Hafele-Keating Experiment Reassessed

Preprint · September 2020

Gianni Casonato
European Organisation for the Exploitation of Meteorological Satellites (EUMETSAT)

Hafele - Keating Experiment Notes - 1.2

Table of Contents

1 Background

1.1 Scope

The note address the experiment conducted in the 70’s by two researchers, Hafele and Keating, for measuring the effect on atomic clocks time of a relative motion of the clocks with respect to a ground based time reference station, and to compare the results against the predictions made by special relativity theory.

In the note the calculation of predicted time shift is re-calculated based on the flight data in [HK-3] and then compared with the measurements made by Hafele and Keating as in [HK-1/HK-2].

1.2 References

The following documents are considered references for the content of this note:

[HK-1] – Around the World Clocks: Predicted Relativistic Time Gains – Hafele J. C., Keating R .E. – Science Vol. 177 (1972)

[HK-2] – Around the World Clocks: Observed Relativistic Time Gains – Hafele J. C., Keating RE. – Science Vol. 177 (1972)

[HK-3] – Performance and Results of Portable Clocks on Aircraft – Hafele J. C., – PTTI, 3rd Annual Meeting, 1971

[HK-4] - Relativistic Time for Terrestrial Circumnavigations – Hafele J. C., American Journal of Physics 40, 81 (1972)

[HK-5] - Relativistic Behaviour of Moving Terrestrial Clocks – Hafele J. C., Nature, Vol. 227, p. 270 (1970)

[SRT] – Special Relativity – French A., 1968

[GRVTD] – Gravitational Time Dilation – Wikipedia

[REL-ITU] – Relativistic Time Transfer – ITU 2018

2 The Experiment

2.1 Description

The Hafele-Keating (HK) experiment on time dilation was done in 1971 with the objectives of measuring time shift on moving clocks with respect to an Earth based reference clock by flying clocks in eastbound ans westbound trips using commercial flights (see [HK1] and [HK2]), and proving the correctness of the prediction made using the theory of relativity.

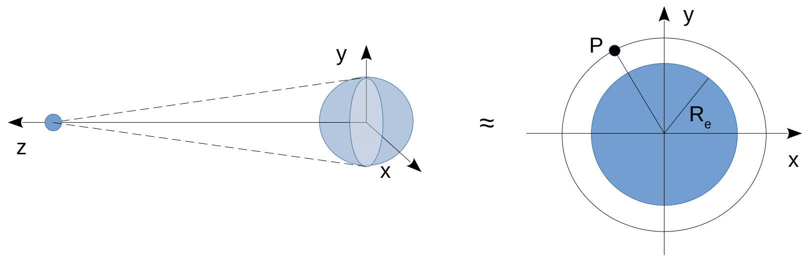
The experiment consisted in using two high precision Caesium atomic clocks, synchronised with a US reference ground station (USNO) for flying them on commercial aircrafts for two separated closed flight trips, one eastward, and one westward. After returning to USNO the clock times were compared with the reference time and the time shifts recorded. The observations were compared with the predicted values calculated applying the general relativity formula and a close matching was found.

2.2 Time Shift Model

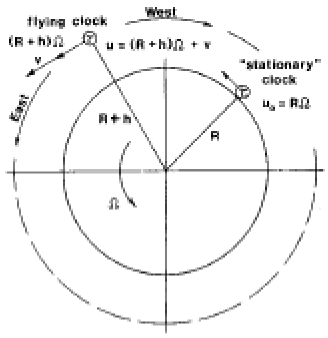
2.2.1 General Expression from GR

As explained in [HK-4] and [HK-5], the formula considered applicable for this experiment can be derived from the GR base expression, by considering a "fixed" reference frame centred in the Earth and with the z-axis passing to the North pole, and three moving reference frames located respectively at each clock location, namely at USNO for the reference clock, and on the various airplanes for the two flying clocks.

To be noted that in spite of this choice for the reference frame (3-dimensional Earth centred), the derivation of the equation done in [HK-4] is in 2-dimensions only, without further explanation. A possible justification could the use of a 3D frame with a z axis aligned with Earth's rotation axis, and a frame origin very far from Earth's centre, such that the problem geometry can be seen as 2D.



Looking at this setup, the experiment can be modelled with some approximation as the relative movement of USNO and flying clocks on circular orbits in the fixed frame.



The relative motion between the USNO located and the flying clocks would create a time shift between them. Starting then from the general GR expression, the differential expression for the metric in the moving frame with respect to the one in the fixed frame is

$$ds^2 = \left(1 + \frac{2\chi}{c^2}\right) c^2\, dt^2 - \left[\frac{dr^2}{1 + \frac{2\chi}{c^2}} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Introducing the specificities of the case, namely:

- weak gravitational field ($\chi \ll c^2$) being

$$\chi = -\frac{GM}{R+h}$$

- low velocity ($v^2 \ll c^2$)

2.2.2 Earth's Based Circular Path Case

Making some simplifications in the expression terms due to above assumptions (2nd order effects negligible), and assuming constant / average velocity v, latitude and altitude h, the final formula for a flight path becomes

$$\Delta_\tau = \tau_b - \tau_a = \left\{ 1 - \frac{GM}{c^2 R(1+h/R)} - \frac{[R\Omega(1+h/R)\cos\lambda + v]^2}{2c^2} \right\} \Delta_t = \alpha(h, v, \lambda)\, \Delta_t$$

where

- c is the speed of light
- h is the flight altitude above Earth's surface – assumed constant
- GM is the Earth' gravitational parameter
- R is the Earth's radius
- Ω is the Earth's angular velocity
- λ is the geographical latitude – assumed constant
- v is the ground speed during the flight path (>0 for eastward flight, <0 for westward flight) – assumed constant
- Δt is the absolute elapsed flight time, i.e. elapsed flight time in the fixed reference frame
- Δτ is the moving clock elapsed time

This expression contains two terms responsible for the offset between reference and moving frame times:

- a gravitational term, depending only on altitude

$$\Delta_{\tau-grav} = -\frac{GM}{c^2 R(1+h/R)}\,\Delta_t$$

- a kinematic term depending on the velocity, both as magnitude and as direction.

$$\Delta_{\tau-kin} = -\frac{[R\,\Omega(1+h/R)\cos\lambda+v]^2}{2c^2}\,\Delta_t$$

To be noted the latter term that the dependency on the direction of the velocity vector v makes it responsible for slower (v>0, i.e. eastward) or faster (v<0, i.e. westward) running clock behaviours.

The above expression is general and valid for both ground and flying clocks, for paths where the assumptions of constant altitude, velocity and latitude holds. In case of variability over time of such parameters an integral expression has to be used, using the expression above as differential and integrating over the whole time interval.

Finally, the expression depends on the elapsed time measurement in the reference frame , which is needed to calculate the net time shift expected on the moving frame but this it's in general not know and not measurable, due to the choice of the reference frame that does not allow to use its origin as part of the experiment.

For the two specific cases of ground and flying clocks, noting that for USNO location both altitude (approximated) and ground speed are zero, the expression becomes

- $\Delta_{\tau-USNO} = \alpha(0,\,0,\,\lambda_{USNO})\,\Delta_t = [1-\frac{GM}{c^2 R}-\frac{R^2\Omega^2\cos^2(\lambda_{USNO})}{2c^2}]\,\Delta t$
- $\Delta_{\tau-east} = \alpha(h_{east},\,v_{east},\,\lambda_{east})\,\Delta_t$

- $\Delta_{\tau-west} = \alpha(h_{west},\ v_{west},\ \lambda_{west})\ \Delta_t$

Being the value of Δt not known and not measurable in the experiment, the expression for the flying case and the ground case has to be rearranged together, observing that even if different, the two paths (ground, eastward or westward flight) considered for each trip start and end at the same point and they are then related to the same Δt value, which can be eliminated by dividing one expression with the other. Following that, it results for each flight path

$$\frac{\Delta_{\tau-flight}}{\Delta_{\tau-USNO}} = \frac{\alpha(h_{flight},\ v_{flight},\ \lambda_{floght})}{\alpha(0,\ 0,\ \lambda_{USNO})} = \frac{1-\dfrac{GM}{c^2R(1+h_{flight}/R)}-\dfrac{[R\Omega\cos(\lambda_{flight})(1+h_{flight}/R)+v_{flight}]^2}{2c^2}}{1-\dfrac{GM}{c^2R}-\dfrac{R^2\Omega^2\cos^2(\lambda_{USNO})}{2c^2}}$$

This equivalent to invert the expression above to calculate the value of the absolute elapsed time Δt from the measured one on ground, namely

$$\Delta_t = \frac{\Delta_{\tau-USNO}}{\alpha(0,0)}$$

From those formulas it may be useful to isolate another parameter delta, defined as the difference in rates of the ground and flying clocks, and which is still independent from the time measurement in the absolute frame, namely

$$\delta = \frac{(\Delta_\tau - \Delta_{\tau_0})}{\Delta_{\tau_0}} = \frac{\alpha(h_{flight},v_{flight})}{\alpha(0,\ 0)} - 1 \quad ==> \quad \delta_{flight/USNO} = \delta_{flight} - \delta_{USNO}$$

Due to the fact the flight path is closed the delta parameter is independent of the reference frame and it provides then a measure for the relative effect of the shift to be used as comparison with respect to the intrinsic RMS accuracy of the clocks (10^{-13} for Cesium based on used in the experiment).

Once known, the absolute values for the eastward and westward trip time shifts can be calculated too. Finally the comparison between USNO and flying absolute time shifts give the expected time shift for the two trips.

$$\Delta_{\tau-east/USNO} = \Delta_{\tau-east} - \Delta_{\tau-USNO} = \left[\frac{\alpha(h_{east},v_{east})}{\alpha(0,\ 0)} - 1\right]\Delta_{\tau-USNO} = \delta_{east/USNO}\cdot\Delta_{\tau_{USNO}}$$

$$\Delta_{\tau-west/USNO} = \Delta_{\tau-west} - \Delta_{\tau-USNO} = \left[\frac{\alpha(h_{west},v_{west})}{\alpha(0,\ 0)} - 1\right]\Delta_{\tau-USNO} = \delta_{west/USNO}\cdot\Delta_{\tau_{USNO}}$$

2.2.3 Simplified Expression for Low Altitude Flight

As explained in [HK-4] and [HK-5], the general expression above can be further simplified assuming h << R and neglecting the high order terms (but not justifying such simplification in terms of order of magnitude of the errors introduced), giving

$$\Delta_\tau = \{ 1 - \frac{GM}{c^2 R} - \frac{[R\Omega\cos\lambda + v]^2}{2c^2} \} \Delta_t$$
$$= [\frac{gh}{c^2} - \frac{(2R\Omega\cos\lambda\, v + v^2)}{2c^2}] \Delta_t$$
$$= \alpha^*(h_{west}, v_{west}, \lambda_{west})\ \Delta_t$$

where, in addition to the parameters defined before for the general expression

- g is the “relativistic” gravitational acceleration on ground, namely

$$g = \frac{GM}{R^2} - R\Omega^2\cos^2\lambda$$

To note that such simplification is valid for ground or airplanes paths, but is not applicable for satellites (h not much smaller than R), where the general expression has to be used.

Similarly the general one for the ratio of the flight and ground delta times become (note: USNO related term seems completely neglected because assumed at sea level (h=0) and due to no ground speed (vgs=0)).

$$\frac{\Delta_{\tau-flight}}{\Delta_{\tau-USNO}} = \frac{\alpha^*(h_{flight}, v_{flight}, \lambda_{floght})}{\alpha^*(0, 0, \lambda_{USNO})} = 1 + \frac{gh}{c^2} - \frac{(2R\Omega\cos\lambda\, v + v^2)}{2c^2}$$

and the actual time drift values can be calculated with the same type of expression found for the general case

$$\Delta_{\tau-east/USNO} = \Delta_{\tau-east} - \Delta_{\tau-USNO} = [\frac{\alpha^*(h_{east}, v_{east})}{\alpha^*(0, 0)} - 1]\ \Delta_{\tau-USNO} = \delta_{east/USNO} \cdot \Delta_{\tau_{USNO}}$$

$$\Delta_{\tau-west/USNO} = \Delta_{\tau-west} - \Delta_{\tau-USNO} = [\frac{\alpha^*(h_{west}, v_{west})}{\alpha^*(0, 0)} - 1]\ \Delta_{\tau-USNO} = \delta_{west/USNO} \cdot \Delta_{\tau_{USNO}}$$

For a generic path at latitude λ, the formula for delta in this case can be expressed in a simple way

$$\delta = \frac{gh}{c^2} - [\frac{(2R\Omega\cos\lambda + v)v}{2c^2}]$$

with the first test the gravitational time shift and the seconf term the kinematic time dilation effect for clock movement w.r.t. Earth’s centre.

Finally, in [HK-1] is also provided an integral expression to be used in case of time variable altitude, velocity or latitude. This expression is the following

$$\Delta_\tau = \int_{t_a}^{t_b} [\frac{gh(\tau)}{c^2} - \frac{2R\Omega\cos\lambda(\tau)v(\tau) + v(\tau)^2}{2c^2}] d\tau$$

and has to be used for integrating the different delta time samples along the path for the time frame measured at USNO. That implies in particular that the flight time to be used has to be the actual flying time, i.e. during which velocity and altitude where not zero, and that "stop" periods between one flight leg to another has to be excluded in the calculation.

2.2.4 Relativistic Model for Earth Flights

A more generic relativistic model for time transfer, with an Earth specific customisation, in provided in [REL-ITU]. Specifically condering a clock close to Earth's surface, the corresponding delta coordinate time is given by

$$\Delta t = \int_A^B \left[1 - \frac{\Delta U(\mathbf{r})}{c^2} + \frac{1}{2c^2} v^2\right] d\tau + \frac{1}{c^2} \int_A^B (\boldsymbol{\omega} \times \mathbf{r}) \cdot \mathbf{v} \, d\tau \,,$$

where:

- dτ is the proper time measured in the clock
- r and v are the clock frame position and velocity in ECEF
- ΔU is the gravitational positional difference between the clock at position r and one on gne geoid, including the centrifugal potential
- ω is the Earth angular velocity

2.2.5 Predicted Values

As reported in [HK-1], the predicted time shift in [ns], calculated by the authors (but not specifying using what formula's parameter values for velocity, altitude, latitude and flight time), including gravitational and kinematic components too, is

Effect	Direction	
	East	West
Gravitational	144 ± 14	179 ± 18
Kinematic	−184 ± 18	96 ± 10
Net	-40 ± 23	275 ± 21

From that table it appears that, for a low altitude flight, the gravitational and kinematic terms have the same order of magnitude, in particular for the eastward trip.

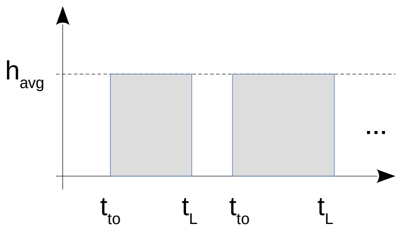
2.3 Flight Data

The indicative flight data path is provided in [HK-3], with departure and arrival airports and relevant times, but with average speed and altitude information (the first column is for the eastward trip and the second column for the westward one).

Trip time	65.42 hours	80.33 hours
Avg ground speed	243. meters/sec	218. meters/sec
Avg altitude	8.90 kilometers	9.36 kilometers
Avg latitude	34. degrees N	31. degrees N

The precise values for velocity and altitude, as well as for flight path latitude, needed for the calculation of the experiment's expected values, were not disclosed by the authors neither in [HK-3] nor in any other reference document(!). That makes very difficult an accurate recalculation by a third party.

In particular for both trips a "flat" altitude profile is considered, i.e. keeping the nominal flight altitude only and neglecting altitude changes for take-off, landing and in flight manoeuvres, assuming those interval are "small" w.r.t. the whole flight duration.



2.3.1 Eastward Trip

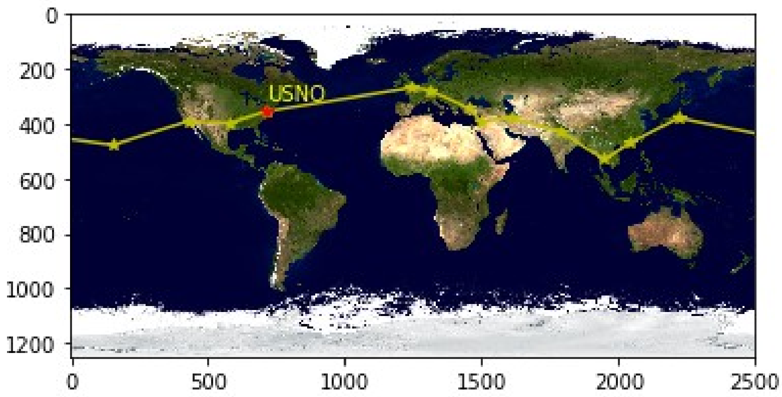
The westward trip started on 04.10.1971 and finished on 07.10.1971. The detailed steps are in the following table.

Date	GMT	Location	Coordinates – Lat [deg]	Coordinates – Lon [deg]	Distance [km]
4. Oct. 1971	07:30:00 PM	USNO D (by car)	38.921674	-77.066884	
5. Oct. 1971	12:12:00 AM	Dulles D	38.9531162	-77.4565388	33.9
	06:56:00 AM	London A	51.4700223	-0.454295499999944	5901.7
	08:14:00 AM	D			
	09:09:00 AM	Frankfurt A	50.0379326	8.56215180000004	653.5
	10:36:00 AM	D			
	12:48:00 PM	Istanbul A	40.9829888	28.8104425	1862.4
	01:57:00 PM	D			
	03:13:00 PM	Beirut A	33.819376	35.491204	990.6
	04:19:00 PM	D			
	06:13:00 PM	Tehran A	35.6899882	51.311241	1458.5
	07:40:00 PM	D			
	10:41:00 PM	New Delhi A	28.5561624	77.0999578	2546.0
6. Oct. 1971	12:00:00 AM	D			
	03:33:00 AM	Bangkok A	13.738007	100.645141	2935.6
	05:13:00 AM	D			
	07:45:00 AM	Hong Kong A	22.308047	113.9184808	1694.8
	08:55:00 AM	D			
	12:16:00 PM	Tokyo A	35.549393	139.779839	2902.1
	02:32:00 PM	D			
	09:10:00 PM	Honolulu A	21.3245132	-157.9250736	6191.5
	11:14:00 PM	D			
7. Oct. 1971	03:50:00 AM	Los Angeles A	33.9415889	-118.40853	4108.0
	04:47:00 AM	D			
	07:13:00 AM	Dallas A	32.848103	-96.851206	2001.3
	07:53:00 AM	D			
	09:59:00 AM	Dulles A	38.9531162	-77.4565388	1869.5
	12:55:00 PM	USNO A (by car)	38.921674	-77.066884	33.9

The summary data for this trip are:

- total flight time (including stops at aiports): 65:25:00 [hh:mm:ss] = 235500 [sec]
- actual flight time (flights only): 41:14:00 [hh:mm:ss] = 148440 [sec]
- total distance: 35183 [km]
- average altitude: 8.9 [km]
- average velocity: 0.149 [km/s]
- average latitude: 34.7 [deg]

Flight Path



2.3.2 Westward Trip

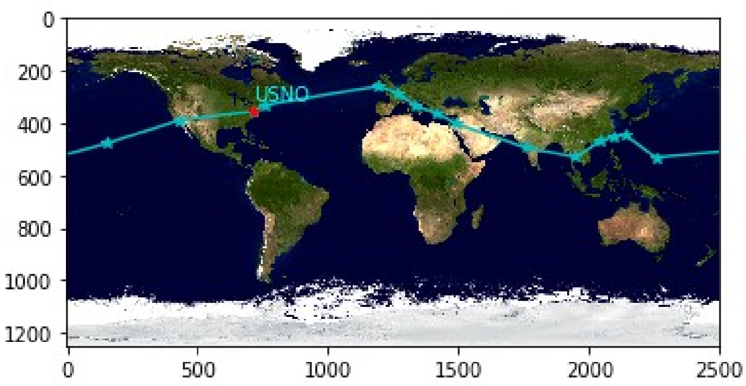
The westward trip started on 13.10.1971 and finished on 17.10.1971. The detailed steps are in the following table.

Date	GMT	Location	Coordinates – Lat [deg]	Coordinates – Lon [deg]	Distance [km]
13. Oct. 1971	07:40:00 PM	USNO D (by car)	38.921674	-77.066884	
	11:22:00 PM	Dulles D	38.9531162	-77.4565388	33.9
14. Oct. 1971	04:00:00 AM	Los Angeles A	33.9415889	-118.40853	3674.0
	05:03:00 AM	D			
	10:14:00 AM	Honolulu A	21.3245132	-157.9250736	4108.0
	01:13:00 PM	D			
	08:15:00 PM	Guam A	13.497036	144.795309	6108.5
	09:13:00 PM	D			
15. Oct. 1971	12:06:00 AM	Okinawa A	26.20935	127.6503	2278.6
	01:07:00 AM	D			
	02:09:00 AM	Taipei A	25.067566	121.552699	624.2
	03:03:00 AM	D			
	04:13:00 AM	Hong Kong A	22.308047	113.9184808	835.5
	12:48:00 PM	D			
	03:14:00 PM	Bangkok A	13.738007	100.645141	1694.8
	04:32:00 PM	D			
	08:06:00 PM	Bombay A	19.09314	72.856753	3019.5
	09:15:00 PM	D			
16. Oct. 1971	04:03:00 AM	Tel Aviv A	32.005532	34.8854112	4045.5
	05:09:00 AM	D			
	06:45:00 AM	Athens A	37.9356467	23.9484156	1193.6
	07:33:00 AM	D			
	09:03:00 AM	Rome A	41.7998868	12.2462384	1086.0
	10:01:00 AM	D			
	11:38:00 AM	Paris A	49.0096906	2.54792450000002	1101.0
	02:25:00 PM	D			
	03:57:00 PM	Shannon A	52.6996573	-8.91469110000003	901.8
	05:06:00 PM	D			
	11:38:00 PM	Boston A	42.3656132	-71.0095602	4646.9
17. Oct. 1971	01:18:00 AM	D			
	02:26:00 AM	Dulles A	38.9531162	-77.4565388	662.8
	04:00:00 AM	USNO A (by car)	38.921674	-77.066884	33.9

The summary data in this case are:

- total flight time (including stops at aiports): 80:20:00 [hh:mm:ss] = 289200 [sec]
- actual flight time (flights only): 48:39:00 [hh:mm:ss] = 175140 [sec]
- total distance: 36048 [km]
- average altitude: 9.36 [km]
- average velocity: 0.125 [km/s]
- average latitude: 32.6 [deg]

Flight Path



2.4 Predicted Time Shift Re-calculation

Based on the expression reported in par. 2.2, and using the flight trip data provided in par. 2.3, a re-calculation of the predicted time shift has been performed.

The following values are used for the USNO located clock:

- h = 0 [km]
- v = 0 [km/s]
- λ_{USNO} = 38.92 deg

2.4.1 Eastward Trip

The average values from flight trip plan in 2.3 are:

- h = 8.9 km
- v = 0.237 km/s
- λ_{flight} = 34.0 deg

The flight time has to be calculated using velocity and path length (calculated using the trip circle divided by ground speed), in order to consider only the actual flight steps, namely

- path length: 35183 km
- flight time = 148440 s (about 41.23 hours vs. 65.4 indicated in [HK-3])

The total time drift as well as the gravitationan and kinematic terms are calculated using average values in both general and simplified expression, as shown below.

SR Model	$dt_{kinematic}$ [us/trip]	$dt_{gravitational}$ [us/trip]	dt_{total} [us/trip]
HK circular low altitude	-196.732	142.470	**-54.262**
HK circular	-211.818	142.618	**-69.216**
SR/GR	-211.818	142.609	**-69.267**
SR/GR Earth	n.a.	n.a.	**-69.209**

The time drift values using an integral expression adding up all the single flight step contributions, as per table in 2.3, is the following. Note that there is not much difference from the approximated case of using single average values.

SR Model	$dt_{kinematic}$ [us/trip]	$dt_{gravitational}$ [us/trip]	dt_{total} [us/trip]
SR/GR	-210.487	142.602	**-67.881**

2.4.2 Westward Trip

The average values from flight trip plan in this case are:

- h = 9.36 km
- v = -0.206 km/s
- λ_{flight} = 31.0 deg

The flight time is calculated with velocity and path length also in this case

- path length: 36048 km
- flight time = 175140 s (about 48.6 hours vs. 80.3 indicated in [HK-3])

The time drift terms are shown in the following table.

SR Model	$dt_{kinematic}$ [us/trip]	$dt_{gravitational}$ [us/trip]	dt_{total} [us/trip]
HK circular low altitude	118.073	176.851	**294.924**
HK circular	90.806	177.061	**267.828**
SR/GR	90.804	177.009	**267.755**
SR/GR Earth	n.a.	n.a.	**267.813**

The time drift values using an integral expression adding up all the single flight step contributions, as per table in 2.3, is the following. Also for this case the steps integration result is quite close to the one calculated with average values.

SR Model	$dt_{kinematic}$ [us/trip]	$dt_{gravitational}$ [us/trip]	dt_{total} [us/trip]
SR/GR	88.749	177.023	**265.763**

2.5 Comparison with Experiment's Observations

The experiment's results data are provided in [HK-3] as initial report and then in [HK-4] as consolidated paper. The results in the two documents are not fully consistent between each other, and the authors' justification is the data post-analysis and consolidation.

The results in the initial report [HK-49 are the measurements performed on the 4 clocks used in both flights, right after the trip conclusion and the clocks move back to USNO.

CLOCK (ser. no.)	R_i (ns/hr)	R_f (ns/hr)	$\overline{R}$ (ns/hr)	ΔR (ns/hr)	$\Delta\tau_i$ (ns)	$\Delta\tau_f$ (ns)	$\Delta\tau$ (ns)
			Eastward flight				
			τ = 65.42 hr				
361	+2.66	+4.38	+3.52	+1.72	1790	1910	-110
408	-1.78	+3.22	+0.72	+5.00	-20	30	+3
120	-4.50	-8.89	-6.70	-4.39	-290	-780	-52
447	-7.16	-8.41	-7.78	-1.25	-1140	-1705	-56
			$\overline{\Delta R}$ = +0.27 ns/hr			$\overline{\Delta\tau}$ = -54 ns	
			$\sigma_{\Delta R}$ = 3.8 ns/hr			$\sigma_{\Delta\tau}$ = 46 ns	
			Westward flight				
			τ = 80.33 hr				
361	+6.89	+3.97	+5.43	-2.93	2880	3390	+74
408	+4.84	+2.16	+3.50	-2.68	490	980	+209
120	-8.88	-4.56	-6.72	+4.31	-2100	-2400	+240
447	-7.17	-9.42	-8.30	-2.25	-2840	-3390	+116
			$\overline{\Delta R}$ = -0.89 ns/hr			$\overline{\Delta\tau}$ = +160 ns	
			$\sigma_{\Delta R}$ = 2.6 ns/hr			$\sigma_{\Delta\tau}$ = 78 ns	

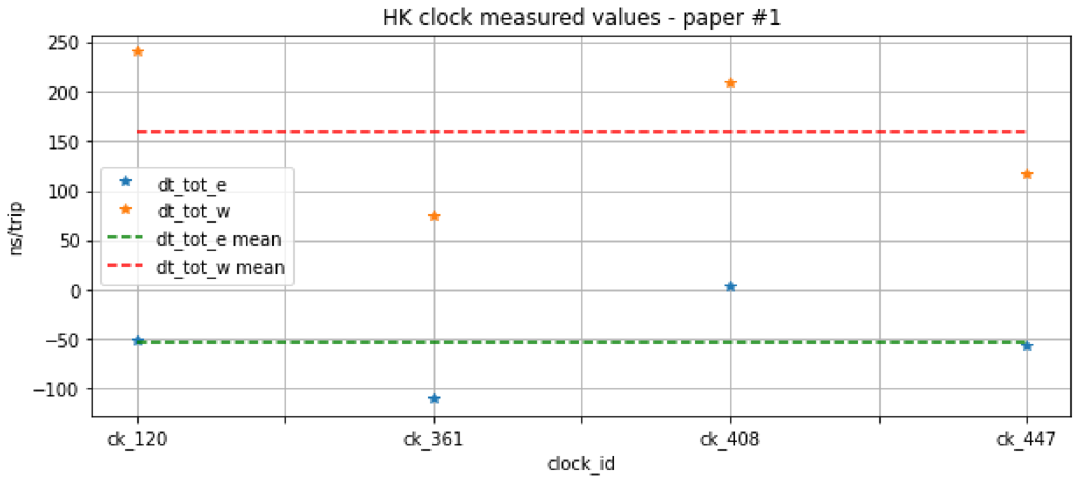
1 ns/hr = 2.78 x 10^{-13} sec/sec

$$\sigma = \left[\frac{\Sigma(\Delta\tau - \overline{\Delta\tau})^2}{3} \right]^{\frac{1}{2}}$$ (standard deviation)

From the table it’s easy to see the following points:

- the 4 clocks have different behaviours with respect to each other, in both E and W cases
- the standard deviations for all clocks are in same order of magnitude of the accuracy expected by the experiment (tenth of ns)

Those differences are explained presenting the table above as “immediate” measurement after the trips, not waiting for a stabilisation of the clock rates. The samples are shown in the figure below.

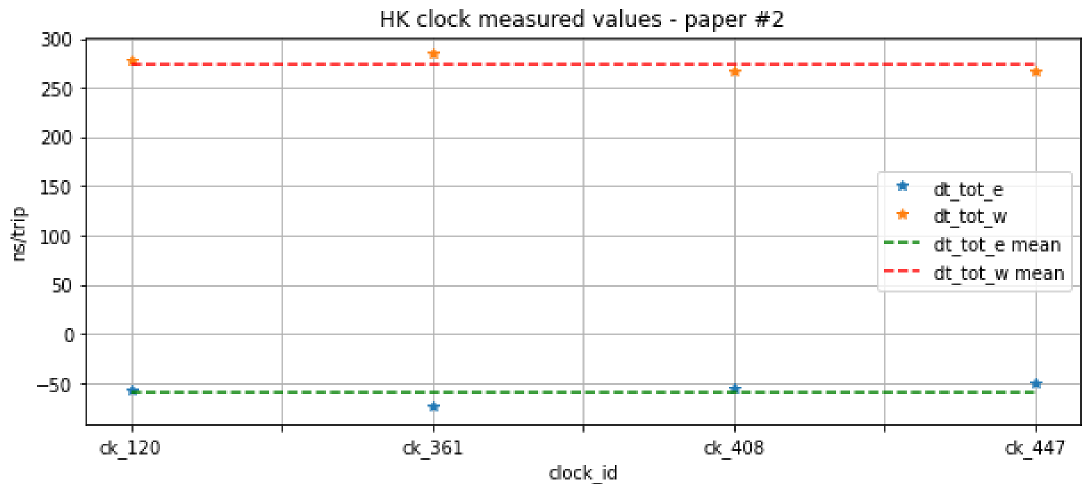


Later, another paper was published, with the final experiment results ([HK-2]), namely:

Clock serial No.	$\Delta\tau$ (nsec)	
	Eastward*	Westward
120	- 57	277
361	- 74	284
408	- 55	266
447	- 51	266
Mean ± S.D.	- 59 ± 10	273 ± 7
Predicted ± Error est.	- 40 ± 23	275 ± 21

* Negative signs indicate that upon return the time indicated on the flying clocks was less than the time indicated on the MEAN(USNO) clock of the U.S. Naval Observatory.

The samples are also shown in the followng plot.



Here the values looks more in line with the expectations from the theory, but they're significantly different from those in [HK-4], in both average values and standard deviations(!).

This aspect, together to the actual differences found and a missing clear explanation of the steps performed, creates further uncertainty in the measurement values considered in the experiment, and it may look as a post-processing "consolidation" of experiment results, which was not documented or even mentioned in the papers.

Here below a final summary table provides an overall comparison between predicted values, re-calculated values (as part of the note), and results from both first and second paper (in red to highlight the differences).

	Eastward dt_{total} [us/trip]	Eastward dt_{total} residual [%]	Westward dt_{total} [us/trip]	Westward dt_{total} residual [%]
HK predicted	-40 ± 23	--	275 ± 21	--
HK measured #1	-54 ± 46	25.9%	160 ± 78	41.8%
HK measured #2	-59 ± 10	32.2%	273 ± 7	0.7%

SR/GR Earth Model	Eastward dt_{total} [us/trip]	Eastward dt_{total} residual [%]	Westward dt_{total} [us/trip]	Westward dt_{total} residual [%]
Predicted	-68		266	
Diff vs. HK predicted	-28	41.2%	-9	-3.4%
Diff vs. HK measured #1	-14	20.6%	106	39.8%
Diff vs. HK measured #2	-9	13.2%	-7	-2.6%

It comes out then that

- observed values have significant changes from release #1 (HK-3]) and release #2 ([HK-2]), with the second being much closer to predicted values
- simplified model's recalculated values are closer to the HK predicted and observed ones than those from the general model, which is supposed to be more precise and then expected to be closer to measured values
- re-calculation is close to the HK predicted values both for eastward and westward trip, and either using the general model and the simplified one, but there is not a perfect matching and the residuals are not small (tenths of nsecs) – a perfect matching would be expected using the simplified model because it's the same used by HK – reason for the difference is unclear(!)
- the final residuals between predicted – recalculated and observed is significant with respect the order of magnitude of the values, namely up to 47% for Eastward case and 41% for Westward one – experiment measurements accuracy looks not adequate to provide a conclusive answer for providing the validity of the GR model

Concerning the sensitivity of the experiment results, noting that no sensitivity assessment was provided by HK in the various papers, the model results are also significantly dependent on the accuracy of the input parameters, velocity in particular but also altitude, namely

- latitude: 1e-9 [s/deg]
- ground speed: 1.7e-6 [s/(km/s)]
- altitude: 15e-9 [s/km]

3 Conclusions

The HK experiment has been reproduced by recalculating the predicted values using flight data provided in [HK-3] using the formulas described in [HK-1] and [HK-4], and compared with expected values provided with HK in their paper in [HK-1] and actual observations in [HK-2].

It is expected that the values re-calculated using theory model matches with the HK predicted ones, as presented in [HK-2] and [HK-5], within the limit of the approximation applied, but that is not the case for both general and simplified models.

Passing to the observed values, the figures provided by HK in their paper significantly change from the preliminary ones in [HK-3] to the final ones in [HK-2] without any clarification of the type of consolidation made in the data post-processing.

The reason could be due to a wrong use of the model expressions or their flight parameters, which are not fully released by HK, and/or to insufficient experiment data accuracy, which did not considered external effects (e.g. environmental) influencing the measurements.

As final conclusion, it is noted that

1. the accuracy of the clocks used for the experiment, namely the rms of their measured times both on ground and in flight, looks of the same order of magnitude of the effect to be measured. That raises doubts on the possibility of using any type of result for the purpose of the experiment's objectives;
2. overall all analysed data, either predicted, recalculated and observed, are within the same order of magnitude (tenths of nsecs for Eastward case and hundreds of nsecs for Westward case), but the residual differences as significantly high (up to 40%), meaning that the accuracy of the experimental measurements was not good enough for providing a conclusive answer to the objective of validating the SR/GR model as the only one valid for time shift.