Luneburg Theory of Visual Geodesics in Binocular Space Perception

An Experimental Investigation

PAUL C. SQUIRES, Ph.D., New London, Conn.

Introduction

Luneburg [14] concluded that the metric of the space of binocular perception may be formulated in terms of negative Riemannian curvature, the curvature being constant. That is to say, he maintained the position that binocular space is hyperbolic in the sense of the geometry of Bolyai and Lobachevsky.

Luneburg developed his equations principally upon the basis of the Blumenfeld [3] alleys. Blumenfeld discovered that in addition to the well-known phenomenally parallel alleys studied by Hillebrand [10] and Poppelreuter [16] there is another alley, called by Blumenfeld the equidistance alley. The latter alley consists in this fact: If two rows of point-like lights are set up in a darkroom so as to appear straight, and parallel to each other, the various pairs of lights do not appear to be the same transverse distance apart throughout the alley's extent.

If binocular space were Euclidean, the parallel and equidistance alleys would coincide both phenomenologically and geometrically. But Blumenfeld found that, for his observers, the parallel alley lay inside the equidistance alley; this fact, according to Luneburg, means that the space metric for these observers was hyperbolic (negative Riemannian). If a case were found where the parallel alley lay outside (geometrically) the equidistance alley, then the observer's binocular space would be elliptical in the sense of the Riemann-Cayley geometry (K being positive).

No previous study has ever been made of parallel and equidistance alleys viewed under photopic conditions. The chief purpose of our investigation has been to test for Riemannian K-sign under photopic conditions and to compare results with those derived under scotopic conditions. For it must be borne in mind that the mathematical conclusions made by Luneburg were primarily based on Blumenfeld's experiment, which was carried out in a darkroom.

Historical Background

Blumenfeld founded his alley experiment [3] upon the work of Hillebrand,[10] whose investigation is a classic. Hillebrand reawakened interest in the vista problem, which had been discussed by Bouger about the middle of the eighteenth century, by Porterfield in 1759, and by Priestly in 1772.* Hillebrand used vertical

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Research Psychologist, Medical Research Laboratory, U. S. Naval Submarine Base.

I have had valuable discussions concerning the mathematics involved with Dr. Hermann von Schelling, mathematician at the Medical Research Laboratory. But it is to be distinctly understood that Dr. von Schelling is not to be deemed responsible for any of the mathematical statements or conclusions set forth in this article.

* Reference 4, p. 309.

threads as stimulus objects. Poppelreuter[16] repeated Hillebrand's parallel alley experiment, using vertical rods. Poppelreuter confirmed Hillebrand in all substantial respects: The loci of the sides of the parallel alley lie somewhere between the constant angle subtended by the fixed pair of stimuli most distant from the observer and the physically parallel lines running through these fixed stimuli at right angles to the vertical plane in which the stimuli lie.

Just what is the real significance of Blumenfeld's experiment? First, he was the pioneer in eliminating the observer's visual frame of reference by using small gas jet flames adjusted to a "blue" level, of faint intensity, in a darkroom. Second, he discovered that the sides of a parallel alley do not appear to be the same distance apart throughout the alley's extent. That is to say, psychological parallelism does not mean equidistance apart; by equidistance we here mean phenomenal equidistance.

The phenomenal parallels of Blumenfeld lay much nearer to the locus of the visual angle subtended by the pair of standard (fixed) stimuli than did the parallels of Hillebrand or Poppelreuter; this would be expected, because secondary criteria were eliminated by darkroom conditions. As Boring remarks, "This is, in Katz's sense, a reduction experiment."†

Blumenfeld's equidistance (frequently referred to as "distance") alley does not have sides phenomenally parallel to each other; this alley is built under the instruction that the apparent transverse distance between a given pair of stimuli (tiny gas jets) must be made equal to the apparent distance between the standard stimuli.

The outstanding significance of Blumenfeld's research consists in the fact that it seems to demonstrate that apparent parallelism does not carry with it apparent equidistance between the parallels, despite the fact of Euclidean plane geometry that two parallel lines are everywhere the same distance apart. Blumenfeld himself regarded this difference between parallel and distance alleys as merely paradoxical; he saw in this difference no psychometrical import for binocular perception.

But Luneburg, a mathematician, who became interested in the distorted rooms devised at the Dartmouth Eye Institute under the direction of Ames, recognized the importance of the Blumenfeld data for the geometry of binocular depth perception.‡

Luneburg was careful to point out that the hyperbolic visual metric cannot be considered as a certain conclusion until precise quantitative tests have been made: "We still assume that the visual space is a space of constant curvature. But whether hyperbolic, Euclidean or elliptic is decided by the experimental result of K being negative, zero, or positive."§ He also said that it "should be possible to find new directives for the design of binocular instruments such as binocular microscopes or range finders. The final test of our theory then rests upon the consistency of such applications."||

When we undertake the construction of a distance alley, we are dealing with the problem of "size constancy," one of the more obscure questions in the history of psychology. Euclid was aware that apparent size does not follow the law of

† Reference 4, p. 296.
‡ References 14 and 15.
§ Reference 14, p. 104.
|| Reference 14, p. 103.

the visual angle.¶ A large literature has grown up around this subject.# Hering (1879) distinguished between three kinds of size: (*a*) visual angle, or retinal size; (*b*) phenomenal, that is to say apparent, size; this does not follow the law of the visual angle; (*c*) estimated size, as determined by factors over and beyond bare sense-perception. There is no trouble encountered in drawing a distinction between (*a*) and either (*b*) or (*c*); the difficulty consists in distinguishing both in theory and in practice between (*b*) and (*c*). Hering said *scheinbare Grösse* for (*a*), *Sehgrösse* for (*b*), and *geschätzte oder gedachte Grösse* for (*c*).*

Boring is of the opinion that Holaday, who worked with Brunswik in Vienna, "has produced the best general analysis of the conditions that underlie this "constancey phenomenon'."† Holaday concluded that the nearest approach to "size constancy" is made when distance clues are provided; conversely, that the falsification of the impression of distance leads to a reduced impression of size constancy. This conclusion goes right back to Berkeley's thesis (1709) that size perception depends on distance perception.

Hardy and co-workers ‡ carrying on the work of Luneburg, which was cut short by his untimely death, claim to have verified his hyperbolic theory. The Hardy experiments were conducted in a darkroom, using "points" of electric light for the stimulus configuration.

The human engineering aspect of the problem before us has to do with the possible relation of visual space curvature to various motor skills and capacities, wherein high degrees of precision are required of the operator in his visuomotor coordinations and adjustments. The Hardy study of visual goedesics was initiated under U. S. Navy auspices; it was believed that such a study might reveal relationship of the curvature (K) constants of a number of observers to their mechanical abilities. "The possibility that a high correlation of mechanical abilities (in drivers, pilots, steersmen, gun sighters, etc.) to the values of K could be established was a principal motivation in undertaking this study."§

Experimental Design

The main technical problem involved was to devise an equipment which would transform the Blumenfeld experiment into terms of daylight vision.

Equipment—The stimulus-pairs were Munsell N-1 paper squares, 5/16 in. on a side. These squares were glued to rectangular methacrylate (Plexiglas) strips $10\frac{1}{2} \times \frac{5}{16} \times \frac{1}{16}$ in.; each strip was clamped vertically into a weighted wooden block grooved to slide along a meter stick set transversely on a table. The transparent methacrylate used was colorless, with a transmission of 92% in the visible range.

Each black square was at the top of a methacrylate strip, the strip being adjustable so that it could be set at such height as to make the top horizontal edge of each square lie in a common horizontal plane. The blocks into which the strips were fastened could be set at any position on the meter stick by means of a thumbscrew.

Distance of furthest (fixed) stimulus-pair from eyes was 350 cm.; transverse

¶ Reference 4, p. 309.

Reference 4, pp. 308-311.

* Reference 4, p. 309.

† Reference 4, p. 298.

‡ References 7 and 8.

§ Reference 8, pp. 54-55.

distance between the fixed pair being 20 cm. The fixed pair were glued to the white cardboard background.

The stimulus squares were so set that a direct reading of the meter sticks gave a highly precise measure. We used seven stations, at the following distances from the observer's eyes as measured along the median line: 65.7, 85.6, 111.2, 145.4, 195.1, 266.1, and 350.0 cm. Station 1 was at 350 cm.

Observer's view of the table top was prevented by appropriate screening. An adjustable chin and head rest permitted obtaining the desired angle of vision with respect to the plans of the stimulus squares. Calibrations of the experimental set-up were frequently made. Illumination was furnished by an overhead L. C. Doane Co. fluorescent Luminaire, 9000-S6401, 73818, Rev. 4-Type, JF-31B; two 8-watt lamps, G. E. Daylight, furnished just the right illumination: Brightness of white background was about 10 fL.

Procedure.—Each observer completed the parallel alley task before undertaking the construction of the distance alley. Instructions for building the parallel alley follow:

"You see a pair of black squares in the white background.

"I ask you to instruct me to set other pairs of squares so that, when they are all set, you can confidently say that (1) the right and left hand arrays of squares *appear* to be parallel to each other and that (2) each array of squares *appears* to lie in a straight line.

"The two lines of squares must be made to appear as if they could never meet no matter how far they might be extended in front of or in back of you. Be careful to avoid even a trace of the appearance you see in railroad tracks which converge in the distance.

"Be sure to view the alley formed by you as a *whole.* If you find at any time that you become unable to grasp the configuration of squares as a *totality,* so inform me.

"You must use *both* eyes at all times when you are making the alley. Let your gaze wander freely over the region of the squares."

Instructions for constructing the equidistance alley were as follows:

"You see a pair of black squares in the white background.

"I will now show you a second pair of squares. You see that the two pairs of squares appear to be at different distances from you. You are to instruct me to set the pair of squares last shown you in such a way that the *apparent* distance between these squares is the same as the *apparent* distance between the fixed pair of squares first shown you; they are to be set symmetric to the median.

"Do not reason about the situation, do not try to estimate width. Simply *sense* the apparent widths as quickly as you can and make your judgment *while holding your depth-perception attitude.*"

Instructions were read by the observer and thoroughly discussed with the examiner. The greatest care was taken to insure that the observer carried out the instructions at all times.

Observers.—There were four observers, including the examiner. Three of the four observers were psychologists, the other observer being unversed in experimental psychology. The observers will be identified thus: P.C.S., C.C., G.L., T.S.

Darkroom Alleys.—T.S. was conducting his own experiments on the Blumenfeld alleys under darkroom conditions at the time of our experiment; his work was performed at the American Optical Company, Southbridge, Mass. Thus, T.S. was tested both under his and our illumination conditions; P.C.S. was likewise tested under both conditions.

Results

Tables 1, 2, 3, and 4 show that for three observers the parallel alleys lie outside the distance alleys; these observers operated strictly under the experimental instructions. For them, binocular space is Riemannian-positive, i.e., elliptical. There is not the slightest evidence indicating a Euclidean visual space.

Table 1.—*Observer P.C.S., White Light*

Station	P-Alley (Cm.)	D-Alley (Cm.)	D-Alley (Nonunitary) (Cm.)
1	10.0	10.0	10.0
2	9.0	8.2	9.3
3	8.2	6.3	8.5
4	7.5	5.6	7.9
5	7.1	4.6	7.8
6	7.2	4.8	7.3
7	6.8	5.1	8.2

Table 2.—*Observer C. C., White Light*

Station	P-Alley (Cm.)	D-Alley (Cm.)
1	10.0	10.0
2	8.8	8.3
3	7.8	6.8
4	7.0	6.3
5	6.5	5.8
6	6.8	6.0
7	6.6	5.5

Table 3.—*Observer C. L., White Light*

Station	P-Alley (Cm.)	D-Alley (Cm.)
1	10.0	10.0
2	9.6	9.0
3	9.4	8.2
4	8.9	7.3
5	8.7	6.7
6	8.8	7.6
7	8.0	7.8

Table 4.—*Observer T. S., White Light*

Station	P-Alley (Cm.)	D-Alley (Nonunitary) (Cm.)
1	10.0	10.0
2	8.7	10.8
3	7.1	9.5
4	5.8	8.4
5	4.9	8.8
6	4.7	9.0
7	4.3	9.4

The measures given in the Tables are for the right side of the alley only, the two sides being substantially symmetrical.

For T.S., the binocular metric in our white-light situation is prima facie hyperbolic, i.e., Riemannian negative; his parallel alley runs inside of his distance alley.

The dihedral angle formed by intersection of the stimulus plane and the plane running through the observer's pupils to Station 1 is approximately one degree. Let us designate any angle of intersection as θ.

Note that the non-unitary D-alleys (distance alleys) in Tables 1 and 4, lying outside of their associated P-alleys (parallel alleys), give presumptive evidence of a hyperbolic metric. "Nonunitary" here means that the two pairs of apparently equal tranverse distances were not apprehended by the observer as a unified depth-Gestalt: This fact is crucial to an evaluation of the Luneburg theory.

Tables 5, 6, and 7 show data derived from the darkroom alley experiments conducted at the laboratories of the American Optical Company; in these Tables, θ equals approximately 2.3 degrees.

TABLE 5.—*Observer P. C. S., Darkroom*

Station	P-Alley (Cm.)	D-Alley (Cm.)
1	10.0	10.0
2	9.4	9.2
3	8.7	7.2
4	8.2	5.4
5	7.5	4.8

TABLE 6.—*Observer T. S., Darkroom*

Station	P-Alley (Cm.)	D-Alley (Nonunitary) (Cm.)
1	10.0	10.0
2	8.8	10.5
3	7.6	10.2
4	7.0	9.6
5	5.7	13.3

TABLE 7.—*Observer T. S., Darkroom**

Station	P-Alley (Cm.)	D-Alley (Nonunitary) (Cm.)
1	10.0	10.0
2	11.1	11.3
3	11.2	11.4
4	10.2	12.3
5	8.1	12.5

* In this Table, $\theta=0$.

Comparison of Tables 6 and 7 reveals that raising of the eye-level above the plane of the stimuli is associated with a wider spread between the P-alley and D-alley, but the mapping is presumptively Riemannian-negative in both Tables.

Only five stations were available in the darkroom observations.

T.S. reported that he built his D-alleys by comparing apparent transverse distances under an *Einstellung* which regarded the two pairs of squares as lying in two different vertical planes which were not unified, integrated, as a depth-Gestalt. He seemed entirely unable to compare any two transverse distances while maintaining a depth attitude. That is to say, he admitted that he could not follow the explicit instructions on this point. The other observers had no difficulty in following the instructions on the matter of depth attitude. Thus, T.S. was avowedly nonunitary in his attitude when forming the D-alleys, and the question arises whether this attitude was basic in producing his so-called hyperbolic metric.

P.C.S. was able to obtain a hyperbolic metric by deliberately assuming the nonunitary attitude in formation of the D-alley (Table 1, last column); his nonunitary D-alley lies outside of his P-alley.

Comparing Tables 1 and 4 with Tables 5, 6, and 7, it is clear that the algebraic sign of the Riemannian K was not affected by illumination conditions. P.C.S. is Riemann-Cayley-elliptical under either white light or darkroom conditions; T.S. is Lobachevskian hyperbolic (prima facie, at any rate) under both conditions.

In our white-lighted alleys it is true that there are various empirical depth-perception clues. One might suppose that in the darkroom alleys, where empirical clues are absent, the depth experience would be markedly less than for the photopic alleys. But for P.C.S. the depth effect in the darkroom alleys at the American Optical Company laboratories was striking, far surpassing the depth effect he had expected to see. In fact, although the stimulus brightnesses of the light points were most carefully made apparently uniform by P.C.S., he could not readily perceive the pairs of light points except as strongly bound together in depth relation.

Discussion

No trace of zero K (Riemannian) having been found in our experiments, we are unable to accept as valid Fry's contention‖ that binocular space is Euclidean.

Hardy and co-workers state: "The most striking evidence that visual space is non-euclidean lies in the distinction in visual perception between apparently parallel straight lines and curves of apparent equidistance. This difference was first reported by Blumenfeld."¶

When the observer is instructed to construct a parallel alley, the sides to appear straight, he is building geodesics. A geodesic may be defined as (1) shortest line or (2) straightest line; definition (1) is that of metrical geometry. Definition (2) is usable both in geometry and in psychology. "Straightness" in a parallel alley is a phenomenological quality, closely akin to a bedrock sensory quality, and, mathematically, the "straightest" line "is not an absolute property of the surface, but depends on the way the surface is imbedded in space."# In a parallel alley, the "shortest" line would be a matter of inference; but the "straightest" line is a matter of direct observation.

In an article by Hardy and co-workers* the K-constants of 15 observers were reported; of these, 6 Ks are positive, indicating elliptical space, the other Ks being negative. However, in the final report issued by Hardy and co-workers the six cases of positive K are not even mentioned[8]; only two sets of alley loci are presented, both representing negative K.

Since only one of our four observers was hyperbolic (prima facie), the others being elliptical, the question at once presents itself as to whether the observer (T.S.) who presumptively manifested negative K rigorously followed out the instructions. The answer is that he did not. We know that the other three observers did scrupulously follow the instructions.

Close questioning of T.S. revealed that when he constructed his distance alley he compared the two transverse distances as separate configurations, not tied together in a depth-Gestalt. Thus, he viewed the two transverse distances in a

‖ References 5 and 6.

¶ Reference 8, p. 20.

Reference 9, p. 221.

* Reference 7, p. 10 of reprints.

nonunitary manner, never as integrated into an over-all whole. The instructions explicity require that the observer make his judgment of apparent transverse equality while holding the depth-perception attitude. T.S. formed his parallel alley correctly enough: He made his settings while regarding the situation as a whole. But he shifted his attitude to a purely analytic one when building the distance alley. An alley cannot be an alley unless it possesses a depth characteristic. Thus, it is plainly evident that T.S. built the two alleys under entirely different attitudes, namely, the totalizing and the analytic. His distance "alley" cannot properly be called an alley at all.

Neither Blumenfeld nor the Hardy group constructed their distance alleys under instructions to the observers to maintain the depth perception attitude or set. Luneburg had the key to the solution of the problem of binocular space metric in his hands, but he threw away the key by failing to recognize that the P-alley, constructed under the totalizing, Gestalt attitude, cannot validly be compared to the D-alley built under the partializing, nonunitary *Einstellung*. The D-alleys we find in the experimental literature are not phenomenal realities but psychological abstractions, because depth is of the very essence of any alley.

The P- and D-alleys of P.C.S., C.C., and G.L. were built upon the firm foundation of depth perception and upon the tenacious adherence to an over-all totalizing attitude; positive Riemannian K resulted. We must carefully note that when P.C.S. shifted his attitude to the nonunitary one while constructing a D-alley he manifested negative K (Table 1, last column). Now, which is the true metric for P.C.S., hyperbolic or elliptic?

The Luneburg hypothesis depends upon the tacit assumption that the P- and D-alleys are validly comparable. We maintain that they are of entirely different psychological constitutions, and hence noncomparable mathematically: The alleys belong to utterly different universes of psychological discourse. The D-alley is not a phenomenal alley: this is the crucial point overlooked by Luneburg. It is our conclusion that the non-Euclidean metric of T.S., as evidenced by his settings in both the daylight and the darkroom situations, cannot be correctly defined as that indicative of a hyperbolic geometry; rather, his metric may be characterized as quasihyperbolic or pseudohyperbolic.

We not only have established the nonhyperbolic nature of binocular space for our observers but we have also ascertained the fact that the sign of the Riemannian curvature does not change for a given observer under change of illumination conditions (darkroom or white light), for the values of θ used in this experiment. Some time after we had formulated our views on the Luneburg hypothesis, we came across some notes by Renshaw, who states: "But of course the question arises: how much does one artificialize the situation by elimination of normal field or peripheral seeing conditions? The point is debatable"†; he also says: "The question of the proper metric for binocular space remains open."‡

Luneburg, although he originated and strongly favored the hyperbolic hypothesis of binocular space, does not seem to have been completely dogmatic about the matter§; also, it must be borne in mind that he died before he could have the opportunity to consider the mathematical implications lying in the phenomenology of recent work. Deductive logic has played too large a part in Luneburg's hypothesis: He was a mathematician, not a psychologist. Luneburg's followers have

† Reference 17, p. 45.

‡ Reference 17, p. 45.

§ Reference 14, p. 104.

carried on the work he began with great zeal, with almost a fanatical fervor; perhaps they have manifested dogmatism more acutely than did Luneburg.

Visual space is a hypothetical space; it is not a physical space. Luneburg's task was to determine the mapping of physical space into visual space and thereby be in a position to characterize the visual space. With the assumption of a constant curvature, K, then Riemann's differential line element expresses the metric of any space. The metric of a space determines all the properties of a space. Spaces represented by the Riemann differential line element are the only spaces in which localization and form are not correlated.

Only by the assumption of point-sources of light for stimuli in a darkroom can we legitimately apply the technique of differentials to the determination of the mapping function. Where gross stimuli are used, as in our experiment, the use of differentials must be replaced by the theory and practice of finite differences.

Hilbert states: "An effective description of physical reality must be based not on ordinary Euclidean geometry, but on a more general Riemannian geometry."|| The same may be said of phenomenal reality.

The "size constancy" phenomenon is the basis upon which the distance alley is derived. This phenomenon is not yet clearly understood. According to Boring, Holaday,[11] who worked with Brunswik in Vienna, "has produced the best general analysis of the conditions that underlie this 'constancy phenomenon'."¶

Recent studies on the Luneburg hypothesis are those by Bank.# Of course Koffka's book[13] has become a modern classic on configurational principles; these principles must be understood by anyone investigating the Luneburg hypothesis. As Boring states the case: "It is to Gestalt psychology that the modern psychologist turns for his systematic framework when he deals with perception."*

Main Conclusions

This experimental investigation of both parallel and equidistance alleys is the first one to be made under white light conditions. The following conclusions have been drawn:

No evidence has been found indicating a parabolic (Euclidean) metric for binocular space.

Binocular space does not evidence the hyperbolic metric for the observers used in this experiment. This fact contradicts Luneburg's hypothesis that negative Riemannian constant curvature characterizes binocular space.

Three of our four observers evidence the metric of elliptic Riemannian space, the curvature being positive.

A fourth observer, who presumptively satisfied the Luneburg conditions for the hyperbolic space metric, should rather be designated as of quasihyperbolic or pseudohyperbolic type.

The algebraic sign of the non-Euclidean metric is not affected, so far as our data reveal, and under the values of θ used, by the differences between white lighted and darkroom conditions.

|| Reference 9, p. 172.
¶ Reference 4, p. 298; Reference 12.
# References 1 and 2.
* Reference 4, p. 299.

REFERENCES

1. Blank, A. A.: The Luneburg Theory of Binocular Visual Space, J. Optic. Soc. America 43:717-727, 1953.

2. Blank, A. A.: An Account of Recent Investigations Relating to the Luneburg Theory of Space Perception, Am. J. Optometry 30:374-379, 1953.

3. Blumenfeld, W.: Untersuchungen über die scheinbare Grösse im Sehraume, Ztschr. Psychol. u. Physiol. Sinnesorg. 65:241-404, 1913.

4. Boring, E. G.: Sensation and Perception in the History of Experimental Psychology, New York, Appleton-Century-Crofts, Inc., 1942.

5. Fry, G. A.: Visual Perception of Space, Am. J. Optometry 27:531-553, 1950.

6. Fry, G. A.: The Relation Between Perceived Size and Perceived Distance, Am. J. Optometry 30:73-77, 1953.

7. Hardy, L. H.; Rand, G., and Rittler, M. C.: Investigation of Visual Space: the Blumenfeld Alleys, A. M. A. Arch. Ophth. 45:53-63, 1951.

8. Hardy, L. H.; Rand, G.; Rittler, M. C., and Blank, A. A.: The Geometry of Binocular Space Perception, final report to the U. S. Office of Naval Research: Project NR143-638, Contract N6onr-27119, 1953.

9. Hilbert, D., and Cohn-Vossen, S.: Geometry and the Imagination, New York, Chelsea Publishing Company, 1952.

10. Hillebrand, F.: Theorie der scheinbaren Grössen bei binocularem Sehen, Denkschr. Akad. Wiss. Wien 72:255-307, 1902.

11. Holaday, B. E.: Die Grössenkonstanz der Sehdinge bei Variation der inneren und äusseren Wahrenhmungsbedingungen, Arch. ges. Psychol. 88:419-486, 1933.

12. Holway, A. H., and Boring, E. G.: Determinants of Apparent Visual Size with Distant Variant, Am. J. Psychol. 54:21-37, 1941.

13. Koffka, K.: Principles of Gestalt Psychology, New York, Harcourt, Brace and Company, Inc., 1935.

14. Luneburg, R. K.: Mathematical Analysis of Binocular Vision, Princeton, N. J., Princeton University Press, 1947.

15. Luneburg, R. K.: The Metric of Binocular Visual Space, J. Optic. Soc. America 40:627-642, 1950.

16. Poppelreuter, W.: Beiträge zur Kenntnis des Sehraumes auf Grund der Erfahrung, Arch. ges. Psychol. 20:101-149, 1911.

17. Renshaw, S.: Object Perceived-Size as a Function of Distance, Engineering Experiment Station News 25:44-48, 1953.