Both values, however, give a critical half thickness below 500 Å. Thus, we see that a film somewhat thinner than 1000 Å *could* break up into a mixed state. Whether it does, however, depends on whether it is energetically favorable to do so. There is, though, one further piece of information. Even though κ is greater than 0.707 in a thin film, the order will probably be constant across the film thickness d, for we can show that $L > d$ for $d < d_{\text{critical}}$. From (35)

$$a/L = \frac{A}{0.91}\left[\frac{H_c(t)}{H_0}\frac{\lambda_L(t)}{\lambda_L(0)}\right](a/\xi_0)^{1/2}. \qquad (42)$$

For diffuse scattering, which gives the slightly larger estimate, we obtain from (42) the result

$$a/L = 0.45, \quad \text{at} \quad T = 0^\circ\text{K}, \quad a = a_{\text{critical}}. \qquad (43)$$

For thinner films, higher temperatures, or specular scattering, a/L is even smaller. Thus we see that the order probably does not vary across the film thickness in a thin film.

To summarize our results, we have shown that a simple nonlocal model previously used with some success to calculate the critical fields of superconducting films can also be used to calculate the Ginzburg-Landau-Abrikosov-Gor'kov parameters such as the weak field penetration depth δ_0, the dimensionless parameter κ, and the range of order parameter $L = \delta_0/\kappa$. We have calculated these quantities in the local limit and have shown that the results are in good agreement with those obtained by other techniques. We have also obtained simple expressions for these parameters which are valid in the thin limit. From these limiting formulas, we show that κ can get large in a thin film and estimate that κ exceeds the critical value of $1/\sqrt{2}$ in indium films somewhat thinner than 1000 Å. We also conclude that in the thin limit, one would expect the order parameter to be constant across the film thickness.

Test of Special Relativity or of the Isotropy of Space by Use of Infrared Masers*

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The highly monochromatic frequencies of optical or infrared masers allow very sensitive detection of any change in the round-trip optical distance between two reflecting surfaces. Hence, comparison of the frequencies of two masers with axes perpendicular to each other allows an improved experiment of the Michelson-Morley type, or a very precise examination of the isotropy of space with respect to light propagation. Two He-Ne masers were mounted with axes perpendicular on a rotating table carefully isolated from acoustical vibrations. Their frequency difference was found to be constant to within 30 cps over times as short as about one second, or to one part in 10^{13} of the maser frequency, which is near 3×10^{14} cps. Rotation of the table through 90° produced repeatable variations in the frequency difference of about 275 kc/sec, presumably because of magnetostriction in the Invar spacers due to the earth's magnetic field. Examination of this variation over six consecutive hours shows that there was no relative variation in the maser frequencies associated with orientation of the earth in space greater than about 3 kc/sec. Hence there is no anisotropy or effect of either drift larger than 1/1000 of the small fractional term $(v/c)^2$ associated with the earth's orbital velocity. This preliminary version of the experiment is more precise by a factor of about 3 than previous Michelson-Morley experiments. There is reason to hope that improved versions will allow as much as 2 more orders of magnitude in precision, and that similar techniques will also yield considerably improved precision in an experiment of the Kennedy-Thorndike type.

OPTICAL and infrared masers make possible and attractive a number of new experiments, and refinements of old ones, where great precision in measurement of length is needed. On type is the examination of the isotropy of space for light propagation, or more specifically the examination of what effects the earth's velocity or various other fields may have on the velocity of light. We have completed the first stages of an experiment with He-Ne masers which can be regarded as equivalent to a Michelson-Morley experiment of improved precision. These preliminary tests show that the effect of "ether drift" is less than 1/1000 of that which might be produced by the earth's orbital velocity.

I. INTRODUCTION

A synoptic treatment of the connections between measurements in coordinate systems in relative motion

has been given by Robertson,[1] whose discussion will be followed here.

In a four-dimensional Euclidean space, one may define the metric as

$$d\sigma^2 = dt^2 - (1/c^2)(dx^2 + dy^2 + dz^2) \qquad (1)$$

in a particular coordinate frame called the "rest system." Clocks and rods are used to measure time-like or space-like intervals, respectively. In a second coordinate system moving along the x axis at velocity v with respect to the first one, the metric becomes

$$d\sigma^2 = g_0^2 dt'^2 - (1/c^2)[g_1^2 dx'^2 + g_2^2(dy'^2 + dz'^2)], \qquad (2)$$

where the intervals dt', dx', etc., are measured by clocks and rods in the moving system similar to those used in the rest system. Robertson[1] shows that

$$g_0 = [1 - (v/c)^2]^{1/2} a_0^0,$$
$$g_1 = [1 - (v/c)^2]^{1/2} a_1^1, \qquad (3)$$
$$g_2 = a_2^2,$$

where a_0^0 represents the transformation coefficient between dt and dt', a_1' that between dx and dx', and a_2^2 that between dy and dy'. Examination of the transformation between two such cordinate systems thus involves determination of three independent quantities, a_0^0, a_1^1, and a_2^2, or g_0, g_1, and g_2. Special relativity predicts, of course, that $g_0 = g_1 = g_2 = 1$.

Usually the "rest system" is assumed to be one which is stationary with respect to the fixed stars and is taken, as in (1), to be isotropic. Actually, in such a system there is no *a priori* reason why at the earth the metric need be isotropic, since matter and fields are not isotropic about the earth, at least on a local scale. Hence even in the "rest" system one must regard the assumption $g_0^2 = g_1^2 = g_2^2 = 1$ in (1) as only empirically established, and probably not exact. Thus an examination with very high precision of the values of these three quantities serves not only as a test of special relativity, but also of whatever other anisotropy there may be in the propagation of light.[1a]

As Robertson explains,[1] the Michelson-Morley experiment, the Kennedy-Thorndike experiment, and the Ives-Stilwell experiment in concert determine experimentally the three independent quantities g_0, g_1, and g_2 since they measure g_1/g_2, g_0/g_1, and g_0, respectively.

The time transformation, or g_0, was measured by Ives and Stilwell[2] with great precision in 1938. This experiment determined the quantity $1 - a_0^0$, or $\frac{1}{2}(v/c)^2$, to an accuracy of about one part in 30. A more recent

experiment[3,4] using the very high short-term frequency stability of an ammonia beam maser has examined g_0 to still greater precision. In this experiment, two ammonia masers were mounted with their beams moving in opposite directions, and changes in frequency upon rotation of the whole system through 180° were examined. Observation of zero change in the relative frequency of the two masers upon rotation can be shown[5] to verify the value of g_0 predicted by special relativity. Each one-day experiment of this type verified the expected value of $1 - a_0^0$ to an accuracy of about one part in 1000. Repetition of the experiment during four seasons throughout the year established $1 - a_0^0$ to an accuracy of about one part in 2000. The same type of experiment was repeated[6] with the two maser beams perpendicular to each other in order to examine isotropies corresponding to a different type of directional dependence, and again a null result was found to within the precision of the experiment, the frequencies remaining unchanged due to reorientation in space by any amount greater than one part in 10^{12}.

The rotating Michelson interferometer used by Michelson and Morley[7] should have produced, on the basis of an ether theory, a shift in fringes due to a change $2La_1^1/[1 - (v/c)^2] - 2La_2^2/[1 - (v/c)^2]^{1/2}$ in the optical path length. Here L is the length of the interferometer arm, and this path length change is the result of a 90° rotation of the interferometer when one arm is oriented along the velocity v. Since no path length change was observed, one can conclude that $a_2^2 = a_1^1[1 - (v/c)^2]^{1/2}$. The most percise measurement with this type of apparatus has been made by Joos,[8] from whose work one can conclude that $(a_2^2 - a_1^1)/a_2^2 = \frac{1}{2}(v/c)^2$ to an accuracy of about one part in 375.

THEORY OF THE EXPERIMENTAL MEASUREMENT

An optical or infrared maser consisting of excited atoms between two parallel reflecting plates oscillates at a frequency given by[9]

$$\nu = (\nu_m Q_m + \nu_c Q_c)/(Q_m + Q_c) \qquad (4),$$

where ν_m is the atomic frequency and $\nu_c = nc/2L$, where n is an integer and L is the plate separation, or more

[1] H. P. Robertson, Rev. Mod. Phys. **21**, 378 (1949).

[1a] *Note added in proof.* For tests of anisotropy of other kinds, see V. W. Hughes, H. G. Robinson, and V. Beltran-Lopez, Phys. Rev. Letters **4**, 342 (1960); R. W. P. Drever, Phil. Mag. **6**, 683 (1961); K. C. Turner and H. A. Hill, Bull. Am. Phys. Soc. **8**, 28 (1963); and Refs. 3, 4, and 6 below.

[2] H. E. Ives and G. R. Stilwell, J. Opt. Soc. Am. **28**, 215 (1938); **31**, 369 (1941).

[3] J. P. Cedarholm, G. F. Bland, W. W. Havens, Jr., and C. H. Townes, Phys. Rev. Letters **1**, 526 (1958).

[4] J. P. Cedarholm and C. H. Townes, Nature **184**, 1350 (1959).

[5] It is shown in Ref. 4 that an assumption of time dilation is needed to produce the observed null effect, and hence the comment is made that the maser beam experiment is more closely related to the Kennedy-Thorndike experiment than to that of Michelson and Morley. However, since the frequency of oscillation is essentially determined by interactions between the vibrating ammonia molecules and the electromagnetic field, length does not enter in any critical way into the experiment. Thus the maser beam experiment is not precisely parallel to that of Kennedy and Thorndike either but measures g_0 and is equivalent in this sense to the Ives-Stilwell experiment.

[6] J. P. Cedarholm and C. H. Townes (private communication).

[7] A. A. Michelson and E. W. Morley, Am. J. Sci. **34**, 333 (1887).

[8] G. Joos, Ann. Phys. **7**, 385 (1930).

[9] C. H. Townes, in *Advances in Quantum Electronics*, edited by J. R. Singer (Columbia University Press, New York, 1961), p. 3.

precisely $2L/c$ is the time for a round trip of the light between the two plates. $Q_m = \nu_m/\Delta\nu_m$, where $\Delta\nu_m$ is the half-width at half-maximum of the atomic resonance and $Q_c = \nu_c/\Delta\nu_c$, where $\Delta\nu_c$ is the half-width at half-maximum of the optical resonance between the plates. Ordinarily, $Q_c \gg Q_m$ so that $\nu \approx \nu_c = nc/2L$.

If the separation between the plates is taken as L_0 and the "ether" is assumed to be streaming parallel to the axis of the maser at velocity v, then $\nu_c(1) = (nc/2L_0) [1-(v/c)^2]$. Similarly, if the ether drift is perpendicular to the axis, $\nu_c(2) = (nc/2L_0)[1-(v/c)^2]^{1/2}$. If two such masers are oriented at right angles, their relative frequencies would hence change on rotation between the two 90° positions by

$$2[\nu_c(2)-\nu_c(1)] \approx (v/c)^2 \qquad (5).$$

Since $(v/c)^2$ for a velocity v equal to the earth's orbital velocity is about 10^{-8}, this represents a frequency change of 3×10^6 cps for infrared light of wavelength one micron, or of frequency $\nu_c = 3\times10^{14}$ cps.

In principle, this change of frequency can be examined with great precision because of the almost monochromatic nature of maser radiation.[9,10] Spontaneous emission produces a frequency spread for each maser oscillator of[10]

$$2\delta \approx 8\pi h\nu/P(\Delta\nu_c)^2, \qquad (6)$$

where P is the power in the oscillations and $h\nu$ is the energy of one quantum of the radiation. For a power of 10^{-3} W and $\Delta\nu_c/\nu_c = 10^{-8}$, which are figures typical of He-Ne masers 2δ is from (7) somewhat less than $1/10$ cps. Thus if a measurement of frequency is only as precise as this theoretical width, the fractional accuracy is about 3 parts in 10^{16}, since the frequency ν_c is near 3×10^{14} cps.

There is an additional frequency spread due to thermal vibrations of the spacers used to hold the two reflecting plates at a fixed separation. The primary contribution to this frequency variation comes from the lowest frequency stretching mode of the spacers, which changes L and hence from expression (5), the frequency. If these spacers are cylindrical, then the frequency spread due to this effect is given by

$$2\delta' = 2\nu(2kT/YV)^{1/2}, \qquad (7)$$

where kT is the Boltzmann energy at the temperature T of the spacers, Y is Young's modulus, and V is the total volume of the spacers.

For a typical case δ' is about 3 cps, and hence somewhat larger than δ. This still allows a precision of one part in 10^{14} if the frequency is determined within this limiting frequency variation. Of course, in principle measurements over times much longer than $1/\delta'$ can give a still much improved precision by allowing a determination of the center of the frequency band.

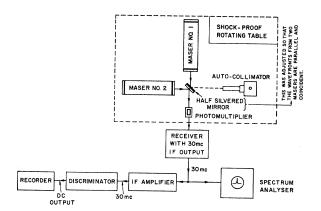For He-Ne masers which are carefully isolated from

FIG. 1. Schematic diagram for recording the variations in beat frequency between two optical maser oscillators when rotated through 90° in space. Apparatus on the shock-proof rotating table is acoustically isolated from the remaining electronic and recording equipment.

acoustical noises and other disturbances, and which are stationary, a precision of definition of frequency within one order of magnitude of the above limit has been achieved, since spectral widths as narrow as about 20 cps have been demonstrated.[11] If this precision is achieved while two masers are rotated 90° for the experiment discussed here, any change in relative frequency as large as one part in 10^{13} could be detected. This would allow a very much improved comparison of g_1 and g_2, and detection of any change with rotation as large as 10^{-5} of the $(v/c)^2$ term which might be associated with the earth's orbital motion.

EXPERIMENTAL ARRANGEMENT

The experimental arrangement is shown schematically in Fig. 1. A photodetector, a discriminator, and a recorder produced a continuous record of the frequency difference between the two oscillating masers. The two masers were mounted on a rotating shock-proof platform with their axes at right angles to each other. The experiment consisted basically in rotating the platform back and forth 90° and observing the change in frequency difference between the two masers.

In order to obtain the best short-term stability and monochromaticity from the masers, it is important to minimize acoustic disturbances, which may change L, the separation of the mirrors. For this purpose the platform and equipment on it, weighing about 200 lb, were suspended from a metal plate on four rubber shock cords about 3 ft long. The metal plate was in turn suspended from a 2-ft length of $\frac{3}{16}$-in.-diam rod of beryllium copper attached to a beam. Resonant periods of the various motions of the platform were of the order of a few seconds, so that direct acoustic coupling to the building was quite low. Acoustic disturbances transmitted through the air seemed usually to be the dominating

[10] A. L. Schawlow and C. H. Townes, Phys. Rev. 112, 1940 (1958).

[11] T. S. Jaseja, A. Javen, and C. H. Townes, Phys. Rev. Letters 10, 165 (1963).
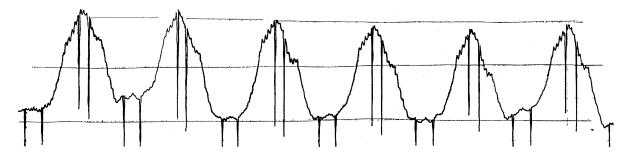
FIG. 2. A plot of frequency variation between masers due to 90° rotation of the table. Vertical scale is such that maximum variation is about 275 kc/sec. Markers indicate rotational angular positions zero and 90°. Double markers appear because the total rotation slightly overshot the zero and 90° positions on each swing.

ones. The assembly was in a basement vault which had previously been a wine cellar, on MIT's Round Hill Estate at New Bedford, Massachusetts. The experimenters and parts of the apparatus which produced acoustic noise were outside the vault, and the building was otherwise unused at the time. Vibrations due to strong winds outside the building or to waves on the shore produced appreciable frequency disturbances at times, so that the experiments were carried out in quiet weather.

Relative stability of the masers indicated, as noted earlier[11], short-term fluctuations of about 20 cps in each maser and a frequency drift of the order of 10 cps/sec under the best conditions.

Torsion of the beryllium-copper rod gave a rotational oscillation period for the platform of about 20 sec, with a decay time for the oscillation of the order of 10 min. Hence the platform and masers could be rotated back and forth about 90° at a frequency near resonance with the application of a very gentle torque. This torque was supplied by a motor-driven oscillating rod beneath the platform and coupled to it by a thin rubber band about one foot long, which again gave very good isolation from acoustic vibrations of the building.

The rotating platform had four legs which cleared the floor by a short distance. On rotation, these legs interrupted a light beam, producing a marker on the recorder which indicated rather accurately a rotation of 90°.

Even if external space is completely isotropic, one must expect that rotation will produce some variation in the frequency difference between the two masers due to local effects. For example, the earth's magnetic field produces magnetostriction in the spacers which separate the maser mirrors and Zeeman effects on the atomic spectra, each of which can affect the frequency of oscillation. Rotation of the table by 90° will vary these effects. Aceleration of the masers due to the oscillatory motion of the table may also produce a relative frequency change if the two masers are not identically constructed.

To discriminate between these local causes of fre-

[11] T. S. Jaseja, A. Javan, and C. H. Townes, Phys. Rev. Letters 10, 165 (1963).

quency variation with rotation and an effect associated with a more basic anisotropy in space, the observations need only to be repeated throughout some large part of a day, as was done in the beam-type maser experiment.[3,4] Thus at noon and midnight the earth's orbital velocity is in the plane of the rotating table and an ether drift effect should be maximum, whereas at sunrise and sunset the orbital velocity is more nearly perpendicular to the table and any effect due to it must be a minimum. Local laboratory effects such as the earth's magnetic field do not, of course, vary systematically with the earth's rotation as would an ether drift.

EXPERIMENTAL RESULTS

The measured variation in relative frequencies of the two masers with rotational oscillation of the platform shown in Fig. 2, which is a recording of the output of the discriminator over a number of periods of oscillation of the table. The markers separated by 90°, which may be seen on this recording, allow determination of the relative frequency change with a rather precise and reproducable 90° rotation. The magnitude of this frequency change is about 250 kc/sec, or somewhat less than that attributable to the earth's orbital velocity on the simple ether theory. The change is mostly associated, as indicated above, with local effects such as the earth's magnetic field, and must be measured throughout some appreciable part of the day to allow detection of any more fundamental spatial anisotropy.

A recording of frequency variations with rotation was made over a period of some minutes at each half-hour interval during a little more than six hours. On each recording, the frequency change between 90° markers was measured for about 5 complete oscillations. From these the mean changes and a probable error were computed. The resulting data from 6:00 a.m. to 12:00 noon on 20 January 1963, are shown in Fig. 3. The lengths of vertical lines indicate the probable errors of each point, which are about ±4 kc/sec.

As indicated above, any sinusoidal variation with a twelve-hour period in the frequency shifts shown in Fig. 3 could indicate an effect of "ether drift" or anisotropy in light propagation. The six-hour period plotted

represents one-half cycle of such a variation. A very detailed statistical analysis of the possible variation revealed by these points is probably not warranted because of the ever troublesome question of possible systematic errors, and because the present experiment is a preliminary one, even though it does represent an improvement over previously available measurements. It is easy to see quickly from this figure that the frequency shift does not appear to change in a systematic way more than a few kilocycles. A somewhat more definite numerical result may be obtained by averaging the six points from 6:00 a.m. to 8:30 a.m. and comparing this with the average of the six points from 9:30 a.m. to 12:00 noon. The difference is 1.6 kc/sec ,with a probable error of 1.2 kc/sec calculated solely from the fluctuations. Similarly, one may compare the average of the seven points between 7:30 a.m. and 10:30 a.m. with the average of the remaining points. The difference between these averages is again 1.6 kc/sec, with a probable error of 1.2 kc/sec based solely on the fluctuations.

From the above, one can conclude that the amplitude of the sinusoidal variation in the frequency shift due to an "ether drift" equal to the earth's orbital velocity—that is, $\nu(v/c)^2$, where v is the earth's orbital velocity and ν is the maser frequency—is not greater than 3 kc/sec, or is hence less than 1/1000 of the effect of an "ether drift" as large as the earth's orbital velocity.

DISCUSSION AND FURTHER EXPERIMENTS

The present preliminary experiment with optical masers already gives a useful improvement over previous precision with which a Michelson-Morley type of experiment could practically be carried out. However, the experiment is still far short of any real limits set by the technique. When the rotating table was at rest under favorable conditions, the two masers changed frequency with respect to each other by no more than about 30 cps during a few seconds time. This is four orders of magnitude less than the background variation due to magnetostriction or other effects when the table was rotated, and about two orders of magnitude smaller than the limit of error set by the present experiment for any more interesting effects of anisotropy.

The next planned step in this experiment is the use of quartz spacers between the mirrors to eliminate magnetostriction effects. Hopefully, this will allow another
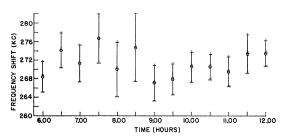

FIG. 3. Plot of relative frequency variation of two masers with 90° rotation as a function of the time of day between 6:00 a.m. and 12:00 noon on 20 January, 1963.

order of magnitude in precision of the search for any anisotropy. It appears likely that great care in this experiment may eventually allow two orders of magnitude improvement, or detection of any effects of anisotropy as large as five orders of magnitude less than the $(v/c)^2$ term associated with an "ether drift" when v is the earth's velocity.

It is clear from the introduction that the Kennedy-Thorndike experiment, or the comparison of time and length, already represents the greatest uncertainty in an experimental test of transformation of the line element in a moving coordinate system. Fortunately this too may be now redone with considerable accuracy by the use of infrared or optical masers.

Equation (4) shows that the frequency of oscillation of masers may be primarily determined by the separation between mirrors (when $Q_c \gg Q_m$), or primarily by the frequency of the atoms involved (when $Q_m \gg Q_c$). In the first case, the frequency depends primarily on a length and in the second primarily on a time. Hence, if one maser of each type is mounted on the rotating table and any change of their frequency difference with rotation observed, one determines $g_0/(g_2-g_1)$. Such an experiment is hence equivalent for our purpose to the Kennedy-Thorndike experiment. This method appears to give an opportunity of improving the precision of the Kennedy-Thorndike result from about $\frac{1}{3}$ of the $(v/c)^2$ term to about 1/1000 of this term, or possibly better. To obtain some improvement, it is not essential to produce two masers with the extreme characteristics $Q_c > Q_m$ and $Q_m > Q_c$, but only to have the two depend on the atomic resonance and the mirror separation, according to expression (4), in some appreciably different quantitative way.