A NEW ELECTRODYNAMICS

BY

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ABSTRACT

So successful have been Maxwell's equations that the electrodynamic formulations of Ampère, Weber, and Riemann have been almost forgotten. The purpose of this paper is to present a rejuvenation of the older theories, based entirely on the force between charged particles. The new formulation leads to the correct formulas for force, induced emf, and radiation. In fact, it may be regarded as an alternative to Maxwell's equations, with the advantages of Galilean relativity and a closer contact with reality (charges rather than fictitious flux lines).

1. INTRODUCTION

In the early part of the 19th century, Ampère (**1**),[3] Gauss (**2**), Weber (**3**), and Riemann (**4**) developed a theory of electrodynamics, free from the magnetic-field fiction and based on an extension of the familiar equation of Coulomb. At that time, the electron was unknown and ideas of metallic conduction were of the haziest kind; yet a true particle-theory was produced, both magnetic flux and the aether being ignored.

The attitude of these early investigators was surprisingly modern: their hard-headed phenomenological approach is much closer to the spirit of modern physics than is the mechanistic pictorialism of Faraday and Maxwell. It cannot be denied that the visualization of magnetic flux lines has been a genuine aid in electrical engineering. But there are advantages, both theoretical and practical, in the direct calculation of inter-particle forces in the pre-Maxwellian manner. It is interesting to see what can be done by a modernization of these older methods.

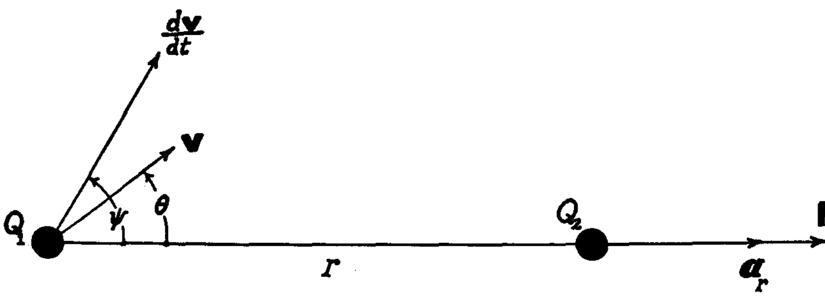

FIG. 1.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

Consider two charged particles (Fig. 1) separated by distance r. If there is no relative motion between the particles, Coulomb's equation applies. In the rationalized mks system,

$$\mathbf{F}_2 = \mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}. \tag{1}$$

This force is called the *Coulomb force.*

If Q_1 is moving at constant velocity $\mathbf{v}$ with respect to Q_2, the *Ampère force* resulting from this motion is

$$\mathbf{F}_2 = \mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left[1 - \tfrac{3}{2}\cos^2\theta\right], \tag{2}$$

where θ is the angle between $\mathbf{v}$ and the unit vector $\mathbf{a}_r$. We have shown (5) that Eq. 2 is the only equation that is consistent with the Ampère experiments and with modern ideas of electronics.

The Ampère force is a substitute for the magnetic field. Equation 2 handles all problems dealing with forces on conductors carrying direct current. Equations 1 and 2 cover the questions that are ordinarily treated under electrostatics and magnetostatics. To complete the theory, however, we must introduce additional terms dealing with the *acceleration* of charge and with the *time-variation* of charge.

Consider all possible forces that can be exerted by charge Q_1 on charge Q_2 (Fig. 1). Evidently the possibilities may be classified as

(*a*) constant Q_1, no relative motion.
(*b*) constant Q_1, uniform relative velocity.
(*c*) constant Q_1, accelerated motion.
(*d*) Q_1 a function of time.

Condition (*a*) gives the Coulomb force, (*b*) the Ampère force. The remainder of the paper will be concerned with (*c*) and (*d*).

2. THE WEBER FORCE

The next step consists in introducing a force caused by an accelerated charge. This force may be called the *Weber force,* since Wilhelm Weber (3) was the first to include an acceleration term in the equations of electrodynamics. Reference to Fig. 1 shows that the force can depend on only five variables: the charges Q_1 and Q_2, the distance r, the acceleration $d\mathbf{v}/dt$, and the angle ψ between $d\mathbf{v}/dt$ and $\mathbf{a}_r$.

The requirement of linearity means that the force must be directly proportional to Q_1Q_2. The Weber force is directly proportional to acceleration because induced voltage is directly proportional to dI/dt or to $Q\, dv/dt$. Dimensional analysis then proves that the Weber force is inversely proportional to the first power of r and inversely proportional to c^2. Without loss of generality, we can employ the constant of Eq. 1 and write for the Weber force,

$$\mathbf{F}_2 = \frac{Q_1Q_2}{4\pi\epsilon c^2 r}\frac{dv}{dt}\cdot\mathbf{F}(\psi), \tag{3}$$

where $\mathbf{F}(\psi)$ is an unknown vector function of the angle ψ.

Consider an element of conductor $d\mathbf{s}_1$ carrying a varying current (Fig. 2). The current is taken in the direction of the vector $d\mathbf{s}_1$, and

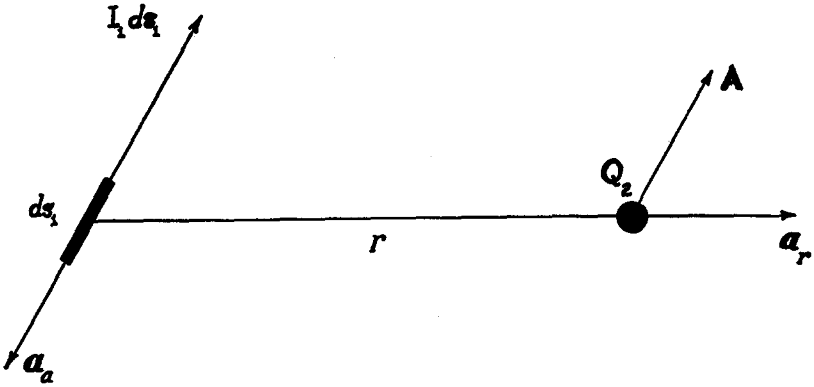

FIG. 2.

$$\begin{aligned} I_1 &= N_1A_1|Q_e|v_1, \\ dI_1/dt &= N_1A_1|Q_e|\,dv_1/dt, \end{aligned} \tag{4}$$

where

N_1 = number of free electrons per unit volume of conductor (m^{-3}),
A_1 = cross-sectional area of conductor (m^2),
$|Q_e|$ = magnitude of electronic charge (coulomb),
v_1 = drift velocity of electrons (m sec^{-1}) with respect to the conductor.

The electrons in ds_1 constitute a charge

$$Q_1 = -N_1A_1|Q_e|\,ds_1.$$

From Eq. 3, the force per unit charge on Q_2 is

$$\mathbf{F}_2/Q_2 = -\frac{N_1A_1|Q_e|\,ds_1}{4\pi\epsilon c^2 r}\frac{dv_1}{dt}\cdot\mathbf{F}(\psi)$$

and from Eq. 4,

$$\mathbf{F}_2/Q_2 = -\frac{ds_1}{4\pi\epsilon c^2 r}\frac{dI_1}{dt}\cdot\mathbf{F}(\psi). \tag{5}$$

Perhaps the simplest way of determining the unknown function $\mathsf{F}(\psi)$ is to compare Eq. 5 with the corresponding expression based on Maxwell's equations. According to Maxwell's theory,

$$\begin{cases} \mathbf{A} = \dfrac{1}{4\pi}\displaystyle\int \dfrac{\mathbf{J}}{r}\,d\mathcal{V}, \\ \mathbf{F}/Q = -\left[\nabla\varphi + \mu\dfrac{\partial \mathbf{A}}{\partial t}\right], \end{cases}$$

where **A** is the vector potential and **J** is the current density. For the element of Fig. 2,

$$\mathbf{J}\,d\mathcal{V} = I_1\,d\mathbf{s}_1,$$

so

$$\mathbf{A} = \frac{I_1\,d\mathbf{s}_1}{4\pi r}, \quad \frac{\partial \mathbf{A}}{\partial t} = -\,\mathbf{a}_a\frac{ds_1}{4\pi r}\frac{dI_1}{dt},$$

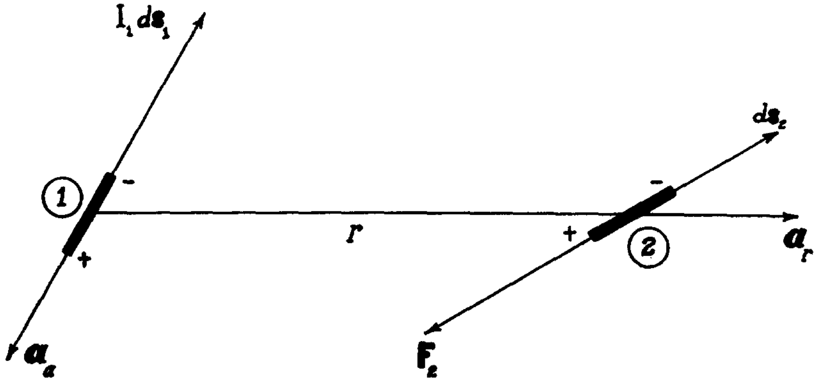

FIG. 3.

where $\mathbf{a}_a$ is a unit vector in the direction of the electron acceleration.

Since the element $d\mathbf{s}_1$ is uncharged, the scalar potential is zero and

$$\mathbf{F}_2/Q_2 = -\,\frac{1}{\epsilon c^2}\frac{\partial \mathbf{A}}{\partial t} = \mathbf{a}_a\frac{ds_1}{4\pi\epsilon c^2 r}\frac{dI_1}{dt}. \tag{6}$$

Comparison of Eqs. 5 and 6 shows that

$$\mathsf{F}(\psi) = -\,\mathbf{a}_a.$$

Thus the final equation for the Weber force for two charged particles is, from Eq. 3,

$$\mathbf{F}_2/Q_2 = -\,\mathbf{a}_a\frac{Q_1}{4\pi\epsilon c^2 r}\frac{dv}{dt}, \tag{7}$$

where v is the relative velocity of charge 1 with respect to charge 2.

3. INDUCED EMF

The difference in potential between the ends of an element $d\mathbf{s}_2$ (Fig. 3), induced by the varying current in $d\mathbf{s}_1$, is

$$dV_2 = -\mathbf{F}_2/Q_2 \cdot d\mathbf{s}_2,$$

where the emf is taken positive for a potential rise in the direction of $d\mathbf{s}_2$. From Eq. 6, the induced emf in $d\mathbf{s}_2$ is

$$d^2V_2 = -\frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{4\pi\epsilon c^2 r}\frac{dI_1}{dt}. \tag{8}$$

Introducing the mutual inductance M, we obtain

$$d^2M = \frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{4\pi\epsilon c^2 r}$$

or

$$M = \frac{1}{4\pi\epsilon c^2}\int\int\frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{r}. \tag{9}$$

This is the equation of F. Neumann (6). Equation 9 applies equally well to self inductance L when $d\mathbf{s}_1$ and $d\mathbf{s}_2$ refer to elements in the same circuit. The result was obtained without reference to magnetic flux, and it is sometimes more convenient than the conventional method of flux linkages. Grover (7) says of Eq. 9:

> This is the most general expression for finding the mutual inductance. It leads quite simply to a *formal* expression for the mutual inductance even though for most cases it is not possible to perform the integrations [analytically]. . . . For inclined filaments . . . the Neumann formula has the advantage, and the formula for the mutual inductance of two straight filaments placed in any desired position has also been obtained by its use.

Thus the Weber force, caused by the acceleration of a charged particle, is given by Eq. 7. The emf induced in an element $d\mathbf{s}_2$ is given by Eq. 8. The mutual inductance between closed circuits of any form is expressed by Eq. 9, and the induced emf is

$$V_2 = -M\frac{dI_1}{dt}. \tag{10}$$

As an illustration, consider the mutual inductance between two coaxial circles (Fig. 4). Take a charge Q_2 at a fixed point P on the outer circle; and determine the force on this charge, caused by a varying current in the inner circle.

According to Eq. 6,

$$d\mathbf{F}_2/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r'}\frac{dI_1}{dt}, \tag{6a}$$

where $ds_1 = a\,d\zeta$, $r'^2 = a^2 + b^2 + z^2 - 2ab\cos\zeta$. The tangential component of force at P, caused by current in the complete loop of radius a, is

$$F_t/Q_2 = -\frac{a}{4\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{2\pi}\frac{\cos\zeta\,d\zeta}{r}$$
$$= -\frac{a}{2\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{\pi}\frac{\cos\zeta\,d\zeta}{[(a^2+b^2+z^2)-2ab\cos\zeta]^{\frac{1}{2}}}.$$

Let $\zeta = \pi - 2\varphi$, $\cos\zeta = -\cos 2\varphi = -(1-2\sin^2\varphi)$, $d\zeta = -2\,d\varphi$. Then

$$F_t/Q_2 = \frac{a}{\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{\pi/2}\frac{(1-2\sin^2\varphi)\,d\varphi}{[(a+b)^2+z^2-4\,ab\sin^2\varphi]^{\frac{1}{2}}}$$
$$= \frac{a}{\pi\epsilon c^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI_1}{dt}\int_0^{\pi/2}\frac{(1-2\sin^2\varphi)\,d\varphi}{[1-k^2\sin^2\varphi]^{\frac{1}{2}}},$$

where

$$k^2 = 4ab/[(a+b)^2+z^2]. \tag{11}$$

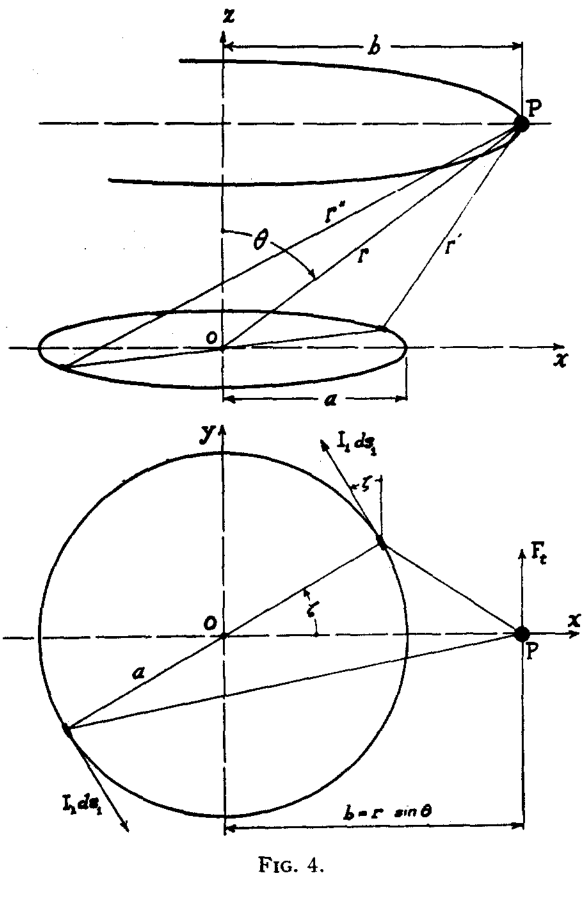

FIG. 4.

Integration leads to

$$F_t/Q_2 = -\frac{a}{\pi\epsilon c^2 k^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI_1}{dt}[(2-k^2)F-2E], \quad (12)$$

where F and E are complete elliptic integrals of the first and second kinds. Equation 12 gives the force per unit charge on a particle at P, produced by a varying current in the loop of radius a.

The emf induced in a complete loop of radius b is

$$\begin{aligned} V &= \oint (F_t/Q_2)\, ds_2 = 2\pi b(F_t/Q_2) \\ &= -\frac{2ab}{\epsilon c^2 k^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI}{dt}[(2-k^2)F-2E], \end{aligned} \quad (13)$$

so the mutual inductance of two concentric circles is

$$M = \frac{1}{2\epsilon c^2}[(a+b)^2+z^2]^{\frac{1}{2}}[(2-k^2)F-2E]. \quad (14)$$

This is exactly the equation obtained by Maxwell. It could have been found, of course, directly from Eq. 9, though the above derivation gives a better physical picture of what is taking place.

4. THE LOOP ANTENNA

The induced emf in a circular loop is considered in Section 3. The equations apply to relatively low frequencies so that the time lag caused by the finite velocity of propagation may be neglected. We now take the case where retardation must be included but where the current is still essentially in phase around the loop.

The unretarded Weber force per unit charge is

$$d\mathbf{F}/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r}\frac{dI(t)}{dt} \quad (6a)$$

and the retarded Weber force is therefore

$$d\mathbf{F}/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r}\frac{dI(t-r/c)}{dt}. \quad (6b)$$

Let $I(t) = \sqrt{2}I^* e^{i\omega t}$ and $F(t) = \sqrt{2}F^* e^{i\omega t}$, where I^* and F^* are rms values (generally complex numbers). Then

$$\frac{dI(t-r/c)}{dt} = i\omega\sqrt{2}I^* e^{i\omega t}e^{-\frac{i\omega r}{c}}.$$

Thus Eq. 6*b* becomes

$$d\mathbf{F}/Q_2 = -\frac{i\omega\sqrt{2}\,ds_1 I^*}{4\pi\epsilon c^2 r} e^{i\omega t} e^{-\frac{i\omega r}{c}}$$

and the tangential force per unit charge (Fig. 4) is

$$dF_t^*/Q_2 = -\frac{i\omega\, ds_1 I^* \cos\zeta}{4\pi\epsilon c^2 r'} e^{-\frac{i\omega r'}{c}}. \tag{6c}$$

Consider two current elements situated diametrically opposite each other on a circle of radius a (Fig. 4). Distances from these elements to point P are r' and r''. The tangential force, produced at P by current in the pair of elements, is

$$dF_t^*/Q_2 = -\frac{i\omega a I^* \cos\zeta\, d\zeta}{4\pi\epsilon c^2}\left[\frac{e^{-\frac{i\omega r'}{c}}}{r'} - \frac{e^{-\frac{i\omega r''}{c}}}{r''}\right], \tag{15}$$

with

$$r'^2 = a^2 + r^2 - 2ar\sin\theta\cos\zeta,$$
$$r''^2 = a^2 + r^2 + 2ar\sin\theta\cos\zeta.$$

If $r \gg a$,

$$r' \cong r\left(1 - \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$r'' \cong r\left(1 + \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$\frac{1}{r'} \cong \frac{1}{r}\left(1 + \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$\frac{1}{r''} \cong \frac{1}{r}\left(1 - \frac{a}{r}\sin\theta\cos\zeta\right).$$

Substitution in Eq. 15 gives

$$dF_t^*/Q_2 = -\frac{i\omega a I^* e^{-\frac{i\omega r}{c}} \cos\zeta\, d\zeta}{2\pi\epsilon c^2 r}\left[i\sin\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) + \frac{a}{r}\sin\theta\cos\zeta\cos\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right)\right].$$

If $\omega a/c \ll 1$,

$$\sin\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) \cong \frac{\omega a}{c}\sin\theta\cos\zeta,$$
$$\cos\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) \cong 1.$$

Thus,

$$dF_t^*/Q_2 \cong -\frac{i\omega a^2[I^*]}{2\pi\epsilon c^2 r}\left[\frac{i\omega}{c}+\frac{1}{r}\right]\sin\theta\cos^2\zeta\, d\zeta, \tag{16}$$

where

$$[I^*] = I^* e^{-\frac{i\omega r}{c}}.$$

The tangential force per unit charge at P, caused by current in the complete loop of radius a, is

$$F_t^*/Q_2 = -\frac{i\omega a^2[I^*]\sin\theta}{\pi\epsilon c^2 r}\left[\frac{i\omega}{c}+\frac{1}{r}\right]\int_0^{\pi/2}\cos^2\zeta\, d\zeta$$

or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_\psi\frac{\omega(\pi a^2)[I^*]\sin\theta}{4\pi\epsilon c^2 r}\left[\frac{\omega}{c}-\frac{i}{r}\right], \tag{17}$$

where $\mathbf{a}_\psi$ is a unit vector in the tangential direction. This equation agrees with the usual formula derived from Maxwell's equations. It shows that the force at P is directly proportional to the area of the loop and directly proportional to the sine of the angle from the loop axis. The first term of Eq. 17, which varies inversely as the first power of the distance, represents the ordinary radio wave. The second term varies inversely as the second power of r and thus becomes negligible at very large distances.

5. THE MAXWELL FORCE

The preceding sections have evaluated cases (*a*), (*b*), and (*c*) of Section 1. There remains the rather unusual case (*d*) of a charge that is not constant. For example, with a linear conductor in the stationary or quasi-stationary state, the conduction current entering an elementary length of conductor is equal to the current leaving it, in accordance with Kirchhoff's second law. But at sufficiently high frequency, this relation no longer holds: the charge on the element depends on both time and position. Because (*d*) is closely related to Maxwell's displacement current, the corresponding force will be called the *Maxwell force.*

Suppose that Q_1 of Fig. 1 varies sinusoidally with time:

$$Q_1(t) = \sqrt{2}Q^* e^{i\omega t}.$$

The corresponding retarded quantity is

$$Q_1(t - r/c) = \sqrt{2}Q^* e^{i\omega(t-r/c)}.$$

According to Maxwell's theory, this varying charge produces a force on Q_2:

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r\frac{[Q^*]}{4\pi\epsilon}\left[\frac{1}{r^2}+\frac{i\omega}{cr}\right], \tag{18}$$

where

$$[Q^*] = \sqrt{2}Q^* e^{-i\omega r/c}, \quad F = \sqrt{2}F^* e^{i\omega t}.$$

This same result is obtained from the new theory if we assume for the Maxwell force,

$$\mathbf{F}/Q_2 = -\mathbf{a}_r \frac{1}{4\pi\epsilon} \frac{\partial}{\partial r}\left[\frac{1}{r} Q_1(t - r/c)\right]. \tag{19}$$

Then for the charged particles of Fig. 1, Eq. 19 gives

$$\mathbf{F}/Q_2 = -\mathbf{a}_r\sqrt{2}\frac{Q^*}{4\pi\epsilon}\left[-\frac{1}{r^2} - \frac{i\omega}{cr}\right]e^{i\omega(t-r/c)}$$

or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r \frac{[Q^*]}{4\pi\epsilon}\left[\frac{1}{r^2} + \frac{i\omega}{cr}\right]. \tag{18}$$

For the special case of $Q_1(t) = Q_1 = \text{const}$, Eq. 19 reduces to the Coulomb force,

$$\mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}. \tag{1}$$

Thus the Maxwell force includes the Coulomb force as a special case, and no separate term for the Coulomb force is needed.

Just as the Ampère force replaces the magnetic field, so the Maxwell force replaces the displacement current, though of course Eq. 19 was not formulated by Maxwell. In fact, none of the previous theories seems to include such a force, which may account for the rather unsatisfactory nature of the theories of Weber, Riemann, Ritz (**8**), and Warburton (**9**).

As an example, take an electric dipole (Fig. 5) with the charges varying sinusoidally with time:

$$Q = \sqrt{2}Q^* e^{i\omega t}.$$

For the upper charge, according to Eq. 19,

$$\mathbf{F}/Q_2 = \mathbf{a}_{r'} \frac{\sqrt{2}Q^*}{4\pi\epsilon}\left[\frac{1}{r'^2} + \frac{i\omega}{cr'}\right]e^{i\omega(t-r'/c)}.$$

But if $l \ll r$,

$$r' \cong r(1 - (l/2r)\cos\theta), \quad r'' \cong r(1 + (l/2r)\cos\theta),$$

$$\frac{1}{r'} \cong \frac{1}{r}(1 + (l/2r)\cos\theta), \quad \frac{1}{r''} \cong \frac{1}{r}(1 - (l/2r)\cos\theta).$$

Then

$$\mathbf{F}^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}} e^{\frac{i\omega l}{2c}\cos\theta}}{i\omega 4\pi\epsilon} \left\{ \frac{1}{r^2}\left(1 + \frac{l}{r}\cos\theta\right) + \frac{i\omega}{cr}\left(1 + \frac{l}{2r}\cos\theta\right) \right\} \times \left[\mathbf{a}_r + \mathbf{a}_\theta \frac{l\sin\theta}{2r} \right]$$

where I is the current between the two spheres:

$$I = \sqrt{2} I^* e^{i\omega t}, \quad Q = \frac{\sqrt{2} I^* e^{i\omega t}}{i\omega}.$$

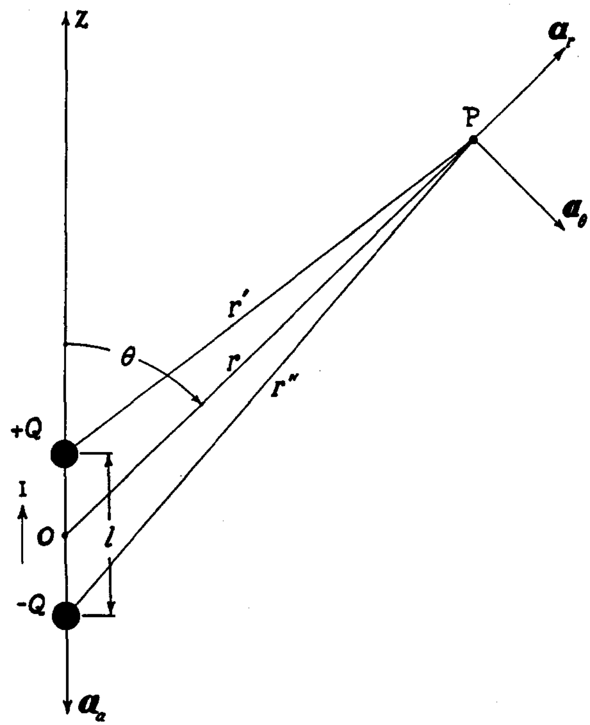

FIG. 5.

Similarly, for the lower charge,

$$\mathbf{F}^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}} e^{-\frac{i\omega l}{2c}\cos\theta}}{i\omega 4\pi\epsilon} \left\{ \frac{1}{r^2}\left(1 - \frac{l}{r}\cos\theta\right) + \frac{i\omega}{cr}\left(1 - \frac{l}{2r}\cos\theta\right) \right\} \times \left[-\mathbf{a}_r + \mathbf{a}_\theta \frac{l\sin\theta}{2r} \right].$$

The sum of the two forces has a radial component,

$$F_r^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}}}{i\omega 4\pi\epsilon} \left\{ \frac{1}{r^2}\left[2i \sin\left(\frac{\omega l}{2c}\cos\theta\right) + \frac{2l}{r}\cos\theta\cos\left(\frac{\omega l}{2c}\cos\theta\right)\right] + \frac{i\omega}{cr}\left[2i\sin\left(\frac{\omega l}{2c}\cos\theta\right) + \frac{l}{r}\cos\theta\cos\left(\frac{\omega l}{2c}\cos\theta\right)\right]\right\}.$$

But

$$\sin\left(\frac{\omega l}{2c}\cos\theta\right) \cong \frac{\omega l}{2c}\cos\theta, \quad \cos\left(\frac{\omega l}{2c}\cos\theta\right) \cong 1,$$

so

$$F_r^*/Q_2 = \frac{[I^*]l\cos\theta}{4\pi\epsilon c}\left[\frac{2}{r^2} - \frac{i2c}{\omega r^3} + \frac{i\omega}{cr}\right]. \tag{20}$$

Similarly, the component of force perpendicular to the radius is

$$F_\theta^*/Q_2 = \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right]. \tag{21}$$

6. THE DIPOLE ANTENNA

Consider the dipole antenna (Fig. 5), assuming that the current in the vertical wire between $+Q$ and $-Q$ is not a function of z. The dipole itself produces the force given by Eqs. 20 and 21. To this must be added the Weber force, Eq. 6, caused by the varying current in the wire. For $I(t - r/c) = \sqrt{2}I^* e^{i\omega(t-r/c)}$, the retarded Weber force is

$$\mathbf{F}/Q_2 = -\mathbf{a}_z \frac{\sqrt{2}i\omega l I^*}{4\pi\epsilon c^2 r} e^{i\omega(t-r/c)}$$

or

$$(\mathbf{F}^*/Q_2)_W = -\mathbf{a}_r \frac{i\omega[I^*]l\cos\theta}{4\pi\epsilon c^2 r} + \mathbf{a}_\theta \frac{i\omega[I^*]l\sin\theta}{4\pi\epsilon c^2 r}. \tag{22}$$

Equations 20 and 21 express the Maxwell force:

$$(\mathbf{F}^*/Q_2)_M = \mathbf{a}_r \frac{[I^*]l\cos\theta}{4\pi\epsilon c}\left[\frac{2}{r^2} - \frac{i2c}{\omega r^3} + \frac{i\omega}{cr}\right] + \mathbf{a}_\theta \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right]. \tag{23}$$

The Ampère force is a double-frequency force because of the second power of the velocity, Eq. 2. Thus the total force at fundamental frequency is the sum of the Weber and the Maxwell forces, or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r \frac{[I^*]l\cos\theta}{2\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right] + \mathbf{a}_\theta \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3} + \frac{i\omega}{cr}\right]. \quad (24)$$

At very great distances, Eq. 24 reduces to

$$\mathbf{F}^*/Q_2 = \mathbf{a}_\theta \frac{i\omega[I^*]l\sin\theta}{4\pi\epsilon c^2 r}, \quad (24a)$$

showing that the force per unit charge at the receiver varies directly as the length of the antenna, directly as the frequency, and inversely as the distance. Equations 24 and 24*a* agree with the classical results obtained from Maxwell's equations.

7. SUMMARY

The paper has developed an electrodynamics that is in agreement with Galilean relativity and free from the aether and the magnetic-field concept. The fundamental equation expresses the force on a charged particle, caused by another charged particle (Fig. 1). The particles may be stationary with respect to each other, or they may be in relative motion. The charges may be constant or they may vary with time. In any case, the force per unit charge at point 2 is

$$\mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2[1 - \tfrac{3}{2}\cos^2\theta] - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{d}{dt}v(t - r/c) - \mathbf{a}_r \frac{1}{4\pi\epsilon}\frac{\partial}{\partial r}\left[\frac{1}{r}Q_1(t - r/c)\right]. \quad (25)$$

The first term represents the *Ampère force*, which is particularly important with direct currents. The second term represents the *Weber force*, which gives induced emf's. The third term represents the *Maxwell force*. It includes the *Coulomb force* as a special case where $Q =$ const., but it is especially useful where the frequency is high enough so that Q is a function of both time and position.

Note that Eq. 25 is inherently relativistic, without the need of Lorentz contraction or Einstein pseudo-relativity. The only velocity is the *relative* velocity $\mathbf{v}$ of charge 1 with respect to charge 2; the only acceleration is the rate of change of relative velocity. Equation 25 may be used as the basis for all of electrodynamics. Thus questions dealing with aggregates of charge are handled by summation (or integration) of F/Q_2, and problems dealing with currents are treated by considering large numbers of electrons in motion. The only limitation of Eq. 25 seems to be that it applies only to ordinary velocities. It is probable that the functions of Eq. 25 will require slight modification if they are applied to electrons at extreme velocities.

The new formulation of electrodynamics is built on the foundations laid by Weber and by Ritz. It differs from all previous theories, however, particularly in the term for the Maxwell force. This formulation may be regarded as an alternative to Maxwell's equations. Maxwell based his work on the closed circuit, on the aether, and on the Faraday visualization of flux lines. The Maxwell equations give the correct answers to a great number of questions; but the application of these equations to open circuits is sometimes ambiguous and uncertain. Particularly in considering the magnetic field produced by electrons in motion, one may find the classical field concepts to be clumsy tools. The new formulation generally leads to exactly the same results as Maxwell's equations, but in some cases it may give a more direct approach and one that is free from ambiguity.

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