

# Precision alignment of the LIGO 4 km arms using the dual-frequency differential global positioning system

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The alignment of the Laser Interferometer Gravitational-Wave Observatory (LIGO) using the global positioning system (GPS) is described. The LIGO project is designed to detect gravitational waves from astrophysical sources by laser interferometry. There are two sites separated by 3002 km that will be operated in coincidence. At each site, laser beams propagate in two orthogonal 4 km long evacuated beam lines 1.2 m in diameter. The subject of this article is the alignment of the 16 km of beam tubes using dual-frequency differential GPS. A maximum deviation from straightness in inertial space of 5 mm root mean square and an orthogonality between arm pairs of better than 5  $\mu$ rad is reported. Analysis of the as-built alignment data allows determination of the geodetic coordinates for the vertices and the arm orientations at both sites. From this information, the baseline distance between the vertices of the Hanford, Washington and Livingston, Louisiana sites was determined to be 3001.8 km. © 2001 American Institute of Physics.  
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## I. INTRODUCTION

The Laser Interferometer Gravitational-Wave Observatory (LIGO)<sup>1-3</sup> is dedicated to the direct measurement of gravitational waves from astrophysical sources. The project is funded by the National Science Foundation and is operated jointly by the California Institute of Technology (Caltech) and the Massachusetts Institute of Technology (MIT).

Gravitational waves are emitted by accelerating masses and are expected to be detectable at the Earth from sufficiently violent events occurring throughout the universe.<sup>4</sup> The formation and collision of black holes and the coalescence of orbiting compact stars, such as binary neutron stars or black holes, are likely sources. In addition, there is possible residue from the primeval universal explosion. Einstein's theory of general relativity predicts that the waves travel at the speed of light and that they cause a distortion of spacetime transverse to their direction of propagation.<sup>5,6</sup>

The direct detection of gravitational waves can provide fundamental evidence for the behavior of spacetime in strong gravitational fields where Newtonian gravitation is no longer a good approximation. Detection may also yield a new view of the universe since gravitational waves emerge from the densest regions in astrophysical processes without attenuation or scattering.

The LIGO facilities consist of two observatories, one

located at the U. S. Department of Energy's Hanford Nuclear Reservation in Washington State [LIGO Hanford Observatory (LHO); see Fig. 1] and the other in Livingston Parish, Louisiana [LIGO Livingston Observatory or (LLO)]. The site-to-site separation is 3002 km. This distance corresponds to a gravitational wave travel time of 10 ms. The interferometers located at the two sites are operated as a network. A network of detectors enables the determination of the position of sources on the sky from arrival time differences of the wave. It also reduces the influence of non-Gaussian environmental noise in the individual interferometers and thereby increases confidence in detection.

LIGO detects the gravitational waves by comparing the time of propagation of light in mutually orthogonal paths in the distorted space between freely suspended test masses separated by 4 km using laser interferometry.<sup>7</sup> The distortions that need to be measured are not expected to be larger than a strain of  $10^{-21}$ . The tubes, aligned by the global positioning system (GPS) described here, provide an evacuated and low scattering path for the laser beams (see Fig. 2).

At the inception of LIGO project construction, GPS surveying techniques had been applied to a number of large scale precision surveys<sup>8-10</sup> and their use in construction had become standard practices. LIGO, however, posed several unique challenges. The beam tubes needed to be aligned along the propagation direction of light in vacuum and not along the direction perpendicular to local gravity on the surface of the Earth.<sup>11</sup> The curvature of the Earth will cause the

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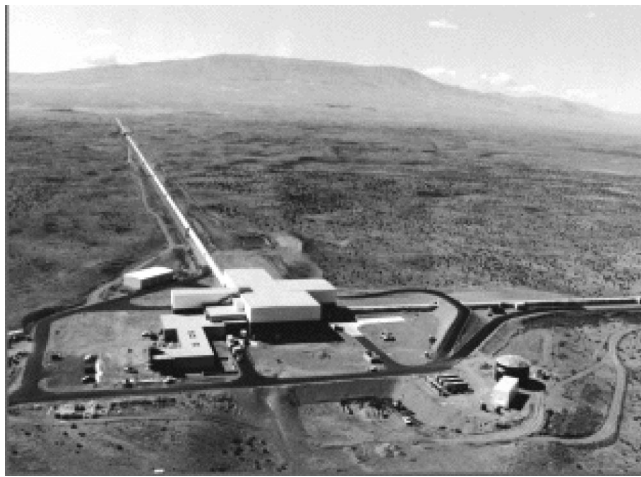


FIG. 1. Aerial view of LIGO Hanford Observatory toward the SW along the Y arm. The arm is 4 km long and has a midstation located at 2 km.

Earth's surface to deviate from the straight line propagated by light in vacuum by 1.25 m over a 4 km path if the line starts out level with the surface. The alignment was, therefore, not the same as that for a level highway or pipeline. A second special consideration, dictated by the nature of the construction project, was that the GPS alignment needed to be carried out concurrently with the construction with no significant opportunity for a check to be made by standard optical techniques until the construction was completed.

**II. METHOD OF ALIGNMENT**

LIGO contracted with CB&I Services, Inc. (CB&I) to design, fabricate, install and align the beam tubes. Proper alignment of the four 4 km beam tube sections was perceived as a major concern for LIGO. The beam lines require a minimum clear aperture of 1 m in order to accommodate multiple interferometers operating simultaneously within the same vacuum envelope. In addition, control of scattering and diffraction of light at an acceptable level for the most sensitive detectors contemplated in the facilities requires a large aperture (refer to Fig. 3).

The beam tubes are fabricated from 3 mm thick, spirally welded 304L stainless steel and have a nominal aperture di-

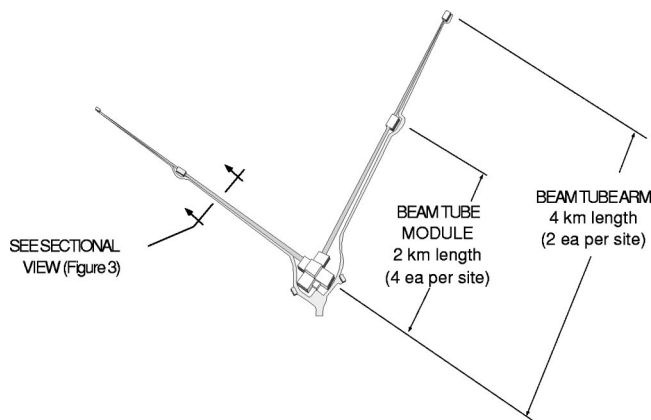


FIG. 2. Perspective schematic view of the LIGO LHO site showing the beam tube arms and the 2 km modules of which they are composed.

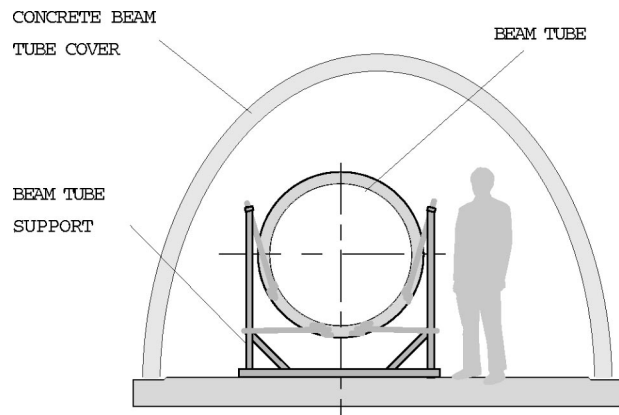


FIG. 3. Sectional view through the beam tube enclosure showing the relative scale of the beam tube and its structure.

ameter of 1.24 m. Optical baffles 9 cm high installed in the beam tube and fabrication and installation tolerances reduce the actual clear aperture to 1 m. These details are listed in Table I. Construction of the beam tubes was undertaken in 2 km sections, called beam tube modules.

**A. Feasibility studies and design**

LIGO had identified in its 1989 conceptual design<sup>12</sup> the use of a high precision dual-frequency, differential global positioning system (DGPS) survey as a technique by which to set reference monuments that could be used as millimeter-level optical benchmarks. However, at the time of the proposal, the GPS equipment and procedures to achieve this precision were not yet widely available to industry.

The introduction of commercially available, real time DGPS systems in 1993 permitted the use of the GPS to be reconsidered by the time construction of the beam tube was to begin. Trimble Navigation's Site Surveyor Real Time Kinematic (RTK) system was identified as an off-the-shelf system with millimeter-level accuracy that could perform in real time as needed in the field.

A field demonstration was performed to evaluate the capability of accurately measuring millimeter displacements

TABLE I. Allocation of budgeted tolerances for the beam tube clear aperture.

Description	Value <sup>a</sup> (m)
Fabricated beam tube aperture, minimum	1.238
Optical baffling system <sup>b</sup>	0.202
<i>Sources of aperture degradation during fabrication and installation</i>	
Straightness <sup>c</sup>	0.010
Concentricity errors	0.010
Ellipticity of beam tube cross section	0.006
GPS measurement error and tube placement tolerance	0.018
Net clear aperture	0.992

<sup>a</sup>Errors are given as peak-to-peak values on the diameter and are added algebraically to compute the total error allowed.

<sup>b</sup>To control stray scattered light, the beam tubes are lined with strategically placed sheet metal baffles which protrude radially 0.090 m into the beam tube. Tolerances on the design result in an allotment of 0.101 m for the total radial projection.

<sup>c</sup>Includes "corkscrewing" due to fabrication (0.006 m), thermal warping, and sagging due to the weight of the tube material (0.004 m).

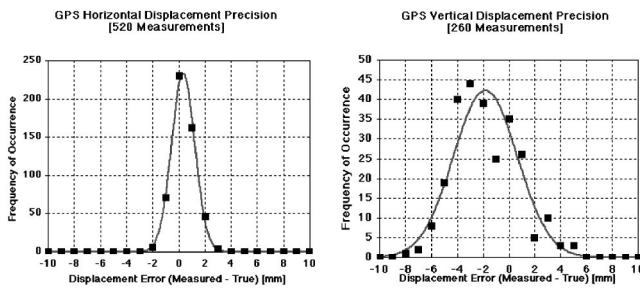


FIG. 4. Measured precision of differential GPS measurements taken during the alignment process. Horizontal:  $\mu=0.00036$  m;  $\sigma=0.0009$  m;  $\text{rss}=0.00096$  m. Vertical:  $\mu=0.0018$  m;  $\sigma=0.0025$  m;  $\text{rss}=0.0030$  m. The horizontal precision was three times better than the vertical precision. The vertical data show evidence of a bimodal distribution (one component with  $\mu' \sim 0.000$  m and one with  $\mu' \sim -0.003$  m). The assumption of a single broader distribution provides a conservative estimate of the precision.

using the dual-frequency system. This test was performed by displacing a GPS receiver antenna vertically and horizontally with a translation stage and then comparing the GPS readout to a mechanical dial indicator. Accuracy and precision tests were performed with an integration time of 10 s. A total of  $\sim 260$  points was measured for each of three displacement directions. Postprocessing using a precise satellite ephemeris yielded  $1\sigma$  (one axis, horizontal) = 0.0009 m and  $1\sigma$  (one axis, vertical) = 0.0025 m (see Fig. 4). The data also exhibited biases (i.e., nonzero means or offsets) in the distribution of residuals. The horizontal performance conforms very well to a normally distributed set of measurements. However, the vertical data exhibit a bimodality which, in the raw measurement-versus-actual displacement data sets, resembles nonlinearity for small vertical displacements. As a conservative estimate of overall measurement uncertainty, the root sum square (rss) of measurement bias and variance was used for both axes. These results were used in the  $\chi^2$  analysis described later.

Similar repeatability tests were later conducted for each arm at both sites to confirm that no systematic effects were present. This also demonstrated that millimeter root mean square (rms) precision was achievable along the 4 km arms.

The early field test results were incorporated into the final design of the beam tube. The nominal beam tube diameter was chosen as a trade-off among materials costs, dimensional control of the fabrication process and beam tube alignment accuracy. The trade study resulted in the allocation of errors presented in Table I.

The beam tube supports were designed to incorporate a heavy stiffener with precision machined inside and outside surfaces to provide a center reference. By incorporating tight tolerances for concentricity of the stiffener rings, their outside diameter could be used as a reference to the beam tube centerline.

## B. GPS equipment

CB&I chose Trimble Navigation's Site Surveyor System for use on this project.<sup>13</sup> The following hardware configuration was used:

- (1) GPS dual-frequency receivers 4000SSi with OTF and 1 MB memory;
- (2) data collector TDC-1;
- (3) geodetic ground plane antennas;
- (4) a TRIMTALK radio system;
- (5) GPSurvey software;
- (6) TRIMMAP software;
- (7) a Pacific Crest 35 W radio system in lieu of the TRIMTALK systems;
- (8) 38 cm (15 in.) Dorn–Margolan choke ring antennas to replace the geodetic ground plane antennas.

The 1 MB random access memory (RAM) in the receiver was insufficient to accommodate a full day of acquired data at the 5 s integration and sampling rate that was used. The field technician was required to halt data acquisition and to download the fixed and roving receiver databases at 4 h intervals. Incorporating a minimum of 3 MB in the receiver would have greatly improved operational efficiency in the field.

## C. Field implementation

### 1. Layout of the global coordinate system

The fundamental coordinate system for the alignment was the Earth ellipsoidal model WGS-84.<sup>14,15</sup> All raw GPS data were referred to this system using geodetic coordinates [height above ellipsoid ( $h$ ), latitude ( $\phi$ ), longitude ( $\lambda$ )]. Geodetic coordinates were transformed to the standard Earth-fixed Cartesian system ( $X_E$ ,  $Y_E$ ,  $Z_E$ ), where  $\hat{z}_E$  is aligned along the Earth's polar axis and  $\hat{x}_E$  penetrates the ellipsoid at the intersection of the Greenwich Meridian with the Equator.  $\hat{y}_E$  is perpendicular to both axes (refer to the Appendix).

A global coordinate system (denoted by subscript  $G$ ) specific to each LIGO site was defined in which the  $\hat{x}_G$  and  $\hat{y}_G$  are aligned along the interferometer arms and  $\hat{z}_G$  is normal to these axes. The interferometer plane was chosen to minimize construction costs (determined by the local topography) such that the global coordinate system lies in a plane that is locally tangent to the WGS-84 model at some point within the triangle defined by the 4 km arms. Deviations between the local zenith and  $\hat{z}_G$  can range up to  $\sim 0.63 \times 10^{-3}$  rad. The global coordinate systems for the two sites are further described in Sec. III.

The ends of the beam tube modules along each arm (i.e., at  $\sim 46$ ,  $\sim 2012$ ,  $\sim 2022$ , and  $\sim 3989$  m from the vertex) constituted controlled interface points. These points were identified by benchmarks (monuments) having measured geodetic coordinates that were provided by an independent surveyor. CB&I used these points together with an array of five other LIGO primary GPS monuments to calibrate their GPS instrumentation and software. The installation and survey of primary monuments were performed prior to the beginning of the construction activities at the sites. At LHO the work was performed by the Washington State Department of Transportation (WADOT) and it was performed by a private surveyor at LLO. The calibration was used by the GPS data acquisition system in real time to provide alignment data in global coordinates.



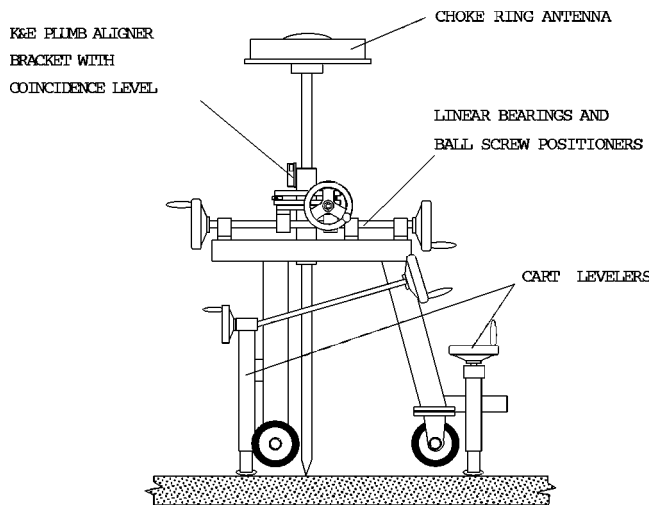


FIG. 5. Schematic of the field layout cart with a positioning system and a choke ring antenna. A layer of ECCOSORB™ material was placed below the antenna to limit spurious multipath reflections from the nearby concrete enclosure and beam tube during the installation and measurement phases.

Using global coordinates, the beam tube centerlines were marked along the foundation slab at points spaced uniformly at ~20 m intervals (the unit length of beam tube sections that were welded together in the field) along both arms.

A straight line in space varies in ellipsoidal height by ~1.25 m over a 4 km baseline. At each of the fiducial points, the design ellipsoidal height of the beam tube centerline was calculated using the WGS-84 model with the latitude and longitude as inputs. These heights were used to perform preliminary alignment of the tube sections as the supports were installed during beam tube fabrication on the slab.

Fiducial points were surveyed using a layout cart (refer to Fig. 5). The cart was equipped with linear bearings and a plumb alignment bracket and modified to support a fixed height antenna rod. Antenna rods, levels and attachment fixtures were periodically calibrated to better than 0.25 mm accuracy to ensure repeatability of measurements. Antenna rods used a leveling device consisting of a coincidence-type bubble level with a sensitivity of 10 arcsec/mm. This arrangement provided acceptable repeatability in a reasonable setup time. After a nominal fiducial point was identified, a 15–20 min static control point measurement was taken to provide a location determination. The nominal fiducial point was adjusted if required and a scribed mark was made on the foundation slab for the tube installation crew to use for rough elevation and centering of the tube section and its support.

## 2. Final alignment

At Hanford, supports were aligned for the final time after installation had proceeded for three to four sections, i.e., ~80 m from the installation activity. This was just before the beam tube became covered by cement enclosures and was thus no longer directly available. At Livingston, the beam tube enclosure sections were core drilled directly over the beam tube supports at ~120 m intervals. This design improvement allowed alignment to be confirmed after fabrication was completed.

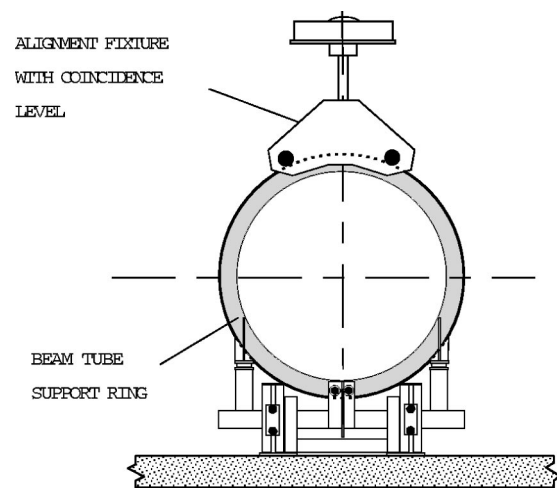


FIG. 6. Schematic showing the final alignment centering fixture with the antenna in place over a supporting ring of the beam tube.

The final alignment fixtures were similar for both sites (refer to Fig. 6). The antenna rod was longer in Livingston to enable the antenna to protrude through the cored enclosure. The fixture was a high accuracy centering head clamped on the machined beam tube support stiffener ring, plumbed using a coincidence level and centered relative to the beam tube support stiffening ring. The GPS antenna<sup>16</sup> was attached to a rod of calibrated length. The reference distance from the beam tube centerline to the antenna was taken as the rod length plus the radius of the beam tube support stiffener.

With this fixture in place, the GPS receiver was set to the RTK “stake-out” mode to position the beam tube center to the desired position relative to the global coordinate system. Once the position was determined to be within a few mm of the desired location, the support was locked down. A 30 min control point measurement was then taken and the raw data were logged by the receiver for subsequent postprocessing. The control measurement was taken in the RTK mode to verify position and to allow real time adjustment. Whenever an adjustment was required, a repeat 30 min control measurement was taken to verify the change.

## 3. GPS reference points outside enclosures

Additional GPS reference points were located (only at LHO) outside the beam tube covers to provide the capability to monitor the foundation slab for long-term height changes due to settling. These reference points were placed at the

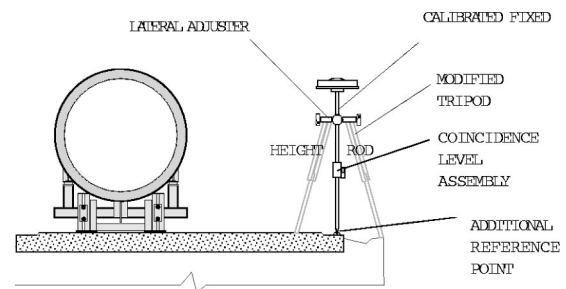


FIG. 7. Schematic of the modified tripod setup for reference point measurements at LHO.

edge of the foundation slab in line with each support ring. A two axis centering tripod designed to hold and adjust a fixed length GPS antenna rod was used (refer to Fig. 7). A 30 min static control point measurement was taken for each point. In addition, a crosscheck was made using Trimble's RTK stake-out procedure. These data serve as a reference for future GPS surveys that are made to assess slab settlement.

#### 4. Postprocessing and data review in the field

A data quality review was incorporated as part of the daily field alignment procedure. Precise GPS satellite ephemeris data were employed to determine postprocessed positions. The ephemeris data were downloaded from the U.S. Coast Guard web site.<sup>17</sup> Satellite residuals for each data point were determined relative to the reference mean for that point. Residuals were evaluated for consistency with the mean to identify and eliminate outliers. For every point surveyed, the time dependence of residuals from all visible satellites was displayed graphically. Typically, outliers arise from data coming from satellites lying closest to the local horizon. Residuals exhibiting nonstatistical fluctuations were eliminated by moving the satellite elevation cutoff to higher zenith angles or by eliminating data from suspect satellites. There is a trade-off between degradation of the signal-to-noise ratio (SNR), which decreases as satellites are removed from the analysis, and better data repeatability, which improves as data from marginally visible satellites are removed. After postprocessing, CB&I determined the geodetic coordinates of all points measured along the beam tubes. These were provided to LIGO for further analysis.

### III. DEFINITION OF GLOBAL COORDINATE SYSTEMS AND ANALYSIS OF BEAM TUBE ALIGNMENT DATA

The final alignment data provided by CB&I were analyzed by LIGO to determine the residuals relative to the best-fit descriptions for each site of the local right-hand coordinate systems that have their  $X$  and  $Y$  axes aligned along the beam tube arms. The methods employed at each site differed and reflected the accrued experience. These are described below.

#### A. Hanford, Washington

##### 1. Definition of the global coordinate axes

The interface benchmarks that are located at the termini and midpoints of each arm were used in a nonlinear regression analysis to first determine the best-fit description for the global coordinate axes. The data consisted of repeated measurements by independent surveying contractors for each of the eight positions located at nominally (46, 2012, 2022, and 3989 m) along the two arms. The data were used in a  $\chi^2$  minimization of the transverse [two-dimensional (2D)] residuals of the benchmark positions to determine the best-fit axes. There are six degrees of freedom for the fit: three translational and three rotational. These were chosen as follows:

(1) **three** coordinates for the vertex ( $X_v, Y_v, Z_v$ );

- (2) **two** vector components for the  $\hat{x}_G$  axis ( $n_{xx}, n_{xy}, 1$ ); the  $n_{xz}$  component was kept fixed; the unit vector was obtained by normalization in a second step;
- (3) **one** vector component for the  $\hat{y}_G$  axis ( $n_{yx}, -(1 + n_{xx}n_{yx})/n_{xy}, 1$ ); the orientation of the  $Y$  axis was constrained to lie in the plane normal to the  $X$  axis; this is achieved by varying only the  $x$  component of the  $Y$  axis direction, constraining its  $y$  and  $z$  components; the unit vector was obtained by normalization in a second step.

Not all errors associated with the measured data points were reported in the surveys. Therefore we used the measurement statistics that are shown in Fig. 4 for all data points.

The rms residual of the input data with respect to the best-fit axes was 0.005 m with a  $\chi^2$  statistic of 1.5 per degree of freedom (DOF) for 75 DOFs. This fit yields the parameters listed in Table II.

#### 2. Results for the as-built beam tube arms at Hanford

The geodetic coordinates for the beam tube center positions reported by CB&I were converted to Earth-fixed Cartesian coordinates using the relationships described in the Appendix. The points were transformed into the global coordinate system of Table II. The values of  $\delta Y_G$  and  $\delta Z_G$  along the  $X$  arm (or  $\delta X_G$  and  $\delta Z_G$  along the  $Y$  arm) are the residual alignment errors for each beam tube support position, and include both measurement errors and actual positioning errors.

Figure 8(a) presents the residuals in the transverse directions as a function of axial position along the beam tube arms. Some periodic and systematic trends are evident; however the magnitudes of the residuals meet LIGO specifications. A small amount of skewness ( $\sim 0.004$  m; see Fig. 9) is evident between the arms (i.e., the lines which best described the two arms individually do not intersect). The  $X$  arm was the first to be aligned and exhibits more scatter in the data. The better statistical characteristics of the  $Y$  arm are due to the improved alignment techniques that evolved with increased experience in the field. Figures 9(a) and 9(b), are scatter plots of the GPS measurements for both arms at Hanford. These are the same data as in Fig. 8(a), viewed along the global coordinate system  $X$  or  $Y$  axes.

Both beam tubes at Hanford are aligned to their respective axes to better than 0.005 m rms (two axis). This quality of alignment comfortably meets LIGO requirements. Table III presents the means and standard deviations for the transverse dimensions along both arms. These results comprise a total of 404 data points taken at 20 m intervals and distributed equally between the arms.

After beam tube alignment by CB&I, LIGO employed the services of a surveying company [Rogers Surveying, Inc. (RSI)] to verify the GPS based alignment data by independent means. Quality checks of the CB&I data were performed for a large number of selected points along both arms. The checks were performed for both vertical and horizontal alignment using a combination of optical and gravimetric techniques. We present data only for the vertical alignment verification because this dimension is the one for which GPS accuracies are worse typically by a factor  $\sim 3\times$ .

TABLE II. Parameters of the global coordinate system for Hanford, WA.

Parameter	Value	Estimated error	Unit
Vertex	Geodetic (h, $\phi$ , $\lambda$ ): [142.555,(N46°27'18.527 841"),(W119°24'27.565 681")]	...	[m,(dms),(dms)]
	Earth fixed ( $X_E, Y_E, Z_E$ ): (-2.161 414 928×10 <sup>6</sup> , -3.834 695 183×10 <sup>6</sup> , 4.600 350 224 ×10 <sup>6</sup> )	(0.006,0.006,0.005) <sup>a</sup>	m
$\hat{x}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (-0.223 891 216, 0.799 830 697, 0.556 905 359)	...	
	Compass direction: N35.9993°W (refer to geodetic north) <sup>b</sup>	2×10 <sup>-6</sup>	rad
	Angle below the local horizontal at the vertex: 6.196×10 <sup>-4</sup>	3×10 <sup>-6</sup>	rad
$\hat{y}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (-0.913 978 490, 0.026 095 3206, -0.404 922 650)	...	
	Compass direction: S54.0007 °W (see footnote b)	2×10 <sup>-6</sup>	rad
	Angle above the local horizontal at the vertex: 1.24×10 <sup>-5</sup>	3×10 <sup>-6</sup>	rad
$\hat{z}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (-0.338 402 190, -0.599 658 144, 0.725 185 541)	...	
	Deviation from the zenith at the vertex: 6.195×10 <sup>-4</sup> , toward $\hat{x}_G$	3×10 <sup>-6</sup>	rad

<sup>a</sup>With respect to the physical location of the benchmarks.

<sup>b</sup>Site drawings call for the arms to run N36.8°W and S53.2°W; these are referenced to the WA State Plane Lambert South Zone NAD 83/91. Geodetic North is 47' 39" (~0.8°) W of grid north at the vertex.

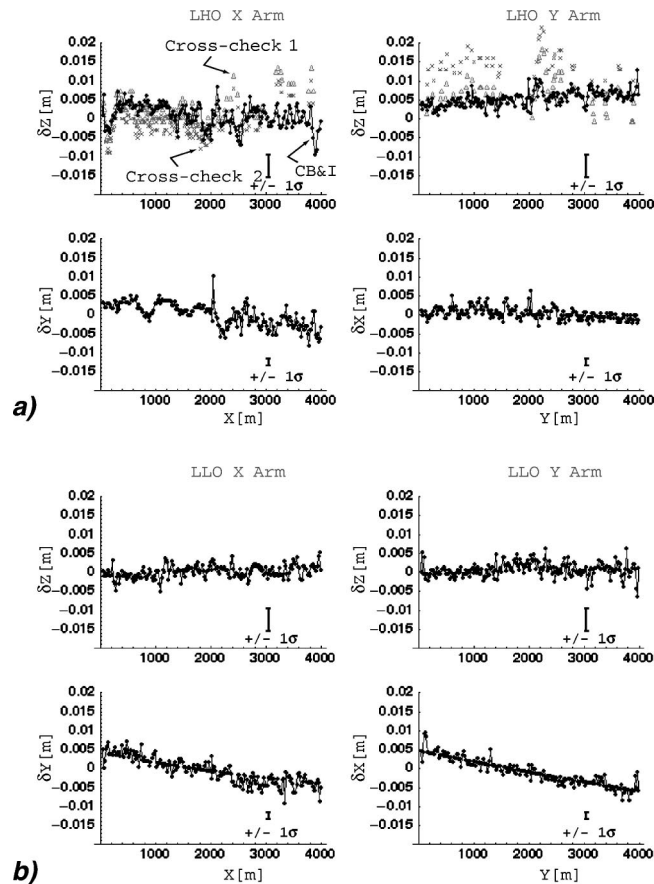


FIG. 8. Plots of the dependence of residuals on distance along the arms: (a) Hanford, WA and (b) Livingston, LA. The  $\pm 1\sigma$  GPS measurement errors are indicated.

We note that the horizontal alignment data in the lower panels of Fig. 8(a) exhibit less variability and less evidence of systematics, which reflects the fact that horizontal positional accuracies are generally better with GPS.

The follow-up surveys provided orthometric heights relative to the geoid (as opposed to the ellipsoidal heights provided by GPS). The geoidal deviations from the WGS-84 at each of the measurement points were calculated with a software package from the U.S. National Geodetic Survey using the GEOID96 model.<sup>18</sup> The orthometric height data provided by RSI were presented as differential data for each arm separately (differences of heights as one proceeds along each arm). In order to obtain absolute data connecting both arms, a choice of reference datum was required. Two different data were selected: (i) the first interface point along the X arm at  $X_G=46$  m as determined by RSI and (ii) the same datum as determined earlier by a third independent surveyor, IMTEC. These crosschecks are presented in the top two graphs in Fig. 8(a).

The RSI reference data are denoted as cross-check 1 and cross-check 2 and are indicated by  $\times$  and  $\Delta$  in the plots. The overall concordance between CB&I's reported alignment data and the quality checks is evident and meets LIGO requirements. This established confidence in CB&I's quality of data so that subsequently fewer crosschecks were deemed necessary.

### B. Livingston, Louisiana

Alignment of the Livingston beam tube arms proceeded in a different manner from what was done at Hanford. Experience with the quality of alignment at Hanford led to the decision to use CB&I's alignment database as the primary basis for determining the global coordinate system without

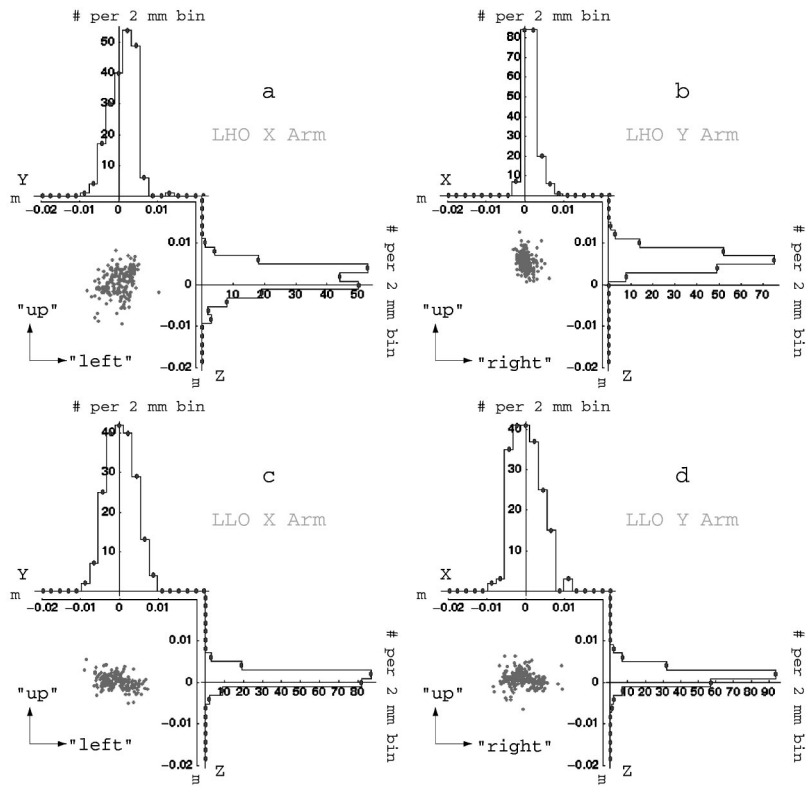


FIG. 9. Scatter plots of transverse residuals from the best-fit global axes for both Hanford, WA (a), (b) and Livingston, LA (c), (d). Refer to Table III for statistics of the distributions. The histograms give the number of points falling within 2 mm bins. The scatter plots correspond to looking down the arms from the vertex. The orientations (up, left, right) of the residuals from this reference point are noted in the scatter plots. The causes of the offsets, especially for the LHO Y arm, are discussed in the text.

relying on independent surveys. Hence, instead of using only eight interface points to determine the global coordinate system, all available alignment data were used.

CB&I core drilled the beam tube enclosures and developed a direct *in situ* final alignment procedure with a contacting fixture at 64 control points where GPS measurements were made directly on the as-built beam tubes. We used the interface points in defining the module ends, just as at LHO, and the rest of the module was defined as being in a straight line between those points. The core-drilled control points were used as a final check of alignment. We used the measurement statistics that were described earlier for all data points.

The rms residual for the best fit was 0.004 m with a  $\chi^2$  statistics of 1.9 per DOF for 186 DOFs. Table IV presents the parameters describing the best-fit coordinate axes in Livingston.

The 404 alignment data points reported by CB&I for every 20 m along the beam tubes were transformed to global

coordinates in the same manner described for Hanford. Both beam tubes at Livingston are straight to better than 0.004 m rms (two axis). This quality of alignment comfortably meets LIGO requirements. Table III presents the means and standard deviations for the two transverse dimensions along each arm.

Figure 8(b) presents the residuals in the transverse directions as a function of axial position along the beam tube arms. Systematic trends in the horizontal error with axial position are evident. This indicates that the beam tube centerlines are slightly nonorthogonal. In fact, the lines which best describe the centerlines subtend an included angle which is greater than  $90^\circ$  by  $5.3 \mu\text{rad}$ . The small nonorthogonality of the arms resulted from the initial benchmark data provided by LIGO to CB&I: subsequent analysis of the larger data set revealed the deviation. If we remove the linear trend from the residuals that is caused by this slight nonorthogonality, then the best-fit (but nonorthogonal) centerlines result in rms residuals of 0.002 m (X arm) and 0.001 m (Y arm). Aside from the (acceptably small) nonorthogonality of the arms, the quality of alignment is better than it was at Hanford: once again, the accrued experience and improved procedures of the CB&I team are evident.

Figures 9(c) and 9(d) show scatter plots of the GPS measurements for both arms at Livingston. There is no evidence of skewness for the Livingston site.

#### IV. DISCUSSION

The data acquisition techniques described above were developed by a CB&I senior field engineer and were successfully transferred to skilled technicians who then executed them repeatedly with consistent results. The subcentimeter

TABLE III. Statistical description of the residuals for both sites.

Quantity	Mean, $\mu$ (m)	Std. dev., $\sigma$ (m)
Hanford, WA		
X arm, $\delta Z$	0.0007	0.003
X arm, $\delta Y$	0.00006	0.003
Y arm, $\delta Z$	0.005	0.002
Y arm, $\delta X$	0.0003	0.002
Livingston, LA		
X arm, $\delta Z$	0.0002	0.002
X arm, $\delta Y$	-0.0006	0.003
Y arm, $\delta Z$	0.002	0.002
Y arm, $\delta X$	-0.0008	0.003



TABLE IV. Parameters of the global coordinate system for Livingston, LA.

Parameter	Value	Estimated error	Unit
Vertex	Geodetic (h, $\phi$ , $\lambda$ ): [−6.574,(N30°33′ 46.419 531″), (W90°46′ 27.265 294″)]	...	[m,(dms),(dms)]
	Earth fixed ( $X_E, Y_E, Z_E$ ): (−74276.041 92, −5.496 283 721×10 <sup>6</sup> , 3.224 257 01610 <sup>6</sup> )	(0.006,0.006,0.004) <sup>a</sup>	m
$\hat{x}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (−0.954 574 615, −0.141 579 994, −0.262 187 738)	...	
	Compass direction: S72.2836°W (reference is geodetic north) <sup>b</sup>	2×10 <sup>−6</sup>	rad
	Angle below the local horizontal at the vertex: 3.121×10 <sup>−4</sup>	2×10 <sup>−6</sup>	rad
$\hat{y}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (0.297 740 169, −0.487 910 627, −0.820 544 948)	...	
	Compass direction: S17.7164°E <sup>b</sup>	2×10 <sup>−6</sup>	rad
	Angle below the local horizontal at the vertex: 6.107×10 <sup>−4</sup>	2×10 <sup>−6</sup>	rad
$\hat{z}_G$	Earth fixed ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ): (−0.011 751 435, −0.861 335 199, 0.507 901 150)		
	Deviation from the zenith at the vertex: 3.121×10 <sup>−4</sup> , toward $\hat{x}_G$ 6.107×10 <sup>−4</sup> , toward $\hat{y}_G$	2×10 <sup>−6</sup>	rad

<sup>a</sup>With respect to the physical location of the benchmarks.

<sup>b</sup>Site drawings call for the arms to run S72°W and S18°E; these are referenced to Lambert grid coordinates, NAD 83/92, Louisiana South Zone (1702). Geodetic North is 17′ 01″ (~0.28°) W of grid north at the vertex.

levels of precision achieved demonstrate the utility of DGPS for *routine* applications in the field. By using the procedures described above the effort needed to align large-scale systems such as LIGO was significantly less than it would have been had we relied on conventional techniques.

The layout of the beam tubes and the orientations determined by GPS in the manner described were subsequently used to erect survey monuments to align the laser beams down each arm by dead reckoning. The monuments were erected in the vicinity of the vertex at each site and the directions to the 2 and 4 km distant mirrors were defined. In defining these directions, only the first few hundred meters of the arms could be used due to practical constraints. Thus the angular pointing accuracy expected in deriving the directions to the distant mirrors was of the order of

$$\sigma_\theta \approx \frac{\sigma_x}{200 \text{ m}} = 25 \times 10^{-6} \text{ rad.}$$

The laser beams were propagated down the evacuated 2 km beam tube module at Hanford and the full 4 km long arm at Livingston. The beam pointing derived from the survey markers resulted in a dead reckoning alignment of the beams to approximately 50  $\mu$ rad. This angular uncertainty is consistent with expectation. However, it does correspond to a transverse error of 20 cm at the end of a 4 km path and this does not approach the alignment precision of 5 mm determined from the GPS alignment. Nevertheless, it was extremely satisfying to see the beam and it established that there were no large systematic errors although the random errors were known to be small. The direct measurement of the alignment using the laser beams will be improved as the

LIGO interferometers are commissioned and this will provide a more stringent comparison of GPS and optical alignment.

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## APPENDIX

The Earth-fixed Cartesian system ( $\hat{x}_E, \hat{y}_E, \hat{z}_E$ ) is used for geodetic work. In this system,  $\hat{x}_E$  pierces the Earth's surface



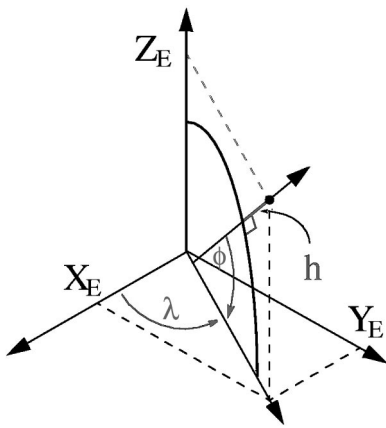


FIG. 10. Schematic showing the relationship between geodetic and Earth-fixed coordinate systems.

at  $(\phi, \lambda) = (000, 000)$ ,  $\hat{y}_E$  pierces the Earth's surface at  $(\phi, \lambda) = (000, 090E)$ , and  $\hat{z}_E$  pierces the Earth's surface at  $\phi = 090N$ . The relationship between the coordinates of a point  $(h, \phi, \lambda)$  and  $(X_E, Y_E, Z_E)$  is depicted graphically in Fig. 10.

The functional relationships are given by

$$X_E = [R(\phi) + h] \cos \phi \cos \lambda,$$

$$Y_E = [R(\phi) + h] \cos \phi \sin \lambda,$$

$$Z_E = [(1 - \epsilon^2)R(\phi) + h] \sin \phi.$$

The Earth model, WGS-84, is described by an oblate ellipsoid with its semiminor axis  $b = 6\,356\,752.314$  m along  $\hat{z}_E$ , a semimajor axis with a value of  $a = 6\,378\,137$  m, and eccentricity given by  $(1 - \epsilon^2) = 0.993\,306$ .  $R(\phi)$  is the local radius of curvature of the ellipsoid at latitude  $\phi$ :

$$R(\phi) = \frac{a^2}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}}.$$

Note that in the geodetic model the vector  $h$  is aligned along the local surface normal. Consequently, its extension to the equatorial plane will not in general intersect the origin.

<sup>1</sup>A. Abramovici *et al.*, *Science* **256**, 325 (1992).

<sup>2</sup>B. C. Barish and R. Weiss, *Phys. Today* **52**, 44 (1999).

<sup>3</sup><http://www.ligo.caltech.edu/>.

<sup>4</sup>K. S. Thorne, in *300 Years of Gravitation*, edited by S. Hawking and W. Israel (Cambridge University, Cambridge, 1987), Chap. 9.

<sup>5</sup>A. Einstein, *Sitzungsber. K. Preuss. Akad. Wiss.* 688 (1916).

<sup>6</sup>A. Einstein, *Sitzungsber. K. Preuss. Akad. Wiss.* 154 (1918).

<sup>7</sup>P. R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, Singapore, 1994).

<sup>8</sup>J. Collins and A. Leick, *First International Symposium on Precision Positioning with GPS*, Vol. 2, p. 667 (1985).

<sup>9</sup>R. Ruland and A. Leick, *First International Symposium on Precision Positioning with GPS*, Vol. 1, p. 483 (1985).

<sup>10</sup>A. Leick, C. Aiken, and J. Kor, *Survey Land Inform. Systems* **52**, 69 (1992).

<sup>11</sup>The deflection of light in a 4 km path on the Earth's surface due to the gravitational field of the Earth is less than  $2 \times 10^{-9}$  m.

<sup>12</sup>A Laser Interferometer Gravitational-Wave Observatory (LIGO), proposal to the National Science Foundation, December 1989.

<sup>13</sup>CB&I, as the contractor, had sole responsibility for the choice of hardware for the project. This information is provided for technical communication purposes only and does not constitute an endorsement by either the National Science Foundation or the California Institute of Technology.

<sup>14</sup>B. Hoffman-Wellenhop, H. Lichtenegger, and J. Collins, *GPS Theory and Practice*, 3rd ed. (Springer, Wien, 1994).

<sup>15</sup>A. Leick, *GPS Satellite Surveying*, 2nd ed. (Wiley, New York, 1995).

<sup>16</sup>One source of systematic error is multipath interference. LIGO's geometry and conducting beam tubes posed a concern. To eliminate this source of error, a 61 cm  $\times$  61 cm square of "ECHOSORB" material was placed 0.095 m ( $\lambda_{L1}/2$ ) beneath the GPS antenna. This material served to define the GPS antenna's field of view to include only the upper hemisphere viewing the sky. This improved SNR from 10% to 25%.

<sup>17</sup><http://www.navcen.uscg.mil/GPS/precise/>.

<sup>18</sup><http://www.ngs.noaa.gov/GEOID/geoid.html>; [http://www.cs.umt.edu/GEOLOGY/classes/Enviro\\_Geophys/geoid96.html](http://www.cs.umt.edu/GEOLOGY/classes/Enviro_Geophys/geoid96.html).

<sup>19</sup>ECCOSORB is a registered trademark of the Emerson Cummings Company, <http://www.eccosorb.com>.