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RECENT RESEARCHES IN ELECTRICITY AND MAGNETISM.

Notes on Recent Researches in Electricity and Magnetism, intended as a sequel to Prof. Clerk Maxwell's Treatise on Electricity and Magnetism. By J. J. Thomson, M.A., F.R.S., &c., Professor of Experimental Physics in the University of Cambridge. (Oxford: at the Clarendon Press, 1893.)

THE supplementary volume to Maxwell's "Electricity" which it was announced the present occupant of Maxwell's chair in the University of Cambridge had in preparation, was looked forward to with keen interest by all electricians. It was sure, of course, to be a work of great scientific importance; but it was awaited with all the more impatience because certain promises and allusions in the new edition of Maxwell's treatise, lately published under Prof. Thomson's editorship, had led to pleasant anticipations that the supplement would be more or less of a commentary on the treatise, and would deal with some of the outstanding difficulties of Maxwell's electromagnetic theory. One promise in particular, made in the notes on the Electricity, we looked forward to seeing fulfilled in the supplementary volume, that of further discussion of the Maxwellian stress in the electromagnetic field. It is just here that the greatest difficulties of Maxwell's theory present themselves to some at least, and that a commentary such as the author could have written would have been particularly valuable.

Any little disappointment which may be felt at first as to the contents of the volume vanishes when their solid scientific value becomes apparent, and it is felt that perhaps the author has done the right thing after all by preferring to give a full account of the great work which has been done in recent years, in confirming and verifying Maxwell's theory, and in answering the questions it has suggested.

The book opens with an account of Prof. J. J. Thomson's method of regarding electric and magnetic phenomena as produced by the motion of Faraday tubes of electric force. This method is fully explained, and applied to the discussion of various physical phenomena, such as electrolysis, the action of a galvanic cell, and so forth. These show, perhaps, to the best advantage the power of the method, which certainly enables a mental image of what goes on in such cases to be more easily and clearly formed.

These tubes, according to Faraday's idea, start from positive and end on negative electricity, and the positive and negative electricities at the extremities of a tube, being merely the two aspects or surface manifestations of the state of strain existing in the medium within the tube, are complementary and equal.

According to Prof. Thomson's specification, a tube is either closed or terminated by atoms. When it has a length of the same order of magnitude as the distance between two atoms in a molecule, the atoms are in chemical combination; when the length is of a higher order of magnitude the atoms are chemically free.

These tubes move through the field when electrical changes take place, and by their motion produce magnetic force, which is proportional to the velocity of the tube moving at the point considered, and at right angles to the plane defined by the tube and the direction of motion. When a tube as a whole reaches a conductor, it shrinks to molecular dimensions.

To account for a steady magnetic field, which occurs without dielectric polarisation, and therefore apparently without the presence of tubes in the field at all, it is supposed that there are passing through a given small area just as many positive as there are of negative tubes, (that is, there is a distribution of oppositely directed tubes throughout the field, such that there is nowhere any preponderance of one kind over the other), but that these two sets of tubes are moving with equal velocities in opposite directions, so that the magnetic forces which they produce reinforce one another.

A quantity which the author calls the polarisation of the medium is defined, for any given direction at a given point, by the excess of the number per unit of area of positive over negative tubes passing through a small plane surface drawn through the point at right angles to the given direction. When the dielectric is not air, the unit area is supposed taken in a narrow crevasse cut in the medium with its walls at right angles to the given direction. Thus the dielectric polarisation is exactly analogous to the magnetic induction in the magnetic field as ordinarily defined.

The momentum of the tubes per unit volume of the field at any point is a directed quantity which is normal to the plane defined by the magnetic induction and the polarisation, and its components are proportional to the rates of transference of energy, according to Poynting's theory, in the directions of the axes across unit area held at right angles to each of them.

When the electric intensity of the field is due solely to the motion of the tubes, they move at right angles to their own directions with the velocity of light.

This theory gives a clear idea of why both polarisation and conduction currents and the motion of charged bodies produce magnetic effects. Everything is due to the motion of Faraday tubes, and in all three cases the Faraday tubes are moved through the field.

It is questionable whether this theory will ever successfully compete in electromagnetic discussions with the reciprocal method, that of the motion of tubes of magnetic force. Both were given by Faraday, who speaks most unmistakably in his "Experimental Researches," in the language of the modern theory of the induction of currents by the motion of tubes of magnetic induction across conductors situated in the field. It is well to have both, and their use will serve to emphasise what is of very great importance, the reciprocal character exhibited very strikingly in the modification of Maxwell's electromagnetic equations, given by Heaviside and Hertz, of the relations between the electric and the magnetic forces.

After these preliminary discussions comes an account of the phenomena accompanying the passage of electricity through gases. This reviews and coordinates to a considerable extent the experimental researches of Crookes, Spottiswoode and Moulton, Hittorf, and others in this field. This part of the work has a value enhanced by the contributions made to our knowledge of this department of electrical science by the author himself, both as regards the actual experimental facts and their theoretical explanation. The action of a magnet upon discharges in tubes or bulbs without electrodes is peculiarly interesting. The discharge being oscillatory in such a case is separated into two distinct portions, consisting of the discharges in the two opposite directions. Thus a ring discharge in a horizontal plane has one part raised, the other lowered by the action of a horizontal magnetic field. Further, as has been observed by the author, the discharge is rendered more difficult when it has to pass across the lines of force of a magnetic field, while it is facilitated when it has to pass along the lines.

The explanation suggested by the author is ingenious. The gas breaks down along the line of maximum electromotive intensity, and a discharge occurs which gives a supply of dissociated molecules, which readily convey subsequent discharges. The magnetic field, when at right angles to the line of discharge, acting on the molecules taking part in the discharge, removes them from the line of maximum electromotive intensity, and thus the instability of electric strength which the discharge tends to set up is continually being annulled by the magnetic action.

In the other case it is suggested that if branching of the discharge from the main line takes place, the dissociated molecules there formed will be brought into the line by the magnetic action, and would thus increase the facility of the discharge, beyond that which would exist if there were no field.

This, as Prof. Fitzgerald has suggested, has an important bearing on the nature of the aurora, and probably explains the streamers which form so remarkable a feature of auroral displays. These may be simply more than averagely bright discharges along the electrically weaker lines of magnetic force in the rarefied air of the upper parts of the atmosphere.

A chapter is next devoted to Conjugate Functions in their applications to the solution of electrical problems. This method is very serviceable for the solution of problems of electrical flow in two dimensions, but it can hardly be applied in a systematic manner to the various problems which present themselves.

The theory of functions of a complex variable has been greatly advanced since Maxwell wrote, and there is certainly much, as has been pointed out, more especially by Klein, that has direct application to the solution of electrical problems. Prof. Thomson has therefore done well to include some of the general transformations of this theory, with their applications to such problems as the effect of the gap between the plate and guard-ring in Lord Kelvin's absolute electrometer or guard-ring condenser, different arrangements of piles of plates, and the like. This theory of the condenser he has himself made use of in his determination of v.

It may be objected that some of the problems solved by the indirect method employed in this chapter have no very distinct practical application; but there can be no question of the value of such a discussion. It places within the reach of students who are able to follow it processes ready to hand by which problems quite unassailable by ordinary methods are discovered and solved; and who can tell when such problems may not become of great practical importance, in the present rapidly advancing state of the science?

We are taken next to the subject of electrical waves and oscillations, which in some form or other is the theme discussed in the remainder of the book. The problem of periodic disturbances is very fully treated in a large number of practically important cases. Throughout the analysis the method of representing a simply periodic function in the form $Me^{(mz+pt)i}$ where $i = \sqrt{-1}$ is adopted. This tends greatly to condensation, and the results are always interpretable at will by properly "realising" the solution.

First is taken the extremely important case of waves along a cylindrical wire surrounded by a coaxial coating of dielectric, outside which again is an infinitely extending cylindrical conductor; and this is treated with special fulness. The solutions are expressed in terms of Bessel's functions; and this part of the book ought to lead to a more general study of the properties of such functions, and their applications to physical problems. They had their origin in a physical problem, and their importance to physicists has gone on increasing with the development of physical mathematics which has been brought about by the problems disclosed by scientific progress in recent times.

The theoretical solution of the problem of waves along wires is mainly due to Lord Kelvin, Mr. Oliver Heaviside, and Prof. J. J. Thomson. The solution given long ago by Lord Kelvin of the more limited problem which was then the practical one, appears as a particular case of the general solutions which these physicists have since obtained. The conclusions they have reached are of the utmost interest in connection with telephony, and seem likely to point the way to a more extended use of telephonic communication than has hitherto seemed possible. Lord Kelvin's early solution, it is not too much to say, gave for the first time light on the vexed question of the conditions of success in signalling through submarine cables and, together with the marvellously delicate and simple instruments which he also invented, rendered signalling through such cables commercially possible. Even now the question of ocean telephony has come to the front, and if it succeeds (and who will venture to say its difficulties will not be overcome?) it will be in great measure a result of the patient researches of men like Lord Kelvin, O. Heaviside, and the author of the work before us.

The complex variable treatment is adhered to, and contributes greatly to brevity of expression. The treatment of the subject is very complete, and though it involves some rather complicated work seems very accurately printed. The author has apparently pressed forward from point to point, taking the path which presented itself at the time, and hence, to one coming after, it is possible to suggest some shortening and smoothing of the way. For example, the values of the electromotive and magnetic intensities are perhaps more compactly investigated by first specialising the fundamental equations for the case of symmetry round an axis, noting that the electromotive intensity reduces to two components, one P along the axis, and the other R at right angles to the axis, in a plane through the axis and the point considered, while the magnetic force H, say, has a single component perpendicular to the plane. Thus two differential equations are got connecting P, R, and H, from which (P having first been found from the differential equation involving P alone) R and H are found at once in the forms $M\partial P/\partial r$, $N\partial P/\partial r$ where M and N are multipliers, and r is the radius drawn from the axis to the point considered.

It is to be noted also that the sign between the two groups of terms into which $K_0(x)$ is divided in (2), p. 263, should be the same as that before $\log x$ in the first group in brackets and that C should be taken with the same sign as $\log x$, and $\log 2$ with the opposite sign. This involves a correction likewise in the table of approximate values of the functions given lower down on the same page. Again, the same constant C, which has the value

$$\underset{n=\infty}{\text{Lt}}\left(\Sigma\frac{1}{n} - \log n\right)$$

is called Gauss's constant at p. 263, while the quantity $\gamma = e^{C}$ is called Euler's constant at p. 430. The established usage seems to be to call C Euler's Constant from its discoverer, who gave its value (to sixteen places of decimals) in his *Institutiones Calculi Differentialis.*

The "throttling" of the current in wires subjected to rapidly alternating electromotive forces is fully considered for a cable with inner and outer coaxial conductors, and for two flat strips in parallel planes with a stratum of insulating material between them. In this connection the author first introduces Mr. Oliver Heaviside's word *impedance.* Writing E for the external electromotive force, I for the total current, and R and L for the effective resistance and self-inductance, we have (p. 272)

$$E = L\frac{\partial I}{\partial t} + RI.$$

R is called by Prof. Thomson the impedance. According to Heaviside's proposal it is $\sqrt{R^2 + n^2L^2}$ that should be called the impedance, where $n = 2\pi/T$, T being the period of the alternation.

The manner in which the damping out of the vibration is taken account of by the complex analysis is well worth remarking. The eating up of the energy and consequent tapering off of the amplitude according to an exponential function of the distance from the starting end by the impinging of the oscillations in the dielectric on the conductors bounding it, and the lowering of speed of propagation of phase in the dielectric below the natural speed, that of light, all come out in the most beautiful manner.

Mr. Heaviside's careful synthetical explanations of such phenomena are well worth reading in this connection.

The author next passes to his own most valuable investigations regarding the effect of subdivision of iron on the dissipation of energy in the iron of a transformer, to electrical oscillations on cylinders and on spheres, and other problems of the greatest interest to all students of the later developments of Maxwell's great theory carried out by Hertz, now, alas, to be continued entirely by other hands.

The concluding portion of the book consists of a most valuable account of the work of Hertz, and forms the most appropriate supplement to Maxwell's great work that could have been written. The idea of Faraday tubes is well applied to picture the action of a Hertzian resonator in its different positions relatively to the vibrator in the experiments on direct radiation, and those on waves along wires. Not only is Hertz's own work fully described and explained, but the vast amount of fine work that has been done at Dublin, Liverpool, at Cambridge, and on the continent, is discussed, and much of it submitted to careful mathematical analysis.

Space does not permit of even a summary of the topics here treated, and we can only say that the reader who wishes to know these things well, and who shrinks from the labour of digging them out of *Proceedings*, *Annalen*, and *Berichte*, here, there, and everywhere, ought to read Prof. Thomson's work. Such a work is worthy not only of the author, but of the researches of the master and his great disciple who have passed away. A. Gray.

GREENHILL'S ELLIPTIC FUNCTIONS.

The Applications of Elliptic Functions. By Alfred George Greenhill, F.R.S., Professor of Mathematics in the Artillery College, Woolwich. (London: Macmillan and Co., 1892.)

It would be difficult to exaggerate the part which the study of elliptic functions has played in the pure mathematics of the present century. And this was to be expected; for whether we regard natural science as the application of common sense to the material needs of life, or as the outcome of the need for expansion in the mental world, and whether we consider mathematics as that exact basis without which progress was not permanently possible, or esteem it to be those higher Alps—

Where we can ever climb, and ever
To a finer air—

in either case we must see that a development of integral calculus—a development which was competent to fill so large a part of Legendre's life, which suggested such magnificent algebra as we find in Jacobi's Fundamenta, which promised, too, in Abel's hands such generalisations as are not even yet brought to perfection, such a theory, surely, was well worthy of persevering pursuit. And if we attribute the present extent of the theory of curves and of the theory of functions to the day when Riemann stood best man to the ideas of Cauchy and the suggestions of hydrodynamics, we must admit it was because his methods were employed upon the materials left by Abel that such results have come.

The importance of the present work lies in its recognition that the theory of elliptic functions arose as a development of integral calculus, and as such may be expected to supply a formulation of the solution of many problems of physics otherwise regarded as unfinished. Prof. Greenhill is well known to be a man who has not allowed his unwearied application to such problems to destroy his sympathy with pure mathematical speculation; on the contrary, he has sought, by every means in his power, to fill the difficult position of apostle to the Gentiles in this respect, by making as many of the results