# When and Where is a Current Electrically Neutral?

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Many textbooks of electromagnetism give an example in which a current-carrying wire is alleged to be electrically neutral when at rest in the laboratory. They then show that the Lorentz contraction of moving charge, demanded by special relativity theory, causes a bunching of positive charge and a thinning of negative charge in the inertial system co-moving with the conduction electrons, with a resulting charge density imbalance and non-vanishing electric field measurable in that system. By a more careful application of special relativity theory, we show, on the contrary, that the wire cannot be strictly neutral in its rest system. Therefore the textbook calculations are in error. A consequence is that special relativity, coupled to Maxwell's field theory, predicts inconsistent force exertions upon a test charge, when analyzed in different inertial systems.

## 1. Introduction

Modern textbooks of electromagnetism have come to rely heavily on the special theory of relativity, even for demonstrating logical consistency. It is rather as if the earlier-invented field theory could not have been correct until the advent of the later-invented relativity theory. Examples are to be found, for instance, in the books of Purcell [1], French [2], and Ohanian [3]. A problem treated in all of these books is the following: Consider two inertial systems, $S$ and $S'$,

$S$ = The laboratory inertial system in which an infinite, straight, current–carrying wire is at rest.

$S'$ = Another inertial system moving parallel to the direction of the wire in such a way that the conduction electrons in the wire are (on average) permanently at rest in it.

Let the relative speed of S′ with respect to S be designated $v$, the drift speed of the electrons, and let an external test charge $q$ be situated at rest in S′ at some small distance from the wire.

First, consider the viewpoint of the S-observer. According to all the above-mentioned authorities, the S-observer, with respect to whom the wire is at rest, judges it to be *electrically neutral*; so he measures zero electric field. This electrical neutrality implies exactly equal densities of plus and minus charge in any subsection of the wire. Because the test charge moves with a non-zero speed $v$ in $S$, the $S$-observer also measures a non-vanishing **B**-field, which exerts on the test charge a magnetic force of magnitude equal to the product $qv$B. This force causes $q$ to move in a measurable way (radially, *i.e.*, perpendicularly to the wire) in $S$.

Next, consider the viewpoint of $S'$. For this observer the test charge is at rest. Hence $v = 0$, and the magnetic force vanishes, even though the magnetic field **B** need not vanish. Whatever the value of **B**, it cannot be measured with this particular "test charge." But we said above that the test charge *moves* perpendicularly to the wire in $S$, which means it must move similarly in $S'$. If the force is not magnetic, what is it? Maxwell seems stumped. But now on a white charger up comes galloping the special theory of relativity with salvation to offer in the form of the Lorentz contraction. Since the positive lattice ions of the wire are moving in $S'$ in the "backwards" direction at speed $v$, the current observed in $S$ and $S'$ is the same (invariant because of the double sign change, motion direction reversed and charge sign altered). Since the lattice plus charges are moving with speed $v$ in $S'$ they are Lorentz contracted by a gamma factor dependent on $v^2$. This means that the plus ions are bunched together a bit, so that their density is increased. The result is a net preponderance of positive charge in the wire, so that it ceases to be electrically neutral. Consequently a non-vanishing radial **E**-field is present in $S'$, due to Lorentz contraction of the plus ions. This **E**-field acts on the charge $q$ and (when the calculation is made) is found to exert *half* the force needed to account for the radial motion observed in $S$.

The other half of the needed force is rationalized as follows: We said that the wire was electrically neutral in S. But the electrons were moving in $S$, and we did not allow for their consequent Lorentz contraction, which would have resulted in a charge imbalance similar to that just noted in $S'$. Why didn't this happen? Good question. If the moving plusses contract in $S'$, why don't the moving minuses contract in $S$? For a very simple reason: The authorities say so. We have their word for it. They insist that the wire is electrically neutral in $S$, and that can only mean that the Lorentz contraction (of the moving electrons) does not occur there. But doesn't the Lorentz contraction *always* occur, as a matter of kinematics, for all moving objects? Well ... how could it fail to occur for the electrons? In only one way: That is, if *in the rest system* $S'$ of the electrons they undergo a "Lorentz expansion" [by a factor (one over gamma)] that exactly and providentially cancels their Lorentz contraction (by gamma) in $S$. And that, believe it or not, is exactly what happens, so the authorities tell us with one voice. How do we know? Because of the electrical neutrality of the wire in $S$. And how do we know that? Because they say so. It always comes down to that and no more.

Very well, in $S'$ we have it on authority that there is this *Lorentz expansion* of the electrons that reduces their charge density, producing an additional excess density of plus charge of just such a magnitude as to supply the missing other half of the electric force on the test charge, needed to explain its motion in both $S$ and $S'$. So, we not only have the word of authority, we have the *right answer*, to justify both the Lorentz expansion and the original claim of charge neutrality in $S$. Right answers are very important to relativists and to physicists in general. Once they sink their teeth into a right answer they never look back.

## 2. Is the Wire Indeed Electrically Neutral in its Rest System?

Consider a third inertial system S″, defined as

$S''$= An inertial system moving in the same direction as $S'$ at about half-speed, $v/2$, so that the plus charges of the lattice move in one direction at exactly the same magnitude of speed as the minus charges of the electron cloud move in the opposite direction.

By symmetry, we see that the motions of plus charges in one direction precisely match those of minus charges in the opposite direction, so their Lorentz contractions (if, in agreement with the Lorentz transformation, relative motion indeed causes a Lorentz contraction) are identical. We do not need any authorities to tell us about this. We can see it without expert help: the symmetry speaks for itself. In the wire the plus charges are precisely as dense as the minus charges, so there is exact electrical cancellation, with the result that zero electric field is sensed in $S''$. *It is S″ and only S″ in which there is true electrical neutrality.*

This can only mean that the authorities are wrong about S. And they are vociferously wrong, very sure of themselves. Purcell[1], for instance, assures us that, in the laboratory system S, "A test charge at rest near this wire experiences no force whatever." But we have shown here that $S$ is not the inertial system of true charge neutrality. There is only one inertial system of true charge neutrality, and that is $S''$. The Lorentz expansion in S′, which was logically deduced from the assumption of charge neutrality in $S$, is therefore hokum. So there goes the famous *right answer*, and all the tortured reasoning that led to it. Actually, a rational mind would feel relieved by this species of outcome. But modern physics has never been guided by such. It is the Jesuitical mind for which the rewards of tenure are reserved. And that is the type of mind to which the whole of special relativity, replete with its Lorentz contractions and expansions, appeals.

## 3. Consequences for SRT-Maxwell Consistency

What is the implication of this recognition that $S''$, not $S$, is rightly judged to be the inertial system of true electrical neutrality by anyone who believes in the Lorentz contraction? Suppose a test charge $Q$ is at rest in system $S''$. Because of the rigorous electrical neutrality there, $Q$ senses no electric field. Also, because $Q$ is at rest in $S''$, the test charge velocity $\vec{\mathbf{v}}$ in the Lorentz force law, $(Q/c)\vec{\mathbf{v}}\times\vec{\mathbf{B}}$, vanishes, so $Q$ can sense no magnetic force there. So, in $S''$ the test charge $Q$ detects zero total force. In other inertial systems, however, it sees nonvanishing current (hence a nonvanishing $\vec{\mathbf{B}}$-field) and must detect some magnetic force, since $\vec{v}\neq 0$, and no doubt some electric force as well, thanks to the Lorentz contraction. (The two types of force will not cancel each other. That they work in the same direction is proven by the textbook calculations [1, 2, 3] that show electric force in one system acting in the same direction as magnetic force in another system.) Hence SRT fails the test of consistency of force predictions in different inertial systems. This seems to be a general failure of field physics, since it applies also to alternatives to Maxwell's version of field theory that have been examined. `

## 4. Conclusion

The symmetry argument just given seems to me conclusive regarding the electrical non-neutrality of a current-carrying wire in its rest system. It also exposes the Gedanken gymnastics to which relativity authorities have had recourse, beginning with the "Lorentz expansion," which accomplishes nothing except to rid mankind of the Lorentz contraction, when and only when the latter is unwanted (is *de trop*, as the French say). Suppose an actual experiment were resorted to in the final stages of desperation, betokening a belated return of physics to its empirical roots. Then some hypothetical real physicist would sweep the cobwebs from his laboratory and test with great sensitivity whether or not a stationary current-carrying wire produces an external electric field. If so, the present analysis, based on the assumed occurrence of the Lorentz contraction, would be confirmed (provided other effects did not intervene to deprive the observations of meaning). If not, the physical occurrence of the Lorentz contraction would be counterindicated. Neither outcome would appeal to the authorities I have cited, nor to those who have been higher-educated to think like them. Therefore I predict it will be a long time before anybody tries the experiment. And still longer before a first-line physics journal accepts the results for publication. The fact that we have been forced here to recognize a logical inconsistency lurking in the combination of special relativity and field theory is perhaps our most significant finding. It was this inconsistency that all the mental athletics [1, 2, 3] were invented to avoid. Back to the drawing board.

## References

[1] E. M. Purcell, **Electricity and Magnetism**, Berkeley Physics Course, Vol. 2, 2nd Ed., pp. 192-199 (New York: McGraw-Hill, 1963, 2004).

[2] A. P. French, **Special Relativity**, The MIT Introductory Physics Series, pp. 256-262 (New York: Norton, 1966).

[3] H. C. Ohanian, **Classical Electrodynamics**, pp. 219-229 (Boston: Allyn and Bacon, 1988).