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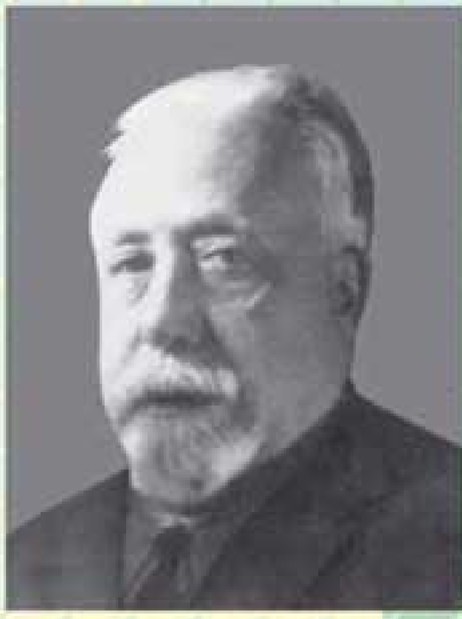

Italian Mathematics Between the Two World Wars

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Preface

During the first decades of the last century Italian mathematics was considered to be the third national school due to its importance and the high level of its numerous researchers. The decision to organize the 1908 International Congress of Mathematicians in Rome (after those in Paris and Heidelberg) confirmed this position. Qualified Italian universities were permanently included in the *tour* organized for young mathematicians' improvement. Even in the years after the First World War, Rome (together with Paris and Göttingen) remained an important mathematical center according to the American mathematician G. D. Birkhoff.

Now, after almost a century, we can state that the golden age of Italian mathematics reduces to the decades between the 19th and the 20th century. In the centre of interest stood the algebraic geometry school with Guido Calstelnuovo, Federico Enriques and Francesco Severi acting as key figures. Their work led to an almost complete systematization of the theory of curves to the complete classification of the surfaces and to the bases of a general theory of algebraic varieties. Other important contributions came from the Italian school of analysis. Its main representative was Vito Volterra – an outstanding analyst with a strong interest in mathematical physics – who produced important results in real analysis and the theory of integral equations and contributed to the initiation of functional analysis.

Guiseppe Vitali, Guido Fubini and Leonida Tonelli were well known in the integration theory and the calculus of variations. At the beginning of the century Tulli Levi-Civita's scientific adventure started: He became one of the most recognized and esteemed Italian mathematicians abroad. There also was a strong connection between the authority in the scientific disciplines and the role they could play for the future and the modernization of Italy. In chapter 1 we describe this thrilling season of Italian mathematics.

The golden age however, is only the prologue of our history. We will focus our attention to the years between the two World Wars. The turning point during those years was marked by the Great War – it was an epochal change. Nothing remained as it was before. The ingenuous hope that the war could simply be a gap of time and afterwards one could come back to the *belle époque* were illusions. In chapter 2 we analyze the changes in Italian mathematics.

From a strict mathematical point of view the twenties and the thirties were less stimulating for Italy than the previous ones, but from the context of the whole century they were attractive on other sides: The social and political situation suddenly changed with the raise of the fascist regime (chapter 4). Also structural scientific aspects changed with the creation of new institutions which should play an important role in the development of Italian science and mathematics for the rest of the century. In chapter 3 we describe the birth of UMI (Italian Mathematical Union) and of CNR (National Research Council); in the chapters 6 and 8 we deal with the consecutive presence of INAC and of Severi's INDAM.

Naturally the next step is to consider whether there was any link between the changes in the political and scientific spheres and if these influenced the organization of the mathematical research, its contents and its quality level.

We can describe the main problems dealt with by our analysis in a more detailed manner. In the period between the two World Wars the leading actors of Italian mathematics were rather the same as before. Perhaps the most relevant difference was the arrival of Mauro Picone on the scene. His presence was particularly noticeable in a numerical and applied perspective and also in the ideas that guided the creation and the development of INAC – an absolute novelty in the international mathematical panorama. Even if the names were rather the same, their role had changed. Volterra's brilliant career was stopped by fascism, and so the old liberal generation was marginalized by the new government. Severi became piece by piece the head of the mathematical group. We dedicate the chapter 3 and 5 to this leadership change. Like Volterra, Severi was an outstanding mathematician and a broad-minded man, and his personality was charismatic, even if different in the coherence of his behaviour patterns.

His leadership should remain till the Second World War. There should be some tensions – see chapter 6: the alternative of the CNR – but in the thirties Italian mathematics grew with a sufficient continuity (chapter 7). It needed another external event, the tragical experience of the Second World War to induce a new discontinuity in the Italian mathematical life (chapter 8).

The mathematical research itself was always at a good level. The influence of Italian researchers on algebraic geometry was a strong one. Enriques contributed some important historical studies to his research in this field. The "old lion", Volterra, wrote a last relevant chapter in his scientific career by analyzing population dynamics. *Tonelli's Fondamenti di Calcolo delle Variazioni* were published and his esteem – about all for the use of direct methods – was high in the mathematical world. Picone's influence has already been described. Some other young brilliant scholars joined the already acknowledged researchers: Renato Caccioppoli, Lamberti Cesari, Francesco Tricomi and others. Another young man, Bruno de Finetti, increased the suspense of the probabilistic studies and anchored a research directed towards economic and social applications. Not to forget the undoubted authority of Levi-Civita and the role he played by corresponding with Einstein and many younger colleagues.

Nevertheless this survey makes a clear statement: for Italian mathematics the golden age was on the retreat. Its potential did never return after the First World War. Not quite a crisis but rather the difficulty to maintain the previously excellent level and to continue in playing a role in originality and creativity. On the contrary the orthodox respect towards a still young tradition and the acceptance of a level just achieved seemed to prevail.

The new abstract and algebraic languages did not speak Italian any more. They were born in situations where the weight of tradition was lower and we could speak of a decline of Italian mathematics with respect to its level 30-40 years before compared to the new languages that were developing in the other countries between the wars.

This is the point where the two histories – the Italian and the mathematical one – met. The conditions and the progress of Italian mathematics are analyzed by focusing on both the inner and the external influences. Is there any link between the establishment of a dictatorial regime and the decline of Italian mathematics? Can we find this possible link in the most repressive fascism facts – the 1931 oath and the 1938 racial laws – or rather in its politics towards science and particularly in its attitude in favour of the applied sciences?

In the following pages we will try to give an answer to these questions by analyzing the most important works of the Italian mathematicians living in the period, the life of the Italian mathematical community, some correspondence of the most representative members of it and their positions outside the research or educational fields. But our interest goes beyond the historical facts of the period between the two World Wars and its influences on the present problems. So the previous questions have a "modern" version too. Can the scientific world accept – and at which conditions – a confrontation with the political power or is it necessary to avoid these contaminations? Which are the possibilities of the political sphere to orient the trends of the scientific developments? And in the particular case of mathematics? How can a political will overcome the constraints imposed by the economic structure? In the light of the episode of the oath and the silence of too many mathematicians at the sight of the racial laws, which are the ethic and political responsibilities of a researcher?

As one can see, the questions are numerous. We just hope to give a contribution in answering them through the analysis of Italian mathematics between the two World Wars.

Angelo Guerraggio
Pietro Nastasi

Contents

Chapter 1

Prologue

1. The Risorgimento generation

The history of modern Italy starts in 1860. In that year the various nation-states into which the Italian peninsula had been politically and administratively divided were unified in a process called *the Risorgimento*. Under the leadership of Piedmont (the north-western region of Italy on the French border, whose capital is Turin) and its hegemony, a remarkable idealistic and democratic impulse with significant popular support led to the unification of the country. Yet, a number of uprisings, two wars of independence against Austria (1848–9 and 1859, the latter of which was fought with crucial help by France), and intense diplomatic activity, were still necessary to achieve this goal.

Some of the mathematicians whom we shall shortly present, who will figure prominently in this prologue, participated in the military mobilisation for these wars of independence, particularly in the years 1848 to 1859[1]. Enrico Betti was a volunteer in a student battalion from the University of Pisa. In 1848, Luigi Cremona participated in the defence of Venice, which had rebelled against Austrian rule and was given the rank of corporal and later that of sergeant. Francesco Brioschi participated in 1848 in the insurrection of Milan against the Austrians and in 1870 in the storming of Rome.

In 1860 the peninsula had not yet been completely unified. The Veneto region (in the north-east of Italy) was still under the sway of Austria. It would only be annexed to the new Italian state after the third war of independence (1866). In particular, Rome was still ruled by the papacy. In this case, public, political and diplomatic issues were of much greater complexity. It would only be in 1870 that the Italian government could overcome the temporal power of the papacy. On this occasion it exploited the opportunities offered by the difficulties faced by the Vatican's erstwhile ally, France, in the aftermath of the Franco-Prussian war, the fall of Napoleon III and the end of the Second Empire. The annexation of Rome by the Italian state would usher in a long period of difficulties in its relationship with

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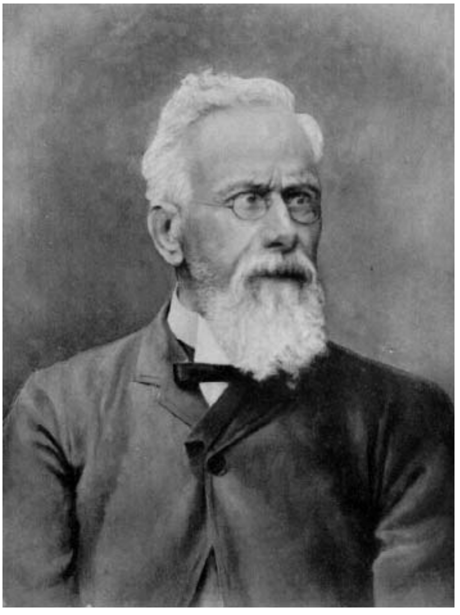
[1] As for the commitment of Italian mathematicians during the Risorgimento, see: *Universitari Italiani nel Risorgimento* (ed. by L. Pepe), Bologna, CLUEB, 2002.



Enrico Betti

Brioschi-statue

Catholicism, and would cause at the same time a cooling in the Franco-Italian alliance and Italy's entry into the sphere of influence of the central European states.

Vittorio Emanuele II thus became the first king of Italy. The capital of the newly founded state, initially established in Turin, was subsequently moved to Florence following the Italo-French agreements of 1864. Actually, this was a step towards making Rome the capital, as it was considered the historical and ideal centre of the Country. Rome was finally made the capital of Italy in 1870.

The next fifty years, before Italy's entry into the First World War, can be characterized as full of intense efforts to weld the country into a *nation*, with infrastructures, *standards* of living and vital statistics as close as possible to the more developed nation states. The initiatives taken to modernize agriculture and to industrialize the economy were held back markedly by the great differences between different regions of the country. The situation in the South, the so-called *southern question*, was particularly critical from the social and economic viewpoint. The north of Italy benefited from a much more solid economic and social base. In the last two decades of the 19th century, a bourgeoisie began to develop here which would progressively influence city styles and customs. The consolidation of a bourgeoisie would be accompanied by a similar development of the working class, mainly concentrated in the Milan-Turin-Genoa triangle.

The mathematicians were in the front-line of this process of nation building, occupying significant political and administrative positions. Just to mention some of the names cited above, both Brioschi and Betti would become parliamentarians, senators and undersecretaries in the Ministry of Education (in the years 1861–2 and 1874–6, respectively). Cremona was appointed the Minister for Education in 1899, even if only for one month. Brioschi, in particular, was a key protagonist in establishing an education system that reflected the outlook of the new entrepreneurial bourgeoisie, which was consolidating in the north of Italy in opposition to a lazy and passive landowning class. The expectations of this emerging class that scientific progress and its technological fallout would nurture and accelerate the industrial development of Italy are paralleled in the mindset of scientists and the culture of scientific research. In particular, they were reflected in the perspectives mathematicians envisaged for their teaching and research. This common world view formed the motivating factor behind the establishment of the Polytechnic of Milan, founded in 1863 by Brioschi with the intention of creating a class of qualified technicians indispensable for the rise of Italy's industrial initiatives.

In short, during the first half-century of its existence as a unified nation state, Italy went through a period which was in many respects similar to that of other European countries. Unlike them, however, it had to race to make up for its late start because of the backwardness and uneven progress of the vast underdeveloped areas surrounding its limited industrial base. For a time, Italy was blessed with political stability accompanied by a gradual, albeit not straightforward and not altogether peaceful, widening of its democratic base. It survived the economic crisis of the last quarter-century. Later it could not resist the siren call of colonial adventure in East Africa. Its agricultural and industrial development gathered pace over the last years of the 19th century with constantly increasing rates of production which sometimes attained considerable heights before 1908. By a remarkable coincidence the boom characterizing the decade before this date also involved mathematics, given that 1908 was to be the year of the fourth International Congress of mathematicians in Rome.

We can now introduce Italian mathematics over the first half-century more systematically by describing its structure and protagonists starting with the generation of the *Risorgimento*. This era precedes a period on which we will focus later. We have already mentioned Enrico Betti (1823–1892) and Francesco Brioschi (1824–1897) in terms of their participation in political and military events. Together with the young Felice Casorati (1835–1890) both these mathematicians visited the universities of Göttingen, Berlin and Paris in 1858 to learn of the most significant advances in European mathematics both from the scientific and organizational point of view. They were able to meet, amongst others, such distinguished mathematicians as R. Dedekind, P. C. L. Dirichlet,

Felice Casorati

B. Riemann, L. Kronecker, K. Weierstrass and C. Hermite. Tradition has it that this voyage marked the birth – almost from nothing – of Italian mathematics. The theory that the *Risorgimento* also caused a new starting point in mathematics naturally derived from a patriotic ideology which emphasized the view that unification had set wings to the aspirations and enthusiasm of the best minds in the country, including science[2]. Actually, it cannot be argued that the mathematical school had sprung up from nothing (nor simply through a fact-finding mission). Even so, extreme as it may appear, this view can still be taken as a suitable starting point.

The collaboration between Betti and Brioschi can be considered the true driving force behind this rebirth of Italian mathematics, which was to be extremely fruitful both in terms of organization and quality of research. Betti's meeting[3] with Riemann in Göttingen and their intense cooperation during the latter's stay in Pisa (from 1863 to 1865) was a turning point. Following Riemann's death in 1866, Betti became a reference point for all European mathematicians interested in further investigating this German mathematician's works. Betti was a physicist-mathematician and the author of significant research (which was also translated into German) into the theory of potential and elasticity.

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[2] In the inaugural speech of the International Congress of mathematicians in Rome, in 1908, Volterra asserted: "Hence, I would not be surprised if, following scientific development, there were a sudden transformation in the Italian thought, brought about by its quick progress and dissemination, and by the new enriching features it took in the years following the period of the political Risorgimento".

[3] About Betti, the mathematical school of Pisa, and more in general about Italian mathematics after the Unity, see U. Bottazzini, *Va'pensiero. Immagini della Matematica nell'Italia dell'Ottocento*, Bologna, Il Mulino, 1994.

In this field his most well-known contribution is the so-called *reciprocity theorem*. This held that, if for an elastic solid one can consider two states of equilibrium consequent on the action of two different force systems, the work carried out by the first system (with respect to the deformations involving the second) is equal to the work of the second system with respect to the deformations involving the first. He also investigated complex variable functions, elliptic functions, and had even earlier looked into a number of issues concerning algebra and algebraic topology. It is no coincidence that Poincarè would coin the expression: *Betti numbers* as a means of measuring the different connection orders in *n*- dimensional figures. Betti was amongst the first in Europe to realize the value of Galois and Abel's research for the resolution of algebraic equations, arriving at original results which subsequently were rediscovered and praised by Hermite. Finally, Betti was appointed director of the *Scuola Normale di Pisa*[4] (from 1865 to his death), making the first contribution to the establishment of what was to become the most important research centre in Italy.

We have already mentioned Brioschi's[5] "political" involvement and his contribution to the education of a ruling class in Italy which would step over the limits drawn by an exclusively legal – literary schooling. In his case, from a more strictly mathematical point of view, it is difficult to single out a particular discipline with which to identify him. Brioschi's research ranged from algebra, analysis, geometry and mechanics, to mathematical physics. In analysis he made important contributions in the field of elliptic functions and differential equations, and particularly in that of differential invariants (associated with singling out the class of differential equations referable to constant coefficients equations). However, it was in algebra where he made his most lasting contributions, with innovative research into the theory of determinants and algebraic forms. By the time he embarked on his "European trip" in 1858, Brioschi was already a highly regarded mathematician. His book on *La teorica dei determinanti e le sue principali applicazioni* (published in 1854) had already been translated into French and German by 1856. His reputation derived in particular from his resolution of fifth and sixth degree algebraic equations (after Galois had demonstrated that it was impossible to solve for radicals equations that were greater than the fourth degree). Brioschi's works accompanied others results by Hermite and Kronecker for the solution of general equations of the fifth degree through elliptic functions, and all three mathematicians were accorded merit for their solution of sixth degree equations through hyperelliptic functions. Finally – as in the case of Betti – one must mention Brioschi's efforts in founding and then successively promoting the *Annali di Matematica pura e applicata* destined shortly to become one of the most prestigious journals in the sector.

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[4] The *Scuola Normale*, founded in 1813, prepared the future school teachers in the Napoleonic Kingdom of Italy. Napoleon's fall caused its *closing* (as well as that of other Napoleonic institutions) in 1814. The Grand Duchy of Tuscany reopened it in 1846, always with the same objective. After the Unity of Italy, besides this "old vocation", it developed as a research centre, different from the university, and as a training centre for future researchers.

[5] On F. Brioschi see U. Bottazzini, Francesco Brioschi and the "Annali di Matematica", in C.G. Lacaita, A. Silvestri (eds.), *Francesco Brioschi e il suo tempo (1824–1897)*, Milano, Angeli, 2000, pp. 71–84; A. Brigaglia, Brioschi, Cremona e l'insegnamento della Geometria nel Politecnico, ibidem, pp. 403–418.

During their trip to the European capitals of mathematics in 1858, Betti and Brioschi were accompanied by the young Casorati[6] (who was then only 23, having been one of Brioschi's students). In his case, it is easier to single out a specific area of research to discuss: this was complex analysis. Casorati was the one who disseminated in Italy the ideas of Cauchy, Riemann and Weierstrass, also by publishing a monograph titled *Teorica delle funzioni di variabili complesse* (1868), containing original results which often preceded similar discoveries usually wrongly attributed to Weierstrass, Mittag-Leffler and Picard.

Aside from Betti, Brioschi and Casorati, few other names need to be mentioned to give a fairly complete picture of the first generation of mathematicians in the recently unified Italy. Among these, the most important were Luigi Cremona (1830–1903) and Eugenio Beltrami (1836–1900).

The former is considered the founder of the Italian school of algebraic geometry[7]. His commitment, and the role he intended to play in the field of geometry, can already be seen in his *Prolusione* published in 1860 at the University of Bologna where he wrote very clearly about the absence of "modern" geometry in Italy although it was already an essential part of teaching in France, Germany and Great Britain. Cremona moved from Bologna to the Polytechnic of Milan (where he held a course of static graphics) and then to Rome, to the School of Engineering, where the appeal of his teaching among students can be considered one of the first indications that the study of mathematics was coming into its own in Italy. In particular, two of his monographs[8] (published in 1861 and 1867) marked the peak of projective studies and introduced a method for the geometric treatment of numerous algebraic problems, in the belief that synthetic geometry, with its clear supremacy, was the only system that could ensure the application of a methodology both rigorous and intuitive. Cremona's main contribution (in which he showed he could appreciate the ideas already expressed by Riemann and the German school) was the introduction of the concept of the *birational transformations* of planes and space. These are a generalization (later called *cremonia transformations*) of the classic concept of linear transformations, and can be expressed through rational functions, usually invertible with functions of the same type. It was by using this concept, as well as the analysis of algebraically invariant properties with respect to birational transformations, that the study and classification of algebraic curves and surfaces starts. This research, in particular his synthetic study of cubic surfaces, won him, together with Charles Sturm, the *Steiner prize* of the *Academy of Sciences of Berlin* in 1866 (considered at the time the most prestigious award in the field). He received this prize again in 1874 without participating in any preliminary examination, in recognition of all his publications on geometry.

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6 On F. Casorati see U. Bottazzini, Alla scuola di Weierstrass, in *Va' pensiero*, op. cit., pp. 195218; A. Gabba, Il carteggio Brioschi-Casorati, in C.G. Lacaita, A. Silvestri (eds.), *Francesco Brioschi e il suo tempo (1824–1897)*, op. cit., pp. 419–429.

7 His great interest for the history of geometry and his many international relationships can be appreciated in his correspondence, being printed by a research group coordinated by G. Israel.

8 See L. Cremona, Introduzione ad una teoria geometrica delle curve piane, *Mem. Accad. Sci. Bologna*, 12 (1861), pp. 305–436; Preliminari di una teoria geometrica delle superficie, *Mem. Accad. Sci. Bologna*, n.s., 6 (1867), pp. 91–136 e 7 (1867), pp. 29–78.

GLI

ELEMENTI D'EUCLIDE

CON NOTE,
AGGIUNTE ED ESERCIZI

AD USO DE' GINNASI E DE' LICEI

PER CURA DEI PROFESSORI

ENRICO BETTI E FRANCESCO BRIOSCHI.

FIRENZE.
SUCCESSORI LE MONNIER.

1867.

Beltrami[9] is mainly remembered for his research on differential geometry, undoubtedly influenced once again by Riemann's ideas and their dissemination during the latter's Italian sojourn[10]. Riemann's studies kindled his interests in non-Euclidean geometry and the creation of their first model on the pseudosphere. Beltrami's research can hence be situated between differential geometry and mathematical physics. With the publication of his monographs: *Saggio di interpretazione delle geometria non – euclidea* (1868) and

[9] As a young man, Beltrami was very active, given his Risorgimento ideals. As a result of these in 1856 he had to suspend his studies at the University of the Pavia before graduation and start working as a humble clerk. After the Kingdom of Italy was founded, Brioschi had him appointed without a public examination (on Cremona's recommendation) as visiting professor in algebra and analytical geometry at the University of Bologna in 1862. Beltrami could at last devote himself to research and teaching, swinging for two decades between the Universities of Pisa, Rome and Pavia. He finally decided to settle in Rome, where he succeeded Brioschi as president of the *Accademia nazionale dei Lincei*. On Beltrami, see R. Tazzioli, *Beltrami e i matematici "relativisti". La meccanica in spazi curvi nella seconda metà dell'Ottocento*, Bologna, Pitagora Editrice, 2000.

[10] Due to health reasons, Riemann spent the winter of the year 1862 in Sicily. From October 1963 until July 1965 he stayed in Pisa.

Teoria fondamentale degli spazi di curvatura costante in the following year, Beltrami's work took its rightful place in the history of non-Euclidean geometry. These works, together with Beltrami's proof of the coherence of Gauss, Lobatchevsky and Bolyai's hyperbolic geometry, lend credibility to their reassessment of the privileged status which Euclidean geometry had hitherto enjoyed.

So far, we have dealt with Betti, Brioschi, Casorati, Cremona and Beltrami. We can also add Giuseppe Battaglini (1826–1894) to this group. Battaglini is essentially a geometrician, a self-educated mathematician whose main concerns were the more analytical "neo-geometry" of Plucker and the geometric theory of algebraic forms by Clebsch. We owe to Battaglini also the Italian translation of Todhunter's classic manual on Calculus and the publication, from 1863, of the *Giornale di Matematiche* (known precisely as "Battaglini's Journal"), which promoted the education of young researchers through the dissemination and explication of major research programs and their results[11]. However, our list of mathematicians stops here. It was this small group which worked towards the mathematical modernization of the country by taking as its model the most advanced European situations. These close links with other countries would remain a constant feature in all the programs established in this period, together with a strong public and political commitment by mathematicians, an almost inevitable consequence of the great ideals and the fervent aspirations expressed in previous decades. Hence, mathematicians can be numbered amongst the most impassioned intellectuals committed to finding solutions for the many problems which afflicted the Italian education system in the period following unification. Foremost amongst these problems were the great differences between the Italian regions.

The development of the Italian education system can be seen from the right perspective when one realizes that it was only in 1877 that the first two years of primary school became compulsory (after a long struggle against the most intransigent sectors of the Catholic church which sought to maintain family prerogatives). Indeed, at the time Italy was united, about 70% of the population was illiterate and this percentage would only decrease slowly in successive decades (from 69% to 62% in the 1871 and 1881 censuses respectively) reaching the threshold of 50% only at the beginning of the 20th century. In Europe a similar situation could be encountered only in Spain (and an even worse one in the Russian Empire). By the mid -19th century the other European countries had just under 58% illiterate people (Austrian Empire, Belgium, France) or even less (Great Britain 25%, Prussia 20%, Sweden 10%). Given that this proportion of educated people form the base of the educational pyramid, one should not be surprised by the small number of university students. Indeed, there were little more than 12,000 in 1871 and they doubled over the next 30 years, with a particularly accentuated progression in the period 1881 to 1901 also because of the prolonged economic crisis at the time (as always one of the variables with the greatest impact on the length of schooling). About one-third of university population attended the polytechnics or scientific degree courses. Here, amongst the teaching staff, the presence of mathematicians was prepon-

[11] A collection of his letters, from 1854 to 1891, can be found in M. Castellana and F. Palladino (eds.), *Giuseppe Battaglini*, Bari, Levante ed., 1996.

Luigi Cremona

derant for the half-century this prologue is dealing with. In 1881, for example, mathematicians held 69 positions, which was slightly less than half of the total number of positions assigned to the scientific faculties.

The *boom* in mathematics (in terms of students numbers and quality of the curricula mentioned above) can be appropriately explained in terms of the initial situation (at the beginning of unification), which we described as being extremely inadequate, making what happened later appear extremely positive by comparison. The same, in particular, can be said for any type of research which did not require great expense or investments and which could therefore develop rapidly even in a country with severe social problems. Also fundamental was the cohesion of the small group of mathematicians introduced above and the atmosphere in Italy during the last decades of the 19th century. At the time, Positivist thought was in the ascendancy and it informed the values of the growing bourgeoisie. The mathematical and physical sciences (not to mention economics) were seen as instruments for its affirmation, as was the development of a prevalently technical education in opposition to the literary and artistic curricula considered as antiquated and typical of a backward social organization. The mathematization of the social sciences also met with a certain measure of success because of the widespread belief in the objectivity of economic laws, contrasted with any attempt to subject economics to moral or ideological priorities.

2. The golden age. The Italian school of algebraic geometry

The virtues of the Risorgimento generation (Betti, Brioschi, etc.) are to be seen, however, in terms of the creation of the conditions which made possible the *second generation* to transform Italian mathematics into a great power, second only to France and Germany. This corresponds to what we can call the *golden age* of Italian mathematics. It is of greater interest to us because it was during this period that some of the future protagonists of the years between two world wars began their careers. The levels of excellence that this group attained set the standards the following generation would have to measure up to.

First we deal with the school of algebraic geometry (which we already mentioned when we spoke of Cremona). The *Premio Bordin* of the *Académie des Sciences*, was awarded to Italian mathematicians[12] on two occasions in 1907 and in 1909 for research in this field, and the prestige of the Italian school is reflected in the epithet: *italienische Geometrie* attributed to algebraic geometry.

One student of Cremona's[13] was Giuseppe Veronese[14] (1854–1917) who worked in Berlin and Lipsia in 1880 and 1881 where he met Felix Klein. It was certainly an important encounter: the structural approach of the German mathematician encouraged Veronese to study the foundations of non-Archimedean geometry and the projective geometry of hyperspaces, to the extent that he would be recognized as one of the fathers of projective geometry in *n*-dimensional spaces. Battaglini, too had a student, Enrico D'Ovidio (1843–1933) who, after arriving in Turin, began to work with the young Corrado Segre (1863–1924). Their collaboration would bring, either by their own efforts or through those of their students, Italian algebraic geometry to full maturity. It was in this school that the study of algebraic surfaces would develop to become the greatest achievement of the Italian mathematical tradition.

Segre[15] started his career with an outstanding dissertation on hyperspatial quadrics and some studies regarding their geometry, following the concepts of Veronese. Soon, these projective techniques would be placed "at the service" of other research allowing him to 'import' and develop A. Brill and M. Noether's program regarding the geometry of an algebraic curve, or in other words, the study of the properties of algebraic curves which are invariant with respect to birational transformations. In addition to these studies which represented the core of his scientific efforts, Segre also investigated such fields as: the ruled surfaces in hyperspaces, enumerative geometry, algebraic topology, and the initial elements of a theory of algebraic surfaces (with the intention of rigorously demonstrating Noether's theorem for the existence of a smooth birational

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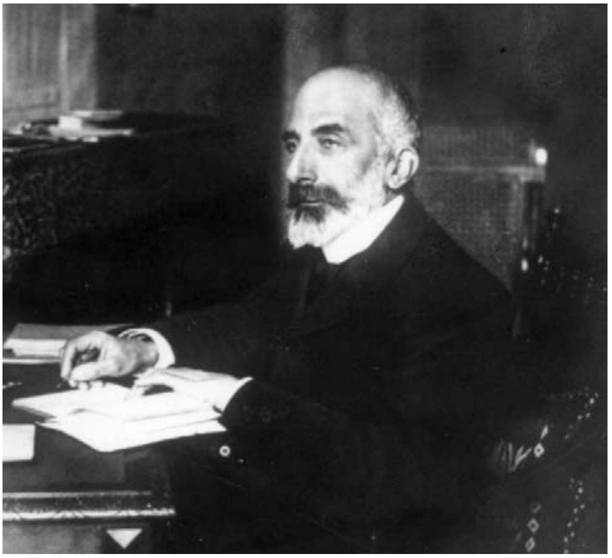
[12] The prize was awarded in 1907 to Federigo Enriques and Francesco Severi and in 1909 to Giuseppe Bagnera (1865–1927) and Michele de Franchis (1875–1946).

[13] Among other pupils of Cremona, we should cite at least Eugenio Bertini (1846–1933).

[14] Veronese graduated in Rome in 1877. From 1897 to 1900 he was Member of Parliament, and later town counsellor in Padua and (from 1904) Senator.

[15] Part of this correspondence (in particular 270 letters and postcards exchanged with Castelnuovo from 1891 to 1898, almost all regarding his early studies on the geometry over a surface) has been published and analysed in P. Garzio, "Singolaritá e Geometria sopra una superficie nella corrispondenza di C. Segre a G. Castelnuovo", Archive for History of Exact Sciences, 43 (1991), n. 2, pp. 145–188.



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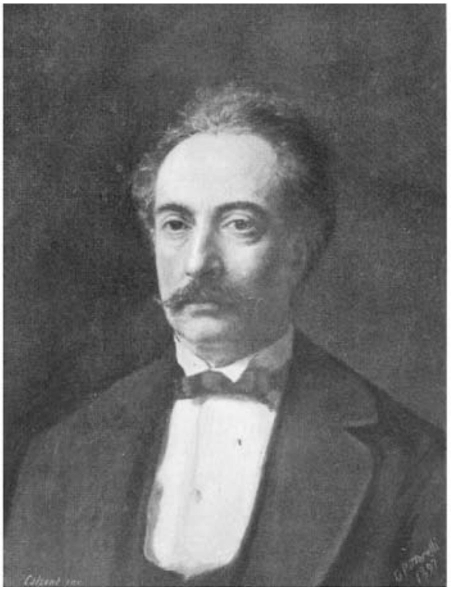

Eugenio Beltrami

model for every algebraic surface) and the varieties described by families of projective spaces. His research on the relations between surfaces submerged in a projective space and partial differential equations would make him the most distinguished mathematician in Italy in the fields of differential geometry of curves, surfaces or varieties submerged in a projective space without a metric structure.

Of equal import was the incisiveness of his teaching in his famous courses on “Superior geometry” held between 1888 and 1924. The radical contraposition between geometry and analysis which can be seen in Cremona’s “purism”, was in some respects overcome whilst still remaining within a framework highlighting the supremacy of synthetic methods. Their elegance and productiveness would become a model for the entire mathematical edifice. Students of Segre’s were Guido Castelnuovo (1865– 1952), Federigo Enriques (1871–1946) and Francesco Severi (1879–1961)[16]. The Italian school of algebraic geometry is generally identified with them. It is worthwhile having a closer look at their role and activities: they debut brilliantly at the turn-of-the-century but we will find them again – maybe in other fields of academic endeavour – also in the 1930s. Severi in particular would become one of the key figures of Italian mathematics between the two world wars.

Immediately after graduating in Padua under Veronese, Castelnuovo began postgraduate study in Rome in 1886 where he heard Cremona’s lessons. The following year he went to Turin, where he began what was to be his lasting and friendly collaboration with Segre. His research mainly concerned algebraic curves, for which he elaborated a rigorous proof of Riemann-Roch’s theorem and the formula of maximum genus, with the subsequent determinations of maximum genus curves. This result generalized a discovery made by G. Halphen and M. Noether, but previously valid only for three dimensional projective spaces.

The techniques used by Castelnuovo to elaborate his proof were original and still striking today for their simplicity and elegance. The turning point came a few years later, in 1891 when he was given a professorship in geometry at the university of Rome. Henceforth he focused on a new study of algebraic surfaces, but we should not neglect results such as those obtained in 1901 when he formulated the first rigorous proof of the theorem for which each cremonian flat transformation can be seen as the product of quadratic and linear transformations. In Rome, Castelnuovo met Enriques, with whom he was to write fundamental works in the history of the theory of algebraic surfaces. Until then the points of reference had been E. Picard’s transcendent and M. Noether’s geometric approach. The former had studied simple integrals of total differentials of the first kind annexed to an algebraic surface, coming to the result that these only existed on particular surfaces, for example hyperelliptic ones. The latter introduced the invariants constituted by the geometric genus p_g, the linear genus $p^{(1)}$ and the numerical genus p_a. It could be $p_a = p_g$, as always happens in the case of curves, or it could be $q = p_g - p_a \neq 0$. Cayley had verified the second possibility in the case of the ruled ones. Since then it had been hypothesized that q was null, with the exception of the ruled ones. However, already in 1891, after studying certain particular types of surfaces, Castelnuovo had built

[16] We should not fail to mention Gino Fano (1871–1952).

the first example of an irregular unruled algebraic surface for which $p_a = p_g$. In 1896 he made his most important discovery[17], with Enriques already working at his side, formulating a famous counterexample, where he extended the Riemann-Roch theorem of curves and the determination of the criterion of rationality. The condition valid for the curves – their rationality is linked to a null genus – was generalized: a surface is rational if and only if: $q = P_2 = 0$, where q is the surface irregularity index and the plurigenus P_n is a new birational invariant, introduced by Enriques.

Enriques had graduated in 1891 at Pisa university. He had wanted to undertake postgraduate study in Turin with Segre. Instead, he managed to find a position in Rome. Here he immediately changed from the team led by the then elderly Cremona to the one of the promising Castelnuovo, who would direct him towards the study of algebraic surfaces. Already in 1893 and in 1896, when he had been in Bologna for two years, Enriques published two fundamental memoirs where he laid the basis for the organic theory and the classification of algebraic surfaces. Enriques would never completely abandon this field of research, unlike Castelnuovo, who would practically stop publishing on algebraic geometry in the early years of the 20th century. However, within this field he would soon dedicate significant attention to elementary mathematics (developed also thanks to his personal acquaintance with Felix Klein) and to the philosophy and history of mathematics.

His meeting with Castelnuovo, their friendship (further strengthened when Castelnuovo married Enriques's sister) and their scientific plans have been documented by an exceptional collection of correspondence containing almost 700 letters written by Enriques to Castelnuovo between 1892 and 1906[18]. Their personalities appeared to be complementary: Enriques was exuberant and possessed an extraordinary power of intuition. Often he would appear already certain of an outcome before securing it with successive formulation. But he was less interested in proofs and their rigour; he was impatient and often superficially read articles by colleagues. In contrast, Castelnuovo was perhaps less brilliant but original as well. He also sought nonetheless to refine and channel his brother-in-law's genial intuitions into more suitable and productive outcomes. Their twenty year collaboration would develop a new method of formulating the theory of algebraic surfaces leading to a particularly simple classification, with the elimination of all the special cases. Consequently, the study of algebraic surfaces would involve now only those of curves lying on the surface. Amongst these, particular attention was dedicated to linear systems and to nonlinear continuous systems (existing only on irregular surfaces, for which the difference $p_g - p_a$ is positive). In two notes[19] written in 1914 Enriques presented almost definitive results on the theme of classifications: the surfaces were subdivided into classes of birational equivalents according to the values assumed by the plurigenera and the geometric genus. In the same year, the publication of a long article[20], written together

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[17] G. Castelnuovo, Alcuni risultati sui sistemi lineari di curve appartenenti ad una superficie algebrica, *Mem. Soc. It. Sci. XL*, 10 (1896), pp. 82–102.

[18] The whole correspondence is published in U. Bottazzini, A. Conte, P. Gario (eds.), *Riposte Armonie. Lettere di Federigo Enriques a Guido Castelnuovo*, Torino, Bollati Boringhieri, 1996.

[19] F. Enriques, Sulla classificazione delle superficie algebriche e particolarmente sulle superficie di genere lineare $p^{(1)} = 1$, Note I e II, *Rend. Acc. Lincei*, 23 (1914), pp. 206–214 e 291–297.

with Castelnuovo, in the *Enzyklopädie der Mathematischen Wissenschaften* was to represent the crowning achievement of their research and the official recognition of its importance by the international mathematical community.

We shall discuss Severi in the coming pages, both to illustrate his research in algebraic geometry and to reveal his rich and complex personality, together with his cultural and philosophical interests. His political role between the two world wars as the undisputed leader of mathematicians will also be examined. Even younger than Enriques, Severi graduated in 1900 at the university of Turin under Segre, with an outstanding thesis on enumerative geometry. Two years later he was in Bologna working with Enriques, who encouraged him to investigate the theory of algebraic surfaces. Severi will win a professorship already in 1905, first in Parma and soon after in Padua. From 1903 onwards, in particular, he concentrated on irregular surfaces (after the counterexample by Castelnuovo who had proven that the conjecture according to which ruled surfaces were the only irregular surfaces was groundless). There is already a glimpse – always within the school – of a strong and original personality, with a marked attention towards topological and functional aspects.

In particular, Severi "retrieved" transcendent methods[21] as a means of determining the link between irregular surfaces and surfaces endowed with total differential integrals of the first and second kind. It is thus proved – also thanks to an algebraic-geometric proof of Enriques, which Severi will not see fit to aprove, though – that irregular surfaces and those with Picard's integrals of the first kind are the same set, and the existing relation between q and the number of integrals of the first and second kind (linearly independent) is stated. At that time, his relationship with Enriques was excellent and their collaboration continued: in 1907 both mathematicians, as mentioned above, received the *Premio Bordin* for their research on the classification of hyperelliptic surfaces by finishing G. Humbert's work. In particular, Severi is awarded the prize *Medaglia Guccia* at the International Congress of Rome, in 1908, by a committee formed by M. Noether, E. Picard and C. Segre. He tries to extend those results and methods, that had proved so effective in the case of surfaces, to the study of varieties. In the same year he is appointed member of the *Accademia dei Lincei*, that in 1913 will award him the *Premio Reale*. In 1912, Severi and Enriques collaborate again, publishing a work on the foundations of enumerative geometry, which B. L. van der Waerden would consider of fundamental importance as a rigorous basis for algebraic geometry. In this work a solid basis was given to enumerative methods and in particular to Schubert's *principle of the conservation of number*, according to which, if an enumerative problem had in the general case a finite number of solutions, then the same number of solutions (unless they become infinite) can also be found in particular cases.

We shall now leave Severi and his studies on algebraic geometry to briefly deal with differential geometry. In reality, these two fields of research are not so distinct (although for clarity's sake we discuss them as if they were) and the protagonists in-

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[20] G. Castelnuovo, F. Enriques, Die algebraischen Flächen vom Gesichpunkte der birationalen Transformationen aus, in *Enzyklopädie d. Math. Wissensch.*, III (1914), 2, 1, C, pp. 674–768.

[21] One can see C. Houzel, *La geómetric algebrique*, ed. Blanchard, Paris, 2002.

volved were often the same. However, two names are new to the scene described above: Luigi Bianchi (1856–1928) and Gregorio Ricci-Curbastro (1853–1925). Both graduated at the university of Pisa and spent a period of postgraduate work in Göttingen together with Klein. It is not the first time that we encounter this German mathematician; in fact, Klein played a similar role to that of Riemann with the first generation of Italian mathematicians, confirming the appeal that German mathematics and its organisational methods exercised over their Italian counterparts.

Bianchi spent his whole mathematical career in Pisa, where he was to become the director of the *Scuola Normale* between 1918 and 1928. He also wrote[22] on subjects such as analysis, algebraic number theory and one of his most important first contributions was his activity as a writer of treatises. Whole generations of Italian mathematicians would study from his book *Lezioni di geometria differenziale.* Of equal merit was his teaching work in algebra, with monographs (on finite groups and the theory of Galois, on continuous groups, and on the arithmetic theory of quadratic forms) which disseminated in Italy the arithmetic techniques formulated by the German school, in particular, by L. Kronecker, R. Dedekind, H. Weber and D. Hilbert. Regarding differential geometry, in his doctoral thesis of 1879, Bianchi introduced the so-called "complementary transformation" for surfaces submerged in the ordinary space. The result was applied in the theory of partial differential equations and in particular in nonlinear equations which we today term *sine – Gordon*. A few years later, the Swedish mathematician A. E. Bäcklund generalized Bianchi's transformation, and in turn Bianchi integrated Bäcklund's theory with the so-called "permutability theorem", which allowed their transforms to be found using only algebraic and derivative calculations (after Bäcklund's transforms of an initial pseudo-spherical surface are all known). Other notes examined the general theory of Riemann's spaces. In a paper published in 1898, Bianchi with greater simplicity demonstrated the result (already known to Riemann) according to which n-dimensional spaces with constant and equal curvatures can be mapped isometrically to each other. In a successive work[23] (dated 1902) he obtained the famous *Bianchi identities*, satisfied by the covariant derivatives of Riemann's four index curvature symbols. However, despite the use of the covariant derivatives, as L. Pizzochero observed[24], Bianchi was substantially unfamiliar with the methods of absolute Calculus.

The true "Master" of this field in Italy was Ricci-Curbastro, who on his return from Göttingen, finally settled in Padua. Here, in the decade from 1885 to 1895 he studied the calculus of tensors, finding his main source of inspiration in the invariant theory of Riemann's varieties, developed in research carried out by E. B. Christoffel, R. Lipschitz and of course, B. Riemann himself. As early as 1886 one of his notes introduced what he would later call *covariant derivatives* of a function (without, to tell the truth, quoting either Lipschitz or Christoffel, who had both already analysed the same operation). This expression appeared for the first time in a work published in the following year: Ricci

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22 His writings, collected in *Opere* (10 volumes), were published in 1952 (Roma, Cremonese).

23 L. Bianchi, Sui simboli di Riemann a quattro indici e sulla curvatura di Riemann, *Rend. Acc. Lincei*, 11 (1902), pp. 3–7.

24 L. Pizzocchero, Geometria differenziale, in S. Di Sieno, A. Guerraggio, P. Nastasi, *La Matematica Italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, Milano, Marcos y Marcos, 1998, pp. 321–379.

Curbastro studied multiple index covariant systems, applying to them the classic law of transformation after changes in coordinates. In a memoir of 1888, with the emergence of systems with many counter-variant indices, he practically announced the birth of absolute Calculus, with the change of the ordinary procedures of differential calculus proposed so that formulas and results keep the same form, whatever system of variables is used. The expressions *absolute differential Calculus* and *absolute systems* appear in the memoir, *Méthodes de calcul différential absolu et leur application,* written together with his student Tullio Levi- Civita (1873–1941) and published in 1900 in *Mathematische Annalen* on F. Klein's invitation. The memoir expounds Riemann's geometry, with the new terms, and physical applications (to elasticity, to electrodynamics, etc.). The usefulness of the new methods would only be realized after some time. International aknowledgment for Ricci-Curbastro would arrive only on the eve of the First World War. In 1913, Einstein would adopt absolute Calculus as the basic mathematical language for the theory of general relativity which he was developing at the time.

The infinitesimal methods of differential geometry were 'exported' to projective geometry. Finally Guido Fubini[25] (1879–1943) dedicated some notes to the construction and the analysis of metrical structures in projective spaces and, in particular, to the description of the metrical structure induced by a hermitian form over a complex projective space (of any dimension). We shall discuss Fubini again when we deal with the Italian school of real analysis. The line element ds^2 in the projective space is still today indicated with his name (together with that of the German mathematician E. Study).

3. The golden period. The mathematical physics

We should now give due recognition to one of Ricci-Curbastro's students, Levi-Civita, one of the most creative Italian mathematicians in the first half of the century. We will mention him often in this book.

Tullio Levi-Civita[26] (1873–1941) graduated in Padua, where he received his entire education, if we except a brief period of postgraduate study in Bologna (where he met Enriques, becoming his lifelong friend) and some teaching in Pavia. In 1918 he was appointed at the University of Rome as professor of Superior analysis and successively Rational mechanics. Levi-Civita was in essence a mathematical physicist whose interests ranged from electromagnetism to analytical mechanics, from celestial mechanics to Relativity, from hydrodynamics to the theory of heat. Throughout his work, as observed by L. Dell'Aglio and G. Israel[27], there was a close correlation between innovation and tradition. He explored new and original perspectives without weakening his steadfast attachment to a method which oriented analytical investigation according to results emerging from the preliminary use of geometric models.

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25 His writings are collected in three volumes in *Opere* (Roma, Cremonese, 1961–1963).

26 His writings, edited by the *Accademia dei Lincei*, are gathered in six volumes in *Opere matematiche* (Bologna, Zanichelli, 1954–1970).

27 See the article by Dell'Aglio-Israel in *La Matematica italiana tra le due guerre mondiali* (A. Guerraggio ed.), Pitagora ed., Bologna, 1987 .

In the first years of his career, Levi-Civita expanded the research on stability according to Liapounov, ; in 1901 he developed his theorem on stationary movement and began his study of the theory of wakes in hydrodynamics that he would later deepen more fully in Rome. In the case of celestial mechanics he focussed on the classic problem of the three bodies, starting from P. Painlevé's results and deducing a regularization of motion equations (he was able to predict and therefore to eliminate their singularities). Levi-Civita's first essential contribution to absolute Calculus dated back to 1896[28]. It was also the first time Ricci's Calculus was adopted in a context outside of metric differential geometry, to solve a problem of analytical mechanics. The memoir confronts the issue, already raised by K. Appell in 1852, of the mutual transformability of "two systems of dynamic equations with the same number of variables". The problem, in the case of forces independent of speed, was to be re-examined by Painlevé, who "by an opportune modification" had revealed that it could be applied to the determination of all systems (called *correspondents*) that have common trajectories. Hence, the invariant character of the problem emerged, and it was reduced to the singling out of all the correspondents of a given system. This suggested quite naturally that Ricci's Calculus could be applied. It was by using this Calculus that Levi-Civita came to the conclusion, for the most general pair of correspondent dynamic systems (having the same number of degrees of freedom and not stimulated by other forces) that "*n* perfectly determinate types" were possible.

Another, but no less significant proof of the fruitfulness of Ricci's Calculus was provided in a memoir[29] published shortly afterwards (1899), containing research on the types of potentials that can be made to depend on only two spatial coordinates. The analytical evaluation of the problem from Riemann onwards had led to differential systems, which were so complex as be intractable. Levi-Civita took as his starting point the observation that all those potentials that allow "infinitesimal transformation in themselves" were independent of one coordinate. From here Levi-Civita went on to consider the infinitesimal transformation to allow by the Laplace $\Delta_2\, y = 0$ equation, finding five categories of infinitesimal transformations to which corresponded five types of binary potentials. *Ricci's Calculus* was used at this point to show (also following advice by F. Klein) that the binary potentials found in this manner are the only ones possible. In the same year – as we already said – F. Klein invited Ricci-Curbastro to arrange a whole and systematic explanation of the calculus of tensors, to be published in *Mathematische Annalen*. In the writing of the article, later considered as the *manifesto* of tensorial algebra, Ricci-Curbastro let the young Levi-Civita, whose contribution would be fundamental especially for its applications to mathematical physics, join in. Tensorial relationships are not modified by the change in the coordinate system, therefore their language is particularly useful to express the properties that are naturally independent of the chosen reference.

The works cited above, published at the end of the century, were written by an extremely young Levi-Civita. Over the same period, the reputation of another Italian

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[28] T. Levi-Civita, Sulla trasformazione delle equazioni dinamiche, *Ann. Mat.*, 24 (1896), pp. 255–300.

[29] T. Levi-Civita, Tipi di potenziali che si possono far dipendere da due sole coordinate, *Atti Acc. Torino*, 49 (1899), pp. 105–152.

mathematician, Vito Volterra[30] (1860–1940) was nearing its zenith. Volterra was the undisputed *leader* of Italian mathematicians in the first decades of the new century. He graduated in Pisa in 1882 after having studied under Betti. He was then appointed professor at the University of Pisa and successively Turin. In 1901 he moved definitively to Rome to become Beltrami's successor. This move to the capital increased Volterra's public profile and his involvement in positions of increasing responsibility in determining the scientific and cultural policies of the nation. At the beginning of the century, Volterra was elected president of the *Società Italiana di Fisica*. In 1905 the Italian Prime Minister, Giolitti, appointed him to the Senate. In 1907 he founded the SIPS (*Società Italiana per il Progresso delle Scienze*) – becoming its first president – on the model of similar societies already existing in France, England and other industrialized countries. His objective was of establishing a meeting point among scientists from different backgrounds as well as giving them a chance to disseminate their research. We shall deal with Volterra again later in this book. His presence influenced 50 years of Italian scientific research and makes us possible to deal not only with mathematical physics and analysis, but also with mathematical economics and mathematical biology.

The age difference with Levi-Civita (who represented an interesting balance between innovation and tradition) is less than 15 years, but it was enough to place Volterra in a more classicist "19th century" perspective, where one feels the powerful pull of a strongly cohesive research, capable of describing the complexity of macroscopic physical phenomena by using only a few basic equations. As regards mathematical physics, the most important contributions, over the turn-of-the-century, regarded the propagation of light in birefractable equipment, the movements of the terrestrial poles (or, to be more precise, the movements of the Earth's surface with respect to the Earth's rotational axis), hereditary phenomena and what in modern terms is called *dislocation theory*. This last subject, which Volterra called *distorsioni (distortions)*, constitutes part of his theory of elasticity which, according to Klein had become a "national issue" for the Italians[31]. In 1901, L. G. Weingarten had proven that a state of tension can exist in an elastic body without being subjected to external forces (occupying a non-simply connected dominion). The first example that comes to mind is that of a ring which after being cut transversally, removing a slice of matter, is then re-attached. Volterra's studies, which were to have a significant impact on the theory of elasticity in non-simply connected dominions, began from this point. His findings, the classification and theory of distortions which derived from his research, were collected in a sizeable memoir dated 1907 (published in the *Annales scientifiques de l'Ecole Normale Supérieur*) "Sur l'équilibre des corps élastiques multiplement connexes". Other authors would continue this research, including

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30 Volterra's *Opere matematiche* were issued in five volumes in 1962, edited by the *Accademia dei Lincei*.

31 Among the several works on elasticity, we would like to point out *Introduzione alla teoria matematica dell'elasticità* (Turin, Fratelli Bocca, 1894) by the Neapolitan Ernesto Cesàro (1859–1906), who died tragically at sea while trying to save his son in danger. Particularly influenced by Beltrami in his works on mathematical physics, Cesàro is still remembered today for his works on analysis and for his classic method of summation of series, and stands out also for his results in the field of intrinsic-geometry and of asymptotic arithmetic.

Carlo Somigliana (1860–1955) who was a friend and colleague of Volterra's, a descendant of Alessandro Volta and the author of a general theory of distortions.

Volterra deserves a final mention as a physicist and mathematician for his researches on hereditary phenomena or on systems with memory, quoted above. His studies, starting from the observation that the deformations of an elastic body depend also on previous deformations, investigated those bodies which maintained the memory of their history and whose future state subsequently depended on their present as well as previous states. Once again an interesting convergence emerged between experimental data and mathematical instrumentation: the equations are no longer differential but integral-differential equations (which would be applied in particular to electrostatics and hereditary elasticity) given that heredity is expressed by functions that are integral with respect to time, of linear combinations of deformation components.

4. The golden age. The analysis.

By briefly referring to Fubini and in particular Volterra, we have been able to bring the study of analysis into the discussion. In Italy, this third great discipline of 19th-century mathematics was developed particularly in Pisa. The leader of this school was Ulisse Dini (1845–1919), who graduated under Betti in 1864 with a thesis on differential geometry. His name[32] is universally known among mathematicians and students of mathematics for his theorem of implicit functions and for the "Dini derivative", in which the customary passage to the limit is generalized through the notion of upper or lower limits. Also deserving mention are his studies on numerical and trigonometric series, complex variable functions, and differential equations. But the greatest impact that Dini had on the Italian mathematical scene (and not only the Italian) was due to the publication of his monograph: *Fondamenti per la teorica delle funzioni di variabili reali* (1878), in which he developed his rigorist program. For the objective was not to discover new results so much as to place already known ones on more solid foundations by completing them and specifying the dominion of their validity.

Giuseppe Peano (1859–1932), from Turin, was another protagonist of the rigorist turning point. His contribution was to present the axioms of arithmetic, to give some counterexamples – some of which were ruthless in their simplicity, with which he ridiculed unsubstantiated hypotheses, mistakes and approximations (some contained in the most widely used manuals) and to obtain a precise and general formulation of a number of fundamental notions of analysis (limits, area of a region, Taylor's formula, partial derivatives, maxima and minima for functions of several real variables, etc.). He is a particularly well-known mathematician[33]: his importance in the axiomatization of math-

[32] Dini's *Opere*, edited by the *Unione Matematica Italiana*, were published in three volumes in 1955 (Roma, Cremonese)

[33] See H.C. Kennedy, *Life and work of Giuseppe Peano*, Dordrecht, D. Reidel Publ. Comp., 1980. Peano's *Opere scelte* have been published in three volumes, edited by the *Unione Matematica Italiana* (Roma, Cremonese).

ematical theories is undeniable; his non – recursive definition of a derivative of order *n* is still used today in some research on analysis and *non-smooth* optimization. His contribution (in the second half of the 1880s) to the theorem of existence for the differential equation $y^1 = f(x,y)$, proven with the sole condition of the continuity of function *f*, is specifically mentioned in many manuals. Equally well known is his role in devising a system of axioms for vector spaces described in his monograph dedicated to the dissemination of Grassmann's ideas. Peano's Curve (1890) more than 100 years on, still remains one of the most amazing and least intuitive conclusions which deductive rigor has brought to set theory and has played a truly significant role in the history of the concept of dimension: it is possible to find a curve, expressed by two continuous functions $x = f(t)$ and $y = g(t)$, which goes through all the points of the unity square whilst t varies over the interval [0.1]. In other words it is not always possible to enclose a continuous curve within an arbitrarily small area.

Indeed, it was on the issue of scientific rigour that Peano engaged in a lively dispute in 1891 with Segre (and Veronese). Segre had backed a less rigid and absolute position by distinguishing the period of *discovery* from that of *rigour*. Peano instead retorted tersely that a theorem can be considered as discovered only when it is proven and that in the absence of the only – absolute – rigour that mathematics comprehends, one may write poetry, but not mathematics. Peano had another, much harsher, dispute with Volterra. Mathematical content[34] concerns the motion of … a cat, allowed to fall in a vacuum upside-down and more generally the internal movements of a body (and the possibility of modifying their orientation) that Volterra had analysed in specific reference to the terrestrial globe subjected to the action of internal forces. Paradoxically in this case, Peano stood accused for the lack of rigour and originality of his conclusions. The dispute increased his isolation. Given the almost forgone outcome of his battle in favour of mathematical rigour, Peano gradually left his research in analysis and began to develop his ambitious plan of reconsidering all of the propositions of classical mathematics, breaking them down and analysing them in their smallest parts so as to be certain that they contained nothing less and nothing more than what was necessary. The same propositions were rewritten using combinations of algebraic and logical signs which leave no scope for misunderstanding and allow their precise and succinct formulation.

In referring to Volterra we can return to Pisa, which we have depicted as the main centre of Italian analysis. Dini's influence on the young Volterra can be seen in the latter's early but famous contribution of 1881 at the young age of 21. Volterra was engaged in the process of completing the Riemann integration theory. One of the main issues of interest were the so-called two *fundamental theorems* of Calculus, that is, the study of the relationships between the operations of derivation and integration. It was here that Volterra devised the now classic example of a function derived in an interval, with a limited but not integrable derivative. At this point, Volterra's research horizons widened beyond the strictures of a rigorist program. Also thanks to Betti and his competence in physics, Volterra was attracted by the possibility of applying analytical tools,

[34] See A. Guerraggio, Le Memorie di Volterra e Peano sul movimento dei poli, *Archive for History of Exact Sciences*, 1984, pp. 97–126.

of course in a sophisticated and adequate manner, to the exigencies of the problem to be faced.

These were the characteristics which we can find in his work on functional analysis. Volterra can rightly be considered one of the founding fathers of this discipline and its independent development, although his pioneering ideas would attract greater appreciation in other countries (in France, for example, thanks to the attention and sensitivity displayed by Hadamard) with slightly different characteristics. We have already spoken of Volterra's 'classicist' outlook when dealing with his research in physical mathematics. Whilst using an abstract language in functional analysis, which was very distant from concrete applications, Volterra still kept his sights on "practical" objectives, such as real problems in physics or other mathematical issues. It would be these "practical" issues that suggested the specific abstraction to implement and which constituted a means of validating the significance of the formalization adopted. His first notes on functional analysis were published at the end of the 1880s. Within a few years, Volterra had introduced the concept of a functional with its associated calculus (up to its development using Taylor's polynomial) and carried out his first research about linear functionals on a given functions space. Actually, he did not use the term *functional* (which would be suggested later by Hadamard) but the term *function of a line* to indicate a real number which depends on all the values taken up by a function $y(x)$ defined over a certain interval, or the configuration of a curve. A functional can be considered as a limit case, for $n \to +\infty$, of a function with n variables. In this manner, the first coherent research was carried out in spaces of infinite dimensions and the whole edifice of classical analysis was generalized to some specific functional spaces. The evolution of such an extension, starting from n-dimensional spaces, was highlighted and took on both an explanatory and reassuring role at the same time. Hence the derivative of a functional (defined on the set $C[a,b]$ of continuous functions over a given interval) is what today we would call a *directional derivative*, or a Gâteaux-Lévy directional derivative. This is obtained by passing from an initial value f_0 to an incremented one: $f_0 + \varepsilon h$, making ε tend to 0 and hence reducing to the customary concept of derivative for a real function (adopting a procedure well-known to Calculus of variations). Volterra is not so interested in studying the functional properties of 'his' derivatives, so much as their actual calculus. And in defence of his approach he reminded those who accused him of giving a too specific definition (with respect to the ensuing "differential according to Fréchet"), such as Hadamard and especially Fréchet, that maximum generality is not the ultimate value to be sought after, but rather the most adequate generality for the problem being dealt with[35]. One should remember that Volterra's first results took place at the end of the 19th-century and that M. Fréchet's *thesis* is dated 1906. Although his initial works still considered specific functional spaces, they already did so from the perspective of general theory. Hence, they would enable and encourage unifying studies of metrical and topological structures.

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[35] Fréchet would not give up either. Still in 1965, in a letter to P. Lévy from the 30th July (published in *Cahiers du Séminaire d'Histoire des Mathématiques*, 1980, n. 1), he clarified that "si je considère que Volterra a réalisé un grand progrès en donnant au moins une définition de la différentielle d'une fonction dont l'argument est une fonction, d'autre part, je considère que sa définition est *mauvaise*".

Volterra's other well-known contribution in this period were integral equations, inserted for the first time into a general theory, later taken up and developed by E. Fredholm, D. Hilbert and others. Volterra investigated integral equations of the first and second kind with a triangular kernel. Here too, the procedure for their resolution was accompanied by the formulation of the principle of the *passage from discrete to continuous*, for which an integral equation of the first kind is the limit case (for $n \to +\infty$) of a system of n algebraic equations in n unknowns.

Giulio Ascoli[36], Cesare Arzelà[37] and Salvatore Pincherle (1853–1936) all graduated in Pisa. The first two names can be seen in every text on functional analysis for their studies on the concept of equicontinuity and the extraction of a converging subsequence from a sequence of equilimited and equicontinuous functions. After graduating, Pincherle studied in Pavia (with Casorati) and Berlin where he studied under the guidance of K. Weierstrass[38]. His stay in Germany is fundamental for an understanding of how his research developed. Pincherle is considered another pioneer of functional analysis thanks to his theory of analytic functions. The remark that each of these functions can be singled out from a countable infinity of parameters, which could be interpreted as its coordinates, led Pincherle to investigate functions spaces of infinite dimension and the abstract study of the linear functionals acting on these spaces. He sought to create a calculus for these functionals similar to the already well known one for the functions of a complex variable. Over the next few decades, these concepts would be developed along different pathways to an extent which was unthinkable at the turn of the century. Instead, the route taken by Pincherle would not be as well trodden, as he himself would serenely come to recognize.

After his brief stay in Berlin, Pincherle moved definitely to Bologna, that would become, together with Pisa, a new important research centre in analysis. The most representative exponent of the school in Bologna was Leonida Tonelli (1885–1946), whom we shall encounter as one of the foremost protagonists of Italian mathematics in the period between the two world wars[39]. He had studied at Bologna under Arzelà and Pincherle, graduating in 1907. His academic career as full professor would begin only after the war for a number of reasons (first at Bologna and later at Pisa). Nevertheless even before 1915, Tonelli had written a number of very important works, numbered among his most significant, in the field of real analysis and Calculus of variations. In 1908 he published a note[40] on the length of rectifiable continuous curves with particular reference to the case in which the functions representing the curve are absolutely continuous. In the

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36 G. Ascoli (1843–1896) graduated at the *Normale* in Pisa in 1868. Then he taught at the *Polytechnics* in Milan.

37 Also C. Arzelà (1847–1912) graduated in Pisa, at the *Normale*, in 1869. Later, he taught at the Universities of Palermo and Bologna. His *Opere*, in two volumes, have been issued in 1992 (Roma, Cremonese) and edited by the *Unione Matematica Italiana*.

38 Pincherle's *Opere scelte*, in two volumes, edited by the *Unione Matematica Italiana*, were published in 1954 (Roma, Cremonese).

39 Tonelli's *Opere scelte* were issued in 1961 (Roma, Cremonese) and edited by the *Unione Matematica Italiana*.

40 L. Tonelli, Sulla rettificazione delle curve, *Atti Acc. Sci. Torino*, 1908.

following year, he published a note[41] where in generalizing the integration formula by parts to the functions of two variables, he provided a criterion for the integrability (according to Lebesgue) of a measurable function $f(x,y) \geq 0$ which admits a pair of successive integrals. It can be affirmed that this article completed the well-known result proven by Fubini in 1907 according to which the double integral (assuming it existed) of $f(x,y)$ can be calculated by two successive simple integrals, independently of the order of integration.

Fubini, whom we have already mentioned for the originality of his studies in projective differential geometry, was another leading figure in the Italian school of real analysis. He is mainly remembered for his theorem on double integrals but he was also the author of other important works in the theory of integration, the minimum principle, automorphic functions and integral equations.

But let us return to Tonelli. His fundamental memoirs on Calculus of variations were published in 1911, 1914 and 1915[42]. Calculus of variations took its rightful place in functional analysis by the systematic use of *direct methods*, already used in particular cases by B. Riemann, D. Hilbert, J. Hadamard, H. Lebesgue, C. Arzelà etc., based on the notions of compactness and semicontinuity (generalizing the definition given by Baire for real functions). It was through direct methods that Tonelli proved some theorems of the existence for the so-called simplest problem in Calculus of variations, avoiding the passage through Euler's equation and hence avoiding difficulties about the calculation (and the existence) of the solution of a boundary value problem, the strong limitation imposed on the functional class by the consideration of differential equations, the privilege given to the relative extrema and then the search for suitable sufficient conditions.

Giuseppe Vitali (1875–1932) was the other main exponent of the school of Bologna, even if he graduated in Pisa (after having studied in Bologna under Arzelà and Enriques)[43]. The year 1905, in particular was a "magical" one in terms of his scientific endeavours. After having proven the necessary and sufficient condition for Riemann integrability of a limited function over a limited interval (depending on the measure of the set of its discontinuity points), in the same year Vitali published a series of notes in which he proved the so-called Lusin's theorem on the almost continuity of measurable functions, giving the famous example of non-measurable sets (according to Lebesgue). Moreover, he characterized the integral functions of not necessarily limited functions by inventing the term, of *absolutely continuous* functions (and studying the class of these functions in relation to those of bounded variation). Many of these results were more or less obtained over the same period by H. Lebesgue. Nevertheless, they were obtained

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41 L. Tonelli, Sull'integrazione per parti, *Rend. Acc. Lincei*, 1909.

42 L. Tonelli, Sui massimi e minimi assoluti nel calcolo delle variazioni, *Rend. Circolo Mat. Palermo*, 1911, pp. 297–337; Sur une méthode directe du calcul des variations, *C. R. Acad. Sci. Paris*, 1914, pp. 1776–1778 and pp. 1983–1985; Sur une méthode directe du calcul des variations, *Rend. Circolo Mat. Palermo*, 1915, pp. 233–264.

43 Vitali's *Opere sull'Analisi reale e complessa*, edited by the *Unione Matematica Italiana*, were published in 1984 (Roma, Cremonese); the publication of the letters addressed to him would follow (edited by M.T. Borgato and L. Pepe).

wholly independently. At this junction one should remember that Vitali was unable to find a university position and for many years was forced to teach in high schools in distant locations removed from the customary channels of scientific communication. At the same time, Lebesgue could also complain that at Poitiers, where he taught from 1906 to 1910, he was not able to consult any Italian journal. This may explain the partial overlap of Vitali's results with those of Lebesgue, without diminishing the originality and value of his research, in particular that regarding absolutely continuous functions. In this instance, Vitali's priorities were clear, not so much because he introduced the term or for his generalization to the functions of two variables, but because of the central position he accorded to such a concept in his theory of integration.

Pisa was also where Eugenio Elia Levi studied. He was born in 1883 and died in 1917 in the war[44]. With him we introduce the topic of complex analysis which we touched upon when discussing Pincherle. His brother[45] Beppo (1875–1961) was also a mathematician and at the same time as Vitali engaged in a brief controversy with H. Lebesgue regarding the cogency of some proofs by the latter. Nevertheless, he would mainly concentrate his efforts on algebraic geometry, number theory, logic and the foundations of geometry. In the complex analysis, Eugenio Elia's research focused on the singular point sets of a holomorphic function of several variables. However, he also wrote on issues relating to: differential geometry, Lie's groups, partial differential equations and the minimum principle. E. E. Levi would also demonstrate the falsity of Weierstrass's conjecture according to which given an open A of C^2, a merophormic function will always exist in A which has essential singularities in each point of the border of A, providing further evidence in favour of the differentiation between the theory of the single complex variable and the theory of more than one complex variable. His research followed Hertogs's theorem (1906) which signals the rise of multidimensional complex analysis as an independent research field.

This *springtime* in Italian mathematics at the beginning of the 20th century was not confined to geometry, mathematical physics and to analysis but also involved the "new" disciplines. We have already mentioned how Peano went on to study logic after embarking on his rigorist struggle and his search for extreme precision in definitions and proofs, also for teaching purposes. Around him and his publishing plans and the *Rivista di Matematica* (founded in 1891), a school of young and combative scholars would rapidly coalesce. Their presence would enliven many conferences which were still an innovation at the beginning of the century. Bertrand Russell would remember his meeting with Peano at the International Philosophy Congress in Paris in 1900 as being a particularly significant event for the formulation of his program. In partial contradiction with the

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[44] E. E. Levi had graduated from Pisa in 1904. He had been Dini's assistant and then taught at the University of Genoa. His *Opere*, edited by the *Unione Matematica Italiana*, were printed in two volumes in 1959 (Roma, Cremonese).

[45] B. Levi graduated from Turin in 1896. After a short period as assistant and as secondary school teacher, he taught geometry in Cagliari and then in Bologna. After the racial laws of 1938, he was forced to emigrate to Argentina, contributing to organize the mathematical activity in that country. His *Opere*, edited by the *Unione Matematica Italiana*, have been printed in two volumes in 1999 (Roma, Cremonese).

new ideas of the period, mathematical logic for Peano did not involve the application of algebraic techniques to traditional logic (and hence was not – nor could be – an independent mathematical discipline) but a tool and a language which were essential to mathematical activity, allowing concepts and proofs to be expressed with the greatest clarity.

With even less traditions a group of young mathematical economists also formed. The "lesson" taught by Walras approach had been adopted by Vilfredo Pareto (1849–1923) who, despite teaching in Lausanne, became the founder of the Italian school of mathematical economics and the true disseminator of the theory of general economic equilibrium. Mathematical economics was already an independent discipline but it had not yet expressed those distinguishing features typical of its full maturity. This field continued to entertain a close exchange with other areas of mathematical research and with those sectors of Italian culture and society interested in mathematizing a science which had traditionally been considered part of the social sciences. Economics was thereby endowed with quantitative and "objective" foundations[46]. The most active season of Italian mathematical economics was brief, very much associated with Pareto's commitment to it. Indeed, in 1909 with the publication of the French edition of the *Manuale di economia politica,* Pareto would in practice cease his research in economics. This would not stop an economist and an economic historian such as Joseph Schumpeter to consider Italian economic research in 1915 (thanks to the mathematical economists) as second to none.

5. External interests

As representative of the Risorgimento generation, we have dealt with a small group of mathematicians of great ability, tempered and selected by the political and military events of the period. These mathematicians associated their research with their public lives and were inspired by the most advanced research of the time in Europe. This small group had now grown. In the next generation we have met almost all the protagonists of our history: Volterra and Levi-Civita, Enriques and Severi, Tonelli. University positions in mathematics was increasing as was the number of young students aspiring to a university career. Before a national society of mathematicians was established, a number of scientific associations and academies had already developed (and they often published their journals and "bulletins"). In 1870, with the taking of Rome, the historical *Accademia dei Lincei* was reorganized. In 1884 the *Circolo matematico* of Palermo was founded; its *Rendiconti* would soon draw international attention, and it would be given the task (together with the mathematical section of the *Accademia dei Lincei*) of organizing the fourth International Congress of mathematicians in Rome in 1908. By that date the *Circolo* would number 924 members, of which 618 were foreigners, and its international prestige would be universally recognized. Also in 1908,

[46] On this issue, see A. Guerraggio, Economia e matematica in Italia tra Ottocento e Novecento, *Scientia*, 1986, pp. 13–39.

Poincarè publicly declared that the *Circolo* was the most important world mathematical organization![47]

Although this increase in the scientific community would make it difficult for a collective mind to formulate and carry out a single plan, the picture that we have before us at the beginning of the new century is sufficiently consistent to be considered as a whole. The efforts made for greater prominence in the international scene continued (as proof we have the exemplary story of the Circolo matematico of Palermo), and were still associated with strong engagement in public affairs. Increasing social complexity, however, meant that this engagement was not only expressed in terms of participation in government and parliament. A greater number of steps and a longer march through society became mandatory.

Such a trek for mathematics could not begin but in the school system. The grave problems in education, noted immediately after unification, would not be solved. The process of homogenization of the different regional situations would be slow, and the modernization of the country placed an added burden of tasks and objectives on the educational system. Tertiary education faced the problem of having too many universities, inherited from the various Italian states before unification, which brought to the fore the problem of the quality of teaching. In the secondary schools, the need to increase levels of education led to many calls to reduce and simplify programs (particularly and especially in mathematics).

Despite this difficult situation the teachers of mathematics would react positively by displaying strong individual commitment, founding (in 1895) a society, called *Mathesis*, which published the *Periodico di Matematiche*, and attracted the collaboration of a substantial number of university lecturers and professors. Unfortunately, results did not always match efforts, as the crises which this association would have to cope with testify to. Nevertheless, a distinguishing feature of *Mathesis* in this period was its great faith in active and direct involvement by members and in the establishment of a grass-roots reform movement. All the main educational issues were expressed and subjected to consultation amongst teachers in a positive fashion. From the point of view of *Mathesis*, the strength of this representation and logic would almost inevitably transform the resulting solutions into a reform project.

The relationship of Italian mathematics to the rest of society was not confined to establishing and disseminating scientific culture among the youngest generations. At the same time, its intention was to "export" the language and rationality it considered distinguishing features of its research, particularly by influencing traditionally closest scientific disciplines. Starting in 1895, *Il nuovo cimento* became the official publication of Italian physicists with Volterra as a member of the scientific committee for the journal. Two years later, the *Società italiana di fisica* was founded, with Volterra becoming its president, as we have already seen.

Even more surprising were the mathematical "incursions" into fields traditionally occupied by the "other" culture. It must not be forgotten that Italian mathematicians developed a strong historical consciousness and also expressed their opinions on philo-

[47] See A. Brigaglia, G. Masotto, *Il Circolo Matematico di Palermo*, Bari, Dedalo, 1982.

sophical issues. This happened thanks to the presence of a strongly interconnected general culture. The rigid separation channels which would bound the disciplines in 20th century thought had not yet been fully excavated.

The most prominent Italian mathematician in this sense was Enriques (although he was not the only one). The first examples of his interest in philosophy date back to the mid -1890s, if we except his first encounter with it as a secondary school student. However, it is in the 20th century that the activities of Enriques as a philosopher acquired public significance. In 1906 he published a volume titled *I problemi della scienza*. He began by philosophically analyzing the construction of geometrical systems and the problem of space. Enriques faced several problems which had also been studied by mathematicians such as F. Klein and H. Poincaré: what is the nature of geometrical postulates? How can the different geometries be explained from this perspective? Enriques stressed the importance of intuition and of the interaction among real space, space intuition and geometry postulates, refusing to consider the latter as a purely formal system. He saw geometry postulates as conceptual abstractions, but based on the different ways in which space is perceived. That same year Enriques founded the *Società filosofica italiana* (*SFI*), becoming its president. In 1907 he founds the review *Rivista di scienza*; in 1911 it would adopt the name *Scientia* turning into an international journal of scientific synthesis, in an attempt to counter tendencies towards excessive specialization. In 1907 he participated in the second congress of the *SFI* presenting a paper titled: "Il rinascimento filosofico nella scienza contemporanea". In the next congress, he even approached Hegel in a paper titled: "La metafisica di Hegel considerata da un punto di vista scientifico." By now it had become clear that his work could no longer be ignored by "professional" philosophers, in particular by Benedetto Croce (1866–1952) and by Giovanni Gentile (1875–1944) who at the time were the leading exponents of Italian idealism. Already at the beginning of the century, they had become exponents of a plan to extend their philosophical hegemony over the culture of the whole country. The event for the *redde rationem* was to be the fourth International Congress of Philosophy (1911). Since it was to be held in Italy it was organized and chaired by Enriques (in his capacity as president of *SFI*). The clash with Croce and Gentile began immediately, during the preparation of congress events. The congress then went smoothly. It was only once it was finished that Croce publicly attacked Enriques, in a newspaper interview, by directly accusing him, coupling ironic comment with harsh judgment, of being an amateur and for encroaching on a field which he knew nothing about. Croce's severe criticism was emblematic: by declaring its incomprehension and hostility, 'official culture', or rather that more closely rooted in the traditions of the country, handed down its negative sentence (destined to "count" for many decades to come) on the enthusiastic attempt by mathematicians to link their extremely qualified professional capacities to active participation in the cultural and social life of the country.

Although the Croce – Enriques controversy is perhaps the most well- known event of the period, the most "political" incident saw the participation of Volterra, with the establishment of the already mentioned *SIPS*. This association was founded with a double objective, which we have already noted in regard to *Mathesis.* The internal objective addressed the scientific community by advocating consciousness of one's intellectual role. Although specialization in academic research was considered as a positive necessity, it should not lead to fragmentation and isolation into small sectors, inspired only by technical perspectives. Consciousness of a greater mission to fulfil, as well as a more attractive image (which was to be achieved by publicising the character and work of scientists) were considered the prerequisites for applying strong pressure to combat the inertia of the political establishment, encouraging it to recognize the usefulness of science by according it a rightful place in society. This was the second objective of the *SIPS*: to participate in the development of a modern country, which recognized the social function of science by following in the footsteps of the more developed European countries. This message was lucid and strong. Volterra suggested that both he and the scientific community should take on a leading role in the development of the country by expressing a model of rationality and organization powerful enough to control and resolve the contradictions of its own growth. We must keep this in mind when describing Italian mathematics in the years between the two world wars.

Chapter 2

Nothing is as it was before

1. Introduction

In the *Prologue* we introduced Italian Mathematics as a young discipline, but certainly growing fast. At the beginning of the 20th century it was extraordinarily exuberant. Its contributions to different research fields, the level it had reached in international *ranking*, and, again, the quickness with which such a position had been achieved (starting from a relative obscurity), were all strongly positive elements.

Mathematics was beginning to clearly distinguish its different research areas, so we must be very careful in the difficult task of identifying a unique leader with maximum influence and authority. And yet, in Italian mathematics the figure of Volterra stands astride the 19th and the 20th centuries. His scientific authority in analysis and in mathematical physics, his international contacts, his prestige even outside national boundaries and, finally, his public activity, turned him into the main icon in the Italian mathematical world. Volterra's work was the best expression of the so-called *19th century* tradition, whose brilliant examples have illumined the story of Mathematics. His physical-mathematical approach was traditional, as was the relationship between the physical world and the mathematical formalism, but he showed as well a remarkable skill in pushing this tradition towards forms of a great modernity (we have seen this skill at work in functional analysis and in the theory of integral equations). Volterra represented the most advanced edge of tradition, both in science and in his values and cultural-political position: "enlightened" conservator, keenly fond of the *Risorgimento*, from which he took his faith in the scientific internationalism – he developed intense relationships mainly with the French mathematical world – and the sensibility to understand the social role of science. Of course, he was also – as we have seen – a man of power who in the years of our study would further develop his public dimension. Beside him, but independently, grew a generation of younger researchers who, at the beginning of the 20th century, left their stamp especially in the real analysis areas: mathematicians such as Tonelli, Vitali, Fubini, etc.

The other pole of Italian Mathematics at the beginning of the 20th century was algebraic geometry and the triumvirate Castelnuovo, Enriques and Severi, whose authority and scientific prestige (even at an international level) deserve recognition similar to that given to Volterra. Castelnuovo was the oldest one, but in the years considered in the *Prologue* he had actually not reached his fifties yet. In the period between the two World Wars he will be a researcher (and supporter of the studies) of probability and an authorial exponent of the Roman mathematical group, though in a less central position. Enriques and Severi will have a bigger role. The former, Castelnuovo's pupil and later his acquired relative[1], had already displayed his intellectual talent in those years. He was an extremely intelligent, cultivated, brilliant man, and set quite naturally a working style and manner concentrated on "great ideas", to the detriment of what he considered simple details. After his controversy with Croce and Gentile he was also known to a wider public. In a more moderate way than Volterra, he also undertook collective enterprises – the example of *Scientia* is enough – but always strictly cultural ones. In contrast, Severi, who did not hide his socialist ideas from the local administration benches of Padua, was interested in politics and in more general contexts. It was easy to see in Enriques' pupil a rising star. His relationship with the master was still good, even if already strained by an "incident" which showed that Severi was "champing at the bit". He felt shackled not only because of the politics of Padua. Some events, such as those regarding the *Associazione Nazionale Insegnanti Universitari* and Severi, who became its president, following Enriques, some years before the war, could be interpreted symbolically too.

Neither is the young Levi-Civita to be forgotten, whose memoir of 1901 and whose contribution to the problem of the three bodies and to relativity theory had attracted international attention. Besides, his correspondence with Einstein confirmed the importance of his research[2]. Levi-Civita's character was different from Volterra, Enriques and Severi. He came from a progressive educated bourgeois family and would never hide his socialist stance. But he would never mix the political sphere with the professional one – as almost any Italian mathematician of this generation would – neither would he add other commitments to the scientific and academic one. Levi-Civita would support his political ideas – very resolutely – but in a private sphere. He was a meek and quiet character (who would gradually show traits of a great humanity) but he could defend his own beliefs with determination.

Finally, we must remember that the liveliness of Italian Mathematics at the beginning of the 20th century existed not only within but as projections into another disciplines, indeed as "field invasions" that were characterized by their originality; Enriques' invasion into the philosophical culture was the most clamorous of them. This phenomenon requires some further comment.

The *golden age* of Italian Mathematics actually ended with World War I. Some warning signals could have been seen before, perhaps. These were not just isolated and specific events, such as the controversy between Enriques and Croce where because of

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1 Castelnuovo had married one of Enriques' sisters.

2 The correspondence is reproduced in P. Nastasi, R. Tazzioli, *Calendario della corrispondenza di Tullio Levi-Civita (1873–1941) con appendici di documenti inediti*, Palermo, Quaderni Pristem, No. 8 (1999), pp. 204–238.

the reaction of the philosophical world the former was seen to have been defeated by the latter. Rather it was a question of raised expectations, lack of confidence in commitment, and few proper results from attempts to reach out. Disappointment was spreading, as well as fatigue. Commitment to and enthusiasm for intellectual progress were decreasing.

The war was, after all, a real *turning point* for a whole society. This was not a war between armies but between peoples. Violence and social upheaval became the first dramatic experience directly lived by millions of people. The end of the liberal age came with the *October Revolution* and the American intervention, which broke off the USA's long isolation period. The tsarist regime and the German, Austro-Hungarian and Ottoman empires collapsed. Twenty-six new nations arose. Relations among the main industrialized countries changed irreversibly. New York and the dollar replaced London and the pound sterling. Stability of prices and of the value of gold and silver coins was only a memory. States looked for larger sharing in their economies; business executives saw their power strengthened, while Parliaments and other governmental bodies lost it.

After World War I, nothing was as before. It was impossible to set out again with a simple *heri dicebamus*.

2. Italian mathematicians take sides.

The *great war* broke out in summer 1914, when the Austro-Hungarian empire declared war on Serbia: on the 28th June the archduke Francis Ferdinand of Austria and his wife had been murdered in Sarajevo. On the 1st August Germany opened hostilities against Russia; two days later it invaded Belgium and "inaugurated" the French front. On the 4th of August it was England's turn to declare war on Germany.

At the time Italy abstained. It would go to war a year later, on the 24th May 1915. In those months the debate in the country was extremely fervent, as one can easily guess. Pacifists and interventionists from different places and with different attitudes confronted each other with great vehemence.

The mathematical world was quite homogeneous, and, on the whole, sided with the Allied powers in favour of a democratic interventionism, against the "German barbarity". The work that had aimed at developing a tighter and tighter network of relationships between Italian and French (but also English and North American) mathematics, especially interwoven by Volterra, had begun to pay off. It is not by chance that Volterra himself was the most committed one in the world of mathematics, to urge an explicit alignment with France, England and Russia. He wrote to Gaston Darboux[3]:

> Très honoré Monsieur e cher Maître
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> j'ai reçu de plusieurs côtés vos nouvelles et j'ai appris de la part de M. Appell que vous êtiez dans les Pyrénées et que vous êtes rentré à Paris dès le commencement de la guerre. Permettez-moi de vous dire que ma pensée est toujours tournée, avec

[3] *Accademia dei Lincei*, Rome; Archives Volterra.

le plus profond attachement, vers vous, vers les maîtres, les confrères, les amis que j'ai en France.
Votre noble et grande nation lutte pour la cause de la justice et de la civilisation. Tous mes voeux sont per le succès et le triomphe de la France.
L'acte par lequel les deux empereurs ont déchainé la guerre et la destruction en Europe a été regardé par moi, ainsi que par la majorité de mes compatriotes comme un crime abominable. Les innombrables actions barbares que les allemands ont accomplies pendant la guerre n'ont fait qu'accroître l'horreur et l'indignation du premier moment.
A mon avis l'Italie doit prendre sa place à côté de sa soeur latine: la France, et de ses alliés contre l'Autriche et l'Allemagne. C'est son rôle et sa mission. Elle ne doit pas y manquer. J'éspère de tout mon coeur que cela arrivera.
Voilà mes voeux et mes espérances. Puissent ces voeux et ces espérances, si repandues dans toutes les régions et parmi toutes les classes sociales d'Italie, être réalisées et puissent nos deux pays être unis toujours davantage pour la liberté et la civilisation.

Volterra's letter was dated 7th September 1914. A month later, on the 4th October, 93 German intellectuals – among which were Felix Klein and Max Planck – created and spread a manifesto to defend with very resolute tones the reasons for their own patriotic commitment, against what they called the *distortions of* western public opinion. Volterra received the manifesto from O. E. Staude, Klein's pupil in his teaching period in Lipsia, and soon after he received a similar opposite declaration from his French colleagues. On the 16th of October, for example, É. Borel wrote to him[4]:

Mon cher ami,

vous avez sans doute lu l'appel adressé par les intellectuels allemands au monde civilisé. Je désirerais publier dans la Revues du Mois quelques-une des appréciations ou réponses relatives à cet appel, dues à des neutres. Il me semble que il sérait préférable, si possible, d'avoir des textes déjà publiés dans les pays neutres, et non pas écrites spécialement à l'instigation de français ou d'anglais.
Pourriez-vous me signaler et me procurer au besoin des textes de ce genre parus en Italie? Nous voyons bien ici quelques journaux italiens, mais pas tojours régulièrement et nous sommes parfois forcés de nous borner à y lire les nouvelles directes de la guerre de source allemande ou autrichienne, qu'il est toujours intéressant de connaître, mais qui ne nous enlévent pas notre confiance dans le succès de notre cause.

Volterra answered quickly (24th October[5]), confirming his whole support to the Allied cause.

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[4] *Accademia dei Lincei*, Rome; Archives Volterra.
[5] The letter, unpublished, is kept in the Archives Volterra at the *Accademia dei Lincei* in Rome.

Mon cher Ami

Je viens de recevoir votre très-chère lettre du 16 Octobre et je m'empresse à vous repondre. Je vous ai écrit vers la moitié de Septembre une lettre où je vous demandais des nouvelles de M. Gateaux[6], de M. Pérès, de M. Boutroux et d'autres jeunes amis Français. Je n'ai reçu aucune réponse et, comme vous ne faites aucune allusion à cette lettre, elle doit avoir été perdue. A un mois de distance je ne peux que confirmer ce que je vous écrivais alors, c'est à dire mes félicitations pour les succès de la France, ma plus vive sympathie pour votre noble pays qui lutte pour la justice et la liberté et pour la cause de la civilisation contre la violence du plus brutal et odieux impérialisme. Je vous disais que le rôle de l'Italie est, à mon avis, celui de s'unir à la triple entente. Je puis ajouter aujourd'hui que la confiance dans cette union n'a fait qu'augmenter, car la sympathie pour la France l'Angleterre et la Russie s'est accrue chéz-nous. D'autre part la persuasion que tous nos intérêts au point de vue moral ainsi que politique sont en opposition avec l'Autriche et l'Allemagne n'a fait que se raffermir.
Vous avez raison de désirer un article déjà publié. Je vous envoie un article remarquable qui a paru dans la "Tribuna" du 6 Octobre, le jour après celui où l'appel des savants allemands a paru. L'article est signé "Rastignac" c'est à dire Vincenzo Morello très-connu dans notre monde littéraire et un des meilleurs écrivains parmi les journalistes Italiens. Il est aussi très-apprécié comme auteur dramatique.
Je crois que l'article de Rastignac est justement ce que vous cherchez et ce qu'il vous faut. Je chercherai cependant s'il y en a aussi d'autres qui pourraient vous convenir.
Nous avons eu une foule d'articles de toute sorte qu'on a appelé la "polemica nazionale" sur la question de la guerre et beaucoup de protestations contre les barbaries des allemands, leurs violations des conventions de la Haye et des traités ainsi que sur les distructions qu'ils ont accompli en France et en Belgique. A ces protestations se sont associés des Universités des Académies des hommes politiques des savants etc. J'ai toujours adhéré à ces protestations, mais il est presque impossible de suivre et de recueillir toutes ces protestations qui sont répandues un peu partout dans les journaux de Rome et de la province et qui ont paru la plupart sous forme des télégrammes et d'ordres du jour.
Vous savez sans doute que M. Richet[7] a été très-fêté chez-nous. Ses conférences, auxquelles j'ai assisté avec beaucoup d'intérêt, ont eu un grand succès et un grand nombre d'auditeurs. Il a aussi très-bien réussi dans la polémique contre l'appel des savants allemands. Nous avons adhéré à ses protéstations.

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[6] René Gateaux died during the first months of the war. Just in February 1914 he had held a conference titled: "Une face du développement du calcul fonctionnel", at the *Seminario Matematico di Roma*. Volterra would personally commemorate him on the 19th December 1914 during a session at the same *Mathematical Seminar*.

[7] Charles Robert Richet (1850–1935), Nobel prize for Physiology in 1913. As it can be inferred from the text, his conferences (one of them titled: "Science and civilization today") encouraged Italy's entering into the war on the Allies' side.

> Vous avez parfaitement raison d'être sûr du succès de la cause de la France et des alliés. Tout le monde civilisé est contre l'Autriche et l'Allemagne.
> Je suis parmi ceux chez-nous, qui sont les plus impatients de sortir de la neutralité, mais je ne doute pas que même ceux qui ont un peu moins d'impatience ne peuvent pas manquer d'avoir les mêmes espérances et les mêmes aspirations que moi.
> Mon voeux aujourd'hui est le même que celui que j'ai fait dépuis le commencement de la guerre. L'Italie, la France et ses alliés doivent être unies contre l'ennemi qui a accompli le crime de déchaîner la guerre et qui voulait asservir l'Europe.

And some days later (on the 29th October)[8]:

> Mon cher Ami
>
> J'espère que vous aurez reçu ma lettre du 24 Octobre en réponse à votre lettre du 16 Octobre et le journal que je vous ai envoyé dans un pli à part.
> Je vous envoie aujourd'hui un autre article qui est paru dans le "Messaggero" le plus populaire de nos journaux. L'article est de M. Edoardo Cimbali professeur de Droit International et il est aussi une réponse à l'appel des intellectuels allemands.
> J'espère vous envoyer aussi quelque autre article.
> En vous exprimant encore une fois mes voeux les plus chaleureux pour le succès des armées alliées et pour l'union de nos deux pays, je vous envoie l'expression de tout mon dévouement et de l'amitié la plus sincère.

English intellectuals mobilized against German propaganda too. Already on the 21st of October, 150 English scholars drafted a counter-manifesto which denounced Germany as "the common enemy of Europe and of all peoples". Volterra confirmed his position immediately (in a letter to the physician Joseph Larmor)[9].

> Ho ricevuto il suo biglietto e la ringrazio dei suoi auguri che ricambio cordialmente. E vivamente li contraccambio e li estendo anche a tutto il vostro grande paese verso il quale le simpatie già così grandi presso di noi sono ancora maggiormente accresciute nelle circostanze attuali.
> Ho ricevuto da Sir Archibald Geikie vari opuscoli relativi alla guerra in cui si parla delle ragioni che hanno spinto l'Inghilterra nel conflitto, e della sua condotta verso il Belgio. fra essi vi è anche la lettera colla quale gli scienziati inglesi hanno risposto ai tedeschi.
> Ho già risposto a Sir Archibald Geikie che per parte mia divido pienamente le idee manifestate dagli scienziati inglesi nella loro risposta e che è viva in me l'ammirazione per la condotta dell'Inghilterra sia verso il Belgio sia nel voler salvare l'Europa dall'aggressione dei due imperi tedeschi. E che la guerra sia derivata da una

[8] *Accademia dei Lincei* in Rome; Archives Volterra.
[9] *Accademia dei Lincei* in Rome; Archives Volterra.

> aggressione voluta e preparata dalla Germania e dall'Austria lo mostrano tutti i documenti comparsi.
> Tali idee sono lieto che siano divise dalla gran maggioranza dei miei compatrioti e io non dubito che esse serviranno di norma alla condotta dell'Italia il cui intervento io mi auguro possa condurre a notevoli conseguenze.
> Sono ben lieto di inaugurare l'anno 1915 col voto che i vincoli fra i nostri due paesi si facciano sempre più stretti[10].

Castelnuovo and Enriques had the same stance as Volterra, though not so overt. The discussions and controversies of the years 1914–15 about Italy's position in the war brought Enriques to resign from *Scientia*, the journal he had founded in 1907 and that he directed with Eugenio Rignano. It all started with an inquiry upheld by Rignano himself, that considered it unacceptable for a scientific journal "to shut itself up in the ivory tower of the abstract synthesis" and "to remain impassive in front of the tragic reality of the present hour". The inquiry, which was intended to be an "objective, serene research – that is a scientific one – of the war causes and factors", slipped though towards positions that – at least so thought Enriques – risked straying from the initial objectives. An article by Rignano, *I fattori della guerra ed il problema della pace*, was specially worrying; in it the author – Italy had already entered the war with the Allies – had no hesitation in speaking freely about the imperialistic aims of the England, its allies, and their responsibility for the war. Enriques asked Rignano explicitly to withdraw the article, which was not an expression of a free scientific opinion but a real political act, and could offend patriotic feelings. Faced with Rignano's refusal, Enriques precipitously left the editorship – together with many of the founders – to go back to it only in 1930, after Rignano's death.

Enriques' position on war did not emerge from the letters he wrote then to Rignano, but it can be easily inferred from his correspondence with Xavier Léon, director of the *Revue de Métaphysique et de Morale*[11]. At Easter 1914, Enriques and Léon had

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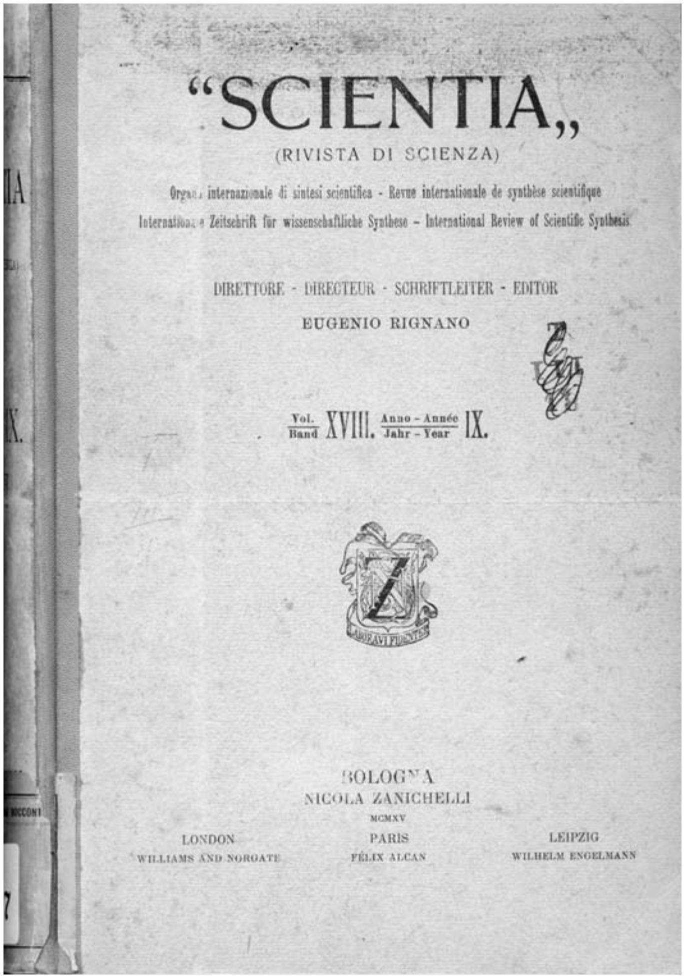
[10] I received your note and I thank you for your wishes, which I heartily return. I warmly return and extend them also to your great country, towards which our great sympathy has, in the present circumstances, further increased.
I have received from Sir Archibald Geikie some pamphlets concerning the war, about the reasons which have driven England to war, and about its behaviour towards Belgium. Among them there is also the reply of English scientists to Germans.
I have already told Sir Archibald Geikie that I fully share the ideas English scientists display in their reply, and that I deeply admire England's behaviour both towards Belgium and towards its will to save Europe from the aggression of the two German empires. And the fact that the war was due to a wilful aggression prepared by Germany and Austria has been demonstrated by the shown documents.
I am glad these ideas are shared by most of my compatriots and I do not doubt that they will guide the behaviour of Italy, whose intervention will lead to remarkable consequences, I hope.
I am really pleased to begin the year 1915 with the wish that the bonds between our two countries become tighter and tighter.

[11] The letters, which come from the Léon "papers" kept at the *Bibliothèque Victor Cousin* of the Sorbonne in Paris, have been published in L. Quilici, R. Ragghianti: Il carteggio Xavier Léon: corrispondenti italiani, *Giornale critico della Filosofia Italiana*, 1989, p. 295–368.



organized in Paris a congress of Philosophy of Mathematics, held a *latere* of the *Commission on the Teaching of Mathematics* (presided over by F. Klein, with Henri Fehr as secretary). The congress had approved the proposal of creating an *International Society of Mathematics' Philosophy*, was actually swept away by the war. On the 25th August 1914 Enriques wrote to Léon:

> Cher Ami
>
> Je ne veux pas rétarder plus longtemps à vous exprimer mes sentiments de sympathie chaleureuse pour votre pays dans cette heure tragique pour l'Europe.
> Je me trouvais en Suisse, à Zürich, lorsque l'orage est éclaté (…). J'ai passé des heures d'angoisse avant qu'on eut proclamé la néutralité italienne; en rentrant en

> Italie j'ai compris que toute autre décision du gouvernement aurait été impossible, puisque le sentiment de tous les Italiens, de toutes les classes et de tous les partis, est unanime contre les aggresseurs.
> Si vous pourriez voir avec quelle anxieté on attend ici les nouvelles de la guerre et comment le peuple entier fait de voeux pour le salut de la France, vous en seriez touché. D'ailleurs on a ici le sentiment que l'acte d'indépendance accompli vis-à-vis de l'Allemagne, va nous coûter cher au cas où les Allemands seraient les vaniqueurs. On est préparé à être attaqué à notre tour; mais si la paix ne peut être mantenue, que nous nous trouvions du côté de la civilté et du droit! C'est la pensée intime du peuple italien tout entier, dont la calme et le pacifisme ne cachent en somme que le propos de contribuer nous aussi – lorsque l'heure sera sonnée – à l'oeuvre de libération.
> Cher Ami, veuillez participer mes sentiments aux communs amis auxquels s'adresse une pensée au moment où la France offre au monde un si beau spectacle d'unité, de fermeté et de dignité. Comme vous, j'ai confiance dans le succés final, coûte ce qui coûte.

And some months later, on the 4th February 1915, he added:

> Quant à l'évenement de cette guerre, personne ne sait bien à quoi s'en tenir. Vous savez quelles sont nos sympathies et nos aspirations, il n'y a peut-être qu'un petit nombre de personnes qui ne les partagent (malhereusement la phlilosophie hégélisante ne se fait pas honneur, elle est du petit nombre des sympathisant pour l'Allemagne). Mais je crois que la presse italienne ne donne pas un'idée juste de la situation lorsqu'elle semble ne s'occuper guère du traité d'alliance. Personne ne connaît bien nos engagement, mais il est à craindre que le gouvernement ne soit pas entièrement libre, sauf dans le cas où les Allemands eux-mêmes commetteraient la méprise d'attenter à nos droits ou de nous ménacer.

The reference to "hégélisante" philosophy, discredited because of its pro-German sympathies, was obviously directed against Benedetto Croce. In an interview given to the *Corriere d'Italia*, on the 13th of October 1914, when the journalist asked whether he had, "in Italian and foreign journals, kept up with controversies about the relationship between Italian culture and French and German thought", the Neapolitan philosopher curtly answered that he considered those controversies "manifestations of the state of war. It is no more a matter of rational questions, but of clashes between passions; not of logic solutions, but of assertions of interests, which, even if rather high, are national, that is, particular; not of reasoning, but of fake reasoning, built by imagination"[12]. There was also an explicit attack on that expression – *German barbarity* – on which Volterra and the democratic intellectuals so much insisted:

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[12] Cf. B. Croce, Giudizi passionali e nostro dovere, in B. Croce, *L'italia dal 1914 al 1918. Pagine sulla guerra*, Laterza, Bari, 1950, pp. 11–12.

> Credo che, a guerra finita, si giudicherà che il suolo d'Europa, non solo ha tremato per più mesi o per più anni sotto il peso delle armi, ma anche sotto quello degli spropositi. E Francesi, Inglesi, Tedeschi e Italiani si vergogneranno e chiederanno venia pei giudizî che hanno pronunciati, e diranno che non erano giudizî ma espressioni di affetti. E anche più arrossiremo noi, neutrali, che molto spesso abbiamo parlato, come di cosa evidente, della "barbarie germanica". Fra tutti gli spropositi, frutti di stagione, questo otterrà il primato, perché certo è il più grandioso[13].

About the war, Severi's ideas are similar to those of Volterra and Enriques. He could be placed in that sector of political thought which historiography called *revolutionary interventionism*, that joined people with interventionist positions belonging to different revolutionary movements (socialists, anarchists, trade-unionists, republicans, etc). Severi was an *official socialist*, as he defined himself in a speech that appeared in the journal *L'Adriatico* on the 9th March 1915. This public stance laid down the beginning of his moving away from the socialist party, siding increasingly with neutralist positions.

> Io spero e credo che l'atteggiamento degli organi direttivi del mio partito, in questo grave momento, sia l'espressione dello stato di angoscioso dissidio in cui ogni socialista d'intelletto e di cuore si trova fra gli imperativi ideali della propria fede e la percezione delle necessità ineluttabili dell'ora presente; piuttosto che indice di un proposito d'azione maturato e metodicamente perseguito.
> Ma se così è, e se è pur vero, secondo io penso, che il partito socialista, come organismo politico, non potrebbe mai farsi promotore di un intervento guerresco, assai meglio parmi si provvederebbe, se la protesta socialista contro la guerra fosse, in ogni occasione, contenuta nel campo puramente ideale, riconoscendo nello stesso tempo la ineluttabilità d'una situazione che non ci è dato oggi di modificare, appunto perché deriva da condizioni sociali che il partito nostro non può cambiare di colpo.
> Porsi da un punto di vista di assoluta negazione di problemi che esistono e che reclamano una soluzione indifferibile, significa lasciarsi cullare dalla ingenua illusione di poter violentare lo svolgersi dei fenomeni storici, e venir quindi, in ultima analisi, a contraddire a quello che è lo spirito animatore della dottrina socialista.
> Un atteggiamento meno assoluto della Direzione del nostro Partito, sarebbe importantissimo anche dal punto di vista politico, giacché lascerebbe ad ogni inscritto la libertà di valutare gli elementi reali della situazione, secondo la propria coscienza di cittadino italiano, e nello stesso tempo *consentirebbe ad ognuno di noi* di conti-

[13] I am sure that, when the war ends, it will be said that European ground has shivered during several months or several years not only under the weight of weapons, but also under that of blunders. And French, English, German and Italian people will shame and beg pardon for the judgements they gave, and will say that they were not judgements but expressions of feelings. And we neutrals, who so often have talked, as of an evidence, about the "German barbarity", will blush even more. Of all blunders, product of the time, this will hold the record, because it is certainly the most striking one.

nuare la propaganda socialista fra le masse, additando loro quali disastri immani conseguono dall'ordinamento capitalistico della società.
Io, che sono convinto della necessità dell'intervento dell'Italia a fianco della Triplice Intesa, sento di non aver mai provato un odio così implacabile contro la guerra – la quale non crea, ma sfrutta valori morali già esistenti; – né di aver mai desiderato, con altrettanto ardore, profondi rinnovamenti sociali, come da quando assistiamo alla spaventosa ecatombe di vite umane, all'enorme distruzione di ricchezza, all'acutizzazione del disagio economico del proletariato, al dispregio del diritto e delle bellezze dell'arte che la guerra europea trascina con sé.
Ma come non v'ha uomo cui la violenza ripugni, che ad essa non possa contro ogni sua voglia essere costretto; come non v'ha socialista che, vivendo e vestendo panni in questa società borghese, non s'adatti, nelle pratiche contingenze della vita, a ciò che l'ambiente gli impone senza che per questo egli rinunci a dar l'opera sua per un migliore domani, così non trovo vi possa essere contraddizione sostanziale fra la fede nei nostri ideali e l'azione che oggi cagioni storiche superiori alla nostra volontà possono prescriverci.
Vi sarà è vero, per chi ami dilettarsi in così tragico momento di quisquilie dialettiche, una contraddizione formale; ma sciaguratamente le più angosciose situazioni sentimentali si sciolgono di rado alla stregua della logica pura.
Eppoi il partito socialista non ha forse riconosciuto che nella pratica quotidiana conviene adattarsi ad un programma minimo e non evitare talvolta contatti con le frazioni più illuminate della borghesia, quando occorra, ad esempio, contrastare la vittoria di partiti i quali minaccino di prevalere in modo pericoloso per le libertà politiche, che costituiscono il presupposto delle conquiste economiche del proletariato?
E perché dovremmo racchiuderci in una formola d'intransigente negazione, proprio in una questione che di gran lunga trascende la importanza della minuscola politica d'ogni giorno, e che è in fondo ancora una questione vitale di libertà?
Giacché è ben vero che le cause di questa guerra sono giustamente capitalistiche, ma non si può disconoscere che, sia per le brutali violazioni del diritto naturale dei popoli compiute dalla Germania, sia per l'esistenza di molte questioni insolute, sia infine per *l'interesse* di alcuni belligeranti, e soprattutto dell'Inghilterra, affinché vengano rispettate le nazionalità minori ("L'interesse e il dovere spingono l'Inghilterra nella stessa direzione", hanno scritto i professori dell'Università di Oxford), la guerra è andata acquistando, in modo prevalente, il carattere d'un conflitto fra due opposte concezioni dei diritti e delle forze, che debbono prevalere nel mondo moderno.
Inoltre, secondo la lettera e lo spirito della dottrina marxista, il socialismo potrà e dovrà succedere agli attuali ordinamenti, soltanto allora che la civiltà sia passata per tutte le fasi del suo sviluppo, tra le quali vi è appunto la conquista delle unità e delle autonomie nazionali. Di guisa che, per dirla con una frase scritta in questi giorni nell'*Avanti* da Enrico Leone, la Nazione diventa la porta d'ingresso dell'Internazionale.
E quando si parla della Nazione non ci si appiglia ad un "diversivo borghese", poiché la Nazione è una formazione storica naturale, la quale vive nelle tradizioni

The retreat (1917)

di lingua, di arte, di cultura, di ciascuna razza, e sta al disopra e al di fori delle iniquità degli ordinamenti statali.
Riconosco come vi siano purtroppo molti, i quali, per le condizioni di inferiorità culturale e materiale in cui si trovano, non certo per colpa loro, non possono sentire tutto il valore spirituale dell'idea di Nazione; ma essi comunque non dovranno disconoscere che il dominio straniero rappresenta sempre un altro sfruttamento, da Nazione a Nazione, che viene ad aggiungersi allo sfruttamento del capitalista sul salariato.
Eppoi in qual modo si concreterebbe la solidarietà internazionale se, fino a quando non sarà più diffusa la coscienza della disastrosa follia degli armamenti, di fronte a tentativi di sopraffazione imperialistica a danno di altri popoli, non si fosse disposti anche a sacrifici di sangue?
D'altronde i socialisti, predicando l'avversione alle spese militari, hanno sempre presupposto la sincerità e l'efficacia della propaganda antimilitarista negli altri paesi, ed hanno inteso con ciò di cercar di diminuire la possibilità di conflitti armati fra i popoli, ma non già di negare le idealità nazionali. Allorché la Patria sia in pericolo, ancor più impellente sorge quindi per noi socialisti il dovere di difenderla, avvalorando agli occhi di chi ci considera utopisti, la nostra persuasione che dalla coscienza di un buon diritto possa – ove occorra – sprigionarsi la più grande delle forze.
Ed io credo per certo che sul riconoscimento di questo dovere, la stragrande maggioranza dei socialisti italiani sia senza esitanze concorde, anche se qualche eccesso polemico possa a taluno far supporre il contrario. Non imprigioniamoci dunque nell'adorazione di formule assolute, giacché il pericolo per il nostro Paese

è insito nella grave situazione internazionale, la quale potrebbe trascinarci più tardi, anche nolenti, ad una guerra disastrosa per l'Italia e più particolarmente per il Veneto.

Prof. Francesco Severi, dell'Università di Padova

P.S. – Ragioni varie hanno fatto ritardare per circa due settimane la pubblicazione di questa mia lettera. Non ho ora nulla da mutare, ma di fronte al fatto – segnalato anche ieri in queste colonne dall'amico Gino Piva – che le condizioni economiche del proletariato veneto vanno di giorno in giorno aggravandosi, in modo veramente doloroso e allarmante, desidero di aggiungere una parola di viva deplorazione per l'inerzia del Governo, il quale sembra non abbia capito e non capisca che la preparazione non deve limitarsi alle sole provvidenze militari. Come si potrebbe sperare che le masse popolari offrissero la necessaria resistenza morale e materiale, se la nostra regione dovesse essere assoggettata, dall'intervento dell'Italia nel conflitto europeo, ad altre e ben più dure prove?
Provvedimenti eccezionali (lavori e sovvenzioni dello Stato ai Comuni) urgono qui nel Veneto per fronteggiare la grave crisi. Altro che proibire i comizi!

9 marzo 1915 F.S[14].

[14] I hope and believe that the behaviour of my party's directive organs, in this serious moment, is the expression of the state of painful disagreement in which each socialist by intellect and by heart finds himself, between the ideal imperatives of his faith and the perception of the inescapable needs of the present moment; rather than sign of a matured and methodically persecuted intention.
But if it is so, and if it is also true, as I think, that the socialist party, as a political organ, could never promote a warlike intervention. It would be better, I think, if the socialist protest against war was always limited to a purely ideal field, while acknowledging the ineluctability of a situation which can not be modified today, since it arises from social conditions that our party cannot suddenly change.
The absolute denial of existent problems which claim an urgent solution means to cherish the ingenuous illusion that it may be possible to force the development of historical events, and therefore to contradict, in the end, the inspiring spirit of socialism.
A less absolute attitude of our Party Direction would be very important also from a political point of view, because it would leave to each member the freedom to value the real elements of the situation, according to the conscience of each Italian citizen, and at the same time *it would allow to each of us* to continue the socialist propaganda with the masses, showing them to which immense disasters leads the society's capitalistic order.
Even if I am sure of the need of Italy's intervention on the side of the Triple Entente, I have never experienced such an implacable hate towards war – which does not create moral values, but exploits the already existent ones – nor have I ever wished deep social changes so fervently as since we witness this horrible hecatomb of human lives, this enormous destruction of richness, the acuteness of the economic hardships of the proletariat, the scorn of law and of the art's beauties that the European war drags with itself.
But as there is no one who, disgusted by violence, could not be obliged to it against his will; as there is no socialist who, living and breathing in a bourgeois society, does not, in the practical circumstances of life, adapt to what the environment imposes on him, without renouncing thus to work for a better future, so I do not find any substantial contradiction between the faith in our ideals and the action that present historical reasons, superior to our will, order us.

[14] (continued)

Those who in such a tragic moment delight in dialectical trifles will find a formal contradiction, it's true; but unfortunately, awfully distressful sentimental situations release themselves rarely according to the criteria of pure logic.

And then has maybe the socialist party not recognised that in daily practice it is better to follow an abridged programme and not to avoid sometimes contacts with the most enlightened bourgeoisie, when, for example, it is necessary to contrast the triumph of parties which threaten to prevail in a dangerous way for political freedom, which are the premises for the economic achievements of the proletariat?

And why should we withdraw into an intransigent negation, just in a question that goes beyond the insignificant daily politics and which after all is still a vital question of freedom?

Since it is absolutely true that the causes of the war are rightly capitalistic, but one cannot deny that, because of Germany's brutal violations of the natural right of peoples, because of the existence of many unsolved questions and, lastly, because of the *concern* of some belligerents, specially England, for the respect of minor nationalities (Oxford University professors have written: "Interest and duty push England in the same direction"), war has mainly acquired the nature of a conflict between two opposite views of the rights and forces which must prevail in modern world.

Besides, in the letter and spirit of Marxist doctrine, socialism can and must succeed the actual order, only when civilisation has passed through all stages of its development, among which there is precisely the achievement of national unions and autonomies. So that, to say it with a sentence written in these days in *Avanti* by Enrico Leone's, the Nation becomes the entrance door of the International.

And when we talk about Nation we are not following a "bourgeois diversionary", since Nation is a natural historical formation living inside the traditions of language, arts and culture of each race, and it is above and outside the iniquities of the state order.

I know that unfortunately many people, because of the conditions of cultural and material inferiority in which they find themselves, certainly not because of their fault, cannot wholly feel the spiritual value of the idea of Nation; but they should not deny, though, that foreign domination is another kind of exploitation, from Nation to Nation, which adds to the exploitation of the waged by the capitalist.

And then, how would international solidarity be achieved if, until the consciousness of the disastrous craziness of armaments is not further spread, we were not ready even to blood sacrifices in front of the imperialistic attempts to overwhelm other peoples?

On the other hand, socialists, while urging to despise military expenses, have always believed in the sincerity and efficacy of antimilitaristic propaganda in other countries, and they have tried to diminish thus the possibility of armed conflicts between countries, but not to deny national ideals. If our country is in danger, we socialists feel even more impellent the duty to defend it, reinforcing, in front of those who consider us utopian, our belief that, if necessary, a greatest force can be released from the consciousness of a good right.

And I firmly believe that the vast majority of Italian socialists will without hesitation support this duty, even if some polemic excesses may let suppose the contrary. Let's not get caught, then, in the veneration of absolute formulas, since the danger for our country is in the serious international situation, which could drag us later, even unwillingly, to a war disastrous for Italy and specially for Veneto.

Professor Francesco Severi, University of Padova

P.S. – Several reasons have delayed almost for two weeks the publication of this letter. I do not intend to change anything, but – as my friend Gino Piva remarked yesterday in these columns too – seen that economic conditions of the proletariat in Veneto grow worse day by day, in a really painful and alarming way, I wish to express my vivid disapproval for the inertia of the Government, that seems not to have understood and not to understand that preparation does not mean only military provisions. How could we hope that popular masses would resist morally and materially to much harder proofs, if, following Italy's intervention in the European war, our region were to be subdued?

Extraordinary provisions (State works and subventions to Municipalities) are urgently required here in Veneto so as to face this serious crisis. Other than banning meetings!

9 March 1915 F.S.

There were obviously divergent positions from this majority line even among mathematicians, but they were mostly embodied in little gestures in professional everyday life or in private and personal communications. For example, in the first months of 1916 Volterra asked Somigliana to invite J. Hadamard to deliver a speech in Turin, and Somigliana was obliged to confess that C. Segre had raised doubts[15].

> Ne ho parlato a Segre, perché facesse lui l'invito, come preside. Ma mi ha sollevato parecchi dubbi. Intanto vuol sapere quando Hadamard potrà essere a Torino; poi sotto qual forma dovrà esser fatto l'invito. In conclusione la mia impressione è che egli è preoccupato del pensiero di dover fare una qualsiasi dimostrazione che non sia quella di una corretta accoglienza al matematico Hadamard.
> Purtroppo l'ambiente della nostra Facoltà è così; Segre poi lo intensifica per conto suo. Il concetto predominante è che si debba vivere come nel limbo dei Santi Padri, ignorando la guerra, privi di qualunque antipatia o simpatia per alcuno, salvo il dovuto rispetto ai tedeschi.
> Ora francamente io penso, che Hadamard sarà venuto in Italia per qualche cosa di più che una semplice esposizione di teorie analitiche; e che il metterlo a contatto con questi elementi potrebbe fargli riportare un'impressione del nostro paese, che non è quella che desideriamo[16].

The Paduan socialist Levi-Civita, was, together with Segre, out of the "chorus" – the only real discordant note in the interventionist positions of Volterra, Enriques, Severi, etc. His pacifism would never fade during the whole war, causing a remarkable cooling in the relationship with Volterra: their usually very friendly correspondence, in the war years, took on a formality more eloquent than any speech. Levi-Civita never broke off his relationship with German scholars, and he asserted his neutralism and his pacifism every time the opportunity came. On the 23rd of August 1916, for example, he wrote to G. D. Birkhoff[17]:

> Comme vous l'imaginez aisement, on ressent en Europe, bien plus qu'en Amérique, l'influence deprimente de la guerre sur l'activité et sur la collaboration scientifique: efforts, aspirations, jeunes énergies, et, en général, toute forme d'énergie

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[15] *Accademia dei Lincei* in Rome; Archives Volterra.

[16] I have asked Segre to make the invitation, as dean. But he has raised several doubts. First, he wants to know when could Hadamard be here in Turin; then, which form should the invitation take. In short, I think he is worried about having to display more than a right welcome to the mathematician Hadamard.
Unfortunately so is the faculty's atmosphere; Segre on his side intensifies it. The leading idea is that we have to live as in the limb of Holy Fathers, ignoring the war, with no sympathy or antipathy for anybody, except for the due respect for the Germans.
Now I sincerely believe that Hadamard has come to Italy for something more than a simple exposition of analytic theories; and that to get him in touch with these elements would give him an undesired impression of our country.

[17] The letter, unpublished, comes from the *Harvard University Archives*, Cambridge (Mass.). We seize the opportunity to thank Brian A. Sullivan, Reference Archivist, for putting at our disposal Levi-Civita's letters to Birkhoff.

sont sensibilisés par les évênements du jour; et malheuresement aucun symptôme ne laisse pas encore soupçonner une détente prochaine, capable d'assurer la justice en rétablissant la fraternité des peuples.

Noting an excessive enthusiasm towards the Allies' cause in a paragraph of a letter from Birkhoff dated 3rd December 1916 (probably lost), he replied gentlemanly but resolutely[18].

Il m'est bien agréable d'apprendre par votre lettre du 3 Décembre dernier que vous participez au concours de l'Istituto Veneto pour les mathématiques, expirant le 31 Décembre 1917. En m'en faisant part vous ajoutez des considérations qui témoignent de la délicatesse de vos sentiments. Je me regarde autorisé d'or et déjà à les faire connaître si par hasard quelque circonstance imprévue dût m'en montrer l'opportunité. A présent une telle opportunité n'existerait pas. L'Istituto a tenu, jusqu'à l'heure actuelle, à fonctionner, pour tout ce qui se rapporte à son activité strictement scientifique, comme dans les temps normaux. Si les choses continueront à se passer ainsi, il ne me paraît pas le cas de les compliquer avec des déclarations ou des réserves de votre part, malgré la noblesse d'esprit et la sympathie pour la cause des alliés, qui les inspirent.

Just in those months the United States got ready to take part in the war – they would declare war on Germany on the 5th April 1917 – and Birkhoff felt the need to clarify his position with as much elegance and firmness[19].

You refer in your letter to my sympathy for the Allies. I would be ashamed of my country if I did not believe that sympathy of the very deepest kind for their cause is felt by almost all Americans. The vote of Congress the other day is a testimony of this fact. Of all my colleagues at Harvard only two not of pure German blood and German born incline toward the other side; and even these keep very quiet. (I might say that I am entirely of Dutch descent, all of my great grand parents being born in the Netherlands.) Unless President Wilson vastly misunderstands American sentiment he will proceed at once to arm our ships and take other necessary steps to uphold our rights upon the seas which the Central Powers have so flagrantly violated. Personally I favor even more rigorous participation on our part. The Germans are a great people of course, but their success would be the defeat of civilization and the best interests of mankind.

On the 10th of April 1917, a few days after the USA entered the war, Levi-Civita closed the argument with a new profession of pacifism[20]:

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18 Letter to Birkhoff, 3rd January 1917, in the *Harvard University Archives*.

19 The letter, unpublished, comes from the Archives Levi-Civita at the *Accademia dei Lincei* in Rome. It is reproduced in P. Nastasi, R. Tazzioli, *Aspetti scientifici e umani nella corrispondenza di Tullio Levi-Civita (1873–1941)*,Palermo, Quaderni Pristem, No. 12 (2000), p. 201.

20 In the *Harvard University Archives*.

Mon cher Collègue,

J'ai bien reçu votre lettre du 7 Mars. Elle est extrêmement intéressante au point de vue scientifique, et une profession de foi sur la guerre, qui sévit depuis trois ans, clairvoyante, élevée, équitable. Les évenements vous ont donné pleine satisfaction. Il n'est point douteux que les principes, si noblement proclamés par votre Président, trionpheront dans et après la fin de la guerre. On doit seulement souhaiter que ce soit au plus tôt.

3. Mathematicians at the front

World War I was not a technological war, at least in the sense we usually give to this expression from World War II onwards (with the use of missiles and radar technique). It was obviously also a submarine and aerial war, with problems caused by the incredible production of guns and munitions, but it was above all a war of position and of wear and tear. It did not depend as much on the acquisition of chiefly new knowledge as on the ability to organize mass production. So researchers took part in the war not as such, but rather as *ordinary* citizens and patriots defending their country. They often were sent to the front. As scientists, they acted at the most as *referees* in the several projects presented by the various "inventors". In short, the consciousness of the utility of science and modern technology to win a war was raised in the tragic experience of 1914–1918, but not yet their first systematic application. And of course this newborn awareness was quite far from imagining the close relationship between science and defence that would be established twenty years later.

As with the rest of the world, the time of a planned involvement of scientists as such had not come for Italy either. War mobilization developed in a framework of great confusion and disorganization, with spontaneous research into a more rational use of resources. In July 1915, a *Comitato Nazionale Esami invenzioni attinenti al materiale di guerra*, legally recognized only in March of the following year, was established at Milan's Polytechnic. This was one of the very few research centres linked to the military apparatus through the testing of steel, wood, cement, compressed gas recipients, projectiles, parts of aeroplanes, etc. Other centres (apart from Medical and Pharmaceutical Institutes) were only the *Istituto di Chimica farmaceutica e tossicologica* in Naples and the *Istituto Geografico Militare* in Florence. The former produced the chloropicrin (nitro-chloroform), used as asphyxiating tear gas in the retaliation against the Austrian enemy, which in June 1916 had attacked the Italian positions using poison gas. The second saw to the realization and updating of cartographic material from the war theatres and to the training of artillery officers on the trigonometric-topographic procedures for fire direction.

We mustn't be surprised at the scarce use of mathematical knowledge. Perhaps the most relevant exception – certainly the most advertised one afterwards – was Mauro Picone (1885–1977), whom we will meet in the next chapters as one of the chief protagonists of Italian Mathematics in the years between the two World Wars. At the outburst of the first war, he was a young educated at the *Scuola Normale* of Pisa, where he graduated in 1907, having studied under Bianchi, Dini and E. E. Levi, among others. He was in Pisa until 1913, as Dini's assistant. Then he moved to Turin, teaching later at the Universities of Cagliari, Catania, Pisa again, and Naples, where he stayed from 1925 until 1932. Then he moved to Rome, where he settled himself definitively. As we said, in the pre-war years Picone was still a young mathematician. But he already had to his credit numerous publications with important and original contributions to ordinary and partial differential equations (with a few "concessions", also, to differential Geometry and to the strong influence that Bianchi's personality had). His most known memoir of the time was the dissertation for his teaching qualification (1910), in which he proved the so-called *Picones identity* for ordinary linear differential equations of the second order depending on a parameter:

$$\frac{d}{dx}\left[\theta(x,\lambda)\frac{dy}{dx}\right]+Q(x,\lambda)\,y(x)=0\ ,$$

repeatedly quoted and appreciated because of its simplicity and of the many results to which it led in several situations. This is how Picone himself spoke about his war experience in an autobiography[21] published only five years before his death.

> Chiamato alle armi, con la mia classe (del 1885), nell'aprile del 1916, fui assegnato al 6° Reggimento di Artiglieria di Fortezza, il cui Deposito era a Torino, col grado di sottotenente della territoriale, senza che io avessi mai prestato, in precedenza, servizio militare e avessi mai visto, da vicino, un cannone. Nel luglio del 1916, dopo aver perso un tempo prezioso a fare la scuola a piedi, fui inviato alla fronte di combattimento e assegnato alla I Armata, operante sulle montagne del Trentino. In ciò il caso, il puro caso, fu fortunato, poiché bastava che il Comando del Deposito di Torino, anziché alla I Armata, mi avesse inviato ad una di quelle operanti in pianura, sull'Isonzo, perché, come si vedrà fra poco, le mie qualità di matematico non avessero avuto modo di rivelarsi subito utili ed io fossi rimasto, forse per sempre, nella concezione puramente speculativa della Matematica.
> Presentatomi al Comando d'Artiglieria della I Armata, vi fui accolto con un freddo discorso, come questo: "*I depositi seguitano a mandarci ufficiali su ufficiali, dei quali non abbiamo bisogno. Non sappiamo, per ora, cosa farne di lei. Torni a presentarsi fra otto giorni. Cosa faceva da borghese?*". Io risposi che ero libero docente di *Calcolo infinitesimale* all'Università di Torino e me n'andai mogio e deluso. Allo spirare dell'ottavo giorno mi presentai a detto Comando e mi fu comunicato che il colonnello Federico Baistrocchi (…) si era dimostrato interessato ad avere alle sue dipendenze un ufficiale esperto in Calcolo, e che perciò ero stato assegnato a quel Raggruppamento che, con mezzi di fortuna, dovevo raggiungere in giornata.
> Dopo un viaggio, quanto mai fortunoso, arrivai, a notte inoltrata, al Comando al quale ero stato destinato e fui subito ricevuto dal Comandante, Colonnello Bai-

[21] M. Picone, *La mia vita*, Roma, 1972.

Mauro Picone in 1917 during the I World War

strocchi, che mi aspettava. (…) Io risposi al Colonnello Baistrocchi, forse anche non riuscendo a celargli il mio stupore, che non possedevo nozione alcuna di artiglieria e, tanto meno, del suo impiego tattico. Ma questi, e con ciò dimostrò di essere all'altezza della situazione, mi disse: "*Si tratta di risolvere un problema di calcolo e lei deve essere in grado di farlo, si tratta di calcolare i dati da fornire alle nostre artiglierie d'assedio, per il tiro contro bersagli per i quali le tavole di tiro regolamentari, che esse possiedono, non sono sufficienti*". Ma io, aggiunsi, non ho neppure nessuna nozione di Balistica, sulla quale, suppongo, devono fondarsi quei calcoli. Allora il Colonnello tirò fuori da una cassetta d'ordinanza un ingiallito voluminoso libro e mi disse: "*Qui c'è il trattato di Balistica di FRANCESCO SIACCI, le dò l'ordine di studiarlo e di ricavarne, entro un mese da oggi, il calcolo dei dati di tiro per le nostre artiglierie d'assedio, contro i capisaldi dello schieramento nemico*". E mi congedò.
Mi misi febbrilmente all'opera, dedicandovi anche la notte, all'incerto lume di una candela e presto riconobbi la giustezza delle opinioni del Colonnello Baistrocchi, pervenendo anche a spiegarmi le difficoltà, nel calcolo dei dati di tiro, incontrate dai nostri artiglieri, che non potevano essere da essi superate. Ecco come stavano le cose. Per il tiro d'artiglieria in montagna era previsto, nel precedente periodo di pace, l'impiego di cannoni del più piccolo calibro, detti appunto da montagna, trasportabili a dorso di mulo sulle più alte creste montane, cannoni che tiravano senza calcolo, a puntamento diretto, laddove, per la possibilità recente di costruire rapidamente solide strade, anche nell'impervio terreno montano, e di impiegare potenti autotrattrici che potevano trainare, anche su strade di forte pendenza, pezzi d'artiglieria di qualsiasi calibro e peso, si pensò – da noi e

dal nemico – di valersi, anche in alta montagna, del concorso del tiro di cannoni di medio e di grosso calibro e per distruggere le resistentissime fortificazioni dell'avversario ed anche, con un nutrito e preciso fuoco, per precedere, nell' offensiva, le proprie truppe d'assalto avanzanti o per sbarrare, nella difensiva, il passo a quelle nemiche dinanzi alle nostre linee. Senonché, le tavole di tiro regolamentari, in dotazione alle artiglierie di medio e grosso calibro, fornivano i dati di tiro per bersagli posti nello stesso piano orizzontale della batteria, consentendo lievi correzioni, dei dati stessi, ove si fossero verificati dislivelli, fra batteria e bersaglio, che non dovevano però superare certi limiti. Ora fra le gole del Trentino, questi limiti erano di regola sorpassati, ed anche sovente sorpassati fino a tal punto da essere il dislivello fra batteria e bersaglio dello stesso ordine di grandezza della loro mutua distanza orizzontale. Ciò constatato, potei facilmente determinare le cause dei disastri provocati dal tiro delle nostre artiglierie, che veniva, spesso, per fatali inevitabili errori di calcolo, centrato sulle nostre difese, anziché su quelle dell'avversario. Occorreva, senza indugio, rifare, con criterii tutti diversi, le tavole di tiro per le dette artiglierie, fondandosi su taluni perfezionamenti non immediati della Balistica razionale classica, ciò che non poteva essere conseguito che da un matematico.

Li ottenni nel mese prescrittomi e a cominciare dal successivo mese di settembre 1916 tutte le artiglierie del 21° Raggruppamento d'assedio tiravano correttamente con dati calcolati da me. (…)

Si può immaginare, dopo questo successo della Matematica, sotto quale diversa luce questa mi apparisse. Pensavo: ma, dunque, la Matematica non è soltanto bella, può essere anche utile[22].

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[22] Called-up with my class (1885's) in April 1916, I was assigned as second-lieutenant of the Territorial Army to the 6th Regiment of Fortress Artillery, whose depot was in Turin. I had never served before nor seen a cannon close by. In July 1916, after having wasted a precious time going to school on foot, I was sent to the front and assigned to the I Army, working on the mountains of the Trentino. This was a happy chance, a happy sheer chance, because if the Command of Turin's depot had sent me to one of the Armies working on the plain, over the Isonzo, instead of sending me to the I Army, my qualities as mathematician, as we will see soon, would not have had the possibility of turning out useful immediately and I would had been left, maybe forever, with a purely speculative conception of Mathematics.

When I reported to the I Army Artillery Command, I was received with a cold speech, such as: *"The depots go on sending us official after official, whom we don't need. We don't know what to do with you, now. Come back to report in eight days. What did you do as civilian?"*. I answered that I lectured in *Infinitesimal Calculus* at the University of Turin, and went away dejected and disappointed. By the end of the eighth day I reported to the said Command and was told that Colonel Federico Baistrocchi (…) was interested in having at his service an official expert on Calculus, and that for this reason I had been assigned to that Group, which I had to reach that same day by whatever means of transport was available.

After a quite eventful journey, I arrived, late at night, at the Command to which I had been posted and was immediately received by the Commandant, Colonel Baistrocchi, who was waiting for me. (…) I answered the Colonel Baistrocchi, maybe not being even able to conceal my astonishment, that I had knew nothing about artillery, let alone about its tactical use. But, proving to be able to cope with the situation, he told me: *"It is a calculus problem which you should be able to solve: to calculate the*

Actually, other mathematicians – for example Severi, E. E. Levi, Tonelli and Volterra himself – were involved in similar experiences too. They solved several problems on external ballistics and telemetry brought about by the quick development of artillery – the rifled cannon, which permitted longer and more precise fire; the adoption of curved-fire guns; etc. – and by the adaptation of the fire tables to the specific geographical setting of some war stages. The last sentence of the quotation turns Picone's experience into a significant one – beyond the personal and nationalistic emphasis of the narration. The sudden revelation of a mathematics which was *not only beautiful* but *can be useful too*, happened in years which were decisive for the building and completion of his scientific personality and career. The discovery of mathematics' utility was not a parenthesis which was to be closed when returning to civilian clothes and resuming an already well thriving activity, but the feature on which Picone decided to bet so as to get a specific qualification within the mathematical world. The foundation of the *Istituto Nazionale per le Applicazioni del Calcolo* would realize the intuition born during the war years.

Picone's contribution to the enhancement of the ballistic tables is to be valued as a *rara avis* in a war that opened up the world's collective imagination regarding

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[22] (continued)
data our siege artilleries need for the fire against targets for which the actual regulation fire tables are not enough." I added that I knew nothing about ballistics either, on which, I supposed, that calculation should be built. Then the Colonel took out from a regulation box a yellowed voluminous book and told me: *"Here you have FRANCESCO SIACCI's treatise on ballistics, I order you to study it and to calculate, within a month from today, the fire data our siege artilleries need against the strongholds of the enemy formation."* And he dismissed me.

I began to work feverishly, also overnight, under the changeable candlelight, and I soon admitted Colonel Baistrocchi was right, reaching even an explanation for the difficulties our artillerymen had with the calculation of the fire data, and that they could not overcome. It was like this. In the previous peacetime it was set for the artillery's mountain fire the use of small calibre cannons, so-called mountain cannons, which could be transported on the back of a mule over the highest mountain peeks and fired with no calculation, by direct pointing. But now, thanks to the recent possibility of building solid roads quickly even in the inaccessible mountain terrain and of using powerful auto-tractors that could tow artillery pieces of every calibre and weight even in strong sloped roads, we – and the enemy too – had thought of using, even in high mountain, middle and big calibre cannons, which, thanks to their fire range, could destroy the enemy's highly resistant fortifications and, with an intense and accurate fire, could during the offensive precede the advancing assault troops or block the way to the enemy troops before our lines in the defensive. Regulation ballistic tables, with which middle and great calibre artillery were equipped, gave fire data for targets positioned on the battery's same horizontal level, allowing to slightly correct the data itself in case of steeps between battery and target, but only to some extent. Within the Trentino's gorges these limits were usually overstep, often so much than the steep between battery and target was as long as their mutual horizontal distance. Once this was established, I could easily determine the reasons for the disasters caused by the artillery fire, which, because of fatal unavoidable calculation errors, often stroke not the enemy but our defensive works. The artillery ballistic tables had to be redone without delay, with quite different criteria, based on some improvements of classical rational ballistics, which were not direct and could be obtained only by a mathematician.

I got them within the prescribed month and from the following month, September 1926, all Artilleries of the 21° Siege Group fired correctly using the data I had calculated. (…)

After this success of mathematics, one can guess I saw things in a different light. I thought: but, then, mathematics is not only beautiful, it can be useful too.

the future and "modernity". A marginal experience within an event with quite a different meaning but an experience that would deeply influence his scientific personality and the organization of Italian mathematics. We find the same features in Volterra, whose leadership within the mathematical world was further strengthened by the *great war*.

A keen interventionist – as we have seen – when Italy went to war Volterra immediately volunteered. In 1915 he was 55! In the following months, he distinguished himself in risky deeds on Zeppelins, whose optimization he helped also from a technical point of view: he suggested the use of helium instead of inflammable hydrogen, prepared a means for photo-telemetric surveys and personally experimented with the installation and use on board of a 65 mm calibre cannon so as to fire upside downwards. His commitment can be retraced in a letter to Mittag-Leffler[23] of May 1916[24].

> Vous me parlez d'un congrès de mathématiques en Suède cette année et d'un voyage en Suisse pendant le printemps. Je vois que vous ne vous faites pas une idée de l'état d'âme en Italie. Ce n'est pas le moment de voyager. Toutes nos pensées sont tournées à la guerre que nous combattons avec le plus grand enthousiasme à côté de nos alliés et nous ne pensons qu'a rapprocher l'instant de la victoire définitive contre nos ennemis. Nous sommes sûrs de la victoire et nous espérions dans un avenir heureux pour notre patrie qui n'a hésité à se placer du côté de la justice et de la liberté.
> Je suis engagé dans l'armée et je suis officier du Génie. Mes occupations militaires et techniques dans le corps d'aéronautique absorbent maintenant toute mon activité. Mes connaissances de mathématiques et de physique me sont utiles dans ce moment.

A year later, he wrote to an Italian physicist[25]:

> Chiarissimo Professore,
> in risposta alle sue lettere del 1 e 4 maggio, sono lieto che Ella dimostri tanta attività e spero che l'opera sua potrà essere di efficace aiuto alla difesa del Paese.
> In modo speciale hanno vivamente interessato gli studi che Ella ha intrapreso di un microfono subacqueo, per la segnalazione di navi e sottomarini[26]. Spero che la pros-

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23 About his efforts to create a better climate between German and Allied mathematicians after the war, it is possible to read: W. Dauben, "Mathematics and world war I: the international diplomacy of G. H. Hardy and Gösta Mittag-Leffler as reflected in their personal correspondence", *Historia Mathematica*, 1980, pp. 261–288.

24 The letter, unpublished, comes from the *Archives of Mittag-Leffler Institute*.

25 The letter, addressed to Michele La Rosa, has been published in P. Nastasi (ed.): *Lettere a Michele La Rosa (1903–1932)*, Science History Seminar of Palermo's Faculty of Science, 1991.

26 In 1917 the problem of the localization of submarines (both with magnetic surveys and through ultrasounds, as the French physicist P. Langevin proposed) was on the agenda of almost all belligerent countries. The works developed on this subject by R. A. Millikan and the American physicians are well-known.

> sima chiusura dei corsi di quest'anno scolastico, possa permetterLe di proseguire questi studi che sono particolarmente importanti nelle presenti circostanze.
> Per quanto riguarda la proposta di un congegno per lo scoppio dei proiettili sott'acqua contro i sottomarini, la quale pure sembra notevole, la miglior cosa sarebbe che Ella mandasse senz'altro, in doppia copia, una nuova relazione ed i disegni relativi, senza tener conto dei precedenti che non hanno rapporto col nostro Ufficio.
> Infine per quanto riguarda lo scudo-corazza Pagano, esiste già in questo Ufficio un parere sfavorevole di uno scudo-corazza-zaino Pagano, presentato il 10 ottobre 1916 ed esperimentato dalla Scuola di Applicazione di Fanteria di Parma. Se si tratta della stessa invenzione, e non di omonimia, non potrebbe esser presa in considerazione, a meno che, nel modello costruito nel suo Istituto, non siano state apportate modificazioni e correzioni a quello già esaminato a Parma.
> Nella speranza che le ricerche così felicemente da Lei iniziate, possano presto portare un contributo attivo e fortunato ai lavori di questo Ufficio, Le porgo i miei distinti saluti[27].

There is a good summary of this stage in Volterra's activity in Edmund Whittaker's commemoration (published in 1941 among the *Obituary Notices of Fellows of the Royal Society*).

> Before describing the scientific work of the last twenty-five years of Volterra's life, let us take up again the thread of his personal history. In March 1905 he was created a Senator of the Kingdom of Italy – a great honour for a man still comparatively young – and about this time he was appointed by the Government as Chairman of the Polytechnic School at Turin, and Royal Commissioner. The way was open for him to become a great figure in political and administrative life: but he preferred the career of a pure scientist, and took an active part in public affairs on only two occasions – the Great War of 1914–1918, and the struggle with Fascism.

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[27] Most distinguished Professor,
replying to your letters from the 1st and 4th May, I am pleased You are so dynamic and I hope your work will effectively help to defend our country.
Above all, your studies about an underwater microphone to signal ships and submarines are certainly interesting. I hope the next closure of this academic year's courses will allow you to continue these studies, especially important in the present circumstances.
Regarding the proposal of a device for the underwater detonation of projectiles against submarines, which is remarkable too, it would be better if You could send, in a double copy, a new report and the concerning designs, without considering the previous ones which have no relation with our Office.
Lastly, concerning the shield-armour Pagano, this Office gave already an unfavourable opinion of a Pagano shield-armour-rucksack, presented on the 10th of October 1916 and experimented by the Infantry School of Application of Parma. If it is the same invention, and not an homonymy, it won't be considered, unless the model built in your Institute has been modified and corrected in relation to the one already examined in Parma.
Hoping that the researches You have so satisfactorily begun could soon bring an operative and successful contribution to this Office's works, I send You my kind regards.

> In July 1914 he was, according to his custom at that time of year, at his country house at Ariccia, when the war broke out. Almost at once his mind was made up that Italy ought to join the Allies: and in concert with D'Annunzio, Bissolati, Barzilai and others, he organized meetings and propaganda which were crowned with success on the 24th of May in the following year, when Italy entered the war. As a Lieutenant in the Corps of Engineers he enlisted in the army, and, although now over fifty-five years of age, joined the Air Force. For more than two years he lived with youthful enthusiasm in the Italian skies, perfecting a new type of airship and studying the possibility of mounting guns on it. At last he inaugurated the system of firing from an airship, in spite of the general opinion that the airship would be set on fire or explode at the first shot. He also published some mathematical works relating to aerial warfare, and experimental with aeroplanes. At the end of these dangerous enterprises he was mentioned in dispatches, and decorated with the War Cross.
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> Some days after the capitulation of Gorizia he went to this town while it was still under the fire of Austrian guns in order to test the Italian instruments for the location of enemy batteries relative by sound. At the beginning of 1917 he established in Italy the Office for War Inventions, and became its Chairman, making many journeys to France and England in order to promote scientific and technical collaboration among the Allies. He went to Toulon and Harwich in order to study the submarine war, and in May and October 1917 took part in the London discussion regarding the International Research Committee, to the executive of which he was appointed. He was the first to propose the use of helium as a substitute for hydrogen, and organized its manufacture.
>
> When in 1917 some political parties – especially the Socialist – wanted a separate peace for Italy, he strenuously opposed their proposal: after the disaster of Caporetto, he with Sonnino helped to create the parliamentary bloc which was resolved to carry on the war to ultimate victory.

Whittaker's commemoration introduces what will be later seen as Volterra's biggest contribution during the war years. We have seen that the relationships between science, industry and military apparatus were still slender. In the allied countries, which had more developed military and state structures, the need was felt to go towards an explicit involvement. France, for example, already had, since 1894, a Commission for the examination of militarily interesting inventions. The physicist Mascart, the chemist Moisson and the mathematician P. Appell belonged to it. In 1914, just at the beginning of the war, the Commission – transformed to *Superior Commission of Inventions for National Defence* – was reinforced by the presence of technicians, members of Parliament and academicians. Presided over by Paul Painlevé, who would later become Minister[28] and would be replaced by E. Borel, it used the work of prestigious mathematicians (such as E. Borel, J. Hadamard, H. Lebesgue and P. Montel) and of physicists such as Cotton, Langevin,

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[28] Between 1915 and 1916, P. Painlevé was Minister of Education, Fine Arts and Inventions for National Defence (this Ministry was just created for him). In 1917 he was Minister for War.

Francesco Severi (1879–1961)

Vito Volterra

Perrin and Weiss. It used as well university and industry laboratories to develop the research on: anti-gas protection systems, enhancement of fire tables, sound localization of enemy batteries, submarine detection, etc. Volterra had the merit of understanding at once the importance of such initiatives (in France, but also in England and in the United States, for the war's outcome and a peaceful, hopefully near future) and of using many personal contacts to enter this "network".

In January 1917 he presented to the War Ministry a project for an *Ufficio Invenzioni e Ricerche*, following a mission in France, where he went "to pick up information and news on the present relationships between French scientific laboratories and military administration organs". The project was accepted two months later, but within a year Volterra would make over the *Ufficio Invenzioni e Ricerche* – the seed of the future *Consiglio Nazionale delle Ricerche* (C.N.R.) – into a scientific and technical consultancy organ directing the creation of an autonomous center of applied research[29]. The delay with respect to the allied countries (and their means) was sensible, but it was anyway the first achievement of a research institution on applied problems of national interest.

The first *Conferenza interalleata sulla organizzazione scientifica*, in which delegates of the Scientific Academies of the allied countries and of some neutral countries

[29] Cf. G. Paoloni, *Vito Volterra e il suo tempo (1860–1940)*, Roma, Accademia Nazionale dei Lincei, 1990, c. IV.

took part, was held in London at the end of the war, from 9th to 11th October 1918. In Italy, the *Accademia dei Lincei* delegated Volterra, also in his capacity as director of the *Ufficio Invenzioni e Ricerche*. Before the Conference, the American delegate G. E. Hale – Volterra's old friend since 1909, when he had gone to Rome to lecture on "Solar Vortices and Magnetic Fields" – had presented a proposal which aimed at transforming the several organizations that had appeared in the allied countries during the war to coordinate the research world and the industrial production world (on the model of the *National Research Council*). The "research national councils" thus created, should have, in turn, set up with their own delegates an "international council". The London Conference approved the proposal and created a Commission, formed by É. Picard, the English physicist A. Schuster (secretary), Hale, Volterra and the Belgian astronomer G. Lecointe, entrusted with studying which methods could be used to realize this council. In a second meeting in Paris (26th to 29th November), a provisional "*International Research Council*" was founded by instituting an executive committee which among other things had the task of preparing the draft for the statutes and of organizing the following conference in Brussels, from the 18th to the 28th of July 1919. It was on this occasion that the *International Council*, the general organ of the *International Unions* concerning the different disciplines, was officially presented. In each country, adhering to a *Union*, a corresponding *national Union* or *Committee* was to be formed, and all these would join together in a "Research National Council".

Volterra would tenaciously work in both directions: to constitute the *Unione matematica italiana* and to found the *C.N.R.*

Chapter 3

Volterra's leadership

1. Introduction

Everything had changed with the war, significantly. The deep transformations which had taken place in the relations between States, in the political scene and in the social field itself, involved also the mathematical community of Europe. Jean Dieudonné offered a synthetic but most effective view[1]:

> Jusqu'à la guerre de 1914–1918, les écoles française et allemande, dominées par leurs plus illustres représentants, H. Poincaré et Hilbert, génies universel d'une rare envergure, restent les plus nombreuses et les plus variées, et excercent en mathématiques une prépondérance incontestée. A leurs côtés, les foyers de recherche mathématique comptant les plus nombreux et les plus actifs participants sont l'Italie et l'Angleterre. La première brille surtout par ses écoles de Géométrie algébrique (Castelnuovo, Enriques, Severi), de Géométrie différentielle (Levi-Civita, E. E. Levi) et d'Analyse fonctionnelle autour de Volterra, et ne subira une éclipse (dont elle n'a commencé à sortir que récemment) qu'à partir de 1935 environ; tandis que qu'après Cayley et Sylvester, l'école anglaise, changeant de cap, se groupe à partir de 1910 environ autour de Hardy et Littlewood et va entrer pendant 30 ans dans une féconde série de découvertes sur l'Analyse classique et ses applications à la Théorie des nombres, avant de céder la place, à l'époque actuelle, à une brillante phalange d'algébristes et de topologues.
>
> Après 1918, la France, dont la jeunesse scientifique a été saignée à blanc per l'hécatombe, va se replier sur elle-même pendant 10 ans, et, à l'exception de E. Cartan et de Hadamard, l'école mathématique française se cantonnera dans le domaine restreint de la théorie des fonctions d'une variable réelle ou complexe; dont le développement considérable aux aléntours de 1900 avait d'ailleurs été surtout son

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[1] Cf. J. Dieudonné, Introduction to *Abrégé d'histoire des Mathématiques: 1700–1900*, Paris, Hermann, 1978, I–II, p. 1–17.

> oeuvre (avec Picard, Hadamard, E. Borel, Baire, Lebesgue, puis Montel, Denjoy, Julia). L'Allemagne, au contraire, qui a su mieux préserver la vie de ses savants, conserve intactes ses traditions d'universalité; en outre, elle voit éclore une remarquable école d'Algèbre et de Théorie des nombres (E. Noether, Siegel, Hecke, E. Artin, Krull, R. Brauer, Hasse, van der Waerden (d'origine hollandaise), qui inaugure la tendance axiomatique déjà en germe dans les travaux de Dedekind et de Hilbert. Entre 1920 et 1933, ces mathématiciens assurent aux Universités allemandes, où se pressent les étudiants de tous les pays (notamment les jeunes Français, venus renouer aves les traditions oubliées chez eux), un éclat et un rayonnement exceptionnels, qui seront malheuresement brisés brutalement par l'ère hitlerienne. Il faudra attendre ensuite les environs de 1950 pour que l'école allemande se reconstitue, influencée cette fois (par un curieux renversement de la situation) par les mathématiciens français de tendance «bourbachiste».
> Toutefois, le phénomène le plus marquant après 1914 est l'apparition sur la scène mathématique de vivaces écoles nationales dans des pays qui n'avaient guère connu jusque-là qu'un petit nombre de savants ayant atteint une renommée internationale. Dès avant la fin de la première guerre mondiale, il faut d'abord citer l'U.R.S.S. et la Pologne, d'où surgit brusquement une pléiade de mathématiciens de premier ordre (Lusin, Souslin, puis Urysohn, P. Alexandrov, Kolmogorov, Vinogradov, Pontrjagin, Pterowski, Gelfand en U.R.S.S.; Sierpinski, Janiszewski, Kuratowski, Banach, puis Hurewicz, Eilenberg, Zygmund, Schauder en Pologne); c'est à leurs efforts que l'on devra surtout le développement des fondements de la Topologie et de l'Analyse fonctionnelle modernes. En U.R.S.S., l'élan ainsi donné ne s'arrêstera ps, et a continué à produire de très nombreux mathématiciens de grande valeur; quant à la Pologne, dont la moitié des mathématiciens ont été massacrés par les nazis, elle n'a commencé que récemment à combler ses vides et reprendre sa marche en avant.

Some of the elements presented by J. Dieudonné had already been displayed by A. Denjoy in his speech at the *Réunion Internationale des Mathématiciens*, held in Paris in 1937 on occasion of the *Exposition internationale*[2]:

> Depuis la guerre mondiale de 1914–1918, la production mathématique a cru en intensité dans de très forte proportions. Le fait a été moins sensible dans les régions appartenant à des pays constitués avant 1914 que dans celles dont les nouveaux Etats ont été formés. Dans ces derniers, un nationalisme très vif, mais de la nature la plus louable, a poussé les gouvernements et les peuples à la fondation de nombreuses universités dont le personnel professoral s'est pris d'une très noble émulation pour rivaliser avec les représentants des écoles mathématiques étrangères les plus réputées, et pour tenter, souvent avec succès, de les surpasser.

[2] Cf. A. Denjoy, Aspects actuels de la pensée mathématique, *Conférences de la Réunion Internationale des Mathématiciens*, Gauthier-Villars, Paris, 1939, pp. 1–12.

The "great war" brought about conspicuous changes in the mathematical world (too): an unusually rapid generational imbalance occurred in some of the most developed countries (due to the especially tragic war) and the birth of new national schools which often followed a strong and aggressive course. Fortunately, the first element affected Italy only slightly[3]. More than 2000 university students died in the war – we don't know how many of them had begun studying Mathematics – but among the researchers mentioned in the *Prologue*, who before the war had already won public favour, only E. E. Levi died in the front, during Caporetto's retreat (1917)[4].

Mathematicians on the Italian scene after the war were essentially the same as before it, but the atmosphere had certainly changed. The end of war made the resumption of normal life possible, but in a disenchanted atmosphere in which institutions and people felt acutely the (material and moral) injuries of a lethal war. The doubt that modernity could slide into barbarity, subverting the consolidated hierarchy of custom and power, had entered society. The happy dream of the beginning of the century had ended, and now one had to reckon with a much harsher reality. That dream was of universal progress thanks to scientific achievements; in reality one woke up wondering whether or not to resume scientific meetings and relationships with colleagues from the defeated countries.

The question of relationships with German researchers blocked the mathematical community for almost ten years. International relationships in general returned to a balanced if precarious normality only in 1928, with the Congress of Bologna. There were "hawks" and "doves". Among the former were most French mathematicians (particularly E. Picard) and some Italian ones of Volterra's "calibre". Picard, in his address for the closing meeting at the VI Mathematicians International Congress (Strasbourg, 23–30.9.1920), stated[5].

> En ce qui regarde spécialement notre Congrès, nous n'avons jamais dissimulé que nous entendions lui donner une signification particulière, en le réunissant à Strasbourg. Aussi avons-nous été extrêmement touchés de l'empressement avec lequel nos amis étrangers ont répondu à notre appel. (…) Des liens plus intimes ont été formés, qui resteront précieux. Nous continuerons ainsi, entre peuples amis, nos travaux scientifiques, apportant dans cette collaboration nos qualités diverses,

[3] The Italian situation was better than the French one. According to J. J. Gray (*The Hilbert Challenge*, Oxford Univ. Press, Oxford, 2000) more than 40% of students in Mathematics or in Sciences were killed or blessed in France during the war.

[4] E. E. Levi took part in the war as a volunteer.

[5] The quotation is taken from the *Comptes Rendus du Congrès International des Mathématiciens* (Strasbourg, 22–30 September 1920), Toulouse, 1921, p. XXXI–XXXIII. Among French mathematicians, E. Picard (1856–1941) was one of the most resolute nationalists (he wrote among other things a book with the significant title *L'Histoire des Sciences et les pretensions de la Science Allemande*, Perrin, Paris 1916). He had lost a son in the war and took an almost racist position. Hadamard, despite having lost two sons in the war, had a more moderate position. French nationalist mathematicians decided that it would be symbolically appropriate for the disputed Strasbourg (newly passed from German into French hands) to host the International Congress of 1920, from which German mathematicians had been excluded.

> sans qu'aucun prétende exercer une insupportable hégémonie et sans nous soucier de certaines menaces, qu'avec une impudeur qui ne nous étonne pas, on a osé proférer.
> Quant à certaines relations, qui ont été rompues par la tragédie de ces dernières années, nos successeurs verront si un temps suffisamment long et un repentir sincère pourront permettre de les reprendre un jour, et si ceux qui se sont exclus du concert des nations civilisées sont dignes d'y rentrer. Pour nous, trop proches des événements, nous faisons encore nôtre la belle parole prononcée pendant la guerre par le cardinal Mercier, que, pardonner à certains crimes, c'est s'en faire le complice.

The view coming from the other side of the Rhine was in stark contrast with this one, as documented in the letter[6] of the German mathematician R. Rothe to M. de Franchis (second president of the *Circolo Matematico di Palermo*, after the founder, G. B. Guccia), dated 24th March 1921.

> Sono stato ripetutamente da Lei invitato di indicare il mio indirizzo. Io Le confesso francamente che sono stato per molto tempo in dubbio al riguardo, perché del Circolo Matematico sono ancora soci persone come E. Picard, Ch. de La Vallée-Poussin ed altri, che fino al momento attuale hanno manifestato il loro odio contro tutto ciò che è tedesco. Come esempio cito soltanto l'istituzione d'un così detto congresso internazionale di Matematica nell'autunno 1920, da cui erano intenzionalmente esclusi i Matematici tedeschi e che in segno di speciale sarcasmo contro di loro è stato tenuto nella vecchia città tedesca di Strasburgo nell'Alsazia.
> Però dopo che Ella mi ha ripetutamente e in maniera così amichevole invitato a pronunciarmi, non voglio più a lungo tacere, e invece voglio manifestarle i miei più sentiti ringraziamenti per la Sua gentilezza, ed esprimere la mia speranza che il Circolo Matematico possa riuscire mediante i suoi sforzi a ripristinare le antiche pacifiche relazioni fra i Matematici, per il bene della nostra scienza, che dovrebbe unirci tutti[7].

The President of the *Circolo Matematico di Palermo* opposed the request of French mathematicians and some of the Italian ones, that German members be expelled from

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6 Archives of the *Circolo Matematico di Palermo*. The letter is obviously written in German, but it is translated in Italian on the margins. Rothe taught at the time at the *Technische Hochschule* in Berlin.

7 You have often asked me to give my address. I sincerely confess You that I have hesitated for a long time, because people as E. Picard, Ch. de La Vallée-Poussin and others, who have until now manifested their hate against everything which is German, are still members of the Mathematical Circle. As an example I quote only the creation of a so-called International Mathematics Congress in Autumn 1920, which intentionally excluded German mathematicians and which, with special sarcasm against them, was held in the former German city of Strasbourg, in Alsace.
But as You have frequently and friendly asked me to pronounce on the matter, I won't keep silent anymore; I wish to warmly thank you for Your gentleness, and to express my hope that the Mathematical Circle will restore the old peaceful relationship among Mathematicians, through your effort, for the good of our science, which should join us all.

the *Circle*. He also refused the resignation that M. Noether offered "in gentleness". In the meanwhile, some – like Bianchi – proposed an intermediate solution[8].

> Riguardo ai Soci tedeschi, la questione è molto complessa e delicata. Intanto, per evitare attriti e proteste, abbiamo deciso di rinviare ancora l'elezione del nuovo Comitato di Redazione dei Rendiconti. Personalmente, io ritengo che, come non vorrei essere tenuto responsabile di una impresa abominevole eventualmente condotta da una minoranza di faziosi col concorso del popolo suggestionato abilmente con falsi miraggi, così non posso addossare su tutto un popolo e tanto meno su una eletta schiera di studiosi il peso degli orribili misfatti dei quali si è macchiato l'imperialismo germanico e dei quali, tra parentesi, sarebbe capace di macchiarsi qualsiasi altro imperialismo spinto alle ultime conseguenze da un manipolo di persone interessate. Per me, il principale nemico dell'umanità è l'imperialismo il quale costituisce ai nostri tempi un anacronismo e solo per il tornaconto di un esiguo gruppo di furbi che si arricchiscono e godono sui lutti e sulle miserie della grande maggioranza. E ritengo che inconsciamente i popoli che hanno lottato contro l'imperialismo siano ora scaltramente incanalati verso di esso; un indice è lo stato di inestinguibile odio che si vuole perpetuare non fra umanitari e imperialisti ma fra popoli. Converrei quindi nell'idea di radiare quei soci che avessero firmato il manifesto dei 93 intellettuali, perché li ritengo indegni di coltivare relazioni con gente civile, ma non posso convenire, per esempio, che sia giusto radiare un Hilbert, che fin dal principio della guerra ha fatto sapere di disapprovarla, solo perché Hilbert è nato in terra germanica. Insomma che non si voglia avere contatti con persone di sentimenti ignobili è giusto, ma che debba esserci anche il *peccato originale* del luogo di nascita, non mi pare che possa sostenersi. Ma queste sono idee mie e posso anche sbagliare. Ciò che però è fuori di dubbio è che le distinzioni che si vogliano fare tra gli scienziati a seconda del paese di origine, toglieranno per lungo tempo alla Scienza il carattere internazionale, togliendo ad una parte dell'umanità i frutti del lavoro di un'altra parte (…). E badi che dopo ciò, tra qualche anno, la collaborazione scientifica è fatale che si riattivi, ma intanto la nostra Società sarà morta[9].

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8 Bianchi's letter to de Franchis, dated 1st March 1919, is kept in the Archives of the *Circolo Matematico di Palermo.*

9 The question about German members is quite complex and delicate. We have decided to postpone again the election of the new Reports Editing Committee, meanwhile, so as to avoid conflicts and protests. I would not like to be held responsible for an abominable deed led, if anyone, by a minority of factoids supported by the folk, which has been ably persuaded with false mirages; at the same time, I think that an entire folk, and certainly not some elected scholars, cannot be blamed for the terrible misdeeds with which Germanic imperialism has stained itself and with which, incidentally, every imperialism driven to its last consequences by venal people could stain itself. In my opinion, mankind's main enemy is imperialism, which is in our times an anachronism and fits only a few crafty people who enrich and enjoy themselves thanks to the mourning and miseries of the vast majority. And I think that unconsciously nations which have fought against imperialism now move shrewdly towards it; the intent to perpetuate an everlasting hate not among humanitarians and imperialists but among peoples is a sign of this. I would agree then to the idea of striking those members who signed the 93 scholars manifesto off, because I think they do not deserve to have any contact to civilized people, but I don't

Bianchi's proposal involved, obviously, a researcher of the value of F. Klein. De Franchis, of course, did not know the reasons that could be adduced to defend the behaviour of other German mathematician, whom André Weil remembered in his autobiography[10].

> To conclude with the "manifesto", I will note here what I learned much later, in Germany. First of all, Hilbert, who always behaved with the utmost dignity throughout the war, refused to sign it – though I do not think that his name was familiar enough to me in 1922 for me to notice that it was absent from the list of signatories. Second, I have been told that many of those who signed, including Felix Klein, had not seen the text; they had simply been asked over the telephone to support what was put forth as a patriotic duty. Only those who have no inkling of how petitions, protests, and declarations of all sorts are peddled among the intelligentsia would find this surprising.

In the *Prologue* and at the beginning of Chapter II, Volterra has been identified as the main *leader* of Italian Mathematics in the pre-war time. His image was strengthened by the conflict. He had "bet" on intervention and on a "democratic interventionism", against German "barbarity". During the war, besides his military actions – already mentioned – and a remarkable commitment to convey and use scientific knowledge, he promoted also cultural initiatives which were to consolidate a social alliance that he thought of as much more than military. Thus, for instance, in 1916 he promoted an *Associazione italiana per l'Intesa fra i Paesi alleati e amici* to favour the exchange of information, teachers and students among the Universities of the allied countries. The facts and the outcomes of the battlefield proved he was right. His "philosophy" was strengthened too. It favoured development of mathematics within the boundaries of an abstract and deeply innovative research, which in the way to its formalizations, however, had to refer always to applications. He realized that the interaction between science, on one hand, and social and productive fields, on the other, would become a driving force for industrialized countries. The role of science, both on the economic and on the cultural level, would lead to defining and experimenting with new organizational forms of research and of scientific communities, with more funds, more power and more social responsibility. In 1915 Volterra, during H. Poincaré's commemoration held at the *Rice Institute*, affirmed that science was no longer the exclusive product of a few highly privileged sci-

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9 (continued)
think it right, for example, to strike off Hilbert, who since the beginning of the war said he disapproved of it, only because Germany is his native soil. In short, I think it right to avoid having contact with ignoble people, but the idea of the birthplace *original sin* cannot be approved of. However, this is my opinion and I may be wrong. There is no doubt, though, about the fact that if we discriminate between scientists because of their mother country, science will be deprived of its international character for a long time, as a sector of mankind will be deprived of the results of another sector (…). And listen to me: scientific collaboration will be inevitably resumed within some years, but in the meanwhile our Society will be dead.

10 Cf. A. Weil, *The Apprenticeship of a Mathematician*, Birkhäuser, Basel-Boston-Berlin, 1992, p. 38.

entists; it had become a collective enterprise which needed, besides conceptual thinkers, a host of specialists who would lead research in an organized and coordinated manner. Lastly, Volterra's personal prestige would come out of the war further consolidated. He was, actually, the representative of Italian Mathematics in all organisms and assemblies that, during the war and in the months immediately following its end, organized Europe's new scientific assets on an institutional level.

The first accomplishments of Italian Mathematics and science after the war – the *Unione Matematica Italiana* (U.M.I., 1922) and the *Consiglio Nazionale delle Ricerche* (C.N.R., 1923) – were published in his language and took his name. Before exploring this topic, however, we would like to dwell upon an apparently minor episode which happened in 1921 and concerned Rome University. It was a "usual" conflict of interest in the academic world but, later on, it would turn out to be anything but negligible, due to its consequences.

2. Rome, 1921

In our *Prologue* we had occasion to cite the leading role played by Rome on a cultural and scientific level by virtue of its political standing as the capital of Italy. With regard to mathematics, this politics began to function early on with the "call" to the capital of personalities such as Cremona, Beltrami, Castelnuovo and Volterra. At the beginning of the 20th century, the same politics prevailed, though it had its ups and downs; a few first-rate researchers did not accept the transfer because they found the city's life (and scientific atmosphere) to have hard rhythms, and some second-rate researchers took their places. One refusal from an "excellent" choice was that of Levi-Civita, who in 1909 cold not be persuaded to leave the quiet Paduan environment and his family for what became known as the "big leap". Anyway, it was the pair Castelnuovo and Volterra that ran the operations there and represented the largest academic power centre within the Roman Institute.

After the war, and encouraged by the nationalist and patriotic climate, the political pull "towards the capital" resumed more vigorously. For a professor, the "call" from Rome was always a moment of great prestige, often the culmination of his career. Levi-Civita himself – in the meanwhile married (1914) and temporarily in Rome after Caporetto's defeat of 1917 – thought it over and moved there in 1918. Then, on the 29th December 1920, an analyst died[11] and the race for the succession began. Castelnuovo's letter to Levi-Civita, dated 4th January 1921, that is, a few days after his colleague's death[12], reports it.

> Carissimo amico,
>
> Ghigo [Federigo Enriques] mi ha parlato del suo desiderio di venire ad occupare una delle cattedre lasciate vacanti dal povero Tonelli (l'Algebra). La notizia mi ha fatto molto piacere e mi ha fatto anche piacere che egli, risparmiandomi dei passi

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[11] Alberto Tonelli, not to be confused with Leonida Tonelli.

[12] The letter has been published in P. Nastasi, R. Tazzioli, *Aspetti scientifici e umani nella corrispondenza di Tullio Levi-Civita (1873–1941)*, Quaderni Pristem, 12, 2000, p. 263–264.

Enriques

Francesco Severi

penosi per la mia posizione delicata, abbia discorrere discorso con vari colleghi di Facoltà. Sembra che egli abbia trovato buone disposizioni anche presso il Volterra, del quale, per ragione di scuola, si poteva temere qualche contrasto. Mi par di capire che il Volterra, il quale mi ha parlato della cosa, accoglierebbe volentieri Ghigo all'Algebra, purché accettassimo la sua proposta riguardo il Calcolo. Sai che il Volterra, anche prima che tu esprimessi il desiderio di venire a Roma, aveva pensato per l'Analisi al nome di Leonida Tonelli; ed ora, negli ultimi giorni di malattia del nostro collega, mi ha ripetuto lo stesso nome per le cattedre che si sarebbero rese vacanti. Più che alla persona il Volterra tiene all'indirizzo; egli tiene che a Roma sia rappresentata la teoria delle funzioni di una variabile reale nei suoi ultimi perfezionamenti; ed io, pur non avendo speciali simpatie per questo indirizzo, non so dargli torto nell'aspirazione che ogni importante indirizzo di matematica abbia il suo rappresentante nella nostra Facoltà. Perciò credo che egli non rinunzierebbe, se non forzato, a desistere dal suo proposito, e dubito se ci convenga forzarlo, dato pure che si riuscisse, quando, come in questo caso, egli adduce delle ragioni plausibili della sua idea.

È quasi superfluo dirti quanto piacere mi farebbe la venuta di Severi, insieme a quella di Ghigo. Son legato al Severi da cordiale amicizia, mentre conosco appena il Tonelli; l'indirizzo del Severi lo giudico molto più importante di quello del Tonelli; e, a parte l'indirizzo, ritengo molto superiore il valore del primo rispetto al secondo. Per tutte queste ragioni, a cui tu pure alludi nella tua lettera, dovrei sostenere la candidatura Severi, e lo farei con tutto il cuore se non sentissi in Volterra una opposizione, per pregiudizio di scuola, che non credo opportuno di

> combattere in questo momento. Bada *per pregiudizio di scuola* e non per la persona, giacché, avendo detto a Volterra, prima di ricever la tua lettera e di conoscere la *specifica* aspirazione del Severi, che questi desiderava alla prima occasione di venire a Roma, il Volterra mi disse che sarebbe ben lieto di appoggiarlo appena l'occasione si presentasse.
> Ti prego dunque, nell'esprimere al Severi il mio rincrescimento per le difficoltà che incontra il suo trasferimento a Roma nel momento attuale, di dirgli che il suo nome è molto ben accetto tra i colleghi di qui, matematici e non matematici, e che, appena si renda scoperta una cattedra che ci sembri a lui adatta, terremo conto del suo desiderio che è anche il nostro.
> Quanto al Bianchi non credo sia il caso di pensarvi, perché a Pisa ha la direzione della Scuola Normale, e perché egli più volte mi disse che a Roma egli non si trova bene, per la vita tumultuosa e agitata che vi si conduce[13].

So the "candidates" for the suddenly free chair were three. Actually, the Roman Faculty had also suggested some older authoritative colleagues like Bianchi and Pincherle. The "short-list", though, soon shrank to Tonelli (Leonida), Enriques and Severi. The first one had already distinguished himself before the war for his works on real analysis and his important contributions to the Calculus of Variations; his candidacy was supported by

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[13] My dearest friend,
Ghigo [Federigo Enriques] has told me about your wish to occupy one of the professorships left vacant by poor Tonelli (algebra). I was very pleased by the news and by the fact that he has talked to several colleagues of the faculty, sparing me painful steps because of my delicate position. Even Volterra, who could contrast it because of school reasons, seems favourably inclined. I think that Volterra, who told me about it, would willingly admit Ghigo in algebra, if we accept his proposal regarding calculus. You know that Volterra, even before you expressed your wish to come to Rome, had thought of Leonida Tonelli for analysis; and now, during the last illness days of our colleague, he often named him for the shortly vacant professorships. For Volterra, the branch is more important than the person; it is important that the function theory of a real variable and its last progresses are represented in Rome; though I do not have special sympathy for this branch, I cannot blame him for his ambition to have in our Faculty a representative of each important branch of mathematics. This is why I think that, if he can, he will not abandon his intentions, and I don't think it worthwhile to force him, even if we managed to, as in this case his reasons are praiseworthy.
I don't need to tell you I would be very pleased if Severi, as Ghigo, comes. Severi is a good friend, while I hardly know Tonelli; I consider Severi's branch much more important than Tonelli's; and, apart from it, I think the former is quite above the second. For all these reasons, which you mention in your letter too, I should support Severi's candidature, and I heartily would if I did not suspect that Volterra, because of a school prejudice which I do not feel like fighting right now, is against him. Mind that I say due to *school prejudice* and not due to the person, for when I told Volterra, before receiving your letter and knowing Severi's *exact* aspiration, that he wished to come to Rome at the earliest, Volterra said he would be very glad to support him as soon as the chance came.
Please inform Severi that I am sorry for the trouble he finds now in moving to Rome; tell him that he is welcomed by the colleagues here, mathematicians and not, and that, as soon as a professorship suited to him is vacant, we will consider his wish, which is also ours.
As for Bianchi, we don't need to think about it, since he is director of the *Scuola Normale* in Pisa, and he told me several times he does not feel good in Rome, because of the tumultuous and restless life of the city.

Leonida Tonelli

Volterra himself, who wanted absolutely an analyst to obtain the professorship in analysis. Enriques was partially supported by Levi-Civita, as well as by his brother-in-law Castelnuovo; Levi-Civita, however, "spent" his authority above all on Severi (with whom he had an excellent relationship of esteem and friendship, consolidated in the familiar surroundings of Padua).

It was a delicate situation in which several aspects had to be considered: analysis and geometry; particular situations like Enriques', who wanted to leave Bologna for some of the usual personal reasons; family relationships (Enriques and Castelnuovo) and school relationships (Severi had been a pupil of Enriques), in a framework rendered even more delicate by the unquestionable quality of the three competitors. These were researchers of great prestige who were being measured by an international standard; to complicate matters, one of them, Severi, had become in the meanwhile chairman of the *University Association*, which grouped university professors for professional and syndicalist purposes, and was about to become a member of the *Superior Council of Education*.

This pressing debate among the Faculty actually engaged Roman mathematicians for almost two years. Out of "gentleness" reasons, Castelnuovo did not participate openly, so that Volterra and Levi-Civita are the main actors in the "scene". The former organized[14]

[14] Here Volterra was helped by Giovanni Vacca, whom we'll find in Chapter V, when talking about historical studies.

for the first time (only he could afford it) a national lobbyist action of analysts. With several letters and other personal interventions, he denounced to colleagues the "rapacity" of geometers. He turns it into a matter of principles – e.g., teaching posts in analysis had to remain with analysts – which obviously Levi-Civita did not share[15].

> Al prof. Volterra segue il prof. Levi-Civita che rileva che il prof. Volterra ha impostata la questione da un punto di vista, degno indubbiamente della massima attenzione, ma che, secondo lui, non è il solo. Egli ritiene che, in linea di massima, vi sia un criterio preponderante cui ispirare la nostra condotta e le nostre decisioni. Coprire le cattedre vacanti cogli elementi di maggior valore che si possano nel momento attuale attrarre a Roma, assicurandosi soltanto che all'altezza scientifica si accompagni l'eccellenza didattica, onde rimangano soddisfatte in maniera ineccepibile le esigenze molteplici dell'insegnamento[16].

In the end, Levi-Civita and Severi "succeeded". In Rome nobody would talk about Tonelli anymore, at least for some years. Enriques instead arrived almost immediately, "reintegrated" with the "sacrifice" of Castelnuovo who voluntarily relinguished the chair in superior geometry he had held for twenty years[17].

> Il Preside legge una lettera inviata dal prof. Castelnuovo, il quale dichiara che non è intervenuto alla seduta giacché si tratta di una questione che lo riguarda personalmente; ma che, se fosse stato presente, avrebbe consigliato i Colleghi di valersi della cattedra di Geometria superiore per assicurare alla Facoltà uno scienziato del valore di Federigo Enriques, benché egli non si distacchi senza dolore da questo insegnamento e senza la speranza di riprenderlo un giorno[18].

Volterra had lost his battle. The minutes of the Faculty Councils tell of his perseverance, but also of the many defeats he encountered, on this occasion, within *his own* Faculty. Severi's "call" from Rome is a minor episode, pointed out because of the "unsuccess" of Volterra and the first manifestations of Levi-Civita's personality, who, just arrived in Rome, did not hesitate a moment to oppose – with placating but resolute tones – a charismatic and powerful *leader* such as Volterra. Levi-Civita went on thinking

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15 From the minutes of the Science Faculty Council of Rome University, dated 18.3.1921.

16 After Prof. Volterra comes Prof. Levi-Civita, who notes that Prof. Volterra has set out the question from an undoubtedly very interesting point of view, which is though not the unique one, according to him. He thinks that, on the whole, there is a preponderant criteria on which to base our behaviour and our decisions. That is, to cover the vacant teaching posts with the most valued elements who can currently be attracted to Rome, making sure only that scientific quality goes with didactic excellence, so as to perfectly satisfy the multiple exigencies of teaching.

17 From the minutes of the Science Faculty Council of Rome University, dated 15.2.1923.

18 The Dean reads a letter sent by Prof. Castelnuovo, who explains that he had not participated in the meeting because it deals with a question that regards him personally; but that, if he had been present, he would have advised his colleagues to make use of the teaching post in superior geometry to assure the Faculty of a worthy scientist such as Federigo Enriques, though he detaches himself from this teaching with sadness and hopes of recovering it one day.

to Volterra as the *princeps mathematicorum* – as he writes in a letter of 13.11.1918 to Volterra – but, anyway at the time Levi-Civita was a well-known researcher himself. His paper "Sur la regularisation du probléme des trois corps" received important acknowledgements. G. Mittag-Leffler stated that the article had pathed the way to a long series of studies. G. D. Birkhoff writes that "your treatment of the problem of regularization is definitive".[19] Later, Levi-Civita published the well-known memoir on parallel displacement where – as E. Cartan will say – he succeeded in bringing the fundamential notions of the tensorial calculus "jusqu'alors purement analytique dans le domaine de la Géometrie". In a 1938 report on his works, sent to E. Borel and to E. Picard for his election as a *associé étranger* of the *Académie des Sciences*, Levi-Civita wrote:

> A vrai dire cette notion de parallélisme et son application presque immediate á la dérivation des vecteurs avaient été conçues par l'auteur comme un couronnement ou mieux (si l'on peut s'exprimer ainsi) comme le vernissage du développement de la géométrie différentielle des variétés, telle qu'elle s'était constituée en corps de doctrines, depuis Gauss et Riemann (á travers Lamé, Christoffel, Darboux, Beltrami et beaucop d'autres) jusqu'á Ricci. Mais ce vernissage a été un point de départ pour des nouvelles conceptions géométriques, dont M. Cartan a su tirer des théories d'une rare élégance et des liaisons des plus fécondes avec la théorie des groupes de Lie. Je fais allusion aux transports des vecteurs le long d'une courbe, qui a été, par Weyl, Schouten, Veblen er surtout par Cartan, détaché de la méthode de l'espace ambient, et s'est développé comme géométrie différentielle des connections, l'existence d'un ds^2 riemannien constituant pour ces investigations une hypothése tout á fait particuliére et souvent inessentielle.

In the same year T. Y. Thomas explained Levi-Civita's idea as reported down below:

> In the case of a two-dimensional surface S in a three-dimensional euclidean space, where the idea of Levi-Civita has its strongest appeal, the producedure, in brief, may be described as follows. Let F be a developable surface tangent to S along C (envelope ot the one-parameter family of tangent planes to S along C.) Let ξ_P be a vector in the tangent plane to S at a point P on C. Roll F on a plane so that C becomes a curva C′ in the plane, P a point P′ and C′, and ξ_P a vector $\xi_{P'}$ at P′. Displace $\xi_{P'}$ along C′ by parallel displacement in the ordinary sense (parallel displacement in a euclidean plane). We thus define vectors $\xi'(t)$ along C′ parallel in the ordinary sense. Now wrap the plane about the surface S along C to secure the original developable surface F. Thereby the vectors $\xi'(t)$ go to into vectors $\xi(t)$ tangent to S along C. We define the vectors $\xi(t)$ which are in the surface S, that is, in the tangent plans in S along C, to be parallel with respect to C and in fact to result from the original vector ξ_P at P by parallel displacement along C. Levi-Civita's definition of parallel displacement of a vector in a surface S generalizes the ordinary euclidean

[19] The letter from G. D. Birkhoff to Levi-Civita (26.10.1916) is kept in the Archives of the *Academia dei Lincei.*

> concept of parallel displacement in the sense that if S is a plane (in general an n-dimensional euclidean space) the parallel displacement is identical with the ordinary euclidean displacement.

The concept of *parallel displacement* play an important role in the development and in the strengthing of some ideas on the relativity. We quote the 40 notes written by Levi-Civita just on the relativity but an equally important contribution can be found in the letters with Einstein from March to May 1915: "a such important correspondence never happened to me – Einstein wrote to Levi-Civita on 02.04.1915 – you would have to see how anxiously I wait for your letters". It was the German physician M. Abraham (1875–1922) – who had met Levi-Civita at the International Congress of Rome in 1908 – who introduced the two scientists each other. The Italian mathematician had pointed out a mistake in an important proof of *Entwurf*; from this the covariant properties of the gravitational tensor would be deduced. Einstein tried many times to oppose himself to Levi-Civita's critics but at the and he was obliged to admit that the colleague was right. Then, after a long and hard period of research, he was able to publish the note on *Preussische Akademie der Wissenschaften* where he proved the right version of gravitational equations. He admitted: "they represent a true triumph of Ricci's Calculus". The difficulties of this method were balanced by the originality of the results that it allowed to get in the theory of the relativity.

The episode we told is a minor event that would bring about important consequences, though. Severi had arrived in Rome as a valuable mathematician, educated man (even if he did not have Enriques' prominence and cultural interests) and ex-socialist who was just beginning to enter the centres of power. Volterra had found the future alternative to his *leadership* at home.

3. The foundation of the Unione Matematica Italiana

There was obviously nothing which at once seemed to give to Severi's move to Rome and to his presence in the country's political centre any more importance than a routine university change. Volterra went on carrying through the institutional program which he had further developed during the war years.

His first achievement, at the beginning of the 1920s, was the *Unione matematica italiana* (U.M.I.), which appeared not so much as a drive from within the mathematical community (as it had been for other Italian professional societies, such as, the physical or chemical societies), but due to the vote expressed in Brussels by the *International Research Council* at the meetings held between the 18th and the 28th July 1919. As indicated at the end of Chapter II, the wish to have a *Union* for each discipline in every allied country had emerged in Brussels. The U.M.I. appears concretely first in a circular written by Volterra in March 1920.

> È vivo desiderio di molti studiosi di costituire una unione italiana la quale raccolga i matematici, analoga a quelle già fondate per le scienze chimiche, per le scienze

> astronomiche e per l'Oceanografia.
> L'Unione si propone:
>
> a) l'incoraggiamento alla scienza pura –
> b) il ravvicinamento tra la matematica pura e le altre scienze –
> c) l'orientamento ed il progresso dell'insegnamento –
> d) l'organizzazione, la preparazione e la partecipazione a congressi nazionali ed internazionali.
>
> Questa unione si propone altresì di ricongiungere e coordinare le forze delle altre associazioni nazionali esistenti[20].

Volterra intended the circular to be signed by about ten mathematicians (among them Bianchi, Pincherle, Somigliana and obviously Volterra himself). At the end, some of the expected signatures were not obtained, maybe due to the same reasons appearing in a letter[21] received by Levi-Civita in October 1920.

> So che ti sei meravigliato di vedere il mio nome tra le firme della circolare d'invito per una unione dei matematici. In verità mi sono meravigliato anch'io. Non prestai sufficiente attenzione a quanto mi disse rapidamente il Sen. Volterra, e non pensai che l'unione potesse mirare – come ora mi si dice – ad una azione antitedesca. Aderii verbalmente senza dare importanza alla cosa e senza credere d'impegnarmi con una firma. Questo ti dico confidenzialmente e pregandoti di non farne parola al Volterra, perché infine non posso incolpare che me stesso. E te lo dico perché tu abbia direttamente la conferma di quanto avevi felicemente intuito sulla portata della mia azione. Ma io credo e spero che l'Unione resti un pio desiderio, e che la germanofobia sia, per questo lato almeno, un male passeggero[22].

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[20] It is a strong wish of many researchers to create an Italian union which gathers mathematicians together, a union similar to those already founded for chemical sciences, astronomical sciences and Oceanography.
The Union intends to:
a) encourage pure science –
b) draw pure mathematics and other sciences closer –
c) guide and improve teaching –
d) organize, prepare and participate in national and international congresses.
This union also undertakes to gather the forces of other current national associations together and to coordinate them.

[21] The letter, dated 5th October, and written by a mathematical physician from Bologna, Pietro Burgatti, is kept in the Archives of the *Accademia dei Lincei*.

[22] I know you were astonished at seeing my name among the signatures in the circular inviting to a union of mathematicians. Actually, I was astonished too. I didn't pay much attention to what Senator Volterra said to me quickly, and I didn't think that the union could aim – as now they say – to an anti-German action. I agreed verbally without giving much importance to it and without thinking to commit myself with a signature. I say this to you confidentially, so please do not to tell Volterra, because in the end I am the only to blame. I only want to confirm you what you had rightly guessed about the importance of my action. But I believe and hope that the Union will be a pious wish, and Germanophobia a fleeting evil, at least on this side.

The excessive political anti-German portrayal was the main difficulty the U.M.I. had to face in its gestation period. But there were also other difficulties, such as the unavoidable rivalry which arose with the present mathematical and scientific associations (especially with the *Circolo matematico di Palermo*) and the resulting need to show the usefulness of the new association, distinguishing its content and scope. The same could be said about the project of *Bollettino*, with which the *Union* should provide itself, within a journalistic-mathematical scene in which there were several other headlines, many of them with a common generalist tendency.

It is necessary to return to Pincherle, mentioned in the *Prologue*, so as to talk about the problem of the presidency. A first-class analyst and researcher in the theory of analytic functions, Pincherle was one of the "founding fathers" of functional analysis. In fact, his "golden age" were the decades straddling the two centuries. After the war, the decline in scientific activity forced him to concentrate mainly on institutional roles and organizational tasks. The underlying theme of his research remained, however the study of linear functionals through their intrinsic operational properties, independently of particular analytic expressions.

His two posthumous memoirs summarize the research he did in his last twenty years. Pincherle thought, with clear satisfaction, that a number of studies he had developed during decades of research, which had not always been really appreciated, were being reintroduced by modern Physics. In a Note of 1926, for instance, he wrote: "the triumph of the discontinuous in natural Philosophy, with statistical mechanics, with the new views of the constitution of matter, with the theory of the quanta, etc, cannot help but bring about a revaluation of that branch of analysis which stands out exactly because of its discontinuous changing of subject".

The U.M.I.'s presidency represented another difficulty for the birth of the association because it came from above and it was unilaterally chosen by the *Accademia dei Lincei*, maybe so as to avoid any discussion. Volterra informed Pincherle officially about his successful appointment on the 18th March 1921.

> Mi pregio di comunicarLe che si è costituita la "Unione Matematica Italiana" la quale entra così a far parte della "Unione Matematica Internazionale" che insieme alle altre Unioni Scientifiche, compone il "Conseil International de Recherches". Sono lieto di aggiungere che la Presidenza della "Unione Matematica Italiana" è a Lei affidata; ed a Lei è pure connessa la nomina del Segretario della Unione stessa; nomina della quale, a suo tempo, Ella vorrà dare comunicazione al prof. Emilio Picard Presidente del "Conseil International"[23].

Pincherle intensified his pursuit of recovering dissidents or anyway of the "half-hearted", with particular care for Levi-Civita. The following letter, dated 12th April 1922,

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[23] I am glad to inform you that the *"Unione Matematica Italiana"* has been founded, being therefore member of the *"Unione Matematica Internazionale"*, which along with the other Scientific Unions forms the "Conseil International de Recherches". I am pleased to add that the chairmanship of the *"Unione Matematica Italiana"* has been entrusted to you; and related to you is also the appointment of the Union's Secretary, which, to its due time, you will communicate to Prof. Emil Picard, Chairman of the "Conseil International".

was addressed to him. A few days afterwards, when writing to Volterra, Pincherle would regretfully notice that "among the list of the sometimes enthusiastic adhesions, Rome is quite sparsely represented".

> E poiché ho occasione di scriverti, mi permetto di rivolgerti una preghiera; ed è di non fare cattivo viso all'iniziativa che, a preghiera d'alcuni colleghi e quasi *à son corps défendant*, ho dovuto prendere, di costituzione dell'Unione Matematica Italiana. Si tratterebbe di aver fra noi qualcosa d'analogo alla *Deutsche Math. Vereinigung* o alla *Société Mathématique de France* e a quanto hanno da noi i fisici, i chimici, i geodeti, ecc. La mia azione è affatto provvisoria, ma ci ho messa tutta l'anima, coll'intendimento, appena si raggiunga un numero sufficiente di soci, di lasciare l'impresa nelle mani d'una presidenza che verrà eletta dai soci stessi e dovrà essere formata d'elementi giovani e fattivi: la mia è dunque pura opera d'*éclanchement*, e per riuscire, conto sul ben volere e sull'appoggio dei colleghi. Spero dunque che non vorrai rifiutare la tua adesione – che va mandata alla ditta Zanichelli di qui – non solo, ma che vorrai darci, per uno dei primi numeri del futuro bollettino, un tuo scritto, sia pure brevissimo, ma che porti la tua firma. Sei così versato nell'argomento che ora interessa sopra tutti il mondo scientifico, che lo scrivere due righe su qualche problema connesso alla teoria della relatività, deve essere per te cosa di nessuna fatica, e ti saremo estremamente grati d'una risposta favorevole.
> Ti prego pure di volere interessare a fare parte dell'Unione i cultori della Matematica, anche applicata, fra i quali conti un così grande numero d'amici e di ammiratori. Uno degli scopi che, secondo me, l'Unione si deve proporre, è d'abbassare, per quanto è possibile, la barriera fra la scienza pura e le applicazioni. E se il promotore dell'impresa ti sembra troppo impari alla riuscita, pensa che formata che sia e posta in mani autorevoli, l'Unione Matematica potrà rappresentare un organo importante in avvenire, per la Scienza che coltiviamo[24].

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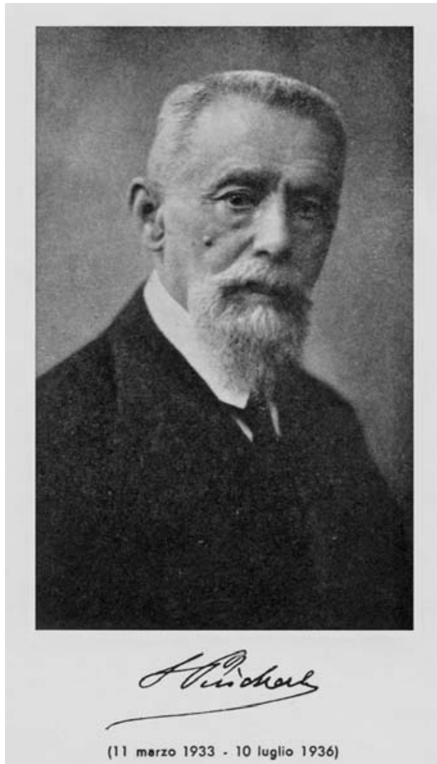
[24] And as I have the opportunity to write to you, I take the liberty to ask you not to oppose the initiative that I have had to take, by request of some colleagues and almost *à son corps défendant*, of constituting the *Unione matematica italiana*. The point is to create here something similar to the *Deutsche Math. Vereinigung* or to the *Société Mathématique de France*, similar to what our physicists, chemists, geodeticists, etc. already have. This move is absolutely provisory, but I have given myself up to it, and, as soon as there are enough members, I intend to leave it in the hands of a presidency elected by the members themselves and composed of young and efficient people: mine is a pure work d'*éclanchement*, whose success relies on my colleagues' esteem and support. I hope then that you will give us not only your support – to be sent to the company Zanichelli here – but also, for one of the first numbers of the upcoming bulletin, one essay, even if very short, but with your signature. You know so well the subject that now chiefly interests scientific world, that writing two lines on any problem related to the theory of relativity will be no effort for you, and we will be extremely thankful for a positive answer.
I also ask you to drag researchers of mathematics, and applied mathematics, among which you have so many friends and fans, into becoming members of the Union. I think one of the aims of the Union is to lower, as far as possible, the barrier between pure science and applications. And if the promoter of the undertaking seems to you too unfit for the success, think that, once it has been created and put in authoritative hands, the Mathematical Union will be, in the future, an important organ for the science we cultivate.



Salvatore Pincherle

In this letter Pincherle included an especially important passage: his determination to reach not only a simple representation in international projects, but to create a real professional association, with an autonomous structure, similar to that of other existing societies (the *Société Mathématique de France*, founded in 1872, the *Deutsche Mathematiker-Vereinigung*, founded in 1890, and the *American Mathematical Society*, founded in 1891). It was a decisive moment in the history of Italian Mathematics. When numbers increase, the professional association makes the functioning of well organised disciplinary communities possible, both by creating an agreement around *standards* of scientific quality and by regulating the distribution of resources according to scientific status. Finally, changing its direction by partially and progressively smoothing the most critical positions[25], the U.M.I. set out on its voyage. Volterra had made it! On the 31st March 1922, Pincherle made up his mind and sent to his colleagues a circular introducing the programme of the proposed society.

[25] Levi-Civita agreed at the end of 1922. In 1926 – perfectly coordinated with the deliberation of the *International Research Council* of abrogating the exclusion clause against German scientists – most Italian mathematicians were members of the U.M.I.

The first years of the *Union* were not very fascinating. Pincherle handed the management of the didactic aspects over to *Mathesis* (and to Enriques) and receded into the background with respect to the *Accademia dei Lincei*, as regards the school's reform (presided over by Giovanni Gentile). The only wide-ranging act was the organization of the *Congresso Internazionale*, held in Bologna from the 3rd to the 10th of September 1928. In this Congress the intransigent work for the scientific internationalism found finally a resolute affirmation.

The series of International Congresses of mathematicians was resumed (after the war) with the ones in Strasbourg (1920) and Toronto (1924), from which mathematicians of German, Bulgarian, Austrian and Hungarian nationality had been excluded. But already in Toronto – with France's strong opposition – the delegates of the United States, Great Britain, Italy and other countries had proposed the abolition of the preclusion. Pincherle did everything he could to reach a normalization of the international situation. On the 1st September 1925 he wrote to Volterra thus:

> Carissimo amico,
>
> ricevo ora la tua gradita lettera del 29. Non ho qui il testo esatto della mozione presentata a Toronto, il 15 agosto 1924, dalla Delegazione americana, e fatta propria dalle delegazioni di altri 8 o 9 Stati, fra cui l'Italia, ma era formulata all'incirca così: "La delegazione degli U.S.A., all'unanimità, invita il Consiglio internazionale di ricerche ad esaminare se non sia giunto il momento di modificare in senso meno restrittivo l'ammissione di altri Stati al Consiglio stesso".
> Ora una lettera del Polya, che è stato parecchi mesi a Cambridge e che ha conferito coi maggiori matematici inglesi, mi avverte che questi, e in particolare la London Math. Society, in accordo in ciò colla American Math. Soc., sarebbero propensi a dichiararsi ostili ad un Congresso internazionale che mantenesse le esclusioni.
> Specialmente se la Germania verrà ammessa alla Società delle Nazioni, l'atteggiamento del Consiglio Internazionale, più che poco simpatico, sarebbe addirittura puerile, a giudizio del pubblico anglosassone. (…)
> A giorni scriverò una lettera circolare a tutti i componenti del Consiglio di presidenza dell'Unione, per prendere accordi circa alla prima preparazione del futuro congresso, e per sentire se la maggioranza è d'accordo di riunirsi, a Parigi o a Ginevra, per trattare di questi accordi. (…) Ma prima sarebbe bene che la questione dell'ammissione venisse ripresa e risoluta in senso liberale, diversamente, la crisi è indubbia[26].

[26] My dearest friend,
I have just received your letter of the 29th. I don't have here the exact text of the motion presented in Toronto by the American Delegation on 15th August 1924, which was approved by the delegations of 8 or 9 countries, among which Italy, but it said something like: "The U.S.A. delegation, to the unanimity, invites the International Research Council to consider whether the moment has come to modify in a less restrictive sense the admission of other countries to the Council itself".

At the meeting of the 26th June 1926, the *International Research Council* decided to abolish every restriction. The U.M.I. was given the all- clear and could invite German mathematicians to the Congress in Bologna. However, the sequels of particularly conflictual years were long lasting. On the German side, some were unwilling to accept the olive branch and, a few days before the opening of the Congress, the Dutch L. E. J. Brouwer published an open letter, recalling the words with which years before Painlevé had justified ostracism towards German scientists and polemically wondering: "according to these words, how can a mathematician think of participating in the planned congress without deriding the memory of Gauss and Riemann, the cultural nature of mathematical sciences and the independence of human spirit?" Thus the German mathematical community split into two groups: the first one around Hilbert, supporter of the participation and coherent upholder of scientific internationalism[27], the other around Brouwer, diametrically opposed. With him sided mainly Berliner mathematicians (Erhard Schmidt, Ludwig Bieberbach and Richard von Mises), representing the most resolutely nationalistic tendencies and German science's values as typical.

The Congress was regularly held, at last, as Italian mathematicians intended. The *Minutes* registered the participation of 836 mathematicians, among them, 76 Germans (the most numerous group after the Italians). In the German delegation, representing – with R. Courant and E. Landau – Göttingen University and its *Gesellschaft der Wissenschaften*, appeared also David Hilbert, who was appointed Chairman of the Congress and gave the first general lecture: *Probleme der Grundlegung der Mathematik*.

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[26] (continued)
A letter from Polya, who has been in Cambridge for several months and has conferred with the greatest English mathematicians, warns me that these, and in particular the London Math. Society, agreeing with the American Math. Soc., would be inclined to oppose themselves to an International Congress that would keep the exclusions.
The behaviour of the International Council, in the opinion of the Anglo-Saxon public, would then be not only unpleasant, but even childish, specially if Germany will enter the Nations' Society. (…) In a few days I will write a circular to all members of the Union's Presidency Council, to make arrangements for the first preparation of the upcoming congress, and to see if the majority approves to convene in Paris or in Geneva, in order to discuss about this accordance. (…) But first it would be better to take up again the question of the admission and to solve it in a liberal sense; otherwise there will certainly be a crisis.

[27] During the spring of 1928, Hilbert wrote: "We are convinced that pursuing Herr Bieberbach's way will bring misfortune to German science and will expose us to all justifiable criticism from well disposed sides. (…) The Italian colleagues have troubled themselves with the greatest idealism and expense in time and effort. (…) It appears under the present circumstances command of rectitude and the most elementary courtesy to take a friendly attitude towards the Congress" (cf. O. Lehto, *Mathematics Without Borders. A History of the International Mathematical Union*, Springer, New York, 1998, p. 46). One can read S. L. Segal, *Mathematicians under the Nazis*, Princeton University Press, Princeton, 2003, pp. 349–355, for a detailed historical reconstruction of the polemics of German mathematicians about the Bologna Congress.

4. The foundation of the Consiglio Nazionale delle Ricerche

The C.N.R. (*Consiglio Nazionale delle Ricerche*) was Volterra's second great achievement in the first half of the twenties. It was the culmination of a long activity and expressed the deep tuning existing between the still – in those years – uncontested *leader* of Italian mathematical world and the trends of European research.

At the end of Chapter II we mentioned the *Ufficio invenzioni e ricerche* (accomplished by Volterra in 1917), the inter-allied Conference on scientific organization (London, October 1918) and G. E. Hale's proposal for the launching of the *International Research Council* (taken place at Brussels conference in July 1919). The foundation of the C.N.R. emerged out of this process: the C.N.R. was to be the Italian expression of the *International Research Council*, and its aim was to rationalize and develop the first achievements on scientific organization, which arose with the war.

The birth date was 18th November 1923, day in which the royal decree that founded the C.N.R. in Rome as a non-profit making company was published. The gestation period turned out to be particularly long, if we consider that Volterra began working on a first draft of the statute in February 1919. The causes of such a delay are evident[28] at once if we consider red tape slowness, the continual changes of government and the very delicate time that Italy was going through just in those months. The statute – finally approved in October 1924 – underlined the national role of the new organism, which was not only the Italian ramification of the *International Research Council*: the

Middle thirties CNR building in Rome

[28] The difficulties of that period were not inferior in the other countries. In France, for example, *CNRS* was created just in 1939.

C.N.R. had to "coordinate and stimulate the national activity in the different branches of science and its applications; be in touch with the different state centres for scientific matters; manage and eventually establish general and special research laboratories".[29] The self-government board was represented by a Council which elected, among its members, the President and the Secretary General. The Council itself chose the Administrator.

On 12.01.1924 Volterra was elected unanimously as chairman of the C.N.R. It was his moment of greatest prominence. That same year he was elected president of the *Accademia dei Lincei*, of which he had been vice-president in the previous three-year period. He was the "grey eminence" of the U.M.I. (which acted as the C.N.R.'s *mathematical Committee*), and the most authoritative person of the S.I.P.S., without considering presences and presidencies in other scientific societies and a really considerable political weight in several international organisms.

The statutes of the C.N.R. reflected the great expectations of its promoters. The conditions of academic research had to be improved. It was nominally free, but in reality heavily constrained by scarcity of resources and the compartmentalisation imposed by the university administration, and by the personalities of its faculty. At the same time, research had to be focused on the great national problems, overcoming the modest aspirations of the few existing public boards and laboratories. During its first years, the C.N.R. certainly operated beneath the level implied by such ambitious expectations. The determining factor was the scarcity of financial resources. The support provided by the government was hardly enough to cover current expenses, which included the functions of the executive organs and payment of dues to international organizations. At least until 1925, the C.N.R. contributed especially to the basic expenses of the U.M.I., undertaking also the burden of the "missions" of Italian mathematicians involved in several international congresses. With such scarce means, the plan for an extra-university research structure could never be executed; those plans were for the creation of a great national technical-experimental laboratory that would overcome the chronic dysfunction of the existing small and dull university laboratories, which were always weighed down by scarcity and the need for didactics.

Four great Institutes had been planned: one for chemistry and the industries depending on it; a second one for physics, electronics and mechanics; a third one for biological sciences and a forth one for those sciences closely related to agriculture. But money would never be found. Most professors opposed the project; though they were obviously favourable to greater resources, they preferred the "indiscriminate distribution" system to a politics that would have seriously risked marginalizing many university institutes.

But other tiles must be introduced into the mosaic to wholly understand this progress. Volterra's chairmanship in the C.N.R. ran from 1923 to 1926. These years – as recalled in the next chapter – saw a radical change in Italy's political structure: the C.N.R. and its chairman would no longer be circumscribed by the cultural atmosphere and changing political choices of newly appointed executives.

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[29] For what concerning CNR's history – in particular its first years – one can read: *Per una storia del Consiglio Nazionale delle Ricerche* (R. Simili, G. Paoloni eds.), I–II, Laterza, Bari, 2001.

5. Volterra's scientific activity

Volterra was born in 1860. After the war he was then in his sixties. His personal habits and his institutional activity, developed with a feverish rhythm, make almost predictable the observation that his study activity had not the intensity and the originality of the preceding decades.

During the 1920s and 1930s, Volterra abandoned gradually the research field, and engaged instead in summary conferences, expository and review articles or monographs that subsumed organically a whole series of previous studies (adding sometimes some significant element). Many were the issues of interest: *Le calcul des variations, son evolution et ses progrès, son rôle dans la physique mathématique*[30], for instance, is a highly instructive article which reconstructed some stages of the history of calculus of variations. The methodological elements that distinguish Volterra's style come up again, almost with obstinacy. The "old lion" did not stop having his say. He reintroduced the passage from the discrete to the continuous and from real analysis to functional analysis; he defended his definition of the derivative for a functional in opposition to Hadamard's and Fréchet's outlook, and above all he continually reminded his younger and more "modern" colleagues that "n'est qu'en donnant droit de cité à des éléments formels que plusieurs branches de l'Analyse ont pu avancer". Volterra, specially in the first twenties, was mostly interested in the theory of the composition of functions:

$$\overset{*}{f}\,\overset{*}{g} = \int_x^y f(x,z)\, g(z,y)\, dz\,.$$

About this argument he wrote, together with J. Pérès, some notes and a monograph, that resume some pre-war studies in which the theory of composition (firstly motivated by the solution of integral equations and by the study of some questions of hereditary mechanics) was totally autonomously developed as first study of an algebra of operators.

Volterra's minor presence on the most strictly scientific side was very natural – the "great anomaly" were his biological studies, examined in the next paragraph – and just as natural was the fall of the whole "old guard" of the analysts introduced in the *Prologue*. The only remarkable exception was Tonelli, who will be discussed in more detail later. The period of greater commitment and originality of Peano, disappeared in 1932, was already far away; from the beginning of the century his main interests were others. About Pincherle we have already spoken[31].

Vitali was in the same situation as Peano, Pincherle and Volterra, even if through a different path. For long years clientele and insensibility of the academic world had not allowed him a university career, to the point that H. Lebesgue could be ironically satisfied about the Italian situation, so prosperous as to "forget" in high school teaching valued mathematicians such as Vitali. After the war, finally settled at University, Vitali's scientific attention was gradually attracted by the research on absolute differential calculus and

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[30] It is the text of the lectures given in 1931 at Prague University "Charles" and Brno University "Masaryk".

[31] G. Ascoli and C. Arzelà died, respectively, in 1896 and 1912.

on differential geometry. His production on analysis was some ten notes and a few didactic or popular speeches, to conclude with a monograph on real functions. It was not, as for Volterra or Pincherle, a matter of "simple" re-elaborations of previous studies, maybe requested and prompted by a younger colleague. There was commitment and originality. But Vitali's publications in analysis usually had not the same impetus, which is confirmed by the number of publications and by the change of his research barycentre. He resumed with the usual ability and neat elegance the speeches left almost suspended at the beginning of the 20th century. In the meanwhile, though, the discipline had significantly developed and Vitali, in spite of his effort to "keep up to date", had inevitably lost his position and centrality. As for his commitment to follow the most recent contributions, the references to some works of S. Banach and the contacts established with the Polish school stand out (in the view of the Italian analysis of the time). In the recently published[32] correspondence there are letters of O. Nikodym and W. Sierpinski. Vitali became a member of the *Société Polonaise de Mathématique*; one of his papers appeared in the Proceedings of the *I Congrés des Mathématiciens des Pays Slaves*[33], and he had also some articles published in *Fundamenta Mathematicae* and in the *Annales de la Société Polonaise de Mathématique*.

His most committed memoirs of the time were "Analisi delle funzioni a variazione limitata" of 1922 and "Sulle funzioni continue" of 1926. In the first one Vitali studied the structure of bounded variation functions of one variable, starting from the established result according to which such a function could always be written as the sum of its jump function (that "absorbs its discontinuities") and a continuous bounded variation function: if the latter "is absolutely continuous, that is an integral function, the structure of $f(x)$ can be considered to have been adequately identified". Otherwise, Vitali introduced a discard function, "which in some way shows to what extent the given function diverges from absolute continuity", and came to the main representation theorem: each bounded variation function can be written as the sum of its jump function, an absolutely continuous function and a linear (eventually infinite) combination of peculiar, so-called elementary, discards. The letter of M. Fréchet dated 30th March 1923 referred to this memoir: he observed how its central part "semble indiquer que vous n'avez pas eu connaissance des travaux où une telle décomposition a été déjà obtenue", and quoted the classic book of C. de La Vallée Poussin (whose third edition was from 1914!) and one note of his from 1913. Fréchet's request was very polite, but explicit enough: "au cas où vous jugeriez utile de publier tout ou partie de la présente lettre, je n'aurais aucune objection". Vitali took up the invitation promptly. His answer was particularly "amiable" and Fréchet, at this point, did not skimp praises. He underlined the "historical" importance of Vitali's example of a continuous bounded variation function but not absolutely continuous, and appraised "la décomposition d'un "scarto" en une somme dénombrable de "scarto" élémentaires que vous établissez dans votre dernière mémoire (…) nouvelle et intéressante".

[32] *Lettere a Giuseppe Vitali* (M.T. Borgato, L. Pepe eds.) in G. Vitali, *Opere sull'Analisi reale e complessa,* Cremonese, Roma, 1984.

[33] Warsaw, 1929.

A similar "incident" involved his second main memoir of the time. In this case results were so synchronized that Vitali had no special responsibility, though. He proved the remarkable proposition commonly known, still today, as a theorem of *Banach-Vitali*: given a real function (of one variable) which is continuous from (a, b) into (c, d) and specifying that $\Gamma_r \subseteq (c, d)$ is the set of values which have at least r counter-images, the necessary and sufficient condition for f to be a bounded variation function is that it converges the series of measures of the sets Γ_r; the sum of this series gives the total variation of f, anyway. As far as the last terms in the series are concerned, a new definition of total variation was seen: even if it didn't match the usual one when applied to discontinuous functions, anyway it allowed one to rewrite the definition of absolute continuity and, above all, to extend such concepts to two variable functions. For Vitali it was really a pity that W. Sierpinski, when accepting the memoir for publication in *Fundamenta Mathematicae*, remarked how some results were to be found also in a note by S. Banach published in the journal's previous volume. Hence the statement opening Vitali's Memoir:

> mentre correggo le bozze ho da segnalare la recente importantissima nota di Stefan Banach (…) nella quale l'autore con analoghi intendimenti sebbene con procedimenti diversi, ha conseguito risultati che collimano con quelli del presente lavoro. Sono lieto che l'opinione dell'illustre collega dell'Università di Leopoli e la mia concordino nell'indicare la via per estendere alla superficie i noti risultati sulla rettificazione delle curve ed in particolare il bel teorema di Leonida Tonelli[34].

Fubini's case further confirmed the *old guard*'s gradual estrangement from the research *mainstream*. Its exponents, who obtained great achievements in real analysis at the beginning of the century, considered that time almost definitely closed and went towards new stimulating research (to which they could apply their mastery of tools and their highly developed expertise). Hence, in the years between the two wars, Fubini was mainly engaged in projective differential geometry. He wrote only about ten notes in analysis, mostly occasional papers derived from recent publications, encouraging a deeper or an easier proof; he planned no systematic and articulated research program. It goes without saying that Fubini was always Fubini and that, although he was "distracted" by other subjects, his works reached nonetheless the usual high quality. The discussed matters concern complex analysis and differential equations (ordinary and partial ones). He obtained, with a very simple proof, a comparison theorem for ordinary linear equations of the second order or took again, for the equation:

$$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial^2 u}{\partial t^2} = 0 ,$$

[34] As I am correcting the drafts I have to point out the recent and extremely important note by Stefan Banach (…) in which he with similar intention but through different processes, has reached results which are analogous to the present ones. I am glad that the opinion of my illustrious colleague from Leopoli University and mine agree on indicating how to extend to surfaces the widespread results on rectification of curves and specially Leonida Tonelli's remarkable theorem.

an unicity theorem proved by Levi-Civita. Before his "American" note of 1940 (in which, developing an article of K. Menger of the previous year, he verified a necessary and sufficient condition for a differential to be exact), he published his two most significant memoirs of the time (1937). In "Studi asintotici per alcune equazioni differenziali" he expounded a method for homogeneous linear equations (formally considered as nonhomogeneous), which was essentially based on arbitrary constants variations that turned them into integral equations of Volterra; solutions could be expressed then through absolutely and uniformly convergent series, which were specially useful in the asymptotic study of solutions, because of their nature as asymptotic series. Actually written in May 1938 (a few months before the promulgation of the racial laws, that would strike also Fubini), "Sopra una nuova classe di problemi al contorno" started from some mean theorems which E. E. Levi, Volterra, Vitali and Tonelli had proved to be distinctive features of harmonic functions. Similar theorems were true for harmonic functions in a non-Euclidean space and for the solutions of an elliptic homogeneous linear equation of the second order, so that some integral features which synthesize the theorem of the mean matched some differential features (summarized in the differential equation). The memoir at issue inverted this correspondence, proving that functions with such integral features solved problems which were similar to those accomplished by the previous solutions.

Fubini loved great studies and wide horizons rather than limited research. When reading his writings, one is pleasantly impressed by sentences which often emphasize his open and conjectural research. Fubini stood out in the Italian context also because of his real interest in applications – he, a pure analyst! – and in the engineering world he knew and in which he moved, due also to his son Gino's studies. His natural interest and his scrupulous personality, which "took seriously" the didactic location within Turin Polytechnic, brought him to positions which differed a great deal from the bombastic rhetoric typical of the time, and which turned into a concrete sensitivity and just as concrete dissemination work. Mathematics was not to be introduced as a chess game or, better, it had to avoid confusing "the art of the chess-player with the game's rules". Mathematicians did not choose by chance crazy hypotheses so as to deduce useless results, but rather they developed useful tools which Fubini persisted in presenting and explaining in some lectures (on functional and on symbolic calculus) given to engineers. In the meanwhile, he did not renounce at all to the typical features of the mathematical enquiry, although they were often a "safe promise of deep bore". As vehemently did Fubini claim the pleasure to study "for the pride of human spirit alone" and the qualities of the abstract procedure: it was not "a defect; but on the contrary a rare credit and an important richness for our doctrines". This is – in the Italy of the 1930s, which would officially grab any chance to take advantage of any scientific speech – why he tried to avoid that polytechnic teaching be nullified and reduced to mere practice and that "the young never forget that the greatness and the economic and technical independence of a country go together with the love for knowledge, with the pleasure in scientific research".

6. Volterra and Ecology

So far in this chaper we have focused on Volterra's leadership in the first part of the 1920s. Let us now expand this period by some years to introduce his most important scientific contribution in the period between the two world wars. We refer, particularly, to 1926 (when the *Accademia dei Lincei* published "Variazioni e fluttuazioni del numero d'individui in specie animali conviventi"[35]) and to 1931, when the already classical *Leçons sur la théorie mathématique de la lutte pour la vie*[36] were printed in Paris. Volterra's studies on the dynamics of populations, which were original applications of mathematics beyond the classical physical context and which produced important results, were a worthy milestone for the birth of the field of mathematical biology, as it was outlined in the first half of the 20th century.

The story is wellknown. One of Volterra's sons-in-law – Umberto D'Ancona, zoologist – asked his father- in- law for a theoretical explanation of a piece of data fairly evident in the fishing statistics of the North Adriatic Italian ports, regarding the years 1905–1923; according to this data, the percentage of great fishes (*predators*) had considerably increased in the total amount of the fish caught during the years 1915–1918 and the following years. The exogenous elucidations – essentially based on the minor fishing activities during the war years – did not convincingly explain the different behaviour of prey and predators.

Obviously, Volterra was not a biologist, but these questions were not wholly alien to him, either. In 1901, newly transferred to Rome University, he had had the honour of giving the *inaugural lecture* to the academic year and he had chosen as subject *Sui tentativi di applicazione delle matematiche alle scienze biologiche e sociali*[37]. In 1911 he had been appointed vice-president of the *Regio Comitato Talassografico Italiano*. Later he had supported a project (linked to the *Comitato Talassografico*) that contemplated the creation of an "Istituto Oceanografico Nazionale", analogous to those of Monaco and Paris. In 1916 he had inaugurated in Messina the *Istituto Centrale di Biologia marina* at the presence of Prince Alberto I of Monaco (creator of the oceanographic Institutes of Monaco and Paris) and Louis Joubin and Odon Bouen, directors of the oceanographic Institutes of Paris and Madrid, respectively. On these occasion he had declared himself proud of the creation of the Messina Institute, which could be "honourably compared" to "Naples Station, unquestioned property of the Neapolitan city, today at last free from German subjection and purely Italian"[38]. In 1923, again, he was appointed member of the Italian delegation in the "International Commission for the study of the Mediterranean".

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35 *Mem. Accademia dei Lincei*, 1926, p. 31–113.[36] The lectures, written by M. Brelot, were published by Gauthier-Villars.

37 The text was printed separately and reissued in the *Giornale degli Economisti* (1906) and in V. Volterra, *Saggi Scientifici*, Roma, 1920.

38 Actually, the glorious Neapolitan Institute of marine biology was just short-term acquired: in 1920, after a harsh parliamentary debate, Benedetto Croce – new Minister of Education – returned it to Rinaldo Dhorn, in spite of the vibrant protests of Volterra, of some Italian biologists and scientists of the allied Countries, among which were a group of English oceanographers.

Unaware of A. Lotka's contributions[39], Volterra began to study the problem raised by his son-in-law at the end of 1925. He opened his *Leçons* thus: "à la suite de conversations avec M. D'Ancona, qui me demandait s'il était possible de trouver quelque voie mathématique pour étudier les variations dans la composition des associations biologiques, j'ai commencé mes recherches sur ce sujet à la fin de 1925". He wrote the chosen model as a system of ordinary differential equations of the first order:

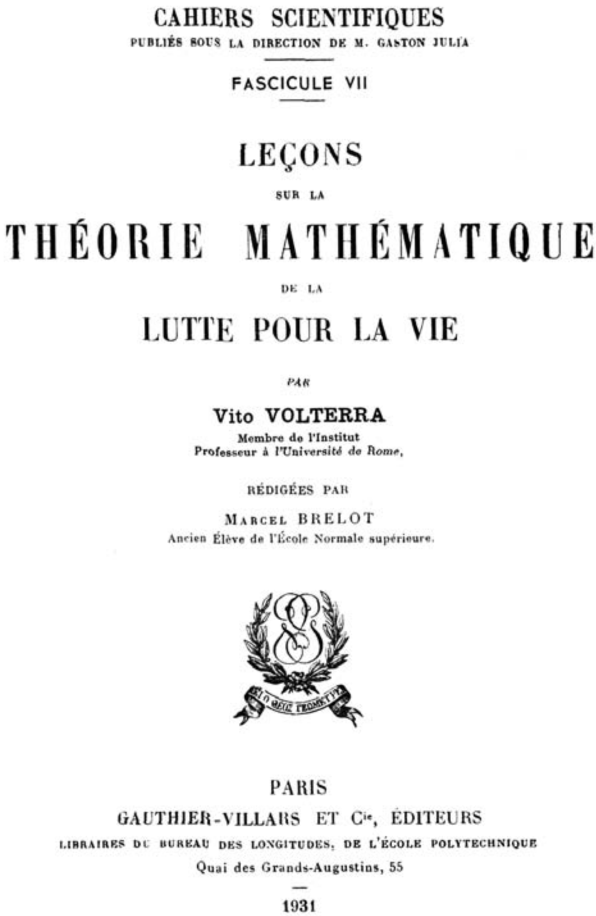
$$\begin{cases} x' = ax - bxy \\ y' = -cy + dxy \end{cases}$$

where $x = x(t)$ e $y = y(t)$ represented, respectively, the evolution in time of the population of prey and predators and $a,\ b,\ c,\ d \in \Re$. From a first hypothesis on the isolated evolution of both species (in terms of constant percentage rates of their growths x'/x and y'/y), followed a behavioural one on the principle of the encounters, according to which predation effects depended on the possible encounters xy in the time unit. The system's solution (with the suitable initial conditions) was set in an explicit way through a clever method that used a reference system with four axes. Out of it Volterra would derive the three laws that govern the model's biological fluctuations: the law of the periodic cycle (which proved the endogenous character of fluctuations), the law of conservation of the mean and the law of perturbation of the mean, which answered the initial problem. A perturbation due to external causes – for example, to the fishing action or to a change in its intensity – would bring new average values and the comparison with the previous ones would justify the experimental observation according to which the diminishing of fishing activity favoured, in a sense, minor species.

Volterra would be engaged in bio-mathematical research until his final years. After having studied the mentioned biological fluctuations' model (which would be expanded several times, starting with the consideration of n species, to the introduction of memory terms) he studied the analytic mechanics of biological associations and later logistic curves. His research on theoretical ecology would not arouse in Italy particular reactions, at least straightaway; there were only, within the social-economic field, some "resumptions" of themes of population dynamics. Then, at once, the most complete ostracism arrived. But these are the years of his *leadership* and his greatest prominence. This story, still to be told, has been brought forward only to explain why Volterra had to ask Borel to sponsor in the *Comptes Rendus* some notes on the analytic mechanics of biological associations, in the same year in which A. N. Kolmogorov published in the *Giornale dell'Istituto Italiano degli Attuari* his famous article "Sulla teoria di Volterra della lotta per l'esistenza".

In the *Prologue* we stated that Volterra was a worthy exponent of the great 19th century tradition, so as to emphasize both his familiarity with several research fields – he was analyst, mathematical physicists, bio-mathematician, and wrote worthy papers on mathematical economics – and that his scientific personality was shaped during the period

[39] For the comparison between the formulation by Lotka and by Volterra and the question of priority, see A. Millán Gasca, G. Israel *The Biology of Numbers. The Correspondence of Vito Volterra on Mathematical Biology*, Birhäuser, Basel, 2002 and the essay of P. Manfredi and G. Micheli in S. Di Sieno, A. Guerraggio, P. Nastasi, *La Matematica italiana dopo l'Unità. Gli anin fra le due guerre*, quoted, pp. 671–733.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FASCICULE VII

LEÇONS
SUR LA
THÉORIE MATHÉMATIQUE
DE LA
LUTTE POUR LA VIE

PAR
Vito VOLTERRA
Membre de l'Institut
Professeur à l'Université de Rome,

RÉDIGÉES PAR
MARCEL BRELOT
Ancien Élève de l'École Normale supérieure.

PARIS
GAUTHIER-VILLARS ET Cie, ÉDITEURS
LIBRAIRES DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE
Quai des Grands-Augustins, 55
—
1931

Cover of Volterra's book on Mathematical Biology

of greatest influence of *physical analogy*, a concept considered as the basis for the new theories. Bio-mathematical research was part of this framework. We could speak of his reductionist project. It was one of the most brilliant applications of the methods of classical mechanics, which marked one of the "strong" moments of scientific thought in the 20th century. "The turning point of the rise of mathematical ecology as an autonomous science is not to be sought then in Lotka's or Pearl's work, but in Volterra's, and in the wholly Eurocentric cultural *humus* in which the big shift of interest from man to mass nourished Mc Kendrick, Ross and Thompson's new population mathematics and Le Bon and Sorel's mass psychodynamics, so close to the great political upheavals of the second quarter of the century"[40].

[40] P. Manfredi, G. Micheli in S. Di Sieno, A. Guerraggio, P. Nastasi, La Matematica italiana dopo l'Unità. Gli anni fra le due guerre, quoted, p. 723.

Chapter 4

Fascism: somebody rise, others fall

1. The march on Rome

We have talked about the founding of the U.M.I. and the C.N.R. and about the consolidation of Volterra's *leadership*. These events cannot be, however, interpreted as a sign of a normal restarting of mathematical and scientific activity after the 1915–18 break. In Italy, the restarting of a *normal* life was quite problematic.

The title of the second chapter, "*Nothing is as it was before*", is meant to emphasize the fact that the war experience was much more than just an additional element in a changing scene. The young democracy had to face – after only 60 years from the Nation's Unification – vaster and more complex problems. The old *establishment* had hoped for some sort of continuity, even if slow and difficult, but reality dashed these hopes almost immediately.

With the advent of fascism in 1922 the political-institutional scene changed. Italians would realize it on the 27th October, when a statement by the leadership of the *Partito Nazionale Fascista* announced that its own militants were marching on Rome to seize power. The *march on Rome* found no sizeable resistance, partly because of the incredible about-face of the King, who refused to sign the proclamation of martial law (previously agreed upon with the government). Benito Mussolini had founded in 1919 the *Fasci italiani di combattimento*, which in 1921 became the *Partito Nazionale Fascista*. He was entrusted with forming the new government as early as the afternoon of the 29th October. His opening speech to Parliament left no doubt about his intentions. Formally, and temporarily, he led a coalition government, but his direction was clear: "potevo fare di quest'aula sorda e grigia un bivacco di manipoli. Potevo sprangare il Parlamento e costituire un governo esclusivamente di fascisti. Potevo, ma non ho, almeno in questo primo momento, voluto"[1].

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[1] "I could have made out of this deaf and grey hall a bivouac of squads. I could bar Parliament and establish an entirely fascist government. I could, but I have not wanted, at least initially". This is the *bivouac's speech*, pronounced before the Camera on the 16th November 1922, on occasion of the vote of confidence for the new executive.

The march on Rome

A moral and political judgement on fascism has already been stated, so we will not expatiate upon its antidemocratic, anti-popular and repressive character. It is difficult, though, to present fascism briefly in a manner useful to the understanding of the course of Italian mathematics between the two world wars.

Fascism, is based on an ingrained malleability of beliefs and inner contradictions that defy labeling it as a well-defined political philosophy. These attributes are moreover boasted of as exalting the personal prestige of the man who has to mediate its needs and aspirations. Fascism is at one and the same time a popular movement, a political party, and an *authoritarian* regime, each aspect merging imperceptibly into the others, and believing economic forces to be the best cure for the kind of popular unrest that had led to such phenomena as the occupations of the factories and the "red biennium" of 1919–1920. Fascism was also considered the most suitable and safest channel through which to build industrial capability. It also offered hope to an Italy disappointed by a war that did not fulfill nationalistic aspirations that it felt it deserved and had earned on the field. A wide spectrum of middle and working classes supported Fascism for a variety of reasons: they were disappointed at the resumption of some negative aspects of the civil life they knew and the failure of the "red biennium", and they saw it as the guardian of their own humble interests.

In short, fascism was not, not even in its folkloristic or tragically ridiculous traits, foreign to the Italian nation, simply imposed by landowners and industrial organizations. It was not just an interval in history. It was rather – to use another popular metaphor – an illness that incubated germs that were already present in the Italian social fabric. It is a mixture of modern and anti-modern drives, or better stated, an "old" recipe (and anyway different from the modernity that would be asserted later), often cooked up by a new political staff. We will examine the importance of the generational question for Italian mathematics. War had brought to the fore a new generation, confident in its leadership ability, in the awareness of its own strength and in a sense of comradeship which grew out of the sharing with their peers of a trauma that had divided sons from their fathers, the new from the old generation. Social and cultural changes following the war brought about a further division also in style and behaviour. Fascism was the channel through which the new generation came to the political forefront. In this sense, it became the "natural" instrument through which new protagonists asserted themselves and their perceived right to change the *status quo*.

The course of many events involving fascism, new generations and modernity, culture and Mathematics can be traced through the image and actions of Giovanni Gentile. As we have seen, in the first years of the century Gentile and Croce were engaged in a heated debate against mathematicians, especially Enriques. Of course, the dispute focused not on mathematicians as such, but on the consolidation of an idealist hegemony in Italian culture.

2. Giovanni Gentile and school reform

From the turn-of-the-century onwards, Gentile had further enlarged his sphere of influence. Above all, he had openly affirmed his independence in regard to Croce in 1913, with a controversy. This did not lead to conflictual behaviour, but it certainly cooled down their relationship. He was called to Rome University on the 24th October 1917, the same day of the defeat of Caporetto. This "call" consecrates his authority, which was favoured by his new studies and philosophic publications, his contacts within the academic world and his particular civil commitment. Gentile was a member of the *Superior Council* of Education since 1915. In the years immediately following the war, he intensified his "public appearances" as a columnist for several national journals. He supported ideas which brought him politically closer to nationalists, even if he did not share their ideology.

His ability in positioning the scholastic question as of national importance certainly helped in raising his popularity and the move toward cultural hegemony. He posed it as a political, not just a financial, question. He put forward an over-all proposal for school reform – above all for the level that Gentile called *scuola media* (the first three years of the secondary school) – through a series of articles, speeches in conventions and publications. The reform was based on the simple recipe of *few but good* (state schools). There would be a quality state school for a small, elite number of students, and the majority would have the chance to enter private schools. In this way he realized a pragmatic *embrassons-nous* with the catholic world, which was always sensible to the issue of private school and teaching "freedom". They particularly agreed on the proposal of introducing a state exam at the end of secondary school. With it, Gentile meant to ascertain the quality and rigor provided by these schools; Catholics saw it as a further step towards the equalization of state schools to private ones, which already passed this final test.

On the 16th of June 1920, in one of the frequent government changes, Croce became minister of Education. It was the right moment to strengthen again a personal and political relationship. Gentile's public tasks increased. He had a well-known influence on Croce, who was certainly less committed on the pedagogic side. But Croce's ministry lasted only a few months. The "march on Rome" was only a few months away.

Gentile was minister of Education during the first Mussolini government. He immediately set to work in order to fully use the 13 months of full powers (until the end of 1923) which fascism had to carry through the proposed financial and administrative State reforms and to legislate organic school reform. Croce had not been able to start it, although he tried through the *escamotage* of several gradual partial measures. The reform would obviously involve mathematics and mathematicians too. But in order to follow their vicissitudes, we have to see first the development of Gentile's political path. He embraced fascism on the 31st of May 1923, with an open letter to Mussolini[2].

> Caro Presidente, dando oggi la mia formale adesione al Partito Fascista, La prego di consentirmi una breve dichiarazione, per dirLe che con quest'adesione ho creduto di compiere un atto doveroso di sincerità e di onestà. Liberale per profonda e salda convinzione, in questi mesi da che ho l'onore di collaborare all'alta Sua opera di Governo e di assistere così da vicino allo sviluppo dei principi che informano la Sua politica, mi son dovuto persuadere che il liberalismo, com'io l'intendo e come lo intendevano gli uomini della gloriosa Destra che guidò l'Italia del Risorgimento, il liberalismo della libertà nella legge e perciò nello Stato forte e nello Stato concepito come una realtà etica, non è oggi rappresentato in Italia dai liberali, che sono più o meno apertamente contro di Lei, ma per l'appunto, da Lei. E perciò mi son pure persuaso che fra i liberali d'oggi e i fascisti che conoscono il pensiero del Suo fascismo, un liberale autentico che sdegni gli equivoci e ami stare al suo posto, deve schierarsi al fianco di Lei[3].

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2 Cf. G. Gentile, *La riforma della scuola in Italia*, Le Lettere, Firenze, 1989, pp. 94–95.

3 Dear President, as I formally join the Fascist Party today, I beg You to let me make a brief statement. I would like to tell You that I see my support as a necessary act of sincerity and honesty. I am a firmly fervent liberal. In these months I have had the honour to collaborate to Your high government work and to follow at close range the development of the principles on which Your politics are based. And I have persuaded myself that liberalism, as I see it and as the men of the glorious Right that guided Italy during the Risorgimento saw it, is not represented in present Italy by liberals, who are more or less openly against You. Indeed, it is You who represent that liberalism, the liberalism of freedom in law and therefore in the strong State, in the State conceived as an ethic reality. Hence I am also persuaded that, among contemporary liberals and the fascists who know Your fascism, an authentic liberal who hates misunderstandings and knows his place must side with You.

Mathematicians began to line up too. Some joined the fascist Party or the fascist syndicate of University professors[4]. Gentile's motivations to join the fascist Party were enthusiastically approved of by Mauro Picone, who would define himself as "a black shirt since the very beginning"[5].

> Illustre e venerata Eccellenza, Catania, li 5/VI/1923
>
> mi consenta di esprimer Le tutto il mio vivissimo intimo compiacimento per l'adesione che Vostra Eccellenza ha voluto dare al Partito nazionale fascista al quale anch'io appartengo.
> La nobile lettera del 31 maggio da Vostra Eccellenza diretta al Presidente del Consiglio rimarrà memorabile nella storia di questi tempi. Quest'ultima adesione al partito fascista – così cospicua – e le meditate affermazioni contenute in quella lettera, vinceranno le esitazioni di tanti colleghi e porteranno ancora nuovo purissimo sangue nelle robuste vene del partito che ricostruisce e rinnova la Patria!
> Viva l'Italia!
> Con i più rispettosi ossequii, Le invio le sincere espressioni della più profonda mia devozione.
>
> Mauro Picone[6]

The reform of school and of university by Gentile was composed of a set of formal decrees adopted during the whole of 1923, on the strength of the legislative proxy given to the government. At an administrative level the Educational system was organized in a rigidly centralistic way: elective representatives were abolished; headmasters of secondary schools, rectors, Faculty deans, Institute directors and components of the academic Senate would all be appointed by the minister. On the other hand, the control powers of the Institute directors and Faculty deans over the teaching staff were increased, which reinforced the sensitivity of the hierarchic order.

The new structural design of the scholastic system rejected the democratic example of a unique *scuola media*. It rather increased the choices young people had at their disposal, through a precocious channeling that intended to make middle school homoge-

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[4] Among the first to join the fascist syndicate of the professors of Rome University there is Giovanni Vacca, Peano's pupil and Volterra's collaborator, historian of Mathematics and expert sinologist.

[5] The letter is quoted in A. Guerraggio, P. Nastasi, *Gentile e i matematici italiani. Lettere 1907–1943*, Boringhieri, Torino, 1993, p. 185.

[6] Illustrious Excellence, Catania, the 5/VI/1923
let me express You my deepest innermost satisfaction for giving Your support to the fascist national Party to which I belong.
The valuable letter of the 31st May that Your Excellence addressed to the Prime Minister will stand out in the history of our time. Your major support to the fascist party, and the well-considered statements within that letter, will overcome the hesitations of many colleagues and will bring more and new pure blood in the strong veins of the party which is rebuilding and reorganizing our country!
Long live Italy!
With my most respectful regards, and my deepest devotion.
Mauro Picone

neous, as much as possible. This selection at the source also helped to develop the *Liceo classico*[7]. Instead, the "modern" (scientific) one was abolished and the technical school reformed, transformed and essentially downgraded to a vocational training school. In short, the reform's fundamental axis was the neat division of secondary education along two paths: the classical-humanistic one (which had to educate the ruling class) and the technical one which aimed at spreading professional working skills. The importance and independence of scientific education diminished everywhere. It had to conform to the double formative channel. In the first path the teachings of Mathematics and Physics were paired. This answered to a need for greater oneness and organic unity and reinforced the formative character of scientific education (but the number of hours was actually decreased). The second path underlined the entirely instrumental aspect of mathematical teaching. At a university level, the same logic divided neatly between scientific Faculties, which aimed at the education of researchers, and the Polytechnics, which was to educate engineers.

There were certainly protests and resistance to the reform by Gentile. To start with, students opposed both the increase of taxes and of exams. It was the Autumn of 1923. The spreading of the students unrest obliged Mussolini himself to take the field on the 6th of December. He then labelled the reform as "the most fascist one among those which my government has approved".

The *Mathesis* expressed the first strong perplexities and discontents among mathematicians. Enriques, its president since 1918, was engaged in a process meant to further entrench the association among teachers. The results arrived soon: within five years the *Mathesis* would almost triple its members, and in 1924 it numbered 1161 members. The protest of the *Mathesis* concentrated mainly on the specific aspect of the unification of the teachings of Mathematics and Physics in secondary schools, which the reform provided for. Sometimes the amount of hours scheduled for both teachings were less than Mathematics alone previously had. The vast majority of the teachers in Mathematics denounced the pairing. The reasons were several: the competence needed for both teachings, the difficulties to realize the reform with an untrained teaching body, the unbearable didactic load and, also, the "constitutional" diversity between the mathematician and the physicist (even if since 1922, to show its own openness, the *Mathesis* had become *Società Italiana di Scienze fisiche e matematiche*). The *Mathesis* reacted by appointing a *Commission* which had among its members Enriques and Castelnuovo. The latter asked for an encounter with the Minister to make sure (in the prudent and softened language of educational politics) that the reform would not diminish "the importance of scientific teaching". Gentile declared himself "glad" to dispel the association's worries. But when in May 1923, two months later, Enriques insisted by handing him counter proposals, in a summarizing pro-memorandum of the Society, he just answered that the proposals put forward were "hardly compatible" with the economic situation of the country[8].

The opposition of the *Mathesis* was soon followed by that of the *Accademia dei Lincei*, the maximum expression of Italian science. In 1925 it was "reinforced" by the opposition of many University Science Faculties, owing to a 30% reduction in their equip-

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7 Italian secondary school which underlines classical education.
8 Cf. *Periodico di Matematiche*, S. IV, vol. III, No. 4 (1923), p. 339–341.

ment for scientific laboratories. Volterra – elected President of the *Lincei* – wrote to a Florentine physicist, Antonio Garbasso, in July 1923[9].

> Tutti sono allarmati dalle riforme Gentile. La nostra Facoltà ha votato un ordine del giorno, ma non così vivace come avrei voluto. Né è passata la proposta di inviarlo anche al Presidente del Consiglio. Fui tra i pochi che votai in questo senso. Nell'ultima seduta dell'Accademia dei Lincei fu proposto e accolto il voto di incaricare una commissione di riferire sulle riforme, giacché l'Accademia non poteva disinteressarsene. Nella commissione sono entrati Scialoja, Pais, Bonfante, Castelnuovo ecc. Avrei voluto mettere anche il tuo nome, ma data la urgenza, e non potendo sperare nella tua venuta a Roma, non ho potuto farlo. Dopo ampia discussione il Castelnuovo relatore ha redatto un rapporto fatto a mio avviso molto bene il quale figura come relazione della Commissione alla Accademia[10].

The conclusions of the analysis carried out by the Commission of the *Accademia dei Lincei* were quite explicit[11].

> Per merito di illuminati legislatori la scuola italiana tra il 1860 e il 1880 era salita ad un alto livello e poteva competere con le migliori straniere. Deplorevoli indulgenze e rilassatezza di disciplina avevano forse negli ultimi decenni diminuita l'efficacia della scuola; ma sarebbe bastata una mano ferma, che avesse rimesso in vigore le norme più austere, per ridare alla scuola l'antico prestigio, pur tenendo conto delle nuove esigenze portate dal progresso culturale ed economico del nostro paese. Una riforma radicale, per quanto ispirata da nobili intendimenti, non sembrava necessaria[12].

As we have seen, several reasons and stances were mixed up in the reaction to the reform by Gentile. The protest of the *Mathesis* did not concern as much the base-elements

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9 The letter is quoted in R. Simili, La presidenza Volterra, in R. Simili, G. Paoloni (eds.), *Per una storia del Consiglio Nazionale delle Ricerche*, Laterza, Roma-Bari, 2001, 2 vol. I, p. 91.

10 Gentile's reforms have alarmed everybody. Our Faculty has voted on an agenda, but not so lively as I would have liked. And the proposal of sending it to the Prime Minister has not passed. I was between the few who voted for it. In the last sitting of the Accademia dei Lincei, the vote of entrusting a commission to report about the reforms was proposed and accepted, since the Academy could not neglect them. Scialoja, Pais, Bonfante, Castelnuovo, and others are its members. I would have liked to add your name too, but since it is urgent, and I could not hope in your coming to Rome, I could not do it. After a long discussion, the speaker Castelnuovo has written quite a good report, in my opinion. This is the report of the Commission to the Academy

11 Sopra i problemi dell'insegnamento superiore e medio. A proposito delle attuali riforme, *Accademia Nazionale dei Lincei*, Roma, 1923, p. 12.

12 The Italian school between 1860 and 1880 had risen to a high level and could compete with the best foreign ones thanks to enlightened legislators. Maybe in the last decades deplorable tolerances and discipline laxity had lessened the efficacy of school; but a steady hand could have restored the old prestige of the school, by reinstating the most austere norms, while considering as well the needs that cultural and economic progress had brought to our nation. A radical reform, although suggested by laudable intentions, was not necessary.

of the reform by Gentile as some specific aspects, which caused also the instant mobilization of the teachers. The *Mathesis* did not question the scientific culture's debasement, as one could have expected. The stance of the *Accademia dei Lincei* was based on a defence of the *status quo*: the Italian school, especially after the war, had undoubtedly some cracks, but they could be rectified just by reinserting a bit of "energy" and the rigour that made possible the post-unitary take-off. Considered the cultural and social atmosphere emerging in the country, the protests of the mathematicians are certainly bold. But they result from a defensive mentality, certainly different from the confident and propulsive behaviour of the beginning of the century. In a word, fascism embraced the change, even if it was a change towards restoration. And it had the charm resulting from proposals presented as new and resolutive of the backwardness that now prevailed.

The opposition of *Mathesis* could be borne, but the judgement of the *Accademia dei Lincei* needed a reply. In the press the Academy was accused of holding itself excessively apart from the country's life life. Gentile himself took part directly and talked about "academic environment, where the sectarians opposing the Minister think they have some followers; they still don't want to yield and recognize that the country's cultural rebirth is one of the issues of the Government presided over by Mussolini"[13]. And in an interview in Milan's newspaper "La Sera" on the 17th August 1923 he tried to distinguish the position of the *Commission* from the one held by the Academy as a whole, forgetting that Volterra was member of the first and President of the second. He continued thus: "if I wished to use a matter *ad hominem*, in the meanwhile I would begin by questioning my critic's competence to criticize me. Obviously, they are all renowned scientists. But how many of them have focused their spiritual work on the academic problem, as I can boast?"[14].

Despite Gentile's presumption, opposition to his projects in the academic surroundings increased. Even Severi, in his capacity as President of the *Associazione nazionale dei professori universitari*, rose up against the threatened purge of the university teachers stated in a decree of March 1923.

3. The battle of the "manifestos"

Anyway, it would not be Gentile who would implement his reform. The political situation plunged quickly and on the 25th January 1924 the Chamber of Deputies was dissolved. On the 6th April there was an election. The atmosphere was well represented by Mussolini's statement: in any event, he would not leave the power, which he had won not through "paper games" but through "revolutionary right". Fascists, though, did not want to run any risk and preferred to make sure that they would win by using pressure, intimidation and violence. They had a majority, with four and a half million votes and 356 deputies, while the opposition (with three million votes) won only 179 deputies, dispersed in several and sometimes tiny lists.

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[13] Cf. G. Gentile, *La riforma della scuola in Italia*, quoted, p. 140.
[14] Cf. *L'Università italiana*, a. XIX (1923), No. 6, p. 73.

Electoral triumph could have announced the constitutional normalization of fascism. It did not happen. In the inaugural speech on May the 24th, Mussolini restated the identification between fascism and the nation with force. The new legislation started off by suppressing the parliamentary Commissions. These were replaced by the system of the Offices, which waived the proportion of parliamentary forces. In the parliamentary debate Giacomo Matteotti had the honour to uphold the reasons of the opposition. In a courageous and potent way, he proved fascist illegalities with documentary evidence. He was a young exponent of the *Partito Socialista Unificato*. On the 10th June he would be murdered, attacked by four hired killers, kidnapped in a car and slaughtered. His corpse would be found only two months later.

Matteotti's murder stirred public opinion. In the parliament, the opposition decided to leave the Chamber and to form the so-called *Aventino*, in the hope of forcing the government, overwhelmed by the moral question and urged to this by the king himself, to hand in its resignation. But it was a vain hope. King Vittorio Emanuele III confirmed his connivance with fascism. The protest of the "*aventiniani*"[15] remained within the boundaries of moral indignation, due also to the actually defeatist behaviour of old liberals. A "reporter"[16] solicited Croce to take up a stance, in June 1924. According to him, Croce said:

> Abbiamo deciso di dare il voto di fiducia. Ma, intendiamoci, fiducia condizionata. Nell'ordine del giorno che abbiamo redatto è detto esplicitamente che il Senato si aspetta che il Governo restauri la legalità e la giustizia, come del resto Mussolini ha promesso nel suo discorso. A questo modo noi lo teniamo prigioniero, pronti a negargli la fiducia se non tiene fede alla parola data. Vedete: il fascismo è stato un bene; adesso è divenuto un male, e bisogna che se ne vada. Ma deve andarsene senza scosse, nel momento opportuno, e questo momento potremo sceglierlo noi, giacché la permanenza di Mussolini al potere è condizionata al nostro beneplacito[17].

The crisis of June 1924 ended with the consolidation of fascism and the relaunch of a fervent extremism, which claimed a new wave of violence. On the 10th July a legislative decree subjugated the press to executive power, which could then intervene with constraints, suspensions and trials of directors and editors.

In the electoral campaign of the spring 1924, Gentile had confirmed and specified the reasons why he supported fascism. This regime embodied the present face of the

[15] The name comes from an historical area in Rome.

[16] G. Levi Della Vida, *Fantasmi ritrovati*, Neri Pozza, Venezia, 1966. Giorgio Levi Della Vida (1886–1967), important Orientalist, would be one of the few university professors who – with Volterra – would have the courage to refuse in 1931 the oath of allegiance to fascism.

[17] We have decided to give our vote of confidence. But it is, of course, a conditional confidence. On our agenda we say explicitly that the Senate expects the Government to restore legality and justice, as Mussolini has promised in his speech, after all. In this way we keep him prisoner, ready to deny him our confidence if he does not keep his word. See: fascism has been a blessing; now it has become an evil, and it has to go away. But it has to go without shocks, at the right time, and we could choose this moment, as Mussolini's power tenure is conditioned to our consent.

Benedetto Croce

Giovanni Gentile

liberalism which meant to safeguard the state's force, "whatever argument be used, from sermon to truncheon". The truncheon of fascist action squads was the necessary surrogate of the state force itself in a revolutionary period. It had been "put away in the attic, in the hope that it would never be needed again" – but it might be necessary to resort to it again – "if all Italians, fascist or not, convince themselves of the need and duty to help in the consolidation of the regime"[18]. His ministerial career, however, finished shortly after the elections. The philosopher, who with his reform had become a cause for contrast and division even in surroundings close to the regime, offered his resignation on the 14th June, four days after Matteotti's murder. It was meant to be a gesture of "national conciliation". His resignation was accepted.

His engagement within the cultural and political fray did not diminish, though. Gentile became president of a study commission which had the task to prepare the legislative reforms. It was composed of five deputies, five senators and five intellectuals. Then, on the 3rd January 1925, Mussolini announced the cessation of any statutory survival: "I declare here, before this assembly and before the whole Italian people, that I, alone, take on the political, moral, historical responsibility for what has happened. (…) If fascism has been a criminal association, I am the head of this criminal association". The following measures – arrests, dissolution of political associations, newspapers distress, etc. – made explicit the decisive transition from fascism to dictatorial regime. Its "normalization", far from reinstating liberal legality, took on without simulations the

[18] In G. Turi, *Giovanni Gentile. Una biografia*, Giunti, Firenze, 1995, p. 336.

face of authoritarian reaction. Gentile applauded the "excellence" of speech Mussolini held on the 3rd January and toed the shift line. His reward was an appointment to the presidency of the *Commissione dei Diciotto*, so-called of the *Soloni*. This commission inherited the tasks of the previous study commission. Also the statistician Corrado Gini (1884–1965), with whom we will deal in chapter VI, took part in it.

In March 1925, Gentile promoted in Bologna the first national Congress of Fascists Institutions of Culture, in which about 250 intellectuals took part. There was to be no "*discussion*, but only written *communications*, previously handed to the organizing Committee" because "the theoretical digressions, besides the fact that they do not arrive to an end, would take the initiative beyond positive and immediate purposes. The achievement of these purposes is the reason why the most elected exponents of Italian thought are gathered in Bologna". Among the communications only the one by Gini concerned science. His title was: "Diffusione all'estero dei risultati della scienza italiana". The *Istituto nazionale fascista di cultura* and the appeal to Italian intellectuals to ask for their support to fascism and to overcome a "little commonplace; the antithesis between fascism and culture", both arose from this Congress of Bologna. Gentile himself was given the task of drawing it up. The appeal appeared on the 21st April in the press and it was soon known as the *manifesto Gentile*. Among mathematicians, only Gini and Pincherle gave their support. The last was obviously not to be underestimated: Pincherle – it has to be remembered – was the president of the new-born *U.M.I.*

"Bets", at least in the cultural world, were not placed yet. There was a will to resist and not to surrender yet to that increasingly dramatic attempt at the fascistization of culture, which fostered the Convention of Bologna and the *manifesto Gentile*. The reply was entrusted to Croce – the two philosophers were already on opposite barricades. He drew up a counter-manifesto, published on the 1st May and supported by the best names of the Italian intellectual elite. Among mathematicians, only Tonelli appeared at first. But on the 10th and on the 22nd of May were published other signatures, among which we find the names of Volterra, Castelnuovo, Beppo Levi, Levi-Civita, Severi. The formulation of the counter-manifesto was based on the old canons of liberal thought, which did not accept an interventionist idea of culture and wanted the intellectual to be "detached", distant enough from the world and almost indifferent to its political results. As we had observed when examining the stance the *Accademia dei Lincei* took up regarding the reform by Gentile, the answer to the aggressive fascist cultural politics was entrusted once again to the high feelings and to the lofty expressions of an "old" world, which left however a resistance statement to the younger generations[19].

> Un gruppo di scrittori, di professori e di pubblicisti ha deciso di comunicare alla stampa una risposta al manifesto degli intellettuali fascisti. Tale risposta non ha pretesa alcuna di rappresentare, e tanto meno di monopolizzare, l'intellettualità antifascista che nessun congresso ha chiamato o chiamerà ad ostentarsi in artificioso schieramento; ma vuole essere, innanzi tutto, una reazione contro quel metodo che pretenderebbe piegare l'intellettualità a funzioni di *instrumentum regni* e

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[19] In E. R. Papa, *Storia di due manifesti. Il fascismo e la cultura italiana*, Feltrinelli, Milano, 1958, p. 92.

> vuole essere in pari tempo la protesta sollevata da alcuni liberi intellettuali contro la versione e l'interpretazione delle cose d'Italia che gli intellettuali fascisti hanno creduto di dover diffondere al di là dei confini d'Italia[20].

The picture of the year 1925 is clear enough. Fascism got off with very quick steps to claim the primacy of politics over any other aspect of individual and collective life, through the dissolution of the "private" into the "public". Being faced with this project, that obviously involved also the cultural world, almost all mathematicians sided with antifascism, which at the moment, expressed regret for the best aspects of the previous tradition. Only Pincherle chose the opposite, due above all to nationalistic feelings[21]. Picone and Enriques were missing in both sides. The controversy's sequels and his aversion to Croce were maybe the reasons behind Enriques' choice, but anyway he was always reluctant to "demean himself" to the level of a political debate. Later on, among mathematicians too, "some deserted" (to use the words of Croce). But Volterra, faithful to his antifascist commitment both in the Senate and in his professional life, did not leave. He was urged by Enriques and by Gentile himself to collaborate – in the name of a higher interest of national culture – to the project of the *Enciclopedia Italiana*, which he had already joined in an earlier stage and in which now was involved also the *S.I.P.S.*[22] Volterra would not deign to answer either interlocutor. This story deserves being told too.

4. Enriques' *rentrée*

The project of a national Encyclopaedia appeared in Italy in the aftermath of World War I, within the nationalistic ferments that the "victorious war" had fomented, and out of the observation of our country's gap in this field. Indeed, the *Nuova Enciclopedia Italiana*, the only editorial realization of this kind, edited by Pomba, dated back to 1835. Hence some intellectuals began to study the feasibility of a national Encyclopaedia, scheduled in 24 volumes to be realized in "six or seven years".

Croce and Gentile's critical attention focused on this project. Their opposition to eclecticism explains their demand for an organic and unitary approach. On the project converged also the will of fascism to take command of all cultural organisms and of national propaganda. In 1924 Gentile took resolutely in hand the situation, depriving the old management of its authority and involving in the initiative instead senator Giovanni

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[20] A group of writers, professors and publicists has decided to give to the press their reply to the manifesto of fascist intellectuals. Such reply does not pretend to represent, least of all to monopolize, the antifascist intellectual elite which no congress has called or will call to parade in affected alignment; it intends to be, instead, and above all, a reaction against the method which would like to force the intellectual elite to act as *instrumentum regni*. It is, as well, the protest raised by some free-intellectuals against the version and the interpretation of Italian things spread beyond the Italian borders by fascist intellectuals, who seemed to consider it their duty.

[21] He was born in Trieste at the East border of Italy. Trieste was obliged to wait for a long time to be considered italian.

[22] We have quoted SIPS in Chapter 1 by describing the "social openness" of Italian mathematicians of the beginning of the century.

Treccani, a textile industrialist who had already previously sponsored other cultural events. On the 18th of February 1925 the *Istituto Giovanni Treccani per la pubblicazione dell'Enciclopedia Italiana* was set up. In his *executive Council* figured prominent personalities and intellectuals, who came from several professional fields and had several ideological stances. There were signatories of the *manifesto Gentile* but also signatories of Croce's counter-manifesto. The name of Enriques stands out.

His presence is surprising, if we think about the debates of the beginning of the century or about the judgement expressed by the *Mathesis* regarding Gentile's reform. But the surprise diminishes if we glance through the letters that Enriques had written to Gentile in this last period[23]. The turbulent relationship of the pre-war years had been replaced by an approaching motion whose psychological hinterland can be traced. Enriques noticed some isolation and certainly a minor centrality for him in the mathematical world. The episode of the "call" to Rome had caused more than an alarm bell ring: when it had come to decide between pupil and master, Roman colleagues have had no doubt about indicating Severi. Enriques had to accept an *escamotage*, made possible thanks to the helpfulness of his brother-in-law. Hence the reason for his will to "return". The reason for a new behaviour which showed willingness, collaboration and great courtesy towards Gentile. There are many examples. We will remark only two: the first one, in 1923, regards Enriques' proposal to invite Einstein to move to Italy, which would allow him to take shelter from the first anti-Semitic demonstrations. The second example, from 1926, regards the *Scuola romana di Storia delle Scienze*.

In order to re-enact the first event and to drag Gentile's name in, we must reassemble the pieces of a brief but noteworthy correspondence. On the 8th of February 1923 Enriques wrote to Einstein and restated the wish to see him settled in an Italian University. He had already manifested this wish in their talks in October 1921, during the week of Bolognese lectures on relativity. At the time Einstein had "in a friendly manner" told him about the motivations that made leaving Berlin not worth his while. "Now – continued Enriques – they say that the conditions of that city have changed and that – because of anti-Semitic reasons – You do not feel at ease anymore and are about to leave that place and Germany too". If it were so, wrote Enriques[24]:

> Rinasce la speranza di poterLa guadagnare, in qualche modo al nostro paese. Questa idea e questo desiderio è in molti e non aspetta che un'occasione e un incoraggiamento per manifestarsi e prendere forma concreta. Io mi sono limitato a parlarne col Ministro della P. Istruzione, che è il filosofo idealista prof. Gentile, ed egli mi ha autorizzato – sebbene in stretta confidenza – a dirLe che è per parte sua disposto ad accogliere molto volentieri una iniziativa in proposito.
>
> Al Ministro ho creduto opportuno di spiegare come Lei, nella Sua situazione, abbia motivo di desiderare soprattutto una grande libertà, ed egli ha compreso perfettamente la cosa e mi ha detto che – se Lei entra nel concetto di accettare una posizione

[23] The letters are published in A. Guerraggio, P. Nastasi, *Gentile e i matematici italiani*, quoted, p. 142–167.

[24] Enriques' letter, kept in the *Archives Einstein* in Jerusalem, has been published by S. Miliani in *l'Unità* on the 6th of July 1995, p. 3.

Federigo Enriques and Albert Einstein

in Italia – è disposto a studiare il modo di soddisfarla. E da parte mia aggiungo che, per tale scopo, si potrebbe cercare intanto una occasione di farLa venire qui per qualche conferenza e avere modo di trattare a voce la modalità della cosa.
Voglia frattanto avere la bontà, appena riceverà questa mia (che non so indirizzare se non al suo vecchio indirizzo di Berlino), di rispondermi un rigo, che mi affretterò a comunicare al Ministro. Inutile pregarLa di considerare, nel frattempo, questa mia come *riservata*, perché il Ministro mi ha espressamente pregato di evitare che la stampa possa impossessarsi anzi tempo dell'idea.[25]

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[25] The hope to gain You to our country swells again. This is the idea and the wish of many of us and it waits only a chance and an encouragement to express itself and to take a concrete shape. I have only spoken to the Education Minister, the idealist philosopher Prof. Gentile. He has allowed me to tell You – in strict confidence, though – that on his side he would willingly accept an initiative on that point.
I have thought it right to explain to the Minister how You, in Your situation, have grounds for wishing above all a great freedom, and he has perfectly understood it and has told me that – if You get into the idea of accepting a post in Italy – he would study the way of achieving it. And on my side I add that, to that end, we could in the meanwhile look for a chance to make you come here for a lecture and to discuss the conditions personally.
As soon as you receive this letter (which I don't know where to address if not to your old address in Berlin), please be so kind as to briefly answer me, and I will quickly report to the Minister. Meanwhile, I do not need to ask you to consider this letter as *confidential*, because the Minister has explicitly asked me to avoid that the press knows about it before time.

Einstein's answer arrived more than two months later, on the 11th April 1923, and it was again a kind and reasoned refusal[26].

> Caro Collega,
>
> la Sua lettera mi ha molto commosso, e devo confessarLe apertamente che preferirei la compagnia Sua, e di Levi-Civita, a quella dei miei colleghi di qui. Ma io non soffro alcun danno dall'antisemitismo, sebbene esso sia presente in grado molto alto. Accade al contrario che io sia lasciato più in pace, di quanto avverrebbe se l'antisemitismo non ci fosse. In più sono molto legato al mio abituale *habitat* da rapporti familiari, professionali e di amicizia. Soprattutto, alla mia età non è così facile cambiare ambiente, perché non si ha più elasticità sufficiente per potersi amalgamare con un ambiente nuovo.
> Per tutte queste ragioni, con ogni sentimento di gratitudine e simpatia per Lei e per il Suo paese, a me sempre particolarmente caro, non posso decidere in questo momento di accettare la Sua gentile proposta. Ma se in futuro mi sentirò costretto, per l'intensificarsi del pericolo, ad abbandonare questo mio nido, mi rivolgerò subito a Lei con gioia e fiducia.[27]

It would not be long for that tragic foreboding to arrive but, when in 1933 Einstein would be compelled to leave his "nestle", fascist Italy would not appear to him as the most inviting and safe country. But this is another story. Let's go back to April 1923: Enriques informed Gentile immediately about the answer, referring to Einstein's "deepest gratitude", but also of the difficulties he had in leaving the Berliner circle. "These are the contents of the letter – he followed – and if there will be no development, it is nonetheless nice on Your side to have tried it"[28].

The second event regards the development of the *Scuola di Storia delle Scienze* in Rome. On the 12th January 1926 Enriques sent a letter to Gentile. Beyond its contents, the tones used are extremely revealing of the changed behaviour towards his old opponent.

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26 The letter, in possession of the family of Adriana Enriques, has been published by O. Pompeo Faracovi, *Federigo Enriques: filosofia e storia del pensiero scientifico*, Belforte, Livorno, 1998, p. 279.

27 Dear Colleague,
Your letter has deeply touched me, and I have to confess You openly that I would prefer Your company, and Levi-Civita's, to that of my colleagues here. But I am not damaged by anti-Semitism, although it is quite present. On the contrary, I am left in peace more than I would if there was no anti-Semitism. Besides, I feel very close to my customary *habitat* due to familiar, professional and friendship relationships. Above all, at my age it is not so easy to change environment, as I am not flexible enough to be able to merge with a new environment.
For all these reasons, and with all my gratitude and sympathy for You and for Your country, always particularly dear to me, I cannot accept Your kind proposal now. But if in the future I will feel compelled to leave my nestle, due to the intensification of the danger, I will immediately address to You with joy and confidence.

28 Cf. A. Guerraggio, P. Nastasi, *Gentile e i matematici italiani*, quoted, letter to Gentile from the 15th April 1923, pp. 150–51.

Caro Gentile,
Le lascio la Relazione che presento al Ministro, acciocché – come siamo d'accordo – Lei possa appoggiare la proposta da parte Sua. Sono ansioso del risultato. Non Le dico grazie perchè – se posso ripetere Renzo – Lei lo fa per Uno che paga bene: cioè per un progresso spirituale che Le sta a cuore.
Ma di sentire l'esigenza di questo progresso, così com'io la sento, e insieme di concedermi personalmente la Sua fiducia come operaio della buona opera, Le sono intimamente grato[29].

The appointment of Enriques as a member of the board of governors and as director of the mathematical branch was not then surprising. In such a capacity, he was given the task of contacting Volterra, in order to "recruit" him to the *Encyclopaedia*. The manoeuvre did not succeed. Then Gentile intervened directly, but also his letter from the 8th April 1925 would get no answer: "I don't know if you have seen the enclosed manifesto, to which I shall take the liberty of drawing your attention. As you will see, it is about a national work which intends to gather the intellectual energies of the country, beyond all political parties or scientific branches, in a big structure which would honour Italian studies and would duly represent them worldwide. These studies, at the same time, would increase national culture"[30]. Gentile, by then firmly at the helm of the cultural politics of the regime, was not stingy with "openings". It can be seen in the correspondence with another desirable collaborator, to which he sent the message "that the Encyclopaedia is absolutely extraneous to political fights; that I am involved only in the general *technical* management"[31].

The project of the Encyclopaedia took off quickly and quite efficiently. A promotional manifesto which planned the issue of 32 volumes within 1936 was also published. It is surprising still today to confirm that, in a work of such a size, the project was actually completed: there would be 35 volumes, and the last one would be issued in 1937! The general framework of the *Treccani* was very close to the structure of the *Encyclopaedia Britannica*. Some 60.000 entries provided for its monographic character, while the 240.000 minor entries (which cross-referenced the main ones) presented the *Encyclopaedia* not as a specialized work but as a "great organ of information within the reach of a vast public with a popular culture".

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[29] Dear Gentile,
I leave you the Report I present to the Minister, so that – as we agreed – You can support the proposal on Your side. I am worried about the result. I don't thank You because – if I can quote Renzo [a character in Manzoni's *Promessi Sposi*] – You do it for One who pays well: that is, for a spiritual progress that means a lot to You.
But I am sincerely grateful to You for feeling the need of a progress, as I do, as well as for placing Your trust in me as performer of this task.

[30] The letter is published in G. Paoloni, *Vito Volterra e il suo tempo (1860–1940)*, Accademia Nazionale dei Lincei, Roma, 1990, p. 141.

[31] Cf. A. Guerragio, P. Nastasi, *Gentile e i matematici italiani*, quoted, p. 95. The interlocutor is Vittorio Emanuele Orlando (1860–1952), another exponent of the liberal opposition, who hesitated to collaborate and whom Gentile invited to learn to "distinguish" so as to "overcome" his political reservations.

Enriques organized the mathematical section. A scholar would find something less than he would have read in a special treatise, such as the *Enzyklopädie der Mathematische Wissenschaften*, but at the same time he was offered a concise view of the individual disciplines, as well as a description of their historic development and of their relationships with other cultural areas. Enriques had the chance to reopen some of the questions of the beginning of the century, which therefore revitalized them, thanks to the indulgence of Gentile. It was obviously a controlled indulgence and Enriques was forced to face, even harshly, the editorial staff of the philosophic branch each time scientific voices approached its sphere of influence. In the end, scientific culture had a more substantial presence than one could imagine a priori. But a product of a cultural hegemony does not revolve around the number of pages reserved to mathematics or on the number of pages dealing more or less with the relationship between science and philosophy. The main thing – to Gentile, who was such an "idealist" that he found "millions of lira that positivists were not able to put together"[32] – was not so much the contents of the work but the fact that it was being carried out and that Italian intellectuals of all shades approved of it. He tried to explain this behaviour and this confidence to the extremist wings of fascism, which wanted to drastically limit the presence of people not aligned with the regime within cultural institutions. Hence in the speech held in Campidoglio on the 19th December 1925 on occasion of the inauguration of the *Istituto nazionale fascista di cultura*, he invited his listeners not to disown "an instrumental culture, according to which two and two will always be four, whether you sum caresses or wallops. And the fascist cannot be willing to get rid of this instrumental culture, which is mere knowledge, organization of well-known notions, criticism, erudition, learning". The day following the publication of the first list of contributors to the *Encyclopaedia*, Gentile said something similar[33]: "I would consider myself unworthy of the card the Fascist Party offered to me in May 1923 (…) if I saw in me such a narrow-mindedness not to be able to distinguish politics from technology in a work that will prove to be a great exam taken by thought and by the character of Italians before all civil countries, most of which preceded us in this contest; if, due to my inconvenient wish to withdraw into my comrades' stronghold, I would not use all people and forces which Italy can offer to build this great national monument (…) This is, in my opinion, fascism. A fascism which can assert with well-deserved pride: I am not a party, I am Italy. A fascism that can and must gather all Italians in every national initiative: even the Italians of the anti-manifesto. If they answer the roll-call, they will not come (…) to do anti-fascism: they will come, at least as regards the Encyclopaedia, to bring their expert contributions: to deal with mathematics or chemistry or physics, in a word, with science". In brief, intellectuals were free, but free to work within their specialization. Politics did not filter out ideological contents, but represented the undertaking as accomplishing the will of fascism.

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32 As G. Turi, *Giovanni Gentile. Una biografia*, quoted. pp. 422–23, documents, the fascist government interfered several times in order to save the project of the Encyclopaedia from the economic crash.

33 Cf. G. Gentile, L'Enciclopedia Italiana e il fascismo, letter to the chief editor of *La Tribuna*, 28th April 1926.

Chapter 5

One man alone in the lead

1. The novelty of the Accademia d'Italia

We would now like to review some snapshots from the previous chapters. There is the "old" Volterra, who, at perhaps his moment of greatest renown – he has "won" the war; he has contributed in a crucial way to the foundation of the U.M.I.; he is chairman of the C.N.R.; he is president of the *Accademia dei Lincei*; he has "in mind" the models that will act as a background to the birth of mathematical Biology –, is compelled to step aside in favour of the new regime that has installed itself in Italy. It is the sunset of the liberal *elite*. A whole bourgeois world is asked to settle down. For Volterra – almost 70 – this means farewell to the chairmanship of the C.N.R. and of the *Accademia dei Lincei* or, in general, to every political-scientific appointment.

Severi seems to have gone downhill as well. Before the war he had argued with the idealist philosophers and now he finds Croce and (especially) Gentile at the top of the national culture and education system. Before the war he had not hidden his support for socialist ideals. We have found him again in Rome, when talking about his transfer, soon followed by Enriques'. The comparison between pupil and master begins thus. The first does not feel a reverential fear anymore. Instead, he is conscious that the Faculty of Rome had established a new structure of hierarchies.

With the presence of Castelnuovo, Severi and Enriques, Rome is really the capital of algebraic geometry. Thus writes C. Parikh in the 3rd chapter of Oscar Zariski's[1] biography (dedicated upon the young Russian mathematician's arrival in Rome, in 1921)[2].

> In the fall of 1921 the University of Rome was the most important center of algebraic geometry in the world. What is now known as "the Italian School" had been

[1] Oscar Zariski (1899–1986) will stay in Rome until 1927, to study under the guidance of Enriques and especially of Castelnuovo. Transferred to the United States, he will become (from the 1930s onwards) one of the main supporters of the need for a rigorization of algebraic Geometry.

[2] Parikh C., *The Unreal Life of Oscar Zariski*, Academic Press, Boston, 1991.

> started by Luigi Cremona soon after the unification of the Kingdom of Italy, a generation before Zariski's arrival in Rome. It was only after 1900, however, as a result of the combined efforts of three great Italian mathematicians – Guido Castelnuovo, Federigo Enriques, and Francesco Severi – that the Italians had carried algebraic geometry off in startling new directions.

The presence of Zariski is not an isolated event. Also thanks to the links of Levi-Civita and Volterra with the *Rockefeller Foundation*, a great number of foreign students come to Rome in those years: Griffith C. Evans (USA); Paul Alexandrov, Alexander Weinstein, Oscar Zariski (USSR); Paul Dubreil, Joseph Pérès, Paul and Marie-Louise Jacotin Dubreil, Guérard des Lauriers, Szolem Mandelbrojt, André Weil (France); Octav Onicescu, Giorgio Vranceanu (Romania); Dirk Struik (Holland); Herbert Busemann, Werner Ferdely, Harald Geppert, Hans Lewy (Germany).

But let's return to Severi. His Roman career, at first, is blossoming. He becomes Rector of the University. But then he signs the *Croce manifesto*. Earlier on, to tell the truth, he had already argued with Gentile (as Chairman of the *Associazione nazionale dei professori universitari*) on the occasion of a Decree of the 6th of March 1923 that threatened the "political" purge of the University teachers who were not aligned with the stances of the new regime. Some signatures do not pass unnoticed and Severi finds himself obliged to give his resignation as Rector, in order to hush a ministerial enquiry about some supposed administrative irregularities during his tenure[3]. Severi's support for the *Croce manifesto* is released in mid-May 1925 and the administrative enquiry opens on July that same year!

At the same time, the position of Enriques, dramatically enhanced by the *Enciclopedia* and the assignment to coordinate the whole mathematical section, seems set to strengthen. But the end of the 1920s holds for us a surprise, through the foundation of the *Accademia d'Italia*.

In the previous chapter we talked about the first *Convegno nazionale delle istituzioni fasciste di cultura*, held in Bologna in March 1925. The regime – just emerged from the turmoil of Matteotti's case – faces the problem of re-enlisting to its politics the intellectuals and the university world, regarded as "the environment traditionally most refractory to fascism"[4]. He will use "the stick" and "the carrot", in the direction of a populist culture, however. We will also talk about this regarding Mathematics: culture must not lose itself in abstruse dissertations but remain connected to tradition and to the country's needs.

Mussolini feels that the hesitant intellectuals must be courted so as to win their (at least) nominal allegiance. Thus, on the 7th January 1926 he announces the foundation of the *Reale Accademia d'Italia*, to which he assigns the magnificent Renaissance Farnesina

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[3] The episode is told to Gentile by Severi himself, in some letters published in A. Guerraggio, P. Nastasi (eds.), *Gentile e i matematici italiani. Lettere 1907–1943*, Bollati Boringhieri, Torino, 1993, p. 193–206.

[4] The expression is found in a 1929 *memorandum* of the "normalized" C.N.R.'s general segretary, Giovanni Magrini (cited in R. Simili, G. Paoloni (eds.), *Per una storia del Consiglio Nazionale delle Ricerche*, Laterza, Roma-Bari, 2001, 2, I, p. 151).

Francesco Severi

palace, situated in front of Corsini Palace (seat of the ancient *Accademia dei Lincei*). It is a deliberately provocative choice. The message was clear: the newborn *Accademia d'Italia* was bound to replace those *Lincei* too unwilling to take sides. After three years of arduous preparation (used also to decide, down to the minute details, the academic uniform's ornaments), on the 16th of March 1929 the text of the decree approving the Statutes of the *Accademia* and indicating the name of its President, Tommaso Tittoni (1855–1931), formerly Chairman of the Senate, appears on the *Gazzetta Ufficiale*. The physicist Enrico Fermi figures among the first members of the Academy. It is Mussolini himself who, later, will recall his appointment.[5]

> The most astonished at the appointment to the Academy was professor Fermi. He was proposed by the whole Roman school of Physics : professors, assistants, collaborators of all university ranks. Professor Fermi seemed to me a simple young man, surprised by the choice that, he said, had rewarded him all too highly. I was told we were in front of the highest and scientifically most authoritative voice of Italian culture. I was told, too, that, at least in that moment, Professor Fermi gave, with his presence at the academy, an unrivalled cultural authoritativeness to the entity for which I had worked during three years.

Much less expected is Severi's appointment – as the only mathematician! – It is, actually, a downright surprise! Actually, the designation "crowns" him as *number one* of

[5] Y. De Begnac, *Taccuini Mussoliniani* (F. Perfetti ed.), Il Mulino, Bologna, 1990, p. 317–318.

Italian mathematics (and not only of algebraic geometers), a privileged interlocutor chosen by the regime to represent this community. He, a former socialist and professed antifascist in 1925!

What has happened, in the meanwhile, during the years that go from the *Croce manifesto* to the constitution of the *Accademia d'Italia*? How does Severi set himself up to become the *only man in the lead* and to realize the aspiration of someone who "wanted to be (and, in a certain measure, was) the "master" of Italian mathematics during the fascist period"?[6]

2. Severi as a mathematician, in the 1920s

Severi, first of all, consolidates his fame as a researcher. In the *Prologue* we talked about the great successes of Italian algebraic geometry at the beginning of the century. After the strong impulse received from researchers such as Cremona and Segre, the Italian school dramatically attracted international attention with Castelnuovo, Enriques and Severi. Italian mathematicians had managed to arrive at a definitive reworking of the theory of curves with Castelnuovo and Severi, to the creation of the theory of surfaces with Castelnuovo and Enriques and to their total birational classification again with Enriques, as well as to the individualization of the basics of a general theory of higher dimensional algebraic manifolds, and of their invariants with Severi. From the first years of the century, an abundance of international certificates and survey articles made offical a general recognition of Italian leadership in this field.

The post-war years are different, even for Italian algebraic geometry. The original research, in unexplored fields, slows down its rhythms in favor of a reworking and systemization of everything that had been accumulated during the previous period, perhaps in a messy way and not always according to an accurate plan.

In general, it is necessary to answer the challenge which, progressively, will emerge from young researchers such as Oscar Zariski, Bartel van der Waerden, André Weil and from the new algebraic and topological languages. The subject's progress must be guaranteed once again by fidelity to the projective methods. The consolidation of geometry on a surface and its extension to varieties is brought up. Severi perceives that above all this subject needs a reworking by the glorious methods that had previously driven the Italian school to success. "In substance – as A. Brigaglia and C. Ciliberto write – it was the height itself of the edifice that was being constructed that required there to be more certain and solid foundations"[7].

After what has been said, one is not astonished that in the 1920s Severi's main attention, as a researcher, is turned to the drawing up of some treatises, beginning with the *Vorlesungen über algebraische Geometrie*[8] published in 1921.

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[6] F. Tricomi, *La mia vita di matematico*, Cedam, Padova, 1967.

[7] A. Brigaglia –C. Ciliberto, *Italian Algebraic Geometry between the Two World Wars*, Queen's Papers in Pure and Applied Mathematics, 100, Kingston, Ontario, Canada, 1995.

[8] F. Severi, *Vorlesungen über algebraische Geometrie*, Teubner, Leipzig, 1921.

The German text is the translation of a volume of *Lectures*[9], edited in Padua in 1908, that in its turn was the first organic presentation of the methods and results of the Italian school. It contains an important preface by A. Brill, who recognizes the Italian primacy and invites young German scholars to acknowledge it in order to meet the challenge and return to a top position in research in the field. Finally, the *Vorlesungen* are enriched with some appendixes. Severi's intention was to suggest to the international public a general outline of the discipline – gathering subjects often dispersed in articles written in Italian and conforming them to the new standards of rigor – but he does not want to renounce the possibility wholly of exploring at least partially new questions and methods. It is exactly the role of the *appendixes*. Among these, Appendixes *F* and *G*. will be the most renowned and quoted[10] – because of the information they give about new research. This one includes subjects already introduced in a 1915 Note, with the study and the classification of the twisted and hyperspatial curves and Riemann existence theorem (starting from the one of existence of a Riemann surface given genus with given ramification points). The entangled problems present already its own complexity in the plane case and still the aim is their extension to the twisted and hyperspatial curves.

The *Vorlesungen* are followed in 1926 by the publication of the first volume of the *Trattato di Geometria algebrica*[11], dedicated to the geometry of linear series. Within the systematization work reasserted in the introduction – "to gather, to co-ordinate, to complete where it is necessary, all that is important within the field of algebraic geometry" – rigour becomes the priority aim. All energies are addressed to reducing the disproportion, by now evident, between more and more complex problems and tools that instead, in substance, remain those of some previous decades and must therefore be used with a certain "confidence" if their competitiveness is to be extended.

> A questo primo seguiranno un volume dedicato ai sistemi continui di curve piane, sghembe e iperspaziali ed alle relative questioni d'esistenza, di classificazione, di postulazione; ed altri in cui verranno esposte la teoria riemanniana delle curve e dei loro integrali; la geometria sopra una superficie e sopra una varietà e le teorie degl'integrali ad esse appartenenti, nonché le proprietà fondamentali delle funzioni (abeliane e automorfe) collegate colle funzioni algebriche; e infine la geometria numerativa.
> Il programma è vasto; ma credo tornerà utile anche se riuscirò a svolgerne soltanto una parte.
> Io desidero che il *Trattato* sia metodico e che ogni questione venga sviluppata in modo esauriente e rigoroso, anche perché occorre sfatare la leggenda che nella geometria algebrica la mancanza di rigore e di determinatezza sia quasi una necessità.

9 F. Severi, *Lezioni di Geometria algebrica*, Litografie Draghi, Padova, 1908.

10 O. Zariski, *Algebraic surfaces*, Ergebnisse der Math., III/5, Springer-Verlag, Berlin, 1935 (Chelsea, New York, 1948, p. 167–171).

11 F. Severi, *Trattato di Geometria algebrica*, Bologna, Zanichelli, 1926 (vol. I, 1st part).

Non disconosco tuttavia che il proceder innanzi per rapide visioni, in cui si prospettino le linee essenziali dei singoli problemi, può, sotto altri aspetti, esser vantaggioso.
Io procedo invece coi piedi di piombo e il lettore accurato lo constaterà e noterà quante volte mi vien fatto di arrestarmi a dimostrar proprietà, che si solito si ammetton come evidenti.
Per esempio chi legge il capitolo sulle corrispondenze troverà i fondamenti di questa teoria svolti con ampiezza inusitata; e una volta richiamatavi la sua attenzione, credo si convincerà facilmente della necessità che, in un assetto rigoroso, sia ben fissato il valore dei principi di corrispondenza, nei riguardi delle molteplicità dei punti uniti.
Questo volume si distacca alquanto dalle mie *Lezioni* litografate del 1908 e dalle *Vorlesungen* del 1921, come può verificarsi con un'occhiata all'indice.
Chi voglia esporre la parte essenziale della geometria sopra una curva in breve corso di lezioni, trova qui i mezzi adatti. Lo scioglimento delle singolarità d'una curva e le proprietà fondamentali che culminano nel teorema di Riemann-Roch, si conseguono col «metodo rapido», occorrendo soltanto il concetto generale di trasformazione birazionale e quello di gruppo jacobiano di una serie lineare. Ma anche gli altri metodi algebrico-geometrici (quello iperspaziale e quello più strettamente algebrico) sono esposti in modo che ciascuno di essi possa con facilità isolarsi dagli altri[12].

[12] To this first will follow a volume dedicated to continuous systems of plane, twisted and hyperspatial curves, and to the relative questions of existence, of classification, of postulation, and others in which Riemann's theories of curves and of their integrals, geometry on a surface and on a variety, and the theories of the integrals belonging to them will be exposed, as well as the fundamental properties of (abelian and automorphic) functions linked to algebraic functions, and finally enumerative geometry.
The program is vast; but I think that it will be useful even if I manage to develop only one part.
I want the *Treatise* to be methodical and each question to be developed in a exhaustive and rigorous way, also because the legend that in algebraic geometry the lack of rigour and of determination is almost a need must be exploded.
I do not deny however that to move on through quick visions, in which the essential lines of the single problems are presented, can, under other aspects, be advantageous.
I proceed instead cautiously and the meticulous reader will notice it and will remark how often I stop to prove properties that usually are taken for granted.
For example whoever reads the chapter on correspondences will find the fundamentals of this theory developed to an unusual extent; and once his attention is drawn, I think that he will easily convince himself of the need that, in a rigorous order, the value of the principles of correspondence be well fixed, relating the multiplicities of the fixed points.
This volume draws a little upon my 1908 lithographed *Lectures* and from the 1921 *Vorlesungen*, as one can verify glancing at the index.
Whoever wants to expose the essential part of geometry on a curve in a brief course, finds here the right methods. The resolution of the singularities of a curve and the fundamental properties that culminate in Riemann-Roch's theorem are obtained with the "quick method", so that only the general concept of bi-rational transformation and that of Jacobian group of a linear series are necessary. But also the other algebraic-geometrical methods (the hyperspatial one and the more strictly algebraic one) are exposed so that each of them can be easily isolated from the others.

Experienced expositor as he is, Severi does not undervalue the merits of "wide views" but, actually, he chooses a direction opposite to the one we will see in Enriques' treatises, usually asystematic and always turned from the particular to the general. The *Treatise is* above all methodical and rigorous. These characteristics are just what blocked the initial project, that provided the publication of several volumes with the aim of delineating the basics of a general theory of algebraic varieties. The search for the highest accuracy and attention to details in demonstrations, even in the parts considered "definitively acquired" and anyway evident, highlighted the fact that grey zones are unfortunately more extended than expected and thus raise some doubts about the possibility of continuing to use the available technical tools in a correct manner.

The *Vorlesungen* and the *Trattato*. The choice of synthesizing Severi's mathematical activity in the 1920s through his textbooks must not induce us to forget his numerous notes and memoirs – Severi's research goes on to set, among others, the objective of a complete construction of geometry on a variety – nor to make us think of this as a low-keyed decade. Not in the least! We have already talked about the strategic value for research that Severi gives to the systematization work. This is a building block far from unimportant in the construction of the image of a *leader*. The treatises; their framing, synthesis and deepening work; the numerous historical notes that accompany each important result and present it as an epilogue of a complex historical *iter*; the "proclamations" and the proud claims of the Italian school's merits: it is also through these elements that Severi ascends to the role of unquestioned protagonist of algebraic geometry. If one thinks about the role that the discipline has in the Italian mathematical scene and to this valuation adds Severi's relational ability, one understands how the transition that takes him to the designation of the *Accademia d'Italia* is reached.

And Castelnuovo and Enriques, where are they? Before going back to them, we have to examine other aspects that promote the change of *leadership* within Italian mathematics too, starting with some allusions to the *school* that, in the 1920s, forms slowly around Severi.

His main pupil is Beniamino Segre (1903–1977), not so much for the time passed in direct contact with the master – only four years – as for the coincidence of orientation and of perspectives that takes place in this period, albeit short. Beniamino Segre gets his degree in Turin in 1923 with the other Segre, Corrado, his distant cousin; he spends a year of study in Paris with Élie Cartan and arrives in Rome, as Severi's assistant, in 1927. We talked of a 4 years-period, because in 1931 B. Segre will transfer to Bologna where in the meanwhile he has won the Geometry chair. His most important works are perhaps just those from the successive decades. The intellectual influence exercised by Severi is such that we will find B. Segre engaged in the same areas of algebraic geometry: series of equivalence, theory of algebraic curves moduli, foundational questions, functions of several complex variables.

Senior are Annibale Comessatti (1886–1945) and Giacomo Albanese (1890–1947). The first one graduated at Padua in 1908 under a very young Severi, taught afterwards at the Universities of Cagliari and, from 1922, and definitely, of Padua. His publications concern especially the theory of abelian mainfolds, the problems of reality, the theory of curves – the 1922 *Bulletin des Sciences mathématiques* contains one review article of his[13], that draws attention to the results obtained by Severi – and that of surfaces. In this last research area, in 1922, he publishes a Note[14] in which he proceeds to a classification of irregular surfaces (those for which the inequality $p_g \geq 2 \cdot (p_a + 2)$ is valid especially with the equality sign).

Severi's relationship to Albanese, who graduated at Pisa in 1913, appears to be less intense. Severi undoubtedly inspires one of his first works on algebraic systems of curves on an algebraic surfaces, but afterwards Albanese's research turns to the problem of the resolution of the singularities of a surface and to the theory of equivalences of groups of points on a variety (and, still, to the theory of correspondences). One note of his[15] from 1924 presents a method – extended later to surfaces – to birationally transform a plane curve to a smooth one in the space, without singularities. In other notes from 1924–1927 Albanese proves sufficient conditions in order a surface is rational; he resolves the problem of the base for the curves on a surface and undertakes a general study of the geometry of manifolds. These are the years in which Albanese becomes professor of Geometry at the University of Catania, to pass later to Palermo and then to Pisa. In 1936 he will definitively leave Italy, to move to S. Paolo University in Brazil.

3. Severi: politician

We do not want to go into particular psychological considerations, to explain certain acrobatic "leaps" of Severi "politician", but even so we have to mention his unhappy childhood: a tyrannical father that had eyes only for the eldest son, who one fine day leaves the family to flee to Latin America, the father's pain and the subsequent suicide, also because of a temporary financial ruin, the family's consequent straitened circumstances (three sisters and an energetic and authoritarian mother, to whom he remained particularly tied); the need to continue his studies, tutoring his contemporaries or turning to not always hefty scholarships. From such a context develops also that iron determination that alone can explain such an ability to definitively overcome seemingly insurmountable difficulties

We have already said that during the 1920s Severi continues his geometric research, with the same energy and creativity, starting the arrangement of the huge amounts of material that had been accumulated. The scientific commitment does not prevent him from an analogous commitment to his political-administrative responsibilities. We know the socialist Severi, *interventionist* during the harsh debate that preceded Italy's entering into the first world war. We have seen him, just transferred to Rome, tak-

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[13] A. Comessatti, Sur la classification des courbes algébriques et sur le théorème d'existence de Riemann (à propos d'un ouvrage de M. Severi), *Bull. des Sciences Math.*, 46 (1922), p. 1–48.

[14] A. Comessatti, Intorno alle superficie irregolari con $p_g \geq 2 \cdot (p_a + 2)$ ed un problema analitico ad esse collegato, *Rend. Circolo Matematico di Palermo*, 46 (1922), p. 1–48.

[15] G. Albanese, Trasformazione birazionale di una curva algebrica qualunque in un'altra priva di punti multipli, *Rend. Acc. Lincei*, 33 (1924), p. 13–14.

ing on the rector's office, from which he will have to resign in 1925 as a consequence of the fascist tightening following the Matteotti case and of his signature on the *Croce counter-manifesto*.

In the meanwhile, his controversy with Enriques becomes more and more fiery and goes past the boundaries of a simple private sphere. We found a proof of this contention in the letter to Gentile dated 24th April 1928, with which Severi communicates to his powerful friend the decision to withdraw his collaboration on the *Italian Encyclopaedia*[16].

> Carissimo Gentile,
>
> Leggi l'acclusa forse una recensione ed edificati.
> Il testo cui si allude è quello di Enriques-Amaldi. Del primo nulla mi fa caso; ma l'altro lo ritenevo tanto superiore a lui moralmente. E invece gli ha tenuto il sacco. Che dolorose delusioni nella vita, quando si crede all'onestà, all'obiettività, alla moralità e a tante altre cose in "tà".
> Ma non ti segnalerei la cosa che ti interesserà assai mediocremente (se mai ti puoi compiacere di aver scritto la prefazione a un testo, che pare contenga veramente quello di cui la Scuola oggi ha bisogno[17]), se non fosse per trarne una conclusione. E la conclusione è questa: Con un uomo come Enriques (che non è in materia alle sue prime armi) e che io giudico il più inadatto per compiere un'opera di valutazione obiettiva, come quella che si richiede in un'Enciclopedia, io non posso più avere nulla di comune e tanto meno relazioni di quasi subordinazione. Perciò ti prego cancellarmi dall'elenco dei collaboratori dell'Enciclopedia.
> Non te ne avere a male; ché tu non c'entri. Ti sarò grato se mi darai o mi farai dar atto della cosa[18].

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16 The letter is kept in the Archive of the *Foundation Giovanni Gentile* in Rome.

17 The allusion is to Severi's textbook (*Elementi di Geometria*, Firenze, Vallecchi, 1928) that contains a presentation by Gentile.

18 Dearest Gentile,
Read the enclosed perhaps a review and learn.
The text hinted at is Enriques-Amaldi's. I'm not surprised about the first; but I considered the second much morally superior. And instead he has been his accomplice. What painful disappointments in life, when one believes in honesty, objectivity, morality and in many other things ending in "-ty".
But I would not remark the thing which will interest you quite poorly (if you can ever be pleased with having written the preface to a text which seems to really contain what the School needs today), if not to draw a conclusion. And the conclusion is: With a man like Enriques (who is not a beginner in the subject) and whom I judge most unsuited to carry out a work of objective valuation, as the one that is required in a Encyclopaedia, I cannot have anymore in common, let alone a relationship of subordination. Therefore I ask you to cross me out from the list of collaborators to the Encyclopaedia.
Don't take it the wrong way; you have nothing to do with it. I will be grateful to you if you acknowledge or let me acknowledge the thing.

Less than a year later, a second tirade against the former master begins. The letter is dated 15th February 1929 and is sent from Barcelona where Severi finds himself for a cycle of seminars[19].

Carissimo Gentile,

Avrei avuto desiderio di venirti a salutare prima di partire per Barcelona (dove resterò circa 3 mesi); ma non mi è riuscito. T'invio pertanto di qua i miei saluti e ti prego vivamente di ricordarti di "preparare l'ambiente" presso i membri del G. C. [Gran Consiglio] sui quali tu puoi più facilmente influire, affinché quando, nel prossimo G. C., verrà, com'è probabile, la questione degli "intellettuali", essa possa esser risoluta nel modo migliore e in via definitiva.
Io per parte mia – e nei limiti delle mie scarse possibilità – ho fatto tutto quanto potevo per questo scopo ed ho ragione di ritenere che il Capo del Governo sia ottimamente disposto. Ragione **fondata**: senza di che, naturalmente, non te ne scriverei.
Ma il Capo del Governo non sembra deciso a portare la cosa al G. C.; pare preferisca di farla risolvere dal Consiglio dei Ministri. Su questo punto però le mie informazioni sono meno sicure. Bisognerebbe prospettare o far prospettare al Capo del Governo, quello che già ti dissi a voce. E cioè che una soluzione deliberata dal Consiglio dei Ministri (una più precisa formula del giuramento p. es.) non può esser risolutiva. Perché vi sono troppi interessati a che la questione si perpetui. Ed essi avranno facile giuoco nel tentativo d'isolare il Ministro della P. I. dal Governo, facendo apparire la soluzione come opera sua esclusiva e svalutandone a priori la portata. D'altronde anche una nuova forma di giuramento basterebbe allo scopo, s'essa fosse proposta e **chiosata** dal G. C. come supremo Corpo politico, in cui son rappresentati anche, attraverso al Partito e alla stampa, le ali d'avanguardia; che così assumerebbero la corresponsabilità dell'atto e non potrebbero più avversarlo, apertamente o copertamente. Occorrerebbe che il provvedimento fosse rappresentato come un atto d'intransigenza diretto ad ottenere la tanto richiesta fascistizzazione delle Università; come un appello alla lealtà dei professori, i quali non potrebbero mancare al giuramento senza incorrere in provvedimenti ben più gravi della messa a riposo d'autorità. Ma nello stesso tempo come una sanatoria di atti politici ormai lontani, per guisa che lo Stato, nell'ambito tecnico, potesse giovarsi senza limitazioni di ogni professore che al giuramento si fosse sottoposto; eliminando dunque l'assurda situazione attuale di tanti professori, che lo sono soltanto a metà, non potendo neppure far parte di Commissioni giudicatrici! Situazione che arriva ad assurdità inconcepibili, come la mia, che posso, attraverso un decreto governativo, che mi pone a disposizione del M° degli esteri, rappresentare la scienza italiana all'Estero, ma… non in Italia! Il mio compito qui non è già di fare soltanto conferenze, ma un corso vero e proprio per gli allievi e dirigerne alcuni in ricerche personali. Il meglio che si possa fare per attrarre nell'orbita della cultura italiana la gioventù spagnuola. Ti sarò grato se, quando potrai, mi farai avere qualche notizia. (…)

[19] The letter is kept in the Archive of the *Foundation Giovanni Gentile* in Roma.

> P.S. Negli ultimi giorni in cui fui costà, per *desiderio* dei nominati Enriques e Amaldi e per le insistenze di amici, mi riappattumai formalmente con quei signori. Ma con grande dichiarata ripugnanza; chè io non posso avere due faccie. Così potrò riverire, al mio ritorno, il tuo amico Ghigo novello accademico, *se riuscirà* nei suoi intrecci tittoniani[20].

We know who are the friends about whom Severi talks, because his letter finds an exact correspondence in a passage from another letter, this time Levi-Civita's to Volterra, dated 3rd February 1929[21].

> Come Ella probabilmente ricorda, Enriques e Amaldi da un lato e Severi dall'altro avevano rotto ogni rapporto personale per questioni intrinsecamente poco importanti, ma collegate a interessi economici attraverso i libri di testo. Dopo una serie di

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[20] Dearest Gentile,
I wished to come and greet you before leaving for Barcelona (where I will stay for about 3 months); but I have not managed to. Therefore I send you my greetings from here, and I heartily ask you to remember to "prepare the atmosphere" for the members of the G. C. [Great Council of the Fascism] on which you can more easily have an effect, so that when, in the next G. C., the question of the "intellectuals" comes, as it is probable, it can be resolved in the best way and definitively.
I on my side – and within the limits of my scarce possibilities – have done as much as I could for this purpose and I am right to think that the Head of Government is extremely well disposed. **Solid** reason: without it, of course, I would not write you about it.
But the Head of the Government does not seem determined to take the thing to the G. C.; it seems he prefers it to be resolved by the Council of Ministers. On this point, though, my information is less sure. What I already told you verbally has to be proposed or make to be proposed to the Head of Government. That is, that a solution deliberated by the Council of Ministers (a more accurate formula of oath f. ex.) cannot be resolutive. Because there are too many interested to the perpetuation of the question. And they will have an easy game in the attempt to isolate the Minister of Education from the Government, making the solution seem exclusively as his own and devaluating a priori its reach. On the other hand, even a new form of oath would be enough for the purpose, if it were proposed and **explained** by the G. C. as supreme political Body, in which the avant-garde wings, through the Party and the press, are also represented; thus, they will take on the co-responsibility of the deed and could not hinder it anymore, openly or covertly. The measure should be represented as an intransigency deed aiming at obtaining the so much requested fascistization of Universities; as an appeal to the loyalty of professors, who could not fail to take the oath without incurring in measures quite more serious than the put to rest of authority. But at the same time as an act of indemnity of by now distant political deeds, so that the State, within the technical field, could make use without restrictions of every professor that had submitted to the oath; eliminating then the absurd present situation of many professors, that are so only half way, not being even able to belong to judging Commissions! This situation arrives to inconceivable absurdities, such as mine; I could, with a government decree that puts me to the Foreign M.'s disposal, represent Italian science abroad, but… not in Italy! My task here is not to give only lectures, but a real course for students and to address some in personal researches. The best we can do to attract in the orbit of Italian culture Spanish youth. I will be grateful to you if, when you can, you give me some news. (…)
P.S. In the last days in which I was there, for *wish* of the mentioned Enriques and Amaldi and for insistence of friends, I formally reconciled with those sirs. But with great stated disgust; for I cannot have two faces. Thus I will reverence, at my return, your friend Ghigo, recent academic, *if he manages* in his tittonian tangles.

[21] The letter is kept in the Archive Volterra of the *Accademia dei Lincei* in Roma.

> trattative, la cui prima iniziativa è dovuta per verità al Fubini, sono riuscito iersera a ravvicinarli, sicchè i rapporti sono ritornati, se non cordialissimi, almeno quelli che erano prima della polemica[22].

The importance of Severi's letter is not as much in the reconciliation (more or less formal) with Enriques, as rather in all the rest. We are at the beginning of 1929 and Severi has already chosen to side with those he now considers the winners. In such a capacity, he assumes the role of *counsellor of the prince* and proposes a political line that will prove successful: *intransigence* (for the inveterate like Volterra) *and an act of indemnity* to cancel the "sins" of the ex-antifascists). Also the manoeuvres for the launching of the *Accademia d'Italia* (established three years before) are on their last lap and Severi knows that Enriques has been proposed and that, if such a proposal were carried out, all of his chances of conquering the *leadership* of Italian mathematics would be barred – or however made more difficult. This last aspect is the one that drives Severi to an acrobatic changeover[23].

> Rettore era allora nel '23 Francesco Severi, grandissimo matematico ed energico uomo d'azione, molto legato a Gentile benché avesse fama di antifascista. Mi sia concesso rammentare di passata che non molti anni più tardi il suo antifascismo non seppe resistere alla seduzione dell'Accademia d'Italia, e poiché un primo fallo se ne porta dietro facilmente un secondo e un terzo, si mutò in adesione entusiastica al Regime. Caduto il quale, Severi, dopo aver corso pericolo di linciaggio nella nativa Arezzo, sentì irresistibile il richiamo della grazia (…) e da allora in poi scrisse articoli e fece conferenze per mostrare che la matematica e la fisica forniscono la prova incontrovertibile dell'esistenza di Dio[24].

So, Severi cannot resist the seduction of the *Accademia d'Italia*. But how does he manage to compete successfully with Enriques, the great favourite in the "race" to the *Accademia*?

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[22] As You probably remember, Enriques and Amaldi on one side and Severi on the other, had broken off all personal relations because of not really very important reasons, but connected to economic interests through textbooks. After a series of negotiations, whose first initiative is actually due to Fubini, I managed yesterday night to reconcile them, so that their relations have become again, if not friendliest, at least as they were before the controversy.

[23] The assertion is from Giorgio Levi della Vida in *Fantasmi ritrovati* (Neri Pozza, Vicenza, 1966). G. Levi della Vida, as Volterra, did not want to make the oath and preferred exile. His statement documents as well another Severi's acrobatic jump (favoured this time by the election to the *Pontificia Accademia delle Scienze*).

[24] Then in 1923 the rector was Francesco Severi, greatest mathematician and energetic man of action, very attached to Gentile although he had a reputation of being antifascist. Let me remind en passant that not many years later his antifascism could not resist to the seduction of the *Accademia d'Italia*, and, as a first mistake brings easily a second one and a third one, it turned into ardent support to the Regime. Once it faded, Severi, after having been threatened with lynching in his native Arezzo, felt the irresistible call of mercy (…) and ever after then he wrote articles and lectured to show that mathematics and physics give the absolute evidence of God's existence.

Fermi, for instance, had been suggested to Mussolini by the Roman school of Physics unanimously. For Severi it is not possible to imagine anything like that, remembering that most Roman mathematicians were signatories of the *Croce manifesto*. It is more likely that it was Gentile who exerted a decisive influence on Mussolini. The Duce remembers thus this renowned mathematician – who perhaps joined the freemasons – with "non- moderate" potentialities, who already in January 1929 appears with a clear analysis of the *intellectuals' problem*, so awkwardly faced by the extremist factions of the regime.

We reproduce the whole document.[25] The problem of the intellectuals continues to blaze. The *Istituto nazionale fascista di cultura*, *Gentile manifesto*, *Croce counter-manifesto*, the *Encyclopaedia* and now Severi's proposals: these are the most significant stages of a pathway which in 1931 will lead to the imposition of the *oath*. The *memorandum* is extremely eloquent in demonstrating the wide intellectual and political horizons in which Severi (who addresses directly Mussolini, needing no mediation) moves and in his political trajectory. He was still a socialist some years before. The reference to the *Accademia d'Italia*, in his previous letter to Gentile, gives the … material and opportunistic substrate for the ideological conversion.

= PROMEMORIA = Roma, 31 Gennaio 1929 – VII

La campagna per l'allontanamento dalle nostre Università di un gran numero di Professori, che vengon qualificati per antifascisti o puramente e semplicemente per afascisti, si fa ogni giorno più vivace. Essa è finora contenuta da una parte della stampa fascista, la quale non nasconde la inopportunità e le difficoltà di un provvedimento generale in merito; ma si può prevedere che tali voci verranno presto soverchiate, se non intervenga, anche in questo, la volontà forte e chiaroveggente del Duce, dinanzi al quale tutti piegano rispettosi.

È per questo che io, che già altra volta, due anni or sono, ebbi a constatare personalmente come a Lui non dispiaccia di vedersi sottoporre le questioni con deferente sincerità, anche da chi non sia inquadrato nelle file fasciste, Gli chiedo l'onore di essere ascoltato.

È vero che vi sono tuttora nelle Università troppi Professori irriducibilmente contrari al Regime?

La conoscenza del nostro mondo, in cui entrai giovanissimo, come professore, quasi trent'anni fa, mi consente di rispondere decisamente di no.

Vi sono state grandi incertezze dal principio, dipendenti da quello spirito critico, che non può scompagnarsi dall'abitudine alla ricerca scientifica, e che impedisce di regola di aderire subitamente a un nuovo ordine di idee. Ma le incertezze sono ormai superate dalla enorme maggioranza; e, poiché i nostri sentimenti di devozione alla Patria non posson non esser quei medesimi che nel 1914–15 fecero delle Università le fucine dell'intervento, i Professori non desiderano oggi che di poter

[25] Severi's memoir, unpublished, is personally addressed to the Duce and attests a personal acquaintance began two years before. It is kept in Rome, at the *State Central Archive*, "Segreteria Particolare del Duce, Carteggio riservato (1922–1940), busta 62, Severi Francesco". A notation reads: "Severi Francesco – See also list of Massons 364/R: Chiovendo and other".

cooperare con lealtà ed in un'atmosfera di fiducia, alla grande opera di ricostruzione, di cui il Capo ha posto i saldi piloni.
La verità è che, se le condizioni spirituali intime di quasi tutti son queste, meno facile è la loro esteriore espressione. L'atteggiamento di una parte della stampa, irriducibilmente contrario agl'intellettuali, che non hanno fatto aperta adesione al Fascismo, irrigidisce moltissimi in una condizione di silenzioso isolamento. Ormai da parecchio tempo ogni spontaneo e sincero atto di adesione, da parte di persone, che abbiano una qualche notorietà, non è, in questa atmosfera, possibile.
Osservo il fatto, senza la menoma intenzione di deplorarlo, giacché comprendo quali possano essere le esigenze di una Rivoluzione, nel periodo di più ardente sviluppo, di fronte a coloro che nei primordi non l'hanno compresa o l'hanno, sia pure idealmente, avversata.
Nonostante ciò, io oso sperare e pensare che il Duce non veda lontano il momento in cui potrà avvenire un avvicinamento formale al Regime, di qualche corrente rappresentativa del pensiero e della scienza italiana, che finora è stata in apparenza indifferente od ostile.
Io oso di più pensare e sperare che in un avvenire non lontano il Fascismo s'identificherà, non soltanto sostanzialmente, ma anche formalmente, con la Nazione. Mi sembra invero d'intravvedere che il coordinamento ed il controllo di tutte le attività collettive e singole, le quali, secondo la dottrina unitaria e totalitaria del Fascismo, devono svolgersi nello Stato e non fuori o contro lo Stato, condurrà prima o poi alla costituzione di un'organizzazione politica prettamente statale, inquadrante tutti i cittadini, e simile all'organizzazione sindacale, in cui sono rappresentate tutte le attività del lavoro e della produzione, che volontariamente non se ne appartino.
Un provvedimento generale, non limitato a quanti persistono in un atteggiamento palese od occulto incompatibile con quello che lo Stato Fascista esige dagli educatori della gioventù italiana, e che per contro allontanasse dalle Università i Professori che compirono in passato qualche manifestazione politica, non ortodossa, ma ai quali non si può oggi nulla rimproverare, sarebbe esiziale alla cultura ed alla scienza italiana, e si rifletterebbe in un danno morale e materiale per la Nazione, con gravi ripercussioni vicine e remote.
Poco tempo fa un Professore della Università di Gottinga, il Landau, qui a Roma, al nostro Seminario matematico, diceva di essere onorato di parlare dinnanzi alla prima Facoltà Matematica del mondo[26]. Poniamo che in ciò vi fosse qualche esagerazione convenzionale, non consueta tuttavia nei tedeschi, quando giudican altri popoli. Sta però il fatto irrefutabile che la matematica italiana occupa una posizione di avanguardia nel mondo, per la vastità, la profondità, l'eclettismo della sua produzione.
Ebbene, un provvedimento generale, come quello cui accennavo, priverebbe le Università nostre di moltissimi dei migliori matematici. Della Facoltà di Roma forse quasi nessuno resterebbe. E per quanto vi siano taluni giovani fascisti di valore, cresciuti alle nostre scuole e da noi aiutati e portati innanzi, molte Cattedre

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[26] On the 14th March 1925 Edmund Landau (1877–1938) gave a lecture in Rome on: "Computo asintotico dei nodi d'un reticolato entro un cerchio".

resterebbero scoperte o verrebbero mal coperte, annullandosi in pratica la continuità del risorgimento scientifico, che seguì faticosamente al risorgimento politico. Questo vale per la matematica, come per tante altre scienze.

Alla campagna contro gli universitari, mossa da ragioni d'intransigenza ideale, confluiscono altresì pressioni di interessi personali, che inserendosi (come sempre è accaduto in casi analoghi) nel grandioso movimento storico, cercan di prevalere al di là di ogni equa valutazione obbiettiva. Ciò accade specialmente – e direi quasi esclusivamente – nelle discipline che hanno immediati riflessi pratici e professionali. Tuttavia l'effetto generale di queste previsioni, ben dissimulate, è tutt'altro che irrilevante.

Si dice spesso che bisogna aprir le finestre perché un'aria vivificatrice d'italianità entri nelle aule e nei laboratori e ne cacci la *Kultur* teutonica, che i nostri professori rimasticano da sessant'anni. Questo è un luogo comune, che il fascismo finirà col toglier dalla circolazione, come tanti altri. Il nostro pensiero scientifico è ormai italianissimo in molte branche della cultura; e ne è riprova la frequenza con cui da Università dei varii Stati d'Europa e d'America si chiedono professori italiani per conferenze o corsi sistematici sopra argomenti scientifici, che fioriscono in Italia con atteggiamenti caratteristici. Ne è riprova l'omaggio che gli stranieri rendono all'originalità del nostro pensiero. Prendo un esempio fra mille. A proposito della traduzione tedesca di un'opera in cui ho sintetizzato una parte di quella che si chiama oggi dovunque la "geometria italiana"[27], un grande scienziato tedeco ultraottantenne, di vecchissimo stile, A. von Brill[28], dopo aver ricordato il monito romano *fas est et ab hoste doceri*, dichiara di confidare che la gioventù tedesca, riacquistata dopo la guerra la propria forza di produzione, sappia rimandar la palla che in questo campo le viene dall'Italia (testualmente: "nunmehr den Ball zurückschlagen, den auf diesem Gebiet Italien uns zugeworfen hat").

La condizione dei professori sulla cui testa pende la minaccia del licenziamento è talmente umiliante, che ognuno s'augura vicina una parola decisiva, qual si può attendere soltanto dall'equità e dall'autorità del Capo del Governo. D'altronde la sfera d'azione di questi professori si è a poco a poco ristretta nello stesso campo prettamente tecnico e un'atmosfera di sospetto li avvolge, anche se, per segni che dovrebbero essere indubbi, essi hanno mostrato di avere accettato con piena lealtà e senza restrizioni la realtà storica.

Eppure tanti di loro son da gran tempo spiritualmente vicini al fascismo, assai più di molti convertiti a precipizio, dopo il trionfo decisivo della Rivoluzione; vicini nella visione dei problemi nazionali e sindacali; negli atteggiamenti mentali di sincerità rude e schietta; nella pratica della vita.

Il mio caso è quello di tanti. Uscito nel primi mesi del 1915 dalle file socialiste (con una dichiarazione, che, se ben ricordo, fu, almeno in parte, pubblicata nel "Popolo d'Italia") mi arruolai volontario allo scoppiar della guerra e fui sempre combattente

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27 The already quoted *Vorlesungen über algebraische Geometrie*, Leipzig, Teubner, 1921.

28 Alexander Wilhelm von Brill (1842–1935). Severi's cited sentence is in the *Preface* that, as it has been told, Brill wrote for the *Vorlesungen*.

al fronte. Finita la guerra, a Padova, dove ero Direttore di quella Scuola Ingegneri, fronteggiai coi combattenti il movimento bolscevico. Non aderii al Fascismo, del quale non avevo allora intravisto l'ossatura di mirabile coordinazione delle attività nazionali ed economico–sociali, che mi si è poi rivelata; ma vi sono oggi vicinissimo, anche se su qualche particolare problema, come quello della stampa, le mie idee sieno meno ortodosse, e collimino con quanto poco tempo fa un autorevole fascista Gentile ha potuto liberamente esporre.

Ebbene, ora sento un chiaro giornalista domandare, in un impeto di sincera intransigenza, che si compia la epurazione delle Università, come fece il De Sanctis nel '60, eliminando dall'Università di Napoli gli "ultimi residui borbonici".

Ed è questa sensazione di diventare a poco a poco straniero nella mia terra, che più addolora me, che non volli, né vorrò, qualunque cosa accada, occuparmi stabilmente all'Estero, ripugnandomi – lo dichiarai al Duce due anni or sono – atteggiamenti nocivi all'Italia nei quali sarei stato o sarei fatalmente coinvolto[29].

Più volte sono stato all'Estero per ragioni scientifiche e per lunghi periodi, dopo l'avvento del Fascismo (nel 1924 in America, nel 1925 in Russia, nel 1928 in Spagna ed in Svizzera). Ed ora mi accingo a tornare in Spagna. E mai ho compiuto atti o pronunciato con chicchessia giudizi che neppur lontanamente potessero essere interpretati come contrari al Regime.

Io oso dunque chiedere al Capo del Governo per me e per tutti quanti si trovan nella mia condizione, di poter continuare a servire silenziosamente e fedelmente la Nazione e lo Stato fascista, nell'ufficio finora coperto. Servire con puro disinnteresse, ma servire senza limitazioni nell'ambito tecnico, cosicché la nostra opera possa svolgersi in tutta la sua efficienza; servire, non aspettando premi o distinzioni o posti di comando, che la Rivoluzione ha il diritto di assegnare a coloro che l'hanno fiancheggiata fin dal principio.

E perciò invoco da Lui, rispettosamente, la desiderata parola, che ponga fine ad uno stato di cose doloroso e dannoso.

Francesco Severi[30]

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[29] So Severi, probably soon after his "forced" resignation as Rector, had thought about "finding a permanent job abroad". We have no document confirming the news, but the reader will forgive us if we dare to put forward a little speculation: how would Severi's personal history, Enriques' parallel one and that of Italian algebraic geometry have fared, if Severi had listened to that first (and impulsive) determination!

[30] = Memorandum = Rome, 31st January 1929 – VII

The campaign for the expulsion from our Universities of a great number of Professors, who are qualified as antifascists or merely as a-fascists, gets every day more fierce. It is until now checked by part of the fascist press, that does not hide the inexpediency and difficulty of a general measure about it; but one can foresee that such voices will soon be overcome, if, also here, the Duce's strong and clairvoyant will, before which everybody respectfully bends, does not intervene.

It is because of this that I, who already once, two years ago, have personally ascertained how He does not dislike having put forward questions with respectful sincerity, also by whom is not located within the fascist ranks, ask You the honour to be listened to.

Is it true that at University there are still too many Professors opposed to the Regime in an irreducible way? The knowledge of our world, which I entered very young, as professor, almost thirty years ago, allows me to answer a resolute no.

[30] (continued)
There has been great hesitation right from the start, due to that critical spirit that cannot split up from scientific research habit, and that normally stops from suddenly coming round to new ideas. But hesitations are by now overcome by the great majority; and as our patriotic loyalty feelings cannot be than the same that in 1914–15 made of Universities the conflict melting pots, Professors today do not wish other than to be able to collaborate with loyalty and in a confidence atmosphere to the great work of reconstruction, whose firm basis has been laid by the Chief.
The truth is that, if these are the spiritual innermost conditions of almost everyone, their outward expression is not so easy. The behaviour of a part of the press, resolutely opposed to intellectuals that have not supported Fascism openly, make many stiffen in a condition of silent isolation. In this atmosphere, a spontaneous and sincere support of people with any renown is not possible since long ago.
I remark the fact having no intention of blaming it, as I understand which can be the requirements of a Revolution in the period of most vigorous development, in front of whom at the very beginning has not understood it or has, even if in a idealistic way, opposed it.
And yet, I dare to hope and to think that the Duce does not consider far the moment in which a formal approach to the Regime on the side of a representative current of Italian thought and science, that until now has been apparently indifferent and hostile, will be possible.
I rather dare to think and to hope that in a next future Fascism will identify with the Nation not only substantially but also formally. I really think to glimpse that the coordination and the control of all collective and single activities that, according to Fascism's unitary and totalitarian doctrine, must develop within the State and not outside or against it, will lead sooner or later to the constitution of a purely and typically state political organization, that situates all citizens, and is analogous to labour organization, in which all labour and production activities which do not voluntarily dissociate are represented.
A general measure, not limited to the ones who persist in an evident or hidden behaviour incompatible with what Fascist State demands to the educators of Italian youth, and that on the other hand would banish from Universities Professors that carried out any political, non orthodox demonstration in the past, but who today cannot be blamed for nothing, would be fatal to Italian culture and science, and would be a moral and material harm for the Nation, with serious nearby and remote repercussions.
Not long ago, here in Rome, at our mathematical Seminar, Landau, a Professor of the University of Göttingen, said he was honoured to speak before the first Mathematical Faculty of the world. Let's suppose that in this there were a conversational exaggeration, though not usual to German when judging other peoples. Remains the undeniable fact that Italian mathematics occupies an avant-garde position in the world, because of the vastness, the deepness, the eclecticism of its production.
Well, a general measure, as the one mentioned, will deprive our Universities of most of the best mathematicians. Almost none would remain at the Roman Faculty. And although there are some young fascists of worth, grown up in our schools and aided and supported by us, many of the Chairs would be left vacant or would be unfairly occupied, and the continuity of scientific revival, that exhaustingly followed to the political revival, would practically vanish.
This is true for mathematics, as for many other sciences.
To the campaign against scholars, moved by reasons of idealistic intransigence, add as well the pressures of personal interests, that, inserting (as has always been in similar cases) in the imposing historical movement, try to prevail beyond any fair objective valuation. This happens specially – and I would say exclusively – for the subjects that have immediate practical and professional repercussions. The general effect of this well hidden expectations, though, is other than irrelevant.
It is often said that it is necessary to open the windows so that an invigorating air of Italianism comes into the room and laboratories and expels the Teutonic *Kultur* that our professors have churned out for sixty years. This is a common place, that fascism will manage to take out, as many others. Our scientific thought is now "italianissimo" in many branches of culture; and the confirmation of this is the frequency with which Italian professors are requested in Universities of several European and American countries for conferences or methodical courses on scientific subjects, that flourish in

We think that this "memorandum" and the following letter to Gentile have been decisive to make the Duce listened to Gentile's proposal of appointing Severi as representative of mathematicians at the *Accademia d'Italia*. It is undoubtedly Mussolini's personal choice, because Enriques' name figured in the list proposed to him and given also to the appointed president Tittoni. At the last minute, write J. R. Goodstein and

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[30] (continued)
Italy with characteristic stances. The homage that foreigners pay to the originality of our thought confirms it.
I take an example among thousands. Regarding the German translation of a work in which I have summarized a part of the today called "Italian geometry", a great old-fashioned eighty-year-old German scientist, A. von Brill, after having reminded the Roman stern warning *fas est et ab hoste doceri*, affirms that he hopes German youth, regaining after the war its own production force, to be able to send back the ball that in this field comes from Italy (literally: "nunmehr den Ball zurückschlagen, den auf diesem Gebiet Italien uns zugeworfen hat").
The condition of the professors over whose head hangs the threat of dismissal is so humiliating, that everybody wishes a decisive word, as the one that can be awaited only from the fairness and the authority of the Head of Government, to be near. On the other side, the sphere of competence of these professors has been little by little restricted in the purely and typically technical field itself and an atmosphere of suspicion wraps them, even if, with signs that should be certain, they have showed to have accepted historical reality with full loyalty and without restrictions.
And yet many of them are long since spiritually near to fascism, more than many converted headlong, after the decisive triumph of Revolution; close in the view of national and syndicate problems; in the mental stance of rude and simple sincerity; in the practice of life.
My case is that of many. After having come out of the socialist files in the first months of 1915 (with a statement, that, if I remember well, was, at least in part, published in the "Popolo d'Italia") I enrolled as a volunteer when the war broke out, and I was always fighter at the front. When the war finished, in Padua, where I was Director of that School Engineers, I faced up the Bolshevik movement with the fighters. I did not support Fascism, whose structure of admirable coordination of national and economic-social activities, later revealed to me, I had not still glimpsed; but today I am very close to it, even if on some particular problem, as the press one, my ideas are less orthodox, and agree with what lately an influential fascist Gentile could freely expose.
Well, now I hear a decided journalist ask, in a spur of sincere intransigence, that University depuration be accomplished, as De Sanctis did in 1860, expelling from the University of Naples the "last Bourbon residues".
And it is this feeling of becoming gradually a foreigner in my own country that most hurts me; I did not want, nor will, whatever happens, to find a permanent job abroad, as offensive postures against Italy in which I would have been or would be fatally involved, revolt me – as I told the Duce two years ago.
I have been abroad several times for scientific reasons and during long time, after the accession of Fascism (in America in 1924, in Russia in 1925, in Spain and in Switzerland in 1928). And now I am about to return to Spain. And I never performed deeds or pronounce any judgement that not even for a moment could be interpreted as adverse to the Regime.
I dare then to ask the Head of Government for me and for everyone in my condition, to be able to go on silently and loyally serving country and fascist state, in the office until now held. To serve with pure disinterest, but to serve without limitations within the technical ambit, so that our work can be developed in all of his efficiency; to serve, not awaiting rewards or distinctions or places of power, that Revolution has the right to assign to those that have supported it from the beginning.
And hence I respectfully appeal to Him for the desired word, so that it puts an end to a painful and harmful state of affairs.

Francesco Severi

A. Capristo[31], Enriques' name was erased and Severi's included. All this within the month went by between Severi's memorandum to the Duce, his letter to Gentile and the choice of the first members of the Academy. Severi's malicious assertion, at the ending of the letter to Gentile, on Enriques' "Tittonian attempts" is then to be explained as the worry caused by the awareness that Enriques and Tittoni were both in the executive Board of the *Enciclopedia Italiana*. In a typewritten note, included in Enriques' dossier for his possible elution, it was thought advisable to remember that "Enriques, unlike his mathematical colleagues of Rome University, refused to sign the known *Manifesto degli Intellettuali* and appeared always devoted to the Regime". Tittoni thought of using Enriques' "refusal" to support "Croce manifesto" so as to reinforce his nomination and Severi was informed about it, that alone would be enough to justify Severi's mentioned documents. All this impress for the openness of his information about fascist world. Severi seems to be perfectly aware of Gentile's and the Duce's will to resolve "the delicate and at this point urgent question of the fascistization of Universities" and the debility of a proposal, for whom the new wording of oath had to include the commitment to form "industrious, upright citizens, devoted to the Country and to fascist Regime". The wording seemed to Mussolini not so fitting, because fascist regime had been inserted as additional, "instead of being essential".

Gentile took to himself the criteria that inspired Severi's letter. The new wording of oath is a crucial moment of this new line and the complete tuning that on this subject is established with Gentile is not irrelevant.

4. The difficult presence of Algebra

Severi's mathematical activity during the twenties, mainly addressed to the systematization of the acquisitions achieved by Italian algebraic geometricians, is the answer to the "competition" coming from new languages and to the fascinating appeals coming specially from *modern algebra*. It is, actually, exactly during the twenties that the changeover from the algebra of Dedekind, Kronecker, Hilbert, etc. (still guided in its development by the number theory and by the study of particular sets) to Emmy Noether's abstract one – always *made in Germany* – is accomplished. Her school will produce its most important treatise in 1931, with the publishing of *Moderne Algebra* by van der Waerden, who assembles in a organic way the lectures given by E. Noether at Göttingen and enunciates in a "definitive" way the notions of group, ring, field and other now classical algebraic structures. It finishes thus, with the consecration of the idea of structure and the autonomous study of algebraic entities defined only through non contradictory axioms, that *long march* begun in the preceding century through the theories of algebraic equations, groups, matrices, determinants, forms and their invariants, algebras, etc.

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31 J. R. Goodstein, L'ascesa e la caduta del mondo di Vito Volterra, in G. Battimelli, M. De Maria, A. Rossi (eds.), *La ristrutturazione delle scienze tra le due guerre mondiali*, Roma 1984, I, p. 289–302; A. Capristo, L'esclusione degli ebrei dall'Accademia d'Italia, *La Rassegna mensile di Israel*, Vol. LXVII (2002), No. 3, p. 1–36.

Gaetano Scorza

In Italy this research tradition is not among the most followed ones. We must not think of a total disinterest and neither of a scarce attention. In the *Prologue* we have remembered the important contributes of Betti and of Brioschi. The works of some foreign mathematicians, particularly significant in the history of algebra, are translated[32] and studied with attention. In short, also in the case of Italy one can talk of a *long march* that ends just at the beginning of the Twenties.

In 1921 the first of the three volumes of the *Teoria dei gruppi di ordine finito*[33] by Michele Cipolla (1880–1947), Sicilian, pupil of Bianchi, who will spend in Sicily – between Catania and Palermo – all his academic career, is published in Catania The three parts deal with abstract groups, groups of substitutions and the theory of algebraic equations (according to Galois). The level of abstraction is still relative: the studied groups are always groups of transformations, even if defined on an arbitrary set. Always during the twenties, Cipolla will write other works on the fundamental subgroups of Hölder groups and on several questions of arithmetic and of number theory.

But in the meanwhile, still in Catania, in 1921, Gaetano Scorza (1876–1939), whom we will find again as a not in the least secondary figure in chapter 6, publishes a second

[32] In this work, among the mathematicians quoted in the *Prologue*, we particularly distinguish Battaglini (because of the theory of groups), C. Segre, Peano (because of Grassmann's work) and Bianchi, of whom we will speak in a short while.

[33] M. Cipolla, *Teoria dei gruppi d'ordine finito*. *Parte I: Gruppi astratti*, Ed. Circolo Matematico, Catania, 1920–21; Idem, *Parte II: Gruppi di sostituzioni*, ibidem, 1921–22; Idem, *Parte III: Teoria delle equazioni algebriche secondo Galois*, ibidem, 1922–23.

treatise – *Corpi numerici ed Algebre*[34]. Scorza received his degree in 1899 in Pisa, "suffering" Bianchi's decisive influence too. During the first years of the new century, after a year of assistantship in Turin and then again in Pisa, he dedicated to high-school teaching, role that he will occupy until 1912. A man of great and general culture, he engages during those years in a lively and even harsh controversy with Pareto, regarding the concept of "maximum of an ophelimity". In 1912, following a competitive exam, he won a professorship in projective and descriptive geometry at the University of Cagliari; then, he will teach at the Universities of Parma and Catania; from 1921 to 1934 he is in Naples, and then moves definitely to Rome, where he will direct the new *Istituto matematico* of the University. The grounds of interest in *Corpi numerici ed Algebre* are not missing. The text is immediately acknowledged for its importance, with awards and influential and encouraging quotations even beyond national boundaries. Scorza exposes a general fields theory – for which he uses the term *corpi numerici* – and an associative algebras theory, pointing also to their applications within the geometrical field. The step towards a wholly abstract presentation of algebras structure is done, at this point. The shyness, still present in Cipolla's textbook, is overcome.

> Chi ha conosciuto che teorie concrete distinte, occupantisi di enti toto coelo diversi, schematicamente danno luogo ad una medesima teoria astratta, non ha potuto far questo, se non perché, trattosi fuori da ciascuna di esse, è riuscito a guadagnare un punto di vista superiore da cui guardarle simultaneamente (…) Il matematico, che non possiede la teoria generale di ciò che si dice un corpo numerico, conosce, a traverso l'Algebra, la teoria delle equazioni, a traverso la Teoria dei numeri, quella delle congruenze rispetto ad un modulo primo, a traverso i trattati sui numeri algebrici, quella delle congruenze rispetto ad un ideale primo; tre teorie di cui, se pure ha colto qualche analogia, non vede gli intimi legami[35].

A further reason of interest in *Corpi numerici ed Algebre* is given by the geometrical concepts which are at the heart of this study of algebra. The starting point can be searched out in some Severi's works on correspondences among curves from the beginning of the century. The attempt of building a basis for singular correspondences had made him use transcendent methods, and, specially, resort to abelian integrals. These are the works of Severi that drive Scorza to try to build a general theory of abelian functions and of abelian integrals. Scorza begins thus to work on *Riemann matrices* already during the pre-war years, observing how the theory of the abelian functions and other advanced Algebraic geometry's research fields present numerous points of contact – as if they were

[34] G. Scorza, *Corpi numerici ed Algebre*, Principato, Messina, 1921.

[35] Who has learnt that different concrete theories, dealing with different entities toto coelo, schematically give rise to the same abstract theory, could do this only because, getting out of each of them, has managed to win a superior point of view from which to look at them simultaneously (…) The mathematician, who does not possess the general theory of what is called *corpi numerici* knows, through Algebra, the theory of equations; through number theory, the one of congruencies respect to a prime module; through the treatises on algebraic numbers, the one of congruencies respect to a prime ideal; three theories of which, even if he has caught some analogy, he does not see the innermost bounds.

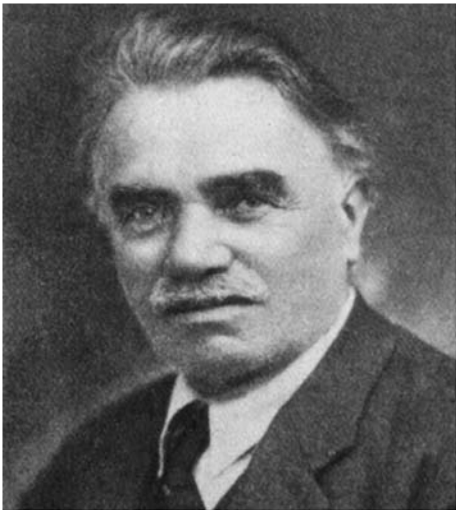
partial aspects of the same general theory – that move around the consideration of a certain "period table". He studies then this "table" (that he calls *Riemann matrix*, in honour of the German mathematician who first found them), managing to find a rigorous demonstrative way for solutions that previously were obtained through arduous and not always transparent proofs. His approach becomes more and more abstract and excludes the initial motives. Still in 1916 he publishes a general study of Riemann matrices – independent from any particular representation – that ends with the complete determination of the group of birational transformations of a hyperelliptic surface to itself.

Scorza becomes thus one of the first protagonists of the modern theory of abelian varieties and of algebras of their endomorphism. In the years following the publishing of *Corpi numerici ed Algebre*, he will continue the researches on abstract algebras, particularly distinguishing himself for the classification of those with dimension 2, 3, 4 on a whatever characteristic field. It can seem strange that – with Scorza, Cipolla and their pupils – the study of *modern algebra* finds its first expression just in Catania. The Sicilian rooting, though, has its logic, if we take into account that here mathematics had a significant tradition – how could we forget the *Circolo matematico di Palermo*? – while, on the other hand, Sicilian universities were considered often a transition place for professors who were "fresh" winners of competitive exam and were waiting for a more central position. It is not by chance then that a young subject, as *modern algebra*, flourishes in places just as "young" where one feels less the burden of some traditions and is therefore more at liberty to experiment new research lines.

The information has only a relative value, since the other growth pole of algebraic researches is Pisa, which is even the first one, as also Scorza and Cipolla studied at Pisa – and at Bianchi's school. Bianchi publishes his *Lezioni sulla Teoria dei numeri algebrici*[36] in 1923. The volume presents a less "advanced" approach compared with Scorza's treatise, but it is certainly not less important (given also the diffusion ensured a priori by the prestige of the author and of his university). The *Lezioni* represent for Bianchi almost the finishing touches of an information work on arithmetic-algebraic arguments, that had led him to the publication of texts on finite groups and the theory of Galois, on continuous groups and on the arithmetic theory of quadratic forms. The *Lezioni* fill another gap as, about the subject in question, Italy could boast only a lithographed course on Fubini lessons from 1917. Bianchi presents Dedekind's theory of ideals, dealing in particular with ideals in algebraic fields, with multiplication and divisibility, with decomposition in prime ideals, with congruencies of numbers with respect to ideals, with the group of composition of classes, with prime ideals in circular fields, with orders in algebraic fields, with regular ideals. Bianchi will continue to deal with algebra and with number theory also later on, introducing specially the concept of *absolute primary ideal*, but differential geometry will remain first within the hierarchy of his research interests. The passion towards number theory is rather didactical-foundational. It is about "contributing to the diffusion of (…) arithmetic theories, too neglected and almost ignored among us", rendering familiar to Italian mathematicians the arithmetical techniques of German school. The question goes beyond a purely technical dimension, so as to propose the *culture* of

[36] L. Bianchi, *Lezioni sulla Teoria dei numeri algebrici*, Zanichelli, Bologna, 1923.

Giovanni Sansone



Luigi Bianchi

the algebraic theory of numbers (and ideals) as unifying element of the mathematical edifice. It is undoubtedly an open window towards the new algebraic-arithmetic languages and the formalist tendencies then developing in Europe.

The first outcomes of this attempt can be valued talking about Giovanni Sansone (1888–1979) who, on the algebraic field, appears as Bianchi's most up-and-coming pupil. The beginning of his career leads us directly to set the difficulties that algebra finds in its development in Italy, after the *long march* and the *outburst* of the treatises at the beginning of the twenties. The attempts to obtain academic spaces for the algebraic field have no result at the moment. In the several competitive exams of these years, Sansone often gets through the first selections but, at the end, the valuing Commission judges his scientific production in the following way:

> É profonda, contiene risultati notevoli ottenuti vincendo non poche difficoltà, ma è di estensione ristretta, toccando argomenti di carattere prevalentemente aritmetico, mentre si tratta di coprire una cattedra di analisi algebrica e infinitesimale in cui devono prevalere i concetti del calcolo infinitesimale: i lavori del Sansone non danno pertanto sufficiente affidamento che egli abbia approfondito almeno le principali parti della Scienza a cui si riferisce la cattedra oggi vacante.[37]

[37] It is deep, it contains noticeable results obtained overcoming not few difficulties, but it has a narrow extension, as it touches prevalently arithmetic subjects, while the question is to cover a chair of algebraic and infinitesimal analysis where concepts of infinitesimal calculus must prevail: one cannot therefore be reliant on the fact that Sansone through his works has broadened his knowledge of at least the important fields of Science to which the vacant chair refers. (It is the competition (1924) for the chair of "Analisi algebrica e infinitesimale" at the University of Florence. Members of the Commission were G. Fubini, S. Pincherle and F. Severi).

Francesco Severi (on the left)

"Narrow extension" or "limited nature" – as one can read in the reports of other competitive exams – are expressions that amount to saying that in analysis or in geometry there was a need to produce. Thus Sansone, who for a while will continue to commit himself to algebra and to number theory (studying specially the *apiristic* solutions of cubic equations in a finite field), will "wisely" steer his researches towards analysis. Many late post-war events of Florence *Istituto matematico* and of the *U.M.I.* will pass through him, and through his long activity.

Sansone's episode is significant. The interest towards number theory and algebraic structures generate noticeably deep researches, but do not succeed in settling, as there is no academic motivation for young people. If to this situation we add Bianchi's demise and the dissolution of the centre of Catania – a transition place, we said – one can understand how algebra soon returns to the ranks of a subject devoid of autonomous prospects, propaedeutic to Algebraic geometry and analysis studies. In Italy, algebra and number theory will remain in this position – with few rare exceptions – until the sixties.[38]

5. Enriques and his school

The reference to Scorza and other geometers that notice more the pull of the new algebraic languages lets understand how Italian geometry during the twenties is quite a rich and composite world. The reference figure is Severi – we have titled this chapter *One man alone in the lead* just to indicate his march towards the *leadership* of the whole Italian mathematics – but beside him there are other researchers and other research currents, among which one has to name at least differential geometry.

The central figure, because of his studies, that make an internationally valued contribution, and are ranked between differential geometry and the theory of relativity, is represented by Levi-Civita, who in 1925 publishes the *Lezioni di calcolo differenziale assoluto*[39], soon after acknowledged as a classic of tensor calculus. We can remember also the Bianchi's researches (on the Levi-Civita' transport along a curve and on sphere congruencies), Fubini, Vitali, Enrico Bompiani (1889–1975), Enea Bortolotti (1896–1942), Sansone (on surface deformations), Alessandro Terracini (1889–1968) and Eugenio G. Togliatti (1890–1977). The two last ones are pupils of C. Segre and continue his research, at least in some ways, plunging into the relations between surfaces immersed in a projective space and linear partial differential equations. In this list impresses the presence of two analysts who, at the beginning of the century, had taken part in the great time of Italian real analysis. Nor is the appearance of Bompiani, whom we will find again in the next chapter when talking of the C.N.R.'s mathematical Committee, to be overlooked.

The presence of Vitali within the field of differential geometry is proved with more than thirty works related to the theory of parallel transport, projective differential geometry and a generalization of Ricci's absolute calculus. He took his degree at the *Normale* in 1899, in the same year of Scorza. After having been for two years Dini's assistant, Vitali had also passed on to secondary school teaching. During the period that goes from 1909 to 1922 he actively takes part to Genoa's political life as town councillor, and in 1923 he becomes professor of infinitesimal Analysis for Modena University. The year after he moves on to Padua, and later on, in 1930, to Bologna. Despite suffering an hemiplegia in 1926, his scientific production shows no sign of a let-up. In 1929 he publishes the *Geometria dello spazio hilbertiano*[40], that represents (finite dimension) Riemannian varieties as sub-varieties of a real infinite-dimensional Hilbert space. In this treatise Vitale reverts to and organizes an idea already present in the Italian school of differential

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[38] The situation was similar in France. See J. Dieudonne', The work of Nicolas Bourbaki, *American Math. Monthly*, 1970, pp. 131–145 and A. Weil, *The Apprenticeship of a Mathematician*, Birkhäuser, Basel, 1992.

[39] T. Levi-Civita, *Lezioni di calcolo differenziale assoluto raccolte e compilate dal Dott. Enrico Persico*, Stock; Roma, 1925.

[40] G. Vitali, *Geometria dello spazio hilbertiano*, Zanichelli, Bologna, 1929.

geometry and in some of his previous works: the consideration of the neighbourhoods of higher orders brings him to an absolute generalized calculus of higher order and to the study of multilinear applications, defined precisely within the infinitesimal neighbourhoods (of any order) of the points in a variety.

In Italy Fubini was the main exponent of projective-differential geometry. Student at the *Scuola Normale di Pisa*, where he graduated in 1900 with a brilliant dissertation (later published) "Sul parallelismo di Clifford negli spazi ellittici", he was educated mainly under Bianchi's influence, from whom he learnt – as he liked to say – "the love for geometrical research". He will teach at the universities of Catania, Genoa and Turin[41]. His strong and lively personality, in singular contrast to his short height, won him the nickname *little giant*. The epithet does not seem to be out of place if Levi-Civita does not think it twice to say about him that "after Volterra, he is considered the most penetrating and ingenuous living Italian analyst, having supplied essential contributions in many vital fields"[42]. In differential geometry, his first studies go back to the beginning of the century, prompted by the elaboration of the theory of automorphic functions. Progressively they acquire a self-autonomy, further characterized by the renounce to any metric structure. The group of automorphisms is no more composed of isometries, but of projective transformations. The objective is to expand to the new set the classical concepts of differential geometry, equally succeeding in developing the theory of surfaces, immersed in a projective space, with the definition of appropriate differential forms. The fundamental result, at least as regards three-dimensional projective spaces, is contained in the first of the two volumes of the treatise *Geometria proiettiva differenziale*[43] (1926–7), written with the Bohemian mathematician Eduard Cech[44] (who had been his pupil in Turin): the assignment of three differential forms (the so-called *projective arc element*, the *quadratic form* and the *cubic form*) determines, at less than a projective transformation, a surface initially described by parametric equations. Cech reminds thus the collaboration with Fubini, in a 1954 letter to B. Segre[45].

> Feci la conoscenza personale di Fubini nel 1921, ossia due anni dopo che io avevo incominciato ad occuparmi di questioni che ora appartengono al quadro della Geometria proiettiva differenziale, e da tale anno, nell'autunno del quale cominciai il mio studio a Torino, data la stampa del mio primo lavoro scientifico.
> Al principio del mio soggiorno a Torino, Fubini mi aiutava moltissimo ascoltando con pazienza in tutti i dettagli le mie descrizioni di ricerche appena cominciate e

41 We will come back to the last part of his career, spent in the United States, when talking about the consequences of race laws.

42 Copy of T. Levi-Civita's letter to O. Veblen (30.10.1938) is kept at the Archive of the Accademia dei Lincei in Rome, *Fondo "T. Levi-Civita"*.

43 G. Fubini, E. Cech, *Geometria proiettiva differenziale*. I & II, Zanichelli, Bologna, 1926–27.

44 With Cech, Fubini will publish in 1931 another treatise, l'*Introduction à la Géométrie Projective Différentielle des surfaces*, Gauthier-Villars, Paris.

45 The letter has been published in B. Segre, Guido Fubini, *Rend. Accademia Lincei*, (8), 17 (1954), 276–294 (p. 286–87).

> seguendo praticamente ogni giorno l'evoluzione delle mie idee. Benché Egli stesso fosse piuttosto analitico, mi incoraggiava moltissimo a sviluppare la facoltà di ragionare sinteticamente; d'altra parte fu Lui a non cessare d'incoraggiarmi ad occuparmi di questioni che richiedevano dei calcoli lunghi ed intricati. Non c'è dubbio che, se più tardi risolsi in alcune Memorie delle questioni analiticamente molte complicate, senza l'incoraggiamento di Fubini non avrei nemmeno osato di pensare a risolverle.
> L'idea di collaborare scrivendo un ampio compendio di Geometria proiettiva differenziale risale a Fubini, e nacque in Lui verso la fine del mio soggiorno a Torino. Io non ritenevo di esser capace a farlo e soltanto con molta fatica Fubini riuscì a persuadermi. Durante la preparazione della nostra "Geometria proiettiva differenziale" e più tardi, ci siamo scambiati centinaia di lettere, talvolta molto voluminose. Non ci sono forse molti esempi di collaborazione così intima non appoggiata che allo scambio di lettere. Eppoi, se ci sono molti libri composti da uno scienziato esperto in collaborazione con un giovane discepolo, raro è invece il caso come il nostro in cui il Maestro pienamente concedeva al principiante di mettere nell'opera comune l'esposizione dettagliata di sviluppi talvolta lontani dagli interessi del Maestro. Credo anche questo punto notevole, e testimonianza significativa della grandezza di Fubini[46].

Finally, Bompiani. In 1910 he received his degree at Rome under Castelnuovo, with a dissertation on "Spazio rigato a quattro dimensioni e spazio cerchiato ordinario"; for a long time he will be Castelnuovo's assistant and in 1913 he will be invited to follow the semester specialization courses run by Hilbert in Göttingen (one on the motion of electrons and another on the foundations of mathematics). During the war, he serves in

[46] I personally met Fubini in 1921, that is, two years after I began dealing with questions that now belong to the field of differential projective geometry, and the printing of my first scientific work dates from that year, in whose Autumn I began to study in Turin.
At the beginning of my stay in Turin, Fubini helped me a lot listening patiently in every detail to the descriptions of just begun researches and following nearly daily the development of my ideas. Although He himself was rather analytic, he strongly encouraged me to develop the faculty of synthetically reasoning; on the other side, He never stopped encouraging me to deal with questions that required long and complex calculus. There is no doubt that, if later I analytically solved very complex questions in some memoirs, without Fubini's encouragement I would not even have dared to think about solving them.
The idea of collaborating in writing a wide summary of differential projective geometry goes back to Fubini, and arises in Him towards the end of my stay in Turin. I did not thought me as being able to do it and only with great effort did Fubini manage to convince me. During the preparation of our tretease on differential projective geometry and later, we have exchanged hundreds of letters, sometimes really voluminous. Maybe there are not a lot of examples of so close a collaboration supported only by letter exchange. And then, even if there are many books written by an expert scientist in cooperation with a young pupil, it is rare instead a case such as ours in which the Master fully allows the pupil to add to the mutual work the detailed description of developments sometimes distant from the Master's interests. I also consider this point to be remarkable, and significant evidence of Fubini's greatness. (The letter is published in B. Segre, Guido Fubini, *Rend. Acc. Lincei*, 1954, pp. 276–294).

the air force with frequent missions in Paris, where in 1918 he gets the degree of aeronautical engineer. After the war (from 1927) he holds the chair of geometry at the universities of Milan, Bologna and Rome, successively. Later, he will be vice-chairman of the *U.M.I.* from 1938 to 1940, chairman since 1949 and honorary chairman since 1952; from 1951 to 1956 he will also be general secretary of the *International Mathematical Union*. His researches (that altogether will give rise to more than 300 publications) concern projective differential geometry in its most analytic line specifically enlarging on the work of Fubini – one *Addendum* of his appears in the second volume of the treatise Fubini-Cech of 1927 – and the geometric theory of differential equations, but also on Riemann varieties and the study of the infinitesimal neighbourhoods of any order (which we have just mentioned in connection with Vitali). In 1931 he will be awarded the *Premio Reale* of the *Accademia dei Lincei* with a long memoir dedicated precisely to Riemann geometry of a higher order.

It makes sense to mention the studies of differential geometry, also because of the significant presences observed within this field, but it is clear that – to become the *only man in the lead* – Severi has to reckon especially with Enriques.

Their stories are, in some ways, analogous. Enriques' name remains mainly bound to the great results of the beginning of the century and, specially, to the whole classification of surfaces. As we have recalled in the *Prologue*, it is a work that was essentially finished before the war with the individuation of the classification's general scheme and the study and the arrangement in this grid of numerous and noticeable particular cases. This is how Castelnuovo, in 1928, sums up the situation[47]:

> Val forse la pena di accennare qual'era il metodo di lavoro che seguivamo allora per rintracciare la via nell'oscurità in cui ci trovavamo. Avevamo costruito, in senso astratto s'intende, un gran numero di modelli di superficie del nostro spazio o di spazi superiori; e questi modelli avevamo distribuito, per dir così, in due vetrine. Una conteneva le superficie regolari per le quali tutto procedeva come nel migliore dei mondi possibili; l'analogia permetteva di trasportare ad esse le proprietà più salienti delle curve piane. Ma quando cercavamo di verificare queste proprietà sulle superficie dell'altra vetrina, le irregolari, cominciavano i guai, e si presentavano eccezioni di ogni specie. Alla fine lo studio assiduo dei nostri modelli ci aveva condotto a divinare alcune proprietà che dovevano sussistere, con modificazioni opportune, per le superficie di ambedue le vetrine; mettevamo poi a cimento queste proprietà colla costruzione di nuovi modelli. Se resistevano alla prova, ne cercavamo, ultima fase, la giustificazione logica. Col detto procedimento, che assomiglia a quello tenuto nelle scienze sperimentali, siamo riusciti a stabilire alcuni caratteri distintivi tra le due famiglie di superficie. Basterà qui citarne uno solo: mentre sopra una superficie regolare ogni sistema continuo di curve algebriche è contenuto in un sistema lineare di curve dello stesso ordine, ciò

[47] G. Castelnuovo, Le geometria algebrica e la scuola italiana, in *Atti del Congresso Internazionale dei Matematici* (Bologna 3–10 September 1928, Zanichelli, Bologna, 1929, I, p. 191–201 (194).

non avviene per le suerficie irregolari, le quali posseggono sempre sistemi continui non appartenenti a sistemi lineari[48].

This is how Enriques had described the classification of surfaces in a 1914 article[49]:

Il problema capitale della teoria delle superficie algebriche è la classificazione di queste, cioè la determinazione effettiva delle *famiglie* di superficie distinte per trasformazioni birazionali, ciascuna famiglia venendo caratterizzata da un gruppo di caratteri interi invarianti e contenendo, entro di sè, un'infinità continua di *classi* dipendenti da un certo numero di parametri (moduli).
Vale la pena di esaminare quali resultati d'insieme si possano trarre dal lavoro dell'ultimo ventennio, in ordine al suddetto problema di classificazione.
Questo è appunto lo scopo della presente Nota, in cui pervengo alle conclusioni che seguono:
La classificazione delle superficie algebriche, conduce naturalmente a considerare il genere d'ordine 12: P_{12}.
Per $P_{12} = 0$ si ha la famiglia delle rigate.
Per $P_{12} = 1$ si hanno le superficie possedenti curve canoniche o pluricanoniche d'ordine 0 (tutti i P_i essendo = 0, 1).
Per $P_{12} > 1$ si hanno le superficie con curve canoniche o pluricanoniche effettive, d'ordine > 0.
Per $P_{12} = 1$ il genere lineare $p^{(1)} \geq 1$ (mentre si può ritenere – com'è noto – $p^{(1)} \leq 0$ per le rigate, cioè per $P_{12} = 0$).
Ad ogni valore del genere lineare $p^{(1)} > 1$ corrisponde un *numero finito di famiglie* di superficie.
Per $p^{(1)} = 1$ si ha un'*infinità numerabile di famiglie* in cui entrano *due interi arbitrari*; tali famiglie sono caratterizzate dal contenere un fascio di curve ellittiche, salvo per $p_g = P_4 = 1$: in questo caso si hanno superficie di generi geometrici $p_g = P_1 = P_2 = \ldots = 1$,

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48 It may be worth mentioning which was the working method then followed to trace the way in the darkness in which we were. We had built, in an abstract sense of course, a great number of models of surface of our space or of superior spaces; and we had distributed these models, so to say, into two windows. One contained regular surfaces for which all went as in the best possible world; the analogy allowed to transport to them the most salient characteristics of plane curves. But trouble began when we tried to verify these properties on surfaces of the other window, the irregular ones, and each sort of exception appeared. In the end the assiduous study of our models had led us to foretell some properties that had to exist, with suitable modifications, for surfaces of both windows; then we put these properties to the test constructing new models. If they stood the test, we looked for the logic justification, the last stage. With this procedure, that resembles the one used in experimental sciences, we managed to establish some distinctive features between both surface families. Here we just need to mention one: while on a regular surface each continuous system of algebraic curves is contained in a linear system of curves of the same order, this does not happen for irregular surfaces, that have always continuous systems not belonging to linear systems".

49 F. Enriques, Sulla classificazione delle superficie algebriche e particolarmente sulle superficie di genere lineare $p^{(1)} = 1$, *Rend. Acc. Lincei*, 23 (1914), p. 206–214 and 291–297.

> e di genere numerico,
>
> $p_a = 1$ o $P_a = -1$,
>
> dipendenti altresì da un intero arbitrario (e da 19 o 3 moduli rispettivamente) che non contengono, *in generale*, fasci di curve ellittiche.
> La costruzione e lo studio delle superficie con $p^{(1)} = 1$ ($p_g = P_4 \neq 1$) dà luogo a sviluppi interessanti in ordine ai valori dei plurigeneri, alla base e ai moduli. Questi sviluppi sono riferiti, per semplicità, al caso delle superficie regolari ($p_a = p_g$). Ma l'estensione al caso $p_a < p_g$ non presenta difficoltà essenziali[50].

The quotations of Enriques and Castelnuovo together give a sufficiently accurate picture. At the *turning point* of the war, the "biggest part" of the work has been done and it is excellent work, that gives expression to the best of the Italian school of algebraic geometry in terms of ideas, of methods and of techniques. Many details still have to be organized and they are not always easy or secondary ones. And it is mainly to this finishing work that Enriques devotes himself in the period between the two world wars. Also in his case, then, we shift to a stage in some way subordinated to the great project of the beginning of the century, but the action of *accomplishment*, rather than of *arrangement*, suggests a more cumulative perspective, less burdened by the need to cope with the competition of new languages. Enriques publishes numerous articles during this

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[50] The main problem of the theory of algebraic surfaces is their classification, that is the effective determination of the surface *families* divided according to bi-rational transformations, every family being featured by a group of integer invariant features and containing within itself a continuous infinity of *classes* dependent on a certain number of parameters (moduli).
It is worth examining which global results can be obtained out of the work of the last two decades, with regard to the above-mentioned classification problem.
This is exactly the aim of the present note, in which I reach the following conclusions:
The classification of algebraic surfaces leads naturally to consider the type of genus12: P_{12}.
For $P_{12} = 0$ we have the family of the ruled surfaces.
For $P_{12} = 1$ we have the surfaces with canonical or multi-canonical curves of genus 0 (each P_i being $= 0, 1$).
For $P_{12} > 1$ we have the surfaces with canonical or multi-canonical effective curves, of genus > 0.
For $P_{12} = 1$ the linear type $p^{(1)} = 1$ (while it can be assumed – as everyone knows – $p^{(1)} = 0$ for the ruled surfaces, that is for $P_{12} = 0$).
To each value of the linear type $p^{(1)} > 1$ corresponds a *finite number of families* of surfaces.
For $p^{(1)} = 1$ we have a *countable infinity of families* in which *two arbitrary integers* come; such families are characterized by their containing a sheaf of elliptic curves, except for $p_g = P_4 = 1$: in this case we have surfaces of geometrical categories:

$p_g = P_1 = P_2 = \ldots = 1$,

and of numerical type:

$p_a = 1$ or $P_a = -1$,

depending as well on an arbitrary integer (and on 19 or 3 modules respectively) that do not contain, *in general*, sheaves of elliptic curves.
The construction and the study of the surfaces with $p^{(1)} = 1$ ($p_g = P_4 \neq 1$) gives rise to interesting developments as to the values of the multi-genus, the base and the modules. These developments, for simplicity refer to the case of regular surfaces ($p_a = p_g$). But the extension to the case $p_a < p_g$ does not present essential difficulties.

period (on the so-called general surfaces; on the characterization of the curves, provided with knots and cusps, that serve as ramification for some multiple plane; on the extension to higher dimensional varieties of the studies that had looked at the classification of surfaces through their projective invariants) but, also in his case, the greatest commitment is the writing up of treatises. Enriques lets some young pupils, for whom the almost daily cooperation with the master becomes a great educational opportunity, join in the project.

The first treatise is composed of the *Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche*[51], written in association with Oscar Chisini (1889–1967) and published in four volumes in 1915, 1918, 1924 and 1934. Chisini, enrolled in the Faculty of Engineering, had been advised by Enriques to take a degree in mathematics at Bologna, and became his assistant. Later, he will be professor of geometry at Cagliari first, then in Milan, where, from 1925 onwards, he will fulfil his career. The *big book* – as Chisini jokingly called the *Lezioni* – deals in its four volumes with the fundamentals of algebraic geometry; the basic theory of algebraic plane curves based on polarity and the study of their singularity; the theory of plane and smooth curves from the point of view of birational transformations; elliptic and abelian functions. In its 2600 pages it is thus exposed, with a wealth of examples and frequent historical notes, the theory of algebraic curves (and of their singularities) presented from different points of view: the synthetic-projective one, the analytic-differential one and the topological-transcendent one. Chisini plunges specially into the theory of singularities. The extension of the geometry of birational transformations – from algebraic curves to algebraic surfaces and varieties – appeared beset with new difficulties and problems. It was necessary to obtain an exhaustive knowledge of the points and singular lines of an algebraic surface in order to construct models provided with simple, or even without, singularities, to work on them without continuous hindrances. The 1920–21 memoir, in which Chisini[52] manages to transform a given algebraic surface into another one provided only with simple singularities, falls within this ambit. The memoir itself uses also the representation of an algebraic surface on a multiple plane (that is the projection of the surface itself from a centre on a representative plane, so as to study the branch curve of the multiple plane thus obtained), tackling the question of birational equivalence of two surfaces having the same branch curve. It will be with such ideas in mind that Chisini will introduce in 1933 the concept of *characteristic braid* of an algebraic curve, composed of a finite number of tracts of spatial curves that (twirling with each other as in a braid) point out – in the particular way of twirling – the essential numerical traits of a curve's singular points. Chisini will be also one of the first witnesses to reveal the original methods with which Enriques worked and argued about the development of the work: not merely formal encounters in his study, but strolling along under the porticoes of Bologna, maybe stopping to write with the tip of his umbrella on the pavement a more complex formal

[51] F. Enriques, O. Chisini, *Teoria geometrica delle equazioni e delle funzioni algebriche*, 4 vol., Zanichelli, Bologna, 1915–1934.

[52] O. Chisini, La risoluzione delle singolarità di una superficie mediante trasformazioni birazionali dello spazio, *Mem. Acc. Sci. Bologna*, 8 (1921), p. 1–22.

development. It is a behavior we will see also in other Enriques' pupils and that will cause us to discuss his "philosophy" and methodology itself.

With his second student, Luigi Campedelli, Enriques writes the *Lezioni sulla teoria delle superficie algebriche*[53]. We are already in 1931 and the book will be the reference point for an up-to-date extensive edition – *Le superficie algebriche*[54] – that Enriques will prepare in the following years and that will be published in 1949 after his death. Campedelli (1903–1978) begins his studies at Pisa, and completes them at Rome, where he receives his degree under Enriques in 1928. Campedelli had been taken to Rome by Enriques himself, who had singled him out in Pisa in one of the recurrent visits he made and that with delighted irony he called *pastoral visits*. Campedelli wins the competition for a chair of geometry in 1935, first at Cagliari and then at Florence (where he will teach uninterruptedly until 1973). His greatest contributions are to the theory of surfaces, with the example of a surface of general type with geometric genus zero, the calculus of the so-called *Zeuthen-Segre invariant*, the classification of elliptic surfaces and the proof of the theorem according to which a surface that has no exceptional curves of the first type is rational or ruled if and only if it contains curves having negative intersection with the canonical curves.

We digress from the period we are dealing with in this chapter – the 1920s – to introduce Fabio Conforto (1909–1954) who, even if not Enriques' direct pupil, is equally an author of the treatise on *Le superficie razionali*[55], published in 1939, that sees Enriques engaged once again. Born in Trieste, Conforto had begun his studies in Vienna, to pursue them later in Trieste, Milan and Rome, where he receives his degree under Volterra in 1931. After a specialization period in Göttingen, he becomes Castelnuovo's assistant and Picone's collaborator. Later, he will draw near Severi and, in 1939, he will be called to succeed Scorza at the university of Rome. The treatise on *Le superficie razionali* is divided into two parts: the first one exposes the salient features of rational surfaces of the first type, while the second one is devoted to the general questions of the theory. But let us cede the podium to Conforto himself who, in the *Preface*, admits explicitly his debt to the master. We are in 1939, a few months after the passing of racial laws that we will duscuss extensively in the next chapters. The independence of judgement of the young Conforto – even regarding Severi's authority – impresses us.

> È mio dovere dichiarare che in questo lavoro di coordinazione e di revisione mi sono valso delle lezioni, che su questi argomenti ha svolto, durante più anni successivi, il prof. F. Enriques alla R. Università di Roma, dalle quali in particolare ho attinto le nuove dimostrazioni che si riferiscono alla classificazione delle involuzioni del second'ordine di Bertini, ai piani doppi razionali, nonché la semplificazione notevole che l'uso del principio di continuità permette di portare alla dimostrazione della razionalità delle involuzioni piane d'ordine qualunque, quale si trova esposta nell'ultimo capitolo del libro.

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[53] F. Enriques, L. Campedelli, *Lezioni sulla teoria delle superficie algebriche*, Cedam, Padova, 1932.
[54] F. Enriques, *Le superficie algebriche*, Zanichelli, Bologna, 1949.
[55] F. Conforto, *Le superficie razionali*, Zanichelli, Bologna, 1939.

> In accordo colle vedute del mio Maestro si riconoscerà una nuova considerazione dei problemi anche nelle parti più elementari di questo trattato[56].

The last treatise we will deal with regarding Enriques and his school, tackles the study of rational surfaces. It is as if Enriques would organize the classic case, represented by such surfaces, to use it later as a springboard in the most general study, with which he will deal in the volume published in 1949.

We have mentioned Conforto's specialization period in Göttingen. We are in 1932. His letters to Bompiani[57] are an extremely valuable and provide direct evidence about the subject (touched on several times previously) of the split that arises after the first World War, concerning especially algebraic geometry, between Italian tradition and the new algebraic language that flourishes particularly in Germany.

> Sono oramai pienamente sistemato ed orientato nell'ambiente universitario di Gottinga e posso quindi scriverle quali sono le mie impressioni e le cose, delle quali mi sono occupato. Quello che più di ogni altra cosa si coltiva qui è la teoria dei gruppi e la così detta Algebra moderna. Di teoria dei gruppi si occupa normalmente il Weyl. Quest'anno fa la Geometria differenziale, ma nella biblioteca si possono vedere le dispense di molti corsi precedenti e questi sono dedicati alla teoria dei gruppi. Del resto nel seminario si occupa anche quest'anno di teoria dei gruppi. Alla teoria dei gruppi sono dedicati anche i corsi del professore Herglotz (funzioni modulari e gruppi di Lie). Per teoria dei gruppi bisogna quasi sempre intendere teoria dei gruppi finiti e con ciò ricadiamo naturalmente nell'Algebra. L'Algebra qui è conosciutissima ed il libro di Van der Waerden fa testo. Del resto anche Van der Waerden è stato un anno qui per un corso. Tutte le cose della Emmy Noether sono anche improntate a questo spirito. Da noi invece l'Algebra moderna non è per niente entrata nell'ambito scolastico.
> E confesso che sono stato alquanto perplesso alle prime lezioni; perché questi algebristi tedeschi possiedono molte cose nuove ma anche usano sempre un linguaggio, irto di termini sconosciuti, per me che non avevo mai approfondito l'Algebra né la teoria dei gruppi finiti. Ma ora mi sono ripreso ed incomincio a capire completamente, quale sia il piano, sul quale si muove questa gente. Sto studiando l'Algebra e vado facendo rapidi progressi. Noto che quasi sempre si rimane nell'ambito dei gruppi finiti, mentre il caso di insiemi infiniti, non è mai

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[56] It is my duty to state that in this work of coordination and revision I have used the lectures that Prof. F. Enriques has carried out, during several consecutive years, at the R. University of Rome, and from which in particular I have drawn the new proofs referring to the classification of Bertini's involutions of the second type, to rational double planes, as well as the remarkable simplification that the use of the principle of continuity allows to bring to the proof of the rationality of the plane involutions of any type, as it is exposed in the book's last chapter.
According to my Master's opinion, a new consideration of the problems, even in the most elemental parts of this treatise, will be recognized.

[57] The letters are kept in the *Bompiani Papers* of the *Accademia Nazionale delle Scienze* in Rome (so-called of the *XL*).

> toccato. Nel testo di Van der Waerden stesso l'argomento è appena sfiorato. Certo la cosa non è facile, ma è possibilissimo che combinando le nozioni che noi possediamo, per esempio nella teoria dei funzionali, con le nozioni, che qui sono di uso corrente si possa trovare qualche facile risultato. Gli è che questo fatto non può venire in mente a nessuno, perché la teoria dei funzionali è completamente sconosciuta[58].

The remarks made in the previous letter dated 24^{th} January are confirmed in a later one dated 18^{th} February.

> Tutto il lavoro viene fatto al solito nello spirito aritmetico algebrico. Ho potuto tastare un poco la psicologia degli studenti di qui. Sono molto unilaterali, scientificamente parlando. A parte che non conoscono cose che non vengono particolarmente coltivate nel loro paese, come la Fisica Matematica e la Teoria dei funzionali, avviene però che anche delle cose loro non sono ovunque buoni conoscitori. Lo studio particolare delle opere di Klein, ad esempio, non è qui una cosa frequente. Certamente tutto l'interessamento, che c'è qui per la teoria dei gruppi e per l'Algebra è in parte opera di Klein, ma Klein come persona, è già dimenticato ed i nuovi algebristi sono veramente quelli che sono conosciuti da tutti. Ed è invece facilissimo incontrare della gente, che non sa nulla di funzioni automorfe o di funzioni ellittiche modulari e di tutte le ricerche di Klein e Poincaré. Naturalmente parlo qui della cultura media degli studenti[59].

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[58] By now I am fully established and well under way in Göttingen's university environment and can therefore tell you about my impressions and the things I have dealt with. Here they cultivate, more than anything else, the theory of groups and the so-called modern algebra. The one who deals with the theory of groups is usually Weyl. This year he teaches differential geometry, but in the library one can see the lecture notes of many previous courses and this are dedicated to the theory of groups. Besides, also this year in the seminar he covers the theory of groups. Also the courses of professor Herglotz (modular functions and Lie groups) are dedicated to the theory of groups. As theory of groups one has to understand almost always the theory of finite groups and consequently we slip back of course into algebra. Algebra is well known here and Van der Waerden's book is important. Also the things of Emmy Noether take on this spirit. Instead, in our country modern algebra has not entered the scholastic area in the least.
And I confess that I was pretty perplexed during the first lessons; because this German algebraists have a lot of new things but they always use a language full of unknown terms, for me who had never plunged into algebra nor into the theory of finite groups. But now I am back on my feet, and I begin to fully understand which is the level at which this people move. I am studying algebra and make quick progress. I notice that almost always one stays within the finite groups, while the case of the infinite sets is never touched. In Van der Waerden's book the subject itself is barely touched on. Certainly the thing is not easy, but it is very likely that combining the ideas we have, for instance the theory of the functionals, with the notions that are in common use here we could find an easy solution. The fact is that this cannot come to mind to nobody, because the theory of functionals is totally unknown.

[59] All the work is done as usual with an algebraic arithmetic spirit. I have been able to sound a bit the psychology of the students here. They are very unilateral, scientifically speaking. Besides the fact that they do not know about what is not specially fostered in their country, as Mathematical physics

The comparison between the Italian tradition, in which Conforto grew up and that he defends because of its richness, and *Moderne Algebra* continues in the letter of the 23rd June.

> Ho iniziato fin dai primi mesi del mio soggiorno in Germania lo studio dell'"Algebra Moderna". Questo impone l'impossessarsi di un vocabolario e di un sistema di denominazioni notevolmente complesso ed intricato, reso anche difficile dalla forma volutamente privata di ogni elemento euristico nella quale i matematici tedeschi sogliono presentare i loro scritti.
> Tuttavia io ho fatto una discreta pratica in mezzo a questa grande quantità di definizioni e di concetti, la cui distinzione è spesso sottilissima. Man mano che acquistavo però la conoscenza della terminologia, mi divenne sempre più evidente come la nuova disciplina potesse solamente avere un valore dal punto di vista metodico. Il contenuto intrinseco invece si trovava semplicemente ad essere il contenuto di quello che è sempre andato sotto il nome di Algebra superiore. Per prendere ad esempio il caso della teoria di Galois, i risultati che sono da trovarsi nelle esposizioni dell'"Algebra Moderna" non differiscono se non per la forma da quelli che si trovano nel nostro trattato del Bianchi. Quello che si può per lo meno sicuramente affermare è che i risultati raggiunti dai matematici tedeschi sono assolutamente sproporzionati alla difficoltà creata con l'introduzione di un frasario così complicato. D'accordo poi con questa mentalità più critica ed ordinatrice che costruttiva, l'Algebra viene coltivata in Germania solamente come fine a sé stessa ed è qui indizio che eventuali nuovi risultati potrebbero dare per ricerche in altri campi (intendo ad esempio il rapporto tra la teoria di Galois e la teoria di Picard, Vessiot per le equazioni differenziali lineari) viene lasciato completamente da parte.
> Però l'"Algebra Moderna" non rappresenta un indirizzo di grande originalità, in un altro campo, il quale pure, a dire il vero, è molto coltivato dai matematici tedeschi, dove esistono dei reali ed importanti problemi. Intendo parlare della Teoria dei numeri, che è argomento generalmente non coltivato in Italia, mentre in tutta la Germania ed a Gottinga in particolare, dopo Gauss e Riemann esso rappresenta una tradizione. Tale campo però è in molte sue parti necessariamente non algebrico e fa uso di mezzi trascendenti in modo da ricollegarsi strettamente con la Teoria delle funzioni. Un'altra parte della teoria dei numeri (e con questa in particolare la teoria dei numeri algebrici) si è spogliata nella esposizione che si da oggi

[59] (continued)
and the theory of functionals, it happens also that they are not experts in any field, not even in theirs. The particular study of Klein's works, for instance, is not a frequent thing here. Of course all the interest that there is here in the theory of groups and for algebra is partly the work of Klein, but Klein as such is already forgotten, and the new algebraists are the ones really known overall. And it is though very easy to meet people who do not know anything about automorphic functions or modular elliptic functions and about all of Klein and Poincaré's researches. Of course I speak here about the students' average culture.

> nella Germania di ogni elemento trascendente e si è accostata alla "Algebra Moderna"[60].

Let's let Conforto talk, this time to come back to Enriques and to the characteristic traits of his research methodology[61].

> Enriques concepiva il mondo algebrico come a sè esistente, indipendentemente e fuori di noi, regolato da una legge suprema che è la legge di continuità, rispecchiante l'analiticità degli enti considerati. Nel cercar di comprendere tale mondo non è tanto da prefiggersi un ideale di perfezione logica; meno che mai è da procedere assiomaticamente, partendo da postulati in qualche modo in nostro arbitrio. Ciò si potrà fare, soleva dire l'Enriques, in altre parti della matematica (…). Il mondo algebrico esiste invece di per sè e l'escludere da esso certi enti, perchè ad esempio eccezionali, è impossibile, perchè contrasterebbe alla legge della continuità. Le eccezioni debbono anzi essere accolte e spiegate al lume della continuità stessa. Il capire dunque il mondo algebrico non è tanto una questione di corretta deduzione, quanto anzitutto e soprattutto una questione di "vedere". Una simile concezione appagava profondamente lo spirito potentemente intuitivo dell'Enriques, il quale spesso arrivava addirittura al punto – e nell'intimità con i Suoi allievi si compiaceva di tale aspetto apparentemente paradossale del Suo pensiero – di non sentire il bisogno di una dimostrazione logica di qualche proprietà, perchè Egli "vedeva"; e ciò lo rendeva sicuro della verità della proposizione in questione e Lo appagava pienamente (…).

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[60] I began, since the first months of my stay in Germany, the study of "Modern Algebra". This imposes to master a vocabulary and a remarkably complex and intricate system of names, made also difficult by the deliberately private form of each heuristic element in which German mathematicians use to present their writings.
Yet I have done a fairly good practice amid this big amount of definitions and concepts, whose distinction is often very subtle. But as I acquired the knowledge of the terminology, the fact that the new discipline could have value only from a methodical point of view became more and more evident to me. The intrinsic content instead was simply the content of what has always been known as superior algebra. If we take as an example the case of the theory of Galois, the results that are to be found in the exposition of "Modern Algebra" do not differ but for the form from the ones we find in Bianchi's treatise. What can certainly be stated is that the results added by German mathematicians are out of all proportions regarding the difficulty arisen by the introduction of such a complex phraseology. According to this way of thinking, more critique and governing than constructive, algebra is cultivated in Germany only as an end in itself and is here a sign that eventual new results could give as researches in other fields (I mean for instance the relation between Galois' theory and Picard's, Vessiot for linear differential equations) is set totally aside.
But "Modern Algebra" does not represent a branch of great originality, in another field, which, to say the truth, is also quite fostered by German mathematicians, where there are real and important problems. I mean to speak of the theory of numbers, a subject not usually fostered in Italy, while in whole Germany, and specially in Göttingen, after Gauss and Riemann it represents a tradition. Such field though is in many parts necessarily not algebraic and uses transcendent means so to be tightly connected to the theory of functions. Another part of the theory of numbers (and with this in particular the theory of algebraic numbers) has got rid of every transcendent element in the exposition today given in Germany and has come close to "Modern Algebra".

[61] F. Conforto, Intuizione visiva degli enti algebrici, *Periodico di Matematiche*, 25 (1947), p. 115–116.

> Eppure nell'Enriques, accanto all'intuitivo per eccellenza, esisteva anche il logico sottile ed il critico profondissimo. Nei riguardi delle questioni critiche, Egli esigeva però sempre che esse fossero questioni effettive e non generate da cavilli o da giuochi di parole. Egli era per un rigore sostanziale, più che formale. Non volle infine mai aderire alla veduta che la matematica sia una costruzione puramente logica. (…) Egli riconosceva bensì che i principi della geometria debbano essere esaminati al lume di un criterio rigidamente logico per stabilire ad esempio la indipendenza e la compatibilità dei postulati; ma conviene sempre tenere altresì presente il criterio psicologico, il quale implica l'indagine delle sensazioni e delle esperienze, che hanno condotto a formulare i postulati stessi[62].

The testimony of Conforto and of Chisini – others, more or less anecdotal, could be added – agree in underlining the characteristic and preponderant role of *intuition* in the formulations of the Italian school of algebraic geometry. Enriques adheres to this system and to this *cliché*, that of course brings him controversies and accusations of poor rigour (even on the part of Severi, as we will see while progressing through the years). O. Zariski ascribes to him the sentence: "We aristocrats do not need proofs. Proofs are for you commoners". True or not, the statement says a lot about his character: relying on the pure fact of having reached the goal and on his excellent cultural credentials – his *aristocracy!* – Enriques appears almost annoyed at having to care for demonstrative details and at the comparison with others who need a proof to reach the truth of a proposition. His production, his exposition on a very wide front and the continuous solicitations of his students inform us immediately that we are not observing a researcher who lives isolated in his "own world". On the contrary. But nobody should burden him with routine demonstrations and exposition that may be formally proper but

[62] "Enriques conceived algebraic world as existing in itself, autonomously and outside us, ruled by a supreme law that is the law of continuity, that mirrors the analyticity of the studied entities. If we want to understand that world we need not as much to pursue an ideal of logic perfection; less than all to proceed axiomatically, starting from postulates in some way to our discretion. This can be done, used to say Enriques, in other parts of mathematics (…). Algebraic world exists instead in itself and to exclude of it some entities for example because they are exceptional, is impossible, because it would oppose the law of continuity. Exceptions must on the contrary be accepted and explained by the light of the continuity itself. To understand then algebraic world is not as much a matter of right deduction, as first of all and above all a matter of "seeing". Such a conception satisfied deeply the strongly intuitive spirit of Enriques, who often reached the point – and in the intimacy with his pupils he was pleased of such an apparently paradoxical aspect of his thought – not to feel the need of a logic proof of some property, because He "saw"; and that rendered him sure of the truth of the proposition at issue and it fully satisfied Him completely (…).
And yet in Enriques, beside the intuitive par excellence, there was also the subtle logic and the highly critical. Towards critical questions he demanded always, though, that they were effective questions and not generated by quibbles or by plays on words. He was for a substantial rigour, more than for a formal one. Finally, he never wanted to adhere to the view that mathematics is a purely logic construction. (…) He recognised indeed that the principles of geometry must be studied in the light of a strictly logic criterion in order to establish for instance the independence and the compatibility of the postulates; but it's always convenient to consider the psychological criterion as well, which implies the study of the sensations and the experiences that have lead to formulate the postulates itself.

are superfluous in substance! Also the exposition of the treatises reflects an unhidden sympathy towards an intuitive and heuristic process. One goes from the particular to the general, through the acquisition of knowledge progressively given by more and more precise examples. The expository style is that of discovery, not the formal statement of its justification. It is his choice to reveal the problems and the diverse paths that have led to the discovery[63].

> Presupposto di tali aspirazioni resta infine l'antico modello classico del trattato, che si riattacca alla venerabile tradizione dell'EUCLIDE: l'idea di una scienza razionale logicamente ordinata come teoria deduttiva, che debba apparire in ogni sua parte chiusa e perfetta, che, discendendo dai concetti più generali alle applicazioni particolari, respinga da sè le incerte e mutevoli suggestioni del concreto, tutto quanto ricordi il passato oscuro della ricerca o scopra nuove difficoltà, rompendo l'armonia del sistema.
> Ma questo ideale del sistema (…) contrasta d'altra parte colla generale filosofia della scienza, frutto della critica moderna. Infatti la critica logica e gnoseologica riesce in ultima analisi a definire il campo della logica ed a riconoscere in ciascuna teoria gli elementi intuitivi di diverso ordine che le conferiscono significato e valore; infine approfondendo la veduta della scienza nel suo divenire, codesta critica oltrepassa l'opposizione fra metodo deduttivo e metodo induttivo, giungendo a considerare la deduzione stessa come fase d'un processo unico, che sale dal particolare al generale per ridiscendere al particolare.
> Vi è luogo a chiedere se questo concepimento dinamico del sapere, che ognor più prende il posto del vecchio concepimento statico, non debba comporre in qualche modo anche l'antitesi tradizionale fra ricerca ed esposizione sistematica, e così fra scienza e storia della scienza[64].

The methodology adopted in this approach to research and in the ensuing stage of exposition is closely connected to a dynamic conception of mathematical thought – not

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[63] F. Enriques, O. Chisini, *Teoria geometrica delle equazioni …*, quoted, I, p. IX.

[64] Prerequisite of such aspirations remains the old classical model of the treatise, that joins EUCLIDE's venerable tradition: the idea of a rational science logically organized as a deductive theory, that must appear close and perfect in every part, that, decreasing from most general ideas to particular applications, rejects by itself the uncertain and changeable suggestions of the concrete, all of which reminds of the dark past of the research or discovers new difficulties, breaking the harmony of the system.
But this ideal of the system (…) on the other hand contrasts with the general philosophy of science, result of modern critique. Actually the logic and gnosiological critique manages in the final analysis to define the field of logic and to recognize in each theory the intuitive elements of different type that give it meaning and value; lastly taking the view of science further in its becoming, this critique exceeds the opposition between the deductive method and the inductive one, going as far as to considering the deduction itself as a stage of a unique process, that rises from the particular to the general in order to descend again to the particular.
There is reason to wonder whether this dynamic conception of knowledge, that always more takes the place of an old static conception, must not settle in some way also the traditional antithesis between research and systematic exposition, and so between science and history of science.

practised by a science philosopher, but practised and made explicit by a mathematician in the heat of his insights – that enhances the role of error and of successive approximations in following a path of discovery. It thus presents integrity of argument and rigor in proof as the final result of a whole process, not as a necessary condition that must be always satisfied. Not, in short, leading to a concise presentation of the theories in a closed and perfect form. Nor, as well, leading to an abstract statement – "where thought vanishes and comes to nothing, as a shadowy, incoherent fog, when the limits of the real are crossed so as to follow only the laws of symbols"[65] – and too general a presentation.

> Ma il gusto della generalità ha ricevuto altra interpretazione presso i geometri contemporanei, specialmente nel nostro paese. Si è eretto a principio di massima che ogni teorema debba sempre enunciarsi nella forma più generale di cui è suscettibile. (…)
> Conviene riconoscere che quest'abito ha diminuito l'efficacia propulsiva di ottimi maestri, e merita di essere seriamente contrastato. Giacché in primo luogo, la forma troppo astratta dell'enunciato riesce ad oscurare il vero significato del teorema nascondendone le origini, e – in secondo luogo – crea nei giovani studiosi la lusinga delle facili generalizzazioni, puramente formali[66].

At this point, Enriques' interest in the history of mathematics is clear. He merges the epistemological sensibility we have mentioned, the philosophical culture – recalled in our *Prologue* and to which his historical activity will be closely linked – with his attention to the didactic questions that had brought him already in 1900 to publish the volume *Questioni riguardanti la geometria elementare*[67] and will lead him to the chairmanship of the *Mathesis* from 1919 to 1932. The *Questioni* will be reprinted (with the title *Questioni riguardanti le matematiche elementari*), in a very enlarged edition, in 1912 (in two volumes) and later in three volumes, from 1924 to 1927. Addressed to the community of teachers, they place themselves as a bridge between the world of research and that of school, as an updating and transposition at a pedagogic level of subjects usually reserved to advanced mathematics. It is not by chance that Enriques opens in 1922 the recently established course of "complementary mathematics" with an inaugural lecture on *Il valore delle matematiche nella cultura e la missione dell'insegnante*. The course, compulsory for the degree in "Mathematics and Physics" of the future teachers, had as an explicit objective "the view of elementary mathematics in the light of advanced mathematics".

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65 F. Enriques, O. Chisini, *Teoria geometrica delle equazioni …*, quoted, II, p. X–XI.

66 But the liking of generalities has been given another interpretation by contemporaneous geometers, specially in our country. It has been established as a general principle that every theorem must always be expounded in the most general form possible. (…)
It must be admitted that this habit has reduced the propulsive force of excellent masters, and deserves to be seriously opposed. Since first of all, the too abstract form of the terms manages to obscure the true meaning of the theorem hiding its origins, and – secondly – it creates in the young scholars the illusion of easy generalizations, purely formal.

67 F. Enriques, *Questioni riguardanti la geometria elementare*, Zanichelli, Bologna, 1900.

What must be intended with such an expression is explained by Enriques in a letter to Gentile dated 23.12.22.

> Caro Ministro,
>
> Le lascio una copia della traduzione tedesca della mia raccolta di “Questioni riguardanti la geometria elementare” che, nella seconda edizione italiana è stata allargata sotto il titolo di “Questioni riguardanti le Matematiche elementari”, colla aggiunta di un altro gruppo di problemi sui numeri, e sui massimi minimi etc. (…)
> E dai volumi di cui si tratta, Ella potrà acquistare un’idea di ciò che forma oggetto del mio corso di Matematiche complementari: corso che – prima di essere istituito per la laurea mista dal Corbino – era stato già richiesto sotto varie forme, dalla società italiana di matematiche “Mathesis” e poi da una relazione al Consiglio superiore del Pincherle (non ricordo se approvata da quel Consiglio) sotto il titolo di “matematiche elementari da impartirsi secondo vedute superiori nel secondo biennio”.
> Aggiungo che la differenza specifica tra questo corso e gli altri due di matematiche superiori del nostro secondo biennio (analisi superiore e geometria superiore) è questa: che qui entrano argomenti precisi – come i problemi della trisezione dell’angolo o della quadratura del cerchio ecc. – intorno a cui si ritiene che l’insegnante debba essere informato, ed a cui non si può costringere i corsi di analisi e di geometria superiore, i soli che mirino preso di noi alla pura scienza matematica! I corsi del primo biennio, a cui anche gl’insegnanti o meglio i laureandi attingono l’istruzione preliminare (che altrove si dà in scuole complementari del Liceo), sono destinati soprattutto agli ingegneri. Inoltre attraverso quei problemi che toccano più da vicino le matematiche elementari e che hanno una storia venti volte secolare, si mira soprattutto a muovere l’interesse dei giovani chiamati all’insegnamento, i quali – anche nel caso che non sieno di proseguire ricerche matematiche originali – debbono essere preservati dal pericolo di diventare ripetitori meccanici di una cultura ricevuta dal di fuori e però estranea veramente al loro spirito[68].

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[68] Dear Minister,
I give You a copy of the German translation of my collection of “Questioni riguardanti la geometria elementare” that, in the Italian second edition has been enlarged under the title of “Questioni riguardanti le Matematiche elementari”, with the addition of another group of problems on numbers, and on the maxima minima etc. (…)
And of the volumes we are talking about, you will be able to get an idea of what forms the object of my course on complementary mathematics: a course that – before being introduced for the mixed degree of Corbino – had already been requested under several forms, by the Italian mathematics society “Mathesis” and later by a Pincherle’s report to Superior Council (I do not remember if approved by that Council) with the title of “matematiche elementari da impartirsi secondo vedute superiori nel secondo biennio”.
I add that the specific difference between this course and the other two of superior mathematics belonging to the second biennium (superior analysis and superior geometry) is that here we deal with precise subjects – as the problems of the trisection of an angle or the squaring of the circle etc. – about which the teacher is supposed to be informed, and which cannot be imposed on the courses of analysis and superior geometry, the only ones to aim at pure mathematical science here by us! The

Similar ideas are expressed in the *Preface* to the third edition of the *Questioni*:

> La raccolta delle Questioni riguardanti la geometria elementare (…) esce ora in una terza edizione interamente rifatta secondo un disegno organico ancora più vasto: che vuole rispondere non soltanto a uno sviluppo di idee, sì anche al posto che quest'ordine di problemi ha preso ormai nella preparazione dei docenti delle scuole italiane. Poiché tutte le riforme recenti della Scuola, nei suoi diversi gradi, – in particolare l'itituzione di un corso di Matematiche complementari per le cosiddette lauree miste –, (…) mettono in valore l'indirizzo dell'opera nostra. E consentono allo sviluppo che essa riceve in quest'edizione, tendente ad appagare il bisogno più sentito di dare alla teoria scientifica una base storica[69].

The *Questioni* open a tradition that will remain lively in Italy and will be later reinforced by the publication, begun in 1930, of an *Enciclopedia delle Matematiche elementari*[70]. During many decades the geometers will be the ones who – in the wake of Enriques – will mainly commit themselves to the foundational problems also linked to teaching.

We talked about the historical Enriques. During these years, his papers are particularly numerous and go from the volume *Per la storia della Logica*[71] (1922) to several articles – above all on Greek mathematics – that will lead to the *Storia del pensiero scientifico*[72], with Giorgio de Santillana[73], published in 1932. The project included most volumes, and aimed to explore the connections between the development of scientific

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68 (continued)
courses of the first biennium, from which also teachers or rather undergraduates obtain the preliminary education (that elsewhere is given in complementary schools to the high school), are addressed above all to engineers. Besides, through the problems that concern directly elementary mathematics and that have a story twenty times secular, one aims above all to move the interest of the young call to teach, who – even in the case they do not intend to follow original mathematical researches – must be defended from the menace of becoming mechanical repeaters of a culture received from outside and really strange to their spirit.

69 The collection of the *Questioni riguardanti la geometria elementare* (…) comes out now in a third edition wholly remade following a wider organic outline: it intends to answer not only to a development of ideas, so that at the place that this type of problems has already taken in the training of the Italian school teachers. As all recent reforms of the School, in their several degrees, – specially the institution of a course on complementary Mathematics for the so-called mixed degrees –, (…) give value to our work's line. And enable the development that it receives in this edition, prone to satisfy the most profound need for giving to scientific theory a historic basis.

70 L. Berzolari (ed.), *Enciclopedia delle matematiche elementari*, Hoepli, Milano, 1930.

71 F. Enriques, *Per la storia della logica*, Zanichelli, Bologna, 1922.

72 F. Enriques, G. de Santillana, *Storia del pensiero scientifico*, Zanichelli, Bologna, 1932.

73 Giorgio de Santillana (1901–1974), Roman of Jewish descent, as was Enriques, died in the United States, where he had sheltered after the race laws 1938, after having long taught at the *Massachussetts Institute of Technology* (MIT). Among his main works (except for those written in collaboration with Enriques) we cite: *The Development of Rationalism and Empiricism* (with E. Zilsel), 1941; *Processo a Galileo*, 1960; *The Origins of Scientific Thought*, 1961; *Reflections on Men and Ideas*, 1968.

thought and the several aspects of civilization. Instead, only the first volume (dedicated to the ancient world, from the dawn of Greek civilization to the Low Latinity) will be brought to its conclusion, and then transferred to the *Compendio di storia del pensiero scientifico dall'antichità fino ai tempi moderni* (1937)[74], that includes also the essential lines of the development planned for the following parts.

Greek science is analysed in a way that enhances its logical-mathematical components and underlines the fruitful links between mathematics and philosophy, to confirm the value and originality of the position taken at the beginning of the century in the cultural battle against Croce and Gentile. In short, the history of mathematics is not a hobby for Enriques, nor is it reduced to an erudite research or to a mere individuation of priority. It is instead the analysis of the concatenations through which the theories and the descriptions of the complex interaction between the changes within the subject and the developments of philosophic and scientific thought are developed[75].

> Una visione dinamica della scienza porta naturalmente sul terreno della storia. La rigida distinzione che si fa di consueto fra scienza e storia della scienza, è fondata sul concetto di questa come pura erudizione letteraria; così intesa la storia reca alla teoria un estrinseco complemento d'informazione cronologica e bibliografica. Ma assai diverso significato ha la comprensione storica del sapere che mira a scoprire nel possesso l'acquisto, e si vale di quello per chiarire il cammino dell'idea, e concepisce questo come prolungantesi oltre ogni termine provvisoriamente raggiunto. Una tale storia diviene parte integrante della scienza, ed ha posto nell'esposizione delle dottrine, per quanto giovi spogliarla – nella misura del possibile – da troppo ingombrante ricchezza di citazioni, che tolga la visione sintetica del progresso nelle sue grandi linee[76].

Enriques passes for an *internalist* and *continuist* historian. Actually, the context is always present in Enriques' historical works, in so far as the consideration of the concatenation of facts leads directly "to the tune of the human motives where they take their own meaning". Then the reference to the context must not be denied. But it must be perfectly defined, because "the cognition of the reasons of human actions takes us always to their ideal representation". Particularly – continues Enriques – when it is about the history of thought, where "the link and the descent of the ideas remains al-

74 F. Enriques, G. de Santillana, *Compendio di storia del pensiero scientifico*, Zanichelli, Bologna, 1937.

75 F. Enriques, O. Chisini, *Teoria geometrica delle equazioni …*, cited, I, p. XI.

76 A dynamic view of science takes of course to the field of history. The strict distinction that is usually done between science and history of science is based on the idea of the last as pure literary erudition; thus understood, history causes to theory an extrinsic complement of chronologic and bibliographical information. But quite a different meaning has the historical understanding of knowledge that aims to discover in mastery the acquisition, and if it uses of the first to clear the way of the idea, and imagines the last as having extended beyond every temporarily reached end. Such a history becomes an integral part of science, and has a place in the exposition of doctrines, although it would be of use to deprive it – so far as possible – of such a bulky richness of quotations, so that it removes the synthetic view of progress in its outline.

ways a characteristic object of historic comprehension". And, to avoid misunderstandings, he specifies[77]:

> ARCHIMEDE potrebbe essere stato spinto ad immaginare le sue macchine dalla costruzione d'una nave gigante per il re Gerone di Siracusa o dalla minaccia romana pendente sopra la patria; in ogni caso tali notizie non ci darebbero affatto le ragioni geometriche e meccaniche delle dette macchine, che sono invece da cercare nelle speculazioni dei geometri precedenti. Le condizioni dell'economia e dell'industria agli inizii del secolo scorso spiegano l'introduzione delle macchine a fuoco e quindi l'attenzione portata dai fisici sopra di esse; ma non spiegano i principii della Termodinamica, cioè quell'insieme di intuizioni e di deduzioni teoriche che si svolge dai lavori di CARNOT, a MAYER, a HELMHOLTZ, a CLAUSIUS[78].

In this sense, Enriques' history of scientific thought is exactly as the one expressed by A. Koyré in his well-known Newtonian and Galilean studies[79]. Regarding the theme of *continuism*, the presence in Enriques' historic works of a strong connection to the "postulate of the unity of human reason" and to a "law of historic continuity" is undeniable. But it would be equally possible to construct a whole anthology of Enriques' texts indicating the presence of breaches, discontinuities and revolutions. How does the "law of historic continuity" match with the evident discontinuities present in the development of scientific thought? Enriques had given an answer in *Problemi della Scienza* (1906) where, talking of the methods of "positive gnosiology" and dissociating from Spencer's "psychological method of evolution", he had stated that the "process of knowledge, even if it comes from a continuous development, has a discontinuous qualitative meaning". So as to explain the exact sense – continuous development at global level, but discontinuous qualitative meaning at local level – that Enriques gives to the dialectic relationship between continuity and breach, in the history of thought, we quote the following passage[80] (taken from Enriques' introduction to the famous three lectures that Einstein gave in Bologna in October 1921).

> Nondimeno Einstein viene presentato al pubblico come un rivoluzionario. La sua dottrina o scoperta ha pôrto nuova occasione per gridare alla bancarotta della scienza. Più d'uno si è rallegrato o doluto che perfino la verità più ferma che da due

[77] F. Enriques, *Significato della storia del pensiero scientifico*, Bologna, Zanichelli, 1936, p. 47.

[78] ARCHIMEDE could have been driven to imagine his machines by the construction of a giant ship for king Gerone of Syracuse or by the Roman threat hanging over the mother country; in any case such news would not give at all the geometric and mechanical reasons of these machines, which are instead to search for in the meditations of earlier geometers. The conditions of economy and of industry at the beginning of the last century explain the introduction of the shooting machines and therefore the attention of physicians to them; but they do not explain the principles of Thermodynamics, that is, that set of intuitions and of theoretical deductions that develops from the works by CARNOT, MAYER, HELMHOLTZ, CLAUSIUS.

[79] A. Koyré, *Newtonian Studies*, Harvard University Press, Cambridge (Mass.), 1965; *Etudes galiléennes*, Hermann, Paris, 1966.

[80] F. Enriques, *Per la scienza* (R. Simili ed.), Bibliopolis, Napoli, 2000, p. 329–332.

secoli abbiamo imparato a riverire come il trionfo della ragione umana, dico la legge della gravitazione universale di Newton, debba ora riconoscersi non esatta. (…)
Chi giudica in tal guisa è lontano, non solo dal pensiero di Einstein, ma dal concetto storico della scienza accolto ormai dalla mente contemporanea e specie dai pensatori matematici. Giacché niuna teoria pretende oggi ad un'assoluta esattezza, ma ciascuna si dà come un grado perfettibile della verità, che si svolge e cresce col progresso della ragione. Così la teoria di Einstein non significa la morte della teoria di Newton, anzi la conquista di una verità più vera, di fronte a cui la precedente figurerà sempre come un grado di approssimazione.
Aver superato questo grado, fino a spiegare le minime perturbazioni or ora accennate, scoprire dunque la legge correttrice di errori appena sensibili, costituisce il più splendido trionfo della ragione umana! Nonostante tutti i sofismi con cui si è tentato di travisarne il significato, questo è anche il vero motivo della commozione suscitata da Alberto Einstein. Egli ci ridà la fiducia nella ragione, proprio in quest'ora tenebrosa in cui essa sembra sommergersi nel cozzo delle passioni oscure. Egli c'invita a distoglierci dal sogno romantico dell'io che s'inebbria della signoria dell'universo, per volgerci alla contemplazione dell'ordine che la mente riesce a scoprire fuori di sé, nella meravigliosa opera d'arte della natura. C'è già in questo invito un alto significato morale. Ma qualcosa di più alto scaturisce da un esame approfondito del pensiero di Einstein.
Avete già udito ch'egli sovverte colla sua critica i comuni concetti dello spazio, del tempo e del movimento. Ai filosofi kantiani che, in nome della ragione, domandano di accogliere taluni giudizii a priori, perché sia possibile la scienza, il Nostro risponde che la ragione non ha limiti necessariamente segnati, che non offre ai dati sperimentali un ordine prestabilito, ma che trova in sé il potere di allargare i quadri in cui si compone l'esperienza familiare delle cose vicine, adattandosi ad un'esperienza più estesa. Certo vi è qualcosa di sorprendente e quasi di pauroso in questo progresso del pensiero che supera i limiti della propria intuizione e foggia a se stesso più alte forme intuitive. Quante volte la vertigine del volo sembra travolgerci nell'abisso dell'assurdo!
Pure tale progresso si presenta come logica conseguenza di una critica che – nella ricerca di armonizzare dati apparentemente contraddittorii – tien fermo ai principii e se ne vale come di chiave per analizzare il significato dei concetti. Per questo aspetto la rivoluzione filosofica che Einstein ha portato a compimento si dimostra come il risultato di un'evoluzione del pensiero, più volte secolare. La quale s'inizia 500 anni innanzi dell'era volgare, da Parmenide d'Elea, primo assertore della relatività del movimento. Non si diminuisce Einstein dicendo che egli conchiude in una più larga sintesi cosmologica il lavoro di una lunga serie di filosofi, di matematici e di fisici, da cui ha raccolto disparati elementi per fonderli nella sua costruzione[81].

Whatever the judgement given to Enriques' work and to its historiographical location is – on the themes of "internalism" and "continuism" and on its supposed isolation

[81] see next page

from the coeval European context too –, the importance of such a strong personality in the Italian community must not be neglected. It is also thanks to *Enriques effect* – nor is to be ignored his presence as coordinator of the mathematical branch of the *Encyclopaedia*, that undoubtedly facilitates the development of the historical voices – that the history of mathematics lives in Italy a rather lively period with several attempts to root the subject at an institutional level and in the university *curricula*. We can name Gino Loria (1862–1954) and Ettore Bortolotti (1866–1947), father of that Enea we mentioned when talking of differential geometry studies, pupil of Pincherle and scholar of Ruffini's works and of sixteenth-century algebra, and also Giovanni Vacca (1872–1953), pupil of Peano and later (from 1922) professor of history and geography of Eastern Asia. Enriques' institutional attempts concern especially the foundation, with the support of *Mathesis*, of an *Istituto nazionale per la storia delle scienze fisiche e matematiche*. The statute of the *Istituto*, established in 1923, assigns him the task of collecting books and documents to

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[81] Still Einstein is presented to the public as a revolutionary. His doctrine or discovery has given a new chance to announce the bankruptcy of science. More than one has enjoyed from or regretted the fact that even the most solid truth that two centuries ago we have learned to venerate as the triumph of human reason, that is Newton's law of universal gravitation, must be recognised now as non accurate (…).
The one who judges in this way is far away not only from Einstein's thought, but from the historical idea of science by now accepted by the contemporaneous mind and specially by mathematical thinkers. Since no theory pretends today an absolute exacteness, but every one presents itself as a perfectible degree of truth, that develops and grows with the progress of reason. Thus Einstein's theory does not mean the death of Newton's, rather the conquer of a truer truth, in front of which the previous one will appear always as an approximation degree.
To have overcome this degree, to the point of explaining the minimum perturbations just mentioned, to discover then the law correcting barely perceptible mistakes, is the most splendid triumph of human reason! Despite all the sophisms with which has been tried to misinterpret its meaning, this is also the true reason of the commotion arisen by Alberto Einstein. He gives us again confidence in reason, just in this tenebrous hour in which it seems to sink in the butt of dark passions. He invites us to turn away from the romantic dream of the self that inebriates with the dominion of the universe, to turn to the contemplation of the order that the mind manages to discover outside itself, in the marvellous work of art of nature. There is also in this invitation a high moral meaning. But something more high arises from a deep exam of Einstein's thought.
You have already heard that he subverts with his critique the common ideas of space, time and movement. To the Kantian philosophers that, in the name of reason, ask to accept some judgements a priori, so that science be possible, Our replies that reason has not necessary indicated limits, that it does not offer to experimental data a predetermined order, but that finds in itself the power to enlarge the pictures in which the familiar experience of nearby things is made, adapting itself to a more vast experience. Of course there is something surprising and almost fearful in this progress of thought that overcomes the limits of one's intuition and shapes higher intuitive forms into himself. How many times the vertigo of flight seems to carry us away into the abyss of absurd!
Yet such a progress presents itself as logic consequence of a critique that – in the search for harmonizing apparently contradictory data – holds to principles and uses them as a key to analyse the meaning of ideas. Under this aspect the philosophical revolution that Einstein has accomplished shows itself as the result of an evolution of thought, several times secular. This begins 500 years before the our age, since Parmenide d'Elea, first assertor of the relativity of movement. Einstein does not debase himself telling that he finalizes in a larger cosmological synthesis the work of a long series of philosophers, mathematicians and physicians, from which he has gathered disparate elements so as to fuse them in his construction.

"proceed with important and long researches"; of diffusing researches and ideas that can "enrich culture and better orientate the studies of science historians"; of organising and publishing manuscripts and notes for the "best knowledge of the development of scientific doctrines"; of publishing "classic or specially interesting works". This, in general, is the *Istituto*'s *mission*:

> anche l'evoluzione della scienza nel mondo moderno e il durevole influsso che essa potrà esercitare nelle epoche a venire, dipenderà in alto grado dal modo come l'epoca nostra saprà comporre le grandi idee che costituiscono il retaggio di un prossimo passato e coglierne il significato filosofico, riguardando alla genesi delle dottrine più semplici ed elementari che aprono la via agli sviluppi contemporanei[82].

The history of the *Istituto* will, not always in a friendly way, intertwine with similar attempts on the side of another science historian, Aldo Mieli (1879–1950). Founder in 1919 of the journal *Archivio di storia della scienza*[83], Mieli will collide, sometimes harshly, with a personality such as Enriques' and will not hesitate to write of him[84]:

> questo direttore era ed è molto noto non solamente per la sua smania di essere infallibile ed il primo in tutte le faccende alle quali si mescola, ma anche per la sua inettitudine a dare vita alle organizzazioni che costituisce coll'unico intento di esserne il capo, senza poi né volere né sapere farle funzionare[85].

Despite the difficulties regarding its management, the *Istituto* – that from 1926 is named *Istituto nazionale di storia delle scienze* – manages to promote the institution of a historical library and to encourage several initiatives and congresses. To the *Istituto* is due, particularly, the foundation of the *Scuola di perfezionamento* in science history, which we mentioned in the previous chapter and in which many of the main historians of Italian mathematics of the late post-war will be educated.

Traditionally, history of mathematics, also for its popular aspects, is a bridge towards society. We move along the borderland between research as such and cultural reasons, communication, didactic considerations, attention to the contexts. It does not surprise, then, if historical studies are between the first – *the* first in our narration – to suffer from the changed social and political situation. The *nationalism* of the 1920s is not

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[82] Even the evolution of science in the modern world and the lasting influence that it could have in the future times, will depend to a great extent upon the way in which our time will settle the great ideas that form the inheritance of a next past and catch its philosophical meaning, regarding the genesis of the most simple and elementary doctrines that open the way to contemporaneous developments.

[83] The *Archivio di storia della scienza* will change its name to *Archeion* in 1923 and from 1929 onwards, when Mieli – antifascist – will leave Italy, it will be published in Paris.

[84] The passage appears in an autobiographic document, written by Mieli when he was about to leave Paris for Argentina: A. Mieli, Passato ed Avvenire, *Archeion*, XXI (1938), p. 1–9 (4–5).

[85] This director had and has a reputation for his itch to be infallible and the first in every event he meddles in, but also for his ineptitude to set up the organisations he founds with the only objective of being the leader, without really wanting to or knowing how to make them function.

a typical Italian feature, but it is certainly one of the main ingredients of fascism's confused ideological substrate. Researches on history of *Italian* mathematics increase in a gradual way, and the tones become decidedly more emphatic and rhetorical in boasting the glories of the national tradition. We will see in the next section the example of another discipline – statistics, and more in particular, demography – depending on the new regime.

6. Castelnuovo, Probability and "social Mathematics"

In this chapter, dedicated for the most part to the studies of geometry, we have not talked, so far, about Castelnuovo, the first of the three "musketeers" with whom the Italian school of Algebraic geometry is usually identified. The reason is simple and we already mentioned it. In the years between the two world wars, in fact, Castelnuovo does not deal anymore with geometric researches[86]. Now he goes in for probability. We do not know exactly the reasons behind this conversion but, after all we have said about the difficulty to obtain the brilliant results of the beginning of the century, we think there is a kind of tiredness and lack of confidence about the possibility of maintaining the same excellence level, holding to the characteristics of the Italian school.

Castelnuovo publishes his treatise on *Calcolo delle Probabilità*[87] in 1919. A following edition in two volumes will appear in 1925 and in 1928 but, compared with the first one, it has only some enlargement concerning the Gaussian law (in two dimensions) and its applications to the theory of correlation, besides some improvement inserted in the proof of the central limit theorem. The 1919 *Trattato* is then the main reference point to understand Castelnuovo's thought within the field of probability, not least because other notes – even if published in prestigious journals – concern specifications and elaborations of questions already dealt with in the volume. The publication of the *Trattato* is soon appreciated and favourably reviewed, because of some original contributions (to probability and statistics, with specific contributions to the problem of the moments and to the theory of dispersion), because of a series of rigorous proofs complete with some classic results and a systematic presentation of the subject that establishes a solid backgound picture, ready to be used also for applications (think, for example, of the chapter dedicated to Maxwell's law).

The *Trattato* is one of the first important moments of the great upsurge in probability at an international level, in the years between the two world wars, with the research on its foundations and on the theory of aleatory processes and the almost definitive arrangement of classic problems. We only need to think about the abstract formulation proposed by Kolmogorov in 1933 and, more in general, to the situation that arises from the *Colloque consacré à la théorie des probabilités* organised at Geneva in 1937. The subject goes beyond the horizons of the simple treatment of gambling games and the so-

[86] In 1921 Castelnuovo published two papers on the abelian functions. Only in 1949 he will write other two notes about the number of the modules for an irregular surface.

[87] G. Castelnuovo, *Calcolo delle Probabilità e applicazioni*, Dante Alighieri, Milano, 1919.

called collective phenomena. It retrieves the pioneer work carried out straddling the two centuries by the French school (Bertrand, Poincaré, Bacheler, Borel) and by the Russian one (Chebyschev, Markov, Liapounov) to give rise to a mathematical theory, based on fairly shared general principles.

Castelnuovo's thought about the meaning of probability falls under the *objectivist* interpretation and, in particular, in the frequentist-empirical line. Once the concept of relative frequency of an event is introduced, probability is defined as the limit of the last one as the number of repetitions of the experiment increases. For Castelnuovo – his approach will influence other writers, such as P. Lévy and M. Fréchet –, the fact that frequency, as the repetition of the experiment increases, tends to become steady around a certain value is merely an empirical fact. Nor is it possible to prove this convergence by using, for instance, Bernoulli's classic theorem. This is how Castelnuovo exposes the *empirical law of the case*: "if an event has a constant probability p in every test, and if it occurs m times in n tests, the ratio, *frequency, m/n* gives an approximate value of the probability p; and usually, the bigger the number n of the tests is, the better the approximation"[88].

Castelnuovo is aided by Francesco Paolo Cantelli (1875–1966) in the redaction of the *Trattato*. Sicilian, Cantelli graduates at Palermo with a dissertation in celestial mechanics. Later, for two decades, he works as actuary in some welfare institutes and this takes him to a definitive conversion towards the actuarial and probabilistic studies. After the first World War, he continues his academic career at the universities of Catania, Naples and Rome, where he will definitely move to in 1931. In Rome, moreover, he founds (1930) the *Giornale dell'Istituto Italiano degli Attuari* that he will edit until 1958 and that will become, under his leadership, one of the most appreciated journals of the sector at an international level. Thus does H. Cramér remember it[89].

> I should mention also another name, the Italian mathematician Cantelli, with whom we also had contact in our Stockholm probabilistic group. Cantelli, like myself, was also working as an actuary. As a matter of fact, he had been, among other things, the actuary of the pension board of what was then called the Society of Nations in Geneva. When he resigned his position, I became his successor. That gave me a contact with Cantelli which I value very much. You know his name from the Borel-Cantelli condition. He had written several very valuable papers on probability, papers which have perhaps not received quite the attention that they really do deserve. He was a very temperamental man. When he was excited, he could cry out his views with his powerful voice: a very energetic fellow. The last time I was in Rome a couple of years ago, he was there, now a very old man. He couldn't come to any meetings, but I had a short conversation over the telephone with him and we remembered together the good old days.

88 G. Castelnuovo, *Calcolo delle Probabilità …*, quoted, p. 3.

89 The quotation, taken from E. J. Wegman: "Some personal recollections of Harald Cramér", *Stat. Sc.*, 1986, p. 528–536, is cited in the essay of E. Regazzini, dedicated to the *Probabilità*, in the volume S. Di Sieno, A. Guerraggio, P. Nastasi, *La Matematica italiana dopo l'Unità. Gli anni tra le due guerre*, quoted.

Cantelli's point of view in the interpretation of probability converges essentially with Castelnuovo's. The need for a formal definition will take him to feel a gradually increasing interest towards the foundations of probability and (in the years coming soon after those we are dealing with in this chapter) to an abstract approach preceding only slightly Kolmogorov's famous axiomatic theory. Both approaches are quite similar, starting from the basic ideas of *elementary event* and of *aleatory variable* (for which Cantelli uses the term *weighted* variable). The abstract theory allows him to formally situate the study of the convergence of the sequence of aleatory variables, to which he had made significant contributions during the war years with the introduction of the concepts of *convergence in probability* and of *uniform convergence in probability* (equivalent to the convergence *in measure* and to the *almost certain* one). With the second, in particular, Cantelli intends to explain the behaviour of the sequence of frequencies in Bernoulli's scheme. It is in this context that he proves the so-called *lemma of Borel-Cantelli*, that appeared already in the quotation of Cramér[90]. The name of Cantelli is instead linked to that of V. Glivenko in the well-known *Glivenko-Cantelli theorem* – called also *fundamental theorem of mathematical statistics* – relative to the convergence of a sequence of empirical distribution functions for independent identically distributed variables. Cantelli manages to simplify the proof given by Glivenko, using the *σ-additivity* hypothesis, and to extend it to the case of cumulative distribution function (not necessarily continuous).

In probability happens what we have already noticed in algebra. The lack of a strong tradition can have a positive role because it facilitates the outburst of new energies that reach in short a remarkable level, not influenced by previous statements. Of course, in the case of probability, the stance of great sensitivity that Castelnuovo always kept, even towards studies that were not always homogeneous with his own positions, is important.

The Italian contribution to the propitious time that probabilistic studies enjoyed during the years between the two world wars becomes more innovative with Bruno de Finetti (1906–1985). With him, who belongs to the generation succeeding that of Castelnuovo and Cantelli, we will sometimes go beyond the time boundaries of this chapter – the second half of the 1920s – but we will actually exhaust the look given to probabilistic studies.

The interpretation – decisively *subjectivist* – of de Finetti[91] is presented in two 1931 memoirs[92] and, later, in a famous 1937 memoir[93] published in the *Annales de l'Institut H. Poincaré*. De Finetti in those years is still very young. He graduates in mathematics in Milan (after having begun engineering) in 1927. He works then in Rome, at the *Istituto Centrale di Statistica*, until 1931. In these years he devises the main ideas within those fields of probability that he will master: subjective interpretation, the study of interchangeable

[90] The previous quotation from Cramér on Cantelli's character is confirmed by his controversy wih E. Slutsky about Borél's contributions and his priority for the strong law of large numbers.

[91] One can read D. M. Cifarelli – E. Regazzini, De Finetti's Contributions to Probability and Statistics, *Statistical Science*, 1996, pp. 253–183.

[92] "Sul significato soggettivo della probabilità", *Fund. Math.*, 1931, pp. 298–329; "Probabilismo. Saggio critico sulla teoria della probabilità e sul valore della scienza", *Biblioteca di Filosofia di A. Aliotta*, pp. 163–219.

[93] "La prévision: ses lois logiques, ses sources subjectives", *Annales Inst. H. Poincaré*, t. VII (1937), pp. 1–68.

Bruno de Finetti (1906–1985)

processes, the theory of processes with independent increments. Later, he works in Trieste, as actuary, in an insurance company. His entrance into the university world will take place only after the war and it sees him engaged at Trieste and Rome universities.

The disappointment at a merely formal theory and the critiques to the objectivist conceptions cause de Finetti to explain probability as a numerical expression of a subjective judgement. The allocation of a probability to a particular event expresses the estimate of a subject that declares the value at issue according to a measure method based on the *scheme of bets*. Of course, it is not about reducing the calculus of probabilities to a set of rules that rationalises behaviour in gambling, but about supplying a conceptual experience that translates in quantitative terms into a judgement of likelihood of a subject about an event. The consequent definition of probability points out, besides, the mere optional nature of the σ-additivity property[94].

> La questione è allora, formalmente, di decidere se le leggi di probabilità che soddisfano, nelle classi finite, il teorema delle probabilità totali si debbano considerare tutte come ammissibili, o se ci si debba limitare a considerare tali quelle sole tra esse che risultano sommabili in ogni classe numerabile. Nel rispondere a tale questione sembra che i diversi autori ritengano di poterne disporre a piacimento, a seconda che risulti a loro più comodo: il raffronto che fa Fréchet con il problema della misura lo mostra in modo lampante. Mi sembra però che il caso sia molto diverso. Se si tratta di introdurre una nozione matematica, potremo certo definirla nel modo più opportuno, e far sì, purché ciò sia possibile, che risultino soddisfatte

[94] "A proposito dell'estensione del teorema delle probabilità totali alle classi numerabili", *Rend. Ist. Lombardo*, 1930, pp. 901–905. De Finetti is engaged in an stimulating dispute with M. Fréchet on the resort to the hypothesis of σ-additivity.

> certe condizioni che ci sembrano utili. Se invece una certa nozione, nel nostro caso la probabilità, ha già un senso ben definito, si tratta non di fare una convenzione, ma di dimostrare un teorema, e non basta che quelle proprietà siano possibili, ma occorre che, per il significato stesso del problema, risultino *necessarie*[95].

It is when linking the subjectivist point of view with the most usual and practical forms of probability reasoning and, especially, when retrieving the relationships between probability and frequency, that de Finetti dwells on equivalent processes – which he, at G. Polya's prompting, will call *interchangeable* in the *Colloque* of Geneva – that already intervene in the paper given at the 1928 international Conference of Bologna. The well-known theorem of representation of interchangeable events dates from 1930.

From the beginning, de Finetti's research in probability has a second scope of inquiry in the *processes with independent increments* that he studies so as to contribute to mathematically reformulate physical laws when a rigidly deterministic point of view is discarded. They are studies whose importance will be soon acknowledged by authors such as P. Lévy and A.N. Kolmogorov.

But de Finetti is not only a probabilist. Other contributions of his concern financial and actuarial mathematics[96], statistics, and economics. We can thus introduce the argument for the interest in and the eventual conditioning of fascism towards mathematical-social sciences.

In statistics, his most important contribution is the one today called the *Nagumo-Kolmogorov-de Finetti theorem*. In 1929, O. Chisini (whom we met in this chapter as a geometer, pupil of Enriques) publishes the note *Sul concetto di media*, in which he critiques the traditional research direction which has turned to introducing specific means, becoming more and more complex, and proposes instead an axiomatic definition: "given a function $y = f(x_1, x_2, \ldots, x_n)$ of some independent variables, $x_1, x_2, \ldots, x_n$, representing homogeneous quantities, we say that the mean of $x_1, x_2, \ldots, x_n$, regarding the function *f*, is that number *M* which, substituted for $x_1, x_2, \ldots, x_n$, gives for *f* the same value as $x_1, x_2, \ldots, x_n$, themselves, that is, that number *M* so that $f(M, M, \ldots, M) = f(x_1, x_2, \ldots, x_n,)$". It is in this definition that de Finetti intervenes, so as to expand it to random, variables.

95 The question is then, formally, whether to decide if the laws of probability that satisfy, for finite classes, the theorem of total probabilities have to be considered as all admissible or if we only have to consider as such just the ones among them that result summable in any countable class. In answering to such a question it seems that different authors consider that they can use it at will, depending on whether it is more convenient: the comparison that Fréchet makes with the problem of measure shows it in an evident way. But I think that the case is really different. If it is a matter of introducing a new mathematical notion, we could certainly define it in the most appropriate way and make that, providing that it is possible, some conditions we consider useful are satisfied. If instead one notion, in our case probability, has already a well defined sense, it is not about making a convention, but about proving a theorem, and it is not enough that such properties be possible, they must be *necessary*, for the meaning itself of the problem.

96 Also during the 1930s, financial and actuarial mathematics experiences a fairly vivid period, and benefits from the encouragement coming from the evolution of the assurance and financial worlds and from the development of the faculty of economics. Besides de Finetti's, we also find works (among the already mentioned mathematicians or the ones we will soon talk about) of L. Amoroso, F. P. Cantelli, C. Gini, F. Tricomi.

Then he extends the application of Nagumo-Kolmogorov theorem to the continuous case, proving that the Chisini means, when satisfying the associative property, correspond to means obtained as monotonic transformed of the arithmetic mean.

Before going on with the examination of de Finetti's contributions to economic analysis, we would like to dwell for some time on statistics, switching our attention to the most applicative front and to demography, daughter of statistics, with which we will deal again when talking about the tragic days of the politics of race.

For Italian statistics, the thirties are a particularly significant period. Its disciplinary location changes with the definitive detachment from juridical-moral sciences and the recognition of the scientific nature of its teaching. In the second half of the 1920s, the number of courses of statistics, applied statistics, demography (and probability) passes from 46 to 71, under the push of the growing request of specialised personnel coming from the public insurance and administrative world. In 1927 two *Scuole speciali* in statistics are founded in Rome and in Padua with a three-years course of study. In 1936, the Roman school will become the first faculty, in Europe, of *statistic, demographic and actuarial sciences*[97]. In 1939, the *Società italiana di Statistica* (SIS) will be instituted. In the meanwhile, the publications of *Metron*, "international journal of statistics", expressly dedicated to statistical methodology, had begun in 1921. *Metron* is followed by the *Rivista italiana di statistica* (1926), *Indice del movimento economico italiano* (1926), *Genus* (1936), *Rivista italiana di demografia e di statistica* (1938). Also the public statistics that has to provide institutions and economists with suitable data on the economic, demographic and social situation, begins to develop again, after the propitious time experienced during the first decades after Unification. Mussolini never conceals his interest in the statistical knowledge of reality and especially in that of the population and already in December 1923 – little more than a year from the seizure of power – he issues three decrees so as to sort out the public statistics service. In 1926 the *Istituto centrale di statistica* (ISTAT) is founded; it receives the inheritance of the pre-existent *Direzione generale di Statistica*, and has greater ambitions of coordination and control of data collection, to the point that in 1929 the *ISTAT* – in which all of the Country's statistical services are now grouped – is put in the Head of Government's employ. And the *ISTAT*, among its various activities, sets out mainly to accomplish the 1921 population census audits (that had tremendously lagged) and to start the 1931 population census immediately, in which a computerized scrutiny will be realized – for the first time.

The figure of Corrado Gini (1884–1965), the real *boss* of Italian statistics during the years between the two world wars, mentioned still today in the textbooks for his *Gini's coefficient*, is at the centre of many of these institutional events. Gini received his degree at Bologna in 1905, and began his academic career in the statistics field in Cagliari, teaching later in the universities of Padua and Rome. He is Dean of the Roman statistics faculty in 1928 (until 1955!); editor of *Metron* since its foundation; Chairman of the *ISTAT* from 1926 to 1932; President of the *SIS* since 1941 (after having been roundly criticised about the expediency of founding such a society).

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[97] It is worth noting the fact that most of these initiatives are promoted by Castelnuovo, Cantelli's real protector since 1918, when Cantelli had asked for the free professorship in probability.

Gini is a fairly good researcher, who moreover has the merit of having "discovered" de Finetti, ensuring him – still a student – a place within the *ISTAT*. Already in the 1910s, he is an internationally valued statistician for his writings on the analysis of variability, the introduction of the simple middle difference and the conviction – polemically confirmed in the *Journal of the Royal Statistical Society* too – of the expediency of using variability indexes. These studies go on also in the postwar period, with an examination of the relationships between the different indexes and the further consideration of series of qualitative data, which contribute to frame a real Italian school of statistics in contention with the one – traditionally hegemonic – of Anglo-Saxon origin. Gini deals also with demographic subjects, elaborating one cyclical theory of populations and repeatedly taking part in the fervent discussion of subjects of Italian demography during the twenties, such as emigration, war as a national reinvigoration factor, and the populationist politics. Exactly these (statistician and demographer) competences, the assonance of judgements with what will gradually qualify itself as fascism's demographic politics, and lastly his quick adhesion to the new regime – with Pincherle, he was the only mathematician to sign the *Gentile manifesto* – draw Mussolini's attention and in short make of him one of the most listened to counsellors on the subject. Author of a paper at the fascism's *Gran Consiglio* on demographic questions, Gini is personally consulted by Mussolini regarding the redaction of the *discorso dell'Ascensione* from the 26th May 1927, that can be considered the foundational act of an explicit nativist politics of fascism. Italy could boast vigour and youth resources that made it rather different from the *old* countries, destined inevitably to a demographic, and thus political, decline. Similarly, it is a "demographic thwarting to the nation", in order to react to the menaces that Mussolini saw as imminent[98]. Mussolini himself, a year later, will write an article with the significant title of *Numero come forza*. The Duce pays particular attention to data and statistics and in 1926 – as we already said – he founds the *ISTAT* with a law that he personally introduced before Parliament and that puts the Institute in his *direct* employ and later (in 1929) even in his *direct and exclusive employ*. Several times Mussolini reaffirms that Statistics and *ISTAT* are for him an essential instrument in the government action and Gini can thus enjoy a very persistent frequentation of the power centres, being welcomed by the Duce – this is known too – twice monthly. Obviously, he is paid back with the significant and prestigious presidency of the *ISTAT* (that on the other side he immediately aligns with the regime's political stances, polarizing the attention on what Mussolini considers the priority problem of birth-rate and of reproductivity) that he will keep until 1931, when for him comes the day of reckoning. His overbearing personality and the powerful positions he had won as President of the *ISTAT*, to the detriment of already consolidated powers and out of control, bring him to quarrel with many officials of the Prime Ministership and even with many Ministers. The resignation demand on the side of Mussolini is at this point almost unavoidable, even if Gini will hold to the regime (and to its most extremist positions) until the end.

In the overview of the relationships with the regime, statistics and demography – advantaged by their specific content – represent an isolated and extreme case, that will

98 Gini's contributions give scientific inspiration for all the fascist legislation that ranges from taxation of the celibates to premiums for numerous families.

not even be repeated by mathematical economics (as we will soon see). Maybe it is excessive to talk about *science of regime*, not least because statistics is not only economic statistics and in any case it is not only Gini[99], but it is true that it has a close relationship with the regime, just as close as the obedience to its stances, especially in the case of demography. Demography becomes – if we can say so – a fascist word not by chance. It is not only the irritating rhetoric (that we will find in many other cases) which presents Mussolini as "founder and moving spirit and supreme controller of Italian statistics (…), who increases studies and national statistical consciousness". Here rhetoric goes with precise events: (applied) statistics and demography turn their researches in good part – objectives; methods; emphasis in underlining some results – according to the precise and explicit demands of political power. In exchange, they obtain such a development, even in terms of chairs, that with no exaggeration we can affirm that statistics sees in fascism the realization of its own expectation of self-assertiveness and demography, even the legitimization of its own existence.

It is an important conclusion, that does not recur in the case of mathematical economics, at least with the same force. Let's follow de Finetti's foorprints. If in probability his innovative charge finds expression in the critique to objectivist ideas, in economics the target of his *polemics* is even such a "classic" as V. Pareto. And, as we will see, particularly heated tones are not missing.

In the *Prologue* we have talked about the big leap forward taken by Italian economic-mathematical studies, actually thanks to Pareto, in the years astride the two centuries. Following his silence about economic matters, the school level will necessarily decrease (just while the equilibrium theory starts its *golden age*). The scholar who takes on the mantle of Paretian inheritance, and with whom mathematical economy identifies for some decades, is Luigi Amoroso (1881–1965). At first a student at the *Scuola Normale* and friend especially of Picone, he graduates at Rome in 1907, soon becoming Castelnuovo's assistant. His friendship with Picone will turn out to be important for the history of scientific institutions too. Soon after the first world war and precisely by virtue of what such a tragic experience had showed, Picone persuades himself of the need to create in Italy an Institute of calculus which would offer concrete solutions to numerical estimate problems. The project needed of course an experimental stage and an initial capital endowment other than paltry, whose achievement could not be considered foregone, considering the boldness of Picone's bet on the potentialities of the numerical calculus. It is Amoroso, in those years economic counsellor of the *Banco di Napoli*, who obtains the necessary financing in order to float the project and it is with his essential contribution that the *Istituto Nazionale per le Applicazioni del Calcolo* (INAC) is established in Naples (to move later to Rome). Previously, in 1914, Amoroso had obtained a chair of financial mathematics in Bari, moving later to Naples (since 1921) and then to Rome (definitively, since 1926).

Already in some pre-war notes we notice some elements that will cause Amoroso to be the most consistent advocate and interpreter of Pareto's thought. Thanks to his work,

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[99] Within economic statistics can be cited, at least, the names of Rodolfo Benini (1862–1956), Giorgio Mortara (1885–1967), Livio Livi (1891–1969), Gaetano Pietra (1879–1966), Marcello Boldrini (1890–1969).

mathematical economics has formed into an autonomous subject, with a scientific *status*, just like analytic geometry and rational mechanics. Now, the Master's guidelines are to be developed in a *dynamic* sense, confirming the choice of mechanics as a privileged model. The *dynamization of the Paretian scheme* is the initial visiting card that, even with significant readjustments, Amoroso will always produce, even as to talk in his autobiography of the "central problem of my scientific activity, about which I have thought for more than thirty years". This project remains the underlying theme of his research, even if it will not produce especially brilliant or original results. It has to be considered, after the war, Amoroso was engaged in an intense administrative activity, that sees him placed in positions of growing responsibility. Besides the already mentioned assignment at the *Banco di Napoli*, in 1925 Amoroso is also appointed vice-commissioner of the *Istituto Nazionale delle Assicurazioni* (of which he is counsellor from 1926 to 1931), to be later managing director of the *Assicurazioni d'Italia* from 1929 to 1944 and counsellor of the *Banca Nazionale del Lavoro*, an appointment that he will hold from 1950 until his death.

More interesting is a 1928 note[100] that fits into the great season of the equilibrium theory and that anticipates in some ways the classic contribution of A. Wald in dealing with the problem of existence and of the uniqueness of the consumer's equilibrium. Equally interesting for the pathway we are following, regarding the relationships between mathematical research and social context, are some indications that become more and more frequent (starting from the paper given at the 1928 international Congress of Bologna) and that will allow Amoroso to develop a series of macroeconomic models: "the task of theorists, today, is not that of building, cold, in theory, general theories, but of developing this practical movement gradually, without a flourish, so as to interpret and to guide"[101]. In this approach, that we could define *empirical-statistical*, the role of theoretical conjecture is obviously not removed; with reference to the subject of market equilibrium, for instance[102]:

> perché la domanda (ed analogamente dicasi per l'offerta) sia determinata, occorre dare, oltre al movimento empirico dei prezzi e delle quantità, delle condizioni complementari. Esse devono essere espresse come postulati. Vi sorprenderà forse, se aggiungo, che tali postulati *non sono suscettibili di verifica empirica*. Ma la sorpresa dilegua, se riflettete che domanda e offerta *non sono un dato sperimentale, sono un'astrazione creata dalla nostra mente*, che ha il suo fondamento non già nell'osservazione dei fatti reali, ma nella presunzione di fatti virtuali, da noi giudicati possibili[103].

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100 "Discussione del sistema di equazioni che definiscono l'equilibrio del consumatore", *Annali di Economia*, pp. 1–19.

101 "Le equazioni differenziali della dinamica economica", *Giornale degli Economisti*, pp. 37–59.

102 *Ibidem*.

103 For the demand to be determined (and the same can be said for the supply), besides the empirical movement of prices and of quantities, complementary conditions have to be given. These have to be expounded as postulates. Maybe it will surprise you if I add that such postulates are *not susceptible of empirical verification*. But the surprise disappears, if you think that demand and supply *are not experimental data, they are an abstraction created by our mind*, that has its ground not in the observation of real facts, but in the presumption of virtual ones that we judge possible".

The dynamization of the Paretian system, though, goes together with more precise references to the economic reality while the research's barycentre is inclined to privilege *macro-economic* models, often conceived with *econometric* purposes.

The fact is that Italian mathematical economics had found itself in the storm-centre. The old accusations of "illicit intromission of mechanics in the study of moral phenomena", of "inapplicability of the mathematical method and of the mechanical mentality to the study of economic reality in its movement, in its life", of "inapprehensible and inconsistent formalism of pure economics" had come back. It is the recall to the primitive state of politics and to the subjectivist forcing that fascism intends to perform towards the *system*. Even if the initial and more radical stage of the protest is already overcome, the cultural environments closest to the regime press strongly to make mathematical economics conform to the new ideological dictates. The choices carried out by Amoroso are the answer to this pressure, to which in part he conforms, committing himself to harmonize and to integrate the new ideas with the classic theoretical system. Taking this position, Amoroso manages to save Pareto's method. At the same time, he does not hesitate now to point out gaps and contradictions. He performs thus a valuable connecting role and manages to approach the problem areas dear to the regime and to many economists who did not intend in the least to reject the principles of scientific rigour and that they put exclusively within the school of general economic equilibrium. The guarantee that his authority offers is complemented by further persuasive elements, coming from the evolution of the productive and distributive process itself. The reality of the big crisis and of the industrial concentration forces a redefinition of a coherent theoretical structure.

De Finetti participates in this conflict with few but significant notes (published during the years 1935–1937). After a first "technical" article[104], in which he develops two specific critical remarks on the utility theory, follow a series of papers that allow one to see without doubt his ideological choices. The one against the liberal-capitalistic system, proclaimed with young aggressiveness, is firm. Fascism is valued from this anti-systemic perspective and its most radical and innovative aspects are supported. De Finetti's verbal violence comes into natural conflict with the decidedly most moderate and academic language of Amoroso. But this is not the only difference. The course taken by de Finetti's critique of Pareto's theory – considered the one which legitimates and inspires the liberal-capitalistic system – is even more interesting. Pareto remains a cornerstone in the history of economic thought, yet it is not enough to update his ideas with some simple modification. Above all, it is not a matter of softening the abstractness and the generalities of Pareto's model, relapsing into empiricism and, when the grounds are still insecure, engaging in "particular researches" or "numerically" solving "detailed concrete problems". This is the de Finetti stance against the tide, the most original critique, "with an opinion opposed to everybody's, upholders and critics". In this sense, the contrast with Amoroso could not be more obvious. Pareto has to be reconsidered in a diametrically opposed sense, depolluting him "of all liberalist remnants": "not to renounce, but to sharpen even more the subtle mathematical acuteness that neatly distin-

[104] "Sui campi di ofelimità", *Riv. It. di Scienze Economiche*, 1935, pp. 5–14.

guishes Pareto from other economists, to loosen instead of tightening the contacts with the historic reality, to make more rigorous and neat, and not to fill up, the distinction between science and the assessment of the end for which it can be utilized"[105]. Pareto's mathematical formulation is to be reinforced with an even neater division between theory and economic facts, that infiltrate the construction and lead to the vicious circle of assuming from the beginning what will be later proved. The theory thus strengthened will observe the passage "from plural to general", from a "naturalistic" point of view (strongly depending on the cognitive aim, that limits the field of the possible hypothesis to those realized until then) to the mathematical one, which, instead, worries about drawing the logic consequences from every set of initial axioms. In short, to arrive at more real conclusions one must not "leave the too abstract character of Pareto's conception so as to observe reality closer, but on the contrary he has to make abstractness more perfectly coherent"[106].

After these premises, through which it is also possible to perceive the awareness and the confidence in the capacities of the axiomatic method, no wonder if the critiques of Pareto's theory are not relegated to the initial ones regarding the orthogonality of the preference lines and the definition of independent goods. The radicalization pre-announced by de Finetti – just think of the mediation line pursued instead by Amoroso! – intends to hit the heart of the model, the characterization of the optimum point. Thus, considerations that anticipate a debate which in short would begin within the welfare economy emerge, starting from the questioning of the uniquenes of the optimum point. The considerations of Paretian optimality sensationally anticipate[107] conclusions usually referring to the second postwar period and formal developments that the welfare economics will experience only through the works by O. Lange, O. Morgenstern, J. von Neumann, T. C. Koopmans.

The "politicization" of research develops in de Finetti in quite a different way from what we have seen in action with Gini and Amoroso. His approach suffers manifestly from the cultural inputs of the time, but lacks any opportunism and any subservience to the regime's stances, to the point that the critiques of the economic system (and of the theories that support it) lead de Finetti towards a direction diametrically opposed to that of an improvident and mannerist utilitarianism.

The choice of utilitarianism and of science's practical value is one that fascism exhibits with more constancy in the first years of its settlement. There is an intensification of forms of nationalism and of technical-scientific claim – we mentioned it, talking of historical researches – of the results obtained by the "italic genius" – emphasizing the Italian scientific superiority – but here the regime "rides" feelings and cultures diffused

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105 "Vilfredo Pareto di fronte ai suoi critici odierni", *Nuovi Studi di Diritto, Econ. e Politica*, 1935, pp. 3–22.

106 *Ibidem*.

107 In "Problemi di "optimum", *Giornale Ist. It. Attuari*, 1937, pp. 3–22, e "Problemi di "optimum" vincolato", *Giornale Ist. It. Attuari*, 1937, pp. 23–37 we find a substantial anticipation of what are usually called *Kuhn-Tucker* conditions for a *multiobjective problem* and that H. W. Kuhn and A. W. Tucker introduce in their 1951 classic work.

in many European countries. There is the direct intervention on statistics and on demography, but it is an isolated case. General enough, instead, is the conviction that politics, through the new regime, can and must have its say on every argument, including that of the organization and contents of scientific research. Public interventionism is not certainly an exclusively Italian case, but fascism however makes a remarkable contribution to it. Mussolini passes a negative judgement on the level of our studies (!) – "scientific research in Italy goes through a period of stasis[108]" – and promises its highest attention to the problem[109]: "fascist government confirms its will to put the problem of science and of scientific research in the foreground of national problems". This ambition and this presumption find a "side" in the emerging generations of researchers, attracted by the advantages they will take of quicker changes, and appear through the gathering and promoting of a certain number of ideas for the foundation of new scientific entities and institutions. When later the attention on the research world generates interest in its contents, there emerges, precisely, the subject of *utility*. Science must not be abstractly useless, but, on the contrary, it must serve the country: "a country does not spend in vain in the research". And still more explicitly[110]:

> bisogna (…) che mi dica se ci sono dei gas ultravenefici, e soprattutto bisogna che mi dica che cosa si deve fare per combattere altri gas. Avete visto quale sviluppo ha avuto la chimica nell'ultima guerra. Come ministro dell'Aviazione, la scienza mi pone di fronte a molti problemi, che sono legati per leggi non tanto misteriose ai fenomeni fondamentali della vita fisica. Ho bisogno che la medicina e la chirurgia mettano a partito tutta quella che è stata la medicina e la chirurgia di guerra[111].

[108] Cf. R. Simili, G. Paoloni (ed.), *Per una storia del Consiglio Nazionale delle Ricerche*, quoted, I, p. 134.

[109] Taken from the speech given in Rome, on the 2nd February 1929, in occasion of the settlement of the "new" C.N.R. (in R. Simili, G. Paoloni, quoted, p. 138).

[110] *Ibidem*, p. 136.

[111] It is necessary (…) that you tell me if there are ultra poisonous gases, and above all it is necessary that you tell me what must be done to fight other gases. You have seen the development that chemistry has had in the last war. As Minister of Aviation, science brings me face-to-face with many problems, that are bound by laws not too mysterious to the basic phenomena of physic life. I need that medicine and surgery invest on what has been the war medicine and surgery.

Chapter 6

The CNR alternative

1. End of decade balance

Although our attention has until now been confined to the first decade after the war, we have sometimes extended our reach into the 1930s to discuss certain topics. In particular, Severi's growing achievements in geometrical research spanned this longer period of time.

The growth and development of the Italian school of Algebraic geometry is exemplary; we will find in particular many of its characteristics when talking about analysis. The profundity of the results and of the system built before the war, and Enriques' and Severi's personal qualities as researchers, provided a base that allowed Italian geometry to retain a primary role. But Castelnuovo's worries and distrust of new techniques are as significant as the need for a meditative pause that Severi perceived (so as to give sounder foundations to results that had already been obtained and enable further steps forward) and the *reservations* appearing now in a researcher such as O. Zariski, who had, perhaps with some trepidation, come to Italy to study. Italian Algebraic geometry does not manage to appropriate the new algebraic and topological methods. It is not only a matter of knowing them and talking about them, but of being able to assimilate them into the centre of own geometrical structure. A generation of researchers – educated with some basic building blocks – can not be asked to support a new research program straightaway. From this point of view, Severi's personal updating work is praiseworthy; but his effort to spread the "new" Mathematics within the community of youngest researchers was small. And the Italian scientific framework, still limited needs a sufficient time for another generation of researchers, independent from the already well-known masters, to emerge.

The big Italian schools of the end of the century – Algebraic geometry, Real analysis, Mathematical physics – begin to have trouble in maintaining their positions, which they had held before the war, in the international sphere. Their production is always influential, but in some cases the research forefront and its propulsive centre are elsewhere. The dynamics of small numbers – which favours the first "outburst", by the absence of obstacles placed by a strong and well-entrenched tradition, but that inevitably slows down the generational exchange – affects also Mathematical physics, whose development in the end depends too much on Volterra and Levi-Civita's personal participation. We have already talked about the first, about his trajectory and his new interests. The second – as we will also see in this chapter's last section – proves to be an absolutely worthy researcher, always generous in helping and giving advice to his younger colleagues but not so interested – by nature and personal choices – in the development of a true school. Instead, algebra and probability studies (besides those of dynamics of populations, in any case closely related to Volterra) take off well – the first outburst we talked about. The first ones return too soon to the ranks, penalised by a school of geometry that does not accept – we repeat it – the innovative charge that maybe would have brought an autonomous development. The second, instead, take immediately hold, favoured by the far-sightedness and "openness" of Castelnuovo and Cantelli; de Finetti moves along different directions, but he is equally aided in finding a position.

In short, while Italy's international prestige reaches its peak with the organization of the Congress of Bologna, we perceive the first rifts in the holding the third place in the international *ranking* that had been unanimously given to Italy at the beginning of the nineteenth century. No crisis; no stasis. Only one more than natural difficulty to repeat a rousing season.

The balance of Twenties, thus is unavoidably transformed in a first moment of comparison with the previous period, at the turn of the century. From a closely scientific perspective, we can only talk of some symptoms predicting a decline. The diffeence between the time periods, however, appears more significant when considering the cultural environment about the mathematical enterprise. In the *Prologue* we referred to the, not always successful, attempts of the mathematicians to impose their culture outside the mathematical community as *external projections*. Now such externalizations are no more. Mathematicians are on the defensive, no longer exporting to other spheres their own *rationale*, their own central ideas or interests. It was all they can do to hold onto their own defined spaces in academic society. The episode of school reform, the *Gentile reform*, is symptomatic. Stances that recognize their authoritativeness within the specific disciplinary field and promise further developments, in return for non-interference take the place of the disinterest of the political class of the beginning of the century. But the same that disinterest had left certain spheres free and allowed them to be filled with the mathematicians' externalization.

The 1920s see also a change of *leadership*. Within the mathematical world. Volterra, too stubbornly linked to the old liberal environment, is ditched; the fascists choose Severi to replace him. We must not think as a clear forward-looking strategy but rather as a confluence of events. We have seen how Severi's friendship with Gentile plays an important part in his appointment as the only mathematician of the *Accademia d'Italia*. It could have gone otherwise, even shortly before the appointment. In the end, it is Gentile who decides. His role is really peculiar: far from – if not opposed to – any scientific and mathematical sensitivity, he appears in a number of different venues in Italian mathematical history several times during this period. We will encounter him when talking about Tonelli and the *Normale*. The pairing of Gentile and Severi will occur again with

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CRONACA DI BOLOGNA

DOPO IL CONGRESSO

I Matematici di tutto il mondo

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I maestri bresciani a Bologna

La festa di Settembre a Minerbio

Chiusura della Colonia estiva della Croce Rossa

Vidimazione delle licenze per il commercio degli esplosivi

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Bancarottiere arrestato

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LA DIREZIONE

Leonida Tonelli

the respect to the matter of the oath (with which we will deal in the fifth section of this chapter).

With Gentile's decisive aid and the attempts to gain credit directly by Mussolini himself, Severi has attained the *leadership* of the Italian mathematical world. Now it has to be defended, not only from his "old" master and friend, Enriques, but also from a new generation that stands out with the Fascism. We will see it in the section 4 of this chapter – dedicated to the *dualism* U.M.I.–C.N.R. But let us take one thing at a time and begin with Analysis studies, to confirm some conclusions we drew when speaking of Algebraic geometry and to introduce Tonelli in more detail.

2. Analysis

In what occasions have we met analysts until now? In the *Prologue*, because the school of Real analysis had represented an extremely qualified presence in the decades following Unification. Later, with Volterra (in the third chapter), we found again what in the meanwhile had become the *old guard*. And, in the same chapter, we talked about Tonelli's failed *call* to Rome.

Once the *old guard* declined, and at the cost of some unavoidable simplification, in the years between the two world wars Italian analysis revolves around two names and two places: Tonelli in Pisa and Picone in Rome – coeval but academically younger – about whom we have talked until now only regarding his corrections to the ballistics tables used by the artillery during the first World War. Gradually, *up-and-coming* young men will join them, even if in the case of Renato Caccioppoli (1904–1959) – Picone's pupil – it is quite different: he will be immediately an important presence for the revival of Italian analysis.

Tonelli is not a new name (besides the episode of the *call* to Rome). He had already drawn international attention at the beginning of the century, with some important notes (cited in the *Prologue*) on the length of continuous rectifiable curves, on the existence and the calculus of a multiple integral and on the calculus of variations. After the war, his main research fields are three: Real analysis, calculus of variations, Trigonometrical series.

As concerns Real analysis, the first remarkable note of the time is "Sulla ricerca delle funzioni primitive"[1]. It deals with Denjoy's integral and with its contribution given to the so-called *second fundamental theorem* of integral calculus.

"Sulla nozione di integrale"[2] is "an attempt to make the theory of Lebesgue's integral easier, I will say more elementary, and thus more easily acceptable by everybody", freed of a preliminary theory of measure ("founded on extremely delicate reasoning") and of the recourse to Zermelo's postulate, and based instead on the basic concept of integral of functions continuous on an interval[3]. With these features, the integral "presents itself as a natural extension of the integral given by Cauchy". Tonelli's article is followed by the almost simultaneous publication of an article by Beppo Levi (with Vitali's following speech) that proposes a new definition of integral (basically approaching a limited function through a sequence of step functions), so as to recover a more intuitive approach and avoid the burdensome theory of measure. Then, Tonelli's approach does not represent an isolated prospect, but evokes a tradition that goes back to Borel and that sees, among others, the contributions of F. Riesz, P. J. Daniell, O. Perron, W. H. Young: the aim is to free the definition of integral from a preliminary theory of measure, identifying the initial class of continuous functions – their role could be similarly performed by the step functions or by the Riemann-integrable ones – and then moving on to a larger class through an extension theorem. The three notes published in 1926 with the title: "Sulla quadratura della superficie", that solve the problem of the area of surfaces written in ordinary form are Tonelli's most "determined" study within the field of Real analysis. Tonelli takes his cue from Lebesgue's definition of surface area and develops a theory generalizing that of rectifiable curves, diverging from it as little as possible. Thus, he proves that the area of a surface, considered in the ordinary form $z = f(x, y)$, with f defined with continuity in the square Q of opposite vertexes (0, 0) and (1, 1), is finite if and only if f is a bounded variation function in Q. The desired generalization is hence obtained through an appropriate generalization of the concept introduced by C. Jordan.

1 *Rend. Acc. Lincei*, 1920, pp. 186–191.
2 *Annali Mat. Pura e Appl.*, 1923–24, pp. 105–145.
3 This concept had already been introduced in the *Fondamenti di Calcolo delle Variazioni* always on the basis of the denial of Zermelo's postulate. Also this time the introduction of Lebesgue's integral is explained through the recognition that "even if by now" such subjects are "essentially important in the field of Analysis, they still have not reached an adequate divulgation in Italy".

For Tonelli, *f(x, y)* is a bounded variation function in *Q* when it is so almost everywhere in [0, 1], considered as a function of *x* alone and of *y* alone, and when the integrals

$$\int_0^1 V_y(x)\, dx \quad \text{e} \quad \int_0^1 V_y(y)\, dy$$

exist and are finite, in which $V_y(x)$ and $V_x(y)$ indicate, precisely, the total variations with regard to the single variables[4]. If *f* is a bounded variation function, then its surface admits a finite area *S* and the following inequality is valid:

$$\iint_Q \sqrt{1+f_x^2+f_y^2}\, dxdy \leq S\,.$$

In particular, the equality holds if and only if *f* is absolutely continuous, i.e. it is continuous, and it is almost everywhere absolutely continuous in [0, 1] regarding the two variables separately considered, with V_x and V_y integrable in [0, 1].

The references to absolutely continuous functions and to bounded variation functions take us directly to the subject of the calculus of variations. The field, still in the last decades of the nineteenth century, had experienced undeniable developments just by centring on the analogy with the ordinary optimization of real functions of real variables. As regards the simplest problem of the calculus of variations, Euler's equation, putting the first variation of the objective functional equal to zero, carries out the same role as Fermat's classic necessary condition. The difficulties in completing the analogy with the ordinary calculus, and in arriving at a theory just as satisfactory, require a critical revision of the classic conditions expressed through the first or second variation, to a more rigorous proof of their necessity or sufficiency and to a more attentive differentiation between *strong* and *weak* extrema. The *direct methods* and the conviction that the difficulties of the calculus of variations force one to take other pathways within the new functional calculus arise here.

All the problematical issues that the classic approach does not manage to overcome will be thus avoided: reduction of calculus of variations to problems of differential equations (and not the other way round) with the consequent calculus difficulty and even before with the existence theorems of the solution of a boundary problem, the limitation given to a functional class by consideration of differential equations, the privilege given to relative extrema, and the search of appropriate sufficient conditions.

The process followed by Tonelli is based on the idea of *semi-continuity* (and on conditions of *compactness*). The functional J [y] $= \int_a^b f(x, y, y')\, dx$, if it poses serious problems from the perspective of continuity, is instead semi-continuous in the uniform topology (under not particularly restrictive conditions of regularity). And this is enough, always under certain assumptions, to assure the existence of a curve $y = y(x)$ that makes it,

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[4] Among others, monotonic functions (concerning single variables) and functions with limited first partial derivative turn out to be bounded variation functions (according to Tonelli). Also bounded variation functions in Arzelà and Hardy's sense satisfy Tonelli's definition, which, instead, does not appear to be comparable with the one proposed, at the beginning of the century, by Vitali.

for example, minimum. Actually, once verified that within the considered functional class is inf J = $\overline{J} > -\infty$, we can construct a minimizing sequence of curves $y = y_n(x)$ such that J [y_n] tends to $\overline{J}$ and from this, through compactness theorems, we can extract a sequence {y_{nk}} convergent to a function $\overline{y}$ for which it results $\overline{J}$ = J [$\overline{y}$]. The *Fondamenti di Calcolo delle variazioni* (published in two volumes in 1921 and 1923) are for their organicity and thoroughness, the embodiment of greatest visibility of the brilliant results which Tonelli achieves by using the direct methods[5]. In the history of the calculus of variations, his name is linked specially to existence theorems. His researches are based on the study of the sign of the integrand function f and of that of its *Weierstrass invariant*:

$$f_1 = \frac{f_{x'x'}}{y'^2} = -\frac{f_{x'y'}}{x'y'} = \frac{f_{y'y'}}{x'^2}$$

(for the parametric case). The separate treatment of the two hypotheses enables us to specify their role. The problem's *regularity*, that is the hypothesis that f_1 has always the same sign, reveals itself as a strong form of convexity of the integrand function, i.e., a strong form of Legendre's and Weierstrass' conditions and assures the functional's semi-continuity (always in the uniform topology). The requirement that f has to be positive allows us to prove the convergence of a minimizing sequence.

In the *Fondamenti,* Tonelli consolidates all he had published during the previous decade. The maturity and the method of the research project are evident. It is no wonder if the following years see a less stress just while the appreciation for Tonelli's contributions begins to spread in other important studies (H. Hahn, C. Carathéodory, G. D. Birkhoff, M. Nagumo, L. M. Graves, E. J. McShane[6] ecc.). We could mention Tonelli's other works but the general tone indicates for the calculus of variations a less propulsive stage after the 1920–23 "high point", mainly distinguished by brilliant observations and elaborations of studies that had appeared in the meanwhile, inspired by publication of the *Fondamenti*. While researches on Real analysis do not undergo significant variations of intensity, during the 1920s the most lively attention shifts progressively towards the study of trigonometrical series.

In 1928 the monograph *Serie trigonometriche* is published. Pointing out that not all trigonometrical series are Fourier series, the volume opens with the study of trigonometrical series, to move later to Fourier series, in one and two variables. Also this essay gathers and organizes all previous results (to which Tonelli will substantially add, in the following years, no new contribution), starting with those concerning the simple and

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5 We find a more synthetic exposition in the fifth chapter of Volterra's and Pérès' *Théorie générale des fonctionelles* (1936). In the preface, Volterra writes: "J'ai l'agréable devoir de remercier ici, en mon nom et de la part de M. Pérès, M. L. Tonelli qui nous a accordé son précieux concours en rédigeant la seconde partie du Chapitre V où est exposée sa méthode pour l'étude de l'extrémum d'une intégrale simple".

6 In his dissertation ("Semicontinuity in the Calculus of Variations and Absolute Minima for isoperimetric problems") Mcshane writes that Tonelli in the *Fondamenti* "has shown the considerable advantages in generality possessed by the method which he designates as the direct method in the Calculus of Variations".

absolute convergence of Fourier series, enriched by a thorough bibliography, including even the latest titles. However, it is regarding double series that Tonelli leaves his most original mark, extending *Parseval's formula* and *Bessel's inequality* to two variable functions in conditions of extreme generality. In "Sulla convergenza delle serie doppie di Fourier"[7], he had already tackled the convergence problem, for which other authors (among them G. Ascoli, E. Picard, G. H. Hardy, W. H. Young, E. W. Hobson) had not managed to avoid the request of restrictive hypotheses.

Gentile's call to move from Bologna to Pisa arrives in 1930 and Tonelli's affairs cross, at this point, with more general events that we will examine in the next paragraph.

The moment has come to return to Mauro Picone, around whom the *pole* of Rome will develop. Compared with Tonelli, there could not be a more different mathematician (and analyst). Tonelli is the last great exponent of the golden age of Italian analysis, astride the last two centuries. He is the last illustrious representative of that Italian school of Real analysis that strongly believes in its own rich peculiarities and has no time and no eyes but for its own territories. The missing entry in the field of functional analysis – due not only to generational factors – and a split, in substance quite rigid, regarding any applicative speech, aid to this characterization. Picone shows quite another sensibility with regard to the new generality made possible by the developments of functional analysis. His speeches on this matter are innumerable. The fact that the realignment of Italian analysis with regard to the new studies arises from his school and from the atmosphere he had managed to create within it, counters the irony or malice about his statements of principle for a "more and more general mathematics". Moreover, Picone's interest in applications certainly overcomes the simple methodological statements.

His most important writings concern differential equations and in particular partial differential equations. The interest that had arisen during the war in numerical calculus problems influences the "pure" researches he starts soon after graduation. It does not matter if the term *numerical analysis* can not be used yet; it does not matter if it is not until 1932 that Picone runs the first Italian official course on this subject (actually an optional teaching, called *Numerical and graphical calculus*, at the *School* of "Statistic and actuarial sciences" in Rome). The research field is fairly well outlined and is quite originally placed *astride* classic analysis and numerical calculus.

In "Nuovo metodo d'approssimazione per la soluzione del problema di Dirichlet"[8], Picone presents some results concerning the problem of approximate integration of totally elliptical partial linear equations of the second order from which "follows a new method of approximate calculus for the solution of Dirichlet's problem, a method that, to my mind, *can be really proposed to the physicist*".

In 1927, after the approximate calculus of solutions, appears another subject that will feature Picone's work: the a priori estimates of the solutions of both ordinary and partial differential equations. But we will talk about this later, because 1927 is first of all the foundation year of the *Istituto di Calcolo* in Naples, that will lead to what can be considered as the main contribution of Picone's scientific life. It is specially his sensibility as

[7] *Ann. Mat. Pura e Appl.*, 1926–27, p. 29–72.
[8] *Rend. Acc. Lincei*, 1922, p. 357–359.

Mauro Picone

Master that prevails and excites admiration still today, together with the creativity of someone who invents an organizational model and discloses, towards it, pioneer manager qualities.

The *Istituto Nazionale per le applicazioni del Calcolo* (INAC) arises thanks to L. Amoroso's interest and to financing from the Banco di Napoli. Picone recalls an "embryonic Institute of Calculus, provided with calculation machines with a moderate power, enough though to begin the experiment". Even if the means of automatic calculation are scarce, a real prototype (and not only for Italy) of the future institutes of numerical analysis and centres of calculus is established. "The most unexpected success crowned it" – to quote again Picone – so much that the INAC follows Picone to Rome, turning from *Istituto di Calcolo* within the chair of infinitesimal analysis at the University of Naples to one of the *Istitut* of the C.N.R., reorganized and presided over (from 1929) by Guglielmo Marconi. The purpose is a synergy with the experimental and applicative disciplines for the "study of mathematical questions of their interest, both with the aim to eventually obtain an initial precise formulation of the questions themselves, and as regards the required numerical valuations, with the necessary approximation"[9]. In

[9] Cf. *La Ricerca scientifica* (già *Bollettino d'Informazioni* del C.N.R.), art. III (1932), 6, p. 644.

the meantime, the I.N.A.C. becomes an auditing organ and takes the control of "already carried out" calculations "concerning projects of civil, mechanical, electro-technical, constructions, etc., with the aim to ensure the exact application of the adopted theoretical formulas"[10]. The organizational structure includes, besides the Director, a vice-director, some assistants and ordinary consultants and a dozen among graduated computists and designers. Success goes on. If during the Neapolitan period the Institute was engaged in "nuclear physics researches along the purely theoretical path followed by Enrico Fermi", now – while the relationship with the Nobel prize-to-be intensifies – the consultancies with the Ministries of the Air, of the Army and of the Navy, ruled by a special convention, and the cooperation with civil construction industries and those producers of electricity (for the projects of big mountain water barrages) are developed too. Around fifty consultancies are operated each year. Reference is made to them, and more in general to the research activity gravitating around the Institute, in nearly 250 publications of the period 1927–1940 that cover an extremely vast front, going from the traditionally most internal questions to analysis to boundary questions between analysis and numerical calculus, to clearly applicative issues on rational mechanics, the theory of structures, the elasticity theory, hydrodynamics, aerodynamics, etc.

We have already mentioned the not so trivial innovations expressed by the INAC's presence in the Italian mathematical panorama. It is the first time that research is organized outside the narrow academic circuit; it is the first time that young people are directed towards a channel that adds a remarkable number of jobs (always in relation to the pre-existent situation); it is the first time that Mathematics becomes subject and object of consultancy, opening itself to new professional relationships and giving rise, almost unavoidably, to a research that it is no longer only individual but involves several figures, from the Director to the recent graduates (arriving, in some measure, at a consistent number of final-year students). This is the INAC's force: a structure linked to the university by several threads but that, with regard to it, can count on a great freedom to move, and on the irreplaceable resource of the enthusiasm of the Director-founder and of a small group of his collaborators.

The changes of which Picone is author are not confined to the structural-organizational aspect, but involve the research contents and the meaning itself of the terms used when *facing and solving a mathematical problem*. It is a new numerical way of thinking that comes to the scene of Italian mathematics. It is not enough to prove a theorem of existence, and eventually of uniqueness, but – in as much an essential way – constructive procedures for the calculus of solutions must be elaborated; in other words, the same attention and the same rigour for the determination of the numerical algorithm, the proof of its convergence and the estimate of the error of approximation is needed. We have recalled the first works on approximated calculus of solutions of a differential equation. Picone of course is not the first one (not even in Italy) to face similar problems, always considered by engineers, physicists, etc.; it is the first time, though, that mathematicians explicitly take this situation on themselves and that theoretical competences and numerical-applicative sensibilities are integrated at a high-level in the same Institute. Lastly,

[10] *Ibidem*.

the age-old debate – already in the 1930s! – on the relationships, the boundaries and the antagonisms between pure and applied mathematics has the possibility not to be based only on considerations of principle but to develop in an effective and significant work experience.

Picone gives a lot to the INAC, in terms of time and attention. The design, construction and development of such an important and valued research and consultancy institute is not simple, not even for one who can avail himself of the imaginable academic and politic supports, essential in this sort of enterprise. The choice is clear. The price that Picone pays in terms of research, of its originality and elaboration, is clear. Without this unavoidably presuming a minor number of works or a reassessment of scientific relationships or a less frequent participation in conventions and conferences. On the contrary, the years from 1927 onward are among the most active in Picone's life. A remarkable commitment and the enthusiasm for the achievements gradually obtained are added to the effects of the transfer to Rome. Picone's eighty publications during this period document, in any case, his presence.

Compared with Tonelli's sober and dry style, Picone "preaches" a lot, also because he feels the need and the exigency to "publicize" a new way of facing mathematical problems. His advice to the "analysts who want to do really useful studies for applications to experimental sciences" is almost wasted. He critiques the introduction of hypotheses that are not "physically acceptable a priori", or rather, that have a "purely analytical meaning" and appear "absolutely not relevant from the physical perspective". He explains any formalization that may seem excessive and, already for Stieltjes integral, he feels almost forced to specify that its use is not due to a "vain desire of analytic luxury" but "it meets a verified need in the treatment of many questions of mathematical Physics". As years go by, precise references to concrete problems put to the INAC by Physics, Engineering or Calculus of probability are added to the methodological requests and statements. With satisfaction, Picone realizes how the abandonment of a "purely speculative" view leads all the same to significant progres for the theory itself: the tasks assigned to the *Istituto* have begun new studies because here often "it is not possible to refer to the classic conditions of existence and singularity of the solution, so new purely existential researches able to support the reasoning and to eventually correct the physicists insight in the construction of the mathematical scheme of the studied phenomenon are needed"[11]. Again:

> l'introduzione di tale trasformata di Laplace mi si è presentata nelle ricerche dell'Istituto Nazionale per le Applicazioni del Calcolo e ciò constato, mi sia consentito il dirlo, col più alto mio compiacimento, potendo così ancora una volta mostrare, nel mondo della Scienza pura, di quali cospicui apporti alla Matematica può essere

[11] The quotation is taken from an ensuing note: "Nuovi indirizzi di ricerca nella teoria e nel calcolo delle soluzioni di talune equazioni lineari alle derivate parziali della Fisica-matematica" (Ann. Sc. Norm. Pisa, V (1936), p. 213–287). The memoir deals in particular with n-hyperharmonic functions, that verify the equation $\Delta^n u = 0$ with $n > 1$. Also the next quotation is taken from a note of the *Academia dei Lincei* of the second half of the Thirties: "Nuove determinazioni per gli integrali delle equazioni lineari a derivate parziali".

> capace il suo incondizionato cimento nei problemi dell'alta Tecnica, diuturnamente in opera presso l'Istituto che ho l'onore e la fortuna di dirigere[12].

The vast memoir – "Sulle autosoluzioni e sulle formule di maggiorazione per gli integrali delle equazioni differenziali lineari ordinarie autoaggiunte"[13] – that begins the study of a priori estimates still concerns ordinary equations. The research context is by now settled: for Picone, all problems put by Mathematical physics lead to a system of functional equations and to the relative problems of *existence*, of *singularity* and of an *approximate calculus* which should be "practically feasible" and allow to know an upper bound of the error. The first problem is obviously fundamental, as a missing reply in terms of existence would render the physicist "already informed that the fundamental hypotheses and the scheme that have taken him to the equations are not compatible as a whole" and the research of calculus procedures, even if these are practically supportable, would lose motivations. But, once the question of the existence (and of the uniqueness) is solved, another question arises in all its centrality: that of approximate calculus, which is of "interest, as someone thinks," not only "for the engineer who has to build, but it is also of vital importance to *natural philosophy* itself, inasmuch as if the calculus of the problem's solution would take to a result of a size order that experience does not confirm, the physicist would be advised again of the fact that the fundamental hypotheses and the scheme and, for instance (…), replacing the increases of functions with their differentials, are not compatible as a whole". The memoir to which we refer gives several approximation formulas for the boundary value problem:

$$\frac{d}{dx}\left(\theta\,\frac{dy}{dx}\right)+A(x)y=f(x)\,,\qquad y(a)=\alpha,\;\; y(b)=\beta\,,$$

only under the hypothesis that unity does not fall under the eigenvalues of the homogeneous associated problem, using minimum properties of auto-solutions. The utility of such formulas – and of their precision – for the unknown function and its derivatives is evident if we think that, besides some quantitative information about the integral, they allow to majorize the remainder of the eventual Fourier series that approximate it and hence to majorize the error. Later[14], Picone considers the boundary value problem:

$$\Delta_4 u+\lambda u=f\,,\qquad u=\frac{du}{dn}=0$$

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[12] The introduction of such a Laplace transformed appeared to me in the researches of the *Istituto Nazionale per le Applicazioni del Calcolo*; allow me to say that I notice it with my greatest satisfaction, being able to prove once again, in the world of pure Science, what substantial contributions to Mathematics can give its unconditioned trial in the problems of high Technique, long at work at the Institute I have the honour and the luck of directing.

[13] *Math. Zeitschr.*, p. 519–555.

[14] "Particolare forma di maggiorazione per le soluzioni di una classica equazione del quart'ordine della fisica-matematica", *Rend. Acc. Lincei*, 1929, pp. 16–20.

for which he manages to give only Hilbertian approximation formulas, that is concerning $\int_T u^2 dT$, however sufficient to approximate the rest of the "series" constructed in Fourier's sense.

Picone comes back to the subject of approximate calculus of the solutions, and to the comparison with Ritz's method, with another memoir from 1928[15], that enunciates the method of the minimum weight power functions as a "spontaneous and, maybe, not infecund" generalization of the classic procedure of minimum squares. The comparison expresses an unequivocal judgement. Where the method of the minimum power functions, revitalized by Lebesgue and properly generalized, approximates "the solution with an error whose infinitesimal order is as large as one wishes, this cannot be said for Ritz's method", moreover much more limited in its applicability field. Not even the last perfections produced by R. Courant justify a role that has to be unavoidably reviewed: "therefore we have to persist in giving, for the approximate calculus of the solutions of differential equations, such an eminent place to Ritz's method, forgetting the old method of the minimum power functions? I dare not to believe it. I hope that this work can help to convince that the method of the minimum power functions has an infinitely bigger range".

With Tonelli and Picone we have several times come close to the subject of functional analysis. When this subject emerged, Italian mathematics had offered major contributers, with Pincherle and especially with Volterra. Then silence, just while functional analysis lives through a decisive phase for its development. The one that we know today as *classic* functional analysis realizes itself just during the 1920s. Banach's dissertation is from 1920; published later in the *Fundamenta Mathematica* of 1922, it gives the definitive axiomatics of normed spaces with the explicit request of completeness, adding itself to the previous contributions, equally significant, of H. Hahn, E. Helly, I. Steinhaus, N. Wiener, etc. Then Banach proves the so-called theorems of Hahn-Banach, of Banach-Steinhaus, of the closed graph, etc., up to the study (1929) of dual spaces and to the methodical exposition of the *Théorie des opérations linéaires* (1932). J. Von Neumann's articles including the axiomatization of Hilbert spaces and the study, within such spaces, of linear operators and their eigenvalues, appear in 1929. In the meanwhile, other contributions extend the classic results of real analysis to functional spaces (for example, the implicit functions theorem), without the characteristics of finite dimension spaces being experienced as a "strait-jacket", that is, without the *passage from discrete to continuous* being binding and restraining the discovery of new and original situations. It is not surprising that in Bologna, on occasion of the International Conference of mathematicians, M. Fréchet gives such a graphic account of the situation: "si, en Italie, l'Analyse générale proprement dite n'a pas encore trouvé d'adeptes, n'oublions pas que cette science nouvelle est née de l'Analyse fonctionnelle, merveilleuse création du génie italien".

Of course, it all has a reason. We have talked about Picone and Tonelli and their choices. Above all, we must add that, from Fréchet's *Dissertation* in 1906, functional analysis develops along lines which differ from those suggested in the same years by

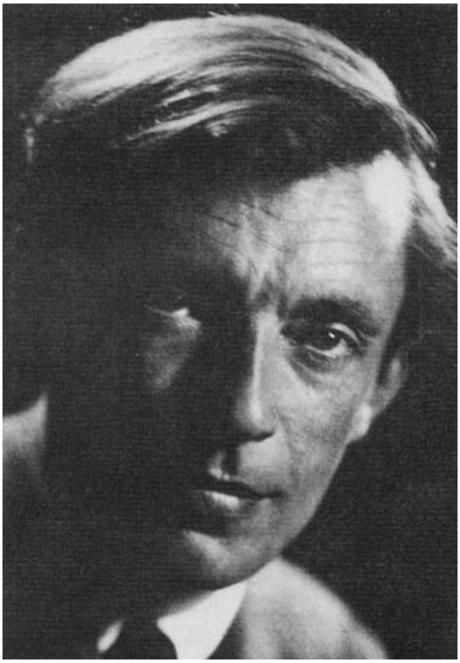
[15] "Sul metodo delle minime potenze ponderate e sul metodo di Ritz per il calcolo approssimato nei problemi della fisica-matematica", *Rend. Circolo Mat. Palermo*, 1928, pp. 225–253.

Volterra and Pincherle's writings. The divergence in the definition of functional derivative is only an indication of a different thinking. There are those who continue to see functional, or *general*, analysis as an instrument – and then the applications will dictate elaborations and their boundaries – and the ones that see the new discipline as independent, as every theory that finds in itself the reasons for further generalizations. No split or repudiation, only a slow and progressive divergence. Tonelli uses thus the direct methods, which reveal certainly a functional approach to the calculus of variations, but he will pay for the inability to situate them in quite larger and stronger frames with slow and limited results when he will try to deal with the bi-dimensional case. A new generation of analysts (C. B. Morrey, S. L. Sobolev, J. W. Calkin, G. Stampacchia, etc.) will be necessary so as to remove the univocal reference of the uniform topology from the direct methods and to ensure, in the Sixties, satisfying conclusions. Picone is more attentive to the new language and he certainly resorts to this generality, but the greatest fruits are reached through his school. Renato Caccioppoli (1904–1959), Picone's pupil in Naples, will be the one to put Italian analysis in touch with the most advanced research vanguard within the field of functional analysis again[16].

Caccioppoli takes up a university chair at a very early age: free lecturer in 1928, he wins the competitive exam in Padua in 1930, moving definitively to Naples three years later. Caccioppoli is certainly the most well-known "product" of Picone's school and owes his popularity to that mixture of genius and intemperance that has made of him the protagonist of books, interviews recalling him and Naples astride the war. Thus the legend of *we dressed after Caccioppoli* is passed down, the legend of the shabby white, dirty *trench*, more and more carelessly carried about along the Neapolitan streets, of the gifted mathematician lost in alcohol, of the educated and cultured intellectual, intransigent and ruthless rival of ignorance and of banalities, that confides his long nights to not always trustworthy companies; of the enlightened, ever since "communist" bourgeois left by the wife who prefers the important party leader.

Caccioppoli is an educated person, a cultured scholar: a great expert and admirer of modern French literature; an enthusiastic and competent expert of cinema; a proficient and skilful pianist, to such an extent that for his whole life he will remain in doubt whether his real talent was music. To the literary, musical, film culture, Caccioppoli adds a strong interest for politics. He is an instinctive and reckless antifascist, equally motivated by aversion towards an antidemocratic regime and by deep irritation towards its grossness. Some events are quite famous. The regime warmly advises against walking the dog. It is not manly enough. Then Caccioppoli gets himself noticed strolling along Caracciolo street with a "manly" cock on the leash. The episode of Autumn 1938 will be much more severe. Then it will be recalled by dating it back to May while Hitler was

[16] Caccioppoli is the son of Giuseppe Caccioppoli and Sofia Bakunin, daughter of the Russian anarchic Michele Bakunin. The father, a doctor, does everything he can so that his children do not follow his footsteps venturing in a profession he thinks as emotionally too difficult and little "dependable", as it has not been able to save the first wife and the first son, both of whom died of an infectious disease. Thus, to Renato at first there is no Lyceum (traditionally the school of bourgeoisie), no University, no Faculty of Medicine. Renato has to "take" privately his high-school diploma, so as to enrol in Engineering. From here he shifts to Mathematics, in which he will graduate at the end of 1925.



Renato Caccioppoli

visiting Naples to confirm the great agreement between the two countries. In a restaurant in the open, Caccioppoli convinces the orchestra to play the Marseillaise, cheering the freedom to do so and remarking on the exhibition with extremely explicit and severe judgements against the regime. A special court was provided for such offences. But the family's agency will manage to deflate the escapade, diminishing it to the gesture of a nutter that will serve his sentence first in the judicial asylum and later in a private clinic. Caccioppoli will still be active during the war, for example when in 1943 he tries to organize the rail and tram workers' strike. Later, he will support the Communist Party without ever getting a membership card, though. Caccioppoli is typical of a southern "enlightened" bourgeois which is deeply "other" from the Italian communism of the Fifties, but that believes in it as the only possibility of democracy and liberation. Caccioppoli is a fellow traveller of Neapolitan communists, constant and never "observant", trustworthy and with a suspicious mind towards the whip. He rises up against Lysenko's "Marxist biology", but gags his dissent for fear that it is mistaken for anti-worker behaviour and may damage the cause. In 1956 he will be deeply distressed because of the Soviet occupation of Hungary, but he will accept the logic of blocs and the need to defend one's own.

The initial subject of Caccioppoli's research – his first publications date from 1926–1927 – concerns the generalization of F. Riesz' theorem on the representation of linear functionals, extended to the functionals dependent on continuous functions with several variables thanks to a proposition that was already expounded in P. Lévy's *Leçons* *d'Analyse fonctionnelle*. The research is still not as wide-ranging as in later times, but it already demonstrates his familiarity with that *embedding* technique which we will see in action, as underlying theme, in many of Caccioppoli's researches.

The 1927 note "Sulla quadratura delle superficie piane e curve"[17], that marks a decisive improvement, concerns the geometric theory of measure. It runs always on the extension, beyond its primitive field of definition, of a functional which must, anyway, keep the lower semi-continuity law "imperiously suggested by geometric intuition". Caccioppoli intends to establish "the principles of a theory of measure of plane and curved surfaces, and more generally of two or more dimensional manifolds imbedded in a linear space", on the basis of an analogy with the theory of rectifiable curves that develops from the (properly generalized) concepts of bounded variation and absolutely continuous function. He will come back to the subject with other notes, up to the memoir "Trasformazioni piane, superficie quadrabili, integrali di superficie"[18] (1930), that he considers definitive "not wanting to ignore thus that in my dissertation survive more than one gap, unavoidable however in a first attempt of synthesis". Certainly one of the reasons that drive him to these elaborations is the reading of some related works[19] of Vitali and Banach. Caccioppoli will repeatedly underline how Banach's definition is, according to him, "unacceptable", as it does not verify the essential law of lower semi-continuity. But the true reply to the priority question is the thought of the *orientated* surfaces and the double specification of *extension* and *orientation*, that has to be attributed to the area element[20].

> Da due punti di vista ben distinti si può prendere a considerare il nostro problema. L'uno, apparentemente più semplice e rigoroso è stato adottato da Lebesgue: l'area di una superficie si definisce come il minimo limite delle aree delle superficie poliedriche di approssimazione. L'altro, più vago certo, ma anche più fecondo, è quello di Peano: ad ogni porzione della superficie si cerca di associare un'area piana *orientata* che rappresenti in qualche modo quello che il vettore della corda è per un arco di curva; si ottengono così per l'area della superficie valori approssimati per difetto, come per la lunghezza di una curva quelli delle poligonali iscritte, e di questi valori si cerca l'estremo superiore[21].

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17 *Rend. Acc. Lincei*, 1927, pp. 142–146.

18 *Rend. Circolo Mat. Palermo*, 1930, pp. 217–262.

19 G. Vitali, "Sulle funzioni continue", *Fund. Math.*, 1926, pp. 175–188; S. Banach, "Sur les lignes rectificables et les surfaces dont l'aire est finie", *Fund. Math.7*, 1925, pp. 225–236.

20 Already introduced by Caccioppoli in the Note "Sulle coppie di funzioni a variazione limitata", *Rend. Acc. Sc. Napoli*, 1928, pp. 83–88.

21 Our problem can be considered from two perfectly different perspectives. One, apparently simpler and more rigorous, has been taken by Lebesgue: the area of a surface is defined as the minimum limit of the areas of approximation polyhedral surfaces. The other, certainly more ambiguous but also more fruitful, is Peano's view: we try to associate to each portion of the surface a plane *orientated* area that in some way represents what the vector of the chord is for the arc of a curve; so approximate values by default are obtained for the surface area, as the values of inscribed polygonals for the length of a curve; we search the supremum of this values.

The comparison with Lebesgue's definition, that he sees as "essentially" identical, can wait[22].

> Per un'altra ragione ancora ho rinunziato ad analizzare qui i rapporti fra la mia definizione e quella di Lebesgue (…): perché ritengo che l'identità che afferma non sia casuale, come potrebbe far credere una faticosa verifica diretta, ma dipenda da circostanze generali. Per una curva non v'ha che una definizione ammissibile di lunghezza: cioè il funzionale *lunghezza* ammette, a partire dal campo delle poligonali, un *unico prolungamento per semicontinuità inferiore*. Un risultato analogo deve potersi stabilire per la superficie, previa opportuna definizione della *proprietà additiva* del funzionale area[23].

This is Caccioppoli. Original, profound, never uninteresting. Not always so clear: the results of the comparison with the descriptive style that, for instance, we can appreciate in Banach's note previously quoted, are extremely instructive. Not always precise in bibliographical quotations, not always informed about the most recent publications, even when these involve an Italian colleague. Almost annoyed by the obligation to give proofs complete with propositions that he judges evident, which it seems he never communicated, not even orally, or by the duty to accomplish a hard direct control so as to support "inferences" that could appear to somebody "maybe a bit superficial".

In 1928 a long memoir[24] opens the "front" of the theory of integration. The general one, that Caccioppoli intends to construct, presents the integral as "a prosecution of Cauchy's integral, in a wider functional field". This is actually the "deep motive" of the researches following Lebesgue's *dissertation*: trying not so much to remove a preliminary theory of measure ("today susceptible of extremely brief and clear expositions"), as to go towards a constructive definition with a natural extension of the primitive nucleus of continuous functions. The direction first pointed at by Borel and followed among others by Tonelli is the one that Caccioppoli intends to develop to avoid circumstances that he still perceives as too particular and contingent. The aim is a more abstract theory, essentially based on that of linear functionals and on the possibility of extending them in the field of discontinuous functions with Stieltjes integral, that naturally arrives to the fundamental theorem of the passage to the limit.

The subjects so far analysed refer to ideas matured in the second half of the Twenties, before the competition exam for a chair in Padua. Briefly, we could say that in Thir-

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22 "Trasformazioni piane, superficie quadrabili, integrali di superficie", *Rend. Circolo Mat. Palermo*, 1930, pp. 217–262.

23 I have renounced, for still another reason, to analyse here the similarities between my definition and Lebesgue's (…): because I think that the identity he asserts is not casual, as an exhausting direct control could let think, but it depends on general circumstances. For a curve there is only one admissible length definition of its: that is, the *length* functional admits, from the field of polygonals, one *unique extension by lower semi-continuity*. It should be possible to establish a similar result for the surface, upon appropriate definition of the *additive property* of the area functional.

24 "Sull'integrazione delle funzioni discontinue", *Rend. Circolo Mat. Palermo*, 1928, pp. 1–29.

ties the great innovation is the "discovery" of functional analysis (that also in Italy draws the attention to a certain literature and to the new languages, filling the *gap* that had arisen). Caccioppoli's first note on the subject contains that fixed point theorem, that remains probably his most famous result in this field. We are in 1930 and some important results on the subject of fixed points for a functional transformation have already been obtained, after the 1912 theorem of L. E. J. Brouwer that affirmed the existence of at least one fixed point for the continuous transformations of an closed sphere of R^n in itself. This theorem is dealt with in Banach's PhD dissertation (1920), which had proved that contractions on complete normed spaces admit at least one fixed point. In 1922 the article of G. D. Birkhoff and O. D. Kellogg, "Invariant points in function space", that generalizes Brouwer's theorem to abstract spaces, is published in the *Transactions of the American Mathematical Society*. However, their generalization is still partial and concerns continuous transformations working on particular sets, such as compact and convex subsets of $C[a, b]$ or of $L^2[a, b]$. In 1927 J. Schauder expands the theorem considering compact and convex subsets of an arbitrary Banach space equipped with a base. This last hypothesis will be then removed in a later note dated 1930. Unaware of these results, in the note: "Un teorema generale sull'esistenza di elementi uniti in una trasformazione funzionale"[25], Caccioppoli proves that a functional transformation on the space of continuous functions on an interval $[a, b]$, uniformly continuous and such that the image of the set $C[a, b]$ is compact, has at least one fixed element. This theorem expands Brouwer's theorem to Hilbert space, allowing one to glimpse new generalizations, since the terms of the previous formulation can be assigned different meanings. The set $C[a, b]$ on which the transformation works is relevant not so much because of its content but for some structural elements that associate it to other functional classes, such as $C^n[a, b]$ o $L^2(a, b)$. The remark is typical of years in which Caccioppoli reports in the first paragraphs the new general definitions concerning algebraic or metric structure but works mainly on some particular functional spaces. He advises then that the obtained results are still applicable in more general contexts, without excessive worry to define them with the maximum precision. The note finishes with the return to the method of step by step approximations (that provide both a theorem of unicity and a calculation procedure) and with the terms of a general criterion of convergence for contractions.

> Se in uno spazio funzionale metrico completo una trasformazione S converte due elementi aventi distanza d in due altri la cui distanza d' sta a d in un rapporto inferiore ad un numero fisso a < 1, S ammette un elemento unito ed uno solo, E; detto E′ un altro elemento qualunque, si ha $E = \lim_{n \to +\infty} S^n [E']$ essendo S^n la trasformazione risultante dall'applicare n volte consecutive la S.[26]

[25] *Rend. Acc. Lincei*, 1930, pp. 794–799.

[26] If in a complete metric functional space a transformation S turns two elements having a distance d into two more elements whose distance d' is to d in an inferior ratio to a fixed number a < 1, S admits one and only one fixed point, E; called E′ another element whatever, we have $E = \lim_{n\to+\infty} S^n [E']$ being S^n the transformation ensued from applying *n* consecutive times S.

Caccioppoli will commit suicide in 1959. We do not know why he shot himself. The grounds for his act is incomprehensible, the possible reasons are many and contradictory : his marital affair, the alcoholism burden, become unbearable, the fear that his mathematical creative inclination would be definitely used up, etc. Giuseppe Scorza Dragoni[27], a friend and colleague, recalls him thus.

> Un'estate ormai lontana fui per parecchie settimane ospite nella sua casa paterna, invitato dalla madre, che mi desiderava vicino al figlio, preoccupata da propositi manifestati. Fu quella la prima volta che ebbi notizia, notizia indiretta, di propositi tristi. Con me, l'accenno preciso ed inequivocabile fu formulato soltanto molti anni più tardi. Non è il caso di riferire qui le parole molto semplici che scambiammo quel giorno. Da allora cercai di incontrarlo anche più spesso. E poiché non mi riusciva di indurlo a partecipare di nuovo alle adunanze di questa nostra Accademia, in loro occasione anticipavo la mia partenza da quella Padova, dove egli mi aveva preceduto ed aveva trascorso, in quattro anni di insegnamento, uno dei periodi migliori della sua vita, e andavo a trovarlo a Napoli, nell'appartamentino che aveva preso, dopo il matrimonio, nel palazzo Cellamare, in via Chiaia, e dove ormai abitava solo. Ed anche il giovedì del 7 maggio 1959 anticipai al solito la mia partenza, per l'adunanza del sabato successivo. Ma quel sabato mi trovai poi a Napoli, in quell'appartamentino. E là seppi che il giorno prima egli era stato visto per via Chiaia fra il mezzogiorno e l'una (l'ora in cui di solito arrivavo); e che si era dato la morte nel pomeriggio inoltrato (quando ormai non sarei certamente arrivato più). E da allora mi domando se ero atteso; e depreco il contrattempo che mi aveva trattenuto a Roma e mi aveva impedito di recarmi dal più bravo e buono e caro, dal più intelligente dei miei amici, indimenticabile per tutti quelli che lo hanno conosciuto[28].

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[27] G. Scorza Dragoni (1908–1996), Gaetano Scorza's son, is another one of Picone's first pupils. He will teach at the Universities of Padua, Rome and Bologna. His research concerns mainly questions of plane topology and the theory of differential equations. The quotation is taken from: "Renato Caccioppoli", *Rend. Acc. Lincei*, 1963, pp. 85–93.

[28] A far Summer by now, I was guest in his father's house for several weeks, invited by his mother, who, worried by manifested designs, wanted me near the son. That was the first time I learnt, indirectly, of such sad designs. With me, the precise and plain allusion was made only many years later. There is no need to refer here the very simple words we exchanged that day. From then I tried to see him even more often. And as I could not persuade him to attend the meetings of our Academy again, each time there was one, I advanced my departure from Padua, where he had preceded me and had passed, in four teaching years, one of the best periods of his life, and I went to Naples to see him, in the flatlet he had taken, after his wedding, in Cellamare palace, in Chiaia street, where he lived alone by then. And also on that Thursday 7th May 1959 I brought as usual my departure forward for next Saturday's meeting. But on that Saturday I was in Naples in that flatlet. And there I knew that the day before he had been seen along Chiaia street between noon and one (the time I usually arrived); and that he had murdered himself late in the afternoon (at a time which was already too late for me to arrive). And from then I wonder if I was awaited; and I condemn the complication that had detained me in Rome and had prevented me from going to see the nicest, the best, the dearest, the most intelligent of my friends, unforgettable for all who had met him.

3. Distinguished Senator, Dear Colleague

We have talked about the two centres – Pisa and Rome – around which Italian analysis mainly develops during the period between the two World Wars. But in the 1920s, Tonelli is still in Bologna. His transfer to Pisa occurs in 1930 and, as anticipated, its orchestrator is Gentile.

At Dini's death, in 1918, Bianchi – another mathematician! – had succeeded him as Director[29] of the *Normale*. Already in this case the philosopher's "hand" had stepped in, as Bianchi's next letter dated 19th November 1918, addressed to the "Dear Professor Gentile", shows:

> So dall'amico Solari quanto ella si è adoperato perchè nella nomina del successore al nostro compianto Sen. Dini, nella Direzione della Scuola Normale, la scelta del Ministro avesse a fissarsi sopra di me. Le assicuro che questa prova di stima e di affetto, venutami da lei, mi è stata subito molto cara, ed avrei voluto ben prima ringraziarla di tutta la pena che si è dato per condurre a tale scopo le trattative. Lo faccio ora sentendo che queste sono giunte a buon punto, ma ancora prima di riceverne la comunicazione ufficiale. Non le nascondo che riconosco tutta la gravità dell'ufficio che vado probabilmente ad assumere, troppo inferiore come mi sento per capacità e per autorità al nostro impareggiabile Dini[30].

Bianchi's direction is crushed by a huge quantity of problems: the old question of the choice between a project of a high professional formation school for secondary school teachers or a different development as a centre of research and research training; a lifeless situation with a low number of pupils; a journal, the *Annali*, published with no regularity; the need for recognition of a special *status*, in terms of an extraordinary financing; the restoration of the update and specialization lessons. When Gentile, at Bianchi's death, in 1928, is appointed Director – first designate and then regular – the situation is not really brilliant. The revival of the *Normale* is still on the agenda. After Betti's and Dini's, Gentile's is the second propitious time in the *Scuola*'s modern history. The strengthening and the leap forward that we observe is in a certain sense comparable with what had happened in the *Normale*'s passage from grand-ducal College to national institution. The increase of the number of pupils (from thirty to a hundred), the institution of postgraduate specialization courses and the restoration of the position of "professor in charge of the seminars" are all signs that point the way out of the tunnel of uncer-

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29 Ulisse Dini had been Director of the *Normale* from 1900.

30 I learn from the friend Solari how much you worked so that in the appointment of the successor to our late Sen. Dini for the Direction of the *Normale School*, the Minister's choice would fix on me. I assure you that this evidence of esteem and affection on your side has soon become very dear to me, and I would have liked to thank you earlier of all the pain you have taken to conduct the negotiations to this purpose. I do it now hearing that these have arrived at a good point, but before receiving the official communication. I do not hide you that I understand all the seriousness of the position I am probably taking, with so much an inferior competence and authority compared to our unrivalled Dini.

tain perspectives that had marked Bianchi's headship. With Gentile, the *Normale* obtains a definitive role[31] as a research centre and a model institution for a formation of the elite. And the revival necessarily involved the group of scientists and the mathematical studies which had brilliantly supported the modern development of the *Scuola*. On the other hand, from Betti onwards, a good number of Italian mathematicians who rose to international prestige had been educated exactly in Pisa. At this point Gentile trusts in Tonelli, hinting also at his wish to work in a Institute in which he would have been unchallenged *leader*.

Actually, the choice is not foregone, nor free of a series of problems. It is true – as we have seen – that at the end of the 1920s Tonelli is a mathematician of unquestioned prestige, unchallenged leader in the field of real analysis and of calculus of variations; it is also true, though, that his *call* to Pisa – on Gentile's side – has undoubtedly political implications, raising not a few perplexities. To Tonelli's early support of socialist ideals was added his failure to subscribe to the fascist party plus his signing of the *Croce manifesto* in 1925. In short, Tonelli is not, from a political point of view, a wholly trustworthy and reassuring figure. But Gentile's is a far-sighted strategy. It is that of a politician who feels himself firmly in power (despite the attacks coming from the catholic world and from some sectors of fascism itself) and that can allow himself even the co-operation of not wholly aligned elements. The brilliant power management will prove to his advantage. Thus he promotes the invitation to Tonelli to move to Pisa and to take an active part in creation of an international and prestigous research group. The different instalments of the negotiation and of the agreement can be followed through two letters that Tonelli and Gentile exchanged within few weeks. The first, Tonelli's, is dated 16th April 1930.

> Illustre Senatore,
>
> (…) Mi sento molto onorato che Ella abbia voluto pensare a me per un posto così importante: e di ciò Le sono veramente grato. Ed il mio sentimento di riconoscenza mi trattiene dal rifiutare senz'altro, in via assoluta, l'offerta che mi viene fatta, pur non dissimulandomi le gravi difficoltà, di vario ordine, che dovrei superare per allontanarmi da questa città e da questa Università.
> Qui a Bologna ho trascorso gran parte della mia vita; qui sono sepolti i miei genitori, e qui ho visto nascere il mio bambino. In questa regione abitano i miei parenti, ed ho anche vari interessi che non posso trascurare. Particolare attaccamento ho poi per questa vecchia e gloriosa Università, che mi vide studente, che mi fece il grande onore di accogliermi come professore, e nella quale godo anche di qualche privilegio. Tutto ciò rende il problema del mio trasferimento in altra Università assai difficile. Ma il desiderio di far cosa gradita a Lei, che tanto amore porta alla

[31] The new statute will be approved in July 1932. It foresees, inter alia, that the *Normale* should be entered only by competition, later free. Women, though, will not be admitted. Gentile thought that "female students are diligent, good at repeating what professors say, but they do not have the critical capacity and the broadmindedness that scientific research needs" (cited in F. Busetta, *Studenti universitari negli anni del Duce*, Padova, il Poligrafo, 2002, p. 151).

Scuola Normale di Pisa, mi indurrà a prendere in esame, con animo ben disposto, le proposte concrete che mi verranno fatte[32].

Gentile's answer is dated the 10th June.

Egregio Collega,

sono dispiacentissimo di non averLe scritto dacché Ella ebbe la cortesia di prendere in considerazione la mia proposta della Sua chiamata a Pisa, ma la mia lontananza da quella sede e la conseguente difficoltà di condurre sollecitamente le necessarie trattative coi professori della Facoltà e con le autorità, insieme con la gravità dei problemi che erano in corso e che sono stati felicemente risoluti per l'assetto della Università e di quella Scuola Normale Superiore – la quale si avvia ormai a diventare un istituto importantissimo a fianco delle due Facoltà di scienze e di lettere – sono state causa di questo lungo ritardo con cui m'è dato di riprendere la proposta che già ebbi il piacere di farLe.

A mia scusa mi permetto di accennarLe che in una piccola minoranza dei professori della Facoltà era sorta una certa esitazione e titubanza pel carattere di talune manifestazioni politiche che di Lei si ricordano: le solite miserie, contro le quali io vengo combattendo energicamente in questi ultimi anni; e a troncare le quali – poiché troppo mi sarebbe dispiaciuto che la Sua chiamata a Pisa non dovesse aver luogo per voto unanime della Facoltà – ho creduto opportuno far intervenire lo stesso Capo del Governo. Il quale ha scritto recentemente al Prefetto di Pisa una lettera che è stata comunicata al rettore e che dissipa, naturalmente, ogni dubbio[33].

The political difficulties mentioned by Gentile are clearly due to Tonelli's support of *Croce's manifesto* (that had won him also venomous attacks and insinuations in the local press, in Bologna). Verbal testimonies explain also Gentile's reference to the interview with the Head of Government: Mussolini only asks Tonelli to have a (.. good!) knowledge of mathematics; having obtained this guarantee, Mussolini writes to the Prefect of Pisa about the instructions of the case. It is always Gentile who mediates to overcome Tonelli's economic requests. Tonelli's demands were surprising in a university

32 Distinguished Senator,
(…) I am highly privileged to have been considered for such an important place: and I am really thankful to You. My sense of gratitude prevents me from immediately refusing, definitively, the offer done to me, even if it does not disguise the several serious difficulties that I should overcome to move away from this city and from this University.
Here in Bologna I have passed most part of my life; my parents are buried here, and here is born my son. In this area live my relatives, and I have several interests that cannot be neglected. I feel a particular attachment to this old and glorious University that saw me as student, that made me the great honour to host me as professor, and in which I enjoy of some privilege too. All this makes the problem of my transfer to another University very hard. But the wish to do You, who loves so much the *Scuola Normale* of Pisa, a service, will induce me to examine, with a well inclined spirit, the concrete proposals that will be done to me.

33 Footnote see next page.

Mussolini presides a sport's show organized by the regime (Rome, beginning of the Thirties)

environment where private business practices were not common, but they cannot be separated from his determination to assume the new task in a business-like manner.

Gentile will always support Tonelli in looking for solutions to the ever-increasing problems: finding appropriate positions for some of the older assistants who were not considered suitable to the new functions of the Institute; softening exasperation at manifestations of intolerance arising from an increasing atmosphere of authoritarian rule espoused by young *normalists* ; preserving the publication of the *Annali*; offering his own agency and defending Tonelli against the meddlesomeness of some colleagues who were nearer to the regime.

At the University, Tonelli holds the chair of Infinitesimal analysis and teaches a course in advanced analysis, taking at the same time the Institute Direction. In such a

[33] Dear Colleague,
I am very sorry for not having written since You kindly considered my proposal of Your call to Pisa, but my absence from that seat and the ensuing difficulty to promptly conduct the necessary negotiations with the professors of the Faculty and with the authorities, together with the seriousness of the problems in hand and that have been happily solved for the order of the University and of that Scuola Normale Superiore – that is now on its way to becoming an important institute alongside with the two Faculties of science and of letters – have been the cause of this long delay with which I resume the proposal I had already the pleasure of doing to You.
As a justification, I dare to mention that in a little minority of the Faculty's professors had arisen some hesitation and dithering regarding the nature of some reminded political statements of Yours: the usual trifles, against which I energetically fought these last years; and to sever them – since it would be a real pity for me to give up the Faculty's unanimity about your moving to Pisa – I have considered it right to ask for the mediation of the Government Head, who has recently written a letter to Pisa's Prefect, communicated to the rector and that of course disperses any doubt.

capacity he lucidly pursues the aim to modernize the cadres, with the ensuing creation of new places for young people to start their research. The running of the *Annali della Scuola Normale Superiore di Pisa* is the other leg on which stands Tonelli's renewal project; to a first series, confined in the six decades 1871–1929 to only sixteen volumes (issued with no regularity, publishing mostly excerpts of dissertations) follows now a journal that experiences a remarkable improvement. Its fame and international prestige rises and solidifies, with Tonelli who in *Normale* finds also time to coordinate two series of lectures and seminars (the first more didactic, the second devoted to research).

It is almost unavoidable that such a work load (and health problems that now show more often) leads to a reassessment of Tonelli's research activity, stressing some signs that had already gradually emerged during the Bolognese years. The period in Pisa adds no noticeable contributions to real analysis and trigonometrical series. The researches on calculus of variations themselves, even if they significantly develop the "front" of the double integrals, proceed though at a different speed, directly demonstrated, however, by the missing publication of the volumes that should have completed the project of the *Fondamenti sul Calcolo delle variazioni*. The memoir: "Sugli integrali del Calcolo delle variazioni in forma ordinaria"[34], resumes the existence theorems for the minimum of the usual integral, always taken in its ordinary form. Tonelli wants to organize the main results that appeared after publication of the *Fondamenti* (with particular regard to E. McShane's works), always, though, sticking to his method that considers integrals in their ordinary form as wholly independent from those in parametric form. In particular, he shows the equivalence between the first of the existence theorems given by McShane and the one with which M. Nagumo had expanded one theorem proved by Tonelli, through a generalization of the *growth condition* (now usually called the Tonelli-Nagumo condition). The memoir, reconstructs the whole theory regarding the existence of the minimum solution for the integrals $\int_a^b f(x, y, y')\,dx$. Besides the general recognition for E. McShane's work (and M. Nagumo's and L. M. Graves') it has continuous references to the *Fondamenti* and to the previous memoirs for a determined assertion of his priorities. It remains thus emblematic of a new atmosphere, in which it is necessary to "reckon with" other fresh forces (indirectly) created by the Italian school itself.

The study that most originally characterizes research in the 1930s is the one concerning double integrals (in ordinary form), expanding the theory of semi-continuity and arriving, in this way and with the same generality, at some existence theorems. The studies on the definition of area according to Lebesgue and on the generalization, for several variables functions, of the ideas of bounded variation and absolutely continuous function are a precious landmark. The first results, regarding semi-continuity, are already included in the note: "Sur la semicontinuité des intégrales doubles des calcul des variations"[35]. After what has been said about the new assignments adopted in Pisa, it is not surprising if the existence theorems are achieved only some years later, specially in the memoir[36] "L'estremo assoluto sugli integrali doppi", the most committed of the time. But at this

34 *Annali Scuola Normale Pisa*, 1934, pp. 401–450.
35 *Acta Math.*, 1929, pp. 325–346.
36 *Annali Scuola Normale Pisa*, 1933, pp. 89–130.

point the narative stops. For the overcoming of some limiting hypothesis, present in the existence theorems about the values taken by a parameter and especially for the too conditioning reference to the uniform topology, the extension to a number whatever of independent variables and the research program presented at Zurigo's International Conference itself – "once established the existence of the minimum, it is important to study the analytical properties of the minimizing function" – remain a project devoid of the strength to maaterialize. This is, in 1940, an honest evaluation of the research on double integrals, outlined by Tonelli himself[37].

> Più arretrata è l'applicazione della nuova teoria agli integrali doppi. Tuttavia anche in questo campo si sono potute precisare e discutere completamente le condizioni per la semicontinuità degli integrali doppi in forma ordinaria e si sono ottenuti dei teoremi di esistenza dell'estremo molto larghi; e pure risultati cospicui si sono avuti in merito all'esistenza dell'estremo degli integrali doppi in forma parametrica. Circa le proprietà analitiche delle superficie estremanti i risultati ai quali si è sino ad ora pervenuti non hanno ancora raggiunto tutta quella generalità che è desiderabile; per altro quanto si è stabilito permette di risolvere completamente problemi classici di grande importanza, come quelli ben noti di Dirichlet e di Plateau[38].

In his last years, Tonelli will be at the centre of a harsh controversy with Picone, and Gentile will offer himself as mediator. Again, the philosopher will be the organizer of a last passage – equally important in Tonelli's career, who at the beginning of the academic year 1943–1944 will take on the headship of the *Normale*. But we will talk about these two events later.

4. The dualism U.M.I. – C.N.R.

The studies of analysis confirm the *trend* we had showed, regarding the *school* of geometry, at the beginning of the chapter, a trend that therefore can be taken as an example of the dynamics of the whole of Italian mathematics. Despite the singular brilliant results we have mentioned, we observe a slow decay or – rather – the inability to repeat and extend the "sparkling outburst" still perceptible at the beginning of the century. The particular and feeble social and scientific structure of the country makes the assertion of new generations of scholars more exhausting and slow than one expected.

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[37] "L'analisi funzionale nel calcolo delle variazioni", *Annali Scuola Normale Pisa*, 1940, pp. 289–302.

[38] The application of the new theory of double integrals is more underdeveloped. However, even in this field, the conditions for the semi-continuity of double integrals in ordinary form have been wholly specified and discussed and very large existence theorems have been achieved; and outstanding results have been obtained also regarding the existence of the extreme of double integrals in parametric form. About the analytic properties of extremant surfaces, the results so far achieved have not reached yet all the desirable generality; on the other hand what has been established allows to completely solve extremely important classic problems, as Dirichlet's and Plateau's well-known ones.

We will give an account of the research situation again, as it develops during the 1930s, but we can say right now that the conclusions will not be very different. The analysis of the different powers within the mathematical world appears to be more moving. Severi's *leadership* is not questioned. We have seen his solidity and we will see, in the next chapter, how this takes him to be one of the privileged interlocutors of the political power each time there is a choice to be made about culture and the university world. The fact is that, with the duopoly U.M.I – C.N.R, the united front of mathematicians that, at the beginning of the century had held despite the tensions coming from the presence of particularly strong personalities and that, after the war, is still protected from Volterra's authoritativeness, begins to split.

The change is therefore represented by the new politics of the C.N.R. We have talked about its foundation, prompted by Volterra who will be its president until 1926. The 3rd article of the *Statute* specified that "the various national Committees adhering to the international scientific Unions which are part of the International Research Council, are under the National Research Council's control". Thus the U.M.I. – that joined the *Unione Matematica Internazionale* – acts as *Comitato matematico* of the C.N.R. while Volterra himself (president of the C.N.R. and authoritative member of the U.M.I.'s executive committee) secured an excellent link between the two institutions. In 1924 the C.N.R. provided the U.M.I. with the financing that would serve to pay the *Unione Matematica Internazionale*'s membership fee of the previous year and the running expenses (including those to finance the Italian participation in the eighth international mathematicians' Conference held in Toronto). The contribution is repeated the year after. In short, it can be deduced that – even if in 1926 and in 1927 there was no other payment made, and the C.N.R. did not finance the Congress of Bologna – its meagre resources gave aid to the consolidation of the young professional association of Italian mathematicians.

The shift comes with Volterra's dismissal and Marconi's presidency. The new legislative procedures[39] lead to a different order within the C.N.R.: new and more substantial financings[40] , resulting from a greater submission to the requirements of the regime, and the creation of twelve independent disciplinary Committees. This last decision creates the suspicion that the cessation of the activities of the "old" previous Committees mean *tout court* the closure of the U.M.I., that had always acted as *Comitato matematico* of the C.N.R. Such was, in the mathematical environment, the overlapping between the two structures! On February the 27th 1927, Pincherle voices this fear and writes to Marconi to remind him that the U.M.I. in the meanwhile had always kept its own independence and legal personality and that therefore the reorganization of the C.N.R. should not interfere with the association's inner life. It is only at this point that the C.N.R.'s presidency realizes the "mess" and makes it clear that the U.M.I., even if not being part of the C.N.R. as *Comitato matematico* any more, obviously keeps its functions by the *Unione matematica internazionale*.

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[39] It is the Royal Decree Law of the 31st March 1927, n. 638 (Official Gazette of the 9. V. 1927, n. 107) turned into the Law of the 20.V. 1928, n. 1347 (Official Gazette of the 2.VII. 1928, n. 152).

[40] The initial yearly financing, set in 1923, is tripled.

Bianchi – we have talked about him several times in the previous pages, in discussions of differential geometry and algebra – is appointed president of the new *Comitato matematico* of the C.N.R. (while Pincherle will maintain his charge as president of the U.M.I.). But his health conditions, actually, will prevent him from performing any function (delegated to the *Comitato*'s secretary, Bompiani). On the 6th of June 1928 Bianchi dies and is succeeded by Scorza, assisted by a presidency executive formed by Berzolari, Bompiani and Picone. This is the *team* that will guide the C.N.R.'s *Comitato matematico* during the four years 1928–1932. We know them. About Scorza we talked in the pages addressed geometry and algebra; about Bompiani, Castelnuovo's pupil, with regard to differential geometry; and, lastly, we found Picone (in this chapter) as author of the second *pole* of Italian analysis in Rome. The only new entry is Luigi Berzolari (1863–1949). Cremona's pupil and expert on algebraic geometry, graduated in Pisa, and professor at the Universities of Turin and Pavia, he will link his name especially to the realization of the *Enciclopedia delle Matematiche Elementari* that, begun in 1930, will go on until the early 1950s. Scorza, Berzolari, Bompiani and Picone will be gradually joined, in the composition of the Committee, by other mathematicians such as Luigi Amoroso, Ettore Bortolotti, Francesco Paolo Cantelli, Guido Castelnuovo, Leonida Tonelli and Giuseppe Vitali.

We do not know about the mechanisms and ways through which Scorza's appointment is reached. The fact is that the head of the U.M.I. does not take the appointment with much enthusiasm, at least judging by the skeletal statement that Pincherle reads at the association's annual assembly[41].

> Il Presidente riferisce poi sulla fondazione del Comitato matematico del Consiglio Nazionale delle ricerche. Dice che fino allo scorso anno funzionava da Comitato matematico la Presidenza dell'Unione, ma che, ora, in relazione col nuovo assetto dato dallo Stato al Consiglio Nazionale delle ricerche, è stato insediato un apposito Comitato cui presiede il prof. Scorza, e funziona da segretario il prof. Bompiani. La nuova istituzione, fornita dal Governo nazionale dei mezzi necessari, sarà certo di grande vantaggio al progredire della scienza matematica in Italia[42].

It is not a question of people – Bompiani, for instance, had been since its foundation a member of the Executive of the U.M.I. – nor of their scientific value. It is not even about the chance to express disagreement and political divergences: Pincherle, together with Gini, had been the only signatory – among mathematicians – of the 1925 *Gentile manifesto*. Rather, the head of the U.M.I. fears that the institution of an independent mathematical Committee within the C.N.R. introduces a dichotomous element in the

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[41] Bologna, 24.2.1929.

[42] The President reports about the foundation of the mathematical Committee of the National Research Council. He says that until last year the Presidency of the Union acted as mathematical Committee, but that, now, relating to the new order given by the State to the National Research Council, a proper Committee presided by Prof. Scorza, with Prof. Bompiani as secretary, has been established. The new institution, provided by the state Govern with the wherewithal, will certainly profit the progress of mathematical science in Italy. (*B.U.M.I.*, a. VIII (1929), n. 2, p. 115).

Enrico Bompiani

professional life of Italian mathematicians, with the risk of modifying the consolidated balances among the different research areas.

In this sense, fears are well-founded. The disagreements, or simply the incomprehensions, begin almost immediately. There is a trace in the correspondence between Ettore Bortolotti and Scorza[43], when the first complains that the C.N.R. has not written a single thank-you line to Pincherle (who only indirectly had known of his replacement), wonders about the nature of the new *Comitati* – “scientific societies, or rather acolytes of a few ones?” – and remarks that the C.N.R. had not assisted nor had cared about the preparation and the outcome of the Congress of Bologna, if not by “replacing, long before the Congress began, the people with charges and functions in the Committee”. Soon, the ruling body of the C.N.R.’s *Comitato matematico* begins to speak another language, and, however, to express different sensibilities and assessments. Bompiani, invited to draw up a report on Italian mathematical institutes, grabs the chance “to remark how these Institutes could be organized both for scientific production (for example by replacing the conferences on detached subjects, such as the ones held here in Rome, with workshops of reading and of study of memoirs, such as Hadamard and Hilbert, to quote two classic examples, do) and for the preparation for professional teaching”[44].

[43] All correspondences mentioned in this paragraph are part of the *Fondo E. Bompiani*, kept at the Archive of the *National Science Academy* (so-called of the *XL*).

[44] Letter from Bompiani to Scorza dated 22.II.1929.

Bompiani's report aimed at "establishing the prevailing streams in the present mathematical research in Italy, at catching a glimpse of the still vague features of those eventually in the making, at fixing the right needs and means, both financial ones and the organization to satisfy them, and lastly to show how the professors carry out the mission entrusted to them". Actually, his analysis succeeds in its purpose of diminishing the merits of Pisa, traditional seat of mathematical "power" and of enhancing the schools of Pavia and specially of Naples, where "particularly noticeable has been the work accomplished by Prof. Picone and by his pupils in the field of mathematical analysis". His 1931 report, addressed this time to the mathematical periodical press in Italy, will be even more critical towards the mathematical *establishment*[45]. Attention is specially drawn to the two most glorious newspapers, the *Annali di Matematica pura ed applicata* and the *Rendiconti del Circolo matematico di Palermo*. Their editorships – writes Bompiani – "exist more by name that in fact and in a whole decade have not suffered any substantial reform or rejuvenation so as to reflect in some way the new life in Italy". The journals – observes Bompiani – are a strong instrument to achieve university chairs: "a refused or delayed Memoir, an anticipation in the publication of a work on the eve of a competitive exam, can exclude one candidate of the chair or of a prize giving etc. and favour another". The editorial Committees must therefore be changed so as not to abandon "every instrument of practical action in the hands of a group perpetuating its potency, beyond the Commissions, in the education of the future professors and more in general in the attribution of rewards".

A letter (dated 28 July 1932) from Bompiani to Picone shows immediately that the dichotomy, and an increasing contraposition, are not elements of a later historic reconstruction but live in the atmosphere and in the documents of the time.

> In gran fretta, prima di partire per la Cecoslovacchia, ti comunico quanto segue:
> S. E. Marconi ha approvato il criterio di dare al Comitato un carattere volto alle applicazioni. Magrini, come ti ho detto per telefono, mi ha dato la lista da modificare ed io gli ho risposto che desideravo che questo lavoro fosse fatto d'accordo con S. E. Parravano e con te come futuro Presidente (io spero) del Comitato Matematico. Stamane ho portato la lista a Parravano; egli ha fatto alcune cancellature e mi ha promesso che la manderà a te.
> Ho esposto a S. E. il pericolo che confinando troppo la scelta nel campo applicativo noi perdiamo ogni possibilità di influenza nel campo rimanente; a noi invece occorre avere un certo numero di amici fedeli che possano aiutarci nel nostro compito di smantellare vecchie posizioni. Così ci conviene avere una certa influenza (per ora poca, ma appunto per accrescerla) sui periodici matematici, che sono gli strumenti con i quali si può favorire o danneggiare un giovane; e perciò bisogna che in questo campo abbiamo persone che stiano con noi. Quindi a mio avviso, criterio applicativo sì, ma inteso con una certa larghezza e in modo da

[45] Letter to Scorza from the 22.VI.1931 in *Fondo E. Bompiani*, Archive of the *National Science Academy* (so-called of the XL).

> non ridurci all'impotenza. Ti ho voluto comunicare questo perché tu possa regolarti[46].

Besides the explicit references to the "other fields" or to "old positions" or still to "sure friends (…) who support us", the letter is significant for two other reasons. It brings to mind that one of the reasons behind the reorganization of the C.N.R. was the will to give more importance to *applications*. The reference to Picone as President-to-be of the *Comitato* deserves some clarification so as not to be taken for Bompiani's underhanded tricks to the detriment of Scorza. In a letter of the 4th May 1932, the latter had told Bompiani about his intention not to accept any executive office within the different orders that were meant to be given to the *Comitato* for mathematics. This of course concerned the evident *diminutio* that mathematicians underwent through their bundling with Physics[47] but also Severi's important intervention about Italy's official representation in the international Conference which was to be held in Zurich that year. The C.N.R., on the 14th April, had appointed the official delegation, approved also by the Foreign Office, composed of Scorza, Bompiani and Picone. The choice sparks off a heated debate (stoked up by Severi, the regime's "official" mathematician) that goes beyond the specific question. Bompiani talks about it in an official report[48] dated 11th May:

> Pregiomi riferire un colloquio avuto il 10-V-1932 con S. E. SEVERI sui due seguenti argomenti:
>
> 1. Delegazione Italiana al Congresso Internazionale dei Matematici a Zurigo.
> 2. Unione Matematica Internazionale.

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[46] In a great hurry, before leaving for Czechoslovakia, I tell you the following:
S. E. MARCONI has approved of the criterion of turning the Committee towards the application field. MAGRINI, as I told you on the phone, has given to me the list to be changed and I have told him that I wanted this work to be done in agreement with S. E. PARRAVANO and with you as future President (I hope) of the *Comitato Matematico*. This morning I have taken the list to PARRAVANO; he has done some crossings out and has promised me that he will send it to you.
I have explained to S. E. the danger that limiting too much the choice within the applicative field we lose all possibility of influencing other fields; we need, instead, to have a number of sure friends that can help us in the task of dismantling old positions. Thus it would be useful to have some influence (a little, for the moment, but just to increase it) on mathematical journals, which are the instruments with which a young can be aided or damaged; and therefore we need to have in this field people who support us. Hence in my opinion, I would say yes to the applicative criterion, if understood with some openness and in such a way so as not to be reduced to impotence. I wanted to tell it to you so that you know how to behave.

[47] As it results from the minutes of the sessions of the Directorate (session of the 2 February 1932), it was the chemist Nicola Parravano (Dean of the Science Faculty in Rome) to put forward the conjecture "that the Committee of Mathematics could be joined to the ones of Physics and of Astronomy, forming a unique Committee".

[48] The report in *Fondo E. Bompiani*, Archive of the *National Science Academy* (so-called of the *XL*).

1°. Il Punto di vista di S. E. Severi in merito al primo argomento si riassume nei seguenti punti:

1) Se c'è una Delegazione Italiana al Congresso di Zurigo essa deve esser unica.
2) Se c'è una (unica) Delegazione Italiana essa deve esser presieduta da S. E. Severi, Accademico d'Italia.
3) Se così non fosse S. E. Severi non andrebbe a Zurigo a tenere la conferenza generale che è stato invitato a fare (unica conferenza generale affidata ad un italiano).
4) Se questo accadesse S. E. Severi porrebbe la questione della delegazione in altra sede per lasciare decidere a chi deve.

S. E. Severi sa che a capo della Delegazione era stato designato il Prof. Scorza e sa anche (da una lettera dello Scorza) che questi rinunzia ad andare a Zurigo.
Non so se Egli conosca i nomi degli altri delegati, che io in ogni modo non gli ho detti.
Egli si è vivacemente lagnato di esser tenuto in disparte, non per i riflessi interni ma per quelli che la sua esclusione, come unico matematico della più alta istituzione culturale del Regime, può avere all'Estero nei riguardi dell'Italia.
Io mi sono limitato ad osservargli che il Consiglio delle Ricerche, dovendo nominare una sua Delegazione ha scelto fra i Membri del Comitato.
Egli mi ha affermato di aver avuto promessa da S. E. Marconi di esser incluso nel nuovo Comitato, ed ha aggiunto di non aspirare (e di dichiararlo palesemente) al posto di Presidente o Vice-Presidente del nuovo Comitato. Sul riordinamento di questo non gli ho dato alcuna informazione[49].

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[49] I am privileged to quote a talk had on the 10-V-1932 with S. E. Severi, on the following two issues:

1. Italian Delegation at the Mathematicians' International Conference in Zurich.
2. Unione Matematica Internazionale.

1°. S. E. Severi's point of view about the first issue can be summed up in the ensuing points:

1) If there is an Italian Delegation at Zurich's Conference, it must be unique.
2) If there is one (unique) Italian Delegation, it must presided by S. E. Severi, Academician of Italy.
3) If it were not so, S. E. Severi would not go to Zurich to give the general lecture he has been invited to deliver (the only general lecture assigned to an Italian).
4) If this happened, S. E. Severi would raise the question of the delegation on some other occasion so that the one who has to decide does it.

S. E. Severi knows that Prof. Scorza has been placed in charge of the Delegation and knows also (thanks to a letter from Scorza himself) that he renounces to go to Zurich.
I do not know if He knows the other delegate's names, which I have not told him anyway.
He has vividly complained about being left out, not because of intern repercussions, but for those that his exclusion, as the only mathematician of the Regime's highest cultural institution, can have for Italy abroad.
I have only remarked that the Research Council, having to designate a Delegation, has chosen between its Committee members.
He has told me that S. E. Marconi promised him he would be in the new Committee, and has added that he does not want (and declares it openly) the President or Vice-President position in the new Committee. I have not given him any information about its rearrangement.

Severi will win the "match" of Zurich's conference, managing even to have the travelling expenses refunded[50]. His is a decisive step in the exclusion of Scorza from the *Comitato*'s executive organs, but will not allow Severi to enter the *Comitato* until the following reorganization in 1937.

But let's come back to the *presidency Scorza* and to his *Council* in the quadrennium 1928–1932: a group of *young colonels* – in the first line are Bompiani and Picone, a little over forty, and a president, Scorza, who has recently entered his fifties – that develops the project of an alternative landmark for the management of the mathematical world. All emerges quite by chance but the changing of the guard between Volterra and Marconi, the reorganization of the C.N.R. and the institution of the twelve disciplinary *Committees* has started an apparently latent project. It is a period in which everything changes in Italy, or seems to, and social and political transformations also favour projects like this. Politics has nothing to do with it, if one thinks of the support of the P.N.F.: sympathies for the authoritarian regime are equally divided between the *establishment* of the U.M.I. – think about Pincherle – and the *novatores*. It has to do, though, if we recall fascism's giovanilistic nature and its will to bring to the forefront new protagonists, making a clean sweep of the generation established in the Giolittian and liberal Italy. Scorza, Bompiani, Picone and Berzolari worry about the stagnancy, the decline elements they glimpse and the Italian research's gap in some apparently strategic areas. They are unhappy with the management of the *old guard*, identified with the U.M.I.'s direction, to which they reproach outwardly reassuring assessments and an attendist/fence-sitting management. They want to give a *good shaking*, that makes the Italian mathematical machine gain speed again, not only through the statements and annual reports, but also through an exact executive program.

There is first of all the editorial *front*, that opens with the publication of Bianchi's *Opere* and of the other volumes of Ruffini's writings (after the first volume printed in 1923, edited by the *Circolo Matematico* of Palermo). Most committed is the second advanced project, the *Bibliografia Internazionale Matematica*, that is the edition of a new publication of mathematical reviews that, overcoming the criteria that shaped the *Jahrbuch über die Fortschritte der Mathematik* and the *Révue semestrielle des pubblications mathématiques*, now out-dated because of the development of new branches of mathematical disciplines, made it possible to "assert Italy's mathematical position within the international field" and above all to "turn it into a weapon against eventual foreign underestimating in any scientific branch". The last two projects will fail because of the scarce financial supplies and because the second project, really challenging, entered on a collision course with a similar German project that emerges in 1932 (it is the well-known *Zentralblatt*), even if "with less technical thoroughness", as Bompiani will say. Instead, a project that will be accomplished – its formulation is due to Scorza – is that of a series of mathematical monographs. Illustrious foreign examples of such series are not

[50] Cf. minutes of the Directorate's meeting of the 23.1.1933: "The President conveys a request from Prof. Severi to get an allowance of L. 1500 for having gone to Zurich to take part to the Conference of mathematicians. The Directorate exceptionally approves." The amount was the one fixed as compensation for each member of the CNR's official delegation.

missing: the *Memorials de Sciences Mathématiques* in France, the *Colloquiums of the American Mathematical Society* in the U.S.A. and the *Grundlehren der Mathematischen Wissenschaften* in Germany. The aim was to gather in light volumes the bodies of doctrines often lost in journals or Academies' *Atti*, readable only by specialists, without a reworking and an organic reorganization stressing the fecundity of the inspiring idea, bringing prestige and "primacy" to the mother country.

There is, then, the *front* of the scholarships and of the relationships with other countries' scientific culture. The scholarships' award serves to avoid the drying up of some research branches and to stimulate those researches that are scantily cultivated in our country.

> Se per qualche indirizzo mancano da noi Maestri che lo facciano conoscere è più difficile che giovani vi si dedichino: e pertanto laddove si presenti tale lacuna essa non può che rendersi sempre più grave col tempo. È all'estendersi di questa lacuna che la Giunta deve opporsi cercando di colmarla tempestivamente. A ciò possono servire le borse di studio quando esse siano assegnate non a persone la cui mentalità scientifica sia stata già determinata ma in modo da influire su questa determinazione. Ammesso questo criterio, la Giunta è passata ad esaminare in quali campi possano venir ravvisate lacune. Ed è apparso che già si manifestano sensibili nell'algebra e nella teoria dei numeri ove il distacco è già notevole fra quanto si fa in Italia e quanto si fa in altri Stati[51].

To this politics, addressed to the young new generations, Bompiani adds in particular an original sensibility towards the US world. He proposes a) survey papers (in English) of Italian mathematical works; b) publication of works in American journals; c) creation of a *Institute of specialization* at Rome able to compete with the Institutes of Paris and Göttingen and to increase the flow of American students that come here to look for currents of thought missing elsewhere; d) summer courses for foreigners in Italian tourist centres; e) Italian professors in America; f) creation of Italian chairs in American universities; g) spread of the Italian book production in the USA. Of course the practical achievements will be positively inferior. As regards the scholarships, the meagre balance of the Committee allows only the award of two specialization scholarships in Italy and of three similar positions abroad (to Gianfranco Cimmino, Picone's Neapolitan pupil, so as to allow him the attend the courses of C. Carathéodory, O. Perron and A. Sommerfeld at

[51] "If Masters who teach certain branches are missing, it is more difficult that young people turn to them: therefore where such gap arises, it can but become worse with time. The Committee has to oppose itself to to the spreading of this gap, trying to bridge it promptly. Scholarships can serve this purpose, when assigned not to people whose scientific mentality has been already determined, but so as to influence on this determination. Once admitted this criterion, the Committee has passed to examine the fields in which these gaps can be identified. And it emerges that they already appear perceptible in algebra and in the theory of numbers where the gap between what is done in Italy and what is done in other countries is already remarkable". (Cf. "Relazione del Comitato Matematico al 31.XII.1931" in "Fondo E. Bompiani", Archive of the *National Science Academy* (so-called of the *XL*).

the university of Munich; to Fabio Conforto, for a six month *stage* in Göttingen, and to Tullio Viola, B. Levi's pupil, for a *stage* in Paris with P. Montel and J. Hadamard).

The participation in international scientific life involves greater energies. In 1930 the C.N.R.'s mathematical Committee takes part in the two International Conferences, both held in Stockholm, of the Actuaries (official delegate is Cantelli) and of Applied mechanics (official delegate is Signorini). The following year, it attends the II International Conference of History of Science (official delegate is A. Perna, who publishes a report[52] in which he underlines the importance of the contributions of the Soviet delegation – guided by Bucharin – on the incidence of social and economic factors in scientific and technological progress). Bompiani himself had taken part in the meeting of German mathematicians and physicians in Praga (1929), in which the "most interesting part of the meeting has been the debate on the theory of knowledge of exact sciences"[53]. Almost all the representatives of the *Circolo di Vienna* (R. Carnap, H. Feigl, P. Franck, H. Hahn, O. Neurath and F. Waismann) and the German epistemologists, with the participation of eminent scientists (P. Hertz, R. von Mises, A. A. Fraenkel, W. Heisenberg and A. Sommerfeld), had discoursed on the subject. And Bompiani is entrusted, on occasion of an invitation to several universities of the United States during the winter of 1930 for a round of lectures and lessons, to:

> trarre occasione dell'invito per indagare quanto in quei paesi siano conosciuti i contributi dell'Italia alle Scienze esatte e per diffondere fra gli americani la conoscenza dell'organizzazione delle nostre Scuole medie e superiori. Il prof. Bompiani (…) con una bella conferenza, tenuta dietro invito, al Congresso della Società matematica americana poté porre in bella luce quanto fra noi è stato fatto e si vien facendo nei vari campi dell'Analisi e della Geometria e che, con interviste pubblicate su giornali americani e con un discorso tenuto in una associazione politica, ha compiuto opera altamente lodevole di propaganda patriottica, sfatando le calunniose leggende sul nostro regime poste in giro dal bieco livore antifascista di rinnegati italiani o di malevoli stranieri[54].

As regards the enhancement of the newest and most vital sectors of Italian mathematics, the C.N.R.'s *Comitato matematico* has no doubt: these are represented by the "nice position that Italy has assumed in the field of calculus of probabilities, of its appli-

[52] Cf. "Bollettino d'Informazioni", a. II, vol. II (1931), n. 9, pp. 239–245.

[53] Cf. "Bollettino d'Informazioni", a. I, vol. I (1930), pp. 17–19.

[54] Take advantage of the invitation to observe up to which point are Italian contributes to exact sciences known in those countries and to spread the knowledge of our middle and high school organization among Americans. Prof. Bompiani (…) with a nice lecture given at the American Mathematical Society's Conference on invitation, could put in a good light what we have done and are doing in the several fields of analysis and geometry and that, with interviews published on American journals and with a speech given in a political association, has done a praiseworthy action of patriotic propaganda, debunking the calumnious stories about our regime noised by the sullen antifascist jaundice of Italian renegades or malevolent foreigners (Cf. G. Scorza, Report to the CNR's Riunione plenaria (21–22 January 1931), in "Bollettino d'Informazioni", a. II, vol. II (1931), n. 1, p. 11).

cations and of all the theories of analysis linked with it" and by the nature of the researches that Picone started in his "Gabinetto di Analisi infinitesimale". But we already spoke of Picone and of his *Istituto nazionale per le Applicazioni del Calcolo*. It is however a story that begins at the C.N.R.'s "home". The first one to propose it is Scorza, as we can read in the U.M.I.'s *Bollettino*[55].

> Il Direttorio del Consiglio Nazionale delle Ricerche si rivolgeva nel Febbraio u.s. ai vari Comitati dipendenti, autorizzandoli a fare proposte motivate per la creazione di Istituti scientifici dei quali l'Italia mancasse e che rispondessero ad una effettiva necessità per il progresso di taluni importanti indirizzi di ricerche. La Giunta esecutiva del Comitato matematico, presieduto dal professore G. Scorza, in seguito a questo invito proponeva al Direttorio la creazione di un Istituto Centrale di Calcolo, per mezzo del quale si potesse conseguire un'intima, continua e non occasionale collaborazione fra i matematici puri da un lato e i cultori di scienze sperimentali e di applicazione dall'altro. Questa proposta veniva ampiamente illustrata da un opuscolo pubblicato dalla detta Giunta esecutiva e diramato, oltreché a tutti i membri del Comitato, a numerose personalità. L'opuscolo porta il titolo: *Sulla necessità, per il progresso delle Scienze sperimentali e Matematiche, della creazione di un Istituto Centrale di Calcolo*. La proposta ha incontrato largo consenso, e la Presidenza dell'Unione Matematica, con lettera diretta al prof. Scorza il 30 luglio 1929, ha espresso la sua piena e cordiale adesione[56].

It is again up to Scorza, on occasion of the SIPS annual meeting (Florence, 18–24 September 1929), to gather the C.N.R.'s *Comitato matematico* to explain "what the Committee plans to do so as to satisfy the needs of the technique that on and on asks the pure mathematician for the solution of many heterogeneous problems". Among the projects and achievements we have looked at, the idea of the *Istituto di Calcolo* is certainly the most far-sighted one, and the one generating the greatest changes. Is it then suprising if, in view of such a far-sighted activism, the U.M.I. answers with cold courtesy? Neither are the signs of a harder answer missing. In the same year, 1929, the elections for the renewal of the social offices within the U.M.I. are carried out. Picone and Amoroso (the mathematical economist who had supported the colleague's project financially) obtain

[55] *BUMI*, a. VIII (1929), n. 4, pp. 231–32.

[56] The Directorate of the National Research Council addressed in last February to the several independent Committees, authorising them to make reasoned proposals for the creation of scientific institutes missing in Italy and answering to an effective need for the progress of some important research lines. The executive Committee of the mathematical council, presided by Prof. G. Scorza, following this invitation proposed to the Directorate the creation of a Central Calculus Institute, through which a close, continuous and non occasional collaboration among pure mathematicians on one side and the scholars of experimental and application sciences on the other, could be achieved. This proposal was widely illustrated in a pamphlet published by the cited executive committee and issued, besides the members of the council, to several personalities. The pamphlet is titled: *Sulla necessità, per il progresso delle Scienze sperimentali e Matematiche, della creazione di un Istituto Centrale di Calcolo*. The proposal enjoys a large approval, and the Presidency of the Mathematical Union, with a letter addressed to Prof. Scorza on the 30th July 1929, has expressed its whole and warm support.

only one vote! It does not go much better with Scorza, president of the C.N.R.'s *Comitato matematico*: two votes!

5. The oath

The C.N.R.'s reorganization represented a search for new balances within the mathematical world. But with the political world in such contention, outer relationships had to change as well. The immediately preceding landmark was the *the manifestos battle.* It was 1925 and the great majority of mathematicians – naturally those who had taken sides – had refused to side with the political and cultural stances of the regime. Only a few years – we are in 1931 – and a different demand, recompensing a silent approval, and we will see the great majority of mathematicians to have a different attitute towards the regime.

The oath's history can be traced starting from 1859's old *law Casati*, that did not impose any special oath for professors, but one that equalized all other civil servants. In the process that adopted the constitution of united Italy, as the different pre-unitary states became annexed to the new state, a political oath of allegiance to the king, the statutes and the laws was required at Universities. The Gentile reform of 1923 provided that:

> i professori di ruolo, prima di assumere l'ufficio, debbono, pena decadenza, prestare giuramento innanzi al Rettore o al Direttore, secondo la formula che verrà stabilita dal Regolamento generale universitario[57].

The Regulations, approved the following year, prescribed (art. 31) that:

> la formula del giuramento che deve prestarsi dai professori di ruolo di prima nomina è la seguente:
> *Giuro di essere fedele al Re ed ai suoi Reali successori, di osservare lealmente lo statuto e le altre leggi dello Stato, di esercitare l'ufficio di insegnante e adempiere tutti i doveri accademici col proposito di formare cittadini operosi, probi e devoti alla Patria*[58].

Whether because the new formulation regarded newly appointed professors, or because – in the first postwar – loyalty to the Monarchy was beyond dispute, it does not appear that anybody had refused to swear. What was implicit in Gentile's philosophy soon emerged during the process of fascistization of the state, after the 1925 exceptional laws. The genesis of the decision to impose a new and more mandatory oath's formula-

[57] Full professors, before taking on their office, must, under penalty of dismissal, swear before the Rector or the Director, according to the formula to be established by the general university regulations.

[58] The oath's formula to be sworn by newly appointed full professors is the following: "I swear to be loyal to the King and to his Royal successors, to faithfully observe the statute and other state laws, to perform my teacher's office and to accomplish all academic duties with the aim to educate industrious, upright and patriotic citizens".

tion for professors is to be traced in that process. This statement[59] of the Secretary for Education leaves no doubts about the changed political atmosphere.

> Che cosa si vuole di più? Io ho imposto ai professori universitari di giurare di non appartenere ad associazioni ed a partiti contrari allo Stato, eliminerò dalle commissioni i professori massoni ed antifascisti. Ma ho fatto di più, onorevoli colleghi, poiché mi sono dato il diritto di escludere dal prender parte ai concorsi e anche dalle cattedre i concorrenti ed i vincitori dei concorsi la cui condotta morale e politica sia tale da offendere la dignità dell'alto insegnamento; e recentemente ho negato la cattedra universitaria a due che erano riusciti primi nella terna. Qui non è in gioco la libertà dell'insegnamento, come qualche piccola congrega universitaria, più o meno massonica, va blaterando. Il fascismo apre tutte le vie all'indagine scientifica; ma noi abbiamo il dovere d'impedire che l'anima dei nostri giovani sia avvelenata[60].

The Royal Decree n. 38 of the 13th January 1927 provided also a new oath's formula (art. 4):

> La formula del giuramento per i professori che siano nominati o trasferiti in Regie università, Regi istituti superiori o Regi istituti superiori di Magistero è la seguente:
> "Giuro di essere fedele al Re ed ai suoi Reali successori, di osservare lealmente lo Statuto e le altre leggi dello Stato, di esercitare l'ufficio d'insegnante e adempiere tutti i doveri accademici col proposito di formare cittadini operosi, probi e devoti alla Patria. *Giuro che non appartengo, né apparterrò ad associazioni o partiti, la cui attività non si concilii con i doveri del mio ufficio*"[61].

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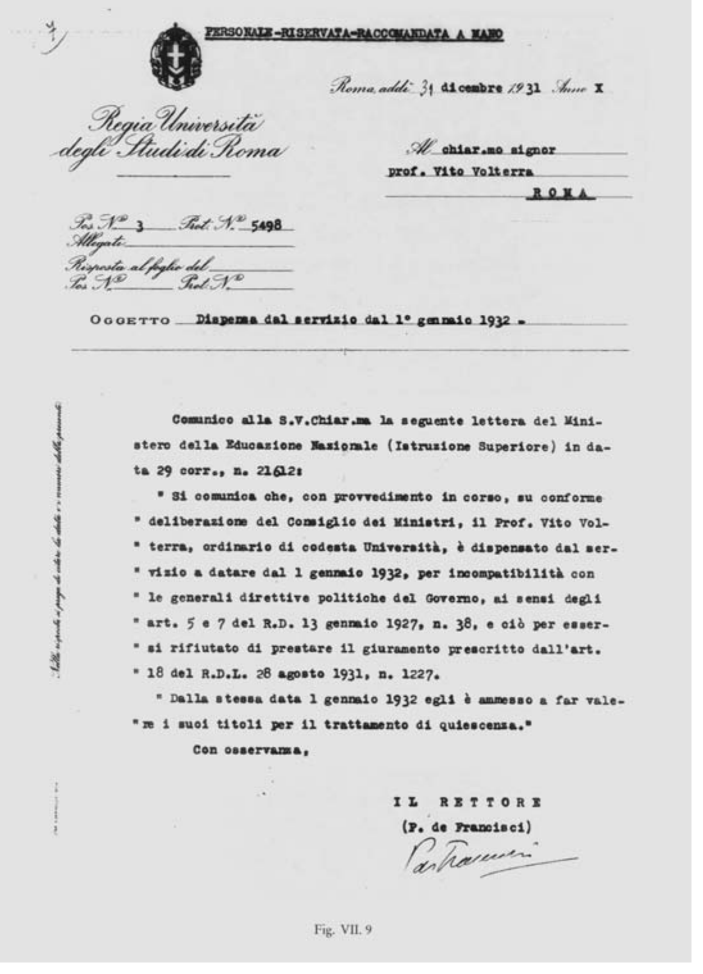
[59] The Minister is Pietro Fedele (1873–1943), author in the years in which he holds the ministry of Education of a real counter-reform of the Gentilian one, abounding in concessions to catholics and aiming to give to the school a more clerical and fascist character. With the decree of the 13th January 1927, Fedele in particular had invoked to himself the appointment of the commissaries of the competition exam. The statement in *L'Università italiana*, vol. XXIII (1927), pp. 161–170 (p. 167).

[60] What else is pretended? I have imposed to professors to swear not to belong to associations and parties opposed to the state, I will erase from commissions mason and antifascist professors. But I did more, honourable colleagues, as I have taken the right to exclude from state examinations and chairs the competitors and the winners of state examinations whose moral and political behaviour is such to offend the dignity of the high teaching; and I have recently denied the university chair to two that had result first in the tern. Here is not in play the freedom of teaching, as some little, more or less masonic, university group chatters. Fascism opens all ways to scientific research; but we must prevent our youth's soul from being poisoned.

[61] The oath's formula for professors appointed or transferred to Royal Universities, Royal High Schools or Royal Teacher-Training High Schools is the following: "I swear to be loyal to the King and to his Royal successors, to faithfully observe the Statute and other state laws, to perform my teacher's office and to accomplish all academic duties with the aim to educate industrious, upright and patriotic citizens. *I swear I do not belong to, nor will belong to, associations or parties whose activity does not harmonize with my office's duties".*



PERSONALE-RISERVATA-RACCOMANDATA A MANO

Roma addì 31 dicembre 1931 Anno X

Regia Università degli Studi di Roma

All' chiar.mo signor
prof. Vito Volterra
ROMA

Pos. N.° 3 Prot. N.° 5498
Allegati
Risposta al foglio del
Pos. N.° Prot. N.°

OGGETTO Dispensa dal servizio dal 1° gennaio 1932 -

Comunico alla S.V.Chiar.ma la seguente lettera del Ministero della Educazione Nazionale (Istruzione Superiore) in data 29 corr., n. 21612:

" Si comunica che, con provvedimento in corso, su conforme " deliberazione del Consiglio dei Ministri, il Prof. Vito Volterra, ordinario di codesta Università, è dispensato dal servizio a datare dal 1 gennaio 1932, per incompatibilità con " le generali direttive politiche del Governo, ai sensi degli " art. 5 e 7 del R.D. 13 gennaio 1927, n. 38, e ciò per esser- " si rifiutato di prestare il giuramento prescritto dall'art. " 18 del R.D.L. 28 agosto 1931, n. 1227.

" Dalla stessa data 1 gennaio 1932 egli è ammesso a far vale- "re i suoi titoli per il trattamento di quiescenza."

Con osservanza,

IL RETTORE
(P. de Francisci)
P. de Francisci

Fig. VII. 9

The dismissal letter of Volterra

1929 is a key year in the history we are telling: the *Patti Lateranensi*, with which the Church closes the fifty-year-old conflict with the Italian State and, actually, legitimates the fascist regime, are signed on the 11th February. And just after four days (15th February 1929) Severi writes to Gentile the letter we have quoted in the previous chapter, in which the mathematician exposes the project of a new oath's formula[62]. In this letter it impresses the presence of many reliable informative elements about the fascist world. As we have said, Severi is about to climb on the bandwagon, that delineates itself more and more clearly. He also arrogates to himself the function of "prince counsellor"

[62] Cf. A. Guerraggio and P. Nastasi, *Gentile e i matematici italiani*, quoted, pp. 211–213.

and proposes to equate old (and new) fascists with old antifascists with an act at once of *intransigence* ("aimed at obtaining the so much claimed fascistization of Universities") and of *indemnity* "of remote political acts", so that the "misdeeds" of ex-antifascists are erased. Gentile makes the criteria suggested by Severi, which had begun to take effect, his own. We find them, for example, during the debate in the House on the budget of higher education for the year 1931–1932[63].

> Il problema politico del personale è ancora posto di fronte alla coscienza dell'Italia fascista. (…) Vi è chi vorrebbe allontanare dalle cattedre tutti coloro che non dettero nei momenti della lotta, adesione piena al Fascismo; e chi, con opposto criterio, sarebbe disposto a una sanatoria generale. La verità è che vi sono discipline – il diritto, la storia, la filosofia – nelle quali lo Stato sorto dalla Rivoluzione, non può conoscere indulgenze o esitazioni. Costituisce un vero pericolo per i nostri giovani l'ascoltare dalle cattedre voci che, nella migliore ipotesi, non riescono a illustrare le leggi, lo spirito, la funzione del nuovo Stato semplicemente perché nelle loro menti e nei loro cuori vive ancora un tipo di Stato e operano idee superate dalla Rivoluzione. In queste discipline il criterio politico deve prevalere su quello scientifico. Nell'attesa che giovani sorti nell'atmosfera arroventata della guerra e della Rivoluzione (…) accumulino i titoli necessari per coprire cattedre statali, si provveda con incarichi. Ma si sfollino le Università dei pesi morti del vecchio regime. In tutte le altre facoltà e scuole, invece – a meno che i docenti non abbiano compiuti atti di aperta e grave ostilità al Fascismo – si possono usare maggiori indulgenze, augurando che anche qui l'onda dei giovani studiosi crei presto gli uomini capaci di salire sulle cattedre[64].

This is the background of the Royal Decree n. 1227 of 28th August 1931 which provides that in advanced education schools, full professors and appointed ones take an oath according to the following formula: *"I swear to be loyal to the King, to his Royal successors and to Fascist Regime, to faithfully observe the Statute and the other state laws, to perform my teacher's office and to accomplish all academic duties with the aim to edu-*

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63 Cf. the speech of the Honourable Lando Ferretti in *L'Università italiana*, vol. XXVII (1931), pp. 135–136 (p. 135).

64 The staff's political problem is still confronted with the conscience of fascist Italy. (…) There are some who would like to remove from the chairs all those who during the fight did not give whole support to Fascism; and who, instead, would agree to a general indemnity. The truth is that there are subjects – law, history, philosophy – in which the State emerged from the Revolution can accept no indulgence or hesitation. It is a real menace to our young people to listen to voices coming from chairs that, in the best of cases, are not able to illustrate the laws, the spirit, the function of the new State, simply because in their minds and hearts still lives a type of State and still work ideas overcome by the Revolution. In these subjects the political principle must prevail over the scientific one. While young people risen in the war and Revolution scorched atmosphere (…) hoard the titles needed to cover state chairs, we can see to it with appointments. But universities should be cleared of the old regime's deadweight. In all the other faculties and schools, instead – unless professors have performed open and serious hostile actions to Fascism – can be used greater indulgence, hoping that also here the wave of young scholars forms soon men capable of holding a chair.

cate industrious, upright citizens devoted to the Country and to Fascist Regime. I swear that I do not belong to, nor will I belong to, associations or parties whose activity does not harmonize with my office's duties".

Like the other professors of Rome University, Volterra receives the call to be up before the rector on November 18th. That same day he expressed, with few and firm words, his objection against the oath[65]:

> Sono note le mie idee politiche per quanto esse risultino esclusivamente dalla mia condotta nell'ambito parlamentare, la quale è tuttavia insindacabile in forza dell'Art. 51 dello Statuto fondamentale del Regno.
> La S. V. Ill.ma comprenderà quindi come io non possa in coscienza aderire all'invito da Lei rivoltomi con lettera 18 corrente relativa al giuramento dei professori[66].

The regime's answer comes soon. On December 12th, "the honourable Prof. Vito Volterra, senator of the Kingdom, full professor of Mathematical physics at the University of Rome" is informed that the refusal to swear has put him "in conditions of incompatibility with the government general political stances", making necessary the sanction of the release from service. On December 29th the measure becomes effective "on conformable resolution of the Cabinet".

The echoes of Volterra's missing oath were wide, both at national and at international levels[67].

> Pregiatissimo professore,
>
> ne avevo sentore, ma solo oggi dal Temps ho avuto conferma che Ella ha rifiutato di giurare. Non può credere quanto io sia soddisfatto di quest'atto, che del resto non fa che confermare la stima che ebbi sempre del suo carattere.
> La Università, con suo disdoro, perderà probabilmente un illustre insegnante, ma per fortuna resta il Maestro, di fama mondiale, e resta un esempio, che speriamo non sia perduto per l'avvenire.
> Ella avrà visto che tra i professori che rifiutarono di giurare ve n'è uno di Pavia. È mio fratello, ed è per me una grande soddisfazione che egli chiuda una carriera

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65 *Historical Archive* of the University of Rome.

66 My political ideas are well-known, although they turn out exclusively from my behaviour within parliament, which is unquestionable on the strength of the Art. 51 of the Kingdom's main Statute.
Your Lordship will understand then how I cannot consciously adhere to the invitation You addressed to me in the letter of this 18th regarding the professors' oath.

67 The letter is Luigi Errera's, dated Turin 19th December 1931 (Archive Volterra, *Accademia dei Lincei*, in Paoloni, cit., p. 178). Luigi Errera was a high railway executive, who seeing the rise of fascism decided to retire so as not to be obliged to join the party. His brother, mentioned in the text, is the chemist Giorgio Errera (1860–1933).

che, se non brillantissima dal punto di vista scientifico, può ben considerarsi esemplare per dignità e fedeltà al dovere.
Mi è grata questa occasione per inviarle i miei migliori auguri[68].

The following letter, from Paul Labérenne, dated Chartres 21st January 1932, is an example of the French echoes[69].

C'est avec un grand étonnementet et une profonde peine que j'ai appris votre révocation. J'avais bien entendu dire que le gouvernement fasciste avait l'intention d'imposer une prestation de serment aux membres des Universités, mais je n'aurais jamais cru qu'un pouvoir qui se dit "italianissimo" en viendrait à traiter ainsi un des plus illustres savants dont puisse s'énorgueillir le pays qu'il prétend représenter. Laissez moi dire, en tout cas, combien je comprends et j'admire votre geste de refus!

Volterra's is not the only critical case[70]. After many hesitations and reconsiderations, Levi-Civita decides not to leave his place to the "new barbarians", and looks for some kind of negotiation. Here is one of his worried drafts of an answer to the rector's call.

Pur rispettando sempre meticolosamente leggi e regolamenti, concepii fin dalla prima giovinezza e seguitai a coltivare, anche dopo il 1922, idealità democratiche e socialiste, le quali, dal punto di vista politico (assai meno nei riguardi economici) discordano da quelle cui si ispirano le direttive del regime. Tali idealità ho potuto finora mantenere almeno di fronte alla mia coscienza e all'ambiente intellettuale. La nuova formula di giuramento, cui si riferiscono la Sua lettera (…) e la sollecitatoria (…), mi sembra precludere persino la semplice, leale affermazione di un dissenso spirituale. Se però Ella, Magnifico Rettore, mi potrà autorevolmente dar atto che ciò non è, mi presenterò senz'altro a giurare entro il termine fissato.
In caso diverso non potrò io violentare il mio sentimento e starò con evidente rammarico, ma con animo sereno in attesa delle sanzioni che l'Autorità Accademica intenderà promuovere a mio carico[71].

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68 Dear Professor,
I had wind of it, but only today did I received confirmation from Temps that You have refused to swear. You have no idea of how much pleased I am with this action, that besides does only confirm the esteem I always had of you.
University, to its discredit, will probably lose an illustrious teacher, but luckily the world-famous Master remains, and remains an example, which we hope is not lost for the future.
You will have seen that among the professors who refused to swear there is one from Pavia. He is my brother, and it is for me a great satisfaction that he closes a career that, if not scientifically brilliant, can properly be considered exemplar in the dignity and loyalty to one's duty.
I am pleased to have an occasion to send you my best regards.

69 Archive Volterra Accademia dei Lincei.

70 While the majority of professors swears loyalty to the Regime, a dozen of them do not want to bend to such an imposition.

71 Even if I was always meticulously respectful of laws and regulations, I conceived since my first youth and continued to cultivate, also after 1922, democratic and socialist ideas, that, from a political point of view (quite less under economical perspectives) differ from those inspiring the government stances.

Finally, Levi-Civita decides to swear. And yet for his whole life the memory of a bullet bitten “for the good of school”, but still insulting his dignity and that, even if it had not silenced the best part of his conscience, had blurred its crystalline lucidity, will be a hard one. In the oath years, though, he expresses disagreement in other ways.

In the meeting of the 3rd February 1932, the Dean of the Science faculty of Rome, reads the release imposed on Volterra. After the communication, Levi-Civita takes the floor.

> Io desidero esprimere, almeno per conto di qualche amico matematico e mio, vivo rammarico per l’allontanamento testé comunicatoci, del Sen. Volterra dalla nostra Facoltà, che si onorò della sua opera scientifica per oltre un trentennio. La Facoltà mostrò ripetutamente di apprezzare le benemerenze del Volterra; non solo, ma ne ebbe indirettamente vantaggio morale attraverso i riconoscimenti, veramente eccezionali per importanza e per numero, che il Volterra ebbe da Università, Accademie e istituzioni scientifiche di tutto il mondo. Ma non è fortunatamente il caso di fare una commemorazione, tanto più che una recente disposizione porge il modo, come i colleghi sanno, di mantenere aggregato alla Facoltà chi vi abbia appartenuto per un certo tempo come professore ufficiale[72].

Needless to specify that the proposal will get no support and that Levi-Civita’s statement will not even enter into the minutes. These episodes, though, are of some use. The news of the oath’s refusal causes a sensation abroad, specially in France and in the U.S.A., where Volterra and Levi-Civita are known also for their lectures and their lessons, and the press returns on several occasions to the episode of Italian professors. Nor is to be forgotten the generous attempt realized by Einstein. On 8th November 1931 an ecclesiastical law professor in Turin – Francesco Ruffini (1863–1934) – writes to him, also on behalf of other Italian colleagues and of his son Edoardo (1901–1982), asking him to intervene, from his high scientific, political and moral authority, against the oath’s imposition[73].

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71 (continued)
I could until now keep such ideas, at least before my conscience and the intellectual environment. The new oath formula, to which Your letter (…) and the reminder (…) refer, seems to preclude even the simple, loyal assertion of a spiritual disagreement. But if You, Magnificent Rector, can authoritatively declare that it is not like that, I will certainly go and take the oath within the established date. Otherwise, I will not be able to force my principles and I will with evident regret, but with serene spirit, wait for the sanctions the Academic Authority will intend to initiate against me.

72 I want to express, at least on behalf of some mathematician friend of mine, deep regret for the just now conveyed dismissal of Senator Volterra from our faculty, which was proud of his scientific work for more than thirty years. The faculty repeatedly showed to appreciate Volterra’s merits; not only, but it indirectly got moral advantage through the appreciations, really exceptional for importance and number, that Volterra received from universities, academies and scientific institutions worldwide. But luckily it is not the case to do a commemoration, all the more that a recent directive, as the colleagues know, offers the way of keeping associated to the faculty who has belong to it for some time as full professor.

73 Cf. L. Polverini, Albert Einstein and the fascist oath of 1931, *Rivista Sorica Italiana*, 103 (1991), pp. 268–280.

> Le Gouvernement italien a rendu une loi d'après laquelle obligation est faite à tous les professeurs d'Université de prêter un nuoveau serment, par lequel ils s'engagent à être *fidèles au régime fasciste* et en outre à exercer leur profession avec le propos de former des citoyens dévoués au *régime fasciste*. A remarquer qu'un pareil engagement des plus graves n'est imposé qu'aux seuls professeurs d'Université, à différence des enseignants des écoles inférieures et de tous les autres employés de l'Etat.
> Ni mon Fils ni moi – mon Fils est professeur d'histoire du droit à l'Université de Perugia – n'entendons prêter un pareil serment. Et le même refus sera opposé à la demande du Gouvernement par quelques uns des professeurs, parmi les plus renommés, des différentes Universités (Volterra, p. e.). Mais la plupart des professeurs devra courber la tête, parce que leurs conditions économiques modestes, et souvent difficiles, ne leur permettent pas d'affronter la sanction qui va suivre le refus, c'est à dire le destitution de leur charge.
> Il ne nous reste qu'un seul espoir, c'est que, si jamais une voix de solidarité et de protestation s'élevait de la part des plus illustres maîtres des Universités étrangères, le Gouvernement se désiste de sa décision inconsidérée, ou tout au moins ne sévisse point contre ceux qui refuseraient de prêter ce serment.
> Je me suis permis de m'adresser à Vous, dont l'autorité est si hautement reconnue dans tout le monde scientifique européen: voyez Vous-même s'il Vous est possible de faire quelque chose pour venir en aide de vos collègues d'Italie; et veuillez agréer …

Einstein's intervention is highly providential: on the 16th November 1931 he writes to the Italian Minister of Justice A. Rocco (that he had already met in 1925, when they met again at the *Commissione internazionale di cooperazione internazionale*) asking him to avoid "the cruel trial that threatens Italian scholars" and "to spare this humiliation to the cream of Italian intelligence". We imagine the trouble and the embarrassment of the Minister, author of the totalitarian state's legal apparatus. Rocco prefers indeed not to answer personally, and hands the answer over to a judge of the Court of Appeal in Rome, Member of Parliament (12th December 1931)[74]: the oath required of professors regarded the "state constitutive laws, without pretending consequently – and I allow myself to underline it in a particular way – that professors join this or that political line (…) and I am pleased to tell you that of over about 1200 full and temporary professors only 7 or 8 have voiced an objection to the proposed formula".[75]

Actually, the political action of the oath, aiming at isolating the explicit antifascism, succeeds completely. It leaves though a chain of irretrievable hate behind, both on the side of the ones that have to renounce teaching and on the side of the ones that must swallow the bitter pill, passing necessarily from one surrender to another (immediately

[74] The letter, written in an excellent German as Einstein will remark, is published in L. Polverini, cit. The counsellor was Giuseppe Righetti.

[75] On the 9 January 1932 Einstein wrote down in his diary Righetti's letter ("Excellent answer in German, but the thing continues to be always a foolish action of ignorant people") together with the unfavourable auspices he saw for Europe's immediate future.

after there will be the request to join the fascist party and then another oath, for the members of the academies), along an exasperating slope.

A mathematician – Severi – had reopened the question and we return to mathematicians to assess the repercussions. Levi-Civita's isolation within the Science faculty of Rome can be measured by the episode of Volterra's replacement. On the 3rd May 1932 the faculty board meets before the dean, N. Parravano, and a score of professors, among whom Castelnuovo, Enriques, Levi-Civita, Bompiani (and Fermi). The minutes reads thus[76].

> Aperta la seduta il Preside comunica che il prof. Severi ha fatto sapere di non poter intervenire alla seduta in conseguenza di un incidente di viaggio aereo occorsogli. Da informazioni che egli ha assunte risulta che le condizioni del prof. Severi non ispirano nessuna preoccupazione. Egli ha già formulato a nome dei colleghi e suoi personali gli auguri più vivi di un pronto e completo ristabilimento. Esprime ora il più vivo rincrescimento per il doloroso infortunio e rinnova gli auguri.
> La Facoltà si associa ai sentimenti del Preside. (…)
> Provvedimenti per la cattedra vacante.
> Il Preside esprime il parere che debba essere coperta la cattedra rimasta vacante in seguito al collocamento in congedo del prof. Volterra.
> A lui si sono rivolti i colleghi titolari di materie biologiche perché la cattedra fosse coperta con un biologo, i professori di discipline matematiche invece hanno espresso il parere che il posto di ruolo vacante debba essere assegnato ad una materia di matematica.
> Egli ha fatto osservare ai biologi che la Facoltà non può accogliere il loro desiderio perché una materia biologica richiede la istituzione di un laboratorio, al quale in questi momenti non è possibile provvedere; e poiché questi colleghi hanno trovato giusta l'osservazione da lui fatta e rimandano ad altra occasione la realizzazione dei loro desideri, così a lui sembra che si possa assegnare la cattedra ad una delle scienze matematiche e precisamente all'analisi superiore.
> Il prof. Levi-Civita esprime il parere che questa questione debba essere rinviata per l'esame ad un'altra seduta e chiede che la Facoltà si pronunci sulla sua proposta di rinvio.
> La votazione sulla proposta Levi-Civita dà i seguenti risultati: contrari al rinvio 17, astenuti 1, favorevoli 1.
> Scartata così la proposta di rinvio, viene messo ai voti il seguente ordine del giorno: "La Facoltà di Scienze della R. Università di Roma udite le dichiarazioni del preside relative alla cattedra vacante, decide di destinare tale cattedra all'insegnamento delle "analisi superiori" e fa voti che a tale cattedra venga trasferito il professore Mauro Picone ordinario di analisi infinitesimale nella R. Università di Napoli, che ha già coperto la cattedra di Analisi superiore nella R. Università di Pisa, quale successore di U. Dini, e che occupa una posizione di particolare rilievo fra gli analisti italiani.
> Il valore e l'energia animatrice del Picone assicureranno alla Università di Roma la continuazione della sua elevata tradizione matematica".

[76] *Historical University Archive*, Minutes of the Science Faculty, vol. n. 11.

La votazione, con schede segrete, di questo ordine del giorno dà il seguente risultato:
Presenti 19 – Votanti 19 – Voti favorevoli al trasferimento del prof. Picone 19.
In base all'esito della votazione, la Facoltà di Scienze della R. Università di Roma si onora di proporre a S. E. il Ministro dell'Educazione Nazionale che il prof. Mauro Picone della R. Università di Napoli venga trasferito alla cattedra di "Analisi superiore" nella R. Università di Roma[77].

We have already said that already in 1921, in the time of Enriques' and Severi's *call* to Rome, Volterra's aim was to have in Rome a real analyst, which he identifies in Tonelli. As we know, Levi-Civita's reasons prevailed then. Now the problem recurs and Levi-Civita would like to honour Volterra's old wish[78].

Illustre e caro Senatore,

Le sono molto grato per la gentilissima, affettuosa Sua lettera del 1 corr., e mi compiaccio delle ottime notizie Sue e della Signora, augurando Loro la più soddisfacente continuazione del soggiorno di Madrid e buona gita in Andalusia.

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77 Once opened the session, the Dean communicates that Prof. Severi has let know he cannot attend the session as a consequence of a plane accident. From some information he has gathered it seems that Prof. Severi's conditions arouse no worry. He has already presented on his and his colleagues' behalf the most deep wishes for a soon total recovery. He expresses now the most deep regret for the painful accident and reiterates the wishes. The Faculty joins the Dean's sentiment. (…)
Dispositions for the free chair.
The Dean expresses his opinion that the chair fallen free following the dismissal of Prof. Volterra should be covered. Professors holding biology chairs addressed him asking for the chair to be covered by a biologist, mathematics' professors have instead expressed their opinion that the chair fallen free should be assigned to a mathematical discipline. He has remarked to biologists that the faculty cannot meet their wish because a biological discipline requires the institution of a laboratory, which presently cannot be supplied; and as these colleagues have thought fit his remark and defer the fulfilment of their wishes, he thinks that the chair could be assigned to a mathematical science and exactly to Superior analysis.
Prof. Levi-Civita is of the opinion that the question should be postponed to another session for examination and asks the faculty to pronounce on his deferment proposal. The voting on Levi-Civita's proposal gives the following results: unfavourable 17, abstained 1, favourable 1. Rejected thus the deferment proposal, the following agenda is put to the vote: "The Science Faculty of the R. University of Rome heeded the dean's statements regarding the free chair, decides to assign the chair to the teaching of "Higher analysis" and vows Prof. Mauro Picone, professor of infinitesimal analysis at the R. University of Naples, who has already hold the chair of Higher analysis at the R. University of Pisa, as successor of U. Dini, and who occupies an outstanding position within Italian analysts, to be assigned to this chair. Picone's value and moving energy will ensure the continuity of the high mathematical tradition of the University of Rome".
The voting, by ballot, of this agenda gives the following result: Attendants 19 – Voters 19 – Favourable votes to Prof. Picone's assignment 19.
On the strength of the voting result, the Science faculty of the R. University of Rome has the honour to propose to the Secretary for Education Prof. Mauro Picone of the University of Naples for the chair of Higher Analysis at the R. University of Rome.

78 The letter from Levi-Civita to Volterra, dated 6 May 1932, appeared in P. Nastasi, R. Tazzioli (eds.), *Aspetti scientifici e umani …*, quoted, pp. 155–156.

> Lunedì scorso c'è stata seduta di Facoltà per provvedere all'Analisi superiore. La grande maggioranza dei colleghi era orientata verso il Picone, e il Preside ne propose senz'altro la chiamata. Io dissi che si potrebbe prima tentare di vincere le presumibili resistenze di due colleghi, che nel momento attuale hanno (per generale consenso e per essere più anziani) una posizione scientifica superiore, alludendo naturalmente a Fubini e Tonelli. E proposi una sospensiva, appoggiata anche ad analogo desiderio di Severi che non potè intervenire alla seduta. La sospensiva, che volli votata formalmente, fu respinta, e allora votai anch'io per Picone, il quale così risulta chiamato all'unaminità (dei 19 presenti). Il Severi non era presente perché si è escoriato un piede (anzi entrambi i piedi) e deve stare qualche giorno ancora in riposo. È questa, fortunatamente, l'unica conseguenza di un incidente di volo, che poteva essere molto grave. Nell'andare (colla signora) a Tripoli in idroplano, furono costretti ad ammarare per guasto al motore e rimasero 5 ore sbattuti da onde violentissime, con pericolo di capottare, a una cinquantina di Km. da Tripoli. Venne una torpediniera al soccorso. Il trasbordo fu drammatico. La signora se la cavò con piccole contusioni, Severi si ferì ai piedi[79].

Picone arrives thus at Rome. With him, the mathematical group most in tune with fascism includes Bompiani and Severi too. It will be a compact group when defending the regime's background politics but strongly divided inside when it comes to the development lines of Italian mathematics, as we will see in the next chapter.

6. Tullio Levi-Civita

The oath's episode has brought us into contact with that Levi-Civita whom we had already introduced in the *Prologue* as a young up-and-coming researcher, pupil of Ricci Curbastro. It was – it is the beginning of the new century – a matter of showing the

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[79] Illustrious and dear Senator,
I am very grateful to You for Your kind and cordial letter from the 1st of the current month, and I am delighted with Your excellent news, wishing You and Your wife the most pleasing prosecution of Your stay in Madrid and a good trip to Andalusia. (…)
Last Monday there was a Faculty session to provide for Higher analysis. The great majority of our colleagues was addressed towards Picone, and the Dean proposed the call without hesitation. I said we could first try to overcome the probable opposition of two colleagues that have presently (by unanimity and by seniority) a superior scientific position, hinting of course to Fubini and Tonelli. And I proposed a deferment, supported also by Severi's similar wish, who could not attend the session. The deferment, that I wanted to be formally voted, was rejected, and then I voted for Picone too, who results thus called by unanimity (of the 19 attendants). Severi was not present because he grazed his foot (rather both feet) and still has to rest some days. This is, fortunately, the only consequence of a plane accident, that could have been very serious. Going (with his wife) to Tripoli by hydroplane, they were forced to land on water because of a motor breakdown and they were for 5 hours flung by very violent waves, in danger of a turnover, about fifty km from Tripoli. A torpedo-boat came to rescue. The transhipment was dramatic. The lady managed with little bruises, Severi hurt his feet.

Tullio Levi-Civita

A young Gregorio Ricci Curbastro

fecundity of the new *absolute calculus*, beyond the confined domain of differential geometry. We have found him again in 1921 – well-known professor, in the meanwhile transferred to Rome – at the time of Severi's (and Enriques') call to the capital. In the war years, Levi-Civita had been one of the few Italian mathematicians to hold to his own internationalist commitment, not being caught up in the general protest against the *Germanic barbarism*. And from this connotation we can start again, to tell about his scientific activity during the 1920s.

We can begin in particular with a letter[80] of T. von Kármán to Levi-Civita dated 12th April 1922, in which the director of the *Aerodynamics Institute* of the Polytechnic of Aachen proposes the idea of organizing an international meeting on the problems of hydrodynamics. The sector is extremely rich in important theoretical and applicative implications but has no institutional position, its researchers being divided among the assemblies of engineering, mathematics or physics. To avoid the predictable refusals that the international situation allowed one to perceive, von Kármán underlines the need to keep the initiative on an informal level, avoiding the sources of tension linked to a more official organization, and suggests an exact division of tasks: he would see to gathering supports in the German language area (with his Master L. Prandtl in front) and in the neutral countries (starting from the Swedish physician C. W. Oseen) while Levi-Civita would develop a similar function on "neo-Latin and English". The background reasons for Levi-

[80] Levi-Civita Archive, *Accademia dei Lincei*.

Civita's choice as partner in such an initiative are clear: the association of a German and an Italian is a clear sign of the authentically international quality that von Kármán means to give to the enterprise, strengthened by Levi-Civita's open (and widely known) position on the subject. The choice of the Italian colleague is significant on a scientific level too: in the transdisciplinary seat that von Kármán intends to prepare, in which "theorists" and "technicians", pure mathematicians and hydraulic engineers would join, the presence of a mathematician of such a prestige and features is a clear evidence of the intended improvement that von Kármán condenses in the effective slogan: "turning engineering design in engineering science". Levi-Civita's answer is as usual ready: he does not only accept enthusiastically the proposal of a Conference, in which both representatives of the theoretical line and of the experimental one of hydraulics and aeronautic technology would bejoined, but suggests this to be considered as a personal initiative of some researchers, open as much as possible to a true international dimension. We are in 1922. The war wounds take a long time to close and the refusals to take part in an Italo-German joint initiative are numerous. Among the "excellent" withdrawals figures Volterra's (and M. Brillouin's and R. von Mises'). Similarly, in September of that same year about thirty scientists – mostly from the German area – gathers in Innsbruck, where it is decided, among other things, to summon periodical conferences – no more restrained to the sector of hydrodynamics alone – spread so as to cover the whole field of applied mechanics. The *International Applied Mechanics Conferences* were born, the first of which will be hold at Delft from the 22nd to the 25th April 1924.

To the *Proceedings* of these two first Conferences, Levi-Civita presents the text of his two speeches: *Sulla velocità di trasporto nel moto ondoso permanente* and *Determinazione rigorosa delle onde permanenti di ampiezza finita*, that indicate the return of his interest towards hydrodynamics. That same year he publishes the note: *Risoluzione dell'equazione funzionale che caratterizza le onde periodiche in un canale molto profondo*[81], including the existence and unicity theorem for irrotational waves. These researches will find an arrangement in the note: *Détermination rigoureuse des ondes permanentes d'ampleur finite*[82], even if it will be M. L. Jacotin – P. Dubreil's wife – the one to complete Levi-Civita's research frame. Here is how Dubreil himself recalls the Roman mathematical environment, experienced in the winter 1930–31[83].

> Après la soutenance de ma thèse, à Paris, à la fin d'octobre 1930, nous sommes partis pour Rome car j'avais obtenu une prolongation de ma bourse Rockefeller pour travailler avec Enriques pendant le semestre d'hiver 1930–31.
> A cette époque, la géométrie algébrique était brillantement représentée, à Rome, par Castelnuovo, Enriques et Severi. Castelnuovo, le plus âgé, était déjà à la retraite et mes relations avec lui se sont limitées à une visite de courtoisie. Chose regrettable, les relations entre mon maître officiel, Enriques, et Severi étaient tendues. Cepen-

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81 *Math. Ann.*, 1922, pp. 256–279.
82 *Math. Ann.*, vol. 93 (1925), pp. 264–314.
83 Cf. P. Dubreil, Souvenis d'un boursier Rockefeller 1929–1931, *Cahiers du Séminaire d'Histoire des Mathématiques*, 4 (1983), pp. 61–73.

> dant, je fus très bien accuelli par Severi en allant me presenter à lui et en lui remettant un exemplaire de ma thèse. Je fus impressionné par l'aspect physique de ce grand toscan (né à Arezzo) et je le fus plus encore par le niveau de sa conversation, en français, au plan humain aussi bien que dans le domaine mathématique. Comme je le quittais, il me dit: "J'irai vous voir" et me demanda de lui laisser mon adresse. Nous avions élu domicile dans le quartier du Pincio, à l'*Albergo Vittoria*, petit hôtel agréable situé à l'entrée de la *via Sardegna* … qui était celle où habitaient, dans un même immeuble, Enriques et Levi-Civita.
> Le nom de Levi-Civita, bien entendu, nous était familier avant notre séjour à Rome. Nous l'avions entendu prononcer souvent par Elie Cartan dans ses cours de géométrie ou par Henri Villat en méanique des fluides. Ma femme se félicitait d'avoir la possibilité de travailler à Rome avec lui. Agrégée en juillet 1929, elle s'était placée aussitôt sous la direction de Villat qui s'occupait surtout de sillages.
> Or, Levi-Civita avait publié en 1925 un mémoire retentissant dans lequel il établissait rigoureusement (c'est-à-dire sans faire d'approximation) l'existence d'une onde irrotationnelle dans un liquide incompressible de profondeur infinie, avec surface libre. Ayant étudié ce travail, ma femme arrivait à Rome avec une remarque assez curieuse: une des conditions introduites par Levi-Civita, "l'absence de transport de masse dans les couches profondes", n'était pas remplie par l'onde de Gerstner, appelée aussi houle cycloïdale, qui est également une solution exacte, mais non irrotationnelle. Elle en fit part à Levi-Civita qui fut surpris, un peu sceptique, mais téléphona dès le lendemain pour dire, après vérification, que la chose était exacte. En rencontrant un peu plus tard, dans une rue de Rome, Mme Levi-Civita, ancienne étudiante de son mari et personne d'un grand charme, nous avons appris que ce que nous pensions être un petit incident n'avait donné lieu qu'à des commentaires sympatiques et élogieux. En fait, cette remarque sur "les couches profondes" fut le point de départ de la thèse de ma femme, où, avec une hypothèse valable à la fois pour l'onde de Gerstner et pour celle de Levi-Civita, elle établit rigoureusement l'existence d'une infinité d'ondes comprenant les deux précédentes (Journal de Math. pures et appl., t. 13, p. 217, 1934)[84].

Levi-Civita's interest in hydrodynamics will end, in the years considered in this chapter, in the memoir: *Attrazione newtoniana dei tubi sottili e vortici filiformi*, published in the volume that opens – in 1932 – the new series of the *Annali della Scuola Normale Superiore di Pisa* entrusted to Leonida Tonelli[85]. The study of the asymptotic form of the Newtonian potential of points next to a thin tube goes back to the first decade of the 20th century. It had been extended to Saturn's rings in 1912 and is now creatively picked up again to be applied to threadlike vortices: "in the light of modern developments of nonlinear dynamics and vorticity", Levi-Civita's work "strikes for modernity

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[84] The title of the work is intentionally similar to Levi-Civita's memoir: "Sur la détermination rigoureuse des ondes permanentes périodiques d'ampleur finie".

[85] The memoir, urged by Tonelli, is really the writing of three lectures (March 1931) given at the Institute of Hydromechanics of the Sorbonne, directed by Villat.

and depth of results" and "it is fundamental for the mathematical formulation of potential theory and capacity theory for slender tubes" [86].

Hydrodynamics is not the only field to which Levi-Civita contributes in an original way during the 1920s. But, before speaking of analytic mechanics, we should review his arrangement work and the publication of treaties and manuals. The *Lezioni di calcolo differenziale assoluto*[87], gathered by the physician Enrico Persico (1900–1969) are published in 1925. It is an answer to the so-called *Italian vector calculus* developed (in the early 1920s) by Cesare Burali-Forti (1861–1931) and Roberto Marcolongo (1862–1943) and to the so-called homographic formalism of Tommaso Boggio (1877–1963). And it represents a winning answer: Levi-Civita manages to win in tensor calculus, among others, Vitali and the analyst Pia Nalli with whom we will deal in the next chapter more widely, and who (on the 14th May 1928) writes him thus[88]:

> tra qualche giorno chiuderò il mio Corso di Analisi superiore: ho trattata tutta la materia contenuta nelle Sue lezioni ad eccezione dell'ultimo capitolo. Mi sono obbligata allo studio del Calcolo assoluto che non avevo mai potuto tollerare per il peso delle formule, ma ora, a studio fatto, mi sono convinta che tale peso non esiste più, grazie alla introduzione della nozione di parallelismo che semplifica tutto in modo meraviglioso. Posso dire che i miei alunni di ambo i sessi, che sono sì ragazzi intelligenti, ma non sono delle aquile, si sono formate senza sforzo delle idee chiare, al punto che hanno studiato da soli, senza mie spiegazioni, gli ultimi due capitoli VIII e IX. Modestamente posso dire che ciò che Ella ha tanto semplificato con la introduzione della nozione di parallelismo, io ho ancora semplificato un pochino per la mia innata qualità di semplificare tutto ciò che è semplificabile[89].

Between the years 1923 and 1927 appear[90] the three volumes of the *Lezioni di Meccanica razionale*, written in association with Ugo Amaldi (1875–1957) – that remains a

[86] Cf. L. Ricca, Rediscovery of Da Rios equations, *Nature*, vol. 352 (15 August 1991), pp. 561–62; L. Ricca, The contribution of Da Rios and Levi-Civita to asymptotic potential theory and vortex filament dynamics, *Fluid Dynamics Research*, 18 (1996), pp. 245–268. Besides his papers, Levi-Civita's interests in the field of flat hydrodynamics are described also through the works he suggests to his students. In this case we quote at least J. Struik (Rockfeller bursar in Rome in 1924–1925) and A. Weinstein (Rockfeller bursar in Rome in 1926–1927), who will become after his exile in Europe and in the U.S.A. because of Nazi purges – a real authority in the field of hydrodynamics.

[87] Stock, Rome

[88] Levi-Civita Archive, Accad. Lincei.

[89] In some days I will finish my course of Higher Analysis: I have dealt with all the material contained in Your lessons but for the last chapter. I have forced myself to the study of absolute Calculus, that I could never endure because of the burden of formulas, but now I have finished, I am convinced that such burden does no more exist, thanks to the introduction of the idea of parallelism that simplifies all in a wonderful way. I can say that my students of both sexes, who are intelligent but not genius, have built without effort clear ideas, to the point that they have studied on their own, without my explanations, the last two chapters VIII and IX. I can humbly say that I have further simplified a bit what You had so much simplified with the introduction of the idea of parallelism, thanks to my innate quality of simplifying all that can be simplified.

[90] Zanichelli, Bologna

fundamental and unequalled work within the Italian mathematical literature of the period. Even spanning an impressive field, the development of the concepts is wide and deep and the formal developments elegant and sober. As M. Borns writes to him[91]: "currently we have in German language no work of equal proportions and of equal importance". In 1924 are issued – in two different editions, Italian and German, the *Questioni di Meccanica classica e relativista* (that gather the texts of a series of speeches given in Spain in 1921) and later the *Fondamenti di Meccanica relativista*, compiled also by Persico, follow. And the work *Sur les chocs dans le problème des trois corps* can also be considered an arrangement work, presented to the *Proceedings* of the II Applied Mechanics Conference (Zurich 1926), in which Levi-Civita, resuming his brilliant researches on the subject, returns to the regularization of the equations of motion of three bodies in a plane[92]. For L. Sedov, one of the fathers of Soviet astronautics, Levi-Civita's works

> in the field of analytical mechanics have contributed to the basis of further development of celestial mechanics and theoretical mechanics in general. His famous works related to regularization of equations in the three body problem, the results connected with transformation and integration of Hamiltonian equation systems and his works establishing some important characteristics of instability of solutions for some interesting cases of canonical systems became classical and were included in manuals on analytical and celestial mechanics and gave the leading ideas for modern research in these fields[93].

The just mentioned two volumes – the 1921 *Questioni di Meccanica classica e relativistica* and the 1928 *Fondamenti di Meccanica relativistica* – offer the chance to return to Levi-Civita's strong interest towards relativity, at which we hinted in chapter III when recalling the meaningful letter exchanged with Einstein in the Spring 1915. Only a few months and Levi-Civita will publish *Sulla espressione analitica spettante al tensore gravitazionale nella teoria di Einstein*[94], where he presents the first Italian version of gravitational field equations.

> L'idea di un tensore gravitazionale fa parte della grandiosa costruzione di Einstein. Però la definizione propostane dall'Autore non può riguardarsi definitiva. Anzi tutto, dal punto di vista matematico, le fa difetto quel carattere invariantivo che dovrebbe necessariamente competerle secondo lo spirito della relatività generale. E anchepiù grave è il fatto, avvertito con fine intuito dallo stesso Einstein, che se ne trae una conseguenza fisica inaccettabile a proposito delle onde di gravitazione. Per questo punto egli trova tuttavia un accomodamento nella teoria dei quanti. In

[91] The letter, dated 22nd September 1923, is kept in Levi-Civita Archive, *Accademia dei Lincei*.

[92] For a more recent analysis of Levi-Civita's (and his student Giulio Bisconcini's) contributions on the problem of the three bodies, cf. J. Barrow-Green, *Poincaré and the Three Body Problem*, AMS, 1997.

[93] Cf. L. I. Sedov, Analitycal Mechanics in Works by Tullio Levi-Civita, in *Atti del Convegno celebrativo del centenario della nascita di Tullio Levi-Civita*, Rome, 1975, pp. 257–268.

[94] *Rend. Acc. Lincei*, v. XXVI (1917), pp. 381–391 (381).

verità la spiegazione è meno riposta: tutto dipende dalla non corretta forma assunta per il tensore gravitazionale[95].

It is the choice of non-tensorial quantities that, according to Levi-Civita, leads Einstein to physically unacceptable consequences. Einstein receives in Switzerland – at A. Hurwitz's home – a copy of Levi-Civita's note, and rushes to write back a long letter (2nd August 1917) trying to explain why the energy of the gravitational field cannot be represented with a tensor. But the effective answer is *Gravitationswellen*[96], in which Einstein openly admits that his previous approach to gravitational waves was not clear and suffering from a regrettable error and expressly dedicates the last paragraph to Levi-Civita so as to acknowledge the importance of the mathematical contribution (even if he still does not share the viewpoint about conservation laws).

Levi-Civita, instead, continues to think that mathematical technique can give an effective contribution to the theory's development and between 1917 and 1919 issues in the *Atti dell'Accademia dei Lincei* a series of notes aiming at integrating in first and second approximation Einstein's gravitational equations. At the same time, he does an intense activity of divulgation starting from the Roman speech *Come potrebbe un conservatore giungere alla soglia della nuova meccanica*[97] – soon translated into French and Spanish. During the 1920s Italian relativists are quite a small number. Even among mathematicians prevail scepticism and diffidence, that a volume as *Espaces courbes – Critiques de la Relativité* (published in 1924 by Burali-Forti and Boggio) rushes to represent with a violent and fierce critique against mathematical and physical aspects of relativity. Levi-Civita and Castelnuovo are among the most illustrious targets of this controversy that sees the so-called *vectorialists* oppose the supporters of *absolute differential calculus*.

Actually, Levi-Civita is the main "responsible party" for the spreading of relativity in Italy. The new theory is almost "reinterpreted" by him according to Einstein general relativity originated from the rule of local equivalence between inertia and gravitation, when it is specified through tensor calculus; to Levi-Civita, instead, relativity is the last step of a logical process that tends to express the laws of physics through variational principles and the use of differentiable varieties as spaces representative of the physical world. Levi-Civita interprets the theory as a mathematician and not as a physicist and this explains his minor attention to narrow relativity, the continuous references to the formal resemblances with the mathematical principles of optics and the continuous comparison with the results and rules of classic mechanics. C. Cattaneo has gone so far as to state that

95 The idea of a gravitational tensor is part of Einstein's great construction. But the definition proposed by the Author cannot be considered as definite. To begin with, from a mathematical perspective, it lacks the invariant nature that should necessarily belong to it according to the spirit of general relativity. And even most important is the fact, perceived with subtle insight by Einstein himself, that an unacceptable physical consequence is drawn in regard to gravitation waves. For this point he finds though a mending in the quantum theory. Actually the explanation is less hidden: all depends on the incorrect form taken for the gravitational tensor.

96 *Preussische Akademie der Wissenschaften Sitzungsberichte*, pt. 1 (1918), pp. 154–167.

97 *Rend. Sem. Mat. Roma*, vol. V (1919), pp. 10–28.

for Levi-Civita exists a principle of correspondence on whose base "every solution of the classic theory of the potential gives way to a well determined Einsteinian solution of first approximation and vice-versa. (…) Between classical solutions and Einsteinian solutions of first approximation there is then a biunique relationship" [98]. Besides, in the already mentioned Roman speech of 1919, Levi-Civita reaffirms that the mission of scientists is that of preserving carefully the well-established intellectual wealth and of examining with severe critical mind all that can lead to a change. The wealth that he would like to preserve is the mechanics of Newton, d'Alembert, Lagrange, with which – using certain valid generalizations – the plausibility "of slight changes of classic laws, quantitatively small but speculatively remarkable, perceived by Einstein and that he himself arranged systematically" is disclosed. This has been called Levi-Civita's *conservative* position – the more relevant if we think about the role he developed in the penetration of relativistic ideas in Italy – that will be adopted by Enriques (and by Somigliana in a much more radical form) too. This, more than to a minimalist inclination, bent on reducing at the maximum the revolutionary range of the new theory, was due to the idea that it was not a "global" revolution, such as to disturb *ab imis* the way to face physical phenomenology, but a new way of describing the phenomena that did not oppose the spirit and the program of rational mechanics, rather it enriched and elevated it (the distinction between "quantitatively small but speculatively big" changes "deposes in this sense").

To analytic mechanics, almost in close relation with the problem of the three bodies, belongs Levi-Civita's other important contribution in these years: the arrangement of the method of adiabatic invariants and their possible application to celestial mechanics. The notion of adiabatic invariant had been introduced by the Dutch physician P. Ehrenfest in atomic mechanics so as to justify the quantum postulates of N. Bohr (and of A. Sommerfeld) and make possible the application to more complex models of atoms. The grounds of the real interest in Ehrenfest's *principle of the adiabatics* are manifested in two works[99] by Enrico Fermi, just returned from a study journey in Germany and Holland, where he had spent some weeks just in P. Ehrenfest's laboratory. There is evidence of Fermi's stay in Göttingen, and of the favourable impression left on M. Born, in a letter of his to Levi-Civita dated 22nd September 1923[100].

> Il semestre scorso è stato da noi un giovane italiano, il Dr. Enrico Fermi, che è eccezionalmente dotato e su di cui vorrei attirare la sua attenzione. Posso prevedere con certezza che il signor Fermi otterrà importanti risultati. La sua amabilità ci ha reso la sua personalità molto cara.

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98 Cf. C. Cattaneo, Leggi classiche e leggi relativistiche nel pensiero di Tullio Levi-Civita, *Tullio Levi-Civita. Convegno internazionale celebrativo del centenario della nascita* (Rome, 17–19 December 1973), Rome, Accad. Naz. Lincei, 1975, pp. 113–125 (122).

99 Cf. E. Fermi, Il principio delle adiabatiche ed i sistemi che ammettono coordinate angolari, *Nuovo Cimento*, 25 (1923), pp. 171–175 (but *Collected Papers*, Rome 1961, pp. 88–91); Alcuni teoremi di Meccanica analitica importanti per la teoria dei quanti, *Nuovo Cimento*, 25 (1923), pp. 271–285 (but *Collected Papers*, Rome 1961, pp. 92–101).

100 Levi-Civita Archive, *Accademia dei Lincei* (our translation).

> Quest'anno ci è impossibile andare in Italia perché la situazione politica ed economica è molto peggiorata ed è al momento molto incerta.
> La nostra visita a Bonn ci ha messo chiaramente davanti agli occhi come funziona l'occupazione francese. Abbiamo visto non solo come gli ufficiali e la truppa d'occupazione vivono nel lusso e nel superfluo mentre la popolazione si trova in grande miseria, ma ci siamo fatti pure un'idea del terrore che esercitano i tribunali militari francesi. Nessuno lì è sicuro che improvvisamente non appaiano in casa sua soldati francesi per portarlo via assieme alla sua famiglia. La gente di solito non può portare con sé nient'altro che la biancheria personale e qualche libro. Tutti i mobili e gli utensili sono portati via dai francesi senza indennizzo. Può immaginare l'angoscia che si impadronisce in questi casi specialmente delle donne. Particolarmente irritante è per noi la vista di molti soldati di colore, specialmente marocchini, occupare la città dove è nato Beethoven e dove Heinrich Hertz ha fatto i suoi famosi esperimenti. Fino a quando il mondo accetterà questa vittoria dell'ingiustizia e della violenza?[101]

Fermi reaches the conclusion that the application of the *principle of adiabatics* cannot be excluded a priori for almost-ergodic systems; it is excluded, though, the possibility of an extension of Ehrenfest's principle to almost-ergodic systems (accurately indicated, except for some exception). The impasse for "old" quantum mechanics is therefore evident and a good number of physicists begin to work so as to overcome it. From this point of view, wave mechanics (that in 1926 had just been announced with the well-known works of E. Schrödinger in the *Annalen der Physik*) seems to reshuffle the cards. It is at this point that Levi-Civita, worried about the premature disposition of Italian physicists towards one of the horns of the dualism wave-corpuscle, takes up a position on the foundational problems of quantum mechanics. His speech (just at the International Congress of Physicists, summoned in Como from 11th to 20th September on occasion of the first centenary of Alessandro Volta's death) *Sugli invarianti adiabatici*, distinguishes itself not so much for the adjustment of mathematical formalism as for the thoughts regarding dualism and for the possible applications to celestial mechanics. Levi-Civita

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101 Last semester a young Italian, Dr. Enrico Fermi, who is exceptionally talented and on whom I would like to attract your attention, was among us. I can certainly anticipate that Mr Fermi will obtain important results. His friendliness has make his personality very dear.
This year it is impossible to go to Italy because the political and economic situation has become quite worse and it is currently very uncertain.
Our visit to Bonn has put us clearly before our eyes how does French occupation works. We have seen not only how officials and the occupation troops live in luxury and in the superfluous while the population is in great poverty, but we have got the idea of the terror that French military tribunals exercise. Nobody there is sure that suddenly French soldiers do not appear at his home to take him away together with his family. People usually cannot take with them nothing more than personal linen and some books. All furniture and utensils are taken away by the French without no compensation. You can imagine the distress that seizes specially women. Specially irritating is for us the view of many colour soldiers, particularly Moroccan, occupying the city where Beethoven was born and where Heinrich Herzt made his famous experiments. Until when will the world accept this victory of injustice and violence?

does not intend to give up his primitive point of view, that associates a unique quantum principle to ordinary mechanics. It is certainly a hybrid order and from this point of view "odious to God and to his enemies", but undoubtedly fascinating as it answers to elementary and concrete forms of physical intuition, and above all it is fit for leading to quantitative relations in the most simple way with the usual methods of analytical mechanics". Levi-Civita returns to the dualism waves-corpuscles in that precious booklet *Caratteristiche dei sistemi differenziali e propagazione ondosa*[102], which collects the text of a series of speeches on characteristics theory organized by the *Seminario matematico* of the university of Rome in the year 1930–1931 and in which – above all in the final chapter, *Il dualismo onde-corpuscoli della Fisica moderna, secondo de Broglie* – continues at a distance the comparison with Fermi, author in 1928 of an *Introduzione alla fisica atomica*[103]. Levi-Civita recalls how the wave hypothesis is not able to explain some phenomena (photoelectric effect and Compton effect) without recourse to Einstein's corpuscular and quantal hypothesis. In its turn, the corpuscular hypothesis by itself seems not to be able to explain the phenomenon of electron diffraction into crystals, discovered in 1927. Instead, the study of differential equation systems in their usual form allows linking them to *characteristic varieties* (wave surfaces) and to *bicharacteristic lines* (trajectories) which offer a scheme large enough to reflect both wave and corpuscular aspects of the same phenomenon: "we have thus a comprehensive mathematical scheme, and, in its agnosticism, wholly satisfactory, of that dualism between waves and corpuscles that inspired de Broglie's genial insights and of which de Broglie himself and others in vain searched a concreter model, really concordant with the examined facts"[104].

Let's return to adiabatic invariants to remark how these are the subject on which Levi-Civita's attention concentrates after the Congress in Como. Thus, for the academic year 1927–1928, he contemplates a course on this subject for the *Seminario matematico* of the university of Rome. He also gives three speeches on adiabatic invariants in the almost-ergodic case at the university of Hamburg and a *talk* at the International Congress of Bologna titled *Applicazioni astronomiche degli invarianti adiabatici*. The applications pertain essentially to the problem of two bodies with variable mass and to the problem of two gravitating and rotating bodies. In 1934 Levi-Civita resumes his (and his scholars') work in an important memoir: *A general survey of the theory of adiabatic invariants*, published in the *Journal of the Massachussetts Institute of Technology*. It is also the right moment for the *Seminario Hadamard* to dedicate one session to Levi-Civita's work on adiabatic invariants, with an acknowledgement justly disclosed by J. Dieudonné[105]:

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[102] Zanichelli, Bologna, 1931 (edited by Giovanni Lampariello).

[103] Zanichelli, Bologna.

[104] Cf. T. Levi-Civita, *Caratteristiche dei sistemi differenziali e propagazione ondosa*, Zanichelli, Bologna, 1931, p. VII.

[105] Cf. J. Dieudonné, The work of Nicolas Bourbaki, *The American Mathematical Monthly*, 71 (1970), n. 2, pp. 134–145 (136).

> Our only opening onto the outside world at this time was the seminar of Hadamard (…). He had the idea (apparently taken from abroad, because this had never been done in France) of inaugurating a seminar of analysis of current mathematical work. At the beginning of the year he distributed to all those who wanted to speak on the subject, what he judged to be the most important memoirs of the past year, and they had to explain them at the black-board. It was a novelty for the time, and to us an extremely precious one, because there we met mathematicians of many different origins. Also, it soon became a center of attraction for foreigners; they came in crowds.

Chapter 7

The 1930s move forward

1. Introduction

This time we have "shifted forward" the time boundaries of the history of mathematics, closing the last chapter with 1932, the year of the Congress of Zurich. The picture that arises portrays Italian mathematics as an ever living and active subject, with a strong international recognition and a numerical basis that is gradually developing, but showing as well some signals of decline. We broached this subject in the last chapter: these "warning signals" are conspicuous if we compare the situation with the one Italian mathematics itself registered at the beginning of the century and if we measure its growth rate – in the considered period – with that of other ("old" or new) national schools. At the beginning of the 1930s nobody wonders about the grounds for the decline yet, so neither does anybody prepare rectifying measures. The awareness of the decline will not take long to unfold. On the other hand, the decline in the number of students that come to Italy to study mathematics will be quite tangible evidence.

The considerations, and the outlines of a policy bent on reversing the trend, which are one of the main grounds of interest for the observer of scientific progress, will begin thus: are there (beyond the greater or smaller brilliance of single researchers) reasons that can explain the changing fortunes of a mathematical school? Are there policies which can actually increase the research quality, even in such a field as mathematics, apparently so little manageable from the outside?

Chapter 5 has pictured the image of Severi's *leadership*, already made offical and further legitimated by his appointment to the Academy of Italy, the only representative of the mathematical world. The regime has made its choice, recompensing for the success of a certainly self-confident personal and political pathway. Severi was a socialist at the outbreak of war, was still a socialist when he arrived at Rome (even if with some tension created by the first world war); he had certainly not aligned himself with the regime when he resigned as rector of the university of Rome, soon after Matteotti's case. Only a few years have gone by, but in this Italy time passes quickly. No sooner has Severi's leadership itself developed and consolidated than, as we already noticed in the last chapter, movements and slippage in the landscape begin to occur.. There is no *golpe* in view. It is much easier to see that, compared to what we could call the previous *Volterra period*, the Italian mathematical world is more differentiated and Severi does not manage to keep it easily under tight control any more. His prominent position arouses now less support (not to mention the old usual envy). Thus, Tonelli and the *Normale* remain outside *Severi's orbit*. Thus, the CNR's mathematical group can put forward a politics of its own, replacing the U.M.I.'s, regarded as static and conservative. Thus, the admirable experience of Picone's INAC – but also the creation of a group of power – starts from the C.N.R. We are always talking about mathematicians wholly aligned with fascism's stances – Picone, rather, boasts that he was a *black shirt* since the very beginning – but the comparison between pure and applied mathematics will be equally lively and not always ascribable to a simple division of sides.

We titled the last chapter *The C.N.R. alternative.* Actually, a tragic external event, such as a war, was necessary to bring about a change in the mathematical world, as well as the world at large. At first the system resists and on the whole, during the 1930s becomes increasingly cohesive. In this chapter, we will make a journey through a period distinguished by the theme of *continuity*.

Continuity seems to distinguish the political sphere as well. For most of the 1930s, the regime registers the highest approval from Italians. The tenth anniversary of the advent of fascism is celebrated, and the difficult years of Matteotti's case, of the cancellation of all political freedom and even of an authoritative tightening within the fascist party, seem to be completely abandoned. Now the image of the regime, and of its achievements, is emblematically portrayed by a triumphant parade along the streets of New York, with which is celebrated in 1933 the transoceanic flight of a fascist party of-

Demonstration of a popular support towards the fascism (Bologna, 1936) ficial, Italo Balbo. It is a time of massive collective public congregations, in which large multitudes hail the Duce. His image and his voice become extremely familiar to all Italians thanks to newsreels and to the new radio machines. It is a time of satisfied appreciation for the great public works done by the regime. Its originality, along a "third path" that neatly overcomes the difficulties of liberal thought and of communism, is theorized and developed within a new economic theory, *corporatism*. Even the Empire arrives, with aggression unleashed toward Ethiopia in 1935–1936. Italians respond to their international political isolation by giving to the Motherland the gold of their families. For the moment, wars are won: in 1936 in Spain and in 1939 by annexing Albania. Everything seems to suggest the beginning of a long era of linear progress upwards. If in our reconstruction of the thought and of the mathematical events of the 1930s we accentuate the tension and particular "discontinuities", it is not because such events are seen at the time to be of significance, but because that significance will gradually unfold. It will give to mathematical Italy a background for alternative choices when the tragedy of war – and before it, of racial laws – will be seen to sweep away existing structures and their only outward steadiness.

2. Geometry

In Chapter 5 we have introduced geometry – especially algebraic geometry – as the *queen* of Italian mathematics. It is the most noble discipline, the one in which we can better see a way of thinking that goes beyond technical aspects and talks also to other cultures. However, the view the 1920s offered was different from the one at the beginning of the century. Above all, geometry had "lost" Castelnuovo and the triumvirate had become bipolar, revolving around Enriques and Severi's strong personalities. Then, while sectors such as differential geometry were developing and interesting horizons were opening towards more specifically algebraic approaches, Italian Algebraic geometry went on meditating on itself considering specially the writing out of some treatises. A need was perceived to organize the material developed through years of hard work and brilliant intuitions; this intuition had to be given a firmer and more rigorous basis so as to bear comparison with the growing use of new languages, a development which did not reject the achievements of the Italian school but affirmed the urgent need to present them as a unique and unified whole.

In essence, this tapestry does not change during the 1930s. Algebraic geometry remains the most practised subject and continues to revolve around Enriques and Severi and their schools. Now for the "internal" reasons recalled earlier, now for other reasons – Enriques is sixty, Severi is over fifty; both are public figures, with many other commitments and activities – their energies are mainly expended in research on foundations and in the writing up of treatises.

During the 1930s, the main landmarks of Severi's foundational research are two considerable works of 1932 ("La serie canonica e la teoria delle serie principali di gruppi di punti sopra una superficie algebrica"[1] and "Un nuovo campo di ricerche nella geome-

[1] *Comm. Math. Helv.*, 1932, pp. 268–326.

tria sopra una superficie e sopra una varietà algebrica"[2]) and the "German" memoir of the following year: "Ueber die Grundlagen der algebraischen Geometrie"[3], coming in 1942 to the publishing of the treatise: *Serie, sistemi di equivalenza e corrispondenze algebriche*[4], that collects the lectures given at the *Istituto di Alta Matematica* during his first two working years[5]. Severi continues with his attempt to extend the classical methods of the Italian school to the geometry of any algebraic variety whatsoever, generalizing the concept of *linear equivalence* (already defined for hypersurfaces of a given algebraic variety) to the subvarieties of any dimension whatsoever. Especially the first of the two 1932 articles mentioned above introduces new invariants that – released from the property of linearity – can happily conform to the extension to varieties of a higher dimension. The second article of 1932 introduces, in this sense, the idea of *series of equivalence*, coming then to the more general idea of *system of equivalence*: the objective is the methodical study of the geometric properties linked to the invariants associated to the series of equivalence. The next step is the use of this structure to face some of the more complex problems of geometry on a variety. And it is not by chance that, among these, is the *problem of the base* for curves on a given surface, already solved by Severi at the beginning of the century: considering the whole of algebraic varieties of a given dimension contained in a variety, is it possible to find a finite number of varieties such that any other variety is *algebraically* equivalent to an integer linear combination of the varieties in the previous finite set? The answer is positive, but the theorem of the base is proved for another type of equivalence that Severi calls *arithmetic* equivalence. A. Brigaglia and C. Ciliberto give a global valuation of the research[6].

> Il punto di arrivo della teoria di Severi dei sistemi di equivalenza, quale appare al termine della sua evoluzione, è quello dello studio comparato dei vari tipi di equivalenza definibili tra sottovarietà pure di data codimensione di una varietà pura assegnata. Precisamente Severi chiama *varietà virtuale* una somma di varietà, cioè un elemento del gruppo abeliano libero generato da tutte le varietà pure di data codimensione. Severi considera poi il gruppo di tali varietà virtuali modulo le equivalenze di vario tipo (algebrica, razionale ecc.) e ne studia le proprietà in relazione con i problemi di classificazione. In definitiva la sua teoria dà luogo ad un ampio apparato difficilmente accettabile da una comunità internazionale che già aveva incominciato a porre in dubbio la solidità dei fondamenti della Geometria algebrica italiana e discuteva perfino concetti basilari come quello di molteplicità di intersezione. Si tenga inoltre conto che le effettive applicazioni che Severi e la sua scuola

[2] *Mem. Acc. Italia*, 1932, pp. 1–52.

[3] *Abhand. aus dem. math. Sem. der Hamburgischen Univ.*, 1933, pp. 335–364.

[4] Cremonese, Roma (edited by F. Conforto and E. Martinelli).

[5] The initial project scheduled the printing of several volumes, which are actually issued but only in 1958 (the second) and the year after (the third). About the *Istituto di Alta Matematica* we will talk in a more detailed way in the next chapter.

[6] Cf. A. Brigaglia, C. Ciliberto, Geometria algebrica, in S. Di Sieno, A. Guerraggio, P. Nastasi (eds.), *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, quoted, p. 214.

riescono a dare si limitano a poco più che un tentativo di estensione al caso delle superficie della teoria delle corrispondenze tra curve, argomento considerato marginale nello stesso ambito della scuola italiana. Molte parti della teoria sono, a giudizio dello stesso Autore, appena abbozzate e non rigorosamente dimostrate[7].

Severi is quite aware of the difficulties he finds, and for a long time he feels unsatisfied about his most recent elaborations, so much that he will continue to propose new definitions and variations. In an article from 1933, for instance, he writes[8]:

> In questa e nelle note successive elaborerò ex novo la teoria delle serie di equivalenza di gruppi di punti sopra una superficie algebrica. L'argomento era troppo importante ed attraente, perché non mi sentissi spinto ad approfondirne con sempre maggiori cure i tratti fondamentali. Le definizioni, cioè i concetti (…) sono i veri elementi costruttori ed operativi nella matematica. Pertanto tutti i miei sforzi sono stati diretti ad approssimarmi successivamente alla più naturale definizione di serie di equivalenza, che rivelasse, sulla superficie, proprietà, per quanto possibile, analoghe a quelle considerate sulle curve per mezzo delle serie lineari[9].

At the same time, he has no doubt about the underlying validity of his geometric-functional approach[10].

> Più volte mi sono occupato dei fondamenti della geometria numerativa. Vi ritorno oggi, perché la teoria dei sistemi d'equivalenza, l'uso sistematico del concetto di

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[7] The endpoint of Severi's theory of systems of equivalence, as it appears at the end of its evolution, is that of the comparative study of the various types of equivalence definable among pure subvarieties of a given codimension in a given variety. Precisely, Severi called a formal sum of varieties, i.e. an element of the free abelian group generated by all the pure subvarieties of a given codimension, a *virtual variety*. Severi then considered the group of such varieties modulo the equivalences of various types (algebraic, rational, etc.) and studied those of their properties related to problems of classification. In total, Severi's theory required a large amount of machinery that was hard to accept by an international community that had already begun to place in doubt the solidity of the foundations of Italian algebraic geometry and was disputing even basic concepts like that of intersection multiplicity. Furthermore one should take into account that the effective applications of this theory that Severi and his school had succeeded in giving were limited to little more than attempts to extend to the case of surfaces some of the theory of correspondences among curves, subjects considered marginal in the sphere of the Italian school itself. Many parts of the theory were, in the judgement of its author, barely sketched out, and not rigorously proved.

[8] F. Severi, La teoria delle serie di equivalenza e delle corrispondenze a valenza sopra una superficie algebrica (7 Note), *Rend. Acc. Lincei*, 1933, 419–425, 491497, 597–600, 682–685, 759–764, 869–876, 876–881.

[9] In this and in later notes I will elaborate ex novo the theory of series of equivalence of groups of points on an algebraic surface. The subject was too important and appealing, thus I felt pressed to analyze thoroughly and with always the greatest care its essential features. The definitions, that is the concepts (…) are the really constructive and effective elements in mathematics. Therefore all my efforts were directed towards bringing myself via successive approximations to the most natural definition of series of equivalence, that would reveal, on a surface, properties as far as possible analogous to those considered on curves by means of linear series.

[10] F. Severi, I fondamenti della geometria numerativa, *Ann. Mat. Pura e Appl.*, 1940, 153–240.

> varietà virtuali, la risoluzione del problema generale della base e la teoria geometrico-funzionale delle corrispondenze, vanno sempre più elevando le questioni enumerative dal modesto rango di determinazioni del numero delle soluzioni di quel problema (determinazioni che tuttavia originarono in più d'un caso progressi sostanziali e concettuali) a quello dei rapporti funzionali e topologici.
> E vi ritorno anche perché mi pare necessario di lumeggiare un'altra volta il valore definitivo e rigoroso dei metodi della geometria italiana. Che la geometria algebrica sia altresì veduta dal punto di vista dell'algebra moderna – come fa per esempio Van der Waerden nei suoi interessanti lavori – è indubbiamente un bene per l'algebra e per la geometria. E c'è da augurarsi che i mezzi penetranti della "Moderne Algebra" siano presto usati per attaccare problemi essenzialmente nuovi, piuttosto che per ricostruire soltanto risultati già scoperti per via geometrica.
> Ma comunque, non bisogna che queste ricostruzioni siano presentate in modo da lasciar credere che quanto si è fatto con i metodi italiani non sia acquisito definitivamente e con ogni precisione[11].

Severi's works are resumed or, in a sense, anticipated by his pupils. It is the case of Comessatti who, in 1932, gives an important contribution to the theory of invariance of systems of equivalence. It is still Comessatti, with B. Segre, who generalizes the notion of canonical system to the varieties of a higher dimension. The work of Albanese[12] of 1934, dedicated to the extension of the classic theory of correspondence among curves to surfaces, introduces the variety that A. Weil in 1950 named *Albanese variety* (which will prove to be the dual of the so-called *Picard variety*). On the contrary, some of B. Segre's contributions concern precisely geometry on a variety and the series of equivalence with the application to non-regular intersection problems between varieties, the fundamental theorem of irregular surfaces and – we are in 1938, the year of racial laws and B. Segre's exile to England[13] – the splitting principle.

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11 I have dealt many times with the foundations of enumerative geometry. I come back to this topic today, because the theory of systems of equivalence, the systematic use of the concept of virtual variety, the resolution of the general problem of the base and the geometric-functional theory of correspondences, have more and more elevated enumerative questions from the modest status of determinations of the number of solutions of that problem (determinations that however developed substantial conceptual progress in more than one case) to that of functional and topological relationships.
And I return to it also because I think it is necessary to illuminate once again the definite and rigorous value of the methods of Italian geometry. That Algebraic geometry is seen from the point of view of modern algebra – as does Van der Waerden in his interesting works, for instance – is undoubtedly a good thing for algebra and for geometry. And one hopes that the penetrating methods of "Moderne Algebra" be soon used to attack essentially new problems, rather than only to reconstruct results already discovered through geometry.
But anyhow, it is not necessary that these reconstructions be presented in such a way as to let one believe that what has been done using Italian methods is not definitely acquired, with all accuracy.

12 Corrispondenze algebriche fra i punti di due superficie algebriche, *Annali Scuola Normale Pisa*, 1934, 1–26 and 149–182.

13 A mention of this sad event in Segre's life can be found in R. Siegmund-Schultze, *Rockefeller and the Internationalization of Mathematics between the two World wars*, Birkhäuser, 2001, p. 200.

We have already recalled how the Thirties for Enriques are above all the years of *treatises*, too: in 1934 the publishing of the *Teoria geometrica delle equazioni e delle funzioni algebriche*, with Chisini, is completed; two years before the *Lezioni sulla teoria delle superficie algebriche* (from which *Le superficie algebriche* of 1949 "will emerge") had been issued, with Campedelli, who in those same years proves the so-called *fundamental classification theorem for surfaces*: the necessary and sufficient condition for a minimal surface (with no exceptional curves of the first type) to be rational or ruled is that it contains curves having negative intersection with the canonical curves; lastly, in 1939 are issued *Le superficie razionali*[14]. The controversy with Severi, that certainly was already simmering, breaks out in regard to treatises and handbooks. From this point of view, the episode of the call to Rome in 1921 (depicted in Chapter 3) is illuminating. Personal rivalries and ambitions, made even more scathing because of the initial master-pupil relationship, confront each other and collide.

It is Enriques who lights the fuse, by publishing in the *Periodico di Matematiche*, in March 1934, a severe review of Severi's *Lezioni di Analisi*, accused of excessive abstractness and generality. Severi's reply arouses a "symmetrical" criticism of lack of rigor against Severi, remarking how in the *Trattato* written in collaboration with Chisini "the proofs of fundamental theorems are often only approximate". In his reply, Severi introduces also the subject of priority and calls B. van der Waerden as a witness to corroborate that the first rigorous proof of the Plücker-Clebsch criterion is his own. Of course Enriques disagrees. The year after, again in the *Periodico*, he insists on his own credit regarding the Plücker-Clebsch criterion. There is another final reply from Severi and the controversy ends. The subject is rigor and the difference, dear to Severi, between the mathematician who discovers fundamental ideas heuristically and the logician who simply has to arrange these ideas and *make figures balance*[15]:

> Se pur ci si fermasse qui l'autore di una trattazione più generale non potrebbe ad ogni modo annullare questo precedente, poiché – quand'anche ci si ponga dal punto di vista del logico che non ritiene acquisita la conoscenza di una verità finché non sia rigorosamente dimostrata – recherebbe soltanto un complemento o una maggiore estensione a ciò che è stato stabilito da altri. Ma (…) questo complemento è pure indicato enunciando la forma geometrica più generale del principio di Plücker-Clebsch e segnando la via della dimostrazione[16].

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[14] Zanichelli, Bologna, 1939 (edited by F. Conforto).

[15] Cf. F. Enriques, O. Chisini, Sul principio di Plücker-Clebsch, *Periodico di Matematiche*, 15 (1935), pp. 276–283 (277).

[16] Even if the author of a more general treatment stopped here, he could not in any case declare void this precedent, since – when one also looks at it from the point of view of the logician who holds that the knowledge of a truth is not acquired until it is rigorously proved – it would provide only a complement or a major extension to that which had been established by others. But (…) this complement is also indicated by enunciating the more general geometric form of the Plücker-Clebsch principle and pointing out the ways to a proof.

It goes without saying that Severi's reply is heavily ironic, strewn with exclamations like: "poor me: the logician must be just me, Severi!"; or, regarding rigor: "I confess to be guilty of this indiscreet demand, but I believe myself to be in good company with many mathematicians, great and insignificant!"; or still, as regards to the "pointing out the way to a proof" in Enriques' previous quotation: "the illuminating rocket, that should be enough to indicate the way to those that know how to fly, but not to the poor in spirit, who proceed with the leaden feet of the logician (…)".

It is Severi then who has the final say in the controversy. It is Severi, on the whole, who continues to pull the strings of the geometric world – and not only there – even if Enriques' figure is confirmed in its respectability and authority (and Italian geometry goes on branching out into numerous research areas). It is Severi who closes the 1930s with the organization of two important international conferences, about which we will talk in the next chapter.

3. Analysis

If we would like to sum up the situation of Italian analysis in this decade in a few lines, we could resort to three indexes: the strengthening of Picone's applicative project and of his *Istituto Nazionale per le Applicazioni del Calcolo*; the crystallization into two blocks – the Roman one, precisely, and Tonelli's in Pisa – with an increasing coldness in the relationships, that will lead to a clamorous controversy between the leaders; the gradual appearance on the scene of a new generation of researchers that will widen, even from a geographic perspective, the framework within which we have moved until now.

We have portrayed Tonelli as the most successful postwar analyst. Within a few years, he will consolidate his position definitively, thanks to his still young age and to a reputation consecrated by the popularity of his researches on real analysis at the beginning of the century, and later strengthened by the publication of *Fondamenti di Calcolo delle variazioni*. With his passage to Pisa in 1930, the Tuscan city becomes the main centre of analysis studies in Italy. The hard organizational work carried on in Pisa, the first signals of poor health and a clear boycott resulting from his political stances – even before his signature at the bottom of the *Croce manifesto* – are the reasons that lead to a slowdown in his scientific production during the 1930s. Tonelli partially divests his researcher's identity to put on, with more and more constancy, that of scientific manager and organizer of the *Normale* and the mathematical Institute of Pisa. He has also the responsibilities of a leader. The commitment is particularly exhausting because the environment of isolation and containment induced by his previous political stances have alienated Tonelli from the ministerial political and administrative power centres. Pupils of Tonelli are the loyal Silvio Cinquini[17] (1907–19), Basilio Manià (1909–1940), Sandro

[17] Reference to Cinquini's scientific production in these years can be found in J. Mawhin's essay: "Boundary value problems for nonlinear ordinary differential equations: from successive approximations to topology" in *Development of Mathematics 1900–1950* (J.P. Pier ed.), Birkhäuser, 1994.

Faedo (1914–2001), Emilio Baiada (1914–1984), Landolino Giuliano (1914–1985) and also Giovanni Ricci[18] (1904–1973).

To Tonelli's school belongs Lamberto Cesari (1910–1990), its most illustrious exponent. His affiliation may seem problematic, and he himself will talk about "two great scientists, Leonida Tonelli and Mauro Picone, (…) of a particular meaning for me, as I have been under the influence of both of them" [19]. The fact is that Cesari graduates with Tonelli, in Pisa, in 1933. The difficulties that Tonelli's pupils met when looking for a place persuade him to spend a specialization year in Munich with O. Perron and C. Carathéodory, followed by another year in Pisa, before making the "big jump" and passing to Picone. In Rome, Cesari assumes the role of vice-president of the INAC. He will later teach at the universities of Rome, Pisa and Bologna, where he will win an appointment to the professorship in Infinitesimal analysis. But in Bologna, and in Italy, he will stay for a time. In 1948, following an invitation of M. Morse and T. Radò, he leaves for the U.S.A. as *visiting professor* and there he will settle permanently, some years later.

The American period is certainly the longest and the most important for Cesari's scientific activity. His international renown has been recognized and celebrated several times by friends, colleagues, and pupils. The three important treatises *Surface area* (1956), *Asymptotic behaviour and stability problems in ordinary differential equations* (1959) and *Optimization theory and Applications* (1983) refer, among others, to the American time.

While seeking for a quickly achieved scientific and professional independence, Cesari acts as a "bridge" between the two schools. At any rate, the tribute that Cesari himself pays to Tonelli is explicit. Equally explicit are, in the first half of the 1930s, the references to the Tonelli who in 1928 had published the *Serie trigonometriche* and had later run several courses on this subject. The covered topics concern the problem of representation of functions through their Fourier series and the double series theory. In particular, starting from his dissertation, Cesari tackles the search for sufficient conditions for a succession to be the succession of Fourier coefficients of an integrable function, generalizing conditions of W. H. Young, S. Szidon and A. Kolmogoroff, and achieving results obtained at the same time by C. N. Moore as well. He passes then to the study of double numerical series and to the comparison between their different ways of converging, so as to apply the results to Fourier's double series. It is here, regarding a possible generalization of Dirichlet-Jordan's theorem that he introduces a new class of two variables functions (which Tonelli will call *generally bounded variation functions*) proving that their Fourier double series converge almost everywhere for rectangles (and therefore also for lines and columns).

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[18] He was already in Pisa, when Tonelli moved there. Ricci's studies concerned differential geometry and analysis, but especially number theory.

[19] Cf. L. Cesari, L'opera di Leonida Tonelli e la sua influenza nel pensiero scientifico del secolo, in *Atti del Convegno celebrativo del centenario della nascita di Mauro Picone e di Leonida Tonelli* (Roma, 6–9 May 1985), Roma 1986, pp. 41–73 (41).

All these topics feel the effects of Tonelli's direct influence. Towards the end of the 1930s, Picone and the INAC, with their passage to Rome, will make their "appearance" with some research linked to technological applications, in particular to the theory of beams, and the study of problems linked to differential equations and to the stability of solutions.

Cesari's name is in any case tied, as regards his Italian years, to the study of bounded variation functions and especially to the analytical solution of the problem of the area of continuous parametric surfaces. Unlike those in ordinary form $z = f(x, y)$ – about which we talked when dealing with Tonelli – here the only general enough result was Radò's (1938), who had proved the weak inequality intervening between area and typical integral. His hypotheses were, though, particularly taxing. Cesari makes his debut by introducing the concept of total variation $W(T)$ for a continuous plane transformation $T: Q \to R^2$, through the consideration of the topological degree of a point with respect to a closed continuous curve. In the note "Caratterizzazione analitica delle superficie continue di area finita secondo Lebesgue"[20], Cesari proves the first fundamental theorem: given the continuous surface $S = \{(x, y, z): x = x(u, v), y = y(u, v), z = z(u, v); (u, v) \in Q\}$ and the three plane transformations T_i ($i = 1, 2, 3$) obtained by considering the preceding ones two by two, for the area L of S we get the inequalities:

$$W(T_i) \leq L(S) \leq \sum_{i=1}^{3} W(T_i) ,$$

from which it follows directly that the surface S has finite area if and only if the T_i are all of bounded variation on Q. In the note: "Sulla quadratura delle superficie in forma parametrica"[21], Cesari introduces the idea of *absolutely continuous* transformation, proving that, in general, the two conditions that characterize such transformations are independent and that the absolute continuity implies a total bounded variation. The last element is the generalization of the Jacobian. Unlike the unidimensional case, the bounded variation hypothesis for the continuous transformation $T: Q \to R^2$ is not sufficient to guarantee the existence of differential elements of the components of T. This remark "forces" Cesari to introduce a *generalized Jacobian* $J(u, v)$ that, always in the case of T of bounded variation, proves to exist almost everywhere in Q and be an integral function. It can thus prove the inequality:

$$L(S) \leq \iint_Q \sqrt{J_1^2 + J_2^2 + J_3^2}\, dudv ,$$

where J_i are the generalized Jacobians of the T_i transformations and where the equality holds if and only if such transformations are absolutely continuous. The frame is completed by extension of the representation theorem: each continuous surface with finite

[20] *Ann. Scuola Normale Pisa*, 1941, pp. 253–295 e 1–42.
[21] *B.U.M.I.*, 1942, pp. 109–117.

area has at least one parametric representation in which x, y, z are derivable, the generalized Jacobians are integrable, having:

$$L(S) = \iint_Q \sqrt{J_1^2 + J_2^2 + J_3^2}\, dudv\,.$$

Just like Tonelli, Picone is quite involved in managerial activity, running his group and managing the INAC. The time for research and the determination to achieve original results certainly diminishes, counterbalanced only partially by the stimulation arising from a close network of relationships, which helps to explain – together with a lively and tireless personality – how he manages to keep a high scientific production. Picone proves to be a specialist of partial differential equations. Many of the research subjects faced during the 1930s are expounded in the *Appunti di Analisi Superiore*[22], certainly the most important book among the numerous volumes published by Picone. The *Appunti* (more than 800 pages) are divided into seven chapters, dedicated respectively to analytic functions, to harmonic functions, to Fourier series, to Laplace functions and series, to some complements of harmonic functions, to the double Fourier and Legendre series and, lastly, to the study of some classical equations from Mathematical physics. The first edition is published in 1940, and contains elaborations and indications which will be, at least for some decades and for his students, a precious landmark, especially as regards the resolutive methods enunciated in the last paragraph of Chapter 7.

Picone used the classic method of Laplace transform for the first time in 1932. Later on will be introduced the particular Laplace transform with *finite* integration interval: $\int_0^t e^{-\lambda t} f(t)\, dt$ which, unlike the usual transform, is an entire function of λ and, even before beginning the integration, it allows us to avoid additional hypotheses on the behaviour ad infinitum of the to-be-transformed function, which is not always a consequence of the problem data and must therefore be tested a posteriori[23]. In the *Appunti*, Laplace's transform method is further considered and is placed, with the functional transformations depending on a discrete parameter, in a more general method of functional transformations that reduce the given problem to the integration of a differential equation containing some parameters, but depending on a less number of variables. Laplace transform, in particular, makes the solution of a boundary value problem rest on the knowledge of other functions, present in the explicit expression of the transformation. Picone, in this work, does not give a general method for the calculus of such functions yet, but shows how in many cases the remark that the transformation is an entire function of the parameters is enough to eliminate the eventual new data introduced, by resorting to a system of integral equations.

Among the resolutive methods, resumed and explained in the *Appunti*, the one that will arouse more interest and a greater number of researches and of results is linked to an interpretation of Green's formula that allows a unitary treatment of the classic boundary

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[22] Rondinella, Napoli, 1940.

[23] M. Picone, Nuovi metodi d'indagine per la teoria delle equazioni lineari a derivate parziali, *Rend. Sem. Mat. Fis. Milano*, 1939, 66–90.

value problems set out by Mathematical physics, considering the elliptic and the parabolic cases, both traced back to Fisher-Riesz integral equations systems[24]. Many of the problems presented in the *Appunti* will be successfully tackled by Picone's pupils. We have already talked about Caccioppoli in Chapter 6. To his name we can now add those of Giuseppe Scorza Dragoni[25] (1908–1996), Gaetano Scorza's son, Carlo Miranda (1912–1982), Gianfranco Cimmino (1908–1989) and later, towards the end of the 1930s, Aldo Ghizzetti (1908–1992), Gaetano Fichera (1922–1996), Tullio Viola (1904–1985) and Luigi Amerio (1912–2004).

Tonelli's and Picone's respective schools are also the symptoms of an increasing crystallization of Italian analysis into two spheres of influence: the Pisan sphere and the Roman one. Actually, rivalries increase[26] and relationships, even personal ones, are not very good. Beginning with the one between the two leaders, who objectively are different personalities, divergent in temper, in culture, in the way of understanding mathematical research and its placement. Tonelli shows serious doubts about the substance of the ballistics tables' episode, during the first World War, on which Picone had built his fame; more in general, he does not understand the versatility of Picone's studies or, at least, he is strongly suspicious. He does not even appreciate in the appropriate measure the richness of the school created by Picone. After much effort, he accepts the abstract approach that Picone tends to give to his works, and is more willing to look at the new results of functional analysis, but he wholly rejects it at a didactical level. Here Tonelli ironically accuses Picone of being the *most general* professor, who starts from an artificial generality to arrive at levels that had no need of those generalizations.

A public and strong polemics breaks out – after some academic disagreement due to academic exam failures of some students – with the publication of the already mentioned *Appunti di Analisi superiore*, which Picone asks Tonelli to review. In the end, the review reveals some grounds for dissension. In particular the procedure for the extension of the concept of integral of bounded Baire functions will be critisized: "such procedure, according to me, is not useful, neither theoretically nor didactically". Picone's reply ap-

[24] The method of calculus that goes through the reduction of a boundary value problem to a Fisher-Riesz integral equations system, as it is presented in "Nuovi metodi per il calcolo delle soluzioni delle equazioni a derivate parziali della fisica-matematica" (*Ann. Sc. Univ. Jassy*, 1940, 183–232), receives a negative review from R. V. Churchill in *Mathematical Reviews*: "the procedure is lengthy and involved". Picone's reply is assigned to a note published the year after in the *B.U.M.I.* It is realized that "the new method, considered as a whole, is not simple. But this is in the nature of things, given the complexity of the problems for which the method itself has been created". But for the problems studied in the memoir, "Mr Churchill knows methods of calculus of the solution, numerically feasible, simpler than those he criticizes?"

[25] With Cinquini Scorza Dragoni has a long and harsh controversy in which both "school" reasons and personal aspects intertwine. About the contents of the controversy, see the already mentioned work by J. Mawhin and also A. Guerraggio, *L'Analisi*, in S. Di Sieno, A. Guerraggio, P. Nastasi (eds.), *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, Marcos y Marcos, Milano, 1998, 1–158 (31–33).

[26] In this atmosphere, especially significant is the Conference of the Accademia dei Lincei, in 1985, to celebrate jointly the memory of both mathematicians. However, almost half a century had to go by from the facts we are talking about.

pears in a long article called "Sull'integrazione delle funzioni", in which (in a note) he defends his procedure of extension of the classic integration, based on successive limit passages in the integral: "about the matter, the colleague Tonelli (…) has voiced his opinion that denies theoretical and practical utility to such procedure. Even after this, I seize the occasion to state my opinion, which is in stark contrast to Tonelli's." Anyway, the most violent attack of the debate is the following reply of Tonelli, who in the note "Sull'integrazione delle funzioni" (1942) does not spare irony, also for the contradictions between the text and the notes in Picone's article: "each of Picone's integrable functions is always the sum of an almost continuous function and of a null integral function. We are therefore faced with an extension of the usual set of almost continuous integrable functions that brings only to a useless extension". In the text, Picone would have brought "with youngish enthusiasm" his "rapturous support" to the theories supported by Tonelli on the most suitable extension to be given to the operation of integration and on the methods to achieve it. It is a pity that in his note he forgets this *conversion* and continues to defend the choices made in the *Appunti*; but "whoever has by chance taken a glance at Picone's lectures on Infinitesimal analysis knows that it is not always easy to agree with him". Clearly, the attack is not left unanswered. There will be an attempt at reconciliation only at the end of the war. Some young colleagues persuade Tonelli to resume personal contacts, in a moment as particularly difficult for Italy as was the reconstruction; others carry out a similar conciliatory action on Picone. In the meanwhile, however, Tonelli dies but – sad irony – not even his death will entirely bring the diatribe to an end[27]. Picone, appointed to the post of his colleague at the University of Pisa and at the *Normale*, chooses for his seminar (despite the opposition of the Pisan mathematicians) just the subject of integration that had sparked the conflict.

Even if Pisa and Rome remain the two main centres of Italian analysis, in the meanwhile other interesting situations arise. In Turin (already for some years) works a skilful researcher, with a strong personality too. He is Francesco Tricomi (1897–1978). His *vis polemica* has been passed down by the colleagues who have met and seen him, through his caustic remarks, but finds also expression in *La mia vita di matematico, attraverso la cronistoria dei miei lavori*[28]. It is a valuable autobiography because of the information it contains, which allows drawing some conclusions about the historiography of mathematics. Tricomi, unlike Enriques, is not exactly a historian of mathematics, and yet in his work there is both strong attention to the context and the courage of a first assessment. The summary of the works is alternated with remarks (written in italics) in which Tricomi tells about the social environment and the circumstances that have caused those publications: the situation of the Institute, the suggestions or the criticism of a colleague, etc., up to the most general socio-historical settings: fascism, war, reconstruc-

[27] Also Tonelli's relationship with Severi is not very good. An example is the episode of the *premio Mussolini* (proclaimed by the *Accademia d'Italia* and financed by the newspaper *Corriere della Sera*) that in 1940 belongs to mathematics. The *Accademia* points at Tonelli as winner and assigns Severi the task of writing the report. His continuous deferrals, just in writing the final document, have the incredible – but unavoidable – consequence that the award goes to physics with the proclamation of Quirino Majorana as winner.

[28] Cedam, Padova, 1967.

Francesco Tricomi

tion. He refuses to think of his own task as (whether "professional" or occasional) historian to be satisfied with a formulation, maybe technically perfect, of a series of results lined up one after the other. This historiographical choice carries a "scandalous" lack of caution and the refusal of a perspective according to which there would be no errors, ideas "poached" from a colleague, *tiredness* periods, polemics, course changes, but only an imposing and gradual elaboration of brilliant and youthful intuitions. Tricomi is different. Certainly favoured by his character, he does not keep his opinions (even heavy, even mean) to himself, even if he – just like the reader – knows well that these are strongly dependent on his own point of view and on a temper he does not manage to silence.

Tricomi begins to study chemistry in Bologna. Some lessons with Enriques, the passage to physics in Naples and the brilliant results in the really mathematical exams are the elements that persuade him to change, in his second biennium, to mathematics. He takes his degree in 1918. His university career begins in 1921 in Padova, as Severi's assistant. The research that will mostly contribute to Tricomi's renown, at the time little over twenty, arrives immediately. It is the famous *Tricomi equation* and, more in general, the study of *the mixed type*, second order partial differential linear equations for which the expression (formed by the coefficients) assigning the type changes sign in the plane (x, y). Only after the war will Tricomi know, through T. von Karman who was visiting Turin, about the great interest that mixed equations aroused in the U.S.A. and in the U.S.S.R., as a mathematical model suited for the description of the phenomena studied by transonic aerodynamics. Some years later, Tricomi will return to *his* equation explaining[29] how it, in first approximation, controls the motion of a fluid – now commonly called *Tricomi-gas* – when close to the sonic speed. Through some substitutions, Tricomi reduces each equation of the mixed type (at least in a neighbourhood of a critical curve) to a canonical form like:

$$y\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}+a(x,y)\frac{\partial z}{\partial x}+b(x,y)\frac{\partial z}{\partial x}+c(x,y)z+d(x,y)=0\ ,$$

which is elliptic for $y > 0$ and hyperbolic for $y < 0$. It is at this point that the study concentrates on the *Tricomi equation*:

$$y\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0\ ,$$

obtained by considering the second-order part of the previous canonical form and grounded on obvious reasons of simplicity, and for which Tricomi tackles the questions of existence and uniqueness, (with suitable boundary value conditions).

After the Paduan period, Tricomi follows Severi to Rome. From here, three years later, he will move to Florence, where in the meanwhile he has won the professorship in Algebraic and infinitesimal analysis.

> Ero lietissimo della cattedra brillantemente conquistata e anche di lasciare Roma e di non essere più assistente di Severi che era, sì, un maestro di eccezionale valore ma anche un 'padrone' un po' pesante, per quanto, in quattro anni, io non abbia mai avuto alcun vero scontro con lui. Ma, forse, perché avevo considerato quel periodo come una specie di prolungamento del mio servizio militare. Più tardi invece dovei, più volte, fargli capire deferentemente ma con la dovuta fermezza, che non ero più alle sue dipendenze (Endnote*, see page 242)[30].

Florence is only a provisory position before the definitive transfer to Turin, where he will strike up an almost fraternal friendship with Fubini.

> Per oltre dieci anni c'incontravamo quasi tutti i giorni, nel tardo pomeriggio, sotto i portici di via Pietro Micca e passavamo un'oretta insieme, passeggiando e parlando di matematica e di non-matematica, con reciproca, illimitata confidenza e fiducia. E ciò nonostante che i nostri temperamenti fossero lungi dall'essere simili, in quanto Fubini era di una straordinaria vivacità e mobilità d'interessi, mentr'io invece sono meno facile ad infiammarmi per questo o per quello, ma poi, quando mi sono interessato ad una cosa, persevero fino al traguardo o all'incontro con ostacoli insormontabili. (Endnote**, see page 242)

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[29] F. G. Tricomi, Correnti fluide transoniche ed equazioni a derivate parziali di tipo misto, *Rend. Sem. Mat. Torino*, 1952–53, 37–52.

[30] F. G. Tricomi, *La mia vita ...*, quoted, p. 15.

The second meaningful research field (after mixed type equations) is opened by two notes of 1926 and 1928. It is the study of the *integral equations* which Tricomi calls *with asterisk*, that is, of Fredholm equations, for which one considers the main value of double integrals. In the meanwhile, Tricomi had become (together with Fubini) the reference analyst for many other disciplinary areas in Turin. If his production is set out thus in a multiplicity of arguments, *the* underlying theme of his interest in applications and the resolution, even numerical, of the problems that these pose remains still quite visible. To this wide spectrum is added (from 1935) the third most important component in Tricomi's scientific production in the years we are dealing with. It is the study of *special functions* and of *integral transforms* which, just as the equations of mixed type, will win him a wide international renown. Precisely these studies are mainly "responsible" for the invitation Tricomi will receive, soon after the second World War, to move to the *California Institute of Technology* for the writing of a handbook about the practical use of special functions and functional transforms (the so-called *Bateman Manuscript Project*). Between 1953 and 1954 five volumes, three dedicated to special functions (*Higher Transcendental Functions*) and two to functional transforms (*Tables of Integral Transforms*), will be issued.

In Turin is his assistant Maria Cibrario (1905–1992), who will marry the Silvio Cinquini we met when describing Tonelli's school. Actually, Maria Cibrario collaborates also with Fubini (with whom she graduates and begins a university career) and with Peano; she will be Peano's assistant until his death. The main theme of her research is partial differential equations. Maria Cibrario studies the almost-linear equations of the second order:

$$A(x,y)\,\frac{\partial^2 z}{\partial x^2} + 2B(x,y)\,\frac{\partial^2 z}{\partial x\,\partial y} + C(x,y)\,\frac{\partial^2 z}{\partial y^2} = f(x,y,z,p,q)\,.$$

and arrives at a total local classification (in six canonical forms) of each equation of mixed type.

Maria Cibrario is not the first skilful woman analyst, in Italy, even if women with a degree in mathematics are in our country, still in the first half of the twentieth century, a small minority.

Decades	Total of graduates in mathematics	Women graduated in mathematics	%
1881–90	2912	21	0.7
1891–00	3681	233	6
1901–10	3951	211	5
1911–20	4622	368	8
1921–30	8279	1188	14
1931–40	11628	2020	17
1941–50	18479	5115	28
1951–60	29871	6542	31
1961–70	35474	13637	38

She is preceded by Pia Nalli (1886–1964), whose most famous publication is probably the *survey* work: *Esposizione e confronto critico delle diverse definizioni proposte per l'integrale definito di una funzione limitata o no*, published in 1914. Soon after the war appear the articles that contain her greatest contributions to the representation of a real, symmetric, L^2 function $K(s, t)$, within the domain $a \le s \le b$, $a \le t \le b$, and in which there is an arbitrary real function $k(s)$, bounded in (a, b). From here originates the study of the integral operator of the third type with symmetrical nucleus:

$$\mathrm{k(s)\,g(s)} + \int_a^b K(s,t)\,g(t)\,dt\,,$$

which can generalize representation theorems previously achieved by D. Hilbert through his fundamental functions. These results are then applied to the resolution of the corresponding integral equation:

$$\varphi(\mathrm{s}) = \mathrm{g(s)} + \tau\left[k(s)\,g(s) + \int_a^b K(s,t)\,g(t)\,dt\right],$$

with τ costant and $\varphi \in L^2(a, b)$ assigned. The results obtained for the third-type integral operator will be later extended to symmetrical and bounded linear functionals, but Pia Nalli's interests will turn soon towards other research fields – tensorial calculus, as we have seen – and after 1928 she will publish no more analysis works.

Besides Turin, other centres host skilful analysts. We point out especially Florence, because here we find again the G. Sansone we met in Chapter 5 regarding the brief Italian *spring* of algebra and of number theory. His preferred research fields as analyst are two (even if we cannot avoid mentioning his excellent work as author of treatises). The first regards ordinary differential equations, with particular attention to linear equations of higher order, in the attempt to extend the results applying to those of the second order; the second is represented by the special functions and by a series of articles, between 1932 and 1937, on subjects such as polynomials Hermite and Legendre, Laplace's series, the developments through series of Legendre's polynomials. The figure of Sansone is significant in the historical progress of Italian mathematics, also because of the way his education follows. Even more than Tricomi and because of the reasons we have mentioned – the "forced" passage from algebra to analysis, without personal landmarks – Sansone is a *self-made mathematician*. We are no longer contemplating a researcher educated *as an apprentice*, in a direct relationship with a Master. Through Sansone we glimpse situations that will become familiar in the following decades. With its *pros*, but also with its *cons*. The absence of a school – the *workshop* – releases more quickly new and original creative energies but intensifies the risk of an education that, based essentially on literature and written communication, loses the drive to elaborate on and make comparisons with the *great ideas*, limiting itself to clarifying and generalizing (even if in a technically perfect way) others' ideas.

Finally, we return to Rome for… a certain Severi. Do you remember the 1921 episode, when he was called to the capital and was preferred to Tonelli and to Enriques?

Officially, then, Severi is an analyst, even if "he was and remained essentially a geometer, even if he had a strong intuition (but scarce culture) in the field of analysis too"[31]. At the beginning of the 1930s, Severi works – within complex analysis – on the problem of the construction of holomorphic envelopes and of the generalization of Hartogs' theorem. Its extension is based on what will be later called *extension method from real to complex*. From Severi the analyst, the *Lezioni di Analisi*[32] should also be recalled. It is a handbook enriched with precious chapters of "complements and exercises", that often contain recent researches and ideas for their elaboration. It resumes, for example, the note[33] of 1934, in which Severi had set the objective of the generalization of the concept of differentiability for functions of *n* real variables. In this note Severi had given, with his geometrical language, the definition of what is commonly called *Bouligand tangent cone* or *contingent cone*[34], an essential research instrument in many fields of analysis, starting with nonlinear programming.

Severi's pupil in Rome is Luigi Fantappié (1901–1956), known for his contributions to complex analysis and the construction of an Italian school of *analytical functionals*. Fantappié graduates in Pisa – at the *Normale* – in 1922. Some of his first works on algebra and on number theory can be certainly explained by the influence of Bianchi. Fantappié appears to come close to Volterra's functional analysis. But the decisive encounter is the one with Severi. With him Fantappié will also share beliefs and political and religious stands[35]; he will develop a particular sensibility towards the most geometrical aspects of the theory of analytic functionals; after having taught at the universities of Cagliari, Palermo, Bologna and Rome, he will occupy the chair of analysis of the INDAM, founded by Severi, with which we will deal in the next chapter; he will take part actively in the cultural agreements underwritten by the fascist regime for the worldwide diffusion of Italian culture, even with a six-year stay in Brazil for the development of the university of S. Paolo.

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31 F. G. Tricomi, *La mia vita ...*, quoted, p. 15.

32 The first volume is published by *Zanichelli* in Bologna in 1933. The second one will be issued in 1942, while the third one, written in association with Scorza Dragoni, will be published after the war (in 1955).

33 F. Severi, Sulla differenziabilità totale delle funzioni di più variabili reali, *Ann. Mat. Pura e Applicata*, 1934, 1–35.

34 For a set $X \subset \Re^n$ and a point $x° \in \underline{\mathrm{el}\ X}$, the set $T(X, x°) = \{x: \exists\{\lambda_n\} \subset \Re_+, \exists\{x_n\} \subset X, x^n \to x°: x = \lim \lambda_n(x^n - x°) \text{ con } n \to +\infty\}$ is called *contingent cone* to X in x°. With the French geometer G. Bouligand, who had never mentioned previously articles that still dealt with the same subject, Severi tones down the polemic: "I don't blame him for this, because not even I manage to follow carefully the bibliography and read rather a memoir or a book after having thought at the subject by myself". On this matter, one can read: A. Ellero, A. Sorato, I coni di George Bouligand e di Francesco Severi, *Lettera matematica Pristem*, n. 37 (2000), 20–24.

35 Fantappié is a mathematician with eclectic cultural interests who does not always come to clear and easily tenable positions. He will also take part in the debate on the foundations of quantum mechanics, underlying how the physics of the twentieth century has accepted in an irreversible way the value of the categories and of theories built independently of mathematics. Mathematical formalism is enough to understand and describe the phenomenal world, and its expressions need no "authorization" (from intuition or classical physics).

Luigi Fantappié

The theory of analytic functionals comes out from a note[36] of 1925, even if – just in the Italian context – Pincherle's previous activity cannot certainly be forgotten. The definition of analytic functional[37] is Fantappié's reply to a problem which is behind many theories of generalized functions: giving a functional interpretation of the operation that matches a given function with the value of its derivative at a certain point. The theory, in the case of functionals dependent on a complex variable, develops quickly and it is soon applied to the solution of questions posed by the study of partial differential equations, such as the Cauchy-Kowalewski problem. Fantappié manages to produce the solution using a *superposition principle* and basing his method on what he calls a *functional indicator*, through which he arrives at the representation theorem. The research finds more trouble when dealing with several variables. Here Fantappié is certainly aided by the scientific sodality with Severi and by his geometric sensibility. But it is not enough. The complexity of problems is real and some decades have to pass for the appropriate theories and instruments for their solution to take shape[38].

Enzo Martinelli (1911–1999), who did all of his work in Rome and whose name is linked to the so-called *Bochner-Martinelli formulas*, found them in a wholly independent

[36] L. Fantappié, Le funzionali lineari analitiche e le loro singolarità, *Rend. Acc. Lincei*, 1925, 502–508.

[37] A functional $F[y(t), z]$, dependent on the function $y(t)$, analytic in Weierstrass' way and with a parameter z, is analytic when – for $y(t)$ fixed – it is an analytic function of z and becomes a function $f(\alpha, \beta, \ldots, \tau, z)$ analytic also with respect to the parameters $\alpha, \beta, \ldots, \tau$ when y depends analytically on the parameters $\alpha, \beta, \ldots, \tau$.

[38] About this topic, see D.C. Struppa, Luigi Fantappié e la teoria dei funzionali analitici, in A. Guerraggio (ed.), *La matematica italiana tra le due guerre mondiali*, Pitagora, Bologna, 1987, 393–429; G. Kato, D. C. Struppa, *Foundations of Microlocal Analysis*, Marcel Dekker, New York, 1997.

Enzo Martinelli

way from the American mathematician S. Bochner (1899–1982)[39]. Bochner-Martinelli formulas "can be held as two essential points in the development of complex Analysis of several variables and they are certainly among the few results to be always mentioned in all the books on the subject"[40]. In order to locate them, it is necessary to go back to theorem and to the proof given by Hartogs himself, based essentially on Cauchy's classic integral formula. Among the several proofs that gradually followed is one by R. Fueter[41], who had first considered regular functions of a quaternionic variable, – for which he had proved a perfect analogy of Cauchy's formula – and had later thought of interpreting the formulas relative to a quaternionic variable as formulas for two complex variables. The study on integral formulas concerning holomorphic functions with several complex variables was thus opened. In the years 1937–1938 Martinelli determines two of these inte-

[39] On Martinelli we can cite: La formula di Cauchy per le funzioni analitiche di due variabili complesse, *Mem. Acc. Italia*, 1937, 33–36; Alcuni teoremi integrali per le funzioni analitiche di più variabili complesse, *Mem. Acc. Italia*, 1938, 269–283; Sopra una dimostrazione di R. Fueter per un teorema di Hartogs, nella teoria delle funzioni di n variabili complesse, *Comm. Math. Helv.*, 1942–43, 340–349; Sulla formula di Cauchy n-dimensionale e sopra un teorema di Hartogs, nella teoria delle funzioni di n variabili complesse, *Comm. Math. Helv.*, 1944–45, 201–208.
On Bochner we mention: Analytic and meromorphic continuation by means of Green's formulaa, *Ann. Math.*, 1943, 652–673; Linear partial differential equations with constant coefficients, *Ann. Math.*, 1946, 202–212; *Several Complex Variables* (with W.T. Martin), Princeton Univ. Press, Princeton 1948.

[40] Cf. D. C. Struppa, "Analisi complessa", in S. Di Sieno, A. Guerraggio, P. Nastasi (eds.), *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, quoted, p. 166.

[41] Cf. R. Fueter, Über die Analytische Darstellung der Regularen Funktionen einer Quaterniunen Variablen, *Comm. Math. Helv.*, 1937, 371–378.

gral formulas[42]: with the first one – the most known – he gives a direct proof of Hartogs' theorem; with the second one, more topological, he tries to fully understand why Hartogs' theorem is not valid in one dimension.

4. Mathematical Physics

During the 1930s, Mathematical physics also experiences a less lively time compared with the previous decades. Most researchers go on publishing in Italian, and, moreover, they use a formalism – the *homographic* one of the so-called *Italian vector calculus* – that the international scientific community finds unintelligible, having by then Ricci's and Levi-Civita's tensor calculus. As we approach the second World War, provincialism and isolation intensify, limited only by the presence of a personality – Levi-Civita's – that continues to produce first-rate results, even in his maturity: "actually, Italian mathematical physics finds itself confined to more and more limited and often quickly obsolete areas, maintaining of course its aristocratic spirit"[43].

The prevailing subject remains the theory of elasticity (to which we can link – for its importance – hydrodynamics). We could begin our review with Antonio Signorini (1888–1963) who, as regards the elasticity theory and the passage to the nonlinear, is the prominent figure during the 1930s. Graduated in Pisa in 1909, after a year of assistantship with Bianchi he enters the "influence area" of Levi-Civita, who starts him on a brilliant academic path: already in 1916 he wins a professorship in Palermo, from where he moves to Naples and then to Rome, to take the place the Master is compelled to leave after the racial laws. The problem that will absorb him wholly from 1930 onwards is the formulation of a *nonlinear* elasticity theory. His work is essential because it tackles the question in quite a general way, beginning to disclose the problem's central point: the concept of constitutive relationship, that is the link that exists between the cinematic quantities and the quantities of intern effort. It is not enough to use nonlinear deformation characteristics for the displacement derivative, as had already been done in the theory of plates, of shells, of beams and of bodies in which one or two dimensions are much smaller than the rest and where the nonlinearity is almost imposed by the body's geometric structure. It is necessary to employ a constitutive theory that allows one to distinguish among them according to their different materials. The nonlinearity of constitutive equations (or, as it is said in some special cases, of the force-deformation links) makes the mechanics of the continuum intrinsically different: two internal forces fields, created by

[42] In "Extension Phenomena in multidimensional complex analysis: correction of the historical record" (*The Mathematical Intelligencer*, 2002, pp. 4–12), R.M. Range remarks: "Note that Martinelli's paper is from 1938, while the footnote on the first page of Bochner's 1943 paper indicates that Bochner had lectured on the formula" as early as in Winter 1940/41, "unaware of Martinelli's earlier paper. Based on the published record, there is thus no question about Martinelli's priority. Still, I follow the commonly used alphabetical order".

[43] F. Pastrone, Fisica matematica e Mecanica razionale, in S. Di Sieno, A. Guerraggio, P. Nastasi (eds.), *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, quoted, 1998, pp. 381–504.

two different deformations, overlapping, do not usually give rise to a field force that adheres to the parallelogram rule. Besides, a correct general constitutive theory cannot avoid taking into account the effects of temperature, that is, the thermodynamic aspect. It is not by chance that Signorini's most important memoirs[44] have, already in the title, the word *thermoelastic*.

Above all, Signorini covers the static case. Equilibrium equations are nonlinear partial differential equations, extremely difficult to deal with when the energy is a general kind: the classic theorems of existence, uniqueness and stability cannot be obtained usually due to given standard boundary value conditions. Actually, the difficulty is, in some cases, a real impossibility: nonlinear theories present situations in which, when the boundary value conditions are equal, a multiplicity of solutions can be reached. It is *Signorini's incompatibility*. The problem arises when, to solve equilibrium equations through series developments, one obtains a succession of differential equations (in vector form), the first of which is precisely that of the linear theory, while the following should provide the terms in the series expansion approaching the exact solution. The apparent contradiction that the solution – unique for the equivalent linear problem – does not behave in this manner emerges already if we stop at the second order, giving therefore rise to a sort of incompatibility between the linear model and the nonlinear one. Even more astonishing is the fact that the lack of unicity depends not on the exact expression of the deformation energy but only on its linear part, in clear contrast with the fact that the same problem – in linear elasticity – admits only one solution. Actually, the contradiction, as has been proved later, is only apparent, as the bifurcation makes singular the correspondence between the nonlinear problem and its linearization.

From 1933 onwards, always within elastostatics studies, Signorini[45] begins the study of the so-called *unilateral* problems, that is of the equilibrium problems of an elastic body whose surface is partly constrained to lean on an even, stiff surface, beyond which it cannot go. He tackles thus, again, quite a complex subject, where usually the portion of surface of the deformed body in contact with the bond is not fixed a priori; it emerges rather as an unknown of the problem. But even if it is fixed, the problem of correctly formulating boundary value conditions and of studying the relative problem still presents substantial difficulties. Its solution will require the use of variational inequalities in suitable functional spaces. Essential contributions to what has been called *Signorini's problem* will be produced in the post-war by analysts such as J. L. Lions, G. Stampacchia, G. Duvant, J. Necas, while the definitive solution (1972) is due to G. Fichera[46].

Another outstanding figure within the field of (linear) elasticity – still ascribable to Levi-Civita's school – is that of Giulio Krall (1901–1971). He graduated in engineering in 1923 at the Polytechnic of milan and in mathematics in 1924 at the university of

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[44] By Signorini see: "Sulle deformazioni termoelastiche finite", *Proc. 3rd Int. Conr. Appl. Mach.*, Stockolm, 1930; "Trasformazioni termoelastiche finite", *Atti SIPS*, 1936; "Recenti progressi della teoria delle trasformazioni termoelastiche finite", *Atti Conv. Mat. Roma*, 1945.

[45] Cf. "Sopra alcune questioni di statica dei sistemi continui", *Ann. Sc. Norm. Pisa*, 1933, pp. 231–252 and "Sopra alcune questioni di elastostatica", *Atti SIPS*, 1933.

[46] Cf. G. Fichera, *Existence Theorems in Elasticity*, Handb. Physik, Springer, Heidelberg, 1972.

Rome. There he will stay as assistant. In 1931 he wins the professorship in tectonics and the one in theoretical mechanics. He chooses the first one and is appointed to the faculty of architecture of the university of Naples where he teaches until 1939, when Severi calls him to Rome. Design engineer and technical consultant, he designs and runs many works regarding the construction of bridges, hydraulic and maritime constructions: the aerial power line on the Straits of Messina, the graving docks of Genoa and Naples, the *Ponte di Mezzo* in Pisa, etc.

The links between the stability of structures and their possible vibrations extend Krall's interest also to vibration mechanics. On this subject he begins to write in 1934 (on commission of the CNR) a powerful treatise[47], finished in 1940. It is this overlapping, between high quality theoretical research and high level technical activity, that makes of Krall a first-rate figure. Classical or recent results of equilibrium and of stability in theory of plates, of beams or of more complex structures, are translated into innovative design techniques. Thus, studying what he calls the elementary problem of bridge dynamics, Krall achieves outstanding results as regards the problem of beams with variable sections, subjected to mobile loads, expressed in terms of integro-differential equations (which he solves numerically too, in an approximate way). Equally significant is his more strictly scientific activity developed under Levi-Civita's direction, concerning especially the theory of adiabatic invariants. Already in 1931 Krall applies them to the case of a system composed by two celestial bodies with gyroscopic structure, in precessional motion, subjected to the adiabatic influence of tides[48]. The same method is then extended to the case of $n + 1$ celestial bodies and to the reciprocal influences of tides. Without imposing restrictions, neither on masses nor on the eccentricity of the orbits, and admitting not rotary but precessional motions with precession axes which are anyhow direct, Krall manages to prove that in the long term orbits tend to become circular and precessional motions rotary, with rotation axes orthogonal to the orbit's plane, while rotation and revolution periods tend to a common equal value.

The topic of the relationship between mathematics and engineering in the Italy of the Thirties necessarily leads to Gustavo Colonnetti (1886–1968). He, too, had graduated in engineering (in 1908) and later, in 1911, in mathematics. After having taught in Genoa and in Pisa, in 1920 he moves to the Polytechnic of Turin, and he will be its Director until December 1925. The new regime begins to bite and Colonnetti must leave his appointment so as not to be forced to join the fascist party. Nevertheless, in 1943 he will emigrate to Switzerland to avoid being subjected to the fascistic Republic of Salò and in Switzerland he will organise the so-called *Campo Universitario Italiano*, where more than 200 refugee students will receive moral and material assistance and will be able to follow a degree later acknowledged in Italy. Returned to Italy in December 1944, he will be deputy of the Constituent Assembly and President of the new CNR (*Consiglio nazionale delle Ricerche e della Ricostruzione*) from 1945 to 1956, contributing in a decisive way to its reconstruction and reorganization. His scientific production develops along three essen-

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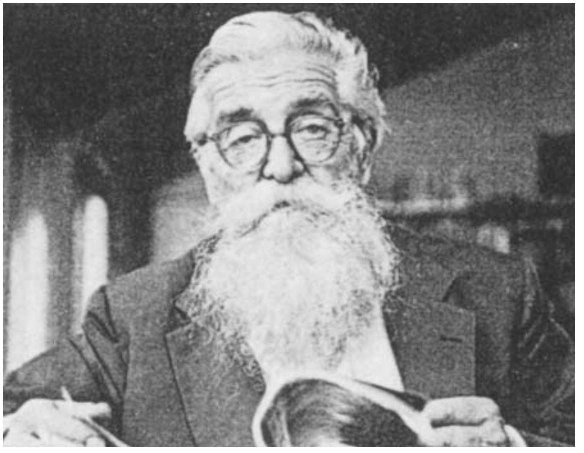
[47] Cf. G. Krall, *Meccanica tecnica delle vibrazioni*, Zanichelli, Bologna, 1940.

[48] Cf. specially G. Krall, Influenze adiabatiche delle maree nel moto kepleriano di due corpi celesti giroscopici, *Rend. Acc. Lincei*, 1931, 270–276.



Gustavo Colonnetti (1886–1968)

tial genres: tectonics, hydrodynamics and the mathematical theory of elasticity where the first outstanding result dates back even to 1912, represented by the theorem he called *second theorem of reciprocity* and that today is often called *Colonnetti theorem*. During his researches, extended quite beyond the second World War[49], Colonnetti has gone back several times to the applications and possible generalizations of this theorem. But his activity has extended to other fields too: for example, the study of the phenomena of elastic hysteresis and that of plastic flows and their influence in the proportioning of beams.

Always with reference to the relationship between mathematics and technique – now especially aeronautics – another outstanding name is Gaetano Arturo Crocco (1877–1968). In 1908 he had founded – with Volterra's decisive aid – the *Istituto Centrale aeronautico*, where the first courses on aeronautics in Italy were run. In 1914 he built an aerodynamic gallery for speed up to 200 kph, that worked until 1935, when Guidonia, the *air city*, arises: Crocco defined its project and promoted its construction. Until 1943 Guidonia will be one of the bigger aeronautical experimental structures in Europe. To him is due as well the organization of the famous *Volta Congress* of 1935, that will see the participation of the greatest researchers and in which the topic of high speed in aeronautics, so dear to the rhetoric of fascism, will be discussed. There are more than 170 scientific publications of Crocco. Author of the first Italian treatise on flight mechanics, *Elementi di aviazione* of 1930, he obtains about thirty patents and realizes about fifty mechanisms. His main contributions to flight mechanics are the researches (previous to the Wright brother's first flight) on transversal plane stability and those on spiral stability, in which he proves for the first time the existence, which had been denied

[49] Such researches are summarized in valuable handbooks, one dating from the end of the Twenties: G. Colonnetti, *La statica delle costruzioni*, UTET, Torino, 1928, the other from the Fifties: G. Colonnetti, *L'équilibre des corps déformables*, Dunod, Paris, 1955.

by Poincaré, of an intrinsic spiral stability. Other problems concern range, take-off and landing, tailspin, helicopter stability and instrument flight. And among the instruments we find the famous heading indicator that bears his name, invented for airships in 1919. Not least are his researches in the field of aerodynamics: the theory of propellers, the already mentioned problem of fast flight, and jet propulsion are due to him. In the last years he will turn to astronautics almost entirely. Crocco is therefore a technician and an organizer, but he is also a theorist with a wide knowledge, working in close contact with Volterra and Levi-Civita. His results in fluid mechanics and in gas dynamics will be resumed in the Fifties by C. Truesdell who has no hesitation in acknowledging that "this work was motivated by earlier results of Crocco"[50]. In particular, in the important note[51] of 1936 (discussed in the *Academy of Lincei*) he introduces a vector – which extension will be called later *generalized convention vector* – which is behind the theoretical body of analysis of helicoidal motions, above all in gas dynamics, in the case of vortical motions, when there is no conservation of the circulation.

Until now we have intentionally left Levi-Civita in the background, preferring that his role appeared indirectly through the research topics assigned to his numerous pupils. The moment has come, though, to give a quick description of his activity in this conclusive decade, in which the Paduan mathematician produces one of his most important contributions regarding the problem of *n* bodies in general relativity[52]. The problem was not a new one. At the beginning of the 1920s Einstein himself, and later W. De Sitter, G. von Droste and K. Schwarzschild had already dealt with it, obtaining positive results only in the case of two bodies. The main problem was due, as M. Brillouin had pointed out in 1922[53], to the essentially nonlinear nature of field equations. So it was not possible to find a relativistic equivalent of the principle of action and reaction that would permit simplification in an essential way of motion equations, and for this reason general methods of approximation needed to be developed. Levi-Civita's first fundamental contribution appears in 1937: "The relativistic problem of several bodies". It is the text of a lecture given on the 4th September 1936 at the *Harvard Tercentenary Conference of arts and sciences*, where "était obtenu de manière cohérente, pour la première fois, le système différentiel régissant les mouvements des centres de gravité de *n* corps et prenant en compte les parties principales des corrections relativistes"[54]. That same year, and in the same review, Levi-Civita publishes the text of the second lecture he gave at the Congress of Harvard: "Astronomical consequences of the relativistic two-body problem" in which he applies his method to the problem of the two bodies, arriving at remarkable conclusions regarding the so-called secular acceleration. Soon after, Einstein, L. Infeld and

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50 Cited in Pastrone, "Fisica matematica e Meccanica razionale", quoted, p. 439.

51 Cf. G.A. Crocco, Una nuova funzione di corrente per lo studio del moto rotazionale dei gas, *Rend. Acc. Lincei*, 1936, 115–124.

52 Cf. T. Levi-Civita, The relativistic problem of several bodies, *Amer. Journ. of Math.*, v. LIX (1937), pp. 9–22, and Astronomical consequences of the relativistic two-body problem, *ibidem*, pp. 225–234.

53 Cf. A. Lichnerowicz, Le problème des *n* corps en relativité générale et Tullio Levi-Civita, *Atti del Convegno celebrativo del centenario della nascita di Tullio Levi-Civita*, Roma, 1975, pp. 127–136 (132).

54 Cf. A. Lichnerowicz, quoted, p. 133.

B. Hoffmann solve the problem through successive approximations[55]. The work is followed by a brief writing of the astrophysicist H. P. Robertson who applies their method to the problem of the two bodies, finding results that apparently differed from Levi-Civita's. It is a miscalculation of the Paduan mathematician, as Sir A. S. Eddington remarks in a letter[56].

> I am sending you a copy of a paper, which I am proposing to publish in the Proceedings of the Royal Society, which treats the problem of the secular acceleration. As you know I was very interested in the problem. I have an exceptionally able student G. L. Clark, and some of the most vital parts of the paper, especially the discovery of de Sitter's error, are due to him.
> You will see that, contrary to your results, we find no secular acceleration; the various terms cancel out as shown in (8.4) of our paper. We think there must have been a numerical slip in your calculation which prevented the cancelling (See our comparison with your results after our equation (8.2).
> I realise that it is not possible to arrive quickly at a definite judgement on an investigation; in which so much depends on accuracy in a very long algebraic calculation; but I should be very glad to hear from you, if you have time to examine it. In any case please call my attention to any point on which I may have misunderstand you, or unintentionally misrepresented you; so that I may remedy it in proof.
> Whatever the result, it is an interesting problem, which deserves the fullest examination – and incidentally it is associated with very pleasant memories of our time at Harvard.
> I have this morning had a visit from Prof. de Mayolo of Peru who was speaking about your visit to S. America.

Levi-Civita's answer[57] is pointed out not so much for its content – Levi-Civita takes his time to understand the origin of his possible mistake – as rather for the announcement that he is already working on a global review of the problem:

> I am sincerely grateful to you for your friendly letter of March 21, and for your very obliging attention to communicate to me a copy of the paper (in collaboration with your distinguished student, Mr G. L. Clark) you are about to publish in the Proc. of the Royal Society.
> As you obviously think, I am enormously interested in this research and in the fundamental discrepancy concerning secular acceleration, which you find. I shall carefully examine all the matter, in order to detect the origin of the difference of our conclusions. Unfortunately I may never exclude some material mistake in my calculations, though I remember well that I have revised the whole investigation

[55] Cf. A. Einstein, L. Infeld, B. Hoffmann, Gravitational equations and the problems of motion, *Ann. Math.*, 1938, 65–100.
[56] The letter, dated Cambridge 21st March 1938, in Fondo Levi-Civita, *Archive Accademia dei Lincei*.
[57] The copy of the reply (from 24th March) in Fondo Levi-Civita, *Archive Accademia dei Lincei*.

> twice, at the interval of one year.
> Just in the next days I intended to resume it in order to prepare a detailed exposition to be printed as a little volume in the collection of Professor Villat "Mémorial des Sciences Mathématiques".
> Of course your paper is a strong stimulus to accelerate this work instituting first at all the comparison between my successive steps and yours.

Following the promise given to Eddington, Levi-Civita prepares a paper: "Secular acceleration of the mass-center in the relativistic problem of two bodies" that he hopes to publish in the volume of the *American Journal of Mathematics* dedicated to the centenary year of G. W. Hill's birth. He also hopes that, publishing all the passages, someone will help him to find the mistake. The following is the text of a letter to Volterra[58] who asks him for details; he answers thus (29th September 1938):

> Le sono molto grato dell'interesse che Ella presta al mio calcolo concernente l'accelerazione secolare del baricentro nel problema relativistico dei due corpi. In questi giorni lo ho rifatto e avrei trovato perfetta conferma del mio risultato numerico. Sto ora redigendo un articolo coi necessari sviluppi in modo che il controllo sia reso non solo possibile, ma anche ragionevole per chi ci si voglia mettere. Dirò nella prefazione che non mi dissimulo la prevenzione a mio sfavore risultante del duplice fatto che Eddington, colla stessa mia impostazione, e Robertson con un nuovo metodo affatto diverso dovuto ad Einstein (ma richiedente interi fascicoli di sviluppi numerici) trovano zero; ma che mi sembra desiderabile localizzare l'eventuale errore. Siccome conterei di inviare la nota allo stesso American Journal, dove avevo pubblicato il risultato con indicazione sommaria del calcolo, trasmetterò il ms. al Robertson (che sta a Princeton) con preghiera di farmi pervenire le sue eventuali osservazioni, trattenendo, se del caso, il ms. anziché farlo pervenire al giornale[59].

Actually, the celebratory volume of the centenary year of Hill's birth is already being printed and the paper cannot be published. Robertson, though, takes the calculation review asked by Levi-Civita upon himself and shows him the part of the work

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58 The letter, dated 29th September 1938, in Fondo Levi-Civita, *Archive Accademia dei Lincei*.

59 I am very grateful for the interest You have taken in my calculus concerning secular acceleration of the barycentre in the relativistic problem of the two bodies. These days I have made it again and have absolutely confirmed my numerical result. I am now writing an article with the essential developments so that the check can be not only possible but also reasonable for whom wants to try. In the preface I do not hide the prevention against me of the double fact that Eddington, with the same approach, and Robertson with a new entirely different method due to Einstein (but requiring whole dossiers of numerical developments) find zero; but that I think it would be desirable to pinpoint the possible mistake. Since I would expect to send the note to the American Journal itself, where I had published the result with a brief indication of the calculus, I will forward the manuscript to Robertson (who is at Princeton) with the request to let me know his eventual remarks, keeping the manuscript, if necessary, instead of sending it to the journal.

hiding the mistake. In his reply, Levi-Civita – besides acknowledging his mistake, obviously – confirms his strong confidence in the adequacy of his method compared with Einstein's[60].

> I am deeply obliged to you for the kind supervision of my paper, which has permitted to you to detect my slip and to re-establish, as a consequence, the agreement with your previous result.
> My procedure, though being quite usual, is perhaps worthy of publication, because it furnishes in a few pages all details of calculation. Therefore I dare to send back to you a manuscript, duly emended and retouched. As you will see, at the end of the introduction, I have anticipated the statement that you will have the kindness to add some formulas and remarks, privately communicated to me: I obviously allude to your last letter. With this hope, I beg you to let forward the paper, after your additions and corrections of any kind, to the Editor of the American Journal, if you do not think, however, that now the thing is quite useless. Of course, I entirely trust to you, thanking you very much for your precious help.

Levi-Civita's work will not be published. Thus, his method, corrected, will be known only in 1950, when the planned volume of the *Mémorial* is issued[61]. But Levi-Civita was right to think that his method was "worthy of publication", as it is stronger and more natural than the one Robertson based his argument on, allowing consideration of the different approximations and the possible influence of the inner structure of the bodies over them It is not by chance that, from 1939, the work of V. Fock, through which relativist celestial Mechanics will be definitely established, is in the direction pointed out by Levi-Civita.

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60 Levi-Civita's letter, dated 12th November 1938, in Fondo Levi-Civita, Archive Accademia dei Lincei.

61 Cf. T. Levi-Civita, *Le problème des n corpsen relativité générale*, Gauthier-Villars, Paris, 1950.

* I was very glad because of the tenure brilliantly won and because I leave Rome and I am not anymore assistant of Severi. Severi was indeed a mentor of exceptional value, but also a harsh master, although I never had, in four years, a true confrontation with him. That is probably because I considered the period I worked with him a sort of continuance of my militäry service. Later on, I was forced to many times make him realize, respectfully but firmly, that I was not at his directions anymore.

** For more than ten years we used to meet almost every day, in the late afternoon, under the arcades of Via Pietro Micca, where we would spend about an hour together, walking and talking about mathematical and not-mathematical topics. Our discussions where always accompanied by a sense of infinite confidence and trust, despite the fact that our personalities were all but similar. Fubini's interests were, in fact, highly dynamic and extraordinarily flexible. On the contrary, I live narrower choices of interests, and only after one particular thing interests me, I am capable of persisting until I cross unsolvable troubles or reach the goal.

Chapter 8

Towards disaster

1. European events

In the 1930s, Italian mathematics apparently settles down and approaches an equilbrium, after the ups and downs of the first decade of fascist rule and its cultural repercussions, also in the mathematical world. Signs coming from the main research areas are quite the same: the cruising speed is high, but undoubtedly inferior to the one reached at the beginning of the century and to that reached by other countries. The country's social and scientific structure and the existing atmosphere make the generational exchange slower than desirable. In order to explain difficulties and to clarify the reference to the new cultural climate, we must recall the different central points in a map of the disciplines that now sees the rise of Theoretical physics and a particular hostility on the part of chemistry, encouraged and strongly supported by a regime that will never reject its utilitarian goals.

Even internal equilibria seemed to stabilize. The old guard, still in command during the 1920s (Pincherle, Volterra, etc.) was deliberately sidelined or removed from the nerve centres. Castelnuovo retired in 1935. Enriques, six years younger, was not even invited to prepare a report for the *UMI*'s first Congress in 1937. Tonelli seemed to be isolated in Pisa – only a few train hours from Rome, but more than ever distant from the institutions based in the capital – wrestling with the problems of the *Scuola Normale*. Scorza, who was anything but an opponent of the regime, was nevertheless obliged to escape from Naples so as to elude the harassment of a colleague, Giulio Andreoli (1892–1969), a fascist exponent particularly faithful to the whip[1]. Scorza, who

[1] Tricomi's irony about clearly fascist mathematicians can be appreciated in his final remark in a letter he wrote to Levi-Civita from Naples on the 27th September 1927: "I have had the *pleasure* to realize here in Naples the remarkable progress attained, even if in non scientific fields, by two prominent colleagues. They are Picone, who has duly joined the fascist party, and Andreoli, who has obtained one of the most important, honorary and sought-after positions to which an Italian citizen can aspire today: he has been appointed Political Secretary of the fascist party in Catania! Mathematics is underway!". The letter is kept in the *Archives of the Accademia dei Lincei* in Roma, "Fondo T. Levi-Civita".

was forced to retreat to Rome and took Castelnuovo's place, would show – after the racial laws of the year 1938 – great humanity and common sense. Also, the duopoly UMI-CNR we portrayed in Chapter 6 (at the turn of the decade, between the reform of the CNR and the Congress of Zurich) returned. After Pincherle's death in 1936 Bompiani – secretary of the *Comitato matematico* of the CNR – revealed himself more and more as the *longa manus* of Severi within the UMI. Equilibria within the mathematical world seemed to revolve around two figures: Severi and Picone who, even if he could not boast an appointment to the *Accademia d'Italia* and a reputation as a keen intellectual, could however put in the balance the steady development of the *INAC*'s activities, his perfect adaptation to the political stances supporting a more and more applied science, and a group of pupils that began to be remarkable for its number and its quality.

The events of 1933 in Germany changed the situation radically. On the 28th January the President of the German Republic, Paul von Hindenburg, called Adolf Hitler to chair a coalition government with the nationalists; these last would be soon dismissed from power – it follows the same pattern as the first fascist government in Italy. Old Hindenburg's death – 2nd August 1934 – formally completed the operation, allowing Hitler to become also the Head of State. Storm-signals, though, were soon noticed, already in 1933: from the "great" fire of the Reichstag, on the 27th February, to the "small" raid (on the 20th March) on Einstein's summer-house – he was then in the U.S.A. – under the pretext of looking for guns hidden by the communists. At the end of the month, when he came back to Europe, Einstein gave his resignation to the Prussian Academy. In the meanwhile, anti-Semitic excesses went on. In front of the main entrance of shops and Jewish offices, Nazi squads invited clients not to enter. Posters exhorted citizens to join the boycott; insults like "German, do not buy from Jews", or *Juden raus!* ("Jews, get out!") were put up in the shop windows.

Italian mathematicians – a small minority, indeed – realized soon the gravity of what was happening. On the 14th April 1933, Fubini suggested to Levi-Civita that they encourage Italian mathematicians who were members of the *German Mathematical Society* to resign[2].

> Carissimo Levi Civita,
>
> Sottopongo a te una questione. Dato l'Hitlerismo imperante, non ti pare che noi dovremmo dimetterci dalla Deutsche Mathematische Vereinigung? Dopo quanto ha subito Einstein, dopo i fatti di Lipsia ecc. a me questo pare doveroso: ma non so se è cosa opportuna, e se ciò può danneggiare i nostri disgraziati colleghi.
> Ne scriverò anche agli altri amici italiani, ma innanzi tutto desidero sentire il tuo parere, a cui senz'altro informerò la mia decisione. E, se tu approvi le nostre dimissioni, ne scriverò, come ti ho detto, a Pincherle, Beppo Levi, Vivanti, Loria. Il Prof. Terracini è d'accordo con me: Fano non è ora a Torino.
> Se tu, sempre così cortese, vorrai dirmi il tuo parere, te ne sarò molto grato.

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2 The letter is kept in the *Archives* of the *Accademia dei Lincei* in Roma, "Fondo T. Levi-Civita".

> Ossequi alla tua Signora anche da parte di mia moglie; a te un cordiale e reverente saluto dal tuo
>
> aff. to G. Fubini [3]

Two months later – on the 14th May – Castelnuovo commented on the German situation with these words, in a letter to Volterra[4]:

> Di ciò che succede in Germania qui si sa poco; i professori profughi a Roma non si son visti. Qualche vaga notizia ha ricevuto il Levi-Civita, qualche altra ne ha portata da Parigi il Fubini che vidi qui giorni fa. Quel che si sa basta già a far prevedere che nessuno andrà più a studiar matematica, e forse nemmeno fisica, nelle Università tedesche. E la scuola matematica di Gottinga, dopo un secolo ininterrotto di gloria, si chiude![5]

In the meantime, the Nazi regime consolidated further. After having left the *League of Nations*, Germany began its re-militarization (both in the air force and in land forces, for which the compulsory draft was restored). The serious violation of peace treaties was denounced in the Italian-French-English meeting in Stresa (April 1934), but always with that leniency deriving from the belief that Nazism was still a rampart against Bolshevik revolution.

Then, we have Ethiopia and Italian anachronistic colonialism which made the climate of international tension worse. The Ethiopian empire, ruled by the *negus* Hailé Selassié, had been admitted since 1925 – on a French-Italian proposal – to join the *League of Nations* so that, despite its backward and semi-feudal structures, it appeared to international public opinion as a sovereign state with full rights. Hence, the Italian aggression of the 3rd October 1935, even if diplomatically prepared by a French semi-approval and by some contact with England, was condemned by the *League of Nations*,

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[3] Dearest Levi Civita,
I will put forward a question to you. Given the rampant Hitlerism, don't you think we should resign from the Deutsche Mathematische Vereinigung? It seems necessary to me, after what Einstein endured, after Leipzig events, etc.: but I don't know if it is right, and if it could damage our unlucky colleagues.
I will also write to our other Italian friends, but first and foremost I would like to hear your opinion, to which I will certainly conform my decision. And, if you approve of our resignations, I will write, as I told, to Pincherle, Beppo Levi, Vivanti, Loria. Prof. Terracini agrees with me: Fano is not in Turin right now.
I would be very grateful to you if you could kindly tell me your opinion.
Best regards to Your wife also from mine; and my best warm and respectful wishes,
Your G. Fubini

[4] The letter is kept in the *Archives* of the *Accademia dei Lincei* in Roma, "Fondo V. Volterra".

[5] Here we know very little about what happens in Germany; there are no exiled professors in Rome. Levi-Civita has vaguely heard something, Fubini, whom I saw here some days ago, has brought some news from Paris. The news we have is enough to foresee that nobody will study mathematics anymore, and maybe not even Physics, at the German Universities. And Göttingen mathematical school is closed, after an uninterrupted century of glory!

The Ethiopic war

In Ethiopia natives learn the fascist ritual with the Roman salute (Popperfoto)

whose intervention the *negus* had sought through several appeals since April. The condemnation was followed by *economic sanctions* (18th November 1935) provided for by the treaty of non-aggression and severely pushed for by the English Foreign Minister Anthony Eden, one of the few English conservatives resolutely hostile to fascism, who felt that Nazi-fascist events had to be scotched. Even regardless of the fact that the economic encirclement was soon broken in two important points – Germany and the United States abstained – sanctions were mildly applied. The *embargo*, for instance, was not extended to all raw materials of great military significance. This ambiguous policy actually allowed fascism to prosecute war undisturbed and to obtain the greatest inner consent, as a consequence of the colonial victory and of the pride with which it pushed ahead – according to the Mussolinian expression that still dominates in faded images on the walls of old houses of Italian cities and towns – despite the alliance of important countries that tried to stop its advance. Italian military operations in Ethiopia ended with the entrance of marshal Badoglio's troops into Addis Ababa on the 5th May 1936. Four days later Ethiopia, to which Eritrea and Somalia were added, was declared *Italian Western Africa Empire*.

The most relevant consequences of the "Ethiopian operation" were in Europe: the spirit of the *League of Nations* was dealt a death blow, there was a further decline in the influence of Western democracies, German imperialism made another step forward, what was more insidious, a convergence materialized of German Nazism and Italian fascism, which soon (October 1936) would become the close cooperation known as the *Rome-Berlin axis*. *Sanctions* enter also in the history of Italian mathematics. We are in 1936, the date planned for the quadrennial International Congress in Oslo.

2. The international Congress of 1936

Before going to Norway, we should stay for a while in Italy just to understand the deterioration process of the professional autonomy of Italian mathematicians. In 1934 the *statute* of the UMI (as happened in most of the associations and academies) was changed, obeying the fascistization process of Italian culture. The new statute, openly in conflict with the UMI's nature of free association, provided that the appointment of the President, the Vice-president and the members of the scientific committee was "legal only after the approval of the National Department for Education". The effects of this change hit the following year, when Volterra and a colleague from Milan[6], both elected among the members of the new *scientific committee*, did not obtain the required ministerial *approval* and were replaced by Fantappié and Fubini (who, with Castelnuovo, had won more votes among the non-elected members). Reporting the voting results, the UMI's *Bulletin* registered only that the names of the elected, "according to the Statute", would have been submitted "to the Ministry for approval". Quite different was the stance of mute but significant protest that on such an occasion adopted the Palermitan de Franchis: the first two pages of the *Rendiconti del Circolo Matematico di Palermo* of 1935 published on one side the composition of the editorial team freely chosen in 1931 (in which we find Richard Courant, Edmund Landau and Vito Volterra) and, on the other, the new fascist statute that went against all the great traditions of the *Circolo*.

At this point the event of Oslo, where the tenth international Congress was held from 13th to 18th July 1936, caused no particular surprises. We have spoken of the international tension. Besides, the Congress of Oslo was a particularly tempting occasion for

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[6] Giulio Vivanti (1859–1949), a mathematician of Jewish descent, expert in the theory of analytical functions.

Italian mathematics, which saw Severi invited to hold a general conference titled *Teorie e questioni nuove nella Geometria algebrica* and to chair the Committee that would have awarded the first two *Fields medals*[7]. The convention of the U.M.I., held in Bologna on the 21st April 1936, examined the advisability of the Italian participation in the Congress; its resolution is especially disappointing.

> Per quel che riguarda la partecipazione della Società al Congresso Internazionale matematico, che quest'anno si terrà in Oslo, il Presidente Berzolari fa osservare che le presenti condizioni politiche impongono stretta aderenza alle direttive del Governo. Perciò occorre anzitutto informarsi se il Governo consentirà che l'Unione Matematica Italiana deleghi qualche suo rappresentante a tale Congresso, la cui sede è presso un paese sanzionista. L'Unione Matematica ha nel suo Statuto la partecipazione a Congressi scientifici; essa ebbe massima parte nel Congresso internazionale del 1928, che fu fatto sotto gli auspici della Università di Bologna, e tenuto in questa stessa città, che era ed è sede della nostra Unione. L'organizzazione e lo svolgimento del Congresso fu opera della Presidenza della nostra Unione. Perciò la partecipazione nostra al Congresso ha significato ben diverso da quel che possa avere la partecipazione di qualsiasi altra Società od Accademia scientifica. Stima perciò opportuno che innanzi tutto si ascoltino le direttive del Governo.
> L'Assemblea, dopo serena discussione, delibera di dare mandato alla Presidenza per ciò che riguarda la partecipazione dell'U.M.I. al Congresso Matematico internazionale di Oslo[8].

The UMI subordinated therefore its decisions to the regime, and chose not to take part in the Congress of Oslo because Norway was a sanctionist country! Thus, only five Italians[9] ennobled for the Congress: among them, an old Vito Volterra, worn-out by the

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7 The decision of this important award had been taken in 1932 during the Congress of Zurich, on the strength of a *memorandum* of the Canadian mathematician John Charles Fields (1863–1932) titled: "An international medal for outstanding achievement in mathematics". As regards the history of the prize, see M. Monastyrsky, *Modern Mathematics in the Light of the Fields Medals*, Wellesley (Mass.), AK Peters, 1997.

8 As regards the participation of our association to the International Mathematical Congress, to be hold in Oslo this year, the President Berzolari points out that the present political conditions require a firm adhesion to the government stances. Therefore we must first and foremost ascertain whether the government will authorize the Unione Matematica Italiana to appoint one representative to that Congress, whose venue is in a sanctionist country. The Unione Matematica in its Statute provides for the participation to scientific Congresses; most members attended the International Congress of the year 1928, held under the auspices of the University of Bologna in that same city, that was and is the site of our association. Our Presidency saw to the organization and the progress of the Congress. Consequently our participation to this Congress has quite a different meaning than the participation of any other scientific academy or association. Hence we think it advisable that first of all the government stances be granted.
The convention, after impartial discussion, resolves to instruct the Presidency with regard to the participation of the U.M.I. to Oslo's International Mathematical Congress.

9 Ernestina Fasciotti from Milan, Elena Palazzo from Rome, Pietro Sciré and Pietro Tortorici from Palermo, and Vito Volterra.

last events and to whom the Congress of Oslo sent a warm greeting telegram. Severi, hence, did not make it, in spite of his personal request for authorization to participate, which was denied by the Minister[10].

> Ministero dell'Educazione Nazionale Roma, 30 Mag. 1936, XIV
>
> Al Rettore della Regia Università di Roma
>
> Voglia la S. V. comunicare a S. E. il Prof. Francesco Severi, di codesto Ateneo, il quale, giusta quanto mi ha fatto presente il Ministero degli Affari Esteri, avrebbe manifestato il desiderio di assistere al Congresso Internazionale di matematica, indetto ad Oslo per il luglio prossimo, che non ritengo opportuna la sua partecipazione a tale Congresso.
>
> Il Ministro di Val Cismon[11]

The absence of Severi, "empêché de venir au Congès d'Oslo" as É. Cartan announced at the closing meeting in which the first two Fields medals were awarded (to Lars Ahlfors and to Jesse Douglass), caused a lot of problems because Severi himself would have had to chair also the executive committee of the board chosen at the Congress of Zurich to study the problem of the international organization of mathematicians. The board, on which sat also G. Julia, H. Weyl, W. Blaschke and C. Carathéodory, had already met a couple of times (in Rome in March 1934 and in Paris in February 1935) without managing to find a satisfactory agreement, and therefore had decided to summon a full board meeting during Oslo's activities. Acknowledging that the present circumstances were not favourable at all to organize an effective International Union, Julia closed the board's activity, strongly complaining about "l'absence de son president, Severi". And the Congress (on proposal of E. Cartan, C. Carathéodory and G. D. Birkhoff) proposes Levi-Civita – despite his forced absence – as member of the board that should have awarded the next two Fields medals.

In short, mathematicians of the "imperial" Italy thought they could do without a shot of that internationalism whose usefulness is instead underlined by C. Stormer in his speech at the inaugural meeting[12]:

> Plus peut-être que dans aucun autre domaine de la culture les résultats acquis par les sciences sont internationaux. Les découvertes scientifiques faites par les hommes

[10] The document is kept in the Historical Archive of the University of Roma, "Fascicolo personale" of Francesco Severi.

[11] National Department for Education Rome, 30 May 1936, XIV
To the Rector of the Royal University of Rome
Please be so kind as to communicate to Prof Francesco Severi, member of this University, who has manifested the wish – a legitimate one, as the Foreign Office Ministry has pointed out to me – to attend the International Congress of Mathematics, to be summoned in Oslo next July, that I do not deem his participation to such Congress advisable.
The Minister di Val Cismon

[12] Cf. Carl Stormer, in *Comptes Rendus du Congés International des Mathématiciens*, I, Oslo 1937, pp. 40–41.

de sciences d'un pays peuvent être aussitôt utilisées dans le monde entier. Ceci s'applique tout particulièrement aux mathématiques. Les vérités mathématiques sont en effet universelles et ses moyens d'expressions internationaux. En conséquence la collaboration par delà les frontières devrait être plus naturelle pour les mathématiciens que pour tous les autres savants. Nous voyons aussi que nos congrès ont toujours obtenu un succès qui témoigne clairement du désir de collaboration internationale des mathématiciens.

Évidemment un mathématicien qui fait une découverte peut en faire part au monde entier en la publiant, mais en en faisant part de vive voix dans un congrès il dispose des moyens plus favorables d'atteindre à un public spécialement intéressé. De ceci témoigne le très grand nombre de conférences qui auront lieu à ce congrès.

Et pourtant il se peut que l'oeuvre la plus importante d'un congrés comme celui-ci n'est pas celle qui résulte de ces conférences, mais de la conversation familière entre mathématiciens des différentes parties du monde. L'échange direct d'idées sous forme de conversation a une importance qui sans qu'on puisse encore en trouver la trace dans les comptes rendus du congrès, se manifestera pourtant dans la littérature mathématique des années qui vont suivre. C'est surtout pour les jeunes mathématiciens que de telles rencontres ont de l'importance, gràce à l'orientation qu'elles leur donnent et à la stimolance qu'ils acquièrent en entendant des paroles encourageantes de vieux et illustres collègues.

C. P. Eisenhart insisted with equal clarity on the banality of the concept of "national" direction in modern Mathematics[13]:

An unusual honor has been conferred upon me in requesting me to say a few words at this dinner for the English speaking countries represented at this Congress. These countries have made great contributions to mathematics, but after all we do not think of mathematics as developed along national lines. When one observes that at this Congress there are representatives of at least thirty nations, and all of them are interested in the history and development of the same science, one realizes that mathematics is international. As such, it does not recognize national boundaries; these have to do with political and economic considerations. Perhaps it is because maps deal with national contours and mathematicians are international in their way of thinking that mathematicians have never been able to solve the four colour map problem.

Racial events and the next positions adopted by the UMI announced a further stage in the gradual international isolation of the Italian mathematical community, besides confirming its definite renunciation of any attempt to defend its own professional autonomy.

[13] Cf. C. P. Eisenhart, *ibidem*, p. 53.

3. The anti-Semitic laws of 1938

Italian Jews of the Kingdom of Piedmont and Sardinia gained civil and political rights in 1848. The events of the Risorgimento, culminating in 1870 with the capture of Rome, had extended these measures to the regions that had been gradually annexed to the new State. Ever after, Italian Jews took part in the life of the country as normal citizens, equal to the others, no longer subject to the restrictions, harassments and banishments that had marked their centuries-old presence in Italy. Ninety years later, between July and September 1938, the almost fifty thousand Jews living then in Italy discovered – and the country with them – that all had been a dream: fascism and the monarchy that endorsed its choice, broke at one single blow the pact of national unity.

We are not only speaking about the sad experiences of a few hundred intellectuals, but rather explaining the importance of the damage these communities suffered. For Mathematics the damage was an ethical one too – to our mind come the words of G. Colonnetti about the "crime of prostituting science"[14] and those of R. Finzi about the "booming approvals" and the "deafening silences"[15] of our intellectuals towards the State's anti-Semitism – and it affected deeply and negatively the stances of the younger generations. But it had also an immediate effect that concerned the productivity of the research. What emerged as the peak of fascist repression, and the beginning of its explicit divorce from the feelings of the great majority, kept researchers of great merit away from study and teaching – in the mathematical field too – with the unavoidable repression of the most brilliant young students.

The analysis of the racial laws of 1938 and of their effects on the Italian mathematical world requires some other general remarks. The first one concerns the – qualitative and quantitative – strength of the presence which Italian Jews reached in every intellectual field, and especially in the scientific area, as the aftermath of the achievement of rights equalization. Within Mathematics we have dealt, among others, with S. Pincherle, V. Volterra, C. Segre, G. Castelnuovo, F. Enriques, T. Levi-Civita, E. E. Levi. This list alone, distinguished by names of unquestioned and acknowledged scientific calibre, would be enough to justify the question about whether the success of such a solid group of Italian Jews must somehow be explained as a result of special attitudes. The most remarkable element of this phenomenon[16] is related to the cultural level of the Jewish community: compared with illiteracy of the Italian people in 1861, not less than 70%, Jewish illiteracy was just above 5%. In the following decades, the number of Italian illiterates had fallen to 50% in 1901 and to 27% in 1927; in parallel, in 1901, illiteracy among Italian Jews had fallen to 5% and in 1927 it had even disappeared.

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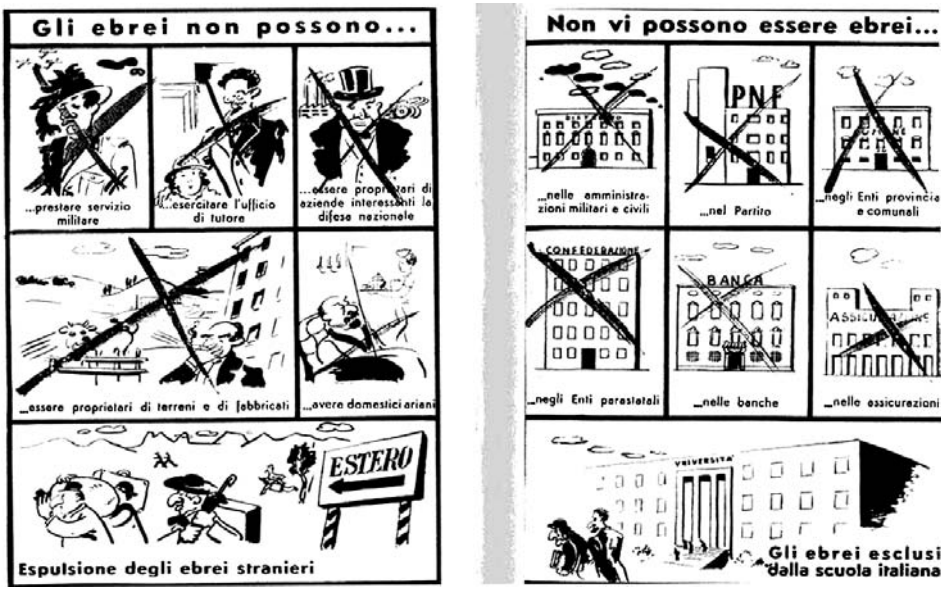
[14] Cf. G. Colonnetti, *Pensieri e fatti dell'esilio (18 settembre 1943-7 dicembre 1944)*, Roma, Accademia Nazionale dei Lincei, 1973, pp. 53–54: "Who among us who has not met biologists that have easily defended racial theories; or economists that have treated as a social progress that official machine that was fascist corporativism, or technicians that have seen autarchy as a progress (…)? To these belong a new type of crime; that of prostituting science. They are to be ineluctably expelled from University, with the whip, as the merchants of the Temple".

[15] Cf. R. Finzi, *L'università italiana e le leggi antiebraiche*, Roma, Editori Riuniti, 1997, p. 20.

[16] For a deeper analysis, see G. Israel, P. Nastasi, *Scienza e razza nell'Italia fascista*, Bologna, Il Mulino, 1998; R. Maiocchi, *Scienza italiana e razzismo fascista*, La Nuova Italia, Firenze, 1999.



The racial laws of 1938

This explains the strong Jewish presence among the still weak Italian *intelligentsia*, in the decades at the turn of the century. These were widely "assimilated" intellectuals, who felt like Italian citizens to all intents and purposes and did not conceive even the possibility of a dual nationality, Italian and Jewish. In the period between the unification of the Nation and the rise of fascism, their posture is modelled on the unconditional and enthusiastic participation in the country's life. Almost all Jews had been patriotic, often more than the rest of the Italians, as if to show their gratitude for the new institutional and political order that had released them from the segregation of the ghettos and from discrimination. They took part in the *great war*, suffered their fallen, won their medals. And with fascism there was no substantial change in their behaviour. Quite a few Jews were blackshirts and continued to participate in the country's life with undiminished fervour. Quite a few Jewish professors taught corporative law and quite a few were active participants in the first drafting stages of the regime's demographic, eugenic and racial policy. In short, Jews – like many other Italians – did not realize at all the outcomes that would result from the regime's policies. They thought and behaved as ordinary Italians and their link to Judaism was more or less the preservation of a surname that indicated their origin. The assimilation process involved also religious feelings. If these were still relatively strong in the middle and lower classes, they were really weak among intellectuals or in the ruling class. It is really difficult, if not to say impossible, to trace back anything Jewish in the way of thinking, writing or behaving of researchers such as Volterra, Castelnuovo, Enriques or Levi-Civita. There is not the slightest trace of issues or subjects linked to Hebraism in their correspondence.

And yet, despite the absence of a strong Jewish identity and consciousness, there were in Italy, quite before the advent of fascism, expressions of anti-Semitic rancour and resentment in a considerable part of the Catholic world and also in that scientific and university world we are more interested in. There is, for instance, a letter exchange among some mathematicians between 1909 and 1924 – that is, before the period we are dealing with. The subject of the correspondence was a usual dispute for professorships, but out of it came to light the existence of an anti-Semitic feeling that identifies the mathematical school of the university of Rome with a den of "Judeans", infiltrated into the state institutions.

In Chapter 6 we talked about R. Marcolongo as a good mathematical physicist. In 1897 he had to compete for a professorship with the rising star of Italian mathematical physics, the twenty-four-year-old Tullio Levi-Civita[17]. The competition's result was such that Levi-Civita got a temporary professorship in Padua while Marcolongo remained in Messina (where he already taught Rational mechanics as temporary professor). Marcolongo's interest in a transfer to Rome is clear in a letter P. Burgatti sent to him in 1909[18].

> Per la Scuola di Roma è un disastro; penso che tutto andrà nelle mani degli ebrei, se tu non saprai farti valere. La cattedra di Roma toccherebbe a te; ed io l'ho scritto al Tonelli[19]; ma temo che i giudei ti preparino una guerra spietata. Comunque bisogna lottare e sperare nella vittoria[20].

Burgatti must be alluding to Vito Volterra and Guido Castelnuovo, as Levi-Civita and Enriques[21] moved to Rome after the First World War. We don't have Marcolongo's answers, but we can get an idea of the situation and of the drift of the conversation from what Burgatti writes[22]:

> dalla tua lettera comprendo che ti sei avvilito, e ciò mi dispiace. Io ti esorto a rimanere sulla breccia; ché la vittoria non può mancarti. L'ostilità degli Ebrei ci

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17 For a reconstruction of this event, see U. Bottazzini, A. Conte, P. Gario (eds.), *Riposte armonie, Lettere di Federigo Enriques a Guido Castelnuovo*, Bollati Boringhieri, Torino, 1996, p. 348.

18 Pietro Burgatti (1868–1938), graduated in 1893 in Roma, where he remained as assistant and full-time teacher (*libero docente*) until 1908, when he won the professorship of Rational mechanics in Messina. Already in 1909 he moved to Bologna, where he lived until his death. His interest went from analytic mechanics to mechanics of the continuum, from complex analysis to partial differential equations. He collaborated with Tommaso Boggio and Cesare Burali-Forti on the writing up of *Analyse vectorielle* (1930).

19 Alberto Tonelli, about whom we have talked in the second section of Chapter 3.

20 For the School of Rome it is a disaster; I think that all will fall into Jewish hands, if you don't assert yourself. You are entitled to Rome's professorship, and so have I written to Tonelli; but I am afraid Judeans are preparing a ruthless war. Anyway, you must fight and hope for victory". Letter from P. Burgatti to R. Marcolongo, Bologna, 3.9.1909, *Fondo Marcolongo*, Department of Mathematics, University of Roma "La Sapienza".

21 The reference to Enriques appears instead in a letter of Burgatti to Marcolongo dated a few months before (13.3.1909): "here we have been on holiday for a month. Enriques has entered politics; yesterday he gave a speech in support of the socialist candidacy, professing himself liberal-conservative, though. He's a one!"

22 Letter from Burgatti to Marcolongo, November 1909, ibid.

> era ben nota; non devi dunque scoraggiarti nel momento della battaglia. Essi speravano nel Levi-Civita; e forse ora lo stringeranno con tali ragioni da fargli accettare ciò che aveva rifiutato; ma se egli persiste nel rifiuto (…), gli Ebrei dovranno rassegnarsi a vederti a Roma. Questa è la mia persuasione; perché infine non dev'esser difficile far capire a chi non è Volterra o Castelnuovo che il Lauricella[23] sarebbe un pessimo insegnante di Meccanica, pur ammesso che sia un valoroso analista[24].

After some ups and downs, Marcolongo refuses:

> molto mi dispiace che tu abbia rinunciato a Roma. Questa tua ritirata farà gongolare di gioia gli Ebrei. Potevi rinunciare in cuor tuo, ma stare sulla breccia per aver almeno la soddisfazione di dare dei grattacapi ai Giudii. Io non ho nessuna voglia di cavarmi il cappello a loro, che non stimo punto"[25].

The Jewish matter – really an obsession in the correspondence between Burgatti and Marcolongo! – also worries other mathematicians such as C. Arzelà, for instance. Burgatti writes[26]:

> la notizia della nomina del Tedone[27] a corrispondente dei Lincei m'ha recato meraviglia e disgusto. Gli ebrei spadroneggiano in modo indegno, e gli altri li lasciano fare. Volterra, che ambisce alla carica di Presidente[28] ha preferito il Tedone a te; ed è naturale; ma che gli altri lo abbiano aiutato a commettere tanta ingiustizia, è cosa veramente incredibile. Tu hai ragione di dolertene; ma non devi dare troppa importanza a coteste bricconate, ché il non essere Linceo nulla toglie ai meriti che tu hai, e che tutti gli uomini dabbene ti riconoscono. A quella canaglia è meglio mostrare il disprezzo, che il proprio dolore. Mi diceva Arzelà un giorno che a cotesta gente (voleva dire gli ebrei) bisogna mostrare apertamente il proprio disprezzo in ogni

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23 Giuseppe Lauricella (1867–1913), analyst and mathematical physicist, who in 1910 obtained the transfer from Rome to Catania.

24 "From your letter I see you are discouraged, and I am sorry about it. I recommend you to be fully active; you cannot miss victory. Jewish opposition was well known; you don't have to lose heart at the time of battle. They hoped in Levi-Civita; and maybe now they will push him with reasons that will make him accept what he had declined; but if he persists in his refusal (…), Jews will have to get used to seeing you in Rome. This is my conviction; as in the end it must not be easy to make understand to anyone who is not Volterra or Castelnuovo that Lauricella would be an appalling professor of Mechanics, even admitting he is a valued analyst.

25 I am very sorry you have renounced to Rome. Your withdrawal will make Jews be overjoyed. You could have given up deep down, but be fully active so as to have at least the satisfaction of troubling the Judeans. I don't feel like taking off my hat before them, I don't appreciate them at all. (Letter from Burgatti to Marcolongo, Bologna, undated, maybe after 1910, ibidem).

26 Letter from P. Burgatti to R. Marcolongo, Bologna, 2.1.1912, *Fondo Marcolongo*, cit. The allusion here is to the Royal Prize of the *Accademia dei Lincei* for the year 1905.

27 Orazio Tedone (1870–1922), mathematical physicist. Tedone had been appointed "*correspondente linceo*" in 1911.

28 President of the *Accademia dei Lincei*. Volterra will actually become its President only in 1923.

occasione; ed in quel modo egli ottenne il premio reale, che volevano dare tutto intero al Castelnuovo[29].

And some years later[30]:

occorrono uomini nuovi. (…) E voglio farti una confessione: se al presente dovessi scegliere un uomo nuovo, sceglierei il Pincherle, il quale è stato in questi ultimi tempi strenuo difensore delle buone tradizioni della Scuola matematica italiana, senza badare a persone e ad amicizie. Oltre ad avere idee buone, è *fermo* nei suoi principi, e sa tener testa ai Volterra, ai Castelnuovo … e compagnia bella. È un *giudio* sì; ma molto diverso dagli altri. Almeno tale a me pare[31].

The illusion lasts very briefly[32]:

l'altro mese fui messo in ballo per il rettorato. In verità era una *bega* che non m'andava; ma spinto da molti amici dovetti accettare la candidatura. Per fortuna non sono riuscito (per 7 voti), e me la son cavata con una votazione lusinghiera. Alla quale fortuna contribuirono molto i nostri *carissimi* (!) matematici; specialmente il Pincherle, che in questa occasione ho pienamente conosciuto, tal quale me lo dipingeva il povero Arzelà. *Non ti fidare, amico mio; gratta, gratta, son tutti uguali* (s'intende i Giudii)[33].

The suspicion and the resentment towards Jewish colleagues became general and spreads with no extenuating circumstances. E. E. Levi's appointment to a chair was seen as an abuse[34]. Castelnuovo was depicted as a professor who served up to students some-

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[29] The news about Tedone's appointment to *Lincei membership* has surprised and revolted me. Jews lord it in a shameful way, and the rest let them do. Volterra, who aspires to the post of President, has preferred Tedone to you; and it is natural; but it is incredible that the others helped him to commit such injustice. You are right to complain; but you must not give much importance to such rascally tricks, as not to be member of the *Lincei Academy* does not deprive you of your worthiness, which every honest man acknowledges. It is better to show contempt than one's own sorrow to that rabble. Arzelà told me one day that one has to show always his own contempt openly to these people (he meant Jews); and in this way he obtained the royal prize that they wanted to give wholly to Castelnuovo.

[30] Letter from P. Burgatti to R. Marcolongo, Bologna, 1.2.1913, *Fondo Marcolongo*, cit.

[31] We need new men. (…) And I want to confess you something: if now I had to choose a new man, I would choose Pincherle, who has lately been strenuous supporter of the good tradition of the Italian mathematical School, regardless of people and friendships. Apart from having good ideas, he is firm in his *principles*, and knows how to face the Volterra, the Castelnuovo… and company. He is a *Judean*, yes; but very different from the others. At least so I think.

[32] Letter from P. Burgatti to R. Marcolongo, Bologna, undated, maybe after 1915, Ibidem.

[33] Last month I was dragged into the election for the rectorship. Actually, it was a hassle that I didn't fancy; but pushed by many friends I had to accept the appointment. Luckily I haven't succeeded (for 7 votes), and I managed with a flattering voting. To this luck contributed a lot our *dearest* (!) mathematicians; specially Pincherle, whom I have got to know on this occasion, just like poor Arzelà portrayed him. *Don't trust, my friend; in the end, men are all the same* (it is meant the Judeans).

[34] Letter from P. Burgatti to R. Marcolongo, Bologna, 1.2.1913, Ibidem.

thing "smattering; stuff taken here and there from popularization journals and books, known to everybody (at least in such a form). He wants to set himself up as a physicist; maybe because he has realized that his geometry is useless"[35]. Severi's or Enriques' call to Rome, to replace A. Tonelli (about whom we talked in the third chapter), became a plot with which "Judeans (…) would certainly like to complete the ghetto and become thus the rulers of mathematics"[36]. It is a pity that Severi was not at all a Jew and, on the contrary, he stood out for his scrupulous application of the racial laws.

One of the more significant fragments of this anti-Semitic chatter is a letter written in 1924 to another mathematical physicist, G. A. Maggi[37]. The writer sent Maggi a Note on an issue of the probability theory and professed to be interested in these topics since "towards the end of 1921 the oddities on relativity that were being printed in books, reviews and journals struck me". He wrote several Notes, but found also systematic resistance to their publication. Then, he wrote confidently to Maggi, who was interested in relativity but was not wholly convinced.

> Ella non è dunque del gran numero di coloro che si sono fatti relativisti, come ora tanti si fanno fascisti, e non ricuserà d'aiutare la pubblicazione di una critica incontestabile. Forse Ella troverà soverchiamente polemiche le mie osservazioni a pag.4bis. Ma, dopo letto un articolo, assai difettoso, del prof. Bouasse in *Scientia*, io gli scrissi per uno schiarimento, ed egli alla risposta aggiunse che la questione in Francia era diventata una questione di religione. Gli israeliti si adoperarono per impedire la divulgazione delle sue critiche. "*Toute la juïverie a donné*". L'anno scorso, nell'estate, mandai un articolo alla *Revue scientifique*, e poi riscrissi per farvi un'aggiunta. Nessuna risposta. Oltre all'*ésprit de corps* degli israeliti (temo molto del Volterra) vi sono tutti coloro che si sono compromessi esaltando la relatività e il suo profeta, e la caduta della meccanica di Galileo e di Newton, e dicendo che capivano le teorie dell'Einstein[38].

The themes are not original. A commonplace expression became a leitmotiv of the anti-Semitic racial campaign, especially in Germany: relativity was a "Jewish science"; Jews had worked especially to create and diffuse this new scientific theory, so as to

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[35] Letter from P. Burgatti to R. Marcolongo, Bologna, 19.2.1914, ibidem.

[36] Letter from P. Burgatti to R. Marcolongo, Bologna, 8.2.1921, ibidem.

[37] The letter, dated Genoa, 19.1.1924, kept in the "Fondo Maggi" of the Department of Mathematics in Milan, is from Giuseppe Erede to Gian Antonio Maggi (1856–1937), a good mathematical physicist, teaching in Pisa and later in Milan.

[38] So You are not among the great majority who have become relativists, as now many become fascists, and You won't refuse publishing an indisputable criticism. Maybe You will find excessively polemical my remarks on p. 4bis. But, after having read quite a poor article by Prof Bouasse in *Scientia*, I wrote to him for an explanation, and he answered me that this had become a religious issue in France. Israelites work towards preventing the publication of his critique. "*Toute la juïverie a donné*". Last year, in summer, I sent an article to the *Revue scientifique*, and later I rewrote to add an extra section. I got no answer. Besides the Jewish *ésprit de corps* (I am quite afraid of Volterra) there are all those who have bound themselves by exalting relativity and its prophet, and the fall of Galileo's and Newton's mechanics, and saying they understood Einstein's theories.

Benito Mussolini

establish their supremacy. The Italian translation of these insinuations depicted the Roman group as a mathematical school ruled by a group of Jews, so attached to their racial-religious identity that they practice a recruitment policy intent on excluding non-Jews. It is not – frankly – a reliable analysis. It is just an invention to talk about a Jewish *esprit de corps* regarding Volterra, who never hinted at his own Jewish identity. The same can be said for figures such as Enriques, Castelnuovo, Levi-Civita, Pincherle, Levi, etc.

Italian racial laws, with respect to mathematics, descended upon this valued, strong and a bit "chattered about" – but above all feared and envied – group, because of a *lobbying* action, in which religious reasons were certainly negligible compared with other social and cultural general values. We are in 1938, the "crucial and terrible" year "for European Hebraism"[39]. On its eve, only Nazi Germany had enacted anti-Semitic legislation; at its end such legislation had permeated the continent: from the after-the-*Anschluss* Austria (13th March 1938) to Poland, from Rumania to Hungary. Mussolini's final decision to pass anti-Semitic legislation and to make the regime officially anti-Semitic was therefore part of a continental process in which fascism, given its political and diplomatic importance, took part as a protagonist. Here the "calendar" of the year 1938:

14th February — Bottai asks the Rector of the Polytechnic of Milan (and presumably also other Rectors) data about Jewish presence among students and Professors[40];

[39] E. Mendelsohn, *Gli ebrei dell'Europa orientale tra le due guerre mondiali*, in *La legislazione antiebraica in Italia e in Europa* (Minutes of the Congress for the fiftieth anniversary of the racial laws, Rome 17th–18th October 1988), Chamber of Deputies, Rome 1989, pp. 343–353.

[40] Cf. A. Galbani, Provvedimenti razziali: un documento inedito del febbraio 1938, *La rassegna mensile di Israel*, V, LVII, n. 3 (1991), pp. 533–536.

13th March	The Great Council of Fascism supports the *Anschluss* (the same day of the annexation);
3rd–9th May	Hitler visits Italy;
31st May	Mussolini, as Minister of the Interior, asks a prefect to ascertain the “professed religion” of a candidate for a ministry’s competition;
4th June	a delegation of the Nazi race department visits Italy;
6th June	a prefect is asked to verify the “professed religion” of a (just appointed) district secretary of a fascist labour union;
18th June	the Cabinet of the prime Ministership receives an “outline disposition” from the *Duce* stipulating that Jews be prevented from participating in International Congresses. The stance is formalized and sent to the Authorities through a circular (21st July) of the under-secretary in the Prime Ministership;
July	Mussolini gives instructions to the parliamentary private secretaries of “his” ministries (War, Navy and Air) not to accept Jews in the military academies;
14th July	the document *Fascism and the problems of race*, better known as the *Manifesto of the racist scientists*, is published in the journal *Giornale d’Italia*. A brief introduction announces that it has been written by a “group of fascist researchers, professors in Italian universities” who have worked “under the aegis of the Ministry for the popular Culture”. The writing establishes “Fascism stance towards racial problems”. Through this document Italians find out suddenly that they belonged to the *Aryan race*;
19th July	the idea of transforming the Central Demographic Office in the Interior Department into *Head Office for Demography and Race*, later known as *Demorazza,* is released;
29th July	the Police General Management asks the Prefects for the member list of Jewish communities and the list of the “dissociates”;
17th August	a circular of the Interior Department orders the Prefects to prevent Jews from being appointed to “official positions”;
22nd August	the *Demorazza*, together with the *ISTAT*, carrys out a special census of Jews;


EDIZIONE DEL POMERIGGIO

CORRIERE DELLA SERA

Le leggi per la difesa della razza

approvate dal Consiglio dei ministri

I matrimoni misti sono proibiti - La definizione di «ebreo», le discriminazioni e l’annotazione allo Stato Civile - L’esclusione dagli impieghi statali, parastatali e di interesse pubblico - Le norme concernenti le scuole elementari e medie e gli insegnanti

25th August	the under-secretary in the Prime Ministership diffuses a circular that prohibited conferring knightly honours on Jews;
5th–7th September	School Decree law against Italian Jewish teachers and students and against foreign Jews;
24th September	the *Demorazza* asks the prefectures to report within two days “about the Jewish question” and about the “situation of Jews in political administrative labour official positions or in commerce or industry”;
29th–30th Sept.	Hitler and Mussolini meet in *Monaco*;
6th October	The Great Council of Fascism decides for persecution of the Jews;
17th November	the law regulating the so-called racial question is issued, absorbing the decrees of the 5th and 7th September.

In a way, fascism does not invent anything new. In the previous centuries, many Italian regions had enacted anti-Semitic laws, later wiped out by the Risorgimento. It is in fact the heritage of the Risorgimento that explains the delay in Italian anti-Semitism, matured all over Europe. In 1938, the last tragic stage of the Spanish civil war takes place, a general testing ground for the second World War and the first great international confrontation between the two opposing ideological groupings. Francoist Spain is one of the grounds on which Mussolini begins to devise a different international posture (until that moment bound to the course of European diplomacy). In that same year, in autumn, the Congress of Monaco seales the destiny of Czechoslovakia with the silent consent of the great European powers that push Germany eastward, against the Soviet Union, so as to delay the threatening European war as much as possible.

But the unavoidable reference to Europe cannot reduce Italian racial laws to a phenomenon deriving directly from Germany. Neither can it be asserted that Italy did not experience a real anti-Semitism, that laws are “mildly” applied, almost dictated by Hitler to Mussolini[41]. It is certainly true that anti-Semitism has never had deep roots in Italy, that can be compared assimilable to German, Polish or Russian ones. The reality of Italian Jewish communities is deeply different: there is not a strong ethnic identity historically rooted and represented by different language and customs, but communities that in the last century had deeply integrated under the Risorgimento. Italian anti-Semitism, essentially political – not biological –, developes especially in the “imperial” years going from 1937 to 1939, when Italy goes through a stage of renewed aggressiveness – openly xenophobic –, where the anti-Semitic element developes together with an element of strong nationalistic revanchism directed at the Mediterranean area. The *Manifesto della razza* is the attempt to give a scientific explanation to an essentially political act. Still, it cannot be denied that the basis of anti-Semitism was also present in our culture, especially in the catholic world. The image of deicidic Jews, of Jews condemned by God or, at best, of Jews as people to convert and straighten out, is present in most of the ecclesiastic tradition. Cunningly, fascism put down roots in this ground. It is not by accident that among the first writings issued to ideologically justify racial laws we find texts like

[41] In many universities, local journals of fascist students publish the names of all Jewish professors of the university, pointing them out with general contempt.

La Chiesa e gli ebrei[42], which recalls the Catholic Church to its traditions, reminding people about the anti-Hebraic past of the Church Fathers and of the *Compagnia di Gesù*. Such calls are not left unanswered by remarkable exponents of the Catholic world, such as father Agostino Gemelli, founder of the *Università Cattolica* of Milan and President of the *Pontificia Accademica delle Scienze* that still admittes (in 1936) "Jewish" scientists, such as Volterra and Levi-Civita.

There is no need to take the *mild* application of racial laws further, as there are several pieces of evidence to deny it. In a mild way or not, racial laws are applied, with a joint liability of the State leadership that do not admit *reservations*. The data is significant especially at a scholastic level: hundreds of university and high-school teachers, hundreds of university students, thousands of high-school students and elementary students are expelled during the academic year 1938–39. In a mild way or not, professional practice is made practically impossible to whoever belonged to the Jewish religion, or rather to the Jewish race. Of course, it is possible to bring evidence of solidarity and help to Jews from single public functionaries that try to circumvent the laws or not to apply them word for word, saving (after 1943) many human lives. But these are single testimonies and individual acts, as noble and commendable as they are random and sporadic. There is no collective and organized solidarity, even among researchers. Their world is one of the first to be damaged. The fascistization of society has to begin with the school. Within this context we find the most grotesque events, such as the "*bonifica del libro*" *(decontamination of the book)*, promoted to *purify* school textbooks of all sorts of Jewish contamination. Thus, in the "historical" reconstructions of the years immediately after 1938, Jewish mathematicians disappeare. See the documentary material for the organization of the *Mostra della Scienza*, which was to be held in 1942 on the occasion of the *Esposizione Universale* of Rome and that was never held because of the outbreak of war; its buildings, partly realized, will be the centre of the Roman district called EUR (*Esposizione Universale di Roma*). In the session[43] of the 17th November 1939, a subcommittee embracing several first-rate mathematicians (among them E. Bompiani, Et. Bortolotti, F. P. Cantelli, G. Giorgi, G. Krall, M. Picone, G. Sansone, F. Sibirani, F. Severi, A. Signorini, L. Tonelli, F. Conforto and R. Marcolongo) is installed for mathematics. Bompiani states that it is necessary, "besides explaining the principles, to claim priority of study for Italians, *when this is possible without distorting science history*" and Sansone immediately offers personally to prove that algebra was born in Italy. In the heat of the moment, those present distribute the several themes, within a historiographical approach tending to the reappraisal of Italian scientific contributions, miles away from the one that distinguished the teaching of Federigo Enriques. Their first result – *Indice e*

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[42] Cf. R. Farinacci, *La Chiesa e gli ebrei*, inaugural speech of the *Istituto di Cultura Fascista di Milano* on the 7th November 1938–XVI, Cremona, Stab. Tip. Società editoriale "Cremona Nuova", 15 pp. By the same author, leading exponent of the fascist squads' activities, see the speech of the 23rd January 1940, radioed to middle schools: *Motivi essenziali della difesa della razza*, in G. Isola, *L'ha scritto la radio. Storia e testi della radio durante il fascismo (1924–1944)*, Milan, Bruno Mondadori, 1998, pp. 40–43.

[43] Universal Science Exhibition, Subcommittee of Mathematics, meeting of the 17 November 1939, ACS, EUR, SOM, file OA D/2-0-0.

Mauro Picone and Francesco Severi (the second and the third from the left)

norme per la presentazione della Matematica nella Mostra della Civiltà Italica – specifys that the index aimed at enumerating the people who "must not be forgotten" and that it had been written with the principle "that Italian contribution to mathematics is, in several essential moments, one of the highest expressions of the intellectual value of the Italian race, to be put, hence, in the front line; all the more because it is invariably ignored in foreign works on history of mathematics". Not a single Jewish mathematician appears in the list (limited to dead mathematicians). It is absurd to see a name like C. Segre erased from Algebraic geometry. But in Bompiani's long historical article titled: *Contributi italiani alla Matematica* we go from ridiculous to obscenity. There are no explicit anti-Semitic statements, but there is the systematic effort to give an image of Italian mathematics depurated of every Jewish contribution. Not a single Jewish mathematician is mentioned, also at the cost of presenting some research fields in a farcical way. In Functional analysis, for instance, Volterra is omitted. Even more egregious, if possible, is the omission of Levi-Civita's contribution to the foundation of tensor calculus (according to Bompiani ascribable to Ricci-Curbastro alone). The limit is reached, lastly, in the presentation of the contribution of the Italian geometric school, which, as Bompiani remarks, held by then "a position of absolute supremacy within the algebraic branch". This position had been achieved also, and above all, thanks to the researches of Jewish mathematicians such as C. Segre, G. Castelnuovo and F. Enriques. Their names are omitted, though. In the volume *Un secolo di progresso scientifico italiano*[44], Comessatti theorized: "the effective force of tradition acts with historical inevitability when, as in the case of the Italian geometric school, that tradition is grafted onto outstanding race qualities, creating even a thinking style, priceless inheritance of intellectual autarchy". But here the "cleansing" operation of Jewish contributions had proved so difficult that, in a full-page foreword published at the beginning of the volume it was stated that[45]:

> per la migliore intelligibilità degli Articoli che seguono, sono citati anche gli apporti più rilevanti di matematici ebrei, che furono professori nelle Università italiane, in quanto l'opera loro, a causa della posizione ufficiale che occupavano, non poteva non determinare reciproci scambi fra i contributi da essi apportati e quelli dei matematici ariani. Lo stesso criterio è stato adottato per gli Articoli di tutte le altre Sezioni[46].

The outcomes of the anti-Semitic legislation on the mathematical world are devastating.

- Guido Ascoli, professor of Analysis, *University of Milan*;
- Ettore Del Vecchio, temporary professor of Mathematics for Economics, *University of Trieste*;
- Federigo Enriques, professor of Higher Geometry, *University of Rome*;
- Gino Fano, professor of Geometry, *University of Torino*;
- Guido Fubini Ghiron, professor of Analysis, *Polytechnic of Turin*;
- Guido Horn d'Arturo, professor of Astronomy, *University of Bologna*;
- Beppo Levi, professor of Analysis, *University of Bologna*;
- Tullio Levi-Civita, professor of Rational Mechanics, *University of Rome*;
- Arturo Maroni, professor of Geometry, *University of Pavia*;
- Giorgio Mortara, professor of Statistics, *University of Milan*;
- Beniamino Segre, professor of Geometry, *University of Bologna*;
- Alessandro Terracini, professor of Geometry, *University of Turin*

were removed from teaching[47]. On the strength of Article 4 of the Royal Decree Law n. 1390 of the 5th September 1938 on the so-called *Measures for the defence of race in fas-*

44 The volume was published (in 1939) by the SIPS that Volterra founded in 1907.

45 *Un secolo di progresso scientifico italiano (1839–1939)*, Rome, SIPS, 1939, Vol. 1, p. 47.

46 For a better comprehensibility of the following articles, the most relevant contributions of Jewish mathematicians, who were professors in the Italian universities, are quoted too, as their work, due to the official position they held, caused necessarily reciprocal exchanges between their contributions and those of the Aryan mathematicians. The same criterion has been adopted for the articles in other sections.

47 To these names, that concern full professors, are to be added those of the temporary professors: Alberto Mario Bedarida (Algebraic analysis in *Genoa*), Giulio Bemporad (Astronomy in *Turin*), Bonaparte Colombo (Infinitesimal analysis in *Turin*) and Bruno Tedeschi (Financial and actuarial mathematics in *Trieste*).

The sign assures that the shop is Aryan!

cist school, all of them were expelled from the Italian educational system and from cultural Institutes and Academies, with effect from the 16th October 1938. Of course, to single out and hit Jews, the Institutes themselves – from which they were to be expelled – must have carried out an internal census. A recent volume[48], from which we learn that Severi in his own card – he filled in at least 11! – wrote that "the undersigned and all his ascendants, family and in-laws, belong to the Aryan race and are Catholic", explains all this immense inquisitorial material. Bompiani underlined, instead, that all his ascendants had "always belonged to Catholicism. The family name and the aristocratic title come from the feud of Castel "Bon Piano", awarded to the founder of the family for his participation in the Crusades". The UMI underwent a strong slimming cure. About 10% of the total, 27 members, were expelled. The behaviour of the professional association of Italian mathematicians was just shameful. Its *Scientific Committee* met on the 10th December[49].

> La Commissione Scientifica della U.M.I. si raduna il giorno 10 dicembre in una sala dell'Istituto Matematico della R. Università di Roma. Sono presenti: Berzolari, Bompiani, Bortolotti Ettore, Chisini, Comessatti, Fantappié, Picone, Sansone, Scorza, Severi. Ha giustificato l'assenza il prof. Tonelli. Presiede il prof. Berzolari, funge da segretario il prof. Bortolotti. Dopo amichevole, esauriente discussione, risulta stabilito quanto segue: Una rappresentanza della U.M.I. si recherà da S. E. il

[48] A. Capristo, *L'espulsione degli ebrei dalle accademie italiane*, Zamorani, Torino, 2002.
[49] "Bollettino U.M.I.", S. II, a. 1 (1939), n. 1, pp. 89–90.

> Ministro della Educazione Nazionale, e gli comunicherà il voto della Commissione 'perché nessuna delle cattedre di Matematica rimaste vacanti in seguito ai provvedimenti per l'integrità della razza, venga sottratta alle discipline matematiche'. Il voto continua osservando che: 'La scuola matematica italiana, che ha acquistato vasta rinomanza in tutto il mondo scientifico, è quasi totalmente creazione di scienziati di razza italica (ariana): Basti ricordare, oltre Lagrangia, fra gli scomparsi, Arzelà, Battaglini, Bellavitis, Beltrami, Bertini, Betti, Bianchi, Bordoni, Brioschi, Capelli, Caporali, Casorati, Cesàro, Cremona, De Paolis, Dini, D'Ovidio, Genocchi, Morera, Peano, Ricci-Curbastro, Ruffini, Saccheri, Siacci, Trudi, Veronese, Vitali. Essa, anche dopo le eliminazioni di alcuni cultori di razza ebraica, ha conservato scienziati che, per numero e per qualità, bastano a mantenere elevatissimo, di fronte all'estero, il tono della scienza matematica italiana, e maestri che con la loro intensa opera di proselitismo scientifico assicurano alla Nazione elementi degni di ricoprire tutte le cattedre necessarie[50].

Soon after – on the 18th December – the Chairman Berzolari rushed to claim from the CNR the due compensation for this unjustifiable act of consent[51].

> Poiché la situazione finanziaria è ancora più meschina per la mancanza delle quote sociali relative ai Soci eliminati (non ariani) non è possibile all'Unione adempiere al suo compito, particolarmente delicato in questo momento in cui bisogna dimostrare all'Italia e all'Estero che la Matematica Italiana può tener alto il suo prestigio anche senza il concorso di forze ebraiche, senza l'aiuto di organi che abbiano nei loro fini la ricerca e la cultura. Il Comitato, per la Fisica e per la Matematica Applicata divide certo con noi questa responsabilità e deve vedere con simpatia il nuovo indirizzo che si vuol imprimere nell'Unione ed aiutare ad attuarlo. Io perciò mi rivolgo a te e al Prof. Bordoni affinché il Comitato voglia concorrere a questa

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50 The U.M.I.'s Scientific Committee meets on the 10th December in a room of the Mathematical Institute of the R. University of Rome. The attendants are: Berzolari, Bompiani, Bortolotti Ettore, Chisini, Comessatti, Fantappié, Picone, Sansone, Scorza, Severi. Prof Tonelli has justified his absence. Prof Berzolari acts as chairman, Prof Bortolotti as secretary. After a friendly, exhaustive debate, what follows is established: a delegation of the U.M.I. will go before His Worship the Secretary for National Education, and will inform him of the committee voting "so that none of the chairs of Mathematics remained vacant due to the measures for the integrity of race, is taken away from the mathematical subjects". The voting goes on remarking that: "Italian mathematical school, that has achieved great renown in the scientific world, was almost wholly created by Italic (Aryan) scientists: we should only remember, besides Lagrangia, among the dead, Arzelà, Battaglini, Bellavitis, Beltrami, Bertini, Betti, Bianchi, Bordoni, Brioschi, Capelli, Caporali, Casorati, Cesàro, Cremona, De Paolis, Dini, D'Ovidio, Genocchi, Morera, Peano, Ricci-Curbastro, Ruffini, Saccheri, Siacci, Trudi, Veronese, Vitali. Even after the removal of some Jewish scholars, it has scientists who, in number and in quality, are enough to keep up, towards foreign countries, the tenor of Italian mathematical science, and masters who with their intense work of scientific proselytism ensure to the Nation elements worthy of holding all of the chairs.

51 A copy of the letter is found in the *Archivio dell'Accad. Naz. Sci.* (so-called of the XL), "Fondo Bompiani".

attuazione con un contributo di L. 5.000 e fin d'ora ti ringrazio, fiducioso, per quanto vorrai fare[52].

The demand was met with double-quick action and already on the 19th January 1939, F. P. Cantelli informed Berzolari that the *Committee for physics and applied mathematics* of the C.N.R. "following the Regime's stances of making closer and closer the co-operation between the researchers of pure mathematics and the researchers dealing with its several applications", granted the required contribution to the *Unione Matematica Italiana.*

The UMI's document of the 10th December was clearly built around two points: Italian mathematics was created by Aryan scientists and, even after its decimation, it had the necessary conditions for progress (and it would have no problem filling the vacant positions). This second statement served the real purpose of the document, which was to ensure that none of the chairs of mathematics that remained vacant through the racial laws was taken away from the mathematical subjects. It is a pity that it was a false hope as false as the first claim, after all: Signorini was called from Naples to replace Levi-Civita; Severi (finally) occupied the chair of Enriques, whom he replaced also as Director of the *Scuola di Storia della Scienza*; Fubini's course in Turin was temporarily given to Tricomi, to whom was also entrusted the direction of the mathematical Library (already held by Fano). The voting of December 1938 gave the clear sensation of being forced, dictated to by the agency of other scientific subjects, subject to the syndrome of encirclement, and by a personal *frenzy of consent*. It is impossible to isolate individual responsibilities but Tonelli's absence from the meeting of the *Scientific Committee* cannot

52 As the financial situation is still more miserable for want of the shares of the erased (non Aryan) Members, it is not possible for the Union to accomplish its duty, specially delicate in this moment in which it is necessary to show to Italy and to the foreign Countries that Italian mathematics can keep up its prestige also without the contribution of Jewish forces, without the help of organs whose purpose is research and culture. The Committee for physics and for applied Mathematics certainly shares with us this responsibility and should see empathically the new direction meant to be given to the Union and help to realize it. Therefore, I address to you and to Prof Bordoni for the Committee to help to this implementation with a contribution of L. 5000, hopefully thanking you in advance.

be suppressed, and it is certainly not by chance that such a meeting was convened by a summons to Rome, Severi's and Bompiani's venue. They appeared the most determined, by belief or opportunism, of those who sided openly with the regime's choice. Nor did Severi or any of the signatories of the UMI's document of 1938 think it necessary, afterwards, to indulge in any important self-criticism.

This evidence cannot, however, go as far as to accept the extremist thesis that Severi is one who[53] attacks, as typically "Jewish", the abstract character of modern Mathematics. Severi had revealed the same embarrassment in a conference given at the Sorbona in 1935[54].

> Dans le titre de ma conférence j'ai posé la question de l'existence d'un esprit de race, en particulier d'un esprit latin, même dans une science comme la nôtre, qui semble au premier abord être dépourvue de tout caractère subjectif et par suite être universelle ou plutôt internationale par sa définition même. Une question analogue, au point de vue teutonique, a été développée récemment par un de nos collègues allemands d'une façon qu'on a jugé un peu passionée et qui a causé plusieurs brouilleries dans le champ mathématique international.

The last statement clearly alludes to the speech of G. Hardy who, in *Nature*, had labelled as nationalistic hysteria the theories of L. Bierberbach and of other mathematicians particularly linked to the regime and to Nazi ideology[55]. Severi was overtly convinced of the intrinsic untenability of Bieberbach's theories, also because of the increasingly international nature of science.

> Chaque savant est influencé dans son travail, plus fortement et plus rapidement que dans le passé, par la pensée des savant d'autres pays: ce qui tend à neutraliser les effets du génie national.

The document of the UMI's *Scientific Committee* was soon known abroad, with easily foreseeable consequences. The international isolation of Italian mathematicians emerged also in other events. In October 1938 both Tullio Levi-Civita and Beniamino Segre were relieved from the co-managership of the *Annali di Matematica*, following Severi's specific suggestion[56]. In October 1938, the editor Springer-Verlag decided to replace Levi-Civita – the only Italian representative – in the editorial team of the *Zentralblatt*; Bompiani and Severi are the Italian mathematicians who took his place! This time the international community rose in revolt. O. Neugebauer asked first Levi-Civita and

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53 Cf. J. Evola, Gli ebrei e la matematica, *La difesa della Razza*, 3 (1939), pp. 24–28.

54 Cf. F. Severi, Peut-on parler d'un esprit latin même dans les mathématiques?, *Revue Scientifique*, 18 (1935), pp. 581–589 (582).

55 Cf. G.H. Hardy, The J-type and the S-type among Mathematicians, *Nature*, 134 (1934), p. 250. On Bieberbach's ideas and the answer by Hardy, one read S. L. Segal, *Mathematicians under Nazis*, quoted (in particular, pp. 248–268 and p. 363.

56 The event is portrayed in a letter from B. Segre to Levi-Civita dated 16th October 1938.

Another sign: the shop is Aryan!

the editor Springer for confirmation of the exclusion[57]. Then – when the racial justification was confirmed – Neugebauer decided to leave the editorial team. With him resigned R. Courant, the Americans O. Veblen and J. D. Tamarkin, the Danish H. Bohr and the English G. Hardy: international scientific solidarity had been severely wounded; the (remaining) threads that linked the international mathematical research world to German and Italian environments had been cut off; the *Zentralblatt* could not be considered a useful scientific enterprise anymore. The review activity would move elsewhere, to the United States. It was the birth of a new international journal, *The Mathematical Reviews*.

The representatives of Italian mathematicians did not stop here. On occasion of the second national Congress of the UMI in 1940, Berzolari sent a letter to the Prefect of Bologna, asking for help in the organization[58].

> Tale Congresso avrà interesse veramente nazionale, poiché sarà una rivista della produzione matematica italiana nell'ultimo triennio, e verrà a dimostrare che, anche dopo la dipartita dei professori di razza ebraica, non è venuta meno la produzione scientifica nel nostro paese, anzi, che nel clima fascista essa ha ripreso nuova vita e vigore[59].

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57 The whole correspondence is printed in P. Nastasi, La Comunità Matematica di fronte alle leggi razziali, in M. Galuzzi (ed.), *Giornate di Storia della Matematica*, Cosenza, Editel, 1992, pp. 332–444.

58 Cf. C. Pucci, L'Unione Matematica Italiana dal 1922 al 1944: documenti e riflessioni, in *Symposia Mathematica*, vol. XXVII (1986), London, Academic Press, pp. 187–212 (210).

59 Such a Congress will really be of national interest, as it will be a journal of Italian mathematical production in the last three years, and it will show that, even after the leaving of the Jewish professors, scientific production in our country has not diminished, rather, that in the fascist atmosphere it has regained new life and vigour.

And the attendants of the Congress applauded the representative of the regime, who opened the sessions by echoing Berzolari's words[60].

> S'affermò in quel Congresso[61] (e in questo se ne avrà di certo, la solare conferma) il primato dell'Italia nella geometria algebrica, nel calcolo delle variazioni, nella geometria proiettiva differenziale; la sua posizione di primissimo piano nelle teorie delle funzioni, delle equazioni differenziali, delle algebre, della relatività, delle trasformazioni termoelastiche, negli studi di calcolo delle probabilità e attuariale, di storia delle matematiche, di storia dei numeri. Più che un trionfo è una rivelazione: la matematica italiana, non più monopolio di geometri d'altre razze, ritrova la genialità e la poliedricità tutta sua propria per cui furono grandi nel clima dell'unità della Patria, i Casorati, i Brioschi, i Betti, i Cremona, i Beltrami, e riprende, con la potenza della razza purificata e liberata, il suo cammino ascensionale[62].

4. Crisis signals

These signals are the ones Severi indicated. But, contrary to what one could expect, they are not related to the consequences of anti-Semitic laws. They concern instead the general condition of Italian research (without any acceptance of its dependence, also, on the racial measures of 1938).

We have already talked about the duopoly UMI-CNR and about how Scorza's and Bompiani's choices had soon reduced it to an opposition between the UMI and (Severi) on one side, and Picone's INAC on the other. The INAC was the prized piece of the CNR's *mathematical sub-committee* and from the INAC started the project of the organization of thematic congresses. The proposal, that today is so natural that it seems obvious, was instead particularly innovative in the Italy of the 1930s. Italian mathematicians had never been summoned to a Congress, gathering only in the SIPS' annual meetings. Before going to the executive stage, the project of a thematic Congress was subjected to the UMI's examination. Thus in its assembly of the 12th May 1935, Bompiani recalled:

> l'azione del CNR ed in particolare quella dell'Istituto di Calcolo che a Roma è diretto dal prof. PICONE. Espone i fini di quell'Istituto, i mezzi che in esso si adoperano, i risultati che si sono ottenuti, quelli che da esso si aspettano; dice che si è

[60] Cf. *Atti del Secondo Congresso dell'Unione Matematica Italiana*, quoted., p. 5.

[61] Bottai is hinting at the UMI's First Congress in 1937.

[62] That Congress stated (and in this one we will certainly have an obvious confirmation) Italy's supremacy in Algebraic geometry, in the calculus of variations, in Differential projective geometry; its first-rate position in the theory of functions, in differential equations, in algebra, in relativity, in thermoelastic transforms, in the study of calculus of probability and in actuarial calculus, in history of mathematics, in number history. More than a triumph it is a revelation: Italian mathematics, no more a monopoly of geometers of other races, finds again its own brilliance and its own many-sidedness for which the Casorati, the Brioschi, the Betti, the Cremona, the Beltrami, were great in the atmosphere of the unity of the country, and resumes, with the strength of a purified and freed race, its upward path.

> riconosciuta la opportunità di indire convegni, nei quali si esaminano le questioni che maggiormente interessano le applicazioni pratiche, anche dal lato puramente speculativo. [63]

Reactions were cold: even if someone (Sansone, especially) agreed about the importance and the advantage of the proposal, the UMI only admitted the possibility (at most) of "strictly mathematical and speculative Congresses". Actually, the President resolved the proposal by inviting Bompiani to write an article for the UMI's *Bulletin* and to insert the proposal into the agenda of a vague next sitting. Of course, all this would never happen. Picone and the CNR mathematicians hurried, promoting on the 3rd and 4th June 1936 (in Rome, at the Istituto di Matematica of the university) the *First Congress of Applied Mathematics* with speeches, among others, of Mauro Picone[64] and Giulio Krall[65]. The *Proceedings* of the Congress would be issued in 1939[66], almost at the same time of the *2nd Congress of Applied Mathematics* (Rome, Istituto di Matematica, 23–25 February), organized in association with the physicists. This second Congress was organized around three themes: 1)*The idea of probability in classic and in quantum physics*; 2) *Homogeneity, similarity, models: theoretical and experimental foundations*; 3) *Methods of calculation in some technical problems*. Of course, there would never be a third Congress (fixed for 1943), considering the "anomalous situation in which some cities, fundamental study centres, as Turin, Milan, Genoa, Naples and Palermo, find themselves because of the enemy attack, and the fact that the shortage of research staff is increasing, due to the calls" [67].

For Picone the difference between the two distinctive levels of the so-called *applied* mathematics – application and research – was clear. In the first one, already proved mathematical propositions (or immediately provable by "any worthy technician whatsoever") were to be adapted to phenomena pertinent to other sciences. In the second one, instead, a more or less complex connection of original mathematical reasoning, similar to those characterizing research in so-called *pure* mathematics, had to be established; the only difference was that for *applied* mathematics, hypotheses and initial mathematical relationships are always considered as approximate, and abstract representations of phenomena

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63 The action of the CNR and specially that of the *Istituto di Calcolo* directed by Prof Picone in Rome. He expounds the aims of such Institute, the used means, the achieved results, the expected ones; he says that it has been allowed to summon Congresses in which the more interesting subjects for practical applications, even from a merely speculative perspective, will be examined.". Cf. *Boll. Un. Mat. It.*, a. XIV (1935), n. 3, p. 196.

64 In "Vedute unitarie sul calcolo delle soluzioni delle equazioni a derivate parziali della Fisica-Matematica".

65 Krall gave two speeches: on "Limiti per le vibrazioni e pel cimento di un ponte qualunque percorso da carichi inerti, molleggiati, mobili con velocità uniforme" and on "Volte cilindriche".

66 We find also the speeches of L. Sobrero on "Estensione di un teorema di elasticità", of C. Ferrari on the "Moti fluidi turbolenti" and of G.D. Mattioli on "Attrito turbolento sulla lastra piana indefinita". The speech of G. Supino on "La propagazione delle onde nei canali" is not found there, though, because its author is a Jew (it will be later published, in French, in the *Revue générale de l'Hydraulique* of the year 1939).

67 Cf. U. Bordoni, Piano di studio per l'esercizio finanziario 1943–44, in ACS, CNR, II vers., b. 972, fasc. 3 "Attività".

belonging to other sciences and conclusions were to be then translated into the language of such sciences and quantitatively (more than qualitatively) established, so as to make possible the comparison with experimental data[68].

> Ora, a mio avviso, non ha senso sezionare la Scienza in "Scienza pura" e in "Scienza applicata" poiché non vi è che una "Scienza" ed una Scienza concepita nel più assoluto rigore logico, e nella sua massima generalità. Vi sono sì, applicazioni della Scienza, ma questa non può essere fruttuosamente applicata che da Scienziati che la posseggano profondamente, in tutta la sua vastità e siano anche in grado di allargarne i confini. Come ho potuto convincermi nella mia trentennale opera di applicazione della Matematica alle altre Scienze e alla Tecnica, un problemino che può sembrare – a prima vista – di limitato interesse scientifico, può trovare una sua soddisfacente soluzione, soltanto in teorie elevate di Matematica già note o che occorre creare ad hoc. Se poi, per "ricerca applicata" si vuole alludere a quella che si deve fare per trovare nei trattati scientifici l'eventuale soluzione di un problema presentatosi al tecnico, essa non può essere considerata ricerca scientifica e deve essere compito di un qualsiasi tecnico rispettabile[69].

Before the CNR's attack (and the success of the Congress of 1936), the UMI was forced to reconsider its positions. In a great hurry, it convened in Florence (in 1937) its first Congress[70] with the excuse that "mathematics, as a pure science, is by now quite neglected in the Congresses of the Society for the Progress of Science, which, at the beginning, had given to it a prominent place". Just on the eve of the Congress, Picone took part in the UMI's *Bulletin*[71].

> Nel concetto del Direttore dell'Istituto vi è che l'Istituto stesso debba presto affermarsi anche come uno dei più efficaci propulsori della ricerca scientifica, non soltanto nel campo delle applicazioni della matematica alle varie scienze sperimentali ed alla tecnica, ma anche in quello della matematica pura. La visione applicativa

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[68] Cf. M. Picone, *Presentazione di pubblicazioni riguardanti l'attività dell'Istituto per le Applicazioni del Calcolo, dal 1927, anno della sua fondazione, al 1960, in cui fu sottratto alla direzione del suo ideatore*, Rendiconti Accademia Lincei, vol. XLIV (1968), fasc. 4, pp. 1–10 of the excerpt.

[69] Now, according to me, there is no point in sectioning Science in "pure science" and "applied science" as there is only one Science, a Science conceived in the most absolute logical rigour, and in the utmost universality. It is true that there are Science applications, but Science can be applied in a profitable way only by Scientists who master it in all its vastness and who are also able to expand its boundaries. As I have realized during my thirty-year-long work of application of mathematics to other sciences and to technology, a little problem which can seem – at first sight – of a limited scientific value, can find a satisfying solution only in high mathematical theories, already known or that are to be created ad hoc. If with "applied research" one means the research to be done so as to find in scientific treatises the possible solution of a problem posed to the technician, it can't be considered scientific research and must be the duty of any worthy technician whatsoever.

[70] The second Congress will be held in Bologna in 1940.

[71] Cf. M. Picone, Organizzazione dell'Istituto per le Applicazioni del Calcolo, *Bollettino dell'Unione Matematica Italiana*, a. XV (1936), n. 5, pp. 231–235.

> degli studi scientifici non può che essere feconda di progresso anche per la scienza pura. Questa massima, che nel clima fascista si è finalmente imposta agli scienziati italiani, è a base di tutta l'organizzazione dell'Istituto per le Applicazioni del Calcolo[72].

Picone's words induced Severi to enter the arena. Some months later he opened precisely the first Congress of the UMI with a speech centred on the applications of mathematics, attacking Picone's position and the existence within the CNR of a *Comitato* that had called itself of "applied mathematics, that is of an inexistent science" out of clear "ostracism to mathematics without adjectives"[73]:

> Il camerata e caro amico che, dopo aver ideato, ottimamente dirige l'Istituto per le Applicazioni del calcolo del Consiglio Nazionale delle Ricerche, ha scritto or ora che "la visione applicativa degli studi scientifici non può che esser feconda di progresso anche per la scienza pura". (…) Bisogna concluderne che la visione applicativa ha da governare e coordinare gli studi scientifici come condizione sine qua non affinché la scienza pura non diventi infeconda? (…) Ancora: il criterio dell'utilitarismo per discriminare e dirigere la ricerca scientifica, va considerato in senso tecnico e strettamente materiale, oppure va inteso anche in senso spiritualmente più elevato, ammettendo che la scienza, specialmente in talune delle sue manifestazioni più astratte, possa adempiere a funzioni sociali nel campo morale e nel campo estetico?[74]

With a broad apparatus of metaphors and quotations, Severi expressed the idea according to which there existed no *applied science*, but only *applications of science and of mathematics*, maybe in the wider sense of possible framings of several groups of phenomena in a single mathematical theory, elaborated though in itself, as a free creation of the human mind, like an artistic creation. The only possible hierarchy was the one that saw *pure* mathematics at the summit. At the most – if we want to consider the fact that the INAC is by now a reality – we can appeal to an equal dignity for *pure* sciences and

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72 The idea of the Director of the *Istituto* is that the Institute itself has to assert itself soon as one of the most efficient propellants of scientific research too, not only in the field of application of mathematics to the several experimental sciences and to technology, but also in that of pure mathematics. The applicative view of scientific studies has to be productive also for the progress of pure science. This principle, which in the fascist climate has finally become successful among Italian scientists, is behind the whole organization of the *Istituto per le Applicazioni del Calcolo*.

73 Cf. F. Severi, Scienza pura e applicazioni della scienza, *Atti del primo Congresso dell'Unione Matematica Italiana*, Zanichelli, Bologna, 1938, pp. 13–25.

74 The comrade and dear friend who, after having conceived it, splendidly directs the *Istituto per le Applicazioni del calcolo* of the *Consiglio Nazionale delle Ricerche*, has now written that "the applicative view of scientific studies must be productive also for the progress of pure science". (...) Should we infer that the applicative view has to rule and coordinate scientific studies as the condition sine qua non so that pure science won't become sterile? (...) Still: is the criterion of utilitarianism to differentiate and direct scientific research to be considered in a technical and strictly material sense, or is it to be intended also in a spiritually higher sense, admitting that science, specially in some of its more abstract manifestations, could fulfil social functions in the moral and aesthetic fields?

applied ones. But it is a merely tactical equal dignity, in deference to the unity desired by the "Chief" and dictated by the circumstances, as the chosen adjectives – artillery-like, engineering-like – used to indicate *applied* mathematics, clearly reveal.

Of course, Severi's reaction to Picone's increased authoritativeness and to the diffusion of the utilitarian *slogans* did not finish with the UMI's Congress of 1937 and his opening speech. Severi got his biggest blow in a year later, with the foundation of the *Istituto Nazionale di Alta Matematica* (INDAM). With a *Memoir* to Mussolini in May 1938 – a document until now unpublished[75] – he asked for his support in the foundation.

> A Sua Eccellenza
> il Dr. Osvaldo Sebastiani
> Segretario particolare del
> Capo del Governo – Roma
>
> Reale Accademia d'Italia — Roma, 25 maggio 1938 XVI
>
> Duce!
>
> Vi chiedo l'onore di esser ricevuto per presentarvi talune mie conferenze ultimamente pubblicate e per sottoporvi un piano per la fondazione a Roma di un Istituto di alta matematica, che, se a Voi piacesse, potrebbe esser inaugurato nel 1939, in occasione del prossimo Convegno internazionale Volta della R. Accademia d'Italia, il quale verterà appunto sulla Matematica.
> Vogliate gradire l'espressione della mia devozione e della mia gratitudine illimitata
>
> Francesco Severi[76]

The Duce, "acceding to the desire expressed to Him", would receive Severi in Palazzo Venezia at "six thirty". Obviously, Severi's project had convinced him; in short (with law 13th July 1939, n. 1129) the INDAM became reality, with the aim to support the *Scuola Normale* of Pisa in the post-graduate education for Italian young mathematicians. Its inauguration would take place on the 15th April 1940 with great pomp and ceremony, before the Duce and the Minister for National Education.

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[75] State Central Archive, Private Secretary of the Duce, reserved correspondence (1922–1940), envelope 62, Severi Francesco.

[76] To His Excellence
Dr Osvaldo Sebastiani
Private Secretary of the
Head of the Government – Rome
Reale Accademia d'Italia — Rome, 25th May 1938 XVI
Duce!
I ask You to have the honour to be received, as I would like to show you some lately published speeches of mine and to propose to you a programme for the foundation in Rome of an Institute of High mathematics, that, if You like it, could be inaugurated in 1939, on occasion of the next Volta International Congress of the R. Accademia d'Italia, that will deal precisely with mathematics.
Please allow me to express my devotion and my infinite gratefulness
Francesco Severi

In the just mentioned *Memoir* to the Duce, Severi alluded to the future Volta Congress. The *Fondazione Alessandro Volta*, annexed the following year to the *Accademia d'Italia*, had been founded (in 1930) on the initiative of the *Società Generale Italiana Edison di Elettricità*. One of its institutional tasks was to organize theme Congresses. Taking advantage of the fact that their alternation provided the *IX Congress Volta* (in 1939) to be up to the *Class of Sciences*, Severi managed to be given the organizational task. The troubled events of the time prevented the realization of the Congress, scheduled for the autumn 1939 and later deferred to 1940, but we can equally get an idea of Severi's project on the strength of his decision (made – take notice – in August 1943, that is after Mussolini's defeat) to publish the reports he had received, divided into three parts[77]: "Algebraic geometry and topology" "Issues of modern analysis" and "Differential geometry according to the most recent directions". If we think that the invited Italian speakers were really few and that, despite the title, there was no scrap of application, the meaning of Severi's act is clear: mathematical Italy needed to resume contacts with the rest of the world, starting just with those fields in which the signals of a missing – or at least not satisfactory – generational exchange were more strongly perceived.

The "virtual" Congress of 1939 was followed by another Congress – this time "real" – that Severi organized in Rome in November 1942 (among the events planned for the twentieth anniversary of fascism). The "international" Congress (actually, congress of mathematicians from fascist or at most neutral Countries) was widely supported by the regime's press and was bestowed the honour of a formal address by Pio XII[78]. F. Conforto was entrusted with the presentation of Italian Algebraic geometry from a very special perspective: that of the researches of Severi, mentioned almost in every page and dominating the bibliography. Enriques was nearly ignored, and the objection that we are in 1942 is not valid, as the *Proceedings* were issued in 1945, when the racial laws had already been abolished (and hence there was a chance to review the text). The Congress, with some unusual references to recent results by mathematicians unrelated to the Italo-Germanic cultural area (W. Hodge, J. A. Todd, S. Lefschetz, O. Zariski, etc.) was a further signal of the need, even if weak, of reopening international relationships and of overcoming the negative consequences following the autarchy so much flaunted by the regime. It was also a signal of the consciousness of a crisis contributed to by – according to Severi – external causes (individuated in the "lack of new, vigorous energies" induced by the race for a career) and inner causes, such as the missing development of fields such as algebra, topology and number theory in Italy. On occasion of his first lesson at the INDAM, Severi returned to the problems of the generational exchange in Italian mathematics and of the crisis of "talents", that went with a lowering of quality in university teaching[79].

> L'Istituto che oggi comincia anche nel campo didattico-scientifico la sua attività (…) è da noi di tipo completamente nuovo. Invero, esso rappresenta qualcosa d'in-

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[77] Cf. F. Severi (ed.), *Matematica contemporanea e sue applicazioni*, Reale Acc. d'Italia, Rome, 1943.

[78] *La matematica scienza di pace*, in Eugenio Pacelli, *Discorsi agli intellettuali (1939–1954)*, Editrice Studium, Roma, 1954, pp. 124–126.

[79] Cf. F. Severi, L'Istituto Nazionale di Alta Matematica ed i suoi compiti, *Boll. Un. Mat. It.*, (2), 2 (1940), pp. 142–146.

> termedio tra l'Università e l'Accademia, intesa, quest'ultima, nel senso migliore, cioè come organo propulsore di ricerche e di discussioni scientifiche, indipendenti da finalità professionali. Anche le organizzazioni similari straniere (per es. l'Institute for advanced study di Princeton, che è forse il più vicino al nostro, l'Istituto Poincaré e il Collège de France di Parigi, l'Istituto Matematico di Göttingen, che è forse quello di tipo più lontano) sono concepiti in modo un po' diverso; sicché il nostro non può trovare nelle esperienze altrui che scarsi barlumi direttivi. (…) L'Istituto ha lo scopo principale di riparare, nel campo matematico, a questa deficienza, sempre più accentuata, della nostra organizzazione universitaria e ad evitare che l'alto livello matematico nazionale venga progressivamente ad abbassarsi (del che si cominciava già ad avvertire qualche segno preoccupante), per difetto di nuove, vigorose energie. Danno questo, che sarebbe grave dal punto di vista del prestigio italiano in un ramo di scienza in cui ci eravamo tanto superbamente affermati da più di mezzo secolo (…). Danno poi, che sarebbe gravissimo per le necessità della tecnica, la quale non può che inaridirsi se ha vicino a sé un pensiero matematico anemico e privo di originalità nazionale[80].

He repeated analogous ideas in the UMI's second Congress (Bologna 1940) – where he underlined how the INDAM had been created precisely to stop "that declining of vigour in our mathematical thought, which was unfolding lately"[81] – and at the beginning of the INDAM's second year of activity[82].

> Ho accennato or ora al fatto che le borse di studio destinate a giovani italiani, pel 1940–41, non sono state tutte coperte. Me ne duole, ma devo dichiarare che ciò è dipeso dalla preoccupante scarsezza di elementi idonei. Esclusi gli aspiranti che risultavano già assolutamente insufficienti dall'esame degli elementi prodotti, abbiamo chiamato quelli dei quali avevamo informazioni buone o ottime. Attaverso a

[80] The Institute that today begins its activity also in a scientific-didactic field (…) is wholly new to us. Indeed, it lies between university and the academy intended in its best sense, that is as a propellant of researches and scientific debates, beyond any professional aim. Foreign similar organizations (i.e. the Institute for Advanced Study of Princeton, which is maybe the most similar to ours, the Poincaré Institute and the Collège de France of Paris, the Mathematical Institute of Göttingen, which is maybe the most different one) are conceived in a different way too; thus, our can find in somebody else's experience only few guiding traces. (…) The Institute has the main object, within the mathematical field, of seeing to this more and more dramatic lack in the organization of our university and of avoiding the gradual lowering of the high national mathematical level (of which there was already some alarming signal), because of the lack of new, vigorous energies. This damage would be serious from the perspective of the Italian prestige in a branch of science in which we had so superbly established ourselves for more than half a century (…). Damage that would be very serious for the exigencies of technology, which can only dry up if by its side there is a lifeless mathematical thought lacking in national originality.

[81] Cf. F. Severi, L'Istituto Nazionale di Alta Matematica e le sue funzioni pel progresso della Scienza Italiana, in *Atti del 2° Congresso dell'Unione Matematica Italiana*, Edizioni Cremonese, Roma, 1942, pp. 26–35.

[82] See Severi F., In occasione dell'inizio dell'anno accademico 1940–41 del Reale Istituto Nazionale di Alta Matematica, *Boll. Un. Mat. It.*, (2), 3 (1941), pp. 130–140.

> colloqui diretti a saggiare la cultura e soprattutto le attitudini potenziali, in vari casi (purtroppo di laureati con pieni voti e lode!) ci siamo trovati dinanzi a desolanti deficienze, in parte almeno imputabili ad un abbassamento di tono dell'insegnamento superiore in qualche Università italiana (…). Debbo ripetere di non mandar qui giovani qualunque, ma elementi di sicura potenzialità. "Vi posso proporre Tizio – mi ha scritto un collega – ma Vi avverto che non è un'aquila". L'ho naturalmente rifiutato. Non posso pretendere di avere qui le aquile che si librano e si libreranno nel cielo matematico italiano, ma non posso accettare quelle che a priori sono classificate in sottospecie libratorie molto più modeste[83].

Severi comes back also to some of the causes he thinks to be behind the alarming decline signals of the Italian mathematical culture:

> Nell'anno corrente 1941 saranno dunque tenuti due corsi da professori provenienti da altre Università; dal prof. Ricci dell'Università di Milano, che svolgerà un corso trimestrale di "Aritmetica analitica" e dal prof. Enea Bortolotti dell'Università di Firenze, che svolgerà un corso sugli "Spazi a connessione proiettiva". Spero che il corso del prof. Ricci possa essere d'incentivo affinché, anche da noi, sia dato maggior impulso allo studio della teoria dei numeri, che è uno dei rami più belli, più difficili ed astratti della matematica, profondamente connesso con tutti gli altri. Spero altresì di poter presto invitare a tenere un corso di algebra moderna, qualche professore delle Università germaniche. (…) A proposito dell'aerodinamica e dell'aerotecnica (…) invito quei nostri discepoli che sentono maggior trasporto per la meccanica e la fisica matematica a volgere l'attenzione verso i problemi aviatori, che richiedono elevate risorse dell'analisi, non sempre possedute da sperimentatori, anche valenti, dei quali, peraltro, noi fortunatamente non manchiamo, nonostante che i laboratori universitari non sieno sempre adeguatamente attrezzati. È un peccato che le nostre alte doti matematiche poco si siano finora cimentate in questo campo, ove abbiamo apportato, sì, pregevoli contributi, specialmente di ordine tecnico, ma nel quale le leggi fondamentali – come quelle geniali e risolutive di Prandtl e della scuola di Gottinga – non sono opera di italiani[84].

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[83] I have hinted right now at the fact that for the year 1940–41 not all scholarships for young Italians have been given. I am sorry, but I must admit that this depends on the worrying shortage of valuable elements. Excluding the candidates who already proved to be absolutely inadequate out of the examination of the produced elements, we have called those about whom we had good or excellent information. Through interviews directed to test culture and above all potential aptitudes, in some cases (unluckily graduates with full marks and a first-class degree!) we have found distressing deficiencies, at least partly due to a lowering of quality in high teaching at some Italian Universities (…). Mediocre young people should not be sent here, but individuals with unfailing potential. "I can propose So-and-So – has written a colleague – but I warn You he's not a genius". Of course I have rejected him. I cannot pretend to have here the actual and future Italian mathematical masterminds, but I won't admit those belonging to much more modest ranks.

[84] see next page.

This wary openness, anyway, can only occur within the warlike and political needs of the regime, which Severi fully shares.

> Quest'anno 1941 avremo anche borsisti stranieri: il che c'impegnerà maggiormente a render più elevato, espressivo e fruttifero il nostro lavoro. L'Istituto deve d'altronde agire fin d'ora sul piano imperiale. Dobbiamo cioè, nella sfera di nostra competenza, cercare i mezzi ed i modi più idonei per legare alla scienza italiana, la scienza e la cultura della nuova Europa, che si sta creando attraverso il genio dei condottieri, l'eroismo dei combattenti, il sacrificio dei popoli dell'Asse. Noi divideremo domani col nostro grande alleato, la responsabilità della direzione politica, economica, culturale dell'Europa, ricostituita su basi più salde e più giuste. Bisogna che ci mettiamo fin d'ora ad un livello di parità con lui. (…) Noi siamo già stati autorizzati a conferire 3 borse di studio dell'Istituto ad un tedesco, ad un ungherese e ad un rumeno. Una borsa di studio del Ministero dell'Educazione Nazionale, d'accordo col Ministero degli Esteri, è stata assegnata presso di noi ad un matematico giapponese; confidiamo infine di aver tra breve due borse di studio del Ministero della Cultura Popolare: una per uno spagnolo ed una per un bulgaro. (…) Così la nostra opera di proselitismo scientifico coprirà quasi tutta l'area delle potenze legate all'Asse politicamente o da vincoli di simpatia e di interesse[85].

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[84] In the current year 1941 two academic courses will be run by two professors coming from other universities; Prof Ricci from the university of Milan will run a three-month course on "Analytic arithmetic", and Prof. Enea Bortolotti from the university of Florence will run a course on "Projective Connection Spaces". I hope that Prof Ricci's course will help to give a greater impulse, also among us, to the study of the number theory, one of the most captivating, difficult and abstract branches of mathematics, deeply linked to all the rest. I also expect to invite soon a professor from a German university to run a course on modern algebra. (…) Regarding aerodynamics and aeronautic technology (…), I invite those of our pupils who feel a greater enthusiasm for mechanics and mathematical physics to put attention to air problems, that require high resources of analysis, which even skilful researchers do not always master; these last, however, are not missing among us, although university laboratories are not always properly equipped. It is a pity that our high mathematical skills have until now been little employed in this field, to which we have certainly given remarkable contributions, specially technological ones, but in which the fundamental principles – as the genial and resolutive ones due to Prandtl and to Göttingen school – are not Italian.

[85] This year 1941 we will also have foreign bursars: this will commit us even more to make our work higher, more expressive and productive. Besides, from now on the Institute has to act on an imperial front. In other words, we must, within our competence, search for the more suitable means and ways to link Italian science to the new European science and culture, which is raising thanks to the talent of the leaders, the heroism of the fighters, the sacrifice of the Axis people. Tomorrow we will share with our great ally the responsibility of the European political, economic, cultural government, re-established on sounder and fairer basis. It is necessary to keep level with him. (…) We have already been authorized to award 3 scholarships of the Institute to a German, a Hungarian and a Rumanian. We awarded a scholarship from the Ministry of National Education, in agreement with the Foreign Office, to a Japanese mathematician; lastly, we hope to have shortly two scholarships from the Ministry for Popular Culture: one for a Spaniard and another one for a Bulgarian. (…) Thus our scientific proselytism work will cover almost the whole area of the powers linked politically or with empathy and interest ties to the Axis.

Even the goals of the polemics correspond to the declared enemies of the regime.

> Bisogna scongiurare il pericolo che si profila all'orizzonte e del quale già diedi l'allarme a Bologna: quello della nostra decadenza matematica. (…) Intanto a noi il compito di impedire la formazione di discontinuità che sarebbero deleterie. Bisogna aver presente in ogni momento la nostra responsabilità nazionale, ripeto nazionale, la cui misura è determinata dalla entità del nostro patrimonio matematico. Varî piloni fondamentali del gran ponte che la matematica protende verso l'avvenire sono italiani. **La geometria rinascente presso i nemici inglesi** è quasi tutta "made in Italy"; **la topologia, tanto coltivata dai non benevoli giudici americani**, ha ricevuto da noi, dopo l'opera del grande matematico tedesco Riemann, i primi fondamentali impulsi attraverso Betti e attraverso i progressi della geometria algebrica, di cui fu maestro Cremona; la moderna geometria tensoriale, come la teoria generale della relatività, derivano dal calcolo differenziale assoluto di Gegorio Ricci e la geometria differenziale proiettiva ha ricevuto in Italia i maggiori impulsi, al pari della classica geometria differenziale metrica, legata alle opere fondamentali di Beltrami e Bianchi, all'ultimo dei quali si deve persino il nome universalmente accolto di geometria differenziale; (…). E potrei continuare a lungo questo glorioso inventario[86].

It seems the speech of a decayed aristocratic, a pathetic recourse to the old family wealth. Still, a fact emerges clearly: even without any self-criticism, the global evaluation of the development of Italian mathematics in this two decades is critical. Severi's evaluation matches with the memories of the young mathematicians that began their studies at the end of the 1930s. For example, this is the declaration of Lucio Lombardo Radice[87].

> Sono stato studente di Scienze Matematiche all'Università di Roma tra il '34 e il '38. Eravamo in pochissimi avviati alla laurea di "matematica pura" (…); tra i

[86] There is a need to ward off the danger on the horizon and of which I already raised the alarm in Bologna: our mathematical decline. (…) In the meanwhile our duty is to avoid the development of discontinuities which would be deleterious. We must always remember our national, I repeat national, responsibility, whose measure is determined by the entity of our mathematical wealth. Several mainstays of the great bridge that mathematics stretches out towards the future are Italian. **The geometry resurgent among the English enemy** is almost wholly "made in Italy"; **topology, that the not friendly American judges developed so much**, has received among us, after the work of the great German mathematician Riemann, the first essential drives through Betti and through the progress of Algebraic geometry, of which Cremona was a master; modern tensor geometry, as the general theory of relativity, come from the absolute differential calculus of Gegorio Ricci, and Projective differential geometry has received in Italy its greatest drives, as well as classic metric differential geometry, linked to the essential works of Beltrami and Bianchi, to whom is due even the universally accepted name of Differential geometry; (…) and I could go on at length this glorious inventory.

[87] Cf. L. Lombardo Radice, Sullo stato della ricerca e della didattica matematica in Italia, *Archimede*, 30 (1978), pp. 97–98. Lombardo Radice (1916–1982) has been algebraist and politician, strongly committed also in the fields of Didactics and History of mathematics.

> cinque e i dieci. Ma, pur lodando il tempo passato per certi aspetti, debbo aggiungere che esso era contrassegnato da una notevole chiusura, tanto rispetto al più vasto mondo, quanto rispetto a nuove teorie e metodi che in quel più vasto mondo si sviluppavano. Non numerosi i viaggi, i congressi, niente professori visitatori ma solo qualche conferenza (non molte), di francesi e tedeschi soprattutto, comunque di europei. Vi era poi uno scarso interesse, e talvolta una certa avversione (p. es. in quel pur grandissimo scienziato che fu F. Severi) per possenti ma pesanti teorie, algebriche e topologiche in particolare, che si sviluppavano in modo impetuoso e sistematico in Francia, in Germania, negli USA, in Polonia, nell'Unione Sovietica. Quando ritornai, nel 1945, Severi, che aveva compreso il grave ritardo algebrico e topologico della nostra ricerca matematica, incaricò me di un Seminario di Algebra (seguivo il van der Waerden), e Michelangelo Vaccaro, matematico allora non meno solitario, di un Seminario di Topologia[88].

Lombardo Radice's statement finds a confirmation in the one by Enrico Magenes, that recalls the atmosphere young mathematicians breathed during the first postwar congress of the UMI (Pisa 1948)[89].

> Quali furono le impressioni lasciate in noi dal Congresso, anche in relazione alle aspettative che, seppure in forma non ancora chiara, avevamo? Mi sembra che esse si possano riassumere sostanzialmente come segue: anzitutto avemmo la conferma che tra i matematici italiani, soprattutto tra i più anziani, c'era un atteggiamento abbastanza diffuso di "diffidenza" verso la tendenza ad un maggiore "astrattismo" nella matematica coltivata all'estero negli ultimi anni. Questo atteggiamento risultò evidente nella pur bella ed interessante conferenza di Severi e così pure nella mancanza di una sezione dedicata all'Algebra, mentre ebbe conferma la tradizione della grande Scuola italiana di Geometria algebrica. Ma anche nel campo degli Analisti italiani, come avevamo già potuto constatare nella Scuola Normale, esso si manifestava nei confronti delle teorie degli spazi "astratti" che erano nate soprattutto in Polonia con S. Banach e in Francia con "Bourbaki"[90].

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88 I have been student of Mathematical Sciences at the University of Rome between 1934 and 1938. Very few of us had started the degree in "pure mathematics" (…); among five and ten. But, even if under some aspects I praise past time, I must add that it was characterized by a remarkable narrow-mindedness, both towards the wider world and towards the new theories and methods that were being developed in that wider world. Few journeys and Congresses, no visiting professors but only some conferences (not many), by French and German above all, European anyway, speakers. There was also a faint interest, and sometimes a certain aversion (i.e. even in the prominent scientist F. Severi was) towards mighty but weighty theories, specially algebraic and topological, that developed in a forceful and systematic way in France, Germany, USA, Poland, Soviet Union. When I came back, in 1945, Severi, who had realized the serious algebraic and topological delay in our mathematical research, entrusted me with a seminar on algebra (I followed van der Waerden), and Michelangelo Vaccaro, a not less solitary mathematician at the time, with a seminar on topology.

89 Cf. E. Magenes, Una testimonianza sul III Congresso dell'U.M.I. Pisa, 23–26 September 1948, *Bollettino U.M.I.*, 1-A (1998), pp. 1–6 (p. 4). The analyst Magenes, born in 1923, studied at Pisa.

90 see next page.

Equally valuable is the declaration of the recent graduate (1949) Giovanni Prodi, who took part in a Congress on "Functional analysis and functional equations" in Parma. There he became keen on the subject, due also to Caccioppoli's charm[91].

> Mi scuso se, come mi capita sempre più frequentemente con il procedere degli anni, attingo ai ricordi personali. Il convegno a cui mi riferisco fu organizzato nella tarda primavera del 1949 dal prof. Antonio Mambriani presso l'Istituto di Matematica dell'Università di Parma. Il tema era quello dell'Analisi Funzionale. Il convegno durò una mattinata soltanto, ma fu vivacissimo. Il momento centrale fu la conferenza di Caccioppoli; era la prima volta che io lo incontravo, anche se mi erano già capitate tra le mani alcune di quelle sue note brevissime e dense di idee.
> Ricordo che parlò della "foresta funzionale", in cui non ha senso studiare un albero da solo, anzi dai caratteri generali della foresta si può risalire a quelli di ciascun albero (…). Ci fu anche un intervento tecnico di Zwirner ancora sul teorema di Leray-Schauder, dove per la prima volta sentii parlare di simplessi e di tecniche topologiche. Ci fu anche un intervento di G. Fichera, allora giovanissimo, che attirò gli elogi di F. Severi, il nume tutelare della matematica italiana. A Sua eccellenza F. Severi veniva spedito un telegramma di ossequio tutte le volte che i matematici italiani facevano qualche importante riunione. (…)
> Io avevo conseguito la laurea da pochi mesi; avevo avuto la fortuna di incontrare, proprio a Parma, Giovanni Ricci, maestro affascinante dotato di uno straordinario gusto matematico. Avevo già deciso di dedicarmi alla matematica, ma non avevo scelto ancora il campo; inoltre la mia cultura matematica, anche per le vicissitudini di guerra, era rimasta ad un livello piuttosto modesto. A conclusione del convegno di Parma mi resi conto che l'analisi funzionale era ciò che cercavo; finite le conferenze, mi avvicinai timidamente al prof. Cimmino per chiedergli da quale libro cominciare. La risposta fu "Dal Banach"[92].

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90 Which were the impressions the Congress left us, also regarding the expectations that, even in a still not clear form, we had? I think they can be essentially summarized as follows: above all we had the confirmation that among Italian mathematicians, specially among the eldest ones, there was quite a diffused posture of "distrust" towards the disposition to a greater "abstractionism" in the mathematics cultivated abroad in the last years. This posture was clear in the however nice and interesting speech Severi gave, and also in the lack of a section dedicated to algebra, while the tradition of the great Italian school of Algebraic geometry was confirmed. But also in the field of Italian analysts, as we could already ascertain in the *Scuola Normale*, it appeared towards the theories of "abstract" spaces born above all in Poland with S. Banach and in France with "Bourbaki".

91 Cf. G. Prodi, Il ricordo di uno dei presenti, in L. Carbone, G. Cardone, F. Palladino, Una conferenza stenografata di Renato Caccioppoli, *Rend. Acc. Sc. fis. mat. Napoli*, 64 (1997), pp. 361–396. Also Prodi, born in 1924, is an analyst that studied with G. Ricci.

92 I apologize if, as it happens more and more often with the passing of time, I turn to personal memories. The Congress I refer to was organized in the late 1949 spring by Prof Antonio Mambriani in the Institute of mathematics of the university of Parma. The subject was Functional analysis. The congress lasted only one morning, but was very lively. The central moment was Caccioppoli's speech; it was the first time I met him, even if I had already had the chance to read some of his short Notes rich in concepts.

We finish the review of "modern" statements with Paolo Salmon, who has amusingly told about the first events of his career in the Turin of the beginning of the 1950s. After having depicted the backwardness of the study program for the degree in mathematics (which still until the beginning of the 1960s included no course on algebra), Salmon continues thus[93] :

> L'arretratezza di tale situazione apparve in modo palese durante il decorso degli anni '50 quando diversi matematici (a partire da Barsotti[94] a Pisa) si erano fatti portavoce delle esperienze di altri paesi avanzati (U.S.A., Francia, Germania). Così, nel 1960, fu introdotto il corso di algebra al posto di quello di chimica e la geometria descrittiva fu praticamente abolita (…).
> Ma a Torino, quella piccola rivoluzione nell'insegnamento era parzialmente iniziata quasi dieci anni prima, nel 1951, coll'arrivo di Aldo Andreotti[95]. Allora appena ventisettenne, Andreotti aveva raggiunto una maturità assolutamente sbalorditiva, derivata in buona parte da lunghi mesi trascorsi negli Stati Uniti vicino a grandissimi matematici quali Weil, Kodaira, Spencer, Lefschetz, Zariski e tanti altri. Gli amori di Andreotti per la geometria algebrica, sorti già a Pisa, ma poi sviluppatisi a Roma dopo il 1947 sotto l'influenza di Severi (grande ammiratore del promettentissimo

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92 (continued)
I remember that he talked about the "functional forest" in which it makes no sense to study one tree alone, rather, from the general features of the forest one can trace those of each tree (…). There was also a technical speech by Zwirner again on Leray-Schauder's theorem, where for the first time I heard of simplexes and of topological techniques. There was also a speech by G. Fichera, then very young, that drew F. Severi's praises, the tutelary deity of Italian mathematics. Each time Italian mathematicians called an important meeting, a telegram of respect was send to His excellence F. Severi.
I had graduated a few months earlier; I had had the chance to meet, just in Parma, Giovanni Ricci, charming master gifted with an extraordinary mathematical liking. I had already decided to devote myself to mathematics, but I had not chosen a field yet; besides, my mathematical culture, due also to the war's ups and downs, had come to a stop at a rather poor level. At the end of the Congress of Parma I realized that Functional analysis was what I was looking for; once the speeches were finished, I shyly approached Prof Cimmino to ask which book I could start with. The answer was "Banach's".

93 Cf. P. Salmon, Un sodalizio torinese degli anni '50, in E. Gallo, L. Giacardi, C.S. Roero (eds.), *Conferenze e Seminari 1994–1995*, Torino, Assoc. Subalpina Mathesis and Sem. St. Matem. "Tullio Viola", (1996), pp. 224–243. Salmon, the youngest mathematician of the last generation studied here (he was born at the beginning of the Thirties), is a geometer strongly interested in foundations.

94 Iacopo Barsotti (1921–1987). He studied at the *Scuola Normale* in Pisa and graduated there in 1942. discharged his military duties, he came back to Pisa as postgraduate specializing in one subject. He was assistant at Rome (with Severi) from 1946 to 1948, when he moved to the United States: at first with a *fellowship* at Princeton and then as *full professor* at the University of Pittsburgh and at Brown University. Returned to Italy, he taught at Pisa as professor of geometry and, afterwards, of algebra. In 1968 he moved to Padua, where he taught geometry until his death.

95 Aldo Andreotti (1924–1980), after having started mathematics at the *Scuola Normale* in Pisa in 1942, took shelter in Switzerland, where he could follow the courses run by B. Eckmann and by G. de Rham. Returned to Pisa, he obtained his degree in 1947, with a dissertation on the problems of conformal representations. He spent in Rome the three following years, first at the INDAM, then as assistant of geometry, having the chance to improve his knowledge under Severi's leadership. After a short stay in Princeton (where he had contact with S. Lefschetz and C. L. Siegel), he was appointed professor in geometry in 1951 in Turin, to be later (1956) transferred to the university of Pisa.

> discepolo), avevano poi risentito dell'esperienza americana, dove i metodi di indagine della pur gloriosa scuola italiana erano stati irrobustiti con altri ingredienti irrinunciabili: algebra, topologia (generale ed algebrica), funzioni di variabili complesse, geometria differenziale, ecc.
> Al momento del suo arrivo a Torino, Andreotti aveva alcuni punti fermi. Tutti i matematici italiani erano un po' ignoranti, ma soprattutto i cultori di geometria algebrica; il vecchio e venerato maestro Severi era spesso di ostacolo all'aggiornamento perché non incoraggiava abbastanza i giovani a confrontarsi con altre scuole giudicate concorrenziali a quella italiana da difendersi ad ogni costo. Era inoltre assurdo continuare ad insegnare nel primo biennio di matematica la geometria descrittiva che veniva da lui sostituita con vari argomenti di algebra: ideali, polinomi, risultante (…)[96].

These statements, already quite revealing because of the narrow-mindedness and the delay they indicate, take on a greater importance if compared with the situation of the mathematically emergent countries, where, in the same years, young students enjoyed the charm of quite another climate and saw logic, abstract algebra, topology and Functional analysis as the most promising areas of mathematical research[97]. The problem of Italian mathematics' "extraneousness" with respect to the most lively trends of the 1930s came again. The activity of Picone's INAC and of Severi's INDAM was not enough. The mentioned statements prove that the "worrying signals" of crisis in Italian mathematics were at a deeper level. There was a preclusion to the new, dictated by an "encircling syndrome", in which the summit of the mathematical community was circumscribed. The economic, political and intellectual autarchy desired by the regime ended by justifying, at an outer level, this choice of isolation. Racial laws, broken out in 1938, and the shameful support of Italian mathematicians to this further wickedness of the regime only made this detachment deeper.

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[96] The backwardness of such a situation appeared clearly as the 1950s went by, when several mathematicians (starting with Barsotti in Pisa) voiced the experiences of other advanced countries (U.S.A., France, Germany). Thus, in 1960, the course on algebra was introduced instead of that on chemistry and descriptive geometry was practically abolished (…).
But in Turin, that little revolution in teaching had began partly almost ten years earlier, in 1951, with the arrival of Aldo Andreotti. Then barely twenty-seven years old, Andreotti had reached an absolutely stunning maturity, coming in large measure from long months spent in the United States close to remarkable mathematicians such as Weil, Kodaira, Spencer, Lefschetz, Zariski and many others. Andreotti's love for algebraic geometry, emerged already in Pisa but developed in Rome after 1947 under Severi's influence (great admirer of the up-and-coming pupil), had suffered from the experience in America, where the research methods of the albeit glorious Italian school had been strengthened with other inalienable ingredients: algebra, (general and algebraic) topology, complex variable functions, differential geometry, etc.
At his arrival at Turin, Andreotti had some certainties. All Italian mathematicians were a bit ignorant, but especially the researchers of algebraic geometry; the old and venerated master Severi was often an obstacle to the updating because he didn't encouraged young people enough to face other schools considered competitive to the Italian one, to be defended at all costs. Moreover, it was absurd to continue to teach in the first two years of mathematics descriptive geometry, which was replaced by him with several subjects of algebra: ideals, polynomials, resultants (…).

[97] For an overview of the American situation towards the middle of the Thirties, cf. G. Birkhoff, Some Leaders in American Mathematics: 1891–1941, in G. Tarwater, *The Bicentennial Tribute to American Mathematics 1776–1976*, The Mathematical Association of America, 1977, pp. 25–78.

Chapter 9

Conclusions

In the previous pages we have narrated a story with several facts. Now we would like, in a more methodical way, to go back to the different threads of the narration we have presented, dropped and sometimes later resumed again, so as to respect the timeline in the history of events.

We have portrayed the situation of Italian mathematics in the two decades that for Italy meant fascism and, for the whole world, meant the passage from the World War I to another even more devastating. World War II began essentially on the 1st September 1939, with the Nazi occupation of Poland. At first Italy looked on, and only in June 1940, when it seemed that the fortunes of war could result in a quick victory for the Axis forces, would it go to war. It was one of the most egregious mistakes of Mussolini: the general incompetence of the armed forces, long anticipated by the military leadership, soon appeared. Results were catastrophic. Unlike during the first World War, the technical-scientific knowledge of mathematicians was not directly employed in war operations. The only exception was Picone's INAC, but the real importance of this commitment is concealed, still today, behind military secrecy.

The Italian adventure ended in 1943. The numerous military defeats and the popular dissatisfaction led to Mussolini's abrupt dismissal, "ditched" by the monarchy and by most of his officials on the 25th July. It was the end of the regime. Alliances were reversed. The armistice with the Allies (who had already occupied Sicily at the beginning of July) was signed on 8th September. Germans punished the Italian "betrayal" by militarily occupying the Country (from Rome upwards) and by creating the so-called *Repubblica Sociale di Salò*, still ruled by Mussolini and allied with the Axis forces. In this part of the country, not freed by the Allies yet, there was an attempt to restore the old administrative structure, including some cultural institutions. The case of the *Academia d'Italia* is an example. Its presidency was offered to Gentile, who accepted the new appointment in the attempt to save an institution created – with his own words – "in defence of the great traditions of the Country". His appeals to "calm things down", his speeches and his philosophical rhetoric were enthusiastically welcomed, among others, by Picone and by Severi, who expressed all their esteem and loyalty towards the old philosopher by taking part in a lifeless organizational meeting of the re-born *Accademia* in Florence. The appeal to "calm things down", while the persuasion of language had ineluctably given way to the power of guns, determined Gentile's demise. On the 15th April 1944, the philosopher was executed in front of his villa's entrance, in Florence.

The difficulty of depicting such a complex period from a mathematical point of view is clear if one thinks of the specializations of the different languages generated by mathematicians. J. von Neumann admitted to knowing less than a third of the whole mathematical *corpus,* and in the view of H. Weyl, nobody could write a history of mathematics of the 20th century. Even so, we have tried to enter into some technical detail because we do not believe in a neat division between history written only for specialists and history devoid of those contents that absorbed its protagonists. But this is not our main point. Even if the implied requirement is some comprehension on the part of the reader, the main point is appreciation of the importance of the mathematical subject. We are convinced that mathematical thought is an important force in generating and expediting social and cultural changes. It has certainly been so in the first half of the last century, and, specially, during the 1920s and 1930s. Some of its conceptual "revolutions" have entered public discourse and help to shape common sense itself. Some of its achievements have been so striking as to underlay our social life. Even from within the subject we bear witness to a stunning progress: a good part of the mathematical twentieth century has developed – not always linearly, of course – acquisition of new knowledge, categorization of broad ideas and realizations of new unities of concepts. The 1920s and 1930s were years of great mathematics. New disciplines and new theories were born and the direct premises for the modern development of many others were laid. We can cite as examples Functional analysis, algebra, biomathematics, topology, logics, probability, Automatic calculation, Mathematical economics, as well as Theoretical physics and the repositioning of the boundaries between mathematics and physics. The way of representing world phenomena (mediated traditionally by physics and by geometry) changed. The development of the concept of *model* was particularly quick. The *certainty* of Mathematics acquired new meanings.

And then Italy. We talked in the *Prologue* about that "third place" which Italian mathematical research was given in the international *ranking* at the beginning of the century. Italy still appears in third place, at the beginning of the 1920s, in the notes[1] of a US mathematician, G. D. Birkhoff, who was especially interested in the European reality. And Rome is even, in his opinion, the second world centre after Paris (and before Göttingen).

We have *pictured* Italian mathematics between the two world wars, outlining the evolution of different subjects but concentrating essentially on geometry, analysis and Mathematical physics. Maybe – we admit to the ritual underlying some choices – it is an unconscious tribute to the classic nineteenth-century trisection into those topics. But Italian mathematics, in the years we have studied, is just this. Moreover, US mathematicians visiting Europe during these years – we continue to refer to R. Siegmund-Schultze's vol-

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[1] These notes are quoted in R. Siegmund – Schultze, *Rockefeller and the Internationalization of Mathematics between the two World Wars*, Birkhäuser, 2001.

ume – repeatedly talk about the school of Algebraic geometry and they add the names only of Volterra and Levi-Civita.

In the 1920s and 1930s the Italian school of Algebraic geometry is Severi, Enriques, their manuals and the need to arrange systematically all the accumulations of original material. In support of them are a great number of distinguished researchers, all however, clearly a step behind the two *leaders*. The inability to extend the state of grace of the beginning of the century, to continue producing new original results, is the clearest indication that the structure erected by Italian geometers needs now a new underlying structure to go on growing. That structure is embodied in the new algebraic and functional languages which Severi knows and understands but is not able to put at the centre of his research, cannot use it as a driving force. From here, an opening out process of the Italian mathematical tradition occurs. From here, the accusations arise of lack of rigor in Italian mathematics and the suggestion that their intuition cannot be trusted. From here, the story that will arrive in 1954 – during the International Congress of Amsterdam – leads to the harsh polemics against Severi by P. Samuel and A. Weil[2].

Italian analysis is no longer epitomized by Volterra , even if no manner of classification can consign to oblivion the value of his contribution given, precisely in the period between the two wars, to the study of the dynamics of populations. Now, Italian Analysis identifies essentially with Tonelli and Picone. Tonelli recalls and extends the features of the great Italian school of Real analysis of the beginning of the century. But Picone – maybe less "classic" and compunctious, in style and character – bursts the bounds of the subject of calculation, of the instruments needed to speed computation, and of numerical analysis as a tool of growing power, resulting in the organization of a body of mathematical work that goes beyond the strictly individual dimension. Attention towards applications and a more complete view of the mathematical process are developed thus with new methods compared with the physical-mathematical tradition dear to Volterra. Caccioppoli, with whom the interest in Functional analysis revives in Italy, studies at Picone's school. It is at Picone's school – this time in "cohabitation" with Tonelli's – that Cesari studied, who particularly in the postwar will be one of the most internationally well-known and valued Italian analysts. Neither can the need to synthesize, that has taken us to identify analysis with Tonelli and Picone, make us wholly forget Tricomi, internationally well-known as well, who (unlike Cesari) will soon return to Italy, though, and will work in Turin, with all his polemical *verve*, up to the years of the student protest at the end of Sixties.

Mathematical physics is the province of Levi-Civita, who, personally, does not show any withdrawal signs. The level of his scientific production is always high, as well as his commitment to help young researchers and colleagues to emerge. With him we have talked about the classic research fields in Rational mechanics and in Mathematical physics, about the extension of the theory of elasticity to the nonlinear case, but also about the fields nearest to engineering and to tectonics or to aeronautical applications. Levi-Civita's commitment to the diffusion and interpretation of the new ideas – think

[2] This event is reported in B. L. van der Waerden, Francesco Severi and the Foundations of Algebraic Geometry, *Symposia Mathematica*, vol. XXII, Academic Press, London-New York, 1986, pp. 239–244.

about Relativity – that in these decades change the aspect/look of physics is also high. His scientific and human presence appears – in international environments and in those movements nearer to liberal and socialist positions – as protagonist for the most direct prosecution and projection in the future of the values still represented by Volterra.

The description of the state of Italian mathematics and of its main results has been accompanied by some words of appraisal. Even in these quick conclusions, we have already used expressions such as *international ranking, inability to extend the state of grace, withdrawal*. We have thus wanted to specify that, in the period between the two world wars, the situation of Italian mathematics experiences clear difficulty in matching the rate of growth of the most advanced schools and of the Italian school itself at the beginning of the twentieth century. Of course we are not talking of a linearly increasing crisis. A good level of all subjects has been attained; outstanding researchers have held their positions; other young brilliant scholars have appeared; and the *exploits* of some domains such as algebra (at the beginning of the 1920s) or probability are manifest. The situation seems rather "leopard spotted": lively enterprises, in full progress, rich in ideas whose importance is often already acknowledged, coexist with other more tired investigations that by now have lost contact with the advanced front lines of research. Such a regression is closely linked to a minor internationalist tension and to some amount of provincialism, that tempts the Italian mathematical world. When we talk of *provincialism*, of course, we are not thinking about mathematicians who ignore what was happening in Europe or in the US. We mean instead the lack of "curiosity" about what happens elsewhere, thinking it unnecessary and trivial to compare and assimilate other research programs, seen that the Italian tradition – think about the language of Algebraic geometry or of mechanics – is able to answer even to new needs in an autonomous way. It is an attitude that can be understood by looking at external elements – the cultural climate; nationalism, which is of course not only an Italian presence; fascism; autarchy, etc. – but about these we will talk soon.

The thread of *description* and *appraisal* of the theory is not the only one we have followed in the previous eight chapters. We have also dwelled upon the main personalities that animate the Italian mathematical world to give a more direct idea of the protagonists and of the group that, somehow, they represent, and of the ways in which inner developments evolve. The mathematical "family" is quite increased – also numerically –, compared with the beginning of the century but, at the cost of some necessary simplification, we have equally built our history around the figures of Volterra, Severi, Enriques, Tonelli, Picone and Levi-Civita. No use hiding – and what for, after all? – that Volterra and Levi-Civita embody the positive protagonists of our history. They are outstanding researchers, with an organic and total view of the development of mathematics and a scientific expertise that still goes through several and differentiated research fields. Unlike Levi-Civita, who concentrates all his energies almost exclusively on science and on the "school", Volterra deals also with the public, and wants to export scientific rationality, in which he strongly believes, also to other cultures. He never forgets that he is a mathematician, but this attachment does not lead him to undervalue other contexts – scientific, cultural, social – in which he is interested as well. His is also the history of a (at least temporary) defeat and of a cruel isolation, which the oath's episode – so to say- makes official. He must cede leadership to Severi. For personal reasons, the handover would not have been delayed much as a normal course of events, but it would not have happened in such a sudden, painful, and clumsy way. It is true that Severi is very similar to Volterra in a sense: in the value of his research and in an utmost intellectual curiosity; as a political “animal”, that desires power and knows how to use it. But morally speaking he is deeply different. Volterra – image of the Italy of the Risorgimento, radical in the defence of some principles – does not stoop to compromises with fascism, does not swear allegiance to the regime and is definitively cast aside by the racial laws. Severi, still a socialist when he arrives in Rome and later when Matteotti’s case breaks loose, has, though, no problem in changing sides and taking the black shirt, in order to enter the *Accademia d’Italia*. And it does not finish here. He will, nimble as a tightrope walker, get through the *purges*, with which the new republican Italy wanted to expel from the power centres the ones who were most implicated with fascism; he will make nothing of setting himself up as a champion of Catholicism. Even Enriques could have played the part of *leader*. He had the makings of a leader: researcher of undoubted worth and originality, educated intellectual known since the beginning of the century in the scientific and philosophical spheres. It is not by chance that he becomes president of the *Società Filosofica Italiana*! What damages him is not the “mishap” with Croce (and Gentile), but an aristocratic – intellectual and temperamental – attitude; the attitude of someone who would not demean himself by looking for approval and measure himself in a political sphere that always means, somehow, a collective. It is others that he requires to come closer to the purity and quickness of his views. Then, Tonelli and Picone. Both are good mathematicians. Within the scientific world, they wield a leadership (barely) inferior to the one of Volterra, Levi-Civita, Severi or Enriques but actually time consuming organizational commitments – the *Normale* for Tonelli and the *INAC* for Picone – add (in Tonelli’s case) to hesitations and political confusion that partially cloud their teaching’s lucidity. Volterra dies in 1940; Levi-Civita in 1941; Enriques and Tonelli pass away in 1946. Actually, only Severi and Picone survive to the second world war. They represent the main line of the new Italian mathematics in the 1920s and 1930s. And, in a sense, theirs will be a disturbing presence. Both are black shirts ; they move without great problems through the short season of political purge, retraining themselves with an unexpected easiness and rapidity, each one good for each season.

Thus, after the description and the appraisal of the state of Italian mathematics, comes unavoidably on the scene the third main theme: the outpourings of the mathematicians and, in general, of the relationships of the subject with the socio-political context of the Italy of the 1930s. We have underlined several times the cultural worth of mathematical thought and its strong presence in the Italian culture and society in the decades we have dealt with. It is therefore natural to introduce here the analysis of these relationships in the more general and often discussed question of the interactions between culture and fascism: there has been a “fascist culture” or, somehow there has been the attempt of the regime to influence cultural choices, to which corresponds the will of some intellectuals to turn their research along directions consonant with the changed political climate? We have not meant to take part in this debate in such a general way – neither do we intend to do it now in the conclusions – but to offer new material and ideas on which to to meditate, remembering that a summary of the relationships between culture and politics in Italy, during the two fascist decades, cannot avoid considering (also) scientific culture.

We have met fascist mathematicians, in the front line when voicing their approval to the regime: Severi and Picone are the most distinguished examples. Others have declared themselves antifascist, whether from liberal positions (Volterra) or following their own socialist ideals (Levi-Civita). Most Italian mathematicians mirror though, quite faithfully, the feelings and the trajectory of the country: at first suspicious towards the noisy, populist and illiberal novelty, they end up conforming to it and accepting it; they lay to sleep their conscience, seeing fascism as the modern – and Italian – evolution of the liberal State, the framework within which they continue to lead their usual lives and to defend their own profession (with some formal change, if necessary). The boundary between necessity, opportunism and real support to the regime is almost invisible. This, for example, is the deposition of one of B. Levi's nephews[3]: "uncle Beppe, with whom I exchanged friendly letters, at first very close to the left-wing democratic groups, had become inclined to nationalistic ideas. I was thunderstruck by one letter of his in which he backed up the claims of Italian expansionism on the strength of the fact that Italian civilization, culture and supremacy in arts and science justified the plan to penetrate the Balkans". Also Tonelli's case is, from this perspective, exemplary. He is a socialist in his youth, antifascist during the whole Bolognese period (and signatory of the Croce manifesto), and in Pisa – some years later – he asks for a fascist membership card just to dispel the insinuations that depicted him as anti-Italian ; irony of fate, he will get it only when a great part of its value has been lost, a few months before July 1943!

Mathematics and belonging to this world is the element that really matter Italian mathematicians. All other factors are only marginal. Mathematics is thought – or however it is asserted – to be able to go its way, barely annoyed by the trivial changes that the political context imposes upon it. A mathematician we have dealt with in Chapters 5 and 7 – G. Sansone – has explicitly affirmed in the postwar period that Italian culture has "always" been "hostile and impervious to fascism; it has continued its way either ignoring it or enduring easily eliminable encrustations". From this group of researchers and professors – fascist or afascist, but always and above all mathematicians – come folkloristic tributes to the Duce, to his innumerable and irresistible talents and to the achievements of fascism, trumpeted in the prologues to books and speeches. Mathematicians soon conform to the repetitive rhetoric of the fascist style, adjusting to the rhetorical code exhibited in the public-ceremonial sphere. What can be inferred if not a loss of reason, ridiculous and actually unaccountable? Severi, for instance, writes: "after the names of the Italian princes that favoured or protected the Renaissance, come soon to mind with Mussolini's one the names of Richelieu and Napoleon in France, of Frederick the Great in Prussia, of Peter the Great in Russia".

In order to pinpoint possible responsibilities of the regime for the slowdown of Italian mathematics during its growth, quite more significant than the ideology and the be-

[3] Cf. D. Jona, A. Foa, *Noi due*, Il Mulino, Bologna, 1997.

haviour of mathematicians is the analysis of the attitude of the regime towards science and mathematics in particular.

To begin with, we have the more explicitly *repressive* aspect, which hits also the mathematical world in 1931 (with the oath's episode and Volterra's dismissal) and in 1938, with enormous proportions and the drama of racial laws. The repression is part of a more general policy that fascism follows towards culture: great sensibility and a new original attention towards the role that intellectuals can play. Their approval can be won, jointly and alternatively, with the "stick" or the "carrot". It finds expression also in that institutional activism that recognizes the need to update scientific structures; this updating is conducted by the researchers of the younger generations, and it often frames institutionally Italian science, even in the second half of the century.

The object of the general debate we have recalled is how much this attention to the culture and to intellectuals has turned out to be a "suggestion" for new contents – in a *fascist culture*, to put it briefly. Some affirm that fascism has been an unlucky period, an illness, a tactical change of capitalist supremacy, a sort of provincial folklore in the hands of the predominant economic forces. In short, there would not be a fascist culture. The regime would have developed in the most absolute ideological void, without any entrenchment. Conversely, fascist culture would not be so singular if fascism is regarded as the logical expression and development of Italian society, as almost a biologic parentage. The omnivorous party-state would have proposed itself as the great author of the culture of the new Italians, extending to a cultural level those *lessons in style* that begin to be propagandized, imposing the *voi* (in place of *lei*)[4] or the Roman salute, instead of the bourgeois handshake. If such a radical thesis of a *fascist culture* is not to be accepted, we should at least talk about *culture of the fascist period*. In the historiography of mathematics there is no explicit coming out for one or the other option, but G. Sansone's previous quotation is clear: mathematics is a real culture and real culture is so strong that it cannot even be grazed by temporary breezes, as fascism has been.

We consider this conclusion to be rash and aprioristic. It deserves a deeper analysis. To which conclusions have we come?

It cannot certainly be said that the regime had ideas, to be later implemented, about a (hypothetical) *fascist mathematics*. Fascism has not had a policy of its own, not even at an institutional level. It has never had a serious and consistent scientific policy. What we have called *activism* is often a confused support for several pressures, which are fulfilled with the main view of keeping and increasing that support. Fascist interventionism is anything but planned. There is no choice. The initial will to impose its own political subjectivity even on science gives way to gentler tones and to a smoother navigation, directed towards domestic shores. Trying to avoid trouble with both sides: applied mathematics and Picone's INAC is supported in order to balance the creation of such a shrine of pure mathematics as Severi's INDAM. Above all, fascism is a "phenomenon" that lasts only twenty years. And soon there are other more urgent priorities. There is no time,

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[4] In Italian there are two courtesy forms: "Voi" is the equivalent to the English "You" and the French "Vous"; "Lei" instead is a little less formal than "Voi"; grammatically "Lei" is the Italian form for the third person singular feminine.

even if there were any intention, to think of mathematics in a different way. The importance of the political dimension in the culture is anyway limited by the consideration of mathematics as a scientific technique, that must remain out of the debate and of the ideological-political choices. Neither do mathematicians themselves do any formulation according to this hypothetical fascist mathematics. In brief, the Italian situation is quite different, from this perspective, from the German one.

The insistence on *useful science* and on *applied mathematics* continues; here the needs (and ambitions) of a regime which goes on claiming to be revolutionary match, with the atmosphere of the time and the cultural trends that, little by little, give rise to rationalistic architecture or to compositional anti-rhetoric, to the cinematography of real life or to a documentary aesthetics. Even in this case fascism – as regards mathematics – chooses the wrong horse, at the wrong time. It chooses applications, in a truly gross and simplistic way, just while the years of the *great theory* develop in many fields of mathematics, including the essential ones. Its boorish way of insisting on applications and the quick end of the fascist experience exclude also the possibility that some tenable choices could have been capitalized on.

In short, the influence of fascism on the contents of mathematics of these two decades – of its research and its teaching – seems to be irrelevant, if we exclude some of the more exposed topics on the socio-economical front (demography, statistics, etc.). The repression of dissent, the alliance with Hitler and war are all tragic experiences, still too close for us to be able to calmly assert that in the history of a country there are more important elements, which have mainly directed its future. We belong to those generations for which fascism represents evil – from the political, civil, ethical point of view – and we are proud of it. And yet the discussion of mathematics and the study of the reasons for its decline, beyond the lesser (casual) brilliance of the mathematicians of the 1930s, induces us to acknowledge that these responsibilities are to be found in deeper structural facts than the presence of a regime that – luckily – had not even time to play a part. During the 1920s and 1930s we were a young country, with a limited scientific wealth, concentrated in some leading elements, devoid of that social and cultural infrastructure essential to coping with an isolated case by a general rule. Of course, nothing impedes great scientists from breaking into the scene, above all in a discipline – such as mathematics – that needed no funding for expensive equipment and chiefly in the aftermath of a period – that of the Risorgimento – that enerated many sources of energy, even bringing them into contact with inspiring developments in other countries. Great mathematicians become *great* in haste, aided by the absence of previous generations with which it would be necessary to enter into competition in order to get *a place in the sun*. The most difficult is though the next step: to turn the pioneer generation to a normality, the *cathedrals in the desert* in an articulated structure, that progressively, by degree, presents every facet. The individual's intelligence is not enough here. The problem concerns the country's scientific wealth and its organization. A wider student support would be necessary to ensure an effective exchange. Cultural and economical incentives would be necessary, but they are missing in an Italy with really few Schumpeterian enterprises and where the industrial structure and small size of the national market do not enable great researches and development, but only to occupy some market niches. An expansion system would be necessary, so that young generations do not see previous ones only as a plug to blow out and so that professorships are not in the hands of the old guard, who always repeat the same things. But such a scientific structure cannot be improvised and Italy in the 1930s – with or without fascism – had to go through this stage inevitably.

From this perspective, the responsibilities of fascism seem smaller, almost confined to the slow-down of this necessary maturation stage. Quite different is the conclusion if we look closer at the established custom. One can understand here how fascism has managed to lengthen its negative presence well beyond July 25th 1943. It invents nothing new in the recent history of the country, but it certainly radicalizes and exasperates the worst sides, concealing them with national pride. Nor should the transfer from the content level to the ethica one, so to say, translate into minor attention and an only rhetorical appraisal. Damages are still to be seen. We have pictured the provincialism to which a nationalistic spirit, steeped in autarchic pride, leads. We have talked about the emphasis on science and on useful mathematics, with emphasis inserted in a strong polemics against the *culturame*[5] of some academic environments. We talked of *carrot* and *stick*: scientists (and mathematicians) will have their reins slackened if they accept to return to the ranks of the "technician", after the "unusual" spring of the beginning of the century. They have to remember that their work is "to do theorems", and forget projections and externalizations that divert them and are not necessary or useful at all. A domineering and servile Italy – domineering downwards and servile upwards – becomes also the model of the academic structure. There is no place for dissent and disagreement. It is always the role of the faithful state functionary to be the composite figure of the university teacher; a functionary in whom authority is always less the almost spontaneous prevailing of competition and becomes an investment from the high thanks to the acquired merits in terms of obedience. Once again no invention, but the legitimization of an academic and intellectual use exacerbated in its worst aspects. The new Italy, at the end of war, will have to reckon with this heritage. And often it is easier to reorganize a school or strike up international contacts again than to eradicate something that tends to be a part of one's own identity.

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[5] *Culturame* is a characteristic Italian word to design the "mass of so-called intellectuals".

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