Phase and frequency shift in a Michelson interferometer

Wolfgang Engelhardt[a)]
Fasaneriestrasse 8, D-80636 München, Germany

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Abstract: Traditionally, the outcome of Michelson's interference experiment has been interpreted as evidence against the existence of a luminiferous medium called "ether." Einstein, however, emphasized in 1920 that an ether must exist in spite of Michelson's null result. In this paper, it is shown that a medium theory—be it for light or for sound—actually predicts the observed null result. Michelson expected a gradual fringe shift when his apparatus was turned in the "ether wind." Such a phase change would, however, require a temporary frequency change in one of the interferometer arms. Since wind does not alter the frequency in the interferometer, a phase shift cannot occur either. © *2014 Physics Essays Publication*.
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Résumé: Habituellement, l'issue de l'expérience sur l'interférence de Michelson a été interprétée comme évidence contre l'existence d'un medium luminifère appelé "éther." Einstein, cependant, a insisté, en 1920, sur le fait qu'un éther doit exister en dépit du résultat nul de Michelson. Dans cet article il est démontré qu'une théorie de medium—que ce soit pour la lumière ou pour le son—prédit, en fait, le résultat nul observé. Michelson s'attendait à un changement graduel des franges d'interférence quand son appareil était tourné dans le "vent de l'éther." Un tel changement de phase exigerait, cependant, un changement temporaire de fréquence dans un des bras de l'interféromètre. Puisque le vent ne change pas la fréquence dans l'interféromètre, un changement de phase ne peut se produire non plus.

Key words: Interferometry; Phase Measurement; Fundamental Tests; Ether.

I. INTRODUCTION

In his *Electromagnetic Theory of Light*,[1] Maxwell derived a wave equation for the vector potential which was supposed to describe the propagation of electromagnetic perturbations. Like air is necessary to carry sound, an analogous medium—called "ether"—was required to transmit electromagnetic waves with a characteristic velocity c. Based on this conception, one could expect an "ether wind" to blow on earth due to the planet's motion through space. Maxwell, who strongly believed in the ether hypothesis,[b)] proposed to measure the strength of this wind by determining the round trip velocity of light.[c)] It was Michelson who took up the idea and invented his famous interferometer which appeared suitable to measure the ether wind by comparison of the light velocity in the two arms of his device. To his amazement, he obtained a null result in 1881,[4] which was later confirmed by himself and Morley,[5] as well as by many other experimenters. A survey of the various attempts to measure an ether drift is given by Galaev.[6] Some experimenters claimed having obtained a positive result which, however, could not be confirmed by others. Galaev himself reported a positive result using a Rozhdestvensky (Mach–Zehnder) interferometer, not a Michelson interferometer. The interpretation relies on a theory of some ether viscosity which is beyond the scope of this paper.

The special theory of relativity (STR) has a simple *ad hoc* "explanation" for Michelson's null result: It postulates the constancy of the light velocity in all reference frames regardless of their motional state. As a consequence, time had to be transformed according to a rule which was proposed by Voigt in 1887.[7] Whereas Voigt considered his transformation as valid for waves in all elastic media including sound, Lorentz applied it only to electromagnetic waves.[8] This enabled him to explain Michelson's experiment in a more general way than by his previous contraction hypothesis.[9] Since then, Michelson's experiment attained more and more the status of an *experimentum crucis*, a cornerstone proving the validity of STR. In 1905, however, Einstein based his own paper on relativity[10] on the Voigt–Lorentz transformation, not so much on the outcome of Michelson's experiment. According to Holton,[11] he did either not know of this experiment or he did not regard it as pertinent. He may have read Voigt's article of 1887[12] or Cohn's paper of 1901[13] which both offered alternative explanations of the null result in terms of classical electrodynamics and the pertaining ether theory.

a)wolfgangw.engelhardt@t-online.de

b)"… Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise."[2]

c)"[I]n the terrestrial methods of determining the velocity of light, the light comes back along the same path again, so that the velocity of the earth with respect to the ether would alter the time [interval] of the double passage…"[3]

In this paper, we show that Einstein's primary intuitiveness was correct: It is not possible to measure any ether wind with Michelson's device. The reason for this is that a phase change can only be brought about by a temporary frequency change, as will be shown in due course. Since the latter does not occur due to wind, as will also be shown, a null result must be expected. This is true not only for light but also for sound, as will be shown explicitly in Sec. III. Before then, in Sec. II, the connection between phase shift and frequency change in a laser interferometer will be studied. Such a device is used as a length measurement apparatus and constructed on the basis of a Michelson interferometer. Finally, concluding remarks will be presented in Sec. IV.

II. PHASE SHIFT AND THE PERTAINING FREQUENCY CHANGE DUE TO A MIRROR DISPLACEMENT IN A LENGTH COMPARATOR BASED ON MICHELSON'S INTERFEROMETER

In 1892–1893, Michelson and Benoît[14] succeeded in determining the length of the standard meter in units of the red cadmium line. They used a Michelson interferometer and counted the number of fringes recorded by a detector when the reflecting mirror in one of the arms was moved. This method is still used in modern laser interferometers where coherence lengths of the laser light with more than 50 m are exploited to achieve an extremely high accuracy (Fig. 1).

The displacement d of the mirror in the horizontal arm is compared to the wavelength of the laser light by counting the number of fringes which are produced by interference of the horizontal with the vertical light beam. There is a simple relationship of the number of fringes m as a function of the wavelength λ and the distance d

$$m = 2d/\lambda. \tag{1}$$

In order to derive this, we describe the light beams in terms of plane waves starting at the beam splitter. When they reunite (interfere) after reflection at the solid mirrors, we may write for the vertical beam

$$A\cos(k2L - \omega t), \tag{2}$$

and for the horizontal beam

$$A\cos(k2(L + d) - \omega t), \tag{3}$$

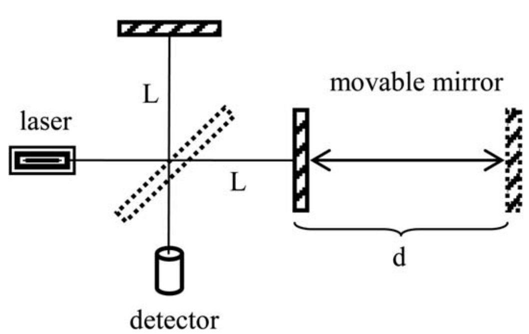

FIG. 1. Michelson interferometer as a length comparator.

where the wave vector may be expressed by the wavelength: $k = 2\pi/\lambda$. Superposition of the two expressions yields

$$2A\cos(kd)\cos(k(2L + d) - \omega t), \tag{4}$$

which is a plane wave with a new amplitude and phase oscillating with the same frequency ω as the waves of the original beams. The amplitude is a periodic function of the displacement d and has an extremum when (1) is satisfied. Thus, by counting fringes, while the mirror is moved one can express the total distance d in units of the wavelength λ.

Note that the frequency of the wave in the horizontal arm cannot stay constant during the motion of the mirror due to the Doppler effect of the light reflected at the receding mirror. This dependence of ω on the velocity of the moving mirror may be included in Eq. (3) by substituting $d = \mathrm{v}t$, which yields

$$A\cos(k2L - (\omega - 2k\mathrm{v})t) = A\cos(k2L - (1 - 2\mathrm{v}/c)\omega t), \tag{5}$$

where we have also inserted the phase velocity $c = \omega/k$ of the wave. This relationship reflects the frequency change $(1 - 2v/c)\omega$ (in first order of v/c) caused by the Doppler effect at the receding mirror. It is important to realize that a change in phase is inevitably brought about by a temporary change in frequency. In modern laser interferometers, this frequency shift is in fact measured, in order to determine the direction of the mirror displacement which would remain ambiguous by counting just fringes.[14]

Hence, it is obvious that a variation of the phase in one of the beams—leading to a fringe shift at the detector—can never occur unless the frequency in one of the beams differs during the time when the phase is shifted. At constant frequency in both arms, the phases would be strictly locked, thus preventing a fringe shift. One may visualize this situation by considering two wheels rotating in phase [Fig. 2(a)].

In order to achieve a phase shift between the two wheels, it is necessary to break the rigid coupling and rotate one of the wheels somewhat faster [Fig. 2(b)]. When a certain phase shift is established, the two wheels are coupled again and rotate at the same frequency as before [Fig. 2(c)].

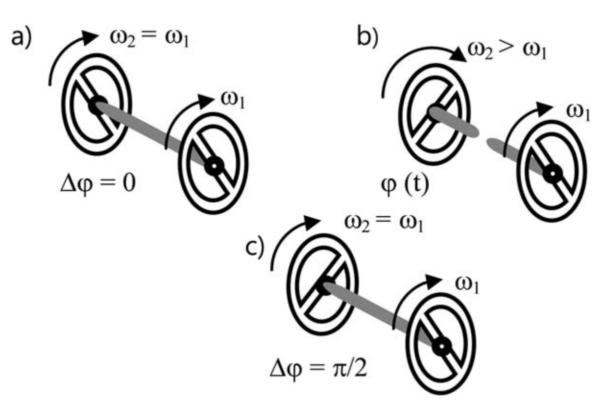

FIG. 2. (a) Two rigidly coupled wheels rotating in phase at frequency ω_1. (b) Wheel 2 decoupled from 1 and rotating at a higher frequency ω_2. (c) Both wheels coupled again after a phase shift of $\Delta\varphi = \pi/2$.

Indeed, precisely this situation occurs when a car travels along a road. The phase relation between the two wheels on one axle remains constant on the straights, as the frequency of the wheels is also equal. As a bend is navigated, the frequency of the outer wheel increases relative to that of the inner wheel, which leads to a phase shift. The phases will be locked again, once the car continues its journey along the next straight. This consideration should be kept in mind when the effect of wind blowing through an interferometer is discussed in Sec. III.

III. DOPPLER EFFECT IN A MICHELSON INTERFEROMETER

Now that the principle of intimate connection between continuous phase change and temporary frequency change has been explained, we turn to the Michelson–Morley interferometer where the distances between the mirrors are kept constant, but an ether wind was expected to blow through the interferometer. First, an acoustic interferometer working with sound waves instead of light is considered, since the physics of sound is well known and entirely understood. A wind blowing along the horizontal arm of this interferometer will be equivalent of both the beam splitter and the mirror moving through a still medium. Therefore, the Doppler effect due to the velocity of beam splitter and mirror through the medium may be considered in order to investigate the effect of such a wind on the stationary interferometer. The important point to note here is that the Doppler effect differs depending on whether the detector (Fig. 3) or the source (Fig. 4) is moving. Let us now assume a wave traveling in still air along the horizontal arm from the beam splitter to the mirror. Using plane wave approximation as before, we have

$$A\cos(kx - \omega_0 t). \tag{6}$$

Initially, the wind effects on the mirror—which is taken as a detector—and the beam splitter—which acts as a source—will be considered separately.

A wind blowing away from the mirror toward the beam splitter is analogous to the detector (mirror) traveling in the wave field on the x-axis with constant velocity v (Fig. 3).

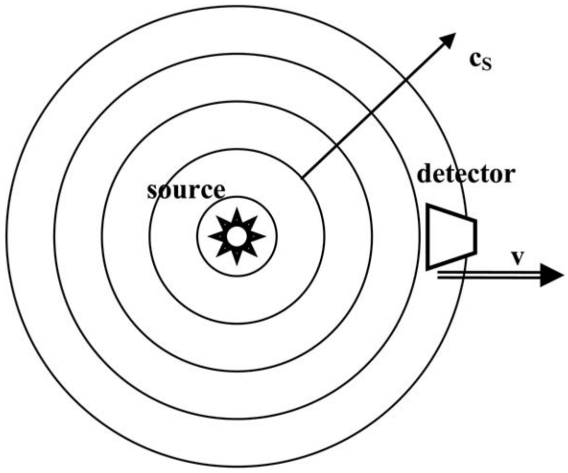

FIG. 3. Doppler effect with moving detector.

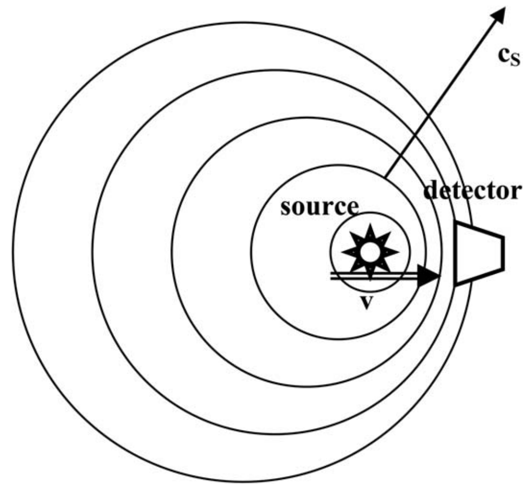

FIG. 4. Doppler effect with moving source.

The frequency the detector experiences may then be obtained by including the velocity term in Eq. (6) and resolving accordingly

$$A\cos(k(x_0 + \mathrm{v}t) - \omega_0 t) = A\cos(kx_0 - (\omega_0 - k\mathrm{v})t). \tag{7}$$

Comparing Eq. (7) with Eq. (6) yields the new frequency

$$\omega_1 = \omega_0(1 - \mathrm{v}/c_S), \tag{8}$$

where $c_S = \omega_0/k$ is the phase velocity of sound. Thus, the moving detector (receding mirror) experiences a wave with a reduced frequency ω_1.

Likewise, the effect of wind blowing from the detector toward the source can be described by considering the case of the source (the beam splitter) moving with velocity v toward the detector (mirror). In this case, the wave fronts are compressed in front of the source as sketched in Fig. 4.

As the source moves a distance $2\pi\mathrm{v}/\omega_0$ between the emissions of two consecutive wave crests, the wavelength is shortened by the amount

$$\lambda = \lambda_0(1 - \mathrm{v}/c_S). \tag{9}$$

In this case, the detector measures the frequency

$$\omega_2 = \frac{2\pi c_S}{\lambda_0(1 - \mathrm{v}/c_S)} = \frac{\omega_0}{1 - \mathrm{v}/c_S}. \tag{10}$$

Since a wind blowing along the interferometer's horizontal arm is in effect the case of both source and detector moving with the same velocity v, the Doppler effects of "moving detector" Eq. (8) and "moving source" Eq. (10) cancel out so that the detector measures simply

$$\omega = \omega_0. \tag{11}$$

Naturally, the same consideration applies to the wave reflected back from the receding mirror, which is now a source, to the forward moving beam splitter which is now taken as a detector. This means that wind has no influence whatsoever on the frequency in the acoustic Michelson interferometer.

Consequently, this result proves that the frequency of the two interfering waves is not influenced when the direction of the wind in the two arms is interchanged by turning the interferometer by 90°. Since we already saw in Sec. II that a phase shift cannot occur under constant frequency [Fig. 2(a)], a change in the interference pattern is also prevented by the physics of the Doppler effect during the turning of the interferometer.

In the optical Michelson interferometer, similar Doppler formulae apply. In Ref. 15, we have derived the relativistically correct formulae under conservation of momentum and energy, when photons interact with matter. The relevant "collision physics" is the same as one employs in the treatment of the Compton effect. We obtained for a moving detector

$$\omega_1 = \omega_0 \frac{1 - (\mathrm{v}/c)\cos\alpha}{\sqrt{1 - \mathrm{v}^2/c^2}}, \tag{12}$$

and for a moving source

$$\omega_2 = \omega_0 \frac{\sqrt{1 - \mathrm{v}^2/c^2}}{1 - (\mathrm{v}/c)\cos\alpha}, \tag{13}$$

where α denotes the angle between the velocity and the momentum vector of the photons (i.e., wave vector in the wave picture). These formulae have been verified in a number of experiments which were quoted in Ref. 15. They were also derived from the Lorentz transformation both in Ref. 15 and by Einstein himself in Ref. 16. In principle, they have the same property as the acoustic formulae (8) and (10): When both source and detector move in the same direction resulting in an ether wind blowing between them, the velocity dependent factors cancel exactly which means that a Doppler effect between source and detector does not arise as there is no relative velocity. Without a temporary frequency change, however, in at least one of the interferometer arms, a phase shift cannot be observed either. This was precisely what Michelson found.

IV. CONCLUDING REMARKS

The usual interpretation of Michelson's experiment—which is reiterated in all textbooks dealing with the subject—follows Michelson's original idea based on a "swimmer" analogy, namely, that the phase shift between the interfering waves can be calculated from the average time a light beam (a swimmer) spends traveling between the beam splitter and the mirror. If there is an ether wind in the horizontal arm, for example, the average travel time between beam splitter and back reflecting mirror was calculated by the relationship

$$\begin{aligned}\tau &= \frac{1}{2}\left[\frac{L}{c-\mathrm{v}} + \frac{L}{c+\mathrm{v}}\right] \\ &= \frac{1}{2}\left[\frac{L(c+\mathrm{v}) + L(c-\mathrm{v})}{c^2 - \mathrm{v}^2}\right] = \frac{Lc}{c^2 - \mathrm{v}^2}.\end{aligned} \tag{14}$$

In first order the velocity cancels, but in second order there is still an influence of the velocity on the average travel time which Michelson attempted to measure. Rather than drawing an analogy with solid particles (swimmers), it is more appropriate in the field of interferometry to consider the average wavelength of the beams traveling forth and back with different velocities due to the ether wind. For the purpose of calculating the interference pattern, it is in fact the average wavelength which is relevant and may be determined from Eq. (9) (which holds also for light)

$$\lambda = \frac{1}{2}[\lambda_0(1 - \mathrm{v}/c) + \lambda_0(1 + \mathrm{v}/c)] = \lambda_0. \tag{15}$$

The cancellation of the velocity is complete so that wind does not lead to a phase difference between the waves in the horizontal and the vertical arm.

Our analysis shows that the outcome of Michelson's interference experiment has no relevance for the existence of an ether. Einstein himself had reintroduced the ether concept in 1920[d)] being well aware both of his own theory of 1905 and of the precise MM-results[5] of 1887. The measurement of the solar system's velocity with respect to the CMB by the COBE satellite[18] also lends strong support for the existence of an absolute system in which light travels with constant velocity in all directions in agreement with Einstein's second postulate of 1905. Einstein's first postulate, however, is at stake when considering that Michelson's experiment cannot disprove the ether in combination with the experimentally proven existence of a distinguished system that may be identified with Maxwell's or Einstein's ether.

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[d)]"Gemäß der allgemeinen Relativitätstheorie ist ein Raum ohne Äther undenkbar; denn in einem solchen gäbe es nicht nur keine Lichtfortpflanzung, sondern auch keine Existenzmöglichkeit von Maßstäben und Uhren, also auch keine räumlich-zeitlichen Entfernungen im Sinne der Physik."[17]. English translation republished in: *Sidelights on Relativity* (Methuen, London, 1922): "According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense."

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