On a New Paradox in Special Relativity

Preprint · June 2018

1 author:

Stephan Gift
University of the West Indies, St. Augustine

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Stephan J. G. Gift
Department of Electrical and Computer Engineering
Faculty of Engineering
The University of the West Indies
St. Augustine, Trinidad and Tobago, West Indies
Email: Stephan.Gift@sta.uwi.edu

Abstract. In this paper a recently published paradox in special relativity referred to as the light speed paradox is discussed. This paradox takes the form of an inconsistency in the Lorentz transformations where light speed determined using the transformations in two different approaches yield two different answers. A more straightforward proof of the paradox is presented and Selleri's resolution of the issue is again highlighted.

Keywords: Lorentz transformations, Selleri transformations, special relativity, light speed invariance postulate, Selleri's paraox, one-way light speed, inconsistency

1. Introduction

In the application of special relativity several paradoxes have been reported including the length contraction paradox [1], the twin paradox [2] and Bell's spaceship paradox [3]. The accepted view today is that these paradoxes have been resolved and the problem is now regarded as one of misinterpretation of concepts [4].

Selleri's paradox [5, 6] however stands unresolved in the relativistic framework [7]. Here the ratio of the speeds of two light signals travelling in opposite directions around a rotating platform is a non-unity value that remains non-unity when the disc radius and angular speed are adjusted such that the system becomes inertial. This contradicts the light speed invariance postulate which requires that this light speed ratio be unity.

In support of Selleri's arguments, a new paradox referred to as the Light Speed Paradox involving the Lorentz transformations was identified [8]. Specifically it is shown that using these transformations to determine light speed in an inertial frame employing two different approaches yields two different answers. In this paper the argument demonstrating the inconsistency is refined such that its validity is more easily verified.

2. One-Way Light Speed Determination

Consider an inertial system S_o with space and time coordinates x_o, y_o, z_o, t_o in which the speed of light is *c*, and another inertial system S having space and time coordinates x, y, z, t which is moving at speed v relative to S_o along the x_o axis. The two systems S_o and S are coincident at $t_o = t = 0$. The Lorentz transformations which relate coordinates in the two frames from S_o to S are given by [9-11]

$$x = \gamma(x_o - vt_o) \tag{1a}$$

$$y = y_o \tag{1b}$$

$$z = z_o \tag{1c}$$

$$t = \gamma(t_o - \frac{vx_o}{c^2}) \tag{1d}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$.

Based on the Lorentz transformations (1), a rigid rod of length Δx at rest along the x-axis in S has a length Δx_o in S_o relative to which it is moving at velocity v given by

$$\Delta x = \gamma \Delta x_o \tag{2}$$

This is the length contraction formula of special relativity [10 (p97), 11 (p62)]. Also based on the Lorentz transformations (1), the time interval Δt between two events at a standard clock fixed in S corresponds to a time interval Δt_o in S_o given by

$$\Delta t = \frac{1}{\gamma} \Delta t_o \tag{3}$$

This is the time dilation formula of special relativity [10 (p100), 11 (p64)] which, according to French [10 (p100)] "is basically the consequence of comparing successive readings on a given clock with readings on *two different clocks*."

Using these transformations the one-way speed of light in frame S is determined for light travelling in a direction opposite to that of v using two different approaches: a differential approach and a kinematic approach.

2.1 Differential Approach

Using the Lorentz transformations (1) the one-way speed of light $c_S(LT) = dx/dt$ in frame S for light travelling in a direction opposite to that of v is found by differentiating the transformations. Hence differentiating equations (1a) and (1d) gives

$$\frac{dx}{dt} = \gamma(\frac{dx_o}{dt_o}\frac{dt_o}{dt} - v\frac{dt_o}{dt}) \tag{4}$$

$$\frac{dt}{dt_o} = \gamma(1 - \frac{v}{c^2}\frac{dx_o}{dt_o}) \tag{5}$$

Setting $\frac{dx_o}{dt_o} = -c$ since the light is travelling in a direction opposite to that of v gives

$$\frac{dx}{dt} = \gamma\frac{dt_o}{dt}(-c - v) \tag{6}$$

$$\frac{dt}{dt_o} = \gamma(1 + \frac{v}{c}) \tag{7}$$

Substituting (7) in (6) gives

$$c_S(LT) = \frac{dx}{dt} = -c\gamma(1 + v/c)/\gamma(1 + v/c) \tag{8}$$

which reduces to

$$c_S(LT) = -c \tag{9}$$

This is the well-known light speed invariance principle of special relativity which is used to derive the Lorentz transformations [9-11].

2.2 Kinematic Approach

For light travelling in a direction opposite to that of v, the one-way speed of light $c_R(LT) = dx/dt$ relative to a receiver fixed in frame S is now found using the kinematic relation speed equals distance over time. Consider a light transmitter I fixed at a point on the x axis of the inertial frame S and a receiver R fixed at the origin of the inertial frame S as shown in fig.1. R therefore moves along the x_o axis in the direction of I with constant speed v relative to S_o. At time $t_o = T_o$ on a clock C_o fixed in S_o and located at the position of the transmitter as shown in fig.2, I emits a light signal in the direction of the receiver. Let the position of the receiver R as measured in S using a rigid measuring rod fixed in S be such that the distance between receiver R and transmitter

I is the proper length Δx. Let the corresponding distance in S_o between R and I at the instant the light is emitted be Δx_o. Equation (2) relates Δx and Δx_o such that

$$\Delta x = \gamma \Delta x_o \qquad (2)$$

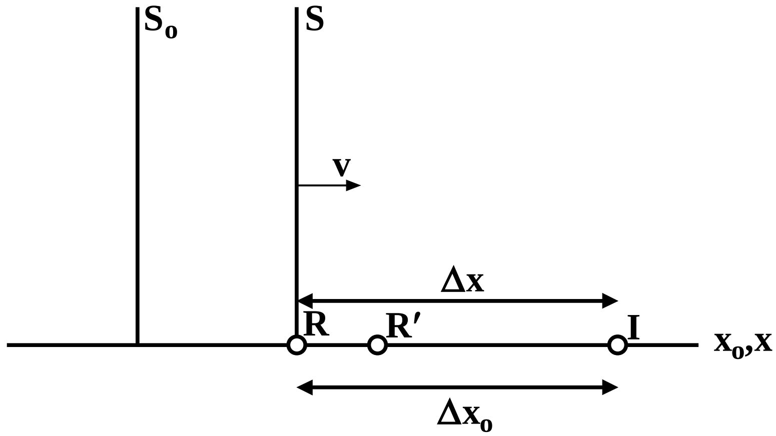

Fig.1 Inertial Frames in Relative Motion at time $t_o = T_o$

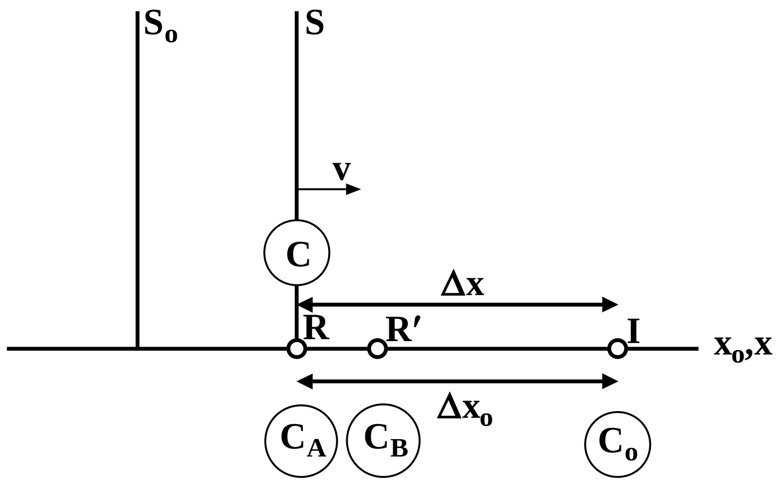

Fig.2 Synchronized clocks C_A, C_B and C_o **in** S_o and **clock** C in S **at time** $t_o = T_o$

As the emitted light travels from transmitter I toward receiver R, the receiver moves to position R' where it receives the light. From fig. 2 let the elapsed time measured by a clock C fixed at the receiver in S for the light to travel from the transmitter I to the receiver at R' be Δt. Measurement of Δt on a single clock is necessary to satisfy the requirements of the theory. In order to accomplish this the instant of light transmission $t_o = T_o$ indicated on clock C_o at the position of the transmitter must be available at the position of the receiver so that the time on clock C at this instant can be recorded. This is easily achieved using a clock C_A fixed in S_o and also located (at this instant) at the position of the receiver R. This clock C_A, because light speed is c in S_o, can be (Einstein) synchronized with the clock C_o in S_o thereby enabling the time $t_o = T_o$ when light emission occurs to be known at the receiver. Therefore at the time of light emission $t_o = T_o$ as indicated on synchronized clock C_A in S_o at the position of the receiver R, the time on clock C at the receiver is recorded. Following this as the receiver travels to position R', time on clock C at the receiver is recorded at the instant of arrival of the light. In this way the proper time interval Δt between the emission and reception of the light is determined by **successive readings on the same clock** C fixed at the receiver in S as required by the theory [10, 11].

Let the corresponding elapsed time measured in S_o between the emission and reception of the light be Δt_o. Time interval Δt_o can be measured using the synchronized clock C_A in S_o at the receiver R to record the instant $t_o = T_o$ and another similarly synchronized clock C_B fixed in S_o at the receiver position R' as shown in fig.2 to record the time the light signal is received. Thus by "comparing successive readings on a given clock [C in S] with readings on *two different clocks* [C_A and C_B in S_o]" we satisfy the conditions of the theory [10 (p100)] and specifically equation (3) which relates Δt and Δt_o such that

$$\Delta t = \frac{1}{\gamma}\Delta t_o \tag{3}$$

Using (2) for the distance Δx in S and (3) for the time Δt in S for the light to traverse that distance and arrive at the receiver, the speed $c_R(LT)$ of the received light relative to the receiver as determined in the frame S of the receiver can be calculated and is given by

$$c_R(LT) = -\frac{\Delta x}{\Delta t} = -\gamma^2 \frac{\Delta x_o}{\Delta t_o} \tag{10}$$

Because the receiver moves from R to position R' during the light transmission, the actual distance in S_o travelled by the light is $\Delta x_o - v\Delta t_o$ where $v\Delta t_o$ is the distance moved in S_o between R and R' in time Δt_o. Since the light speed in S_o is c it follows that

$$\frac{\Delta x_o - v\Delta t_o}{\Delta t_o} = c \tag{11}$$

from which

$$\frac{\Delta x_o}{\Delta t_o} = c + v \tag{12}$$

Substituting (12) in (10) yields

$$c_R(LT) = -\frac{\Delta x}{\Delta t} = -\frac{c+v}{1-\beta^2} = \frac{-c}{1-\beta} \tag{13}$$

The result in (13) deduced here by indirect application of the Lorentz transformations is the same result obtained by direct application of these transformations [8]. This confirms the correctness of (13) as following both directly and indirectly from these transformations in the kinematic calculation.

2.3 Inconsistency

This light speed value $c_R(LT) = -c/(1-\beta)$ calculated in (13) using distance travelled Δx divided by elapsed time Δt must, because of the requirement of consistency, be equal to the light speed $c_S(LT) = -c$ in (9) derived from the Lorentz transformations by differentiation. This requires that

$$\frac{-c}{1-\beta} = -c \tag{14}$$

which for $v \neq 0$ cannot be satisfied and therefore represents an inconsistency [8].

3. Discussion

Thus it has been shown that the Lorentz transformations contain an inconsistency where light speed determined using the transformations in two different approaches yield two different answers. The Lorentz transformations are the accepted transformations in space-time physics and therefore the inconsistency demonstrated in (14) must be removed. We argue that the resolution of this problem involves the approach proposed by Selleri [5, 6] who, on the basis of the examination of all possible linear transformations using his set of "equivalent" transformations, effectively modified the time component (1d) of the Lorentz transformations given by

$$t_L = \gamma(t_o - \frac{vx_o}{c^2}) = \frac{t_o}{\gamma} - \frac{vx}{c^2} \quad (15)$$

by removing the term vx/c^2 in (15) resulting in

$$t_S = \frac{t_o}{\gamma} \quad (16)$$

Here subscripts are used to differentiate between the Lorentz time transformation in (15) and the Selleri time transformation in (16).

Guerra and de Abreu [12] refer to the revised transformations involving (16) as synchronized transformations since the clocks measuring t_S in S can be externally synchronized using synchronized clocks in S_o where the light speed is c. They view the effect in (16) of subtracting the quantity vx/c^2 (thereby resulting in (15)) as delaying these synchronized moving clocks “by a factor that is proportional to their distance x to the reference position $x = 0$”and describe this process as “de-synchronizing” the synchronized measuring clocks which now measure time t_L in S.

These revised transformations (1a-c) and (16) which we shall refer to as the Selleri transformations, also contain the length contraction and time dilation formulas (2) and (3) [5 , 13] and therefore produce the light speed value $c_R(ST)$ of the received light relative to the receiver as determined in the frame of the receiver in (13) given by

$$c_R(ST) = \frac{-c}{1-\beta} \quad (17)$$

While this speed is the same as $c_R(LT) = -c/(1-\beta)$ given by the Lorentz transformations in (13), the two transformations are not equivalent as Guerra and de Abreu believe since they make different light speed predictions based on the differential approach. In particular while the Lorentz transformations using the differential approach predict light speed $c_S(LT) = -c$ as given in (9), the Selleri transformations based on the differential procedure in section (2.1) predict a different light speed $c_S(ST) \neq -c$ [5]. Using the Selleri transformations involving (16) instead of (15) as equation (1d), the one-way speed of light $c_S(ST) = dx/dt_S$ in frame S in a direction opposite to that of v is found by differentiating the transformations giving

$$\frac{dx}{dt_S} = \gamma(\frac{dx_o}{dt_o}\frac{dt_o}{dt_S} - v\frac{dt_o}{dt_S}) \qquad (18)$$

$$\frac{dt_S}{dt_o} = \frac{1}{\gamma} \qquad (19)$$

Again with $\frac{dx_o}{dt_o} = -c$ since the light is travelling in a direction opposite to that of v and using (19) we get

$$\frac{dx}{dt_S} = \gamma^2(-c - v) = -\frac{c+v}{1-\beta^2} = \frac{-c}{1-\beta} \qquad (20)$$

Hence

$$c_S(ST) = \frac{dx}{dt_S} = \frac{-c}{1-\beta} \qquad (21)$$

This light speed value (21) predicted by the Selleri transformations using the differential approach is the same as the light speed value (17) predicted by these transformations using the kinematic approach. These results are summarized in Table 1:

Table 1. Light Speed Calculations

Space-Time Transformations	**Speed of Light: Differential Approach**	**Speed of Light: Kinematic Approach**
Lorentz Transformations	$c_S(LT) = -c$	$c_R(LT) = \frac{-c}{1-\beta}$
Selleri Transformations	$c_S(ST) = \frac{-c}{1-\beta}$	$c_R(ST) = \frac{-c}{1-\beta}$

There is therefore no inconsistency in the Selleri transformations as occurs in the Lorentz transformations and Selleri has shown [13] that these revised transformations predict the confirmed relativistic effects associated with the Lorentz transformations.

4. Conclusion

We have discovered a paradox in special relativity that is the subject of this paper. We have shown using elementary analysis that the Lorentz transformations of special relativity contain an inconsistency since using these transformations, light speed determined utilizing a differential approach is different from light speed determined using a kinematic approach. A critical factor in the kinematic calculation is the existence of (Einstein) synchronized clocks in S_o where light speed is c such that the instant of light emission $t_o = T_o$ is available both at the transmitter and the receiver. This enables the legitimate application of the time dilation formula (3) which, along with the length contraction formula (2) yields the inconsistency represented in (14).

This use of the length contraction formula (2) and time dilation formula (3) in the kinematic calculation is new and represents an indirect application of the Lorentz transformations from which they originate. It produces the light speed result (13) given by $c_R(LT) = -c/(1-\beta)$ which is also obtained by direct application of the Lorentz transformations in the kinematic calculation [8]. The validity of the kinematic result (13) is therefore beyond question. The problem for special relativity is that this light speed result (13) determined using the kinematic approach is different from the light speed result (9) given by $c_S(LT) = -c$ determined using the differential approach.

Removal of this inconsistency requires an adjustment in the temporal component of the transformations such that the term vx/c^2 is excluded. This was done by Selleri who showed that the resulting transformations make predictions that closely accord with observation. These Selleri transformations do not suffer from the inconsistency (14). Moreover they dispose of Selleri's paradox since they predict that the ratio of the speeds of two light signals travelling in opposite directions around a rotating disc is Selleri's non-unity value as required [5, 6, 13]. This is consistent with the light speed values on a rotating platform obtained using the GPS [14, 15] as well as with the fact that one-way light speed constancy is unconfirmed [16]. We therefore advance the Selleri transformations as the correct space-time transformations of modern physics [17].

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