Translation:On the Experiment of F. Harress

On the Experiment of F. HARRESS;

by M. v. LAUE.

In the previous publication, KNOPF reported about an experiment of the late FRANZ HARRESS from the years 1909-1911, in which the propagation of light in a rotating glass-device was observed by means of an interference phenomenon. The one published in 1913 by SAGNAC[1] is closely similar to it; the main difference lies in the fact, that space is the carrier of light-propagation in the latter one (more precisely: air of atmospheric pressure; though it differs too slightly from empty space to make a considerable influence) and that all mirrors and all other apparatuses determining the light path are co-rotating. That in SAGNAC's experiment, the light source and the observing apparatus for the interference fringes share the rotation while they are at rest in HARRESS's experiment, causes no essential difference. In both experiments, the rays that come to interfere, only traverse rotating parts of the device between separation and reunion. The rays have the same fates before and afterwards – this holds for both HARRESS as well as SAGNAC – and common fates are irrelevant for the interference phenomenon.

Both experiments are proving, first, that the optical processes are different in a reference system rotating relative to Earth, as in a system fixed to Earth; we are permitted to consider the latter system with sufficient approximation as a valid system in the sense of the restricted relativity theory. Regarding mechanical processes it's known that every experiment concerning centrifugal forces provides the corresponding confirmation (SAGNAC's interpretation of his experiment as a confirmation of the existence of an "aether" is not decisive at all). That this difference between the mentioned reference systems is according to general relativity not a principal one, but the consequence of their different motions relative to the system of fixed stars, is not required to be considered by us here.

Already in 1911, I gave the relativistic theory for the experiment executed by SAGNAC afterwards,[2] and here I would like to include HARRESS's experiment in the theory. As far as it is about light propagation in moving bodies, the latter lies in the vicinity of the old interference experiment of FIZEAU and the recent one by ZEEMAN.[3] In the following, we want to emphasize particularly the connection of these experiments.

In the calculation that was given to the experiment by HARRESS himself, a mistake was slipped in, making the experimental result incomprehensible at first. This was noticed by HARZER,[4] who published a calculation whose result is in the main correct, and which together with a note by Einstein[5] is capable of interpreting Harress's measurements with sufficient precision. Though full satisfaction we cannot feel also with respect to this form of the theory. First – although this is without practical importance in the light of the present precision of Harress's experiment, but it is of principal importance –, Harzer overlooks that also the parts of the light path traversed in the rotational apparatus, which are lying in the air, are contributing to the phase difference at the interference. Then Harzer with full justification considers that the light ray must be curved in the rotating body. Yet the approach, from which he finds out its shape,[6] is not self-evident in our view, although it can surely be demonstrated from relativity theory. It is also justified, when Harzer changes the reflection and refraction law with respect to the motion of bodies; yet whether the way by which he is doing this, can be justified by relativity theory, was not possible for us to decide. And eventually – and this is the main part – it can be shown very easily, that the curvature of the light-ray and the changes in reflection and refraction (independent from the way by which they are calculated) are irrelevant for this phase difference, as long as we confine ourselves (which is of course valid) to terms of first order in the ratio of the body's velocity to the speed of light. And this makes the theory very much clearer.

For the sake of completeness, and because Einstein's note is for the most professional colleagues not quite near at hand, we repeat its content in §§ 2 and 3.

§ 1. Contrary to the experiment of Fizeau and that of Zeeman, in Harress's experiment light is not propagating in the direction of motion of the body or in opposite direction, but under an arbitrary angle. If ϑ^0 is this angle related to the rest system of the body, if the velocity of the body relative to the valid system upon which the consideration is based – *i.e.*, relative to Earth – has the amount q, and (as usual) c means the speed of light in empty space and n^0 the refraction index of the bodies related to the rest system, then the phase velocity of light according to relativity theory has the amount (relative to the mentioned reference system)[7]

$$V = c\frac{c + qn^0 \cos\vartheta^0}{\sqrt{(c + qn^0 \cos\vartheta^0) + {n^0}^2 \left(c^2 - q^2\right) \sin^2\vartheta^0}}.$$

In the following we always neglect terms of second and higher order in q/c, thus we can write:

$$V = \frac{c}{n^0} + q\cos\vartheta\left(1 - \frac{1}{n^2}\right) \tag{1}$$

where we relate angle ϑ to a system relative to which the body moves with velocity q. Equation (1) shows, that to value c/n^0 which is valid in the case of rest, the component of the body's velocity in the direction of the ray direction[8] multiplied by the dragging coefficient $\left(1 - 1/n^2\right)$, is added.

The transformation formulas for the oscillation number from the rest system to the one fixed relative to Earth, rigorously reads

$$\nu = \nu^0 \frac{c + qn^0 \cos\vartheta^0}{\sqrt{c^2 - q^2}}$$

thus except terms of second and higher order

$$\nu = \nu^0 \left(1 + \frac{qn}{c} \cos\vartheta\right) \tag{1a}$$

§ 2. In Fizeau's interference experiment, light traverses a *stationary* tube of length l, in which water flows with velocity q in the ray direction or in the opposite direction. The time required by the ray traveling in the sense of the stream to traverse the tube, is according to (1)

$$t_+ = l : \left[\frac{c}{n^0} + q\left(1 - \frac{1}{n^2}\right)\right] = \frac{ln^0}{c}\left[1 - \frac{qn}{c}\left(1 - \frac{1}{n^2}\right)\right] \tag{2}$$

However, now the relative oscillation number ν^0 of light relative to water differs from the one relative to the reference system fixed relative to Earth, ν. Since the passage of light into and out of the water happens at *stationary* surfaces[9], then also during the propagation in water the oscillation number relative to the mentioned reference system remains. Consequently it is according to (1a)

$$\nu^0 = \nu\left(1 - \frac{qn}{c}\right)$$

and the refraction index n^0 to be included in (2), is to be calculated from the refraction index n of resting water for the oscillation number ν, by the formula

$$n^0 = n\left(1 - \frac{q\nu}{c}\frac{dn}{d\nu}\right).$$

If we insert this value in (2), we find with H. A. Lorentz:[10]

$$t_+ = \frac{ln}{c}\left[1 - \frac{qn}{c}\left(1 - \frac{1}{n^2} + \frac{\nu}{n}\frac{dn}{d\nu}\right)\right] \tag{3}$$

For the difference existing between the traversing times of both rays, from that it follows

$$t_+ - t_- = \tau = -\frac{2lqn^2}{c^2}\left(1 - \frac{1}{n^2} + \frac{\nu}{n}\frac{dn}{d\nu}\right) \tag{4}$$

This is the formula confirmed by Fizeau, Michelson and Morley, and eventually by Zeeman with a precision of some thousandth.[11]

§ 3. In Zeeman's experiment, the stationary tube with streaming water is replaced by a body moving as a whole with velocity q in the direction of one ray and oppositely to the other one. In order to calculate as to how the traversing time for the first ray is influenced by velocity q, we conveniently introduce the concept of relative velocity of light relative to the body, related to the reference system fixed to Earth. According to (1), since $\cos\vartheta = 1$, it amounts:

$$V - q = \frac{c}{n^0} - \frac{q}{n^2} \tag{5}$$

Because ordinary vector addition also holds according to relativity theory for two velocities related to the same system; in EINSTEIN's addition theory of velocities is is especially essential, that the velocities to be added are related to different systems. The time, in which this ray traverses distance l in the body, is therefore

$$T_+ = \frac{l}{V - q}.$$

However, it is now to be considered, that light (when it enters and leaves at two end-surfaces perpendicular to distance l as in ZEEMAN's experiment) travels a shorter distance in air due to the motion, as when the body would be at rest. Because during time T_+ the body moves a distance $T_+ \cdot q$. Thus if we denote by L the distance between both of them and the *stationary* part of the experimental device being closest to the moving body, then the mentioned ray requires the time (to come from one of these part to another one);

$$t_+ = T_+ + \frac{1}{c}\left(L - (l + T_+ q)\right) = \frac{1}{c}(L - l) + \frac{l}{c}\left(n^0 + \frac{q}{c}(1 - n)\right) \tag{6}$$

Also here, the oscillation number in the reference system connected to the body is different than that connected to Earth. This time, the oscillation number namely remains conserved with respect to the first when light enters and leaves, because the surface at which this happens, share the motion of the body. This time, it is to be set $n = 1$ in (1a), when we want to calculate ν^0 from ν for light propagating in empty space; thus:

$$\nu^0 = \nu\left(1 - \frac{q}{c}\right),\ n^0 = n - \frac{\nu q}{c}\frac{dn}{d\nu}$$

Inserting this into (6) gives:

$$t_+ = \frac{1}{c}\left(L - l\left(1 - n^0\right)\right) + \frac{lq}{c^2}\left(1 - n - \nu\frac{dn}{d\nu}\right)$$

The time difference for both rays is afterwards:

$$\tau = t_+ - t_- = \frac{2lq}{c^2}\left(1 - n - \nu\frac{dn}{d\nu}\right) \tag{7}$$

This is the formula stated by EINSTEIN, ZEEMAN and H. A. LORENTZ[12], which also was essentially confirmed by ZEEMAN.

Now we already come quite near to HARRESS's experiment, when we think of ZEEMAN's experiment as altered in terms of Fig. 1. Within it, light isn't entering and leaving through two

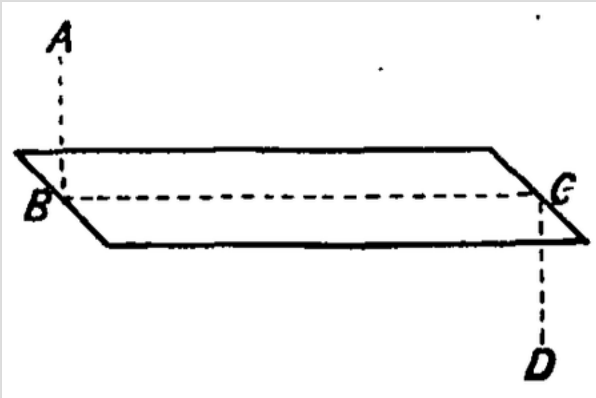

Fig. 1

surfaces perpendicular to velocity, but this happens at two surfaces parallel to velocity. The ray coming from A is reflected to C after its entrance at B at one of the end-surfaces of the body, and there it is reflected at the other end-surface to D. If we ascribe to the body the length $l(=BC)$ and the width b here, then the ray traveling in the sense of motion, needs the time within it:

(8) $$t_{+}=\frac{l}{V-q}+\frac{bn^{0}}{c}=\frac{ln^{0}}{c}\left(1+\frac{q}{nc}\right)+\frac{bn^{0}}{c}$$

The distance to be traversed by it, is not changed by motion. Furthermore, the oscillation number ν^{0} related to the co-moving system, remains conserved in the reflection and refraction processes at the *co-moving* end-surfaces and limiting-surfaces of the body. Since furthermore the ray direction is perpendicular to velocity q[13] before entrance and exit, ν and ν^{0} are in agreement here according to (1a). Thus it is to be set $\nu^{0}=\nu$, $n^{0}=n$ in general, and here we find by (8)

(8a) $$\tau=t_{+}-t_{-}=\frac{2lq}{c^{2}}$$

Contrary to equations (4) and (7), the refraction index is completely dropped out. Also τ and thus the fringe displacement obtain the opposite sign as in Fizeau's experiment. There, the dragging namely accelerates the ray traveling with the motion of matter, so that the stationary distance l is traversed by it in a shorter time as in the case of rest. On the other hand, all parts of the apparatus run away from the mentioned ray, and by that, the contraction is compensated and becomes even reversed into the opposite.

The deepest reason for the independence of the time difference from the refraction index, is shown, however, by the Lorentz transformation. A glass-rod shall be at rest in reference system K', its end-surfaces shall have the equations $x'=0$ and $x'=1$. At time $t'=-n^{0}l:c$ we send a light wave of sinus-form, from any of these surfaces to the opposite surface. Both phases reach their target at time $t'=0$. Now we relate this process to a system K, relative to which K' has the velocity q in the positive x'-direction. The transformation equation for time reads:

$$t=\frac{c}{\sqrt{c^{2}-q^{2}}}\left(t'+\frac{qx'}{c^{2}}\right)$$

Only the second summand in the numerator causes here the time difference τ. Regarding the ray traveling in the positive x'-direction, the times of start and arrival of the considered plate are namely (related to K)

$$-\frac{n^0 l}{\sqrt{c^2 - q^2}} \text{ and } \frac{ql}{c\sqrt{c^2 - q^2}}$$

The traversing time is the difference of this, thus in terms of first order:

$$t_+ = \left(n^0 + \frac{q}{c}\right)\frac{l}{c}$$

The time required by light in Fig. 1 for the paths perpendicular to BC, is not changed by the transformation in terms of first order. Thus if in both rays the relative oscillation numbers are in agreement, then we concluded from the last expression:

$$\tau = t_+ - t_- = \frac{2lq}{c^2}$$

§ 4. Harress's experiment only differs from the one described in Fig. 1 by the fact, that the pure translatory velocity of the glass body is replaced by a rotation. Technical reasons led to this change. To the theory it means a certain impediment, because accelerations are connected with the rotation. But one still doesn't know for sure as to how this is affecting the optical processes in matter. General relativity would of course be capable of giving some statements about it, and we want to show at first that no noticeable influences of acceleration are expected according to it. The occurring centrifugal forces are namely of a magnitude of at most 10 to 100 times as great as Earth's attraction upon the same body. Now, since gravitational acceleration itself has no noticeable influence upon the optical processes even in respect to much mightier celestial bodies than Earth, then such one isn't expected in Harress's experiment as well.[14]

Now, it's of course not necessary to consider the statements of general relativity as unconditionally correct. Then, however, any basis concerning the effects of acceleration is missing, and due to the lack of something better we have to calculate as if there were no effect. (This was also done by Harzer l.c.) In addition, such effects (so far as they influence the path of light rays) come not into consideration for the calculated phase difference. They would only matter when they concern the absolute value of the speed of light, what immediately is to be proven.

Now we begin with the announced proof, that all changes of the optical path relative to the apparatus caused by rotation, don't matter for the interference phenomenon in terms of first order. We imagine the apparatus as being at rest at the beginning, and consider a ray circulating from point P to point Q of the separating plate in the sense of the later rotation. Its path consists of a series of straight distances; the length of any of them we denote by l. $\sum \frac{l}{V} = \sum \frac{ln}{c}$ is the time required for traversing of the path described. When the apparatus is rotated, there is also a ray circulating from point P to the co-moving point Q in the sense of rotation. The rotation first causes a change of the relative velocity V by which the distances l

are traversed (under reservation of the proof, we calculate as if lV were constant upon every distance), but also a change of distance l itself due to the displacement of the relative ray path. The total change of the traversing time amounts:

$$\sum \delta\left(\frac{1}{V}\right)\cdot l + \sum \frac{n\delta l}{c}$$

Herein, the second term is now small of second order according to Fermat*'s theorem of the preferred light path*; because the changes δl are at most of order q/c. And by that the proof is given; it is independent from the way, by which changes of δl are calculated.

However, now a further objection is near at hand: The considered rays have different directions in P and Q. For an observing apparatus adjusted to infinity, in whose focal plane the interferences appear, they are not equally valid.[15] Yet, by that we exceed the area of applicability of geometrical optics. The direction differences discussed are of first order in q/c . For the rim of Harress's apparatus, q is always smaller than $2 \cdot 10^3 \mathrm{cm}/\mathrm{sec}^{-1}$, a value that would correspond to ca. 1000 rotations a minute. Consequently, the angle between both rays is of order of magnitude 10^{-7}. The opening of the observing apparatus employed was surely contained in a square of 3.6 cm side length. Because according to p. 22 of Harress's work, the exit surfaces of the prisms are squares for light. The observing apparatus thus can only resolve angles of order of magnitude 10^{-5} or larger. As it was shown, the ones coming in question here, however, are lying considerably under this limit.

By that we conclude that the displacements of the relative light path are not to be considered for the calculation of τ.[16]

§ 5. After this preparations we state at first: 1. According to equation (1a) for the absolute phase velocity V of light relative to the reference system fixed at Earth, the component of the body's velocity perpendicular to the ray direction doesn't matter, but it is only about the velocity parallel to it. 2. If a line l has the smallest distance r from the rotation axis and if its direction forms with it the angle Θ, then the component of the body's velocity with respect to this direction is equal to $\omega r \sin\Theta$ in any of the points, where ω is the angular velocity of the rotation. Proof: We choose the rotation axis as z-axis, the shortest difference r as y-axis of a rectangular axis-cross. Line l then has the equations $y = r,\ x : z = \operatorname{tg}\Theta$. Velocity $\mathfrak{q}$, however, has the components $\mathfrak{q}_x = \omega r$, $\mathfrak{q}_z = 0$ in an arbitrary point of the line. From that we derive the angle between l and $\mathfrak{q} : \cos(l\mathfrak{q}) = \frac{\mathfrak{q}_x}{q}\sin\Theta$, thus $q\cos(l\mathfrak{q}) = \omega r \sin\Theta$. The phase velocity of light relative to a reference system fixed to Earth, is consequently by (1):

$$V = \frac{c}{n^0} + \omega r \sin\Theta\left(1 - \frac{1}{n^2}\right) \tag{9}$$

Of greatest importance is now the question after the oscillation number ν^0 of light against the glass-body, since (as already in the cases discussed above) the refraction index depends on n^0. We first ask: Is the relative oscillation number changing, as light propagates from one reflecting location to another? The answer is: No. Because in every process, the oscillation number ν relative to the reference system fixed to Earth remains conserved in any case; if there were a difference of oscillation numbers between two points fixed in space, then this indeed would mean a continuously increasing phase displacement of the oscillations within them, which is not possible. However, since the component $q\cos\vartheta$ has the same value upon the total straight ray-path, then also ν^0 is conserved according to (1a). However, the ray is not exactly straight; yet the changes of direction occurring within are in any case small of first order in q/c, thus they don't come into consideration in the second term of (1a) which is proportional to q/c itself. The reflections and refractions are all happening at the co-moving surface, thus they leave the relative oscillation number ν^0 unchanged. Eventually, light (immediately before its entrance and after its exit out of the moving parts of the experimental arrangement) has a direction perpendicular to the velocity present at this place, so that the relative and the absolute oscillation number ν^0 and ν are in agreement. *Thus* $\nu^0 = \nu$ *and* $n^0 = n$ *is everywhere the case in the interferometer*, where under n we understand the refraction index of the glass-body which is to be calculated without consideration of the motion.

§ 6. The calculation of time difference τ can now easily be executed. If we divide every partial section l of the relative ray-path by the corresponding relative velocity, having the amount for the ray circulating with the rotation according to (9)

$$V - q\cos(ql) = V - \omega r\sin\Theta = \frac{c}{n} - \frac{\omega r}{n^2}\sin\Theta$$

then we find for the time required by light from separation until reunion with the other one:

$$t_+ = \sum\frac{ln}{c} + \frac{\omega}{c^2}\sum rl\sin\Theta \tag{10}$$

For the time difference it follows from that:

$$\tau = t_+ - t_- = \frac{2\omega}{c^2}\sum rl\sin\Theta \tag{11}$$

Knopf on p. 440 alluded to the fact, that the sum to be extended over all parts of the ray (also those lying in air) also contains negative terms.

This time difference has two causes. First, the dragging of light by moving bodies, but also the fact that every part of the rotating apparatus runs away from one ray, while it approaches the other one. Both causes together give [according to (11)] a time difference (being the same for all bodies) independent from the refraction index. Even if one (as with Sagnac) choose empty space as the carrier of light propagation, so that dragging drops out, the second purely geometrical cause is producing the same fringe displacement. Harress only accounted for the first cause in his dissertation.

If the value of the dragging coefficient remains undefined at first (denotation: x) and if one accordingly sets according to (9):

$$V = \frac{c}{n} + x \cdot \omega r \sin\Theta$$

then one finds instead of (11) the corresponding formula in the notation of KNOPF:

$$\tau = \frac{2\omega}{c^2} \sum \left[n^2(1-x)\sin\Theta rl\right]$$

$l \sin\Theta$ is the projection of a part of the ray as it proceeds in the case of rest, upon a plane perpendicular to the rotation axis. $\frac{1}{2} lr \sin\Theta$ is thus the triangle surface, which is limited by that projection and the straight connections of their endpoints with the puncture point of the axis through the plane. The center ray returns to its starting point; in respect to it, $\frac{1}{2}\sum lr\sin\Theta = F$ is the surface enclosed by its projection. Thus equation (11) can be written in the form chosen earlier at SAGNAC's experiment:

$$\tau = \frac{4\omega F}{c^2}$$

§ 7. Finally we still have to study, as to how the width of the ray pencil influences the phenomenon. Namely, the unusual case occurs that in both families of parallel rays, united by the telescope in one point, the time difference τ between two mutually corresponding ones is already changed considerably. Already HARRESS alluded to this; he as well as KNOPF gave a formula for the time difference, calculated for a ray parallel to the center ray (of Fig. 1 of HARRESS) whose distance from the center ray, however, has the component δ perpendicular to the rotation axis:

$$\tau = \tau_0 - f(\delta) \tag{12}$$

here, τ_0 means the value of τ for the center ray, f is a positive function of second degree, whose value is derived geometrically in the preceding work. According to HARRESS, the second term amounts to 3% of τ_0 for the quite possible value of 1 cm for δ.

Under these circumstances it is not sufficient, to make the approach (for the light oscillation in the unification point of all these rays):

$$e^{i\nu\left(t+\frac{\tau_0}{2}\right)} + e^{i\nu\left(t-\frac{\tau_0}{2}\right)}$$

but it is necessary to put instead of it:

$$\left\{ \begin{aligned} & \int\limits_0^{\delta_{max}} \left[e^{i\nu\left(t+\frac{\tau_0}{2}\right)} + e^{i\nu\left(t-\frac{\tau_0}{2}\right)}\right] d\delta \\ & = 2e^{i\nu t}\left[\cos\frac{\nu\tau_0}{2}\int\limits_0^{\delta_{max}} \cos\left(\tfrac{\nu}{2}f(\delta)\right)d + \sin\frac{\nu\tau_0}{2}\int\limits_0^{\delta_{max}} \sin\left(\tfrac{\nu}{2}f(\delta)\right)d\delta\right] \end{aligned} \right. \tag{13}$$

So long as

$$\left(\frac{\nu}{2} f(\delta)\right)^2 \ll 1 \tag{14}$$

one can replace (under the integral sign) the cosine with 1 and the sine with its argument, and then finds as the expression for the oscillation:

$$2\delta_{\max} e^{i\nu t}\left[\cos\frac{\nu\tau_0}{2} + \frac{\nu}{2} f \cdot \sin\frac{\nu\tau_0}{2}\right] = 2\delta_{\max} e^{i\nu t} \cos\frac{\nu}{2}\left(\tau_0 - f\right)$$

there, f is the average of f, formed over all occurring δ. It is then, as if time difference τ were invariable within the considered totality of rays, yet as if it had the average formed over the totality.

If condition (14) is not satisfied any more, then FRESNEL's integrals occur in (13), since $f(\delta)$ is a quadratic function. Then the fringe displacement is not proportional to ω any more, but depends on it in a less simple way. One shall not see a contradiction against it, that we otherwise always considered summands only proportional to ω. For the calculation of the time differences τ, the quadratic and higher terms are still neglected. Though the previous calculation shows indeed, that at sufficient width of the ray pencil, one nevertheless observes a fringe displacement being not proportional to ω any more.

§ 8. Now, what can the HARRESS experiment teach us, when it is just executed in total perfection? HARRESS himself and KNOPF calculate the dragging coefficient of glass from it, as to how far the measured value agrees with FRESNEL's formula. We have formed the theory, so that the experiment appears as a replacement of the one represented in Fig. 1, and to the latter we actually ascribed the purpose of being a confirmation of the transformation formulas with respect to time in the Lorentz transformation. Though there is no contradiction between those two interpretations of course. The optics of bodies moving without acceleration, is so uniform and interwoven in itself, that one can confirm or disprove it only as a whole. Any confirmation of any of its statements, is a benefit of the whole.

Thus far, the presupposition of the theory is, that the accelerations connected with the rotation in no way influence the speed of light. General relativity supports this statement. Whether it is correct, can only be shown by a desirable repetition of the experiment.

BERLIN, December 1919.

(Received January 21, 1920.)

---

1. G. SAGNAC, Compt. rend. 157. p. 708 a. 1410. 1913; Journ. d. Phys. (5) 4. p. 177. 1914.
2. M. LAUE, Münch. Sitz.-Ber. 1911. p. 404. See also H. Witte, Verh. d. D. Phys. Ges. 16. p. 142 u. 754. 1914.
3. P. ZEEMAN, Versl. Akademie Amsterdam 28. p. 1451. 1919; P. ZEEMAN a. A. SNETHLAGE, ibid 28. p. 1462. 1919.

4. P. HARZER, Astron. Nachr. 198. p. 378. 1914 u. 199. p. 10. 1914.
5. A. EINSTEIN, Astron. Nachr. 199. p. 9 a. 47. 1914.
6. We mean HARZER's equations

$$\frac{dx}{dt} = -\omega ky,\ \frac{dy}{dt} = v + \omega kx$$

   at p. 379 l.c.

7. See M. V. LAUE, Das Relativitätsprinzip, eq. (257).
8. To be more precise, it is about the direction of the wave normal. However, since the direction difference between both is itself small of first order, this plays no role here. HARRESS uses EINSTEIN's addition theorem of velocities for his conclusions in his dissertation eq. (1), but it's known that this doesn't apply to phase velocities of light waves.
9. Of course, the water-stream doesn't begin immediately behind of the plates through which light is entering and leaving, but light only gradually enters into the area of full velocity q. However, since the stream is stationary, thus the location of that velocity change along the ray path is fixed, then the consideration indicated in the text nevertheless stays correct.
10. H. A. LORENTZ, Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern. Leiden 1895, p. 101. ff.
11. A. A. MICHELSON a E. W. MORLEY, American Journ. of science 31. p. 377. 1886; P. ZEEMAN, Verslagen Akadem. Amsterdam 23. p. 245 1914; 24. p. 18. 1915.
12. See the referenced work of ZEEMAN and SNETHLAGE.
13. This is only strictly correct for the rest system; in the system fixed to earth, the ray direction deviates by a small angle of first order due to aberration. Though it isn't required to be considered, due to our confinement to magnitudes of first order.
14. I owe this remark to a discussion with W. WIEN.
15. HARRESS explains this interference phenomenon as being plan-parallel rings. It must remain undecided, whether he didn't actually adjusted another interference phenomenon.
16. HARZER provides a welcome confirmation of § 4 in the following section of his work:
   "If one assumes with HARRESS, that the rays in the rotating medium conserve their rectilinear shape which they have in the resting medium, and if one accordingly ascribes the change caused by rotation only to the change of velocity caused by dragging upon the invariable paths, then the formula emerges:

$$\delta\varphi = \lambda(1-k)\omega\frac{\Sigma pq}{\lambda v}$$

   whose content differs from the formula given by HARRESS on p. 59, only by the permutation of k with $1-k$....

In the light of the ideal case at which apparatus was aimed as far as possible, a certain similarity of these numbers with the ones that are actually valid, was to be expected from the outset; yet, that the assumption concerning the invariable shape of rays, which is quite incorrect for the individual paths between two optical effective surfaces, and which proves to be correct for the totality of paths to a high degree of approximation enclosing all parts of the calculation, is very surprising. An explanation based upon an analytical investigation of this close approximation was not found by me; thus I have to denote it as accidental in the unspecified meaning of the word."

Thus far HARZER. We only have to add the conjecture, that he would have seen the vanishing of the curvature also in his formula, not only in his number calculation, in case he would have also considered the already mentioned distances in air. The equation mentioned by him, except this difference, is equivalent to our formula (11). Because p and q mean the distances l and r, it is $v = c/n$ and $k = l - n^{-2}$; however, the time difference τ calculated by him is equal to $\delta\varphi\frac{\lambda}{v}$. Eventually, HARZER has always $\Theta = 1$.