of the same kind, the method of calculating matrix elements developed in reference 1 is applicable. The calculations are tedious, however, and the resulting expression for the energy in terms of α, β, and c is rather complicated. Minimization of the energy leads to an optimum value for the binding energy of roughly 0.135 ev, when $\alpha=\beta=(0.5)^{\frac{1}{2}}$ and $c=0.052$, approximately.

In previous calculations considerable improvement could be obtained by a slight improvement in the function. The fact that this is no longer true suggests that we might be near the convergence limit.

If we let ψ_β approximate the quadrielectron wave function, the mean distance between the various particles constituting this cluster is found to be roughly $\bar{r}_{12}=\bar{r}_{ab}=4.5\times10^{-8}$ cm and $\bar{r}_{1a}=\bar{r}_{2b}=\bar{r}_{1b}=\bar{r}_{2a}=3.0\times10^{-8}$ cm, that is, ψ_β gives reasonable relative values of the "repulsive" and the "attractive" distances. Furthermore, using the same function we find the value 1.16 for the ratio of the root mean square value of r_{12} to the mean value of this quantity. For the bielectron in the ground state, on the other hand, the corresponding ratio for the separation of the two particles has the value 1.15.

The values of the various mean distances reveal that the quadrielectron has a considerable size and, consequently, a large breakup probability when passing through matter even of small density.[3]

* The work here reported forms part of a dissertation presented for the degree of Doctor of Philosophy in Yale University.

[1] E. A. Hylleraas and A. Ore, Phys. Rev. **71**, 493 (1947).

[2] S. Weinbaum, J. Chem. Phys. **1**, 593 (1933).

[3] For more details regarding the probability of breakup *vs.* annihilation of light polyelectrons see: J. A. Wheeler, Ann. New York Acad. Sci. **48**, 219 (1946).

The Hyperfine Structure of Atomic Hydrogen and Deuterium†

J. E. Nafe, E. B. Nelson, and I. I. Rabi
Columbia University, New York, New York
May 19, 1947

THE hyperfine structure separation, ν_H and ν_D, of atomic hydrogen and deuterium were measured directly by means of the atomic beam magnetic resonance method.[1-3] For each atom two resonance lines were measured, each at the same value of the magnetic field, and the ν_H and ν_D were evaluated entirely from differences in the frequencies. Neither the value of the magnetic field nor the g values of the atomic and nuclear systems enter into the final result.

In H, where the value of the nuclear spin $I=1/2$ and the atomic $J=1/2$, the π-transitions $(1,1)\leftrightarrow(0,0)$ and $(1,0)\leftrightarrow(1,-1)$ were measured at the same value of the magnet current. The difference between these two frequencies gives ν_H directly (see Eqs. 9–12 of reference 3). For D, where $I=1$ and $J=1/2$, the line $(3/2,1/2)\leftrightarrow(1/2,-1/2)$, $(3/2,-1/2)\leftrightarrow(1/2,1/2)$, an unresolved doublet, and the line $(3/2,3/2)\leftrightarrow(1/2,1/2)$ were measured in quite weak fields of the order of one gauss. The first line gives ν_D almost directly, and the difference in frequency of the two lines gives a small correction of less than 0.01 percent.

The measured values of ν_H and ν_D, in megacycles per second, are

$$\nu_H=1421.3\ \pm0.2$$
$$\nu_D=\ 327.37\pm0.03.$$

The method is inherently capable of greater precision with the improvement of our frequency meter.

Since the theory of the H and D atoms is considered to be complete and exact, these values can be compared directly with calculations. The formula for the hyperfine structure separation of S states was given by Fermi[4] and is

$$\nu=\frac{8\pi}{3h}\left(\frac{2I+1}{I}\right)\mu_N\mu_0\psi^2(0). \tag{1}$$

The nuclear spin is denoted by I, μ_N is the magnetic moment of the nucleus in question, μ_0 is the Bohr magneton, and $\psi(0)$ is the value of the Schroedinger wave function evaluated at $r=0$. $\psi^2(0)$ is proportional to $(1/a)^3$, the cube of the reciprocal of the radius of the first Bohr orbit. Since a is inversely proportional to the reduced mass, the appropriate value of the reduced mass, m_r, has to be inserted. If the values of the quantities in Eq. (1) are expressed in terms of the fundamental constants, Eq. (1) becomes

$$\nu=\frac{4}{3}\left(\frac{2I+1}{I}\right)\frac{\mu_N}{1836.6}\left(\frac{m_r}{m_0}\right)^3\alpha^2CR_\infty. \tag{2}$$

R_∞ is the Rydberg constant for infinite mass, α is the fine structure constant, and μ_N is the nuclear moment in terms of the nuclear magneton, $\mu_0/1836.6$. For μ_P and μ_D we have the accurate values of Millman and Kusch[5]

$$\mu_P=2.7896\ \pm0.0008$$
$$\mu_D=0.85648\pm0.00037,$$

for α^2, R_∞, and C we have the values given by Birge[6]

$$\alpha^2=(5.3256\pm0.0013)\times10^{-5}$$
$$R_\infty=109737.303\pm0.017\ \text{cm}^{-1}$$
$$C=(2.99776\pm0.00004)\times10^{10}\ \text{cm sec.}^{-1}$$

With these values and the value of the ratio μ_P/μ_D given by Kellogg, Rabi, Ramsey, and Zacharias[2] and by Arnold and Roberts[7] as 3.2571 ± 0.001, we obtain the results given in Table I.

TABLE I. The hyperfine structure separation of H and D.

	Measured	Computed from Eq. (2)
ν_H	1421.3 ±0.2 Mc	1416.90±0.54 Mc
ν_D	327.37 ±0.03 Mc	326.53±0.16 Mc
ν_H/ν_D	4.3416±0.0007	4.3393±0.0014

There is clearly an important difference between the measured and calculated values of ν_H and ν_D of about 0.26 percent compared with the probable error of the calculated value of 0.05 percent. The difference is five times greater than the claimed probable error in the natural constants. Whether the failure of theory and experiment to agree is because of some unknown factor in the theory of the hydrogen atom or simply an error in the estimate of

one of the natural constants, such as α^2, only further experiment can decide.

The comparison of the experimental ratio to the calculated ratio is particularly important, since most of the natural constants cancel out. The agreement is much better than for the absolute value but still not exact. The experimental value is about 0.06 percent greater than the calculated value. The error of the calculated ratio arises chiefly from the measured ratio of μ_P/μ_D which is claimed to be accurate to about 0.03 percent. Clearly this interesting deviation is worthy of further study.

† Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] J. M. B. Kellogg, I. I. Rabi, and J. R. Zacharias, Phys. Rev. **50**, 472 (1936).
[2] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, Jr., and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).
[3] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **57**, 765 (1940).
[4] E. Fermi, Zeits. f. Physik **60**, 320 (1930).
[5] S. Millman and P. Kusch, Phys. Rev. **60**, 91 (1941).
[6] R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).
[7] W. R. Arnold and A. Roberts, Phys. Rev. **70**, 320 (1946).

Phase of Scattering of Thermal Neutrons by Aluminum and Strontium*

E. FERMI AND L. MARSHALL
Argonne National Laboratory and University of Chicago, Chicago, Illinois
May 15, 1947

IN a previous paper[1] we have described a method for determining whether neutrons scattered by an atom have the same phase as the primary neutron wave or opposite phase. The method has now been applied to two more elements, Al and Sr. The crystals investigated were Al_2O_3 (corundum) and $SrSO_4$ (celestite). The measured intensities of various orders of Bragg reflections of monochromatic neutrons are given in the following table, which is arranged like Table I of reference 1.

TABLE I. Intensities of reflection of thermal neutrons by Al_2O_3 and $SrSO_4$.

Crystal	Plane	Order	Form factor	Intensity
Al_2O_3	$1\bar{1}0$	1	2Al − 1.44 O	480
		2	2Al − 1.34 O	700
		3	2Al + 2.09 O	5940
$SrSO_4$	001	1	0.44 Sr + 0.77 S + 0.12 O	4351
		2	0.62 Sr − 0.19 S + 0.67 O	3576
		3	0.98 Sr + 0.48 S + 1.01 O	2182
		4	0.24 Sr + 0.93 S − 1.81 O	1682
	210	1	0.78 Sr + 0.66 S − 0.01 O	6021
		2	0.21 Sr − 0.14 S + 0.36 O	413
		3	0.44 Sr + 0.84 S + 1.09 O	1493
	101	1	0.46 Sr − 0.65 S − 0.54 O	702
		2	0.55 Sr − 1.16 S − 2.54 O	3182
		3	1.94 Sr + 0.83 S + 0.50 O	5759

Attempts to fit these data with actual values of the scattering length for aluminum and strontium have not been satisfactory. It seems unambiguous, however, that the sign of the scattering of aluminum is the same as that of oxygen, namely, positive according to our convention. This is proven by the low intensity of first and second order compared with that of the third order.

A similar behavior of the reflection from the (101) plane of celestite indicates that the scattering length of strontium is also positive. From the scattering cross sections of these two elements, 1.4×10^{-24} cm² for Al and 9.5×10^{-24} for Sr, one can calculate the scattering lengths 0.35×10^{-12} cm for Al and 0.88×10^{-12} cm for Sr.

* This document is based on work performed under Manhattan Project sponsorship at the Argonne National Laboratory.
[1] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).

Pressure and Temperature of the Atmosphere to 120 km

N. BEST, R. HAVENS, AND H. LAGOW
Naval Research Laboratory, Washington, D. C.
May 9, 1947

PRESSURES and temperatures of the atmosphere up to 120 km were determined from data taken on the V-2 rocket fired at White Sands, New Mexico on March 7, 1947. The methods used in obtaining these data were similar to those used in a previous flight.[1] The pressure measurements were made with bellows gauges for pressures between 1000 mm Hg and 10 mm Hg. For pressures between 2 mm Hg and 10^{-2} mm Hg, tungsten and platinum wire Pirani gauges were used. A Philips gauge was used for pressures between 10^{-3} and 10^{-5} mm Hg.

Ambient pressures (Fig. 1) were measured up to about

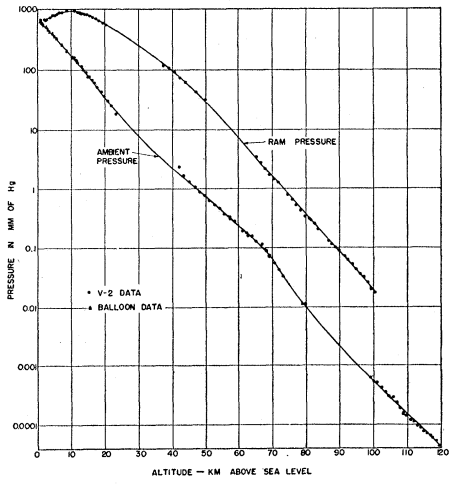

FIG. 1. Ambient and ram pressures as a function of altitude.

80 km with gauges mounted on the side of the V-2, just forward of the tail section. Pirani gauges, mounted in similar positions on opposite sides of the rocket, gave readings which agree within experimental errors, indicating that no appreciable error was introduced by yaw of the missle up to this altitude. A single Philips gauge was mounted on the 15° cone of the warhead. The readings of this gauge were reduced to ambient pressures by use of theories of Taylor and Maccoll.[2] Photographs of the earth made from the missile and gyroscope data indicated a yaw of about 15° at 110 km and a roll period of 40 seconds.