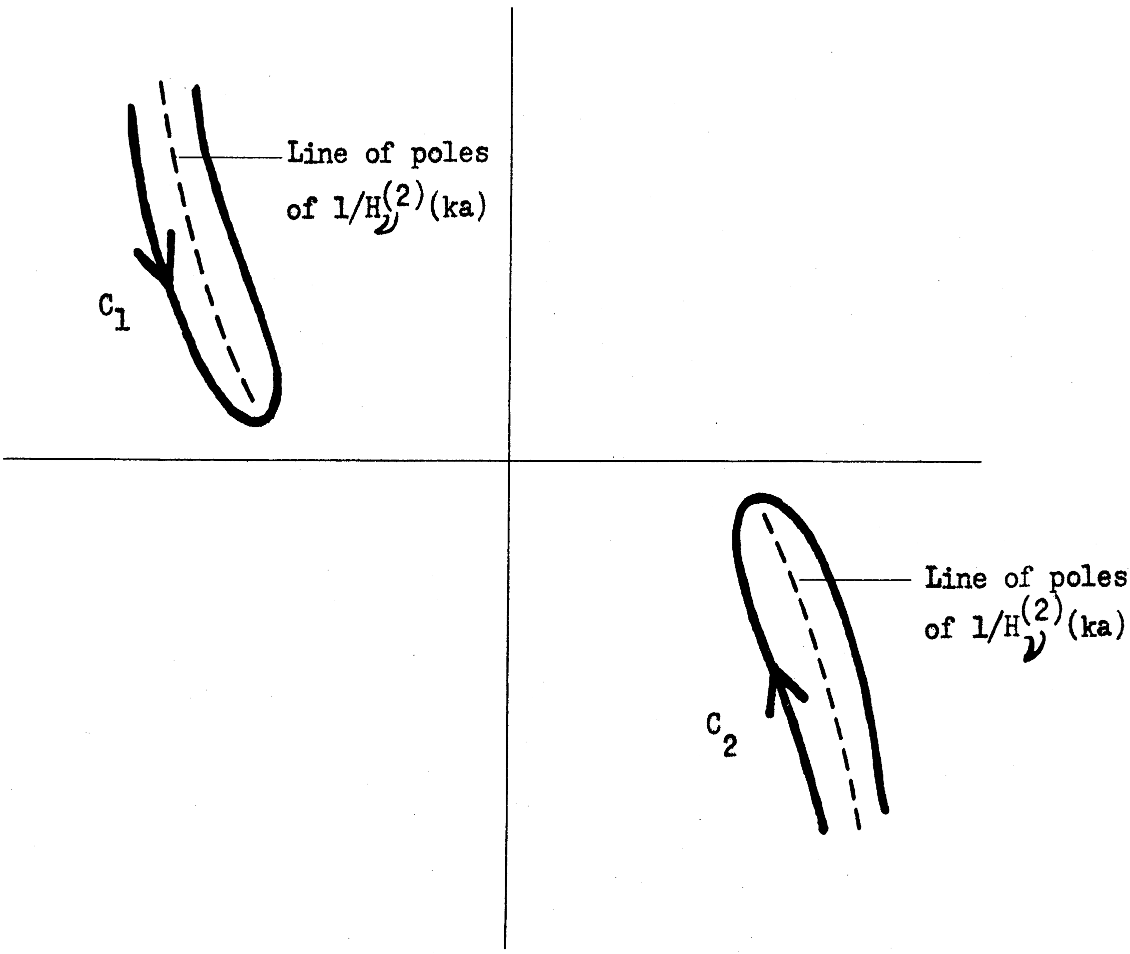

FIGURE 5. PATHS OF INTEGRATION C_1, C_2 ENCLOSING THE POLES OF $1/H_\nu^{(2)}(ka)$ IN THE COMPLEX ν-PLANE

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