CHAPTER 12

On the "Completeness" of Quantum Mechanics

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1. BACKGROUND

No study of the wave–particle dualism would be complete without examination of both the necessity and sufficiency of the mathematical-descriptive formalism that gives rise to it. Concerning sufficiency of the existing formalism, the issue of "completeness" of quantum mechanics as a physical theory was raised most poignantly by Einstein.[1] This matter is usually treated in connection with the Einstein–Podolsky–Rosen (EPR) paradox,[2,3] in the context of proposed "hidden variable" modifications or enhancements. The customary exposition then proceeds to Bell's theorem[4] and its modern developments, both theoretical and experimental[5]—the impression being created that there is a sort of championship-of-the-world fight in progress between clearly identified opponents: in one corner the recognized title-holder, "quantum mechanics," in the other a sequence of all possible (in general more generously parametrized) challengers to quantum mechanics.

As it happens, the assumption of the existence of such well-defined battle lines is a possibly fatal oversimplification. The conceptual fallacy in pitting quantum mechanics against all comers is simply that "quantum mechanics"—or any other product of theoretical physics—is not *in principle* a uniquely defined conceptual entity. Here we have in mind not the many possible versions or "interpretations" of accepted formalism, but substantive variants of that formalism. No theory is defined for purposes of physical description—meaning for purposes of acquiring observational support—except within a penumbra or con-

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gruence of its class of what is termed "covering theories." Every experiment that upholds quantum mechanics upholds also all covering theories of quantum mechanics; for the definition of a covering theory is *any more richly parametrized theory that reduces identically to the "covered" theory for particular fixed values of the extra parameters.*

Any generalization of quantum mechanics that is a covering theory of quantum mechanics must by this definition inherit all the problems of quantum mechanics. Thus, it must encounter difficulties in resolving the EPR paradox to just the extent that ordinary quantum mechanics encounters such difficulties. No matter how richly parametrized they may be, such theories include quantum mechanics and thus include all of that theory's problems of mating to however-defined "physical reality." If we confine attention to covering theories, then in the battle between quantum mechanics and its opponents . . . which is which? (In the words of Pogo, "We have met the enemy and he is us.")

Since one seems thus to be "licked at the start" in any attempt to generalize quantum mechanics via the covering-theory approach, why bring up the subject? The answer is that one is licked only in respect to the answering of certain questions, such as those raised by EPR, and it may be in some sense (knowable only by hindsight from the vantage point of future history of science) that these are the *wrong questions*. In science the questions asked are all-important and nature provides no signposts pointing to the "right questions" for any particular era. The questions to which EPR leads—including metaphysical or ontological ones tending toward a definition of "reality"—may well be the wrong ones.

Perhaps the right ones for our time are more along the line of: *Why does our cherished mechanics of quanta fail to provide a specific dynamics of "nuclear" or "elementary" sub-atomic-scale quanta? Why is it parametrically incapable of describing Einsteinian "point events," not to mention "quantum jumps?" Why does it offer no distinction between the unique facts of history and the ensemble of possible futures? Why should the principle of relativity of physical size (which holds throughout the vast Newtonian range of sizes) suddenly fail at the threshold of subatomic scales?* Such questions lie clearly within the province of the physicist; whereas EPR (at least in its original Einsteinian form) can rapidly lead into territory whose ownership the philosopher may legitimately contest. The plainly "physical" questions just mentioned typify those upon which a study of covering theories of quantum mechanics casts a starkly revealing light.

There is a further, manifestly crucial, aspect of covering theories: invariant covering theories provide a royal road to "new physics," insofar as their extra parameters offer fresh descriptive possibilities. As the reader may have inferred, this chapter will be addressed mainly to covering theories of quantum mechanics—in particular to one most attractive candidate, a "top contender" that seems distinguished from the rest by its simplicity. Some of the implications of this covering theory for physical description will be sketched, and a potentiality will be demonstrated for vastly extending the mechanical descriptive purview. But it must be recognized at its outset that such a study can lead to no "resolution" of the wave–particle dualism. For that dualism, in precise analogy with EPR, poses questions that—though they inspire significant new experiments—may point less toward new physics than toward the clarification and mental integration of old physics.

In fact it is the writer's prejudice that too much attention to dualism distracts from perceiving and enhancing the *unity of mechanics*, which deserves a prominent place among the goals of physical theory. Concerning the *new physics* just mentioned, all experiments are in themselves ambiguous and offer physical insight only in conjunction with theory—so that any experiments, whether or not so intended, may in hindsight be perceived as the precursors of new physics. Humility is in order, for we still play among the pebbles bordering Newton's ocean, and despite much pebble-polishing and knowledge-squirreling have advanced farther in hubris than in wisdom.

2. COVERING THEORIES: AN EXAMPLE FROM ELECTROMAGNETISM

The subject of "completeness" of physical theory in general being intimately bound up with the status of covering theories, it may be instructive to digress briefly from our main subject of mechanics into the neighboring field of electromagnetism—which affords a singularly elegant, though little-known, example of the significance of the invariant covering theory.

In brief, Maxwell's electromagnetism rests upon field equations that are *not invariant* under any known coordinate transformation. This simple statement of fact invites the rebuttal that Maxwell's equations are covariant under Lorentz transformations and that *covariance is "just as good" as invariance*. To this latter contention the obvious response is, "How do you know until you've tried?" That is, the majority of today's physicists have imbibed covariant formalism from their cradle, and have never so much as sampled the flavor of a truly invariant electromagnetic formalism. Having been raised on *ersatz*, how can they render a judgment on *echt*?

The question of what electromagnetic "invariance" means cannot be separated from that of precisely identifying the invariants of kinematics. The subject of higher-order or "exact" description—which has been treated elsewhere[6,7]—lies outside the present purview. Fortunately, at first order in (v/c), Einsteinian and Newtonian identifications of the kinematic invariants are in close enough accord that we can proceed to illustrate here the invariant covering theory idea and physical role without concern about the description of very high-speed motions.

Confining ourselves then to first-order considerations, we invoke history by noting that Heinrich Hertz, the experimentalist who assured Maxwell's fame by confirming the existence of electromagnetic waves, was also a powerful theorist who published[8] a form of Maxwell's equations that was rigorously invariant under

Galilean (inertial) coordinate transformations. To accomplish this, Hertz banished "spacetime symmetry" by replacing Maxwell's partial time derivatives $\partial/\partial t$, wherever they occurred in the free-space field equations, with total derivatives,

$$d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$$

The resulting modified field equations can be shown[7] to be Galilean invariant; in fact, they are readily expressed in manifestly invariant form,[7] such that each symbol in the field equations transforms invariantly under inertial transformations. Thus, there is no occasion to invoke covariance—which in any case could not obtain because of loss of spacetime symmetry (the partial space derivatives appearing in the Hertz equations being not symmetrical with the total time derivatives).

The formal maneuver just described introduces into the field equations a new velocity-dimensioned parameter "$\mathbf{v}$" not present in Maxwell's version of electromagnetism. Hence, the Hertz theory is more richly parametrized than the Maxwell theory. In fact, we see that Hertz's is a covering theory of Maxwell's—because, on assigning to $\mathbf{v} = (v_x, v_y, v_z)$ the fixed numerical values (0,0,0), Hertz's equations reduce identically to Maxwell's (in view of $d/dt \rightarrow \partial/\partial t$). The fact of noninvariance of Maxwell's theory under any coordinate transformation, taken with the fact of Galilean invariance of Hertz's theory, means that Hertz's equations constitute an *invariant covering theory* of Maxwell's noninvariant theory.

Why has the reader never heard about this? Since there is a lesson about physics (sociology of) to be learned, it is worth a brief further historical detour to answer this. First, Hertz used an archaic (nonvector) notation that concealed the simplicity of his total time derivative modification. Second, he gave no proof of "invariance" but simply asserted it. (A modern commentator,[9] on beholding this assertion, failed to check the mathematics but blandly remarked that Hertz must have meant "covariance." Not so; he meant what he said, but took too much for granted about his readers' intelligence. In fact, he could not have meant "covariance" because his equations lack spacetime symmetry.)

Third, Hertz made a genuine mistake on the side of physical modeling or interpretation (the soft underbelly or Achilles' heel of all mathematical physics, as Hertz himself implied in his famous putdown, "Maxwell's theory is Maxwell's equations"): Hertz assumed that "$\mathbf{v}$" measured an ether velocity, and further borrowed an old assumption due to G. Stokes that ether was 100% convected by material bodies. So he interpreted "$\mathbf{v}$" as the observable velocity of such bodies in the laboratory, and consequently termed his theory an "electrodynamics of moving bodies."[8] His equations were thus interpreted[9] as predicting the production of a magnetic field by motion of a dielectric in the laboratory. This effect was looked for and not found.[10] Hence, Hertz's theory (viewed *not* as Hertz's equations but as these plus Hertz's fanciful interpretation) was discredited and discarded in favor of Maxwell–Lorentz theory.

The latter, being spacetime symmetrical, was used by Einstein–Minkowski as the basis for the famous hypothesis of the "metric nature of spacetime." Once that went into the curriculum of the global village there was no turning back from covariance to invariance, and Hertz's important mathematical discovery of an invariant covering theory was lost to history . . . until recently, when several investigators (including S. Kosowski of Poland, F. D. Tombe of Northern Ireland, and C. I. Mocanu of Romania, as well as the present author) independently rediscovered Hertz's invariant mathematics. Naturally, it has been necessary to find a better physical interpretation than Hertz's, and here opinions differ to this day.

This is not the place to go into the subject, which has been treated elsewhere,[(7)] but it may be remarked that by interpreting "**v**" as the velocity in the laboratory of a particular tangible object, the *field detector*, it is possible to avoid both the taint of metaphysics and the trap of false predictions into which Hertz unhappily stumbled. There is—as must be true of all covering theories—predictive agreement of the covering theory (Hertz's) with all observational evidence that supports the covered theory (Maxwell's). The special case in which covered and covering theories become identical is that in which field detector velocity vanishes, $\mathbf{v} = (0,0,0)$, which is precisely Maxwell's case of the *field detector at rest in the laboratory*. In summary: Maxwell got the mathematical physics "right in one laboratory," but not in all variously moving laboratories. That remained for Hertz, who yet tripped at the final step of converting mathematical physics into physics.

The issue of covariant versus invariant description may appear academic, pedantic, or even metaphysical. On the contrary, it relates directly and decidably to observable facts of experience: It is an issue of real physics. According to Einstein–Minkowski *all forces in nature must be expressible in covariant form*, whereas according to Hertz no such requirement can hold for electromagnetism. The discovery of a single example of a noncovariant force in nature would settle the issue in favor of Hertz's invariant covering theory and would disprove spacetime symmetry.

As it happens, there is growing evidence for the existence of noncovariant electromagnetic forces. The original Ampere law[(11)] of force between current elements, for example, obeyed Newton's third law and thus was noncovariant. Ampere's law, though never known to be violated,[(11)] was replaced in the favor of physicists by the Lorentz force law. The two laws differ by a quantity Q that is an exact differential.[(12)] Thus, their predictive differences (together with the Lorentz violation of Newton's third law) vanish when Q is integrated around any closed circuit external to the test current element. Yet if the circuit containing the test element is itself considered, the nontest portion of that circuit forms an open loop. In quantifying action-reaction it is the action upon the test element of the nontest portion—not of the total circuit—that must be considered. The integral of Q around a partial circuit need not vanish and the distinction between the two laws should be measurable. (In this theoretical conclusion we venture to differ from Maxwell.[13])

In fact, the experiment has been done[14] and has indicated that nature votes for Ampere—consequently for Hertz, for invariance, for Newton's third law as acting between current elements, and against homogeneous "spacetime" and its alleged metric nature or symmetry. Earlier evidence of Graneau[11] and others showing support for Ampere's law by exploding wire and railgun-buckling observations at very high pulsed currents is thus confirmed by low-current evidence valid under conditions precluding alternative explanations such as conductor melting.

It is clear from this electromagnetic example that issues of real physics, decidable by crucial experiment, devolve from such seemingly moot questions as covered versus covering theory, invariance versus covariance, etc. We now return to our main topic of mechanics.

3. A SIMPLE COVERING THEORY OF QUANTUM MECHANICS

The foregoing introduction via covering theories to the "completeness" of physical theory was concerned with what might be termed the "sufficiency" of physical description. Let us now address "necessity." In order to reason about this topic it is essential to proceed from *a priori* principles of some sort. Fortunately in the case of mechanics we do not start from a *tabula rasa*: A takeoff point for all other kinds of mechanics is provided by classical mechanics, which is well understood in all its aspects and may lay some claim to being the most broadly successful of all physical theories.

The most highly evolved form of classical particle mechanics is (arguably) the Hamilton–Jacobi form,

$$H = -\frac{\partial S}{\partial t}, \qquad H = H(q_j, p_j, t) \tag{1a}$$

$$p_j = \frac{\partial S}{\partial q_j} \tag{1b}$$

$$-P_j = \frac{\partial S}{\partial Q_j}, \qquad S = S(q_j, Q_j, t) \tag{1c}$$

which we seize on here because of its marked formal resemblance to the Schrödinger and Dirac equations. Two features of Eq. (1) are noteworthy: (1) The huge invariance group of the "contact" (or canonical) transformations under which these equations of motion remain unchanged. This group includes but far exceeds both linear and nonlinear coordinate transformations. (2) The tremendous range of sizes of physical systems to which the equations and their concomitant "point particle" idealization apply.

The second item suggests a physical principle capable of guiding the development of a covering theory of ordinary quantum mechanics—namely,

Principle of Relativity of Physical Size[7]*: The equations of motion of point-particle mechanics are expressible in a form that does not connote absolute largeness or smallness of the physical system described.*

The vital issue is: Over what range of sizes is the Newtonian idealization of the *mathematical point particle* physically permissible? We know that for many purposes it is quite acceptable to treat our sun's planets as mathematical points, and to do the same for baseballs, buckshot, and smoke particles. If Dirac[15] is to be believed, however, all this changes dramatically at the threshold of the atomic world; for, he asserts, *quantum mechanics is the discipline that sets an absolute size scale to the world*. In other words, the grand cavalcade of size relativity comes to a jarring halt right in the province of the chemist. That makes chemists very important people . . . absolutely.

So, anyway, *says* Dirac. Now let us see what he *does*[15]: Heedless of the doctrine of size absolutivity, he applies the idealization of the point particle to the smallest and lightest of the known massive particles, the electron. (Newton could hardly have done more . . . nor less.) Specifically, he cooks up a felicitous point operator form of the function H appearing in Eq. (1a), together with a formal operand Ψ, and thus extends into the smallest physical size range—far below the atomic—the point particle idealization embodied in the size relativity principle. In other words, he ignores size absolutivity and applies Newtonian size relativity . . . and this self-refutation is attended with astounding success! Since we ourselves must humbly decline to succeed better than success, let us follow this great man in doing as he does, not as he says—by applying without stint the size relativity principle.

Note that the principle, as stated above, speaks to "a form" of the mechanical equations of motion. It does not imply that *any* form will do . . . we have to look for a particular form. Fortunately, our task is such an easy one that it practically performs itself. Proceeding from Eq. (1)—since we wish to share Dirac's success in describing electrons—we know that we shall have to supply a formal operand Ψ_f at least to Eq. (1a). This means looking on the symbols of Eq. (1a) in general as operators acting toward the right; and since classically (1b) and (1c) are on an equal footing with (1a) it is natural to think of these additional equations as containing rightward-acting operators equally in need of operands. Being parsimonious, we do not part with operands easily, and so propose sharing the same operand:

$$H\Psi_f = -\frac{\partial}{\partial t} S\Psi_f \tag{2a}$$

$$p_j \Psi_f = \frac{\partial}{\partial q_j} S \Psi_f \tag{2b}$$

$$-P_j \Psi_f = \frac{\partial}{\partial Q_j} S \Psi_f \tag{2c}$$

Although this hypothesis has been arrived at here somewhat in the manner of doodling, it turns out to be quite a satisfactory "form" to represent *equations of motion for all mechanics*, invariant on all size scales in the sense of the size relativity principle. Indeed, we shall now show that not only is it invariant on all size scales of likely interest to physics, but it is a covering theory of all known forms of mechanics. Thus, we have again to deal with an invariant covering theory. Equation (2) is seen to possess three distinct classes of solution:

Class I. Ψ_f = constant. In this case the constant can be canceled from Eq. (2) and what remains is identically the Hamilton–Jacobi equations. Thus, Eq. (2) is a covering theory of classical mechanics, Eq. (1). That classical motions are included among the exact solutions of our postulated generalized equations of motion, Eq. (2), is a fact of profound significance for measurement theory. It alters the relationship of classical and quantum physics, since ordinary quantum mechanics treats classical motion states always as mere approximations to "exact" superpositions of quantum states, never as best-available descriptors in their own right. In dealing with Eq. (2) we have to get used to the idea that *all mathematical solutions are for physical descriptive purposes approximations*. What physical theory offers in any given problem is merely a modest choice between poor and less poor approximation, not a choice between drab *approximate* and gorgeous *exact*. (We describe *things*, and descriptions are not things. Ergo descriptions are never exact, for only things can be exactly things.)

Class II. S = constant = $\hbar/i$. In this case, Eq. (2) reduces to

$$H\Psi_f = -\frac{\hbar}{i}\frac{\partial}{\partial t}\Psi_f \tag{3a}$$

$$p_j \Psi_f = \frac{\hbar}{i}\frac{\partial}{\partial q_j}\Psi_f \tag{3b}$$

$$-P_j \Psi_f = \frac{\hbar}{i}\frac{\partial}{\partial Q_j}\Psi_f \tag{3c}$$

The value $\hbar/i$ of the constant is chosen to agree with experiment. Equations (3a), (3b) are of the form of the Schrödinger–Dirac equations. Equation (3c) is a stranger involving extra parameters (Q_j, P_j) that classically are constants of the motion and that retain the character of constants regardless of the class of solution considered. Obviously, Class II solutions describe quantum (atomic) states of motion. By inspection we see that Eq. (3c) has the solution

$$\Psi_f = \Phi(q_i, t)\exp\left[-\frac{i}{\hbar}\Sigma_j P_j Q_j\right] . \tag{4}$$

The exponential multiplier appearing here is just a constant phase factor, in general absorbed into the wave function normalization factor. Thus, $|\Psi_f|^2 = |\Phi|^2$, Φ being just the Schrödinger or Dirac wave function. Since all the physical predictions of quantum mechanics depend on sums or integrals of mean-value products such as $\Psi_f^* A \Psi_f = \Phi^* A \Phi$, from which the constant phase factor cancels, it is clear that the class of observational agreements of Eq. (3) coincides with that of ordinary quantum mechanics, viz.,

$$H\Phi = -\frac{\hbar}{i}\frac{\partial}{\partial t}\Phi \tag{5a}$$

$$p_j\Phi = \frac{\hbar}{i}\frac{\partial}{\partial q_j}\Phi \tag{5b}$$

This results from canceling the constant phase factors from Eqs. (2a), (2b).

So, Eq. (2), which we saw above is a covering theory of classical mechanics, is now shown to be a covering theory of ordinary quantum mechanics as well. This puts us on familiar ground. Nevertheless, there is something new: The "constant" phase factor of Eq. (4) contains not universal constants but *dynamical constants* (Q,P), which in general "jump" to new values when the descriptive problem (system Hamiltonian) changes . . . thus furnishing an entirely new mechanism of discontinuity and a way of severing that "von Neumann chain" of phase connections which in the Copenhagen view joins all physical descriptive problems into one endless, seamless whole. In short, we acquire a *post facto* way of describing happenings, point events, or what used to be called "quantum jumps"—*without*, however, acquiring any new predictive capabilities. Thus, the transition from covered to covering theory has profound implications for quantum measurement theory. These have been examined elsewhere[7,16] and need not detain us here.

In sum: As a scheme for calculating observable quantities, quantum mechanics is altered not a bit by substitution of the covering theory, Eq. (2). But the richer parametrization of the latter has a great impact on measurement theory. For example: (1) It permits us to view *history as a fact*, not as the sort of statistical ("class of facts") ensemble appropriate to prediction. (2) Through this nontrivial distinction between prediction and retrodiction, it gives substance at the quantum level to "time's arrow." (3) It secures the logical sufficiency of mechanics without need or call for extra axiomatics (e.g., a projection postulate).

Class III. $S \neq$ constant, $\Psi_f \neq$ constant. In this case the mathematical character of the problem changes. Equation (2c) is no longer a "fifth wheel," but becomes a "second equation in the second unknown." That is, in each of the other classes of solution there is only one unknown function, S or Ψ_f. But here both of these functions are unknown and have to be solved for simultaneously. An example of such simultaneous solution has been given.[7,17] It appears to describe nuclear-scale stationary bound states of electron–positrons in terms of states of *imaginary particle momentum* but real mass–energy. Thus, the ability of a covering theory to lead to "new physics" (right or wrong) is reaffirmed. Further evolutionary developments of the theory are needed. Here we confine attention to noting some attributes of the Class III formalism.

The most notable formal features of the Class III solutions are that (a) the classical-analog (CA) operators become in general non-Hermitean and (b) the Heisenberg postulate is violated. (It is for the latter reason that electron–positrons, as noted above, can exist on the nuclear size scale.) The Heisenberg postulate is generalized to

$$(p_k q_j - q_j p_k)\Psi_f = \left[\left(\frac{\partial}{\partial q_k}\right) S q_j - q_j \left(\frac{\partial}{\partial q_k}\right) S\right]\Psi_f = S\partial_{jk}\Psi_f \tag{6}$$

This leads to a tripartite interpretation of the quantity S: On the classical (Class I) scale S is Hamilton's principal function; on the atomic (Class II) scale S is Heisenberg's constant ($\hbar/i$); and on the nuclear (Class III) scale S measures the degree of departure of the commutator of (q,p) mechanical variables from the Heisenberg value. That is, the commutator is no longer universally constant but becomes a space-time variable function subject to the boundary condition that quantum mechanics be recovered far from the scene of nuclear (subatomic) description; i.e., $S(r) \to \hbar/i$ as $r \to \infty$.

If we split off a real function s by the definition $S = (\hbar/i)s$, we see from a well-known theorem (viz., that the product of two noncommuting Hermitean operators is non-Hermitean) that a CA operator such as H is non-Hermitean; for from Eq. (2a) we have

$$H\Psi_f = \frac{\hbar}{i}\frac{\partial}{\partial t} s\Psi_f \tag{7}$$

which exhibits H as the product of a Hermitean operator $-(\hbar/i)\partial/\partial t$ and a Hermitean (real) operator s. The real property is imposed upon s by the physical requirement that the transformations

$$\begin{aligned} \mathcal{H} &= Hs^{-1} \\ \Psi &= s\Psi_f \end{aligned} \tag{8}$$

render the resulting time-conjugate operator $\mathcal{H}$ Hermitean—for these transformations, applied to eq. (2a), reduce it to

$$\mathcal{H}\Psi = -\frac{\hbar}{i}\frac{\partial}{\partial t}\Psi \tag{9}$$

which identifies $\mathcal{H}$ as the operator conjugate to time.

It would in any formalism be disastrous to have the time-conjugate operator turn out to be non-Hermitean . . . and that misfortune is avoided by the simple but crucial formal transformations (8). To confirm the Hermitean property of $\mathcal{H}$, for example in the case of a nonrelativistic one-body problem, we observe that

$$\begin{aligned}\mathcal{H} &= Hs^{-1} = [(1/2m_0)\mathbf{p}\cdot\mathbf{p} + V]s^{-1} \\ &= -(\hbar^2/2m_0)\nabla s\cdot\nabla + Vs^{-1}\end{aligned} \tag{10}$$

which is readily seen to be Hermitean if and only if s is real. (A similar demonstration for the Dirac Hamiltonian is even more immediate.) Here the CA momentum operator $\mathbf{p}$ is seen from Eq. (2b) to be the non-Hermitean product of the Hermitean operator $(\hbar/i)\nabla$ and the real function s. A transformation analogous to Eq. (8) yields a Hermitean momentum,

$$\mathbf{P} = \mathbf{p}s^{-1} = \frac{\hbar}{i}\nabla \tag{11}$$

The transformations (8), by producing Hermitean operators, reduce the Class III formalism to mathematically familiar terms. But it must not be overlooked that the specific form of the Hamiltonian is affected by the transformation, and it is this *specific form* that contains all the physics. Thus, a new theory that will not in the least interest mathematicians may be of considerable interest to physicists.

4. ALTERNATIVE "NECESSITATIONS" OF THE COVERING THEORY, EQUATION (2)

An alternative general principle from which Eq. (2) may equally well be inferred is the following:

Principle of Correspondence Reversibility: The formal correspondence between classical and atomic-scale mechanics shall proceed with equal facility in either direction and shall in either case yield a complete mechanics.

Early in the history of quantum mechanics, Pauli[18] advanced a claim that one could start with the ordinary quantum mechanical equations of motion, Eq. (5), and recover the equations of motion of classical mechanics as a limiting case.

This claim was important at the time for confirming the legitimacy of the new form of mechanics, as it improved its connection with known successful mechanics by making correspondence a two-way street. Unfortunately, Pauli's claim is spurious, as the only links he established were between Eqs. (5a), (5b) on the one hand and Eqs. (1a), (1b) on the other. No mention was made of Eq. (1c), without which no Newtonian mechanics is possible. Recently it was recognized[19] that what one gets by Pauli's route is not Newtonian particle mechanics but Liouville-type statistical mechanics. That is, the formal absence of the constant parameters (Q_j,P_j)—known classically as the "new canonical variables"—from ordinary quantum mechanics deprives that discipline of the specificity needed to describe point events and leaves it with only the capacity to describe (statistical) classes of events. This is a shared disability of quantum mechanics and classical statistical mechanics, as contrasted with classical point particle mechanics. The cause, a parametric deficiency, is likewise shared.

If the correspondence reversibility principle is imposed as mandatory, it is apparent that ordinary quantum mechanics is in violation and must be replaced by some other theory. The need to enrich parametrization on the quantum side in order to improve specificity for *post facto* point event description—and to *prevent any change in number of parameters* during the correspondence transition in either direction—then suggests the covering theory approach, and one quickly gets to Eq. (2) by fairly obvious inferences. There is no need to elaborate here. Equation (2) offers a true point particle descriptive mechanics for all physical size scales.

Finally, setting aside all "principles," there is a direct empirical route to something like Eq. (2) via the reader's personal knowledge. Most physicists have had the experience of observing in a darkened room the scintillations, e.g., of a zinc sulfide phosphor. These are flashes of light that have the appearance of originating at definite times from pinpoint locations. It may be supposed that these attributes of localization would persist if the phosphor were examined under varying magnifications up to the most powerful. We seem thus to experience (and to retain in "historical" memory) personal detection of a specific space-time constellation of point events. Yet since the equations of motion of ordinary quantum mechanics, Eq. (5), lack parameters capable even of after-the-fact description of such a particular event constellation, the Copenhagen interpretation assures us that quantum-level "historical" retrodiction is as futile as prediction and that such an experience, as well as the memory of it, is consequently an illusion.

Physics was chartered to describe human experience, not to denigrate that experience as illusion. The illusion, according to the argument of this chapter, is that Eq. (5) forms the basis for a "complete" mechanical description of nature. The conservative approach to formal completion involves exploiting the wealth of possibilities for parametric enrichment—while "holding fast to the good"—offered by invariant covering theories.

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