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PHYSICS

Sources of Harmonics of Low Order in the External Gravity Field of the Earth

The external gravitational potential, V, of the Earth at a point having the spherical polar co-ordinates, radius vector r, co-latitude θ and longitude λ, with respect to the centre of mass of the Earth, may be written in the form:

$$V = -\frac{GM}{r}\left\{1 - \sum_{n=2}^{\infty} J_n\left(\frac{R}{r}\right)^n P_n(\cos\theta) + \text{terms varying with longitude}\right\}$$

Here G is the constant of gravitation, M is the mass of the Earth, R is a scale factor, commonly the equatorial radius of the Earth, and $P_n(\cos\theta)$ is the Legendre polynomial of degree n.

King-Hele, Cook and Rees[1] have recently determined the coefficients of the harmonics of even order up to J_{12} from the secular changes in the nodes of orbits of seven artificial satellites, while coefficients of the harmonics of odd order up to J_9 have been determined from periodic changes in the orbits. Beyond J_4, the coefficients are of order 5×10^{-7}.

There are now sufficient data to enable some bounds to be placed on the regions within which the sources of these harmonics must lie. A repetition[2] of a calculation originally due to W. H. Munk and G. F. C. MacDonald shows that there is no relation between the coefficients of the harmonics so far determined and the coefficients of the development of the distribution of the continents in surface harmonics; accordingly the sources must be sought below the Mohorovičić discontinuity. An obvious possibility is that the sources may be irregularities in the boundary between the core and the mantle. Now in consequence of the well-known ambiguity in the inference of a density distribution from a gravity field, it is not possible to prove or disprove the existence of such irregularities from gravity data alone; we can, however, ask whether or not there are any irregularities, however great, at a particular depth that could produce the observed field, and if that is not possible, the sources must lie nearer the surface.

Suppose that the variation of density between radii r_1 and r_2 is given by $\sigma_n P_n(\cos\theta)$. Then the coefficient of P_n in the external field is:

$$J_n = -\frac{3}{(2n+1)(n+3)}\frac{\sigma_n}{\rho}\cdot\left(\frac{r_2}{R}\right)^{n+3}\left[1-\left(\frac{r_1}{r_2}\right)^{n+3}\right]$$

where ρ is the mean density of the Earth.

When n is of order 10 or more $(r_1/r_2)^{n+3}$ will be very small unless the two radii are nearly equal, so that the value of J_n is determined by the outer radius of the shell. Then:

$$J_n \simeq -\frac{3}{(2n+1)(n+3)}\frac{\sigma_n}{\rho}\cdot\left(\frac{r_2}{R}\right)^{n+3}$$

A reasonable value of σ_n/ρ is $\frac{1}{2}$.

If we take J_{12} to be 3×10^{-7},

$$\left(\frac{r_2}{R}\right)^{15} \text{ must be not less than } 7{\cdot}5 \times 10^{-5}$$

and hence $\frac{r_2}{R} > 0{\cdot}537$.

This limit is very close to the ratio of the radius of the core to the radius of the Earth, the value of which is 0·545, and further consideration makes it seem probable that whatever irregularities there may be in the core-mantle boundary, they are unlikely to produce the observed 12th order harmonic, the source of which must therefore be sought higher up in the mantle. Let t denote the difference $r_2 - r_1$, so that the formula for the coefficient J_n becomes:

$$J_n = -\frac{3}{2n+1}\cdot\frac{\sigma_n}{\rho}\cdot\frac{t}{R}\cdot\left(\frac{r_2}{R}\right)^{n+2}$$

Taking r_2/R to be 0·545, and σ_n/ρ to be $\frac{1}{2}$, t has to be 150 km, or about 1/20 of the radius of the core, to give the observed value of J_{12}. If such irregularities seem excessive (it is not clear whether present seismic data would reveal them) it follows that additional sources for the 12th and higher order harmonics must lie in the mantle. The value of J_{10}, -5×10^{-7}, requires t to be 65 km if the 10th order harmonic arises from irregularities in the core mantle boundary.

The conclusion appears to be that irregularities in the core-mantle boundary could produce the observed harmonics of order 10 and less, that the 12th order harmonic probably arises from irregularities above the core-mantle boundary and that higher harmonics certainly do. Evidently, then, determinations of the harmonics of order 13 and higher will be of great interest since they are the lowest that must unambiguously arise from density variations in the mantle.

The following list shows the harmonics for which the sources must probably be sought above different levels in the Earth:

Level	r/R	Order of harmonics which must have sources above the level
Boundary of inner core	0·216	> 4
Core-mantle boundary	0·545	> 12
Low velocity layer	0·95	> 113

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Measurement of Relativistic Time Dilatation using the Mössbauer Effect

The Mössbauer effect allows of some of the most direct tests of certain simple relativistic predictions, and in this connexion we wish to report on the progress of some experiments similar in principle to those first reported by Hay, Schiffer, Cranshaw and Egelstaff[1,2]. These authors measured the relativistic frequency shift between a cobalt-57 source of 14·4-keV γ-radiation, near the centre of a rotating disk, and a resonant iron-57 absorber around the periphery of the disk. On spinning the disk the resonance absorption was found to decrease, giving increases in transmission of up to 6 per cent. This was shown to correspond to a relative frequency shift the

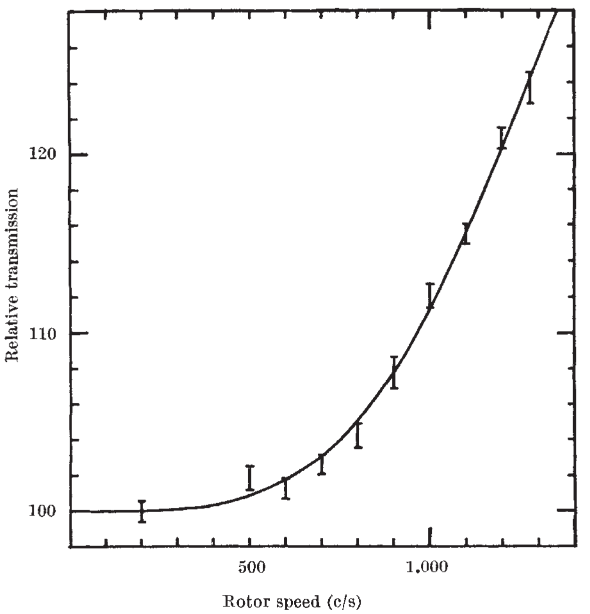

Fig. 1. Graph showing measured transmissions through resonant absorber at various rotor speeds. The full curve is a theoretical curve based on the line shape measured using the first-order Doppler effect

magnitude of which agreed, to within an accuracy of about 2 per cent, with that predicted by the relativistic expression $(\Delta\nu/\nu) = (v_a^2 - v_s^2)/2c^2$, where v_a and v_s are the velocities of the absorber and source. This expression may be obtained either in terms of the time dilatation of special relativity or in terms of the pseudo-gravitational potential difference between source and absorber.

By using higher velocities and more favourable sources and absorbers we have obtained effects of up to 24 per cent. The rotor assembly is similar to that of Champeney and Moon[3], consisting essentially of a hollow tubular rotor, approximately 8 cm long, spun about an axis through its centre perpendicular to its long axis. The source is mounted at the centre, with a resonant absorber across one end and a non-resonant absorber across the other end, the transmissions through the two absorbers being measured on two scalers appropriately gated. The results of a run using a cobalt-57 source diffused into an iron-56 foil in conjunction with an iron absorber foil enriched to 81 per cent in iron-57, fixed to a beryllium backing for rigidity, are shown in Fig. 1. Measurements were also made by mounting the same source and absorber in a device which could move the absorber slowly towards the source at a uniform velocity v, thus inducing a shift of $\Delta\nu/\nu = v/c$ by the first-order Doppler effect. It was found that measurements of transmission as a function of approach velocity could be represented to within the accuracy of the readings by a Lorentzian line shape. The full curve in Fig. 1 was obtained from such a Lorentzian by noting that according to the equations given here the transmission measured on the rotor at a certain speed (with source and absorber velocities of v_s and v_a) should be the same as that on the slow motion device at a speed given by $v = K(v_a^2 - v_s^2)/2c$, with K equal to unity. A method of evaluating our results is to compare the rotor readings with curves calculated using various values of K, and to determine that value of K which gives a best fit using a least squares criterion. Our results so far give a value $K = 1{\cdot}03 \pm 0{\cdot}03$ based on measurements using three absorbers of different characteristics. The error quoted is not due mainly to statistical fluctuations, and a more careful evaluation of systematic background corrections, thermal effects, etc., should in future allow us to decide whether our slight indication of a high value of K is significant. The apparatus lends itself readily to a repeat of the 'tip-tip' experiment, a more accurate repetition of which will provide a sensitive test of the apparatus (for example, with respect to vibrations) besides being of interest in its own right[3,4].

Using a source absorber combination with an inherent line shift one is able to measure the sign of the effect (see Hay[2], Champeney, Isaak and Khan[5]). With a source of cobalt-57 diffused into copper at the centre of the rotor and a stainless steel absorber at the tip, to provide a combination with a relative shift of about half a line width, we obtain a value of $K = +1{\cdot}04 \pm 0{\cdot}15$, the positive sign meaning that spinning the rotor produced the same direction of shift as did a relative approach velocity due to the first order Doppler effect. The chances of K having the wrong value of -1 are thus very remote.

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Note added in proof. Since the time of writing, a related experiment has been described by W. Kündig (*Phys. Rev.*, **129**, 2371; 1963) in which agreement with theory to within an experimental error of 1·1 per cent is obtained. Kündig also found appreciable line broadening which he attributed to vibrations. A consideration of our present results together with those of tip–tip experiments should allow an evaluation of any line broadening mechanism in our experiment.

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Electron Distribution in Low-voltage Arcs

THE plasma properties of a low-voltage arc will depend on the way in which electrons are distributed through it. The electrons, as in other hot cathode discharges[1,2], occur mainly in two groups, one consisting of almost monoenergetic electrons which have fallen through the cathode sheath and acquired random direction in a few mean free paths with little change of energy, the other much slower and near-Maxwellian. As part of a study of hydrogen thyratrons, measurements have been made of the concentrations of both groups in a similar device without control electrode, using Druyvesteyn's probe method[3]. We find that they can be accounted for by diffusion theory and report here briefly some results of general interest which have been obtained for the fast group. Comparable results have been obtained for the slow.

The tubes used were cylinders 6·4 cm in diameter, with anode 6·0 cm in diameter and cathode 2·5 cm in diameter, 5·0 cm apart and perpendicular to the axis. The probe was a sphere 1·0 mm in diameter which could be placed anywhere between the planes of the main electrodes. The numbers of slow and fast electrons were found by electronic double differentiation of the probe current with respect to probe voltage (V) followed by multiplication by $V^{\frac{1}{2}}$, with V zero at space potential, and integration over the relevant ranges of V.

The diffusion equation for the fast electrons was formed by assuming that the diffusion coefficient was given by the electron speed, gas density and the elastic collision cross-