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MAGNETIC MOMENT OF THE ELECTRON

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Recent measurements of the magnetic moment of the electron have cast some doubt either on the validity of quantum electrodynamics in its actual form or on the exactness of the theoretical calculations. In fourth order, the latter are made so lengthy and intricate by the great number of the integrals over auxiliary variables that this possibility cannot a priori be excluded.

The present note explains how the last possibility has been investigated. Owing to the experimental uncertainties, which are at least of the same order of magnitude as the fourth order corrections, many figures in the result are not so interesting. Accordingly, the numerical results for the terms I, IIa—IIe in the work by Karplus and Kroll [1]) have been checked by rigorous upper and lower bounds. Whereas every other term fell well between these bounds, agreement could not be obtained for diagram IIc. Here, therefore, some of the big contributions have been evaluated analytically, the others estimated by analytic upper and lower bounds. The numerical value for this term has been found to satisfy

$$\mu_{\mathrm{IIc}} = (-1.02 \pm 0.53)\alpha^2/\pi^2 \text{ Bohr magnetons.}^{\dagger} \qquad (1)$$

Moreover, to provide some check of the numerous integrations and the tedious algebra, the much less elegant, but safe and fast numerical integrations have been performed for this term. They confirmed every time the bounds derived before. However, here again, analytic integrations have been used to compute the contribution of the logarithmic tails.

It is not claimed here that the numerical results for the terms I, IIa, d, e quoted in Karplus and Kroll are exact, but only that they lie between the bounds found and are not in contradiction with them. If one accepts for these the values given by Karplus and Kroll, the total magnetic moment in fourth order would turn out to be

$$\mu^{(4)} = (-0.81 \pm 0.53)\alpha^2/\pi^2 \text{ Bohr magnetons.} \qquad (2)$$

This result agrees fairly well with the Stanford measurements [2]). As said

† After a term $-(\frac{1}{2}\alpha^2/\pi^2) \log \lambda^2/m^2$ (infra-red term) has been separated out. The value quoted by Karplus and Kroll for this term is $-3.18\alpha^2/\pi^2$.

before, it does not need to be refined as long as the experimental error remains so large (about four times the theoretical uncertainty).

It is of course very desirable that other checks of the fourth order magnetic moment should be performed. Mistakes are easily introduced in such long calculations.

The new value would also change slightly the value of the fine structure constant and increase the Lamb shift by an amount of (0.62 ± 0.16)Mc/s. But to assert a definite value to this effect other fourth order corrections computed by Weneser, Bersohn and Kroll [3]) have to be recalculated. Indeed, these authors used some integrals occurring in the magnetic moment calculation and the result is uncertain.

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References

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