Elasticity and electromagnetism

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Classical electrodynamics can be constructed formally as the theory of a linear elastic continuum. The Coulomb gauge expresses the medium incompressibility. The vector potential corresponds to the medium velocity. The pressure stands for the scalar potential. The electric field is modelled by an external force whose origin is beyond the elastic model. The electric charge corresponds to a medium defect which produces the perturbation $\delta p \sim 1/r$ of the pressure field. The defects interact with each other according to the conservation law in the torsion field of the medium.

A similarity between the electromagnetism and linear elasticity was known for many years (see e.g. [1] and references wherein). However, so far conditions of the exact correspondence between the two theories were not exposed. Recently we have shown [2] that a two-parameter generalization of electrodynamics is isomorphic to the extension of elasticity theory over arbitrary values of material constants of the medium. Thus, classical electrodynamics was found to correspond to incompressible linear elasticity. The Coulomb gauge expresses the condition of medium incompressibility. The vector potential corresponds to the velocity, and the scalar potential – to the pressure of the medium. The shear wave of the medium models the electromagnetic wave in vacuum.

This analogy has a formal character. For, the electric field is modelled by the term of a body force whose origin can not be elucidated in the bounds of the elastic model. Still, a realization of the electromechanical analogy is already known. It was found in a theory of ideal turbulence [3, 4]. So, it is expedient to carry out step by step the programme of the formal constructing of electrodynamics as a theory of elasticity. The present work is just devoted to this task. In accord with what was above said, the consideration will be restricted to an incompressible elasticity. The presentation will be given in a pedagogical style because of the peculiar character of the subject.

The following denotations are used below: $\partial_t = \partial/\partial t$, $\partial_i = \partial/\partial x_i$. The summation over recurrent index is implied throughout.

1. MECHANICS OF A MEDIUM

A medium is characterized by the volume density $\varsigma(\mathbf{x}, t)$ and the velocity $\mathbf{u}(\mathbf{x}, t)$ of the motion of its elements as functions of their position in space $\mathbf{x}$ and time t. Kinematics of the medium is specified by the continuum equation

$$\partial_t \varsigma + \partial_i(\varsigma u_i) = 0 \tag{1.1}$$

In the dynamic equation

$$\varsigma \partial_t u_i + \varsigma u_k \partial_k u_i = \partial_k \sigma_{ik} \tag{1.2}$$

$$i, k = 1, 2, 3$$

the stress tensor σ_{ik} has the meaning of the i-th component of the force, acting on the k-th face of the elementary cube from the adjoining element of the medium. The particular type of the medium or of the motion is specified by the function σ_{ik} of the medium parameters such as the density, velocity, displacement etc.

2. INCOMPRESSIBLE FLUID

The incompressible fluid represents the simplest and at the same time the special case: here the stress tensor does not depend on the strain. In the course of the motion it behaves adjusting itself to the velocity field. In general, we have for the fluid:

$$\sigma_{ik} = -p\delta_{ik} \tag{2.1}$$

where p is an unknown function of $\mathbf{x}$ and t. Then from (1.2), (2.1) the dynamic equation of the inviscid incompressible fluid is

$$\varsigma \partial_t u_i + \varsigma u_k \partial_k u_i + \partial_i p = 0 \tag{2.2}$$

In stationarity, when $\partial_t \mathbf{u} = 0$, this equation can be integrated − along the stream line or along the line of vorticity:

$$\frac{1}{2}\varsigma u^2 + p = \text{const} \tag{2.3}$$

The Bernoulli equation (2.3) demonstrates explicitly what was said before: the pressure function $p(\mathbf{x}, t)$ adjusts itself to the velocity field $\mathbf{u}(\mathbf{x}, t)$.

3. JELLY

We are interested in a medium, whose elements experience only small displacement $\mathbf{s}(\mathbf{x}, t)$ from its initial position $\mathbf{x}$. Notice, that here we passed over to the Lagrange representation, whereas the one used before was Eulerian. Thus, $\mathbf{u}(\mathbf{x}, t)$ in (1.1), (1.2) belongs to $\mathbf{u}$ at the current, or given point $\mathbf{x}$, but not to the initial position of the medium element. For small displacements the distinction between the two representations becomes inessential. One may assume in this case

$$\mathbf{u} = \partial_t \mathbf{s} \tag{3.1}$$

For small velocities we neglect in (1.2) quadratic terms:

$$\varsigma \partial_t u_i = \partial_k \sigma_{ik} \tag{3.2}$$

For a jelly-like medium, which is incompressible though liable to shear deformations, the stress tensor can be in the following way resolved in terms of the Hooke's law:

$$\sigma_{ik} = \mu(\partial_i s_k + \partial_k s_i) - p\delta_{ik} \tag{3.3}$$

$$\partial_i s_i = 0 \tag{3.4}$$

Substitute (3.3), (3.4) and (3.1) into (3.2):

$$\varsigma \frac{\partial^2 \mathbf{s}}{\partial t^2} = \mu \nabla^2 \mathbf{s} - \boldsymbol{\nabla} p \tag{3.5}$$

Remark that one may pass over from the general equation (1.2) to the linearized one (3.2) in a rigor way, not neglecting the quadratic terms. In this event, (3.2) would include the stress tensor in the Lagrange representation. While we have the Eulerian one in (1.2). Strictly speaking, it is for the Lagrange stress tensor that the Hooke's law of the type (3.3) is valid [5]

By virtue of (3.4) the first two terms of (3.5) are solenoidal, while the last one is potential. Hence (3.5) breaks into two independent equations:

$$\varsigma \frac{\partial^2 \mathbf{s}}{\partial t^2} = \mu \nabla^2 \mathbf{s} \tag{3.6}$$

and

$$\boldsymbol{\nabla} p = 0 \tag{3.7}$$

The d'Alembert equation (3.6) describes propagation in the jelly-like medium of the transverse wave, whose velocity c is given by

$$c^2 = \mu/\varsigma \tag{3.8}$$

And (3.7) indicates the invariance of the background pressure:

$$p_0 = \text{const} \tag{3.9}$$

4. LUMINIFEROUS AETHER

Rewrite motion equation (3.5) of jelly-like medium in terms of (3.1) and (3.8):

$$\varsigma\partial_t\mathbf{u} + \varsigma c^2\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{s}) + \boldsymbol{\nabla}p = 0 \tag{4.1}$$

where the elastic term was transformed according to the general vector relation

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot) = \boldsymbol{\nabla}^2 + \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times) \tag{4.2}$$

and the medium incompressibility condition (3.4). Let us define the vector $\mathbf{A}$, $\mathbf{E}$ and scalar φ fields:

$$\mathbf{A} = \kappa c\mathbf{u} \tag{4.3}$$

$$\varsigma\varphi = \kappa(p - p_0) \tag{4.4}$$

$$\mathbf{E} = \kappa c^2\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{s}) \tag{4.5}$$

where κ is an arbitrary constant. Substitute (4.3) - (4.5) into (4.1):

$$\partial_t\mathbf{A}/c + \mathbf{E} + \boldsymbol{\nabla}\varphi = 0 \tag{4.6}$$

With account of (3.1) the medium incompressibility (3.4) gives according to the definition (4.3):

$$\boldsymbol{\nabla}\cdot\mathbf{A} = 0 \tag{4.7}$$

Differentiate (4.5) with respect to t and use in it (3.1) and (4.3):

$$\partial_t\mathbf{E} - c\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{A}) = 0 \tag{4.8}$$

At last, take the divergence of (4.5):

$$\boldsymbol{\nabla}\cdot\mathbf{E} = 0 \tag{4.9}$$

Expressions (4.6) - (4.9) coincide with the respective Maxwell's equations with the Coulomb gauge (4.7) in the absence of the electric charge.

5. INTEGRALS OF MOTION

Equations of motion of a jelly-like medium (4.1), (3.7) and (3.4) are:

$$\varsigma\partial_t\mathbf{u} + \varsigma c^2\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{s}) = 0 \tag{5.1}$$

$$\boldsymbol{\nabla}\cdot\mathbf{s} = 0 \tag{5.2}$$

Let us derive the integral of energy. Multiply (5.1) by $\mathbf{u}$:

$$\frac{1}{2}\varsigma\partial_t\mathbf{u}^2 + \varsigma c^2\mathbf{u}\cdot\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{s}) = 0 \tag{5.3}$$

Integrate (5.3) all over the medium volume and take the second integral by parts. Use in it (3.1). As a result we get the conservation of the energy:

$$\frac{1}{2}\varsigma\partial_t\int[(\partial_t\mathbf{s})^2 + (c\boldsymbol{\nabla}\times\mathbf{s})^2]d^3x = 0 \tag{5.4}$$

Next, let us derive another integral of motion. Take the curl of (5.1):

$$\varsigma\partial_t\boldsymbol{\nabla}\times\mathbf{u} + \varsigma c^2\boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{s})] = 0 \tag{5.5}$$

Multiply (5.5) by $\mathbf{\nabla}\times\mathbf{u}$:

$$\frac{1}{2}\varsigma\partial_t(\mathbf{\nabla}\times\mathbf{u})^2+\varsigma c^2(\mathbf{\nabla}\times\mathbf{u})\cdot\mathbf{\nabla}\times[\mathbf{\nabla}\times(\mathbf{\nabla}\times\mathbf{s})]=0 \tag{5.6}$$

Integrate (5.6) all over the volume. Take the second integral by parts. Use in it (3.1) . This results in a conservation law in the field of the medium torsion

$$\frac{1}{2}\varsigma\partial_t\int\{(\mathbf{\nabla}\times\mathbf{u})^2+[c\mathbf{\nabla}\times(\mathbf{\nabla}\times\mathbf{s})]^2\}d^3x=0 \tag{5.7}$$

Multiply (5.7) by $\kappa^2\varsigma$ and using definitions (4.3) and (4.5), render it into the electromagnetic form:

$$\frac{1}{2}\partial_t\int[(\mathbf{\nabla}\times\mathbf{A})^2+\mathbf{E}^2]d^3x=0 \tag{5.8}$$

The integral in (5.8) is assumed in electrodynamics as the energy of the electromagnetic field.

Substitute (5.1) into (5.7):

$$\frac{1}{2}\partial_t\int[c^2(\mathbf{\nabla}\times\mathbf{u})^2+(\partial_t\mathbf{u})^2]d^3x=0 \tag{5.9}$$

Notice that expressions (5.9) and (5.4) have one and the same structure.

6. THE EXTERNAL FORCE

We are interested in a mechanical system capable to imitate the electrostatic field. For instance, in the case of linear dependence of the stress on the temperature a continual point source of heat creates the stationary pressure field $p\sim 1/r$. Let the input of heat be interrupted. Had we managed to fix the temperature field thus formed we would have received a mechanical analogy of the static electric charge. This can not be done in a solid body because of thermodiffusion. But in an ideal fluid we may have a quasistatic distribution of the turbulence energy. A turbulent fluid is known to possess elastic properties. So, the system sought for can be realized in ideal turbulence [3, 4].

In the elastic model we formally introduce into (4.1) the density $\mathbf{f}(\mathbf{x},t)$ of an external force:

$$\varsigma\partial_t\mathbf{u}+\varsigma c^2\mathbf{\nabla}\times(\mathbf{\nabla}\times\mathbf{s})+\mathbf{\nabla}p-\varsigma\mathbf{f}=0 \tag{6.1}$$

In the thermoelastic model the "external" force corresponds to the gradient of the temperature field: $\mathbf{f}\sim\mathbf{\nabla}T$. In the averaged turbulence a body force arises due to Reynolds "stresses".

A special importance is attached in this system to spherically symmetrical fields:

$$\mathbf{f}=\psi(|\mathbf{x}-\mathbf{x}'|)(\mathbf{x}-\mathbf{x}')/|\mathbf{x}-\mathbf{x}'| \tag{6.2}$$

Any vector function of the form (6.2) is potential. So, with (6.2) equation (6.1) breaks into the solenoidal part (3.6) and potential one:

$$\mathbf{\nabla}p=\varsigma\mathbf{f} \tag{6.3}$$

By (6.3), the pressure field adjusts itself to the field $\mathbf{f}(\mathbf{x})$ of the body force. A unique solution to equation (6.3) is found considering a boundary condition for the pressure field.

7. A PARTICLE AND A CHARGE

We will model a particle of substance by the inclusion into the solid-body substratum of a spherical cavity of the radius R. The pressure on the surface of a cavity should be vanishing. When receding from the inclusion, the pressure tends to the background value (3.9). As a result, we must have for a cavity at $\mathbf{x}'$

$$p=p_0-a\phi(|\mathbf{x}-\mathbf{x}'|) \tag{7.1}$$

where a is the strength of the disturbance center,

$$a\phi(R)=p_0,\quad \phi(\infty)=0 \tag{7.2}$$

The function $\phi(r)$ can be determined from the equation (6.1). With this end, we will consider the process of small oscillation of the cavity in a medium that includes other cavities. Small variation δV of the volume $V = 4\pi R^3/3$ of the cavity leads to the radial displacement of the medium

$$4\pi\delta\mathbf{s} = -\delta V\boldsymbol{\nabla}(1/|\mathbf{x}-\mathbf{x}'|) \tag{7.3}$$

We suppose that a variation $\delta V(t)$ of the volume is coupled with a variation $\delta a(t)$ of the strength of the stress center, the form $\phi(r)$ being conserved. We have from (7.1)

$$\delta\boldsymbol{\nabla}p = -\delta a\boldsymbol{\nabla}\phi(|\mathbf{x}-\mathbf{x}'|) \tag{7.4}$$

Vary (6.1) near the equilibrium (6.3), taking for the sake of simplicity $\delta\mathbf{f} = 0$, and substitute (7.3) and (7.4) into the result:

$$\partial_t^2\delta V\boldsymbol{\nabla}(1/|\mathbf{x}-\mathbf{x}'|) + 4\pi\delta a\boldsymbol{\nabla}\phi(\mathbf{x}-\mathbf{x}') = 0 \tag{7.5}$$

Equation (7.5) implies

$$\phi(\mathbf{x}-\mathbf{x}') \sim 1/|\mathbf{x}-\mathbf{x}'| \tag{7.6}$$

Substituting (7.6) into (7.1), (7.2), we get

$$p = p_0 - a/|\mathbf{x}-\mathbf{x}'| \tag{7.7}$$

$$a = p_0 R \tag{7.8}$$

The disturbance center thus constructed can be taken as a model of a positively charged particle (the proton). Remark that the form (7.6) was obtained considering the process that has no analogy in electrodynamics. With the account of the definition (4.4) we have from (7.7) for the electric charge

$$q = -\kappa a/\varsigma \tag{7.9}$$

By the definition of the charge's sign, it follows from (7.9) that the dimensional coefficient

$$\kappa < 0 \tag{7.10}$$

The mass of the particle can be taken as

$$m = \varsigma V \tag{7.11}$$

Take the divergence of (6.1) accounting for (3.4), (3.1), and substitute (7.7) into the result:

$$\varsigma\boldsymbol{\nabla}\cdot\mathbf{f} = 4\pi a\delta(\mathbf{x}-\mathbf{x}') \tag{7.12}$$

Equation (7.12) describes the generation of the field $\mathbf{f}$ of the body force by a point defect of the strength a.

8. PLASTICITY

Supposedly the defect that creates the perturbation field $\mathbf{f}$ is capable to move freely in the medium. In this event equation (7.12) remains valid so that it is sufficient to take in it the dependence of $\mathbf{x}'$ on time. Relation (6.3) holds no more. Equation (7.12) enables us to determine only the potential component of the field $\mathbf{f}$. Thus, we need an additional equation necessary in order to determine the whole field $\mathbf{f}$. Let us find an equation that describes the evolution of $\mathbf{f}$ in time taking the function $\mathbf{x}'(t)$ for granted. The velocity of motion of the defect is

$$\mathbf{v} = \frac{d\mathbf{x}'}{dt} \tag{8.1}$$

Differentiate (7.12) with respect to time taking into account (8.1):

$$\varsigma\boldsymbol{\nabla}\cdot\partial_t\mathbf{f} = -4\pi a\mathbf{v}\cdot\boldsymbol{\nabla}\delta(\mathbf{x}-\mathbf{x}') = -4\pi a\boldsymbol{\nabla}\cdot[\mathbf{v}\delta(\mathbf{x}-\mathbf{x}')] \tag{8.2}$$

Integrating (8.2) over the space coordinates we find:

$$\varsigma\partial_t\mathbf{f} = -4\pi a\mathbf{v}\delta(\mathbf{x}-\mathbf{x}') + \mathbf{h} \tag{8.3}$$

where $\mathbf{h}$ is a function that is restricted only by the condition

$$\boldsymbol{\nabla}\cdot\mathbf{h} = 0 \tag{8.4}$$

Postulating in (8.3) $\mathbf{h} = 0$, we get a new independent equation

$$\varsigma\partial_t\mathbf{f} + 4\pi a\mathbf{v}\delta(\mathbf{x}-\mathbf{x}') = 0 \tag{8.5}$$

Equations (5.2), (3.1), (6.1), (7.12) and (8.5) give the full description of the system, provided that the function $\mathbf{x}'(t)$ is known.

The motion of defects represents a microscopic mechanism of the plasticity of a solid elastic body. Equation (8.3) corresponds to a convolution of the Prandtl-Reuss model of an elastic-ideal-plastic medium.

9. MECHANICS OF THE DEFECT

Still, the function $\mathbf{x}'(t)$ is not known and it should be determined from the equation of motion of the defect. The translation of a cavity of the volume V is equivalent to the transfer of the mass (7.11) of the medium's material. We will assume that the cavity moves as a classical particle according to the motion equation

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} \tag{9.1}$$

If we know the function $\mathbf{F}(\mathbf{f},\mathbf{s},\mathbf{u},\mathbf{v})$ then the system of equations (5.2), (3.1), (6.1), (7.12), (8.5), (8.1) and (9.1) is closed. Remark that in (9.1) the function $\mathbf{F}$ is taken at the point $\mathbf{x}'$.

In statics the force acting on the center of pressure (7.7) must be proportional to $\boldsymbol{\nabla}p$. We will take it as

$$\mathbf{F} = ka\boldsymbol{\nabla}p \tag{9.2}$$

where k is the coefficient of proportionality. Let us show that (9.2) is consistent with the field equation (8.5). From (9.1) and (9.2) we have the integral of motion of the defect

$$\frac{1}{2}mv^2 - kap = \texttt{const} \tag{9.3}$$

On the other side, let us multiply (8.5) by $\mathbf{F}$ and integrate it all over the medium

$$\varsigma\int\mathbf{F}\cdot\partial_t\mathbf{f}d^3x + 4\pi a\mathbf{v}\cdot\mathbf{F}_{\mathbf{x}=\mathbf{x}'} = 0 \tag{9.4}$$

We have from (9.2) and (6.3)

$$\mathbf{F} = ka\varsigma\mathbf{f} \tag{9.5}$$

Substituting (9.5) in the first term of (9.4) and (9.1) into the second term of (9.4) we may get the integral of motion

$$k\varsigma^2(8\pi)^{-1}\int\mathbf{f}^2d^3x + \frac{1}{2}mv^2 = \texttt{const} \tag{9.6}$$

Substituting (6.3) into (9.6), taking the integral by parts and using in it (7.12) returns us to (9.3).

Expression (9.5) can be generalized adding to it the kinetic term from (6.1) :

$$\mathbf{F} = ka(\boldsymbol{\nabla}p + \varsigma\partial_t\mathbf{u}) \tag{9.7}$$

With (9.7) and (6.1) used in (9.4) the law of conservation of the energy of motion of the defect will look as follows:

$$k\varsigma^2(8\pi)^{-1}\int[(\boldsymbol{\nabla}p/\varsigma + \partial_t\mathbf{u})^2 + c^2(\boldsymbol{\nabla}\times\mathbf{u})^2]d^3x + \sum\frac{1}{2}mv^2 = \texttt{const} \tag{9.8}$$

where equation (9.4) was extended to the case of the interaction of several defects. We suggest everywhere that the energy of self-interaction of the defect is vanishing. Comparing equation (9.8) with (5.9) we see directly that (9.8) is the generalization of the conservation law in the field of the torsion of the medium.

We will get the same result (9.8), if we will add to (9.7) a vector that is perpendicular to $\mathbf{v}$. The general expression for the force acting on a defect that moves in the elastic medium looks as follows:

$$\mathbf{F} = ka[(\nabla p + \varsigma\partial_t \mathbf{u}) - \mathbf{v} \times (...)]_{\mathbf{x}=\mathbf{x}'} \tag{9.9}$$

In (9.9) we indicate explicitly that the field term in the right-hand part of (9.1) should be taken at the point of the defect. It was shown [6] that, at least in the approximation of small velocities v, the exact form of the addition $-ka\mathbf{v} \times (...)$ to (9.7) can be found from the field equations (5.2), (3.1), (6.1), (7.12) and (8.5).

10. MAXWELL'S EQUATIONS

Let us assume instead of (4.5) a new definition

$$\mathbf{E} = \kappa[c^2 \nabla \times (\nabla \times \mathbf{s}) - \mathbf{f}] \tag{10.1}$$

Using definitions (10.1), (4.3) and (4.4), we can render equation (6.1) into the electromagnetic form (4.6). Define the density $\mathbf{j}$ of the electric current by

$$\varsigma\mathbf{j} = -\kappa a \mathbf{v}\delta(\mathbf{x} - \mathbf{x}') \tag{10.2}$$

In terms of (10.1) and (10.2) equation (8.5) looks as the Maxwell's equation

$$\partial_t \mathbf{E} - c\nabla \times (\nabla \times \mathbf{A}) + 4\pi\mathbf{j} = 0 \tag{10.3}$$

Substituting (10.1) and (7.9) into (7.12), we get instead of (4.9)

$$\nabla \cdot \mathbf{E} = 4\pi q\delta(\mathbf{x} - \mathbf{x}') \tag{10.4}$$

11. THE LORENTZ FORCE

Expression (9.9) for the force acting on the defect that moves in the elastic medium has been partly postulated partly substantiated by its consistency with medium's motion equations. The exact form of (9.9) is

$$\mathbf{F} = ka\varsigma[(\nabla p/\varsigma + \partial_t \mathbf{u}) - \mathbf{v} \times (\nabla \times \mathbf{u})]_{\mathbf{x}=\mathbf{x}'} \tag{11.1}$$

Expression (11.1) corresponds to the force acting on the electric charge that moves in the electromagnetic field. Indeed, let us rewrite the definition (10.1) of the electric field using in it (6.1) :

$$\mathbf{E} = -\kappa[\nabla p/\varsigma + \partial_t \mathbf{u}] \tag{11.2}$$

Substituting (11.2), (4.3) and (7.9) into (11.1), we get

$$\mathbf{F} = (k\varsigma^2/\kappa^2)q[\mathbf{E} + \mathbf{v} \times (\nabla \times \mathbf{A})/c]_{\mathbf{x}=\mathbf{x}'} \tag{11.3}$$

The form (11.3) coincides with the respective electromagnetic expression, if we assume for the absolute value of the arbitrary dimensional constant κ:

$$\kappa^2 = k\varsigma^2 \tag{11.4}$$

In terms of (4.3) and (11.2) with the account of (11.4) the integral (9.8) looks as the law of conservation of the electromagnetic energy:

$$\partial_t\{(8\pi)^{-1} \int [\mathbf{E}^2 + (\nabla \times \mathbf{A})^2] d^3x + K\} = 0 \tag{11.5}$$

12. CONCLUDING REMARKS

Thus, classical electromagnetism fits exactly the structure of linear elasticity. The seat and the bearer of the Maxwell's equations appears to be a solid-like medium, which is tough to compression yet liable to shear deformations. Although only some of the features of electromagnetism are realized in the solid elastic medium, all of them can be described consistently in the language of the theory of linear strains and stresses of this medium. In order to map the feature, which is not realized in the bounds of solid medium, it is sufficient to introduce in the dynamic equation the term of an external force. In spite of its simulative character, such a technique is justified by that the source of the unknown force can be attached to a defect, which is really existent in the solid medium.

Macroscopically the electromagnetic substratum possesses the properties of an elastic-ideal-plastic body. The theory of electromagnetism corresponds to the plasticity described in terms of the motion of the defects producing the strained state of the medium.

The mesoscopic mechanism of the substratum elasticity was shown in [3, 4] to be the turbulence of an ideal fluid. It is due to this that the hydrodynamic equation (2.2) can be linearized, and there arises in it the body force (10.1).

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