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Canterbury Ring Laser and Tests for Nonreciprocal Phenomena*

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Abstract

An historic and simple experiment has been revitalised through the availability of supercavity mirrors and also through a heightened interest in interferometry as a test of physical theory. We describe our helium–neon ring laser, and present results demonstrating a fractional frequency resolution of $2 \cdot 1 \times 10^{-18}$ ($1 \cdot 0$ mHz in 474 THz). The rotation of the earth unlocks the counterrotating beams. A new field of spectroscopy becomes possible, with possible applications to geophysical measurements such as seismic events and earth tides, improved measurements of Fresnel drag, detection of ultraweak nonlinear optical properties of matter, and also searches for preferred frame effects in gravitation and for pseudoscalar particles.

1. Introduction

A few years after the advent of the laser, Macek and Davis (1963) demonstrated the first ring laser, and also its unique potential as a rotation detector via the Sagnac effect. The optical lengths of the closed paths for the counterpropagating beams are made unequal by rotation of the whole device. In an active device the frequencies adapt to this, the corotating beam becoming more red and the counterrotating beam more blue (Heer 1964). Both beams take essentially the same path within the cavity, so that when the beams transmitted at any mirror interfere, the resulting beat frequency δf reflects only the difference in optical path length, and not any common-mode effects such as frequency jitter. The beat frequency depends on the angular frequency $\mathbf{\Omega}$ of rotation with respect to the local inertial frame, the area $\mathbf{A}$ and perimeter P of the ring, and the optical wavelength λ (Post 1967; Anandan 1981; Chow *et al.* 1985; Stedman 1985*a*):

$$\delta f = \frac{4\mathbf{A} \,.\, \mathbf{\Omega}}{\lambda P} . \tag{1}$$

Any nonreciprocal effect, i.e. one which distinguishes the optical path lengths of the counterpropagating beams, will give rise to a frequency shift, making the ring laser more versatile than merely a rotation sensor.

Macek and Davis (1963) (see also Macek *et al.* 1963, 1964) used a 1 m^2 square helium–neon ring laser at $1 \cdot 15$ μm to measure beat frequencies down to 5 kHz;

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backscattering from the 16 beam interfaces (the Brewster windows on the four gain tubes) and the corner mirrors caused the two counterpropagating modes to lock at 2 kHz (for discussions of locking, see Aronowitz 1971, Statz *et al.* 1985). If we assume a Rayleigh or $1/\lambda^4$ variation of backscattering with wavelength, the equivalent locking threshold at 633 nm is 22 kHz.

In effect, we have repeated the Macek *et al.* experiment, and on a ring of similar shape and size. The key experimental differences are the use of supercavity mirrors, whose reflectances approach 99·999% — an ultimate, since it demands surface preparation which is smooth over atomic dimensions — and the complete absence of interfaces. This reduces the locking threshold to the point where the beat frequency induced by earth rotation (at 68 Hz) unlocks the ring without the need for any other biassing or dithering systems.

The development of supercavity mirrors has been vigorously driven by the potential for their application as ring gyros, now used for aviation inertial guidance systems in aircraft (e.g. the Airbus 320), and in various missiles (e.g. the Patriot system). An optical gyro with an area of the order of square decimetres has a sensitivity to rotation rates of order $10^{-4}\Omega_E$, where Ω_E is the earth rotation rate. Recently supercavity mirrors have been incorporated in such commercial items as the Newport SR-130 spectrum analyser. The scientific applications of the resolution enhancement inherent in these mirrors have not been fully explored.

2. Theoretical Aspects

(2a) Introduction

At the time of the Macek *et al.* experiment, only the application as a rotation sensor was demonstrated. This itself takes on a new interest with the increase in precision achieved over the intervening years. Some theoretical revitalisation is illustrated by the papers of Forder (1985), Scorgie (1990, 1991), Dieks and Nienhuis (1990) and Hendriks and Nienhuis (1990).

Less obvious physical effects could also generate a nonreciprocal effect in a ring laser (Macek *et al.* 1964; Bilger and Stowell 1977). There has been a renaissance of interest in interferometric tests of fundamental physical theory; the SQUID in the 1960s, the neutron interferometer in the 1970s and 1980s, and more recently atomic interferometry (Levi 1991) have spawned a variety of demonstrations and tests of quantum theory, relativity and related topics. These various interferometers are largely complementary, as discussed by Stedman (1985*a*, 1986) and Bilger *et al.* (1990), although atomic interferometry has much untapped potential (Clauser 1988; Levi 1991) and has demonstrated the Sagnac effect (Riehle *et al.* 1991; see Al'tshuler 1992). It is therefore appropriate to re-evaluate any unique potential that ring lasers may have as probes for old and new physics.

This potential rests partly in the geometry of the closed, as opposed to an open, interferometer, and partly in the unique characteristics of photons as the workhorse of the ring laser (Post 1972; Stedman *et al.* 1991), an electromagnetic system with the capability of detecting a parity-violating effect, in the sense that any frequency difference must be reversed by mirror reflection. Indeed effects which are detectable uniquely in a ring laser are not necessarily chiral (or gyrotropic, Stedman 1991) in the sense used in chemical physics; the relationship depends on the configuration of the ring laser (Stedman 1992). There has been considerable interest over the last decade or so in setting experimental limits on possible parity-violating effects within relativity or gravitation theory on the one hand, and quantum field theory and particle theory on the other.

(2b) Rotation Sensing and Seismology

Since the earth rotation unlocks the Canterbury ring, it is a free device, sensing absolute rotation without the need of any external bias or dither (tricks whose absence prevented the first ring laser from showing the earth rotation, and which are otherwise universally applied for ring lasers). It becomes possible to sense seismic events and earth tides as they affect the observed earth rate, at a precision better than 10 ppm. A ring-laser-based seismometer would measure information on any rotational effects, associated for example with horizontal shear, in seismic waves which would complement that derived from the traditional linear accelerometers, and at comparable precision; the rotation detection sensitivity of 10^{-9} rad/s or 0·01 rad/yr demonstrated in this paper is quite enough for detecting microseisms according to the data of Giazotto (1989); see Robertson (1991). The cave in which our ring is due to be installed is built into an 11 million year old volcanic basalt, 131 km off the very active Indian–Pacific tectonic plate boundary.

In collaboration with Ojai Research, we have shown the possibility of using the polarisation state of a square ring laser to measure mirror tilt with an accuracy of the order of picoradians. This suggests a new technique for linear seismometry (Bilger *et al.* 1990).

According to general relativity, the local Lorentz frame itself can rotate with respect to the fixed stars, or be 'dragged', by a nearby rotating object such as the earth. This Lense-Thirring field exemplifies the gravitomagnetic effects in general relativity which so far have been tested only indirectly (Nordtvedt 1988). An experiment to detect this frame dragging, based on mechanical gyros, is due to be put into orbit by the Space Shuttle in 1996. Scully *et al.* (1981) have considered the possibility of their detection by a land-based experiment using a ring laser. Since such effects arise at $10^{-10}\Omega_E$, rather than the 10^{-5}–$10^{-7}\Omega_E$ sensitivity of the present ring, rings of a larger area/perimeter ratio are required.

(2c) Acceleration in Dispersive Media

Equation (1) for the response of a ring laser to rotation is sufficiently general to hold under linear and angular acceleration to very high precision. However, there has been some interest recently in examining the validity of (1), especially in the presence of dispersive dielectric media, under rotational and linear acceleration (Post 1972; Kuriyagawa and Mori 1979; Takahashi 1985; Fabri and Picasso 1989; Scorgie 1990, 1991; Kowalski *et al.* 1992). We mention also the increasing development of fibre optic gyro systems, e.g. Dennis *et al* (1991); however we consider the lack of solid material in our basic ring to be a potential advantage in reducing field- and stress-induced biases, as well as error and noise sources.

(2d) Fresnel Drag and Special Relativity

A popular application of ring laser precision is in tests of special relativistic effects in the dragging of the speed of light in moving media. The classic experiments of Zeeman (1920) have now been considerably improved by Bilger and Stowell (1977) and by Sanders and Ezekiel (1988), although in each case the analysis and comparison with theory has left something to be desired. In the former case there is a residual small discrepancy between the theoretical estimate and the experimental value. In the latter case some questionable assumptions in the theory need examination, for example the use of the refractive index of the sample with respect to air in lieu of that in vacuo, and various problems of nonreciprocity of beam path in view of the use of nonperpendicular moving medium boundaries. The presence of counterrotating beams which may physically coincide makes the ring laser an ideal instrument for Fizeau-type drag experiments, with greatly increased precision and accuracy. Further experiments of this type would be one obvious test for a ring laser with increased precision.

(2e) Tests of Preferred-frame Theories

Ring interferometry *per se* has a long history in tests of relativity, going back at least to the 1851 experiments of Fizeau on Fresnel drag, but still has a confused status. Stedman (1972, 1973) and Anderson and Stedman (1977, 1992) pointed out some misconceptions arising from a neglect of the logically distinct role of ring interferometry in the kinematical development of special relativity. Tests of relativity have often been motivated at this kinematic level. The test theory of Mansouri and Sexl (1977) has been widely used to motivate Michelson–Morley-like and other experiments with linear lasers (see for example Riis *et al.* 1988; Will 1992*a,b*), albeit still in a form which is flawed by the claim of permitting one-way measurements (Vetharaniam and Stedman 1991, 1992). In the same vein one might well motivate searches for violations of special relativity in the conjugate ring geometry (Post 1972; Stedman *et al.* 1991) by postulating a parity-violating preferred-frame test theory.

For example, let us suppose that a boost, induced by the rotation of the earth carrying the ring laser into a new frame which is approximately inertial, generates a gravitomagnetic field $\mathbf{h}$ ($h_i = g_{0i}$) which is proportional to the velocity $\mathbf{v}$ relative to the preferred frame. It is customary but not necessary to identify the preferred frame as that in which the microwave background is isotropic. Let the proportionality constant be a parameter σ :

$$\nabla \times \mathbf{h} = \sigma \mathbf{v} . \tag{2}$$

From equation (1) in its general relativistic form (Post 1967; Anandan 1981), as applied to the earth-rotation-induced beat frequency in the Canterbury ring laser, we would have a (sidereal) diurnal variation in the beat frequency given by $\delta f = \delta[2f_0 \mathbf{A} \cdot \nabla \times \mathbf{h}/P] = 2f_0 A\sigma v \delta(\cos\theta)/P$, where $\delta(\cos\theta)$ is the maximum diurnal change in the projection $\cos\theta$ of the direction of the ring area $\hat{\mathbf{A}}$ on the direction $\hat{\mathbf{v}}$ of the velocity with respect to the preferred frame, on the assumption that the magnitude v of this velocity is much greater than its diurnal variation. We take the magnitude and declination of the preferred frame velocity from Narlikar *et al.* (1991) as $v \approx 6{\times}10^5$ m s^{-1} and $\delta \approx -26°$ so that, at the latitude of Christchurch ($\lambda \approx -43°.5$), $\delta(\cos\theta) = \cos(\lambda - \delta) - \cos(\pi - \lambda - \delta) = 1{\cdot}30$. Hence $\delta f/\sigma \approx 1{\cdot}6 \times 10^{20}$ m^2 s^{-2} and an experiment searching for a variation in the

Canterbury ring laser beat frequency with the Fourier period of one sidereal day and amplitude $\Delta\omega$ would give an estimate of σ as

$$\sigma \approx P\delta f/2{\cdot}6 f_0 A v; \qquad \sigma(\mathrm{s/m}^2) \sim 6\times10^{-21}\delta f\ (\mathrm{Hz})\,. \tag{3}$$

Equation (3) shows that at a beat frequency precision of 1 mHz we may set a bound on σ to an accuracy of the order of 10^{-23} s/m^2.

Why might we do such a test? While a purely kinematic postulate of such a gross assumption in flat spacetime and in the absence of matter has little appeal, the Mansouri–Sexl (1977) test theory was equally kinematic in motivation, but has still proved useful. One could argue, as did Okun (1988), on the possibility of testing the exclusion principle (whose possible violation, as Okun was concerned to show, is even less credible), that 'in fundamental physics if something can be tested it should be tested.' Franklin (1986) has emphasised, and recent stirring events in condensed matter verify, that novel experiments do not require a supporting theory; physics is an experimental science. Telegdi (1990) has emphasised such points in the context of a warm tribute to the skill of Michelson and Gale (1925) in their historic interferometric measurement of the Sagnac effect from earth rotation.

However, most physicists prefer to have a dynamic model, for example a Lorentz-invariance-violating term in the Lagrangian underpinning any test theory to motivate an experiment. The kinematic approach of Mansouri and Sexl (1977) for linear tests of special relativity has often been replaced (Will 1992*b*) with the dynamic model of the $TH\epsilon\mu$ theory. In this the assumption is made that Lorentz invariance may be broken by differentiating the speed of light and the limiting speed of matter. This still permits an elegant classical Lagrangian formulation and most importantly a quantised form which enables the use of high-precision atomic spectroscopy to place more accurate bounds on the parameters (Will 1992*b*).

Parity-violating models of gravitation have been of renewed interest (Gibbons 1992). Possible effects from a gravitational anomaly have been discussed by Dolgov *et al.* (1988, 1990). Ni (1977) has proposed, and Ritter *et al.* (1990) have tested, an alternative and parity-violating gravity theory in which a postulated constitutive tensor density, dependent in part on a scalar function of the gravitational fields, gives rise to anomalous torques on electromagnetically interacting and polarised bodies. The possible effects of spacetime torsion have been discussed by a number of authors such as Hojman *et al.* (1980) who showed that with torsion, gravitational theories admit a parity-violating term in the action. Hehl *et al.* (1976, 1992) considered the Einstein–Cartan theory of gravity whose non-zero torsion tensor is proportional to the antisymmetric part of the connection, and which gives rise to a non-propagating torsion inside matter. Hojman *et al.* (1978, 1979) developed a theory using Cartan's torsion tensor but departed from the Einstein–Cartan theory by modifying the concepts of gauge invariance and minimal coupling, thus obtaining a propagating torsion even within a vacuum. Propagating torsion was also discussed by Hammond (1990). Moffat (1989, 1990) considered the physical consequences of his nonsymmetric gravitation theory in which the fundamental geometric object is the nonsymmetric connection compatible with a complex, nonsymmetric metric. Others (Mashhoon 1975, 1988, 1989; Gabriel *et al.* 1991) have discussed the electromagnetic effects of gravitational coupling to the rotation of the earth.

Dynamic support for an effect related to that of equation (2) was illustrated within the parametrised post-Newtonian (PPN) formalism by Scully *et al.* (1981). A variation in the apparent earth rotation rate with the period of one sidereal day could be interpreted in terms of a nonvanishing value for the preferred-frame PPN parameter α, which vanishes in general relativity. Presently α is bounded to vanish to 1 part in 10^4 (Will 1992*b*).

Other derivations of equation (2) from the rather more drastic parity-violating modifications referenced above can be expected to yield models that will also justify a search for bounds on the parameter σ.

(2f) Small Material Nonreciprocities

Nonreciprocal physical effects giving absolute phase shifts of 10^{-10} rad between counterrotating beams should be detectable (Stedman 1985; Ross *et al.* 1989; Stedman 1992). This would allow sensitive tests of nonlinear optical effects in gases associated with the chiral effects of electric and magnetic fields. Field-induced magnetic linear dichroism in gases is one obvious candidate. In chemical physics, absolute measurements of such parameters are often very difficult, and the determinations of the hyperpolarisabilities of helium by Buckingham and Dunmur (1968) and the quadrupole moment of CO_2 by Buckingham and Disch (1963) (see also Buckingham 1968) have stood unimproved for many years. Such devices as the ring laser give hope of new results in these directions. Finally, rotation could itself make an atomic gas optically active at a very low level (Silverman 1989; Stedman 1990).

(2g) Anomalous Interaction with Pseudoscalars

The anomaly or triangle diagram occurring in the quantum electrodynamics of pseudoscalar particles has been the focus of an extended literature. Since it is concerned with a parity-violating effect, a ring laser detection system has some relevance.

There has been speculation that even within QED, triangle diagrams provide physically detectable effects (Stedman and Bilger 1987). We are now convinced that such speculation is not well founded (Ross and Stedman 1988; we may note in passing that the work of Maiani *et al.* 1986 is not thereby compromised as Ross and Stedman suggested, since their ϕ does not correspond to measurement of optical activity). However, as discussed above, a novel experiment is worth doing whether or not theorists are sufficiently inventive to get a plausible test theory to justify it.

One can search for upper bounds on electric and/or magnetic field-induced optical properties of the vacuum. The Canterbury ring could set sensitive bounds on such effects. Like the above-mentioned special relativity test, these are particularly clean experiments, requiring no medium and consequently no degradation of cavity finesse. In many respects the vacuum behaves as a nonlinear medium for quantum optical experiments. It can be squeezed, in that some field amplitude fluctuations can be reduced below the levels suggested by the uncertainty principle, at the expense of others. According to QED, the most well-tested of all physical theories, the vacuum is predicted to permit nonlinear processes such as light–light scattering, but at levels below detection by our present ring, although Ni *et al.* (1991) have proposed an interferometric technique for achieving the required sensitivity of 10^{-25} in a refractive index measurement.

The triangle diagram has noncontroversial application when the anomaly is external, and the pseudoscalar particle exists in nature, as for example in the analysis of $\pi^0 \rightarrow \gamma\gamma$ decay. Various candidates for new neutral pseudoscalar particles have been proposed, including the axion (e.g. Sikivie 1992), the arion and the majoron (Fischler 1991). In principle, these if they exist will interact by the axial anomaly or triangle diagram to couple with two photons. Searches have been conducted for axions, in which one photon is that of a magnetic field, and an optical effect is sought to reveal the otherwise invisible particle. The coupling strength is set through the mass of the pseudoscalar. If (as for Hagmann *et al.* 1990) a tunable microwave cavity were included in the ring laser optical path, it would be possible in principle to detect the effects of circularly polarised 633 nm photons created through the anomaly coupling with incoming pseudoscalars. The microwave magnetic field tunability of say 4 GHz is admittedly only a few ppm of the laser photon energy, 2·0 eV, an energy range which nevertheless is not covered by present axion searches (Sikivie 1992).

3. Canterbury Ring Design and Performance

(3a) Noise Limits

The fundamental limits on the resolution attainable with a ring laser are determined by quantum noise (Dorschner *et al.* 1980; Hellwig 1975) and $1/f$ noise. The transition between these two noise sources is clearly evident in the earlier work of Bilger and Sayeh (1983), and when $1/f$ noise takes over, further time averaging does not improve the data. However, both of these are reduced by a factor of order $1/P^n$, where P is the perimeter and $n \sim$ 2–3, by using larger rings through the increasing finesse and the decreasing solid angle in backscatter. When this is done, as in our present ring, $1/f$ noise is essentially eliminated in that the ultimate resolution is restricted by other considerations such as laser stability. Methods such as quantum nondemolition measurement, which extend the resolution of optical systems to break the barrier imposed by quantum noise, are not of prime interest in this application at least at this stage; our strategy is to lower, rather than breach, the standard quantum noise limit itself. The introduction of squeezed light, say by introducing a nonlinear optical element into the cavity, while an exciting theoretical possibility, could in practice risk the very quality of cavity finesse that led to the lowering of the standard quantum limit. Our success in this direction, without resorting to squeezing etc., again illustrates that the potential of the ring laser for precision measurements is still quite underdeveloped.

In confirmation of these comments, we write the power spectral density of the beat frequency fluctuations as $S_{\delta f} = h_0 + h_{-1}/f$, where f is the Fourier frequency. The coefficient for quantum noise is given by $h_0 = 2hf_0^3 3/P_0 Q^2$ with the value $h_0 = 3 \times 10^{-6}$ Hz^2/Hz for a total optical power loss $P_0 = 0{\cdot}1\ \mu$W and the ideal cavity quality factor Q; f_0 is the laser frequency of 474 THz;

$h_{-1} = 8f_0^2/Q^4$ (this is an empirical relation, for which see Sayeh and Bilger 1987). With a measurement time T, the rms frequency fluctuation of the beat frequency is then

$$\Delta f_{rms} = [2hf_0^3/Q^2 P_0 T]^{\frac{1}{2}} . \tag{4}$$

The analysis leading to equation (4) assumes that the cw and ccw beams are uncorrelated; while methods for inducing and exploiting a correlation for noise reduction are now well discussed and physically demonstrated, there is no immediate prospect of using this in systems such as ours. With the above specifications and $T = 1$ h, a frequency fluctuation of 20 μHz can be achieved in principle, corresponding to a frequency resolution $\Delta f/f_0 = 4 \times 10^{-20}$. Other interferometric techniques have been proposed (Ni *et al.* 1991) which could improve on this figure, and which would correspond to the performance that we may expect of a ring laser with an area of say 50 m^2 (as in Scully *et al.* 1981). From this viewpoint, even the earlier ring lasers have already outclassed (Bilger and Sayeh 1986) other precision tools such as the Mössbauer effect (Pound and Snider 1965) and the maser (Vanier 1982).

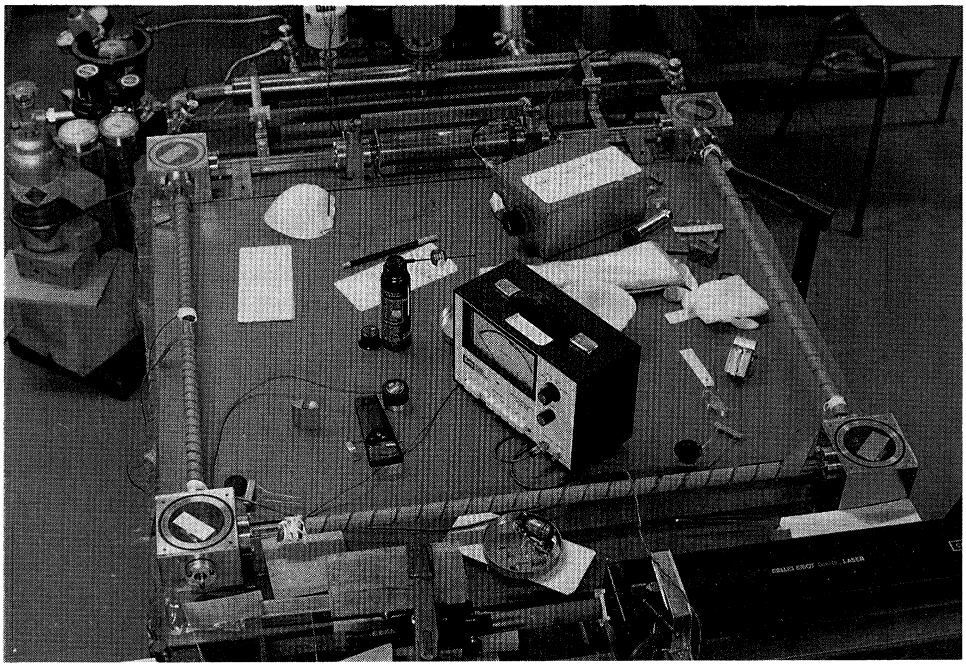

Fig. 1. The Canterbury ring laser. The RF excitation section is in the topmost leg, and the superinvar mirror mounts are visible through the glass lids of the corner boxes. The green helium–neon laser for alignment is in the foreground.

(3b) Construction

The Canterbury ring laser (Fig. 1) is designed to realise the potential of the recent advances. So that the cavity finesse is maximised, the beam paths are entirely in the helium–neon gas; the Rayleigh scattering of neutral neon gas at 300 Pa has a negligible effect on the quality factor. Neither do the beams intersect any interface, although for some applications Brewster windows will be both necessary and tolerable. Supermirror coatings were generously provided by Ojai Research. Zerodur blanks were used which have 0·1 nm rms surface roughness (i.e. smoothness to atomic dimensions), achieved through ion beam milling with argon and nitrogen beams. Ultrahigh vacuum coated SiO_2/TiO_2 $\lambda/4$ layers give losses of 4 ppm due to scattering (TIS), 4 ppm from transmission, and absorption (by difference) of 7 ppm, approaching that of the bulk materials. The observed total reduction in reflection is then 15 ppm, corresponding to a reflectivity of 99·9985%. After extended use in our environment the total losses increased significantly.

The radio frequency excitation mechanism is novel, involving a magnetic coupling at 50 MHz. The laser gas is 7:1 He:Ne with a nominal total pressure of 2·3 Torr (300 Pa); however the latter can successfully be varied over an order of magnitude. A natural mixture of neon isotopes (Ne^{20}:Ne^{22} = 9:1) and natural helium is used at present.

For further mechanical and thermal stability the mirrors are mounted in superinvar holders resting on a 1×1 m^2 Zerodur plate, itself on a granite base. The stainless steel corner boxes avoid mechanical contact with the mirrors; their bottoms are open, and are sealed by Viton O-rings against the Zerodur plate.

A triangular ring was used in earlier lasers such as that of Bilger and Stowell (1977). We have used a square ring primarily to optimise the parameter $G = A/PN$, where A is the area, P the perimeter and N the number of mirrors. From earlier equations, together with the dependence of Q on mirror losses, G is proportional to the signal/noise ratio of the system. Since there is an odd number of reflections, each of which reverses handedness of the beam, a triangular ring cannot be put in circularly polarised mode. A square ring could be put in circularly polarised mode, should applications require it, for example by introducing nonplanarity (Bilger *et al.* 1990), another impossibility with a triangle. The increase in angle of incidence means that backscatter is reduced; indeed, Lambert backscatter formally vanishes at 45° incidence. Together these measures help to avoid locking.

Partly to preserve the advantages sought from a simple open design with mechanical and thermal isolation for the mirrors, and partly on account of cost, no feedthroughs are installed at this stage in the corner boxes. Hence alignment to 20 arcsec in angle and 10 μm in position has to be achieved before adding vacuum sealed covers to the corner boxes, pumping down, gas handling and initiating the lasing. Mirror holders were machined to locate the poles to this precision. A green helium–neon laser beam was overlaid with that of a standard red laser to preserve alignments under a sequence of operations where the mirrors were rotated from retroreflecting configurations, in which tilt about a horizontal axis was adjusted and from which mirrors were rotated about a vertical axis to reach the final positions. The output interferometer was mounted on top of the glass cover for one corner box. Its components were aligned to overlay the emergent beams to a precision of 0·3 mrad.

This device will shortly be taken to an underground cavern giving further thermal and mechanical stability. The results reported here are therefore only preliminary.

(3c) Results

The Canterbury ring generates a nominal beat frequency δf of 68·826 Hz, given the latitude of 43° 29′ South, (vacuum) wavelength $\lambda = 633\cdot 0$ nm, perimeter $P = 3477\cdot 1 \pm 0\cdot 1$ mm (measured from the free spectral range of 86·218 MHz determined from the beat frequency between neighbouring longitudinal modes) and area $A = 0\cdot 748$ m^2 (given with lesser accuracy from the two sides of 898 mm, 837 mm). Since the gain curve for natural neon has a width of order 1 GHz, several longitudinal modes could readily be excited, and of course the intermediate transverse modes. A fused silica tube of length 30 cm and 4 mm internal diameter served both as RF-excited amplifier and as an aperture. Our design aimed to achieve single mode operation by reducing the gain so that (thanks to the variation of gain with wavelength) all longitudinal modes except one would be starved. In principle this required output power reduction to the manageable level of 30 nW. In practice, we were pleasantly surprised at the ease at which such gain control achieved single mode excitation of the ring. Typical RF powers for single mode excitation were 5–7 W, and output beam powers were then indeed of the order of 30 nW. For higher excitation power, say 30 W, output beam powers reached 2 μW, which corresponds (since mirror transmission is 4 ppm) to a circulating power of 0·5 W. Although high circulating powers reduce quantum shot noise, they induce nonlinear effects in mirror media, and as noted by Chow *et al.* (1985) multimode rings give new branches in the beat-frequency–rotation-rate plot. Mode structure was monitored with a Newport SR-130 scanning Fabry–Perot.

The ringdown decay time of the ring was measured to be approximately $\tau = 15\,\mu$s by monitoring lasing output as the RF is turned off using a digital storage oscilloscope. This translates into a quality factor $Q = \omega\tau = 4\cdot 5 \times 10^{10}$, noticeably below the ideal. The causes of this long-term contamination of mirror coatings have been identified, and within our limited budget we are working towards their elimination.

The interference fringes on detection were processed by a Strobes Acquisition-PC data collection system (equivalent to Rapid Systems R380), which permitted runs of 16·384 s with a 2 ms sampling time. A spectrum obtained is given in Fig. 2.

While the 50 Hz signal from mechanical motion induced at the frequency of the New Zealand mains power supply, together with its harmonics, is conspicuous, the spectrum is dominated by the line associated with earth rotation in the region 67–78 Hz. Even the subsidiary features of the spectrum are sharp; the mechanical resonances of the ring support system (at 15 and 19 Hz for example), which appear at those frequencies and as sidebands to the main features, reflect their relatively high mechanical quality factor (measured to be 34 ± 1). Indeed in our current site on the sixth level of a multi-storey building facing the prevailing wind, data of the quality of Fig. 2 can only be obtained under relatively wind-free conditions and late at night, when other uses of the building are minimal.

The earth-rotation-induced signal is almost as sharp as the mains frequency line. Its width is dominated by the windowing resolution of up to 160 mHz (Fig. 3). However, a careful analysis (Stedman and Bilger 1992) has shown that the net optical line width of the beat frequency may be extracted by deconvolution, and by using the second and third harmonics of the earth-rotation-induced signal

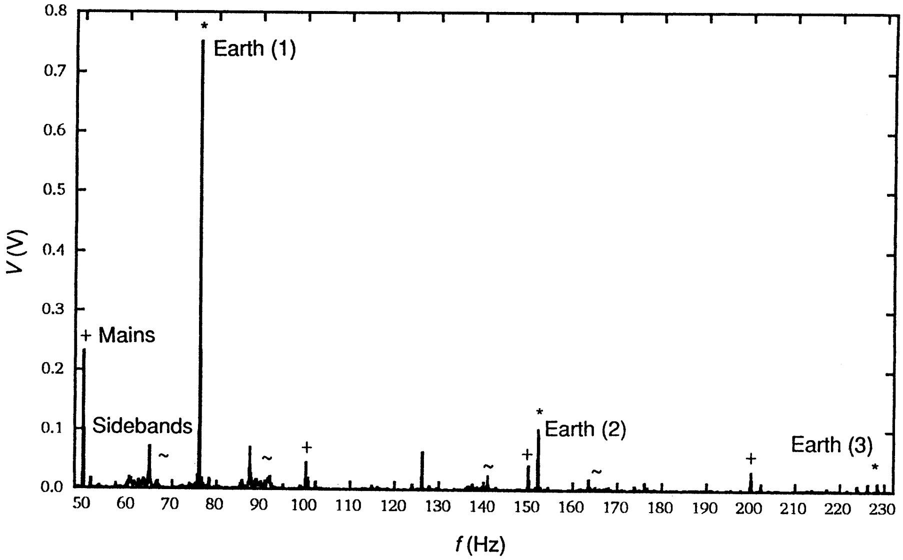

Fig. 2. A spectrum from the Canterbury ring. The beat signal from earth rotation, together with its two higher harmonics, are marked by an asterisk. Sidebands arising from the mechanical resonance of the support are marked by a tilde, and mechanical rotation from the AC mains with its harmonics by a plus sign.

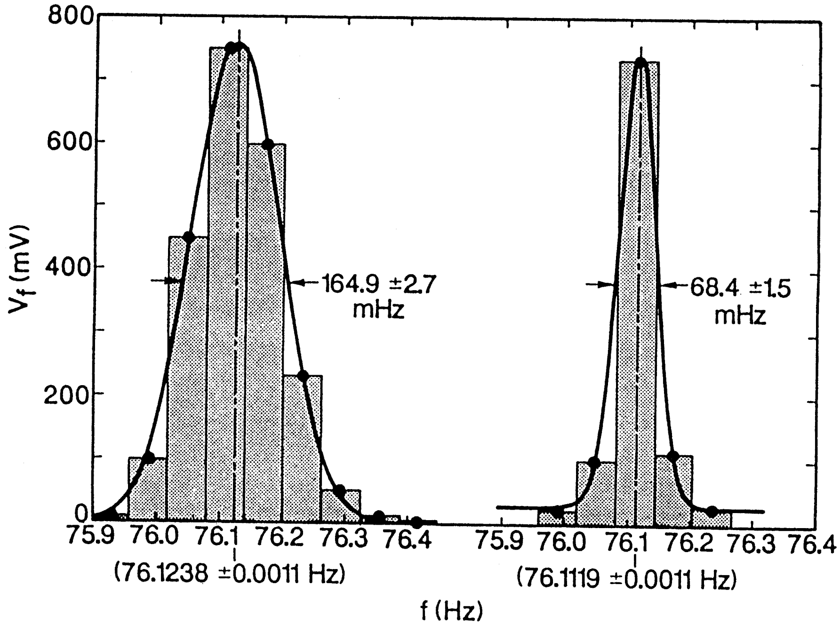

Fig. 3. The effect of windowing on the raw data for the fundamental earth rotation line in Fig. 2; on the left is a Nuttall window (Stedman and Bilger 1992), and on the right a square window corresponding to the time gate of 16·384 s.

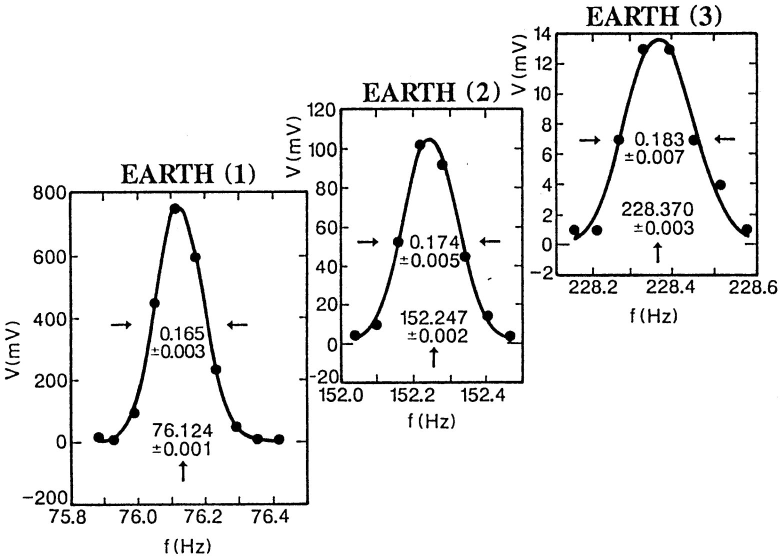

Fig. 4. A comparison of the widths of the raw data for all harmonics in Fig. 2. The fitted curve is a convolution of the known window function and a net laser line shape function, whose width is thus determined consistently to be 33 mHz. As noted in Fig. 3 and in the text, the line positions may thus be determined to 1·0 mHz, or $2{\cdot}1\times10^{-18}$ of the laser frequency.

(Fig. 4) it is possible to check the consistency of this procedure, and obtain at present a laser beat frequency full width at half maximum (FWHM) of $32{\cdot}7\pm1{\cdot}7$ mHz for the runs of 16·384 s duration. This is an order of magnitude larger than the expected quantum noise limit. Most importantly, the line profile can be fitted to a Gaussian and the position of the centre of the line obtained to a precision of 1·0 mHz; this estimate also was reduced by a comparison of harmonics. This justifies our claim for a fractional frequency resolution capability of $2{\cdot}1\ \times10^{-18}$. The earth-rotation-induced line position in our as yet unstabilised ring drifts typically by several hertz in a few minutes. Such drift during the sampling time explains the difference between the observed line width and the quantum noise limit. Shorter sampling times lead to wider windows, and to no improvement in resolution; at this stage our maximum data collection time is (fortuitously) optimal.

We attribute this drift to the well-known effects of the susceptibility variation over the composite neon gain curve when the path length, and with it the resonance frequency, varies, compounded by saturation effects (Aronowitz 1971; Sargent *et al.* 1974; Siegman 1986). Thermal expansion effects even in Zerodur would induce shifts of the cavity modes within the gain curve by several per cent of the free spectral range per Celsius degree, and the counterpropagating waves sample the gain curve at different frequencies. The difference in optical path length for these modes itself depends on the position within the gain curve. We are therefore deferring the proposed test of relativity suggested earlier in this paper until present work on stabilising the cavity modes within the gain curve via feedback is completed. The forthcoming transfer to the cavern will also markedly help reduce drifts and mechanical interference.

The cavity parameters appropriate for the original mirrors and for the results given above are given in Table 1, together with comparative figures for the Newport SR-130 supercavity.

Table 1. Ring parameters

We give the performance limit obtainable in principle given supercavity mirror parameters as measured by the manufacturer, the performance actually attained in our environment, and for comparison the corresponding figures for the Newport SR-130 scanning Fabry–Perot supercavity (Li *et al.* 1990). In all cavities, the quality factor is $Q = f_0/\Delta f_{\frac{1}{2}} = \omega\tau$, where $f_0 = \omega/2\pi = 474$ THz; the finesse is $F = S/\Delta f_{\frac{1}{2}} = Q\lambda/P$, where the free spectral range (FSR) is $S = c/P$, P is the round trip length and $\Delta f_{\frac{1}{2}}$ is the power FWHM of the cavity response. In a Fabry–Perot, $TF = \pi$ where the mirror power transmittance $T = 1 - R$, and $\Delta f_{\frac{1}{2}} = cT/\pi P$; in a four-mirror ring, $TF = \pi/2$ and $\Delta f_{\frac{1}{2}} = 2cT/\pi P$. The nonreciprocal refractive index and rotational velocity limits are estimated as in Stedman *et al.* (1987)

	Ring laser		Fabry–Perot
	Design	Achieved	SR-130
Perimeter P (mm)	3477·1		50·6
FSR S (GHz)	0·086218		6·0
Cavity finesse F	~200 000	30 000	85 000
FWHM $\Delta f_{\frac{1}{2}}$ (kHz)	~0·5	10·6	70
Quality factor Q	~10^{12}	4×10^{10}	7×10^{9}
Resolution δf	~20 μHz	1·0 mHz	70 kHz
averaging time	1 h	16 s	
relative $\delta\nu/\nu$	~4×10^{-20}	$2\cdot1\times10^{-18}$	
rotation (rad/s)	10^{-10}		
refractive index	10^{-20}		

At this stage we estimate the beat frequency to be stable to the order of Hz over a time scale of hours, and that σ in equation (3) vanishes to a precision of the order of 10^{-19} s/m^2. It is hoped to improve this bound significantly in the future. This interferometric determination of the earth rotation rate already makes an interesting comparison with the historic interferometric measurements of Michelson and Gale (1925) using light and Werner *et al.* (1979) using neutrons.

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References

Al'tshuler, V. (1992). *Phys. Today* **45** (Feb.), 13.
Anandan, J. (1981). *Phys. Rev.* D **24**, 338.
Anderson, R., and Stedman, G, E. (1977). *Found. Phys.* **7**, 29.
Anderson, R., and Stedman, G. E. (1992). *Found. Phys. Lett.* **5**, 199.

Aronowitz, F. (1971). *In* 'Laser Applications' (Ed. M. Ross) **1**, pp. 133, 169 (Academic: New York).

Bilger, H. R., and Sayeh, M. R. (1983). *In* 'Noise in Physical Systems and $1/f$ Noise' (Eds M. Savelli *et al.*), pp. 325–8 (Elsevier: Amsterdam).

Bilger, H. R., and Sayeh, M. R. (1986). *In* 'Noise in Physical Systems and $1/f$ Noise' (Eds A. d'Amico and P. Mazzetti), pp. 293–6 (Elsevier: Amsterdam).

Bilger, H. R., and Stowell, W. K. (1977). *Phys. Rev.* A **16**, 313.

Bilger, H. R., Stedman, G. E., and Wells, P. V. (1990). *Opt. Commun.* **80**, 133.

Buckingham, A. D. (1968). *J. Chem. Phys.* **48**, 3827.

Buckingham, A. D., and Disch, R. L. (1963). *Proc. R. Soc. London* **273**, 275.

Buckingham, A. D., and Dunmur, D. A. (1968). *Trans. Farad. Soc.* **64**, 1776.

Chow, W. W., Gea-Banacloche, J., Pedrotti, L. M., Sanders, V. E., Schleich, W., and Scully, M. O. (1985). *Rev. Mod. Phys.* **57**, 61.

Clauser, J. F. (1988). *Physica* B **151**, 262.

Dennis, M. L., Diels, J.-C., and Lai, M. (1991). *Opt. Lett.* **16**, 529.

Dieks, D., and Nienhuis, G. (1990). *Am. J. Phys.* **58**, 650.

Dolgov, A. D., Kriplovich, I. B., and Zakharov, V. I. (1988). *Nucl. Phys.* B **309**, 591.

Dolgov, A. D., Kriplovich, I. B., Vainshtein, A. I., and Zakharov, V. I. (1990). *Nucl. Phys.* B **315**, 138.

Dorschner, T. A., Haus, H. A., Holtz, M., Smith, I. W., and Statz, H. (1980). *IEEE J. Quant. Electr.* QE-**16**, 1376.

Fabri, E., and Picasso, L. E. (1989). *Phys. Rev.* A **39**, 4641.

Fischler, W. (1991). *Phys. Lett.* B **266**, 408.

Forder, P. W. (1985). *J. Phys.* A **18**, 3343.

Franklin, A. (1986). 'The Neglect of Experiment' (Cambridge Univ. Press).

Gabriel, M. D., Haugan, M. P., Mann, R. B., and Palmer, J. H. (1991). *Phys. Rev. Lett.* **67**, 2123.

Giazotto, A. (1989). *Phys. Reports* **182**, 365.

Gibbons, G. W. (1992). Comments on long range parity violating forces. DAMPT R-91/17.

Hagmann, C., Sikivie, P., Sullivan, M. S., and Tanner, D. B. (1990). *Phys. Rev.* D **42**, 1297.

Hammond, R. T. (1990). *Gen. Rel. Gravitat.* **20**, 813.

Heer, C. V. (1964). *Phys. Rev.* **134**, A799.

Hehl, F. W., Lemke, J., and Mielke, E. W. (1992). *In* 'Bad Honnef Lectures' (Eds J. Debrus and A. C. Hirshfeld) (Springer: Berlin).

Hehl, F. W., von der Hyde, P., Kerlick, G. D., and Nester, J. M. (1976). *Rev. Mod. Phys.* **8**, 393.

Hellwig, H. W. (1975). *Proc. IEEE* **63**, 212.

Hendriks, B. H. W., and Nienhuis, G. (1990). *Quant. Optics* **2**, 13.

Hojman, R., Mukku, C., and Sayed, W. H. (1980). *Phys. Rev.* D **22**, 1915.

Hojman, S., Rosenbaum, M., Ryan, M. P., and Shepley, L. C. (1978). *Phys. Rev.* D **17**, 3141.

Hojman, S., Rosenbaum, M., and Ryan, M. P. (1979). *Phys. Rev.* D **19**, 430.

Kowalski, F. V., Murray, J., and Head, A. (1992). Interaction of light with an accelerating dielectric (submitted for publication).

Kuriyagawa, A., and Mori, S. (1979). *Phys. Rev.* D **20**, 1290.

Levi, B. G. (1991). *Phys. Today* **44** (July), 17.

Li, Z., Bennett, R. G. T., and Stedman, G. E. (1991). *Opt. Commun.* **86**, 51.

Macek, W. M., and Davis, D. T. M. Jr. (1963). *Appl. Phys. Lett.* **2**, 67.

Macek, W. M., Davis, D. T. M., Jr., Olthius, R. W., Schneider, J. R., and White, G. R. (1963). 'Symposium on Optical Masers', p. 199 (Polytechnic Press: New York).

Macek, W. M., Schneider, J. R., and Salamon, R. M. (1964). *J. Appl. Phys.* **35**, 2556.

Maiani, L., Petronzio, R., and Zavattini, E. (1986). *Phys. Lett.* B **175**, 359.

Mansouri, R., and Sexl, R. U. (1977). *Gen. Rel. Gravitat.* **8**, 497, 505, 819.

Mashhoon, B. (1975). *Phys. Rev.* D **11**, 2679.

Mashhoon, B. (1988). *Phys. Rev. Lett.* **61**, 2639.

Mashhoon, B. (1989). *Phys. Lett.* A **139**, 103.

Michelson, A. A., and Gale, H. G. (1925). *Astrophys. J.* **61**, 140.

Moffat, J. W. (1989). *Phys. Rev.* D **39**, 474.

Moffat, J. W. (1990). Review of the nonsymmetric gravitation theory, Lectures given at the Summer Institute on Gravitation, Banff, Alberta

Narlikar, J. V., Pecker, J. C., and Vigier, J. P. (1991). *Phys. Lett.* A **154**, 203.

Ni, W.-T. (1977). *Phys. Rev. Lett.* **38**, 301.

Ni, W. T., Tsubono, K., Mio, N., Narihara, K., Chen, S-C., King, S-K., and Pan, S-S. (1991). *Mod. Phys. Lett.* A **6**, 3671.

Nordtvedt, K. (1988). *Phys. Rev. Lett.* **61**, 2647.

Okun, L. B. (1988). *Comments Nucl. Part. Phys.* **19**, 99.

Post, E. J. (1967). *Rev. Mod. Phys.* **39**, 475.

Post, E. J. (1972). *J. Opt. Soc. Am.* **62**, 234.

Pound, R. V., and Snider, J. L., (1965). *Phys. Rev.* **140**, B788.

Riehle, F., Kisters, Th., Witte, A., and Helmcke, J. M. (1991). *Phys. Rev. Lett.* **67**, 177.

Riis, E., Andersen, L.-U. A., Bjerre, N., Poulsen, O., Lee, S. A., and Hall, J. H. (1988). *Phys. Rev. Lett.* **60**, 842.

Ritter, R. C., Goldblum, C. E., Ni, W.-T., Gillies, G. C., and Speaker, C. G. (1990). *Phys. Rev.* D **42**, 977.

Robertson, D. S. (1991). *Rev. Mod. Phys.* **63**, 899.

Ross, H. J., and Stedman, G. E. (1988). *Phys. Lett.* A **132**, 229.

Ross, H. J., Sherborne, B. S., and Stedman, G. E. (1989). *J. Phys.* B **22**, 459.

Sanders, G. A., and Ezekiel, S. (1988). *J. Opt. Soc. Am.* B **5**, 674.

Sargent, M. III, Scully, M. O., and Lamb, W. E. Jr. (1974). 'Laser Physics', Ch. 10 (Addison–Wesley: New York).

Sayeh, M. R., and Bilger, H. R. (1985) *Phys. Rev. Lett.* **55**, 700.

Scorgie, G. C. (1990). *J. Phys.* A **23**, 5169.

Scorgie, G. C. (1991). *Eur. J. Phys.* **12**, 64.

Scully, M. O., Zubairy, M. S., and Haugan, M. P. (1981). *Phys. Rev.* A **24**, 2009.

Siegman, A. E. (1986). 'Lasers', pp. 1193ff (University Science Books: Mill Valley).

Sikivie, R. (1992). Axion searches. UFIFT-HEP-91-29.

Silverman, M. P. (1989). *Europhys. Lett.* **9**, 95.

Statz, H., Dorschner, T. A., Holtz, M., and Smith, I. W. (1985). *In* 'Laser Handbook', Vol. 4 (Eds M. L. Stitch and M. Bass), p. 229 (North Holland: Amsterdam).

Stedman, G. E. (1972). *Am. J. Phys.* **40**, 782.

Stedman, G. E. (1973). *Am. J. Phys.* **41**, 1300.

Stedman, G. E. (1985*a*). *Contemp. Phys.* **26**, 311.

Stedman, G. E. (1985*b*). *Adv. Phys.* **34**, 513.

Stedman, G. E. (1986). *In* 'Quantum Optics IV' (Eds J. D. Harvey and D. F. Walls), pp. 259–66 (Springer: Berlin).

Stedman, G. E. (1990). *Phys. World* **3**, 23.

Stedman, G. E. (1991). *Phys. Lett.* A **152**, 19.

Stedman, G. E. (1993). *In* 'Modern Nonlinear Optics, Part 2' (Eds S. Kielich and M. W. Evans) (Wiley: New York).

Stedman, G. E., and Bilger, H. R. (1987). *Phys. Lett.* A **122**, 289.

Stedman, G. E., and Bilger, H. R. (1992). *Digital Signal Proc.* **2**, 105.

Stedman, G. E., Bilger, H. R., and Kowalski, F. V. (1991). *Aust. & N.Z. Physicist* **28**, 16.

Takahashi, Y. (1985). *Phys. Lett.* A **113**, 5.

Telegdi, V. L. (1990). Mind over matter: the intellectual content of experimental physics. CALTECH ERN-90-09.

Vanier, J. (1982). *Metrologia* **18**, 173.

Vetharaniam, I., and Stedman, G. E. (1991). *Found. Phys. Lett.* **4**, 275.

Vetharaniam, I., and Stedman, G. E. (1992). Significance of precision tests of special relativity (submitted for publication).

Werner, S. A., Staudenmann, J-L., and Colella, R. (1979). *Phys. Rev. Lett.* **42**, 1103.

Will, C. M. (1992*a*). *Phys. Rev.* A **45**, 403.

Will, C. M. (1992*b*). The confrontation between general relativity and experiment: a 1992 update. *Int. J. Mod. Phys.* A (to be published).

Zeeman, H. (1920). *Proc. R. Acad. Sci. (Amsterdam)* **22**, 462, 512.

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