

# How We See Straight Lines

*In looking at a straight line, the eye can detect a lateral break that forms an image only .00001 centimeter wide on the retina. A new hypothesis holds that this ability is due to rapid scanning motions*

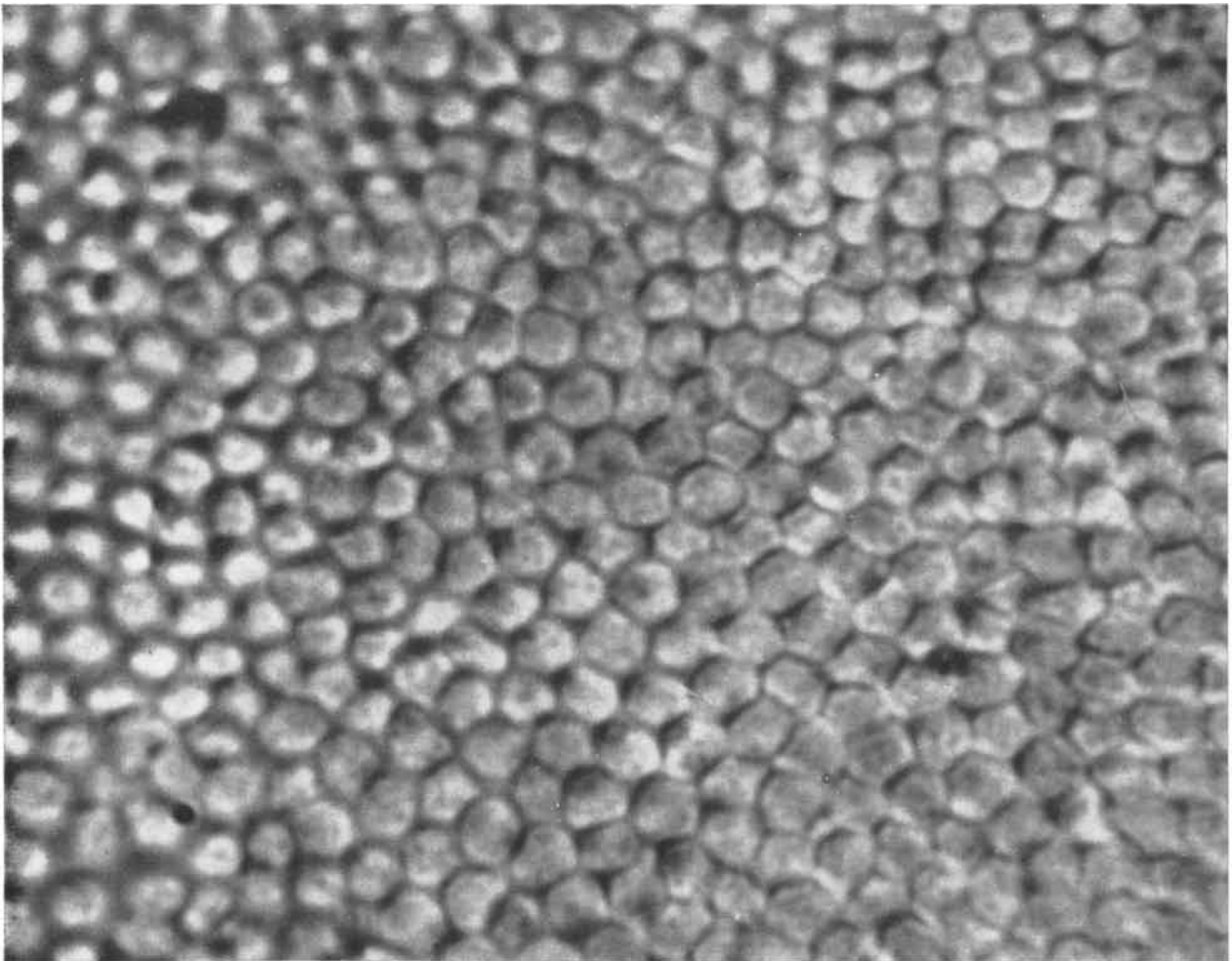
by John R. Platt

On first thought it would seem quite impossible for human beings to see whether a line is straight or not. Our visual mechanism is apparently unsuited to the task.

Consider what happens when we look at a straight line: Its image falls on the

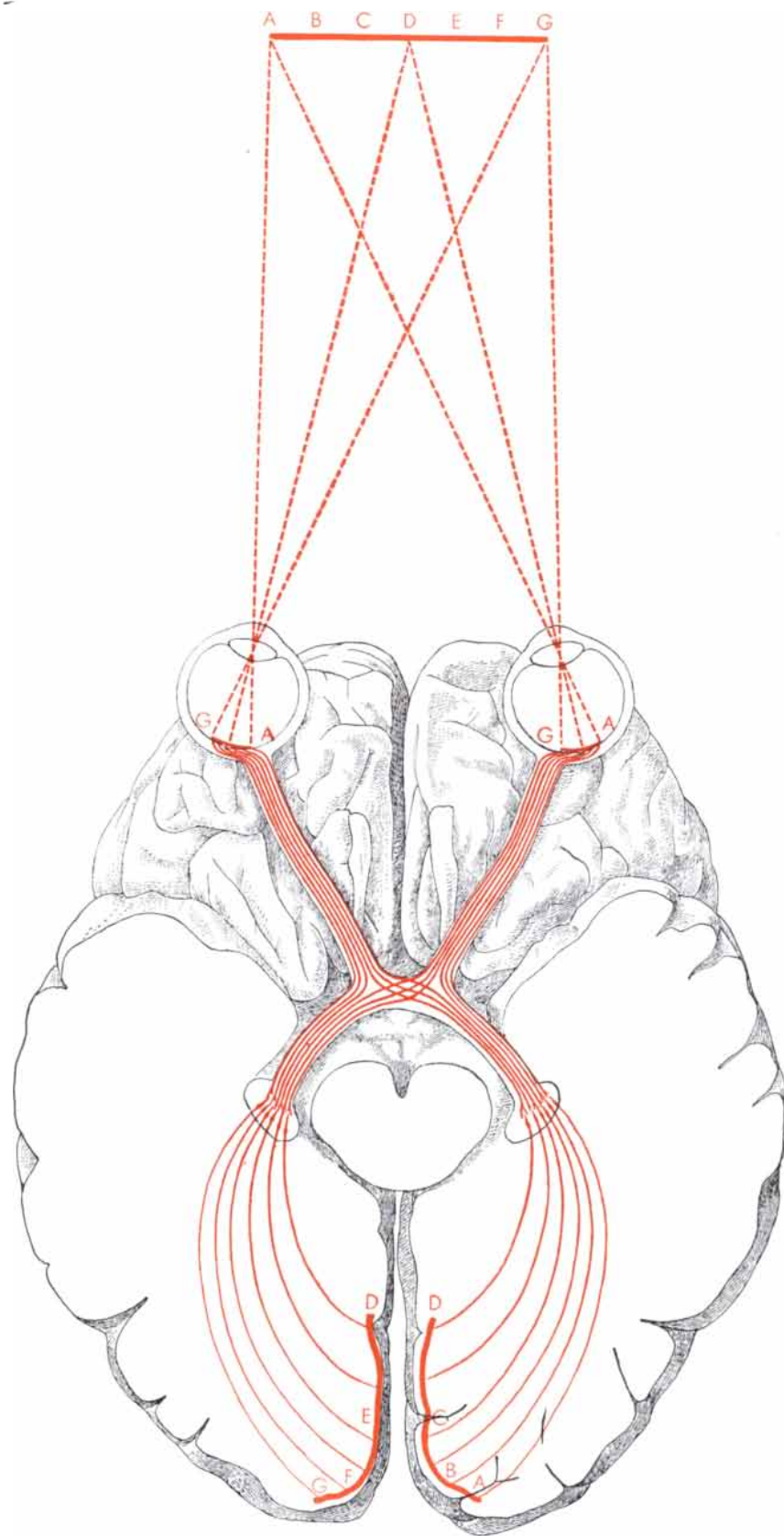
curved surface of the retina at the back of the eye. Here the light pattern stimulates the tiny receptor elements—rods and cones—that lie underneath it, and they fire off a volley of electrical signals to vaguely defined regions on both sides of the cortex of the brain. Surely the

cerebral pattern is not “straight.” It is often argued, however, that the brain somehow knows the location of the rods and cones whose stimulation gave rise to the sensation. If these particular rods and cones lie along an appropriate curve, the object they record is a straight line.



CELLS IN RETINA of the human eye form a mosaic pattern of light-sensitive receptors. This photomicrograph shows the cells en-

larged about 3,000 diameters. It appears in *The Vertebrate Visual System*, by S. L. Polyak, published by University of Chicago Press.



Let us examine this reasoning a little more closely. The retina is a layer of tissue about one inch square, containing something like 10 million receptor cells arranged in a closely packed mosaic. How can the brain know where each cell is? Hermann von Helmholtz, the great pioneer in the theory of vision, thought the knowledge might be provided by specific "local signs"—possibly chemical in nature—from cells at different positions. His idea has recently been confirmed in some elegant experiments performed by Jerome Y. Lettvin and his colleagues at the Massachusetts Institute of Technology. When they cut a frog's optic nerve and allowed it to regenerate, they found that the neuron from each point of the retina grew back to its proper point in the brain.

But this specificity can hardly be indefinitely fine. Seen under a microscope, the mosaic of retinal cells looks random, and, as living tissue, it has been subject to all the accidents and irregularities of biological growth. Surely the cells must be subject to some microscopic uncertainty of location. Thus a line that appears straight to one man should appear full of little wiggles to his twin brother. The amplitude of the wiggles would indicate the limits of accuracy of the genetic or local sign-specification.

Yet the fact is that we *can* tell when a line is straight, and none of us ever sees any such wiggles. Our actual precision in certain visual observations is fantastic. Our vernier acuity, or ability to detect a lateral break in a straight line, is about two seconds of arc. This corresponds to a distance of a little more than a hundred thousandth of a centimeter on the retina, about a 30th of the diameter of a cone cell! Even in mechanical construction this precision is almost impossible; a hundred thousandth of a centimeter cannot be measured in the finest machine shops except by optical methods. In a biological system such as the eye the location of every tissue cell to such an accuracy, 30 times finer than the size of the cell, is quite unbelievable.

I puzzled over this paradox for a long time, until I finally began to wonder if we were not looking at the problem in the wrong way in emphasizing the precise location of the individual cells. We were unconsciously assuming that the brain can somehow examine its associated retina, as if through an external microscope, and locate each of the rods or cones in space.

Thinking about the microscope fallacy, as it might be termed, led me to wonder whether there could not be some high-

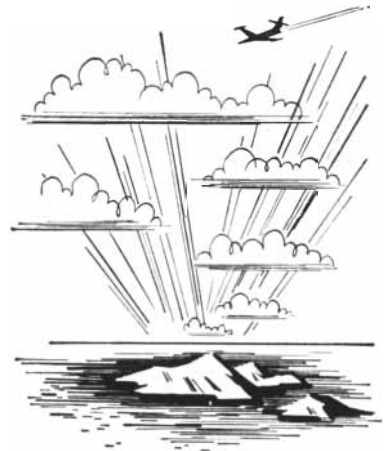
**VISUAL PATHWAYS** of the human central nervous system (solid colored lines) carry stimuli from the eye (center) to the visual areas of the brain. This horizontal cross-section view shows how the image of a straight line (AC) falls on the curved surface of the retina and is projected onto convoluted areas of the visual cortex (heavy colored lines at bottom).



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PARALLEL SCANNING along a straight line (*top row*) or a curved line (*bottom row*) excites a single pattern of receptors on the surface of the retina (*large circles*). Small black circles indicate

excited receptors. If eye scans a broken line (*middle row*), three different patterns of receptors are stimulated. The brain interprets this shift from one pattern to another as a lack of straightness.

precision physical method that would enable a system consisting of 10 million elements to make acute discriminations without knowing exactly where its individual sensory elements were located. I finally found one, a method that I call functional geometry. As its name implies, the method generates spatial relations in the course of the normal functioning of the visual system rather than through the static, point-by-point location of images.

The essence of this functioning is motion, or scanning. Several years ago experimenters discovered that, in order to keep a static pattern steadily in view over even a short period of time, a person must continuously shift his eyes in tiny scanning motions. If he does not, the image fades away. I suggest that the same scanning can provide the sense of straightness.

The basic idea is as follows. Imagine the image of a scene—any scene—projected on the retina. The arrangement of light and dark areas stimulates a particular set of rod and cone cells, which then transmit a specific array of signals to the brain. If the eye scans the scene, moving so as to shift the image slightly, a somewhat different set of receptors is stimulated, and the signal array changes accordingly. But suppose the scene is a straight line, and the scanning is parallel to the line. Then the motion does not change the set of stimulated receptors, and the signal array remains constant. This constancy, or “self-congruence,” after displacement is what the brain recognizes as straightness.

Evidently an ability to detect the sameness of an array is about the weakest demand one could make of a communications network, however it may operate. Moreover, the perception can be made without knowledge of where the individual receptor-cells are located. All that is necessary is an external object that is congruent to itself under a displacement such as the eye can carry out. A straight line fulfills the condition. So do parallel lines. A crooked line, or a set of nonparallel lines, does not.

One of the important features of the method is that the images of these lines on the retina or on the cortex can be as crooked as you please without destroying the self-congruence; all that is required is that the image fall on the same locus after displacement, and it makes no difference what that locus is. The discrimination is therefore for straightness or parallelism in the external field. Clearly the brain does not know—and, if it uses functional geometry, does not need to know—how the images on its

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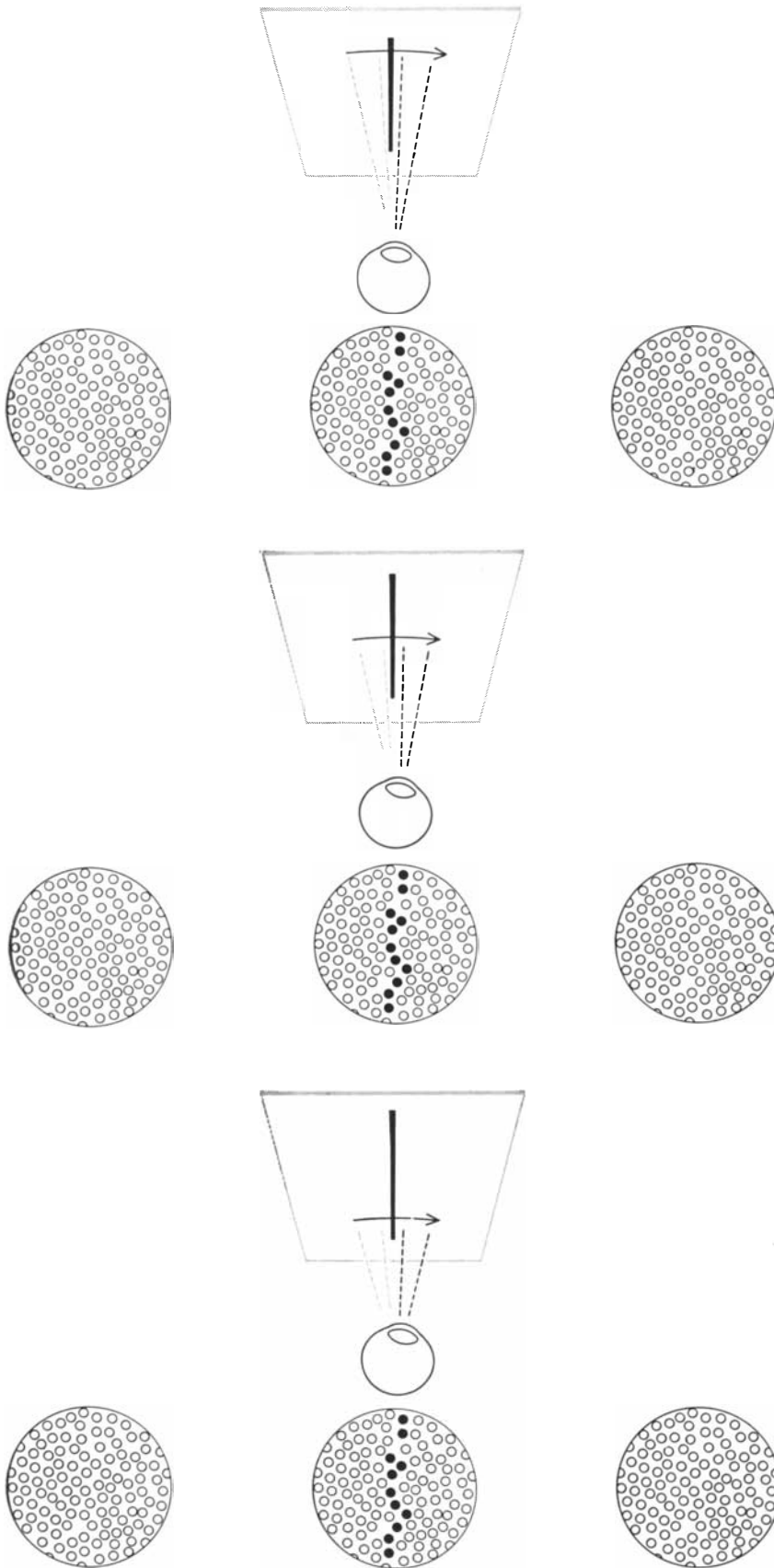
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**TRANSVERSE SCANNING** is another method of determining the straightness of a line. By comparing the patterns produced on the surface of the retina each time the eye sweeps across the line, the brain can form a very accurate judgment of its relative straightness.

cortex would look when observed from the outside. The fact that we see self-congruence in the external field is what makes straightness (and the other self-congruent pattern properties I shall mention) matters for public discussion. We do not see objects, but relationships; and the relationships are public. This is a point of considerable importance in linguistics and in theories of knowledge.

Another important feature of the self-congruent method of perceiving patterns is that it is not affected by damage or loss of receptor cells, or by blind spots. An array of signals can be the same after displacement as before, regardless of what cells have high or low sensitivity. This means we do not have to assume uniform sensitivity in all the cells of the eye. And it is consistent with the fact that we do indeed perceive patterns as passing straight across our blind spots.

In addition to scanning back and forth along a line, our eyes may move transversely across it. By making a series of such perpendicular passes at various points along a line, and by comparing the times at which signals come from different receptors, we can also form a judgment of straightness. Here we are limited only by the time taken in observation. The longer the time, the closer the check on possible discrepancies between time-sequences at various points along the line. It is probably this mechanism by which we make visual judgments of the highest acuity.

Either type of scanning of straight lines, or of parallels, is carried out by combinations of two rotations of the eyeball: around a horizontal axis (looking up or down) and around a vertical axis (looking left or right). However, our eyes are capable of still another motion, though it is sharply limited: rotation around a longitudinal axis pointing along the line of sight [*see bottom of illustration on page 128*]. (To observe this rotation, closely examine a marking on the iris of your eye in the mirror as you tip your head from side to side.) By adding a component of this twisting motion we can scan along a curve, and, if it has constant curvature, keep the image over the same set of receptors on the retina. Thus arcs of circles exhibit the same sort of self-congruence as straight lines, and concentric arcs the same sort of self-congruence as parallels.

As a matter of fact, it is not easy to judge whether a gentle arc is curved or straight. Uniformity of curvature (including the zero curvature of a straight line) is more readily perceived than is straightness or curvedness. If the curvature is sharp enough, however, we



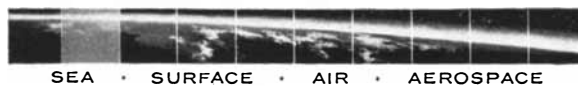
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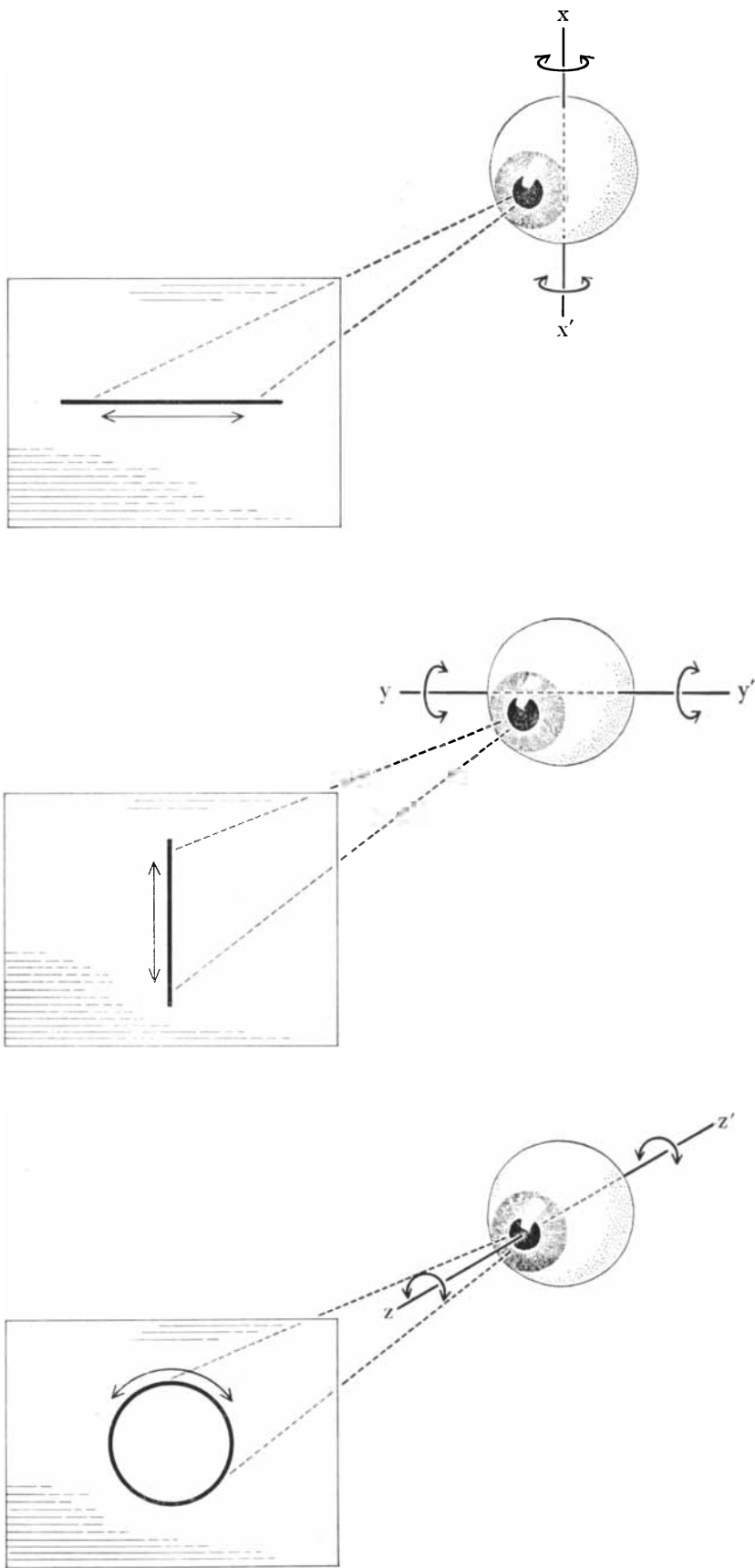
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EYE ROTATES about three axes as it scans lines and geometric shapes. It can turn rather freely about the  $x-x'$  and the  $y-y'$  axes, but can rotate only slightly about the  $z-z'$  axis.

probably become aware of it through muscular cues arising out of the twisting motion.

Both the method of self-congruence and the approach to greater accuracy through repeated trials are central concepts in high-precision optical work. Every amateur who has made his own telescope mirror knows that spherical and plane mirrors and precision screws can be brought virtually to perfection by being polished with a matching tool until they are self-congruent under lateral or rotational displacement. Within a finite time the error can be made less than any preassigned value.

In biology the principle of self-congruence generates perfectly helical elbow-joints and spherical hip-joints, and eyeballs in spherical sockets. The spheres are self-centering; they know nothing about the point centers and fixed radii of Euclidean geometry. This suggests a new approach to the study of geometry. It might be more natural to start not with points, distances, lines and coordinates, but with self-congruences, which are biologically more primitive.

Do we actually apply functional geometry to every judgment of straightness (and other patterns)? The experienced adult eye may not need to scan every new line afresh to determine its approximate straightness. Possibly certain receptors on the retina have been associated so often in past straight-line perceptions that when these elements are excited again and give off the same chorus of signals, we are satisfied of the straightness of the new object without further scanning. It is self-congruence to an old straight line, with a long time-delay. In a sense, the pattern has been learned.

If this is the mechanism of pattern perception, then we should expect to find that an infant or a visually naive adult (for example, a person who has had congenital cataracts removed) would require long scanning and study to determine the straightness of a line. Such, in fact, appears to be the case. The finding is consistent not only with the theory of perception developed here, but also with the doctrine of D. O. Hebb and his school at McGill University. They hold that perceptual organization of even such apparently primitive relationships as straightness or triangularity is acquired—learned—only through visual experience.

Arthropods (such as insects and spiders) can learn almost nothing, and



birds can learn only certain things. It follows that much, if not all, of their pattern-perceiving system must be pre-located and preconnected, determined by genetic information alone. Pattern-perceiving that involves learning, perhaps using methods such as functional geometry, is a way of escaping this genetic limitation. Such an escape is obviously needed for a really big brain with more inputs. This suggests that pattern-learning may be the faculty that grew most rapidly in the sudden evolutionary expansion of our own brain and cortical capacity in the last few hundred thousand years.


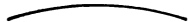

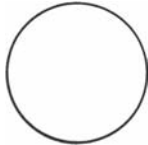

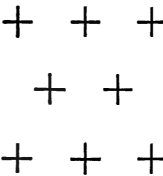
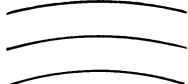
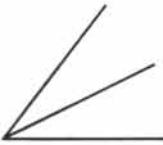

Perhaps the most noteworthy feature of functional geometry with a mosaic system is that it necessarily picks out certain patterns as fundamental or primitive. A mathematician of curved spaces might say that an S-curve in one curved

space is a straight line in another, and that these are equally good descriptions of the line. But a functional mosaic will accept as straight only those Euclidean lines that satisfy self-congruence under displacement. Thus straightness is a primitive and unique category of perception for all mosaic systems. So is parallelism, concentricity and so on. It is interesting to note that the various relationships belong to the "synthetic *a priori*" categories of Immanuel Kant—unique categories that impose themselves on all minds regardless of particular experiences and yet cannot be learned without experiences and comparisons.

I suggest that there is only a small number of unique symmetry categories for a visual mosaic receptor, and that they are determined by the three possible rotations of the eyeball [see table

below]. When the rotations are continuous, we get straightness, parallelism and the like. When the eye moves in discrete jumps, it perceives relationships such as equidistance, congruence and the equality of angles. On this view every visual pattern-relationship that can be perceived is some combination of the primitive elements.

It would be interesting to try to construct artificial mosaic receptors, complete with scanning motions, that might be able to make discriminations similar to those our eyes make. Evidently any such system would have to be able to learn. The receiving network would somehow have to grow or to establish new connections guided by experience. If we could design such a system, it might teach us far more than we now know about how the human eye and brain organize external information.

PATTERN OBSERVED		AXIS OF EYE MOVEMENT	PATTERN OBSERVED		AXIS OF EYE MOVEMENT
STRAIGHT LINE		$xx'$ AND $yy'$			
CURVATURE OF ARC		$xx'$ , $yy'$ AND $zz'$	CONGRUENCE		$xx'$ , $yy'$ AND $zz'$
CIRCULARITY		$zz'$			
PARALLEL LINES		$xx'$ AND $yy'$	EQUIDISTANCE		$xx'$ AND $yy'$
CONCENTRICITY OF ARCS		$xx'$ , $yy'$ AND $zz'$	EQUIANGULARITY		$zz'$
CONCENTRICITY OF CIRCLES		$zz'$			

**PRIMITIVE PATTERNS** are self-congruent under the types of rotation the eye can perform. Patterns at left are perceived

by continuous rotations; relationships at right, by discrete rotations. Axes of rotation are listed in second and fourth columns.

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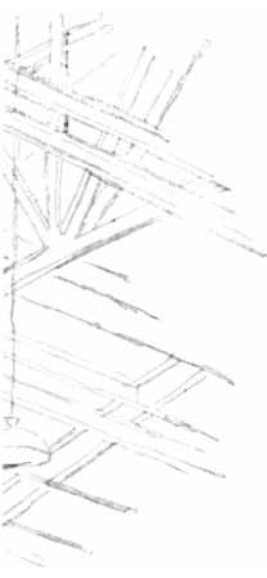


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