Invariant Electromagnetism: Necessity and Sufficiency

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A first-order Galilean-invariant covering theory of Maxwell's equations of vacuum electromagnetism first proposed by Heinrich Hertz is reappraised in modern context. Physically, when properly formulated and interpreted, it is found to offer improved sufficiency for electromagnetic description. Mathematically, its use of the total time derivative instead of the Maxwellian partial time derivative is shown to be logically necessary under the condition that the (inertial) coordinate transformation precedes the application of the field equations.

1. Historical Introduction

The theory of electromagnetism (EM) developed by Maxwell[1] was noninvariant at first order. That is, when subjected to a Galilean (first-order inertial) transformation of coordinates with velocity parameter **v**, Maxwell's field equations yielded surplus terms, proportional to **v**, that were not originally present. False predictions resulted, as of nonexistent fringe shifts in first-order optical experiments. The noninvariant aspects of his formalism apparently did not trouble Maxwell who, as a confirmed etherist, thought naturally in terms of a preferred reference system. But in hindsight it seems that it should have caused some concern, as possible evidence of a first-order flaw in his theory, in view of a growing body of observations (for example those by Mascart[2] in 1872-4, several years before Maxwell's death) that supported universality of the Newtonian relativity principle as a physical fact *at first order in v/c*.

The present analysis is founded on the premise that any theory that makes false predictions at first order is broken at first order and needs to be fixed at first order. Historically, this is not an original thesis. Heinrich Hertz[3] had the same idea, which led him to modify Maxwell's formalism so as to make it rigorously invariant under Galilean (inertial) transformations. This turns out mathematically to be quite easy to do. It mainly requires replacement of Maxwell's partial time derivative operator $\partial/\partial t$, wherever it appears, with a total time derivative operator. The latter is customarily written as *d/dt* by physicists and engineers[4,5], but will here be denoted *D/Dt* to avoid confusion with the usual notation for the derivative of a function of a single variable. Thus

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{w} \cdot \nabla \ , \tag{1.1}$$

where, in physics, **w** is a vector of velocity (of a localized object – such as a test particle – not of a coordinate frame) and $\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\}$. It is also necessary to make a trivial adjustment of the current source term to agree with Galilean velocity addition. The new "convective" velocity parameter **w** appearing here is assumed for simplicity to be a constant independent of space variables (otherwise, one deals with the more general "Helmholtz derivative"(6)).

The role of **w** will be a major focus of attention in what follows. As a "new" parameter not previously appearing in EM description, it requires a new physical interpretation; but the latter is of no significance for the mathematical invariance proof (§2) or for considerations of mathematical necessity (§3). Hertz's was a first-order *invariant covering theory* of Maxwell's equations for the vacuum case – inasmuch as Galilean invariance (asserted(3) but not explicitly shown by Hertz) has been repeatedly proven(7-10) and will be demonstrated again here for completeness. The "covering theory" aspect derives from the fact that the special case $\mathbf{w} = 0$ reduces *D/Dt* to $\partial/\partial t$, thereby recovering Maxwell's theory of vacuum EM identically. This means that the "physics of one laboratory" (in which field-responsive instruments are at rest) is identical in the two theories.

Unfortunately, Hertz's claim of a first-order invariant EM formulation was obscured by the complicated component notation used in his exposition (3) and by a failure to simplify with the help of vector identities. Consequently his formalism was never used by later workers nor was the significance of its first-order invariance recognized. Further, he handicapped its chances of acceptance by proposing a Stokesian ether interpretation of the new parameter **w** [denoted by components (α, β, γ) in Hertz's book(3)], according to which it signified the velocity of an ether 100% entrained (or convected) by tangible matter – so that **w** measured the velocity of just any material "body" in the laboratory. This led to false predictions (*e.g.*, creation of a magnetic field by a charged rotating dielectric) that were considered observationally disproven(11) soon after his death. His formalism of *true invariance* was hence discarded, just in time to make way for Einstein-Minkowski's "universal covariance" – a mathematical substitute readily accepted in lieu of invariance, under the false impression that true first-order invariance was unattainable.

An even-handed application of the principle of *discarding all mathematics proven to be in first-order disagreement with observation* would have led, well before the advent of Hertz's theory, to the discarding of Maxwell's theory. In fact the ether theories of both Maxwell and Hertz were soon forgotten. Still, it seems regrettable that Hertz's first-order invariant covering theory was rejected without a hearing of its mathematical merits, while Maxwell's noninvariant special case was retained, *solely on its mathematical merits*, and was made the basis for all subsequent field physics. To put it another way, Maxwell's spurious ether "physics" was rejected for the sake of keeping his noninvariant mathematics, while Hertz's invariant mathematics was rejected for the sake of discarding his spurious ether "physics." We suggest that it is never too late to consider rectifying this odd turn of the political history of science.

Meanwhile the relativity principle, under Einstein's tutelage [12], received mathematical expression through formal *covariance* rather than invariance. This further historical oddity involved skipping over the first order entirely and going directly to second-order expressions (in v^2 / c^2) affecting kinematics. The underlying "physical" assumption behind this evolution was *spacetime symmetry*, a concept derived directly and solely (in terms of objective supporting evidence) from the formal symmetry in Maxwell's equations of the partial derivative operators $\partial/\partial x, \partial/\partial y, \partial/\partial z$ and $\partial/\partial t$ (or $\partial/\partial ct$ – we here omit factors of *c*). In Hertzian theory that alleged symmetry is spoiled at first order by the requirement of Galilean invariance, which mandates replacement of $\partial/\partial t$ by the *D/Dt* of Eq. (1.1). No mathematical symmetry is exhibited between *partial* space derivatives and such a *total* time derivative, in view of the augmented "fourth-component" parametrization by the 3-vector **w**, with no such augmentation of the spatial components.

Thus, if physics is developed systematically, proceeding from lower to higher orders of approximation and insisting on getting the physics right at lower orders before proceeding to higher ones, then EM theory offers no logical justification for hypothesizing covariance at any stage. It is strict *invariance* that logically expresses relativity at first order, and at that order invariance "breaks" spacetime symmetry. By the nature of orders of approximation, a symmetry broken at first order cannot be restored at higher orders. Therefore it is evident that in a rationally sequenced evolution of EM physics motional relativity would initially have been expressed by (first-order) invariance, not by covariance (which introduces second-order considerations) … and spacetime symmetry would never have suggested itself as an attribute of the physical world. Most of this was implicit in Hertz's work[3] a decade before Einstein's[12]. More will be said in §8 about the implied alternative approach to higher-order EM theory.

In §2 of this paper we address in mathematical terms the *sufficiency* of the Hertzian *D/Dt*-based invariant formalism for physical description via field theory. Sections 3-5 then provide our proof of the *necessity* of total time derivatives from the purely mathematical standpoint. The remainder of the paper treats physical implications and interpretation, which differ nontrivially from those of the Maxwellian theory. In §6 Hertzian and Maxwellian "fields" are contrasted as to their operational definitions, and in §7 the d'Alembertian solution of the Hertzian wave equation is developed. Throughout, attention is restricted to the case of *vacuum* electromagnetism, this being generally viewed as the most fundamental form of EM theory. For the most part, attention is also restricted to considerations of first order, such that Newtonian kinematics applies.

However, in §8 we venture to speculate about higher-order EM description. There, our treatment of phase velocity refers to a proposed higher-order ("neo-Hertzian") form of invariance, with application in §9 to the special topic of stellar aberration, which suggests a crucial experiment. Returning to first-order considerations in §10, we address the invariant form of the field definitions in terms of EM potentials, as well as the implications of an invariance requirement for the EM force law. We deduce that the universally-accepted Lorentz force law may tell only part of the *first-order* EM force story; and cite the so-called Marinov motor[13] as possible evidence of first-order observable departures[14,15] from the Lorentz law. With reference to physics one can never prove the sufficiency of a mathematical formalism, because the last experiment is never done. But one can establish insufficiency … and this, we suggest, experiment may already have done in regard to the Maxwell-Einstein (covariant) EM formalism.

2. First-Order Formal Invariance of Hertz's Field Equations

Confining attention, as stated, to the case of vacuum EM, and arbitrarily choosing the symbols **E** and **B** to designate the field vectors, we may express the familiar electromagnetic field equations (omitting factors of *c*) as

$$\nabla \times \mathbf{B} - \frac{\delta \mathbf{E}}{\delta t} - 4\pi \mathbf{j}_m = 0 \tag{2.1a}$$

$$\nabla \times \mathbf{E} + \frac{\delta \mathbf{B}}{\delta t} = 0 \tag{2.1b}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.1c}$$

$$\nabla \cdot \mathbf{E} - 4\pi\rho = 0 \ . \tag{2.1d}$$

Here all symbols and their physical referents are as in Maxwell's theory, except that $\delta / \delta t$ is employed as a generic notation to denote the partial derivative $\partial / \partial t$ in the case of Maxwell's equations and the total time derivative *D/Dt*, defined by Eq. (1.1), in the case of Hertz's equations. Similarly, $\mathbf{j}_m$ denotes the Hertzian current density, which is a generalization of the Maxwellian **j** to be defined presently. In this section we address Hertz's equations, for which *D* replaces δ $(D \to \delta)$. It should therefore be emphasized that the **E**, **B** appearing here are Hertzian field quantities, not Maxwellian, since *D/Dt* has replaced $\partial / \partial t$. In this connection it is instructive to note that in a typical textbook account[5] of "Faraday's law of induction" the author first introduces *d/dt* (our present *D/Dt*) in order to describe what Faraday *observed* in regard to arbitrarily-moving circuits and then, by arbitrarily forbidding circuits to move with respect to the observer, eliminates *d/dt* in favor of $\partial / \partial t$ in order to recover the usual Maxwell-Heaviside theory.

We now verify the form invariance of Eq. (2.1) [with $(D \to \delta)$] under the Galilean transformation,

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t \ , \qquad t' = t \ , \tag{2.2}$$

which is generally viewed as the correct first-order mathematical description of physical inertial motions. [It therefore suffices for present purposes to consider such irrotational motions, although the Galilean group is less comprehensive than the most general one – the orthonormal group – under which Eq. (2.1) is invariant.] Note that the **v** parameter in (2.2) is conceptually unrelated to the convective parameter **w** appearing in Eq. (1.1). Confusion on this almost self-evident point (which is in fact crucial to our invariance proof) has extended even to a recent two-volume treatise[6] on Hertzian EM. From (2.2) written out in component form we see that

$$\frac{\partial}{\partial x}=\frac{\partial x'}{\partial x}\frac{\partial}{\partial x'}+\frac{\partial y'}{\partial x}\frac{\partial}{\partial y'}+\frac{\partial z'}{\partial x}\frac{\partial}{\partial z'}=\frac{\partial}{\partial x'}$$

and similarly

$$\frac{\partial}{\partial y}=\frac{\partial}{\partial y'}\ ,\quad \frac{\partial}{\partial z}=\frac{\partial}{\partial z'}\ ;$$

which is equivalent in vector notation to

$$\nabla'=\nabla\ . \tag{2.3}$$

Similarly, when (2.2) is written as $x'=x-v_x t$ or $x=x'+v_x t$, so that $\partial x/\partial t=v_x$, etc., we verify that

$$\frac{\partial}{\partial t'}=\frac{\partial t}{\partial t'}\frac{\partial}{\partial t}+\frac{\partial t}{\partial t'}\frac{\partial x}{\partial t}\frac{\partial}{\partial x}+etc.$$

implies

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\ . \tag{2.4}$$

Again we call attention to the different velocity parametrization of Eqs. (1.1) and (2.4), as well as to the distinction between partial and total differentiation. Eqs. (2.3) and (2.4) are well-recognized and accepted in the literature[16]. Eq. (2.4) makes explicit the first-order *noninvariance* of the partial time derivative operator.

The convective velocity parameter **w** is postulated to obey the Galilean velocity addition law,

$$\mathbf{w}'=\mathbf{w}-\mathbf{v}\ , \tag{2.5}$$

(an input essential to the invariance proof, which obviously rests upon the above-mentioned conceptual distinction between **w** and **v**) and the field quantities are postulated to transform invariantly,

$$\mathbf{E}'=\mathbf{E},\quad \mathbf{B}'=\mathbf{B}\ . \tag{2.6}$$

For the moment we set aside the physics of such relations (to be treated in §6) and address only the matter of form invariance. Similarly, we assume for the source terms invariance of charge density and the first-order Galilean velocity transformation property of Maxwellian current density; *i.e.*,

$$\rho'(\mathbf{r}',t') = \rho(\mathbf{r},t) \qquad \text{and} \qquad \mathbf{j}'(\mathbf{r}',t') = \mathbf{j}(\mathbf{r},t) - \rho(\mathbf{r},t)\mathbf{v} \ . \tag{2.7}$$

These are also accepted relations[(16)]. A "convective" current density may be defined by

$$\mathbf{j}_c = -\rho\mathbf{w} \ , \tag{2.8}$$

which by (2.5), (2.7) transforms according to

$$\mathbf{j}_c = -\rho\mathbf{w} = -\rho(\mathbf{w}'+\mathbf{v}) = -(\rho\mathbf{w})' - \rho\mathbf{v} = \mathbf{j}_c' - \rho\mathbf{v} \ ,$$

or

$$\mathbf{j}_c' = \mathbf{j}_c + \rho\mathbf{v} \ . \tag{2.9}$$

Let a "measured" current density $\mathbf{j}_m$, appearing in our field equation (2.1a), be defined as

$$\mathbf{j}_m = \mathbf{j} + \mathbf{j}_c \ . \tag{2.10}$$

We see from (2.7) and (2.9) that $\mathbf{j}_m$ transforms invariantly,

$$\mathbf{j}_m' = (\mathbf{j}+\mathbf{j}_c)' = \mathbf{j}'+\mathbf{j}_c' = (\mathbf{j}-\rho\mathbf{v}) + (\mathbf{j}_c+\rho\mathbf{v}) = \mathbf{j}+\mathbf{j}_c = \mathbf{j}_m \ , \tag{2.11}$$

thus justifying our interpretation of $\mathbf{j}_m$ as a "measured" (objectively physical) quantity. It is apparent that our current density measuring instrument ("detector") – conceptually, a small box with walls permeable to relatively-moving charges – measures the Maxwellian current density **j** when it is at rest ($\mathbf{w}=0$) in the unprimed inertial system, and measures the current density $\mathbf{j}_m$ when it moves relatively to that system with velocity $\mathbf{w} \neq 0$. By this relative motion it generates an additional purely "motional" current density $\mathbf{j}_c$, given by Eq. (2.8), which has a minus sign because a detector moving in one direction is equivalent to a current flowing in the opposite direction. Note that (2.8), (2.10) show that, when $\mathbf{w} \to 0$, $\mathbf{j}_m$ approaches **j**, the Maxwellian current density. In the same limit, $\mathbf{w} \to 0$, $\delta/\delta t$ or D/Dt approaches $\partial/\partial t$. The Hertz equations, (2.1), thus reduce identically to Maxwell's equations in the special case $\mathbf{w} \equiv 0$, and therefore constitute a covering theory. This observation is adequate to establish the *sufficiency* of Hertz's equations for physical description, for all physicists who accept the sufficiency of Maxwell's equations.

A result basic to our proof of first-order invariance of the Hertzian field equations is invariance of the total time derivative. This follows from Eqs. (1.1) and (2.3)-(2.5),

$$\left(\frac{D}{Dt}\right)' = \left(\frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla\right)' = \frac{\partial}{\partial t'} + \mathbf{w}'\cdot\nabla' = \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) + (\mathbf{w}-\mathbf{v})\cdot\nabla = \frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla = \frac{D}{Dt} \ . \tag{2.12}$$

Although explicit appearance of *c* has been suppressed here, it is to be understood that this quantity, which may be construed as a units ratio, is Galilean invariant,

$$c' = c \quad . \tag{2.13}$$

With these preparations we are ready to prove the invariance of the field equations (2.1). Eq. (2.1c) is obvious from (2.3) and (2.6):

$$(\nabla \cdot \mathbf{B})' = \nabla' \cdot \mathbf{B}' = \nabla \cdot \mathbf{B} = 0 \quad . \tag{2.14}$$

Invariance of (2.1d) follows from inclusion of (2.7),

$$\nabla' \cdot \mathbf{E}' - 4\pi\rho' = \nabla \cdot \mathbf{E} - 4\pi\rho = 0 \quad . \tag{2.15}$$

By recalling that for Hertzian theory $\delta/\delta t \equiv D/Dt$, and by use of (2.12), we similarly verify the invariance of (2.1b),

$$\nabla' \times \mathbf{E}' + \left(\frac{D}{Dt}\right)' \mathbf{B}' = \nabla \times \mathbf{E} + \frac{D}{Dt}\mathbf{B} = 0 \quad . \tag{2.16}$$

Finally, the invariance of (2.1a) follows from inclusion of (2.11),

$$\nabla' \times \mathbf{B}' - \left(\frac{D}{Dt}\right)' \mathbf{E}' - 4\pi \mathbf{j}'_m = \nabla \times \mathbf{B} - \frac{D}{Dt}\mathbf{E} - 4\pi \mathbf{j}_m = 0 \quad . \tag{2.17}$$

It will be seen that proof of first-order invariance is no more than a matter of inspection. Since each symbol simply transforms in place between primed and unprimed notations, we may elect to term such a transformation property, involving both formal and numerical invariance, "manifest invariance." This concludes our demonstration that at first order Hertzian EM constitutes an *invariant covering theory* of Maxwellian vacuum EM. The logical *sufficiency* of the Hertzian formalism for purposes of EM physics at the "classical" (non-quantum) level of field theory is thus established. We next turn to proof of the *necessity* (of $D \rightarrow \delta$) in a purely mathematical sense, given first-order form invariance as a physically-mandated precondition. Discussion of physical meaning of the Hertzian formalism will be deferred to §6.

3. Proof of Necessity of the Total Time Derivative

That the operator $\delta/\delta t$ appearing in the EM equations (2.1) is necessarily equivalent to a total time derivative *D/Dt* is a strictly mathematical matter to be addressed in this section. The demonstration will assume invariance and from that deduce $D \rightarrow \delta$. A function *F(x,y,z;t)* is said to be form invariant[(17)] under a transformation T: $(x, y, z,; t)$ $\rightarrow (x', y', z'; t')$ in case its image in the primed space is term-by-term identical with the original function up to a factor dependent solely on the coefficients of the transform.

This is some power, *p,* of the 'modulus' *M* of the transformation, which is the same as the Jacobian:

$$M \equiv \frac{\partial(x', y', z'; t')}{\partial(x, y, z; t)} \quad . \tag{3.1}$$

Thus *F(x,y,z;t)* is an invariant under T in case

$$F' = F'(x', y', z'; t') = F(x', y', z'; t') = M^p F(x, y, z; t) = M^p F \quad . \tag{3.2}$$

In the particular case of a Galilean transformation (2.2) it is seen that $M = 1$ and the factor M^p need not appear.

The functions of present interest are the dependent variables **E** and **B** appearing in the equations (2.1) of vacuum EM theory. Let us presume those equations solved with respect to given mathematical boundary conditions: first in the image space as

$$\mathbf{E}' = \mathbf{e}'(x', y', z'; t') \qquad \mathbf{B}' = \mathbf{b}'(x', y', z'; t') \ , \tag{3.3}$$

and secondly in the direct space as

$$\mathbf{E} = \mathbf{e}(x, y, z; t) \qquad \mathbf{B} = \mathbf{b}(x, y, z; t) \ , \tag{3.4}$$

where the notations $\mathbf{e}'$, $\mathbf{b}'$, $\mathbf{e}$, $\mathbf{b}$ designate the functions of the stated independent variables that resolve the original partial differential equations. The arguments are the same logically for the electric vector as for the magnetic, so we need consider only one or the other; *e.g.,* the electric. The vectors $\mathbf{e}'$ and **e** are to be considered as invariantly related in form if and only if

$$\mathbf{e}'(\alpha, \beta, \gamma; \tau) \equiv \mathbf{e}(\alpha, \beta, \gamma; \tau) \ , \tag{3.5}$$

where $\alpha, \beta, \gamma, \tau$ are merely symbols introduced to emphasize the mathematical relationships without reference to physical content. Equation (2.2) shows that the time variables transform between themselves independently of the spatial variables and that $dt'/dt = 1$.

If, as we assume, $\mathbf{e}', \mathbf{e}$ are form invariant then by (3.3)-(3.5)

$$\begin{aligned} \mathbf{E}' &= \mathbf{e}'(x', y', z'; t') = \mathbf{e}(x', y', z'; t') = \mathbf{e}(x'(x, y, z; t), y'(x, y, z; t), z'(x, y, z; t); t'(t)) \\ &= \mathbf{e}(x, y, z; t) = \mathbf{E} \ , \end{aligned} \tag{3.6}$$

where the implication of the next to last equality is that after the transformation has been manipulated out in terms of *x,y,z* and *t* it reduces to $\mathbf{e}(x, y, z; t)$, with these arguments appearing in it instead of the original x', y', z' and t'.

By elementary calculus, $\frac{\partial}{\partial t'} = \frac{dt}{dt'}\frac{\partial}{\partial t}$, so that from (3.6)

$$
\begin{aligned}
\frac{\partial \mathbf{E}'}{\partial t'} &= \frac{\partial}{\partial t}\mathbf{e}(x'(x,y,z;t), y'(x,y,z;t), z'(x,y,z;t); t'(t)) \cdot \frac{dt}{dt'} \\
&= \left(\frac{\partial \mathbf{e}}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial \mathbf{e}}{\partial y'}\frac{\partial y'}{\partial t} + \frac{\partial \mathbf{e}}{\partial z'}\frac{\partial z'}{\partial t} + \frac{\partial \mathbf{e}}{\partial t'}\frac{dt'}{dt} \right) \cdot \frac{dt}{dt'} \\
&= \frac{D}{Dt}\left(\mathbf{e}(x',y',z';t')\Big|_{\mathbf{x}'=\mathbf{x}'(x,y,z;t);t'=t'(t)} \right) \cdot \frac{dt}{dt'} = \frac{\partial \mathbf{E}}{\partial t} \cdot \frac{dt}{dt'} \ ,
\end{aligned} \tag{3.7}
$$

where the notation D/Dt represents the total time derivative[18]. Since $dt/dt'=1$ we have

$$
\frac{\partial \mathbf{E}'}{\partial t'} = \frac{D}{Dt}\left(\mathbf{e}(x',y',z';t')\Big|_{\mathbf{x}'=\mathbf{x}'(x,y,z;t);t'=t'(t)} \right) = \frac{\partial \mathbf{E}}{\partial t} \ . \tag{3.8}
$$

This is just an application of the familiar chain rule.

The total time derivative is therefore explicitly required. When the problem is thus analyzed in detail, it poses no difficulty whatever. Note that the partial differential operators as well as the total time derivative are interpreted as applied *after* the transformation of coordinates has been made, not before. This is the normal procedure, and it may conceivably be what Maxwell intended by his partial derivative notation – since notations in his day were less standardized than they are now. It is seen that (by virtue of the first-order relationship $dt/dt'=1$) the total time derivative can be used in place of the partial time derivative, in accord with the Hertzian interpretation of the operator we have written as $\delta/\delta t$. The meanings of the partial and total notations in (3.8) in this important special case are equivalent. The question is not a serious one of any real significance apart from the confusion that has resulted from a rather pervasive misunderstanding.

4. Conclusion Concerning Necessity

It is hard to understand why this question has so fretted electricians, but we suppose there is some excuse for so much befuddled thinking in that the notation $\partial/\partial t$ had a dual meaning in the 19th century, which persisted until the mid-20th century. It represented the simple partial derivative as what it is almost universally accepted as meaning today, but it also represented the total time derivative *D/Dt,* as used above, *when a transformation is involved*. The reader, formerly, was just supposed to know this and simply to use the meaning appropriate to the context. It was a bad ambiguity of symbolism, which was actually caused by all available printing fonts of the hand typesetting matrix being used up. There still persist similar confusions of meaning in the symbolism used to designate higher partial and total derivatives that continue to be a mine field over which the wary need to tread lightly. The only reliable course to take through the confusions of partial differentiation is: to be fully and surely aware of what is a function of what, and, if transformations are involved, when they are to be applied, before or after differentions.

Let us clarify the cause of the confusion by a simple, purely mathematical, example summing up the conclusions. Suppose $e = x' + y' + t$; then $\partial e/\partial t = 1$. That is, unless a transformation, such as $x' = 2x + 3t,\ \ y' = y + 5t$ intervenes. But, if it does, then *after* this transformation we obtain the entirely different result,

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial t}\left\{ e\Big|_{\substack{x'=2x+3t \\ y'=y+5t}} \right\} = \frac{\partial}{\partial t}\left[(2x+3t) + (y+5t) + t \right] = 9 \ .$$

Note that the latter result is the same as

$$\frac{De}{Dt} = \frac{\partial e}{\partial x'} \cdot \frac{\partial x'}{\partial t} + \frac{\partial e}{\partial y'} \cdot \frac{\partial y'}{\partial t} + \frac{\partial e}{\partial t} = 1 \cdot 3 + 1 \cdot 5 + 1 = 9 \ .$$

By recognizing the dimensionality of $\left(\frac{\partial x'}{\partial t}, \frac{\partial y'}{\partial t} \right) = \mathbf{w}$ as that of velocity, we convert this last expression to the familiar, physically useful form (1.1) of the total time derivative.

Mathematics knows of this bugbear, but it is not well known. It is not regularly taken up in textbooks, especially those of the pot-boiler class such as have been written as yearly editions by editors, rather than by fully competent senior mathematicians who have in mind another generation that might be aspiring to their level one day. If handled at all, the matter is relegated to a few problems appearing obscurely in some exercise. Courant(19), however, treats of the matter adequately; but, after all, how many students of the calculus, even those intending to become mathematicians, have had the privilege of learning their fundamentals from a bible of the discipline such as his is? Unfortunately, it is now cherished more on the library shelf than in the classroom.

5. The Inertial Transformation Group; Preserving the Metric

The demonstrations of the preceding sections may be easily generalized from the Galilean transforms (2.2) to the (first-order) inertial transformation group defined as $\mathbf{r}' = \mathbf{r} + \mathbf{r}_0 - \mathbf{v}(t - t_0)$. The latter is itself a subgroup of all linear transformations of the general form

$$\text{T:} \qquad \begin{aligned} x' &= a_{11}x + a_{12}y + a_{13}z - v_1(t - t_0) + k_1 \\ y' &= a_{21}x + a_{22}y + a_{23}z - v_2(t - t_0) + k_2 \\ z' &= a_{31}x + a_{32}y + a_{33}z - v_3(t - t_0) + k_3 \\ t' &= f(t - t_0) \end{aligned} \qquad (5.1)$$

where the a's, v's, k's, and t_0 are constants and f is an arbitrary differentiable function. Eq. (5.1) may be compactly written in matrix form as

$$\mathbf{x}' = A\mathbf{x} - \left(t - t_0\right)\mathbf{v} + \mathbf{k}$$
$$t' = f(t - t_0) \ , \qquad (5.2)$$

where the obvious meanings are given the vectors and the matrix A. It is seen on taking A to be the identity matrix that the inertial transformation group is included as a subgroup of (5.2); but additional transforms that are not inertial are contained in the larger group. These other transformations do not preserve the metric and therefore lead to a form of related, but non-Newtonian, mechanics in which the laws of Buridan[20] are changed along with the concept of what is "force."

Let $\mathbf{x}_0$ be any fixed point; then

$$\left(\mathbf{x}' - \mathbf{x}_0'\right) = A\left(\mathbf{x} - \mathbf{x}_0\right) \quad \text{and} \quad \left(\mathbf{x}' - \mathbf{x}_0'\right)^* = \left(\mathbf{x} - \mathbf{x}_0\right)^* A^* \ , \qquad (5.3)$$

where * denotes transposition. The matrix product of these two equations yields

$$\left(x' - x_0'\right)^2 + \left(y' - y_0'\right)^2 + \left(z' - z_0'\right)^2 = \left(\mathbf{x}' - \mathbf{x}_0'\right)^* \bullet \left(\mathbf{x}' - \mathbf{x}_0'\right) = \left(\mathbf{x} - \mathbf{x}_0\right)^* A^* A(\mathbf{x} - \mathbf{x}_0) \ , \qquad (5.4)$$

where $\bullet$ denotes scalar product of two vectors. The right-hand side is a quadratic form which for all $\mathbf{x}$ and $\mathbf{x}_0$ equals

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2 = \left(\mathbf{x} - \mathbf{x}_0\right)^* \bullet \left(\mathbf{x} - \mathbf{x}_0\right) \ , \qquad (5.5)$$

if and only if $A^* A = I$, where I is the identity matrix. In such case

$$\sqrt{\left\{\left(x' - x_0'\right)^2 + \left(y' - y_0'\right)^2 + \left(z' - z_0'\right)^2\right\}} = \rho = \sqrt{\left\{\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2\right\}} \ . \qquad (5.6)$$

Thus the metric is invariant if and only if $A^* A = I$; *i.e.*, the transformation is orthonormal. This was stated but not demonstrated already in §2. The transformation group is therefore inertial if and only if A is orthonormal. The metric distance between any two vectors is then preserved timewise as well; and it does not matter if the transformation be of the alias or alibi type.

Attention may now be given briefly to time and how it affects inertial transforms. It is to be noted that in the Galilean case, by (2.2), $t' = t$, so that $dt'/dt = 1$. This is also true if $t' = \left(t - t_0\right)$. Otherwise there is a change in the rate at which clocks measure time, which would be variable unless *df/dt* is constant. Non-constancy affects the concepts of both velocity and acceleration, changing the relation between force and acceleration that exists in Newtonian mechanics. If *df/dt* is constant but different from unity, a change of time (or clock) rate is implied, which could be compensated by a change in units of the time scale. Unless this is done, it is required that $f(t-t_0) \equiv (t-t_0)$, so as to keep the same force units in the image as in the direct space.

6. Physical Interpretation

We turn now to the interpretive aspects in which all the "physics" of EM theory resides. An essential ingredient in the invariance demonstration of §2 was the assumed invariance of the Hertzian field quantities, Eq. (2.6); *viz.*, $\mathbf{E}' = \mathbf{E}$, $\mathbf{B}' = \mathbf{B}$. What can it mean that there is no covariant "scrambling" of electric and magnetic field components, as prescribed by Maxwell's theory? Is not this scrambling an observable physical fact? The answer lies in the different physical *definitions* of Hertzian and Maxwellian field quantities. (Perhaps it would be wiser to show the distinction by explicit notational differences … but, if the reader will keep in mind the considerations of this section, that complication will be unnecessary.) Maxwell's theory, interpreted in the Einstein way, is physically as well as mathematically specialized: Mathematically, it lacks parameters descriptive of field sink (absorber, detector, etc.) motions in the observer's frame of reference; so such *relative* motions are forbidden to occur. Thus the field sink is always tacitly at rest at the observer's *field point*. Field sources, by contrast, can move, via the **j**-parametrization. Physically, field detection processes describable by a given Maxwell-Einstein inertial observer are restricted to those occurring in instruments, or at "test charges," at rest in that observer's system (at his field point). It is as if each field sink had rigidly attached to it its own "preferred observer," or vice versa. The proof of this somewhat startling proposition is in the manifest lack of parameters: If test charges could move relative to Maxwellian observers, the field equations would contain explicit velocity parameters to describe such motions.

Needless to say, such under-parametrized EM theory provides no foundation for a motional *relativity* theory – nor did Maxwell himself (we may assume) intend it to do so; for he seems to have thought primarily in terms of an ether "at rest." The same "preferred observer" limitation implicit in Maxwell's theory characterizes Einstein's special relativity theory, which took over the Maxwellian EM formalism intact (including the modern "partial" meaning of $\partial/\partial t$). As a result, when different inertial laboratories are in different states of motion, each Maxwell-Einstein "inertial observer" must be equipped with his own comoving set of instruments for EM field or radiation detection. Each instrument is in effect associated with its own preferred observer. As it happens, the symmetrizing of preference among inertial observers (covariance) is logically quite different from the elimination of preference (invariance).

To elaborate, the Einsteinian observer K has his own comoving $\mathbf{E},\mathbf{B}$ meter, stationary at his field point, and observer K' has his own $\mathbf{E}',\mathbf{B}'$ meter, stationary at *his* field point – these primed and unprimed field detectors being entirely different macroscopic instruments in different states of motion. Since different measuring instruments are involved, it may seem surprising that there exists any relationship at all between their readings. It is not surprising that the relationship is somewhat complicated: If the field points comoving with K and K' instantaneously coincide, then at the place and moment of their coincidence the electric *and* magnetic field vector (or tensor) *components* in the two systems are related by a mathematical rule of linear combination termed *covariance*. It is solely because measurements are made by different instruments in different states of motion that a purely electric field in one system "becomes" a mixed electric and magnetic field in the other. And that in turn traces directly to the parametrization of Maxwell's equations (*i.e*., to the failure to parametrize sink motions). Covariant scrambling of field components is thus the hallmark of "fields" construed in the Maxwell way: By definition Maxwell-Einstein fields are *what is detected by instruments comoving with inertial observers (permanently fixed at their field points).*

A tacit assumption implicit in the Maxwellian world view is the exact reproducibility of experiments or observations. The simultaneous presence of many macro-instruments at or near a momentarily-shared field point demands such reproducibility, if the readings of these different instruments are to be mutually consistent and are to yield numbers that accord with the linear-combination prescription of covariance. For the readings of *separate* instruments must represent *physically independent* "experiments" of measurement. As long as the "field" arises from the presence at or near the field point of a superabundance of field quanta, there is no reason to question the validity of such a traditional formulation of EM theory. But there is good reason to do so (as we shall see) in the case of few quanta.

The definition of Hertzian fields is quite different, as to both physics and mathematics. Mathematically, we deal with a covering theory that is more richly parametrized. The total time derivative introduces an extra velocity-dimensioned parameter **w**, for which a physical interpretation is required. We do not wish to repeat Hertz's mistake of interpreting **w** as a descriptor of ether velocity, nor Maxwell's mistake of rigidly attaching sink to observer ($\mathbf{w} \equiv 0$). The reader has probably already guessed, then, that the natural employment for this parameter is as a descriptor of *field-detector velocity*. Thus it seems natural to correct Maxwell's failure to parametrize sink motions by the following:

Interpretation. *The parameter* **w** *describes the field detector, sink, or radiation absorber velocity with respect to the inertial frame of the observer or his field point.*

About the physics of Hertzian fields we can say this: Now that relative motion of sinks or field-detection instruments is permitted through adequate mathematical parametrization, it is no longer necessary for each inertial observer to possess his own "private" field detector. A single "public" instrument will serve, since now each observer has his own independent means of describing the state of motion and predicting the readings of that instrument. Thus a given detector, being in a determinate state of uniform relative motion, will possess its own (different) numerical values of the components of **w** with respect to each differently-moving inertial observer.

This clarifies “invariance” and trivializes interpretation of the Hertzian field transformation equation (2.6), $\mathbf{E}' = \mathbf{E}, \mathbf{B}' = \mathbf{B}$. It means that all differently-moving observers, at any event of simultaneous coincidence of their field points with the given detector, must all read simultaneously from that detector’s digital read-out identically the same numbers. (Of course … no other numbers are there to be read.) This is why form invariance and numerical invariance in this situation amount to the same thing, and we have not chosen to analyze the distinction. Incidentally, at the level of single-quantum detection, the numbers displayed on the readout of the single instrument need not be predictable, but they must be objectively factual, so that all observers agree on them after the fact. That requirement is satisfied by the Hertzian formulation, which therefore fits perfectly with quantum ideology.

We see that the Hertzian and Maxwellian models differ in their physical pictures of the “field” measurement process as profoundly as in their formal mathematical properties of invariance and covariance. In the Hertzian case there is a single “public” field detector or radiation absorber – a unique instrument interrogated by a multitude of variously-moving inertial observers, each of whom assigns a different numerical value to the relative “detector velocity” parameter **w**. (Were we to remove our initial simplifying assumption that **w** is a constant, which could easily be done[6], it is apparent that general relative motions of observers would be allowed – implying EM general invariance. General invariance, throughout physics, follows automatically from a policy of expressing all laws of nature solely in terms of kinematic invariants.) A single event of detection – and of simultaneous coincidence of field points – occurs within this unique instrument. After the fact, it is described invariantly by all observers – who at a given instant all read the same numbers from the instrument’s display.

By contrast, in the Maxwellian case (as we noted) each inertial observer has his “private” field detector, a macroscopic instrument permanently at rest at the field point in his frame, with which he makes his own independent measurements. In this many-instrument case there arises the practical problem that when field points coincide these numerous differently-moving detectors must apparently collide. But of course they can be made small and allowed near misses. As long as there are sufficient field quanta present at the momentarily shared field point to be shared among all these observers, leaving enough quanta detectable by each to allow the law of large numbers to smooth-out statistical discrepancies, the covariant relationships among measured field component values predicted by Maxwell-Einstein must obtain.

Until about 1925 there was thus no information available to physicists that could have enabled them to make an uncontested choice between the Hertzian and Maxwellian formulations. At that time, however, knowledge of the microscopic non-reproducibility of experiments became common, as well as recognition of the *uniqueness* of individual quantum events. At the level of classical field theory the two formulations are equivalent – the Maxwellian and its Hertzian covering theory. But, insofar as quantum realities underlie classical EM theory and are to be recognized as ultimately dominant, or as a non-ignorable limiting case, there can be no question that the invariant Hertzian single-instrument approach offers the only possibility of a consistent EM theory extensible to the quantum level. For if field theory is to be applied to the description of the individual quantum event, such as detection of a single photon, it is obvious that the Maxwellian many-instrument approach is bankrupt: a single-quantum detection event is physically unique and can therefore occur in *at most* a single macroscopic instrument. And only Hertz's theory is parametrically empowered to describe the motion of that single instrument with respect to all inertial observers.

7. Solution of the Hertzian Wave Equation

By taking the curl of Eq. (2.1b), and applying (2.1a) and the condition that the field point is in vacuum (free space), we obtain at first order a Hertzian wave equation,

$$-\nabla^2\mathbf{E} + \frac{1}{c^2}\frac{D^2}{Dt^2}\mathbf{E} = 0 \ , \tag{7.1}$$

as well as a similar equation for $\mathbf{B}$. Let us seek a solution of the form $\mathbf{E} = \mathbf{E}(p)$, where

$$p = \mathbf{k}\cdot\mathbf{r} - \omega t = xk_x + yk_y + zk_z - \omega t \ . \tag{7.2}$$

We find that

$$\nabla^2\mathbf{E}(p) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}(p) = \left(k_x^2 + k_y^2 + k_z^2\right)\mathbf{E}''(p) = k^2\mathbf{E}'' \ , \tag{7.3}$$

where double-prime indicates two differentiations with respect to *p*. Similarly, using Eq. (1.1) we have

$$\frac{D^2}{Dt^2}\mathbf{E}(p) = \left(\frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla\right)^2\mathbf{E}(p) = \left\{\frac{\partial^2}{\partial t^2} + 2\frac{\partial}{\partial t}\left(\mathbf{w}\cdot\nabla\right) + \left(\mathbf{w}\cdot\nabla\right)^2\right\}\mathbf{E}(p)$$

$$= \left[\omega^2 - 2\omega\left(\mathbf{w}\cdot\mathbf{k}\right) + \left(\mathbf{w}\cdot\mathbf{k}\right)^2\right]\mathbf{E}'' = \left[\omega - \left(\mathbf{w}\cdot\mathbf{k}\right)\right]^2\mathbf{E}'' \ . \tag{7.4}$$

From (7.1) and (7.4) it follows that

$$\left\{-k^2 + \frac{1}{c^2}\left[\omega - \left(\mathbf{w}\cdot\mathbf{k}\right)\right]^2\right\}\mathbf{E}'' = 0 \ . \tag{7.5}$$

The vanishing of the coefficient of $\mathbf{E}''$ implies that

$$ck = \left|\omega - \mathbf{w}\cdot\mathbf{k}\right| \qquad \text{or} \qquad \frac{\omega}{k} = \pm c + \frac{\mathbf{k}\cdot\mathbf{w}}{k} \ . \tag{7.6}$$

We may thus introduce a phase *velocity u*, measured with respect to the same laboratory system as is **w**, by means of

$$u = \frac{\omega}{k} = \pm c + \frac{\mathbf{k}\cdot\mathbf{w}}{k} \ . \tag{7.7}$$

The most general d'Alembert solution of the wave equation (for constant **w**) can be written, with the help of (7.2) and (7.7), as

$$\mathbf{E} = \mathbf{E}_1\left(\mathbf{k}\cdot\mathbf{r} + \left[kc - \mathbf{k}\cdot\mathbf{w}\right]t\right) + \mathbf{E}_2\left(\mathbf{k}\cdot\mathbf{r} - \left[kc + \mathbf{k}\cdot\mathbf{w}\right]t\right) \ , \tag{7.8}$$

where $\mathbf{E}_1, \mathbf{E}_2$ are arbitrary vector functions. A similar solution applies to the **B**-field. It might appear that these solutions predict a first-order observable effect of detector velocity on wave propagation speed. But in fact Potier's principle[8,21], derived from Fermat's principle, states that when the expression for phase velocity includes an additive scalar product such as $\mathbf{k} \cdot \mathbf{w}$, regardless of how the "velocity" vector **w** is interpreted physically, it can have no effect on *observable* aspects of EM propagation, such as spatial light paths, interference patterns, etc. This result is well known in connection with an ether wind interpretation(21) of **w**, but is a mathematical theorem completely independent of the physical meaning assigned to that parameter. First-order laboratory-observable consequences of wave propagation are thus the same in Hertz's and Maxwell's theories. Higher-order considerations will be addressed in the next section.

8. Higher-order (Neo-Hertzian) EM Theory

Hertz's invariant covering theory of Maxwell's EM is physically valid only at first order. In this section (alone) we shall explore the subject of higher-order approximation. Classical mechanics expresses itself solely in terms of *kinematic invariants*, and it would seem that in this, as in much else, it could furnish a useful paradigm for all physical science. In seeking an EM theory valid at higher-orders of approximation than the first, then, the investigator's first task must be to identify the higher-order invariants of kinematics. In Einstein's special theory these are the proper space interval $d\sigma$ and the proper time interval $d\tau$. Here we shall review some speculations one of us has offered(8) about a possible approach to higher-order description. First, we notice an oddity: Einstein does not formulate his special theory explicitly in terms of the alleged kinematic invariants σ, τ. Rather, he employs noninvariant quantities in covariant combinations. This made sense in an era when physicists were convinced that the equations of EM could *not* be expressed in terms of manifest invariants. But we have seen that Hertz succeeded *at first order* in doing just that. So, in combination with Newtonian mechanics, Hertzian theory provides a *first-order physics*, complete in respect to both EM and mechanics, wherein the basic laws are all expressible solely in terms of kinematic invariants.

The fact that σ, τ are not self-sufficient in expressing the laws of physics at any level of approximation suggests that at least one of these alleged "invariants" may be falsely identified. Because of their expression of spacetime symmetry, herein discredited ("broken") already at first order, it would be folly in higher-order analysis to accept these symmetrical (alleged) invariants without questioning their empirical antecedents. In the absence of positive empirical evidence for the Lorentz contraction of extended material structures, one is in fact free to try the simplest alternative, which is that the spacelike invariant of kinematics is Euclidean *distance* (length). The timelike invariant we may take to be Einstein's proper time interval $d\tau$ between two events, *provided these events lie on the trajectory of a given particle*; for in this case there is much empirical evidence, such as that of CERN(22), to support the hypothesis. Moreover, proper time τ possesses a simple and convincing *operational definition* (as "pocket watch time of the comoving observer"), whereas σ not only lacks an operational definition but lacks even a conceptual rationalization apart from spacetime symmetry. In this section we shall hence drop σ and assume as higher-order invariants:

Timelike: Proper-time interval along a trajectory

Spacelike: Euclidean length interval.

We wish to re-express the EM field equations (2.1) in terms of these hypothesized higher-order kinematic invariants. The first question about proper time is *proper time of what*? The answer must reflect our identification in §6 of the *field detector* (indubitably the central physical actor in any field theory) as the object that moves with velocity **w** with respect to the observer's frame. We may picture a group of instruments, **E** and **B** detectors, source charge and current detectors, etc., as all moving together (idealized as a single instrument) through the field point at the moment of measurement or "detection." So, how shall that moment be measured? Evidently by a clock comoving with, and part of, that same master instrument (conceived as a "multi-purpose detector"). We may consider the time shown by this comoving clock to be the *proper time of the field detector*, to be denoted as τ_d . In this way time measurement is operationally unified with field measurement.

This τ_d identification might not be obvious, except for the fact that we have already specified our guiding principle to be the use of only kinematic invariants in reformulating Eqs. (2.1), and there is no other proper time than instrument time intrinsic to the problem. The only competitor is proper time of the observer, and this, as can readily be seen, is not intrinsic to the problem. The "field" is "created" not by the human observer, who plays no active role (except in certain highly dubious interpretations of quantum mechanics), but by and at the detection instrument or *test charge* … this being the physical agent that "makes the measurement" of the photon or responds observably to the field.

Our tentative higher-order invariant form of the EM equations, which we shall term *neo-Hertzian*, is identical to Eq. (2.1) but with formal replacement of "*t*" wherever it appears by "τ_d ." Thus *D/Dt* in neo-Hertzian theory is to be understood as $D/D\tau_d$, which we shall (in this section alone) write as $d/d\tau_d$, because of the greater familiarity to physicists of that notation for the total derivative. It may be computationally convenient to employ the familiar defining relationship,

$$d\tau_d^2 = dt^2 - \left(dx^2 + dy^2 + dz^2\right)/c^2 = \; invariant\,, \tag{8.1}$$

expressing the invariance of $d\tau_d$ (in which it must be remembered that the differentials refer to two neighboring events on the trajectory of a single material "particle," here the *detector*). This permits expression of the total time derivative as

$$\frac{d}{d\tau_d} = \frac{dt}{d\tau_d}\frac{d}{dt} = \frac{1}{\sqrt{1-w^2/c^2}}\frac{d}{dt} = \gamma_d\,\frac{d}{dt}\;\;, \tag{8.2}$$

which allows the observer's frame time *t* to be used – a great convenience for field or for many-body calculations. When this is done, and the wave equation is solved in precise analogy with the procedure of the preceding section, the resulting wave solution is found to have a phase velocity

$$u = \frac{\omega}{k} = \pm\sqrt{c^2 - w^2} + \frac{\mathbf{k}}{k}\cdot\mathbf{w}\;\;, \tag{8.3}$$

which generalizes to higher orders our previous Hertzian first-order phase velocity expression, Eq. (7.7). Further discussion and explicit derivation can be found in references[8,10].

The invariance of Eq. (2.6), $\mathbf{E}(p) = \mathbf{E}'(p')$, implies that

$$p = p' \quad \rightarrow \quad \mathbf{k}\cdot\mathbf{r} - \omega t = \mathbf{k}'\cdot\mathbf{r}' - \omega' t' \ , \tag{8.4}$$

which describes a constant phase-value of the propagating field in primed and unprimed frames. Suppose the detection instruments and clock are all at rest in the primed inertial system, so that **v** = **w** and Eq. (2.2) (spatial part) yields $\mathbf{r}' = \mathbf{r} - \mathbf{w}t$. Because of this assumed co-motion, detector proper time becomes identical with t' frame time, so that $\tau_d = t' = t/\gamma$. Then, using these relations to eliminate $\mathbf{r}', t'$, together with the fact that $\mathbf{r}, t$ are arbitrary, so that their coefficients vanish, we find two relations useful for applications,

$$\text{(I)} \quad \mathbf{k}' = \mathbf{k} \tag{8.5a}$$
$$\text{(II)} \quad \omega' = \gamma(\omega - \mathbf{k}\cdot\mathbf{w}) \ . \tag{8.5b}$$

The first of these can be used to describe stellar aberration, the second, the Doppler effect. The latter being fairly straightforward, we shall confine our attention here to the problem of aberration, since relation (I) looks at first glance as if it could not possibly predict aberration. Also, it happens that our first-order Hertzian theory [in which (I) is also true] cannot predict stellar aberration, so this exercise will demonstrate the superiority of neo-Hertzian over Hertzian theory.

9. Stellar Aberration

Eq. (8.5a) states that the direction of light propagation is not altered by inertial transformations – it remains invariant. That is in strong contrast to special relativity theory, which explains stellar aberration by an effect of inertial transformation on the angle of light propagation. How can the present alternative be justified? Let us consider the simplest case of starlight coming straight down from the zenith, on a vertical which we shall suppose to be normal to the plane of the ecliptic. Referring to Eq. (8.3), and ignoring complications such as earth's spin, we see that "detector" (our telescope) velocity **w** is parallel to the earth's orbital velocity and normal to the invariant light propagation direction **k**; so $\mathbf{k}\cdot\mathbf{w} = 0$ and (8.3) (representing vertical phase velocity) reduces to

$$u = \sqrt{c^2 - w^2} \ , \tag{9.1}$$

where we have chosen the plus sign in (8.3) arbitrarily. We know that in the inertial system comoving with the detector the latter is at rest and Maxwellian physics ($|u| = c$ corresponding to the special case **w** = 0) applies. Thus relative to the telescope tube or to the "inertial" observer at rest on the earth's surface the speed of light must be precisely *c*. For the light to move at speed *c* with respect to the telescope tube (*i.e.*, parallel to its axis) and with speed $u<c$ with respect to the vertical, as implied by (9.1), it is necessary that the telescope tube be tilted from the vertical by some small angle α. In this case we can as-

sign to the detector velocity **w** a magnitude equal to v_{orb}, the earth's orbital velocity. By similar Pythagorean triangles it is easily seen that $\alpha = \sin^{-1}(v_{orb}/c)$, which agrees with the observed aberration constant of about 20.5 arc-second. [This argument holds in neo-Hertzian but not in Hertzian theory – since the latter describes phase velocity by (7.7), not by (8.3) or (9.1).]

Actually, it is slightly more complicated than this, since **w** is detector velocity at time t_1 relative to detector velocity at an earlier time t_0, and α is telescope tilt angle relative to tilt at that earlier time – the observability of stellar aberration being wholly dependent on non-inertiality of the earth's motion (a fact that makes it somewhat paradoxical that special relativity, a theory of inertial motions, can predict it at all). Thus all observable quantities are detector-relative. A consequence is that the figure of aberration is traced on the celestial sphere in *phase quadrature* to the earth's orbital motion. The velocity of the light source or of any intervening medium does not come into the analysis.

In summary, in this section we have dealt with modified field equations constituting a possible candidate for higher-order formal EM invariance termed "neo-Hertzian." In this context we have seen that the new convective parameter **w** introduced into EM theory to secure first-order invariance has an influence upon the phase velocity of radiation that itself has no direct first-order effects (Potier's principle), but that does have collateral second-order consequences affecting, for instance, the description of stellar aberration. A more thorough analysis[23] of the latter shows that neo-Hertzian theory predicts the figure of aberration at all angles of light propagation to coincide exactly with a projection of the earth's orbit onto the celestial sphere, to within errors of second order; whereas Einstein's theory predicts first-order departures from such a projected figure. Unfortunately, the first-order angular discrepancy term predicted by special relativity, namely, $(v_{orb}/2c)\sin\theta\cos\theta$ (where θ is the polar angle measured from the normal to the ecliptic plane), is too small to be observed astronomically unless a resolution about 50 times better than the original design value of the Hubble (satellite) telescope can be achieved. The present theory predicts[23] zero for this first-order term. It is to be hoped that in the future crucial observations will become technically feasible. If so, they should prove decisive for or against neo-Hertzian EM.

10. EM Potentials and the Force Law

We return now to considerations of first-order physics. It is a moot point whether the fields or the potentials are to be regarded as more fundamental in EM theory. Maxwell, as reworked by Heaviside (though not in the original[1]), favored the fields. Observations such as the Aharonov-Bohm effect and the Marinov motor (discussed below) seem to favor the potentials. For present purposes this issue need not be resolved. As with the fields, we assume invariance under inertial transformations,

$$\varphi' = \varphi, \qquad \mathbf{A}' = \mathbf{A} \,, \tag{10.1}$$

and define the fields in terms of the potentials by means of the invariant prescriptions

$$\mathbf{E} = -\nabla\varphi - \frac{D\mathbf{A}}{Dt}, \qquad \mathbf{B} = \nabla\times\mathbf{A} \ . \tag{10.2}$$

[We return here to the use of *D* for the total time derivative, as given by Eq. (1.1). The use of the total time derivative, written as *d***A***/dt,* in the context of **E**-definition – on the evidence of Faraday's observations – can be found in some older books; *e.g.*, Smythe(24). It has also received more recent endorsement (25).] Of particular significance is the above invariant *definition* of **E**. It will be useful to clarify its import by means of the standard vector identity(26)

$$\nabla(\mathbf{a}\cdot\mathbf{b}) = (\mathbf{a}\cdot\nabla)\mathbf{b} + (\mathbf{b}\cdot\nabla)\mathbf{a} + \mathbf{a}\times(\nabla\times\mathbf{b}) + \mathbf{b}\times(\nabla\times\mathbf{a}) \ .$$

In this we make the replacements $\mathbf{a}=\mathbf{v}$, where **v** is "test particle" velocity, assumed constant (independent of space variables), and $\mathbf{b}=\mathbf{A}$. We then obtain the simpler identity

$$\mathbf{v}\times(\nabla\times\mathbf{A}) = \nabla(\mathbf{v}\cdot\mathbf{A}) - (\mathbf{v}\cdot\nabla)\mathbf{A} \ . \tag{10.3}$$

Identifying our field detector with the "test particle," so that **w** and **v** become the same, and applying Eqs. (1.1) and (10.3) to the **E**-definition in (10.2), with $\nabla\times\mathbf{A}=\mathbf{B}$, we find

$$\mathbf{E} = -\nabla\varphi - \frac{D\mathbf{A}}{Dt} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} - (\mathbf{w}\cdot\nabla)\mathbf{A} \ , \quad \text{or equivalently} \tag{10.4a}$$

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} + \mathbf{w}\times(\nabla\times\mathbf{A}) - \nabla(\mathbf{w}\cdot\mathbf{A}) = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} + \mathbf{w}\times\mathbf{B} - \nabla(\mathbf{w}\cdot\mathbf{A}). \tag{10.4b}$$

The form (10.4a) is directly useful in computations, as Wesley[13] has shown. The form (10.4b) is more revealing conceptually, as it demonstrates that our invariance requirement does not violate the Lorentz force law based on $\mathbf{w}\times\mathbf{B}$ (usually written $\mathbf{v}\times\mathbf{B}$, which is the same thing, since here the test particle and the detector co-move), but includes the Lorentz law along with an extra term, $-\nabla(\mathbf{w}\cdot\mathbf{A})$.

What can be said about this extra term? First, that as the gradient of a scalar function it will in general not be observable in experiments involving currents flowing in closed circuits – since such a gradient will normally integrate to zero around a closed loop(27). But not all "currents" flow in closed loops. The random charge motions occurring in a plasma, for instance, do not meet either (1) the closed-loop condition or (2) the $\mathbf{w}\cdot\mathbf{A}=0$ perpendicularity condition (obeyed by homopolar generators and motors) needed to eliminate the last (gradient) term in the **E**-field expression, Eq. (10.4b). The extra term should produce some anomaly in the rate of plasma diffusion, but this has never, to our knowledge, been checked empirically.

Since the Lorentz force law is already incorporated in the invariant electric field, expressed in form (10.4b), it is apparent that our insistence on manifest invariance has borne valuable fruit: We have only to multiply the **E** in (10.4b) by the test charge q to obtain a force law:

$$\mathbf{F} = q\mathbf{E} = q\left(-\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{w}\times\mathbf{B} - \nabla(\mathbf{w}\cdot\mathbf{A})\right) . \tag{10.5}$$

According to (10.4a), this can alternatively be written in the simpler form

$$\mathbf{F} = q\mathbf{E} = q\left[-\nabla\varphi - \frac{D\mathbf{A}}{Dt}\right] . \tag{10.6}$$

(In case the reader prefers factors of c in such equations, they can be supplied by everywhere replacing t with ct and **w** with **w**/c.)

We trust this improvement in logical economy is not lost on the reader. In Maxwell's noninvariant theory, as well as in Einstein's covariant apotheosis of it, the field equations do not *suffice* to describe the EM force acting on a test charge. A separate force law, variously attributed to Grassmann, Lorentz, and Laplace(27), must be postulated. Here we see that by simply requiring manifest first-order formal invariance of the **E**-field definition in terms of the potentials (implying use of the total time derivative) we automatically obtain a force law, $\mathbf{F} = q\mathbf{E}$, directly expressible in terms of a solution of the field equations. Thus one of the undocumented features of the Hertzian invariant covering theory of Maxwell's equations (an extra dividend, so to speak) is automatic provision of a force law self-contained within the field equations, without need for extra postulation. Hertz's covering theory of Maxwell's equations thus attains the physical sufficiency attributed to Maxwell's theory, but with improved logical economy.

However, the new force law is not quite the same as the old one. There is that extra term – an enhancement. What evidence suggests that that may have physical content? For one thing, it could be speculated that the extra gradient force term in (10.5) might to some degree account for the *anomalies in plasma diffusion rates* that used to be regularly reported in an era of outmoded honesty, before such reporting became politically inexpedient. Another bit of evidence is furnished by the so-called "Marinov motor." This has been repeatedly observed to operate in a manner very difficult to reconcile with the Lorentz force law(15), and possibly compatible with observable action of the extra force term(13,14) in Eq. (10.5). Unfortunately, it is as yet impossible to give a final judgment on this hypothesis. The empirical observations have shown problems of reproducibility, there is a sign problem, the assumption of **w** constancy may be challenged, and the necessity to choose a preferred gauge for the vector potential may occasion aesthetic discomfort. For such reasons, plus the still not fully resolved physics of the distinction between ponderomotive and electromotive forces, it may prove necessary ultimately to resort to really radical explanations of this type of motor, such as might emerge from a modernization of Weber-type electrodynamics(28). Such problems take us far beyond the purview of the present paper, so will not be confronted here.

11. Summation

The following points merit re-emphasis:

- A first-order (Galilean) invariant EM formulation due to Hertz, by virtue of being a covering theory of Maxwell's equations, *suffices* to describe generally-known (first-order) EM physics.

- It may also contribute to the description of certain less-known empiricisms, such as the Marinov motor, for the treatment of which Maxwell's theory, supplemented by the Lorentz force law, appears to be inadequate. (This remains to be shown.)

- It achieves such possibly enhanced physical sufficiency through improved logical economy, in that the force law, of the simplified form $\mathbf{F} = q\mathbf{E}$, is given by **E**-solution of the (Hertzian) field equations, in contrast to being separately postulated.

- The formal device by which invariance is achieved – the use of a total time derivative – has been shown to be mathematically *necessary*, under the proviso that the coordinate transformation precedes the time differentiations prescribed by field theory.

- The Hertzian form of field theory relates (invariantly) the field component readings of a single instrument – the field detector, sink, or radiation absorber – as seen by a plurality of observers whose field points momentarily coincide with that instrument.

- The Maxwell-Einstein form of field theory relates (covariantly) the field component readings of a plurality of differently-moving field-detection instruments, permanently fixed at momentarily coincident field points. These readings are definite numbers that demand experimental *reproducibility* in order to acquire physical meaning.

- Such reproducibility is attainable only in the many-quantum limit. On pushing field physics in the opposite direction, to the single-quantum limit, it becomes obvious that at most a single macro instrument can be involved in detecting the single quantum. Thus there exists an important physical limit in which the Hertzian one-instrument approach (invariance) is plainly superior to the Maxwell-Einstein many-instrument approach (covariance). This is true independently of the manifest ideological superiority of genuine invariance over mathematical simulacra such as covariance.

- This could explain some of the difficulties that have been experienced in forcing a marriage between quantum mechanics and relativity theory. The Maxwellian paradigm incorporated in relativity theory and hitherto universally endorsed by physicists entails a "spacetime symmetry" (essential to covariance and deriving mathematically from explicit use of the first-order *noninvariant* operator $\partial/\partial t$ in the field equations, where it appears symmetrically with the first-order *invariant* operators $\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z$) that exists at first order neither in nature (operationally) nor in the Hertzian world of invariant total time derivatives.

- If, as we suggest, spacetime symmetry does not physically exist, it becomes possible to formulate all laws of physics explicitly and solely in terms of kinematic invariants. The first step toward progress along this path is *identification* of the higher-order (spacelike and timelike) invariants of kinematics. These must differ from those of Einstein's special theory in that they cannot be presumed spacetime symmetrical.

- An attempt to carry the Hertzian invariant formulation beyond first order, termed neo-Hertzian EM, has been outlined, based on Euclidean length and field detector particle proper time as the assumed higher-order invariants. By this approach all laws of physics are to be expressed solely and explicitly in terms of *kinematic invariants*.

- Observations crucial to the decision between neo-Hertzian and Maxwell-Einstein EM theory, involving stellar aberration, will become possible when satellite-borne telescope resolution is improved by about two orders of magnitude over that of the present Hubble telescope.

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