

## THE EARTH'S GRAVITY

### OUTLINE

#### THE EARTH'S GRAVITATIONAL FIELD

Newton's law of gravitation:  $F_{grav} = GMm/r^2$ ;

Gravitational field = gravitational acceleration  $g$ ; gravitational potential, equipotential surfaces.

$g$  for a non-rotating spherically symmetric Earth;

Effects of rotation and ellipticity – variation with latitude, the reference ellipsoid and International Gravity Formula;

Effects of elevation and topography, intervening rock, density inhomogeneities, tides.

The geoid: equipotential mean-sea-level surface on which  $g =$  IGF value.

#### GRAVITY SURVEYS

Measurement: gravity units, gravimeters, survey procedures; the geoid; satellite altimetry.

Gravity corrections – latitude, elevation, Bouguer, terrain, drift;

Interpretation of gravity anomalies: regional-residual separation; regional variations and deep (crust, mantle) structure;  
local variations and shallow density anomalies;

Examples of Bouguer gravity anomalies.

#### ISOSTASY

Mechanism: level of compensation; Pratt and Airy models; mountain roots;

Isostasy and free-air gravity, examples of isostatic balance and isostatic anomalies.

*Background reading: Fowler §5.1–5.6; Lowrie §2.2–2.6; Kearey & Vine §2.11.*

## THE EARTH'S GRAVITY FIELD

- Newton's law of gravitation is:

$$F = \frac{GMm}{r^2}$$

where the Gravitational Constant  $G = 6.673 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$  ( $\text{kg}^{-1}\text{m}^3\text{s}^{-2}$ ).

- The *field strength* of the Earth's gravitational field is defined as the gravitational force acting on unit mass. From Newton's third law of mechanics,  $F = ma$ , it follows that

gravitational force per unit mass = gravitational acceleration  $g$ .

$g$  is approximately  $9.8\text{m/s}^2$  at the surface of the Earth.

- A related concept is *gravitational potential*: the gravitational potential  $V_P$  at a point P is the work done against gravity in bringing unit mass from infinity to P. A gravitational *equipotential surface* is a surface on which  $V_P$  is constant.

The *geoid* (mean sea level) is an equipotential surface.

- For a mass  $m$  at the surface of a spherically symmetric non-rotating Earth:

$$mg = \frac{GMm}{r^2} \quad \implies \quad g = \frac{GM}{r^2}$$

The equation above shows that the mass of the Earth  $M$  can be estimated from measurements of  $g$ .

## GRAVITY IS A WEAK FORCE!

- Newton's law of gravitation is:

$$F_{grav} = \frac{GMm}{r^2}$$

with  $G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$ .

- The electrostatic force between electric charges ( $Q, q$ ) also obeys an inverse square law:

$$F_{es} = \frac{Qq}{4\pi\epsilon_0 r^2}$$

with  $1/(4\pi\epsilon_0) = 8.98754 \times 10^9 \text{Nm}^2\text{C}^{-2}$ .

*How does the gravitational force between two electrons compare with the electrostatic force?*

The charge on an electron is  $1.60219 \times 10^{-19} \text{C}$ ; the mass of an electron is  $9.10953 \times 10^{-27} \text{kg}$ .  $\Rightarrow$

$$F_{grav}^{e,e} = \frac{553.75 \times 10^{-73}}{r^2} \text{N} \quad F_{es}^{e,e} = \frac{23.071 \times 10^{-29}}{r^2} \text{N}$$

The ratio is  $2.4 \times 10^{-43}$ !

*What about the gravitational and electrostatic forces between two protons?*

A similar calculation shows that the ratio is  $0.809 \times 10^{-36}$ , which is still very small.

James Gleick, in *Genius - Richard Feynman and Modern Physics* (Abacus 1992, p352), quotes Feynman at a conference as saying, "The gravitational force is weak. In fact, it's damned weak." At that instant a loudspeaker crashed to the floor from the ceiling. Feynman: "Weak, but not negligible."

## VARIATIONS IN $g$

The following factors cause  $g$  to vary over the surface of the Earth:

1. the rotation and ellipticity of the Earth bring a systematic variation with latitude;
2.  $g$  varies with elevation and topography, and the density of the intervening rock between an elevated site and MSL;
3. there is a daily tidal variation;
4.  $g$  is sensitive to density inhomogeneities in the crust and mantle.

Variations from (1) are described by the IGF below; those from (2) are compensated by *gravity corrections*; and field procedure corrects those from (3). The variations from (4) are mapped and studied in order to extract information on the crust and mantle.

### THE REFERENCE ELLIPSOID AND IGF (INTERNATIONAL GRAVITY FORMULA)

The theoretical variation of  $g$  with latitude (causes 1 and 2 above) on the surface of an ellipsoidal rotating Earth can be summarised in a formula. The International Gravity Formula (IGF) is the standard formula for the variation of  $g$  with latitude. It gives the variation on the surface of the *reference ellipsoid*, the ellipsoid giving a best-fit with MSL. The 1984 IGF is:

$$g = 9780326.7714 \times \frac{1 + 0.001931851381639 \sin^2 \theta}{\sqrt{1 - 0.0066943999013 \sin^2 \theta}} \text{ g.u.}$$

where  $\theta$  is the geocentric latitude.

The geoid is the equipotential mean-sea-level surface on which  $g =$  IGF value.

THE EFFECT OF SHAPE ON  $g$

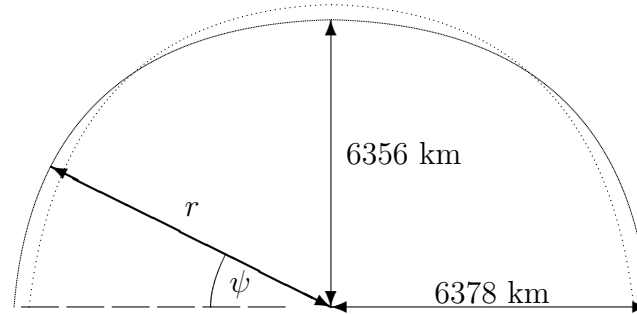
At geocentric latitude  $\psi$ :

$$r = a(1 - e \sin^2 \psi)$$

where  $e = \text{flattening} = 1/298.257$

and  $a = \text{equatorial radius} = 6378 \text{ km}$ .

According to Newton's law, an increase in radius  $r$  will decrease the gravitational attraction and therefore  $g$ .



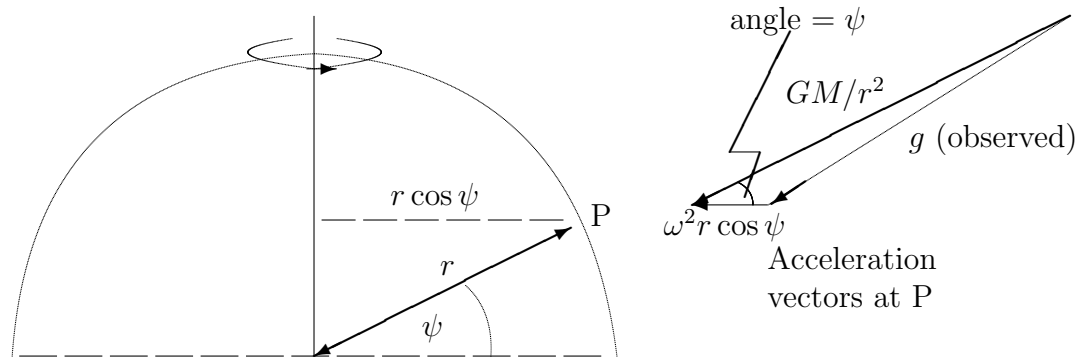
### THE EFFECT OF ROTATION ON $g$

The Earth's rotation diminishes the observed gravitational field because part of the Earth's gravitational attraction goes into supplying the centripetal acceleration that keeps the measuring system in a constant 'orbit' around the Earth. As the diagram below illustrates, the gravitational field from Newtonian attraction is the vector sum of the observed acceleration  $g$  and the centripetal acceleration.

At geocentric latitude  $\psi$ , the distance of the point P from the Earth's axis of rotation is  $r \cos \psi$ . The centripetal acceleration needed to keep P at a constant elevation is  $\omega^2 r \cos \psi$ . The component of centripetal acceleration resolved along the radial direction is

$$\omega^2 r \cos^2 \psi = \omega^2 (1 - \sin^2 \psi)$$

This is the amount by which the observed acceleration  $g$  is reduced by rotation. At the equator the centripetal acceleration is  $0.0033916 \text{ m/s}^2$ , which is  $0.34678\%$  of the equatorial value of  $g$ .



## GRAVITY SURVEYS (1)

### UNITS OF $g$

The SI (Système International) unit is  $\text{m/s}^2$ . A more convenient unit is a millionth of this, the  $\mu\text{m/s}^2$  or gravity unit (g.u.). An older unit, the milligal (mgal) is in common use.

$$1\text{g.u.} = 10^{-6}\text{ms}^{-2} = 1\mu\text{ms}^{-2}; \quad 1\text{mgal} = 10^{-3}\text{cms}^{-2} = 10\text{g.u.}$$

### MEASUREMENT OF $g$

The absolute value of  $g$  can be measured from the period of oscillation of specially designed pendulums or from timing the free-fall of a mass in a vacuum. These experiments can be extremely accurate.

Absolute measurement of  $g$  is unnecessary in most geophysical applications. The usual aim of a gravity survey is to reveal the *gravity anomalies* that are the surface expression of anomalies in density at depth. For this purpose it is sufficient to measure the variation of  $g$  relative a base level.

Gravity surveys use *gravity meters*, or *gravimeters*, which measure not absolute values, but *differences* in  $g$  between one place and another with great accuracy. A gravimeter is essentially a mass on a spring that measures the weight, or gravitational pull,  $mg$  on the mass  $m$  with great sensitivity. Achieving this sensitivity demands elaborate design, commonly involving a zero-length spring, angled arms and optical levers.

## **GRAVITY SURVEYS (2)**

### **SURVEY PROCEDURES**

Surveys over a small area refer the measurements back to a base station. For surveys over a wider area it is usual to establish a base net. On a wider scale still, there are primary base nets of reliably calibrated gravity stations established in many countries. In large scale regional surveys with the aim of investigating deep (crustal and mantle) structure, the measurements would normally be tied to international gravity stations where the absolute value of  $g$  is known to high accuracy. The gravity measurements can then be converted to differences from the theoretical IGF value and any anomalies related to the average deep structure of the Earth.

Because correction of gravity readings for the effects of latitude, elevation and topography is so important, the gravity sites must be located accurately. This would normally entail running a geodetic survey in parallel with the gravity survey. Large scale surveys require widespread accurate geodetic control of the kind available from satellite geodesy.

### **THE GEOID**

Mean sea level, the surface of the oceans after allowance for waves, tides and ocean currents, is an equipotential surface. This surface, and its hypothetical extension through the land, constitutes the geoid. The geoid fluctuates around the reference ellipsoid by a few tens of metres.

Over the oceans the geoid can be mapped directly by means of satellite altimetry (q.v.). Because a spirit level aligns itself in a horizontal plane tangential to an equipotential surface, geodetic surveys on land could in principle track the geoid. In practice geodetic measurements need to be supplemented with gravity readings to map the geoid on land with reasonable accuracy. The geoid is the surface on which  $g$  equals the IGF value.



## GRAVITY CORRECTIONS (1)

### INSTRUMENTAL DRIFT

Creep in the springs causes gravimeter readings to drift with time. The readings at a set of stations are repeated during the progress of a survey in order to estimate a drift curve, which then allows all readings to be corrected.

### TIDAL CORRECTIONS

The gravitational attraction of the sun and moon may cause  $g$  to vary cyclically by a few g.u. in the course of a day. The correction can be found from tidal tables or by monitoring  $g$  at the base station. The drift correction may include a segment of the tidal correction.

### EÖTVÖS CORRECTION

This is required in shipboard and airborne surveys. The Eötvös correction compensates for additional centrifugal forces acting on the mass element in moving vessels. Its value is

$$75.1V_E \cos \phi + 0.041V^2 \quad \text{g.u.}$$

where  $V$  is the speed of the vessel and  $V_E$  is its easterly component, both being measured in knots, and  $\phi =$  latitude. This correction is added to the observed gravity reading.

### LATITUDE CORRECTION

For distances in the north–south direction in excess of 2 km, the latitude correction is obtained from the IGF. For north–south distances up to 2 km, the latitude correction per metre north is

$$0.00812 \sin |2\phi| \quad \text{g.u./m}$$

where  $\phi =$  latitude. Since  $g$  increases with the absolute value  $|\phi|$  of the latitude, this value is *subtracted* from gravity readings at sites that are at a *higher* latitude than the reference station, and *added* for those at a *lower* latitude.

## GRAVITY CORRECTIONS (2)

### ELEVATION OR FREE-AIR CORRECTION

$g$  decreases with increasing elevation above the datum (reference level) by 3.086 g.u. for every metre difference in height. The elevation, or “free-air”, correction (FAC)

$$3.086h \text{ g.u. where } h = \text{height difference in m}$$

is *added* to gravity readings at sites that are *higher* than the reference station.

### FREE-AIR GRAVITY

The *free-air anomaly (FAA)* is the measured gravity after correcting for all the effects listed above. For measurements relative to a local base station, it is obtained from the drift/tide/Eötvös corrected observed anomaly  $\Delta g_o$  using

$$\Delta g_{\text{FAA}} = \Delta g_o - (\text{latitude correction}) + 3.086h$$

where  $h$  is positive for recording sites higher than the reference station and the positive-valued latitude correction is subtracted for sites at a higher latitude than the base station. For drift/tide/Eötvös corrected regional measurements  $g_o = g_{ref} + \Delta g_o$ , obtained by adding to  $\Delta g_o$  values of  $g_{ref}$  at international (and probably national) stations,

$$g_{\text{FA}} = g_o - (\text{IGF correction}) + 3.086h$$

### TOPOGRAPHIC CORRECTIONS

These correct for the gravitational attraction of the topography around the observing station relative to the datum level of the base station. In country that is fairly flat, the topography correction is the sum of the *Bouguer* and *terrain* corrections. In hilly and mountainous regions, it requires digitised elevations of the surrounding country.

Since the topographic correction measures the additional attraction of the slice of rock between the observing site and the datum, it is *subtracted* from gravity readings at sites *above* the base station and *added* at sites *below* the base station.

*Topographic corrections imply some means of estimating the density.*

### GRAVITY CORRECTIONS (3)

#### TOPOGRAPHIC CORRECTIONS (CTD)

##### (a) BOUGUER CORRECTION

The *Bouguer* (note the two 'u's) correction compensates for the gravitational attraction of the slab (or slabs) of rock beneath the recording site down to the datum level. Each slab is assumed to be horizontal and of constant thickness and to extend to a distance well in excess of that thickness all around the recording site. Its value is  $2\pi G\rho h = 0.0004191\rho h$  g.u. where  $\rho$  is the density of the slab and  $h$  is its thickness.

##### (b) TERRAIN CORRECTION

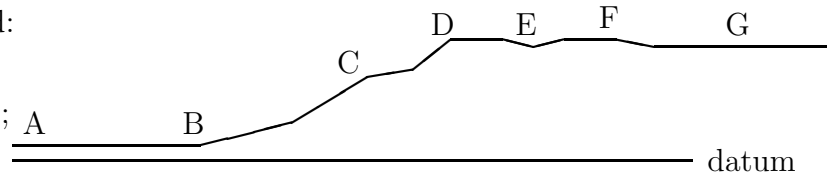
The terrain correction compensates for local relief around the recording site. It is calculated in a similar way to topographic corrections but is significantly smaller. It is unnecessary in gently undulating country. It has the form  $T\rho$ , where  $T$  is a factor computed from a chart laid over a map of the local topography. The net vertical component of the gravitational forces arising from the terrain acts *upwards*. The correction is therefore *added* to the observed gravity.

Example of corrections needed:

topographic at B, C, D;

Bouguer + terrain at E and F;

Bouguer at A and G.



#### BOUGUER GRAVITY

The *Bouguer anomaly* ( $BA$ ) is obtained from the drift/tide/Eötvös corrected observed anomaly  $\Delta g_o$  using

$$\Delta g_{BA} = \Delta g_o - (\text{latitude correction}) + 3.086h - 0.0004191\rho h + T\rho$$

For regional measurements  $g_o$  is tied to international gravity stations,

$$g_B = g_o - (\text{IGF correction}) + 3.086h - 0.0004191\rho h + T\rho$$

Bouguer gravity *estimates* the gravity that would have been recorded at the datum, had all intervening rock been bulldozed away.

## SATELLITE ALTIMETRY

Surfaces of equal geopotential (gravitational potential) become more closely spherical with increasing distance from the Earth. Since satellites tend to follow equipotential surfaces, their orbits are much smoother than the geoid. A radar altimeter mounted on a satellite can therefore register, with appropriate corrections, the height of the sea surface. Tracking stations determine the exact orbit of the satellite.

Typical accuracies from satellite altimetry over the oceans are:

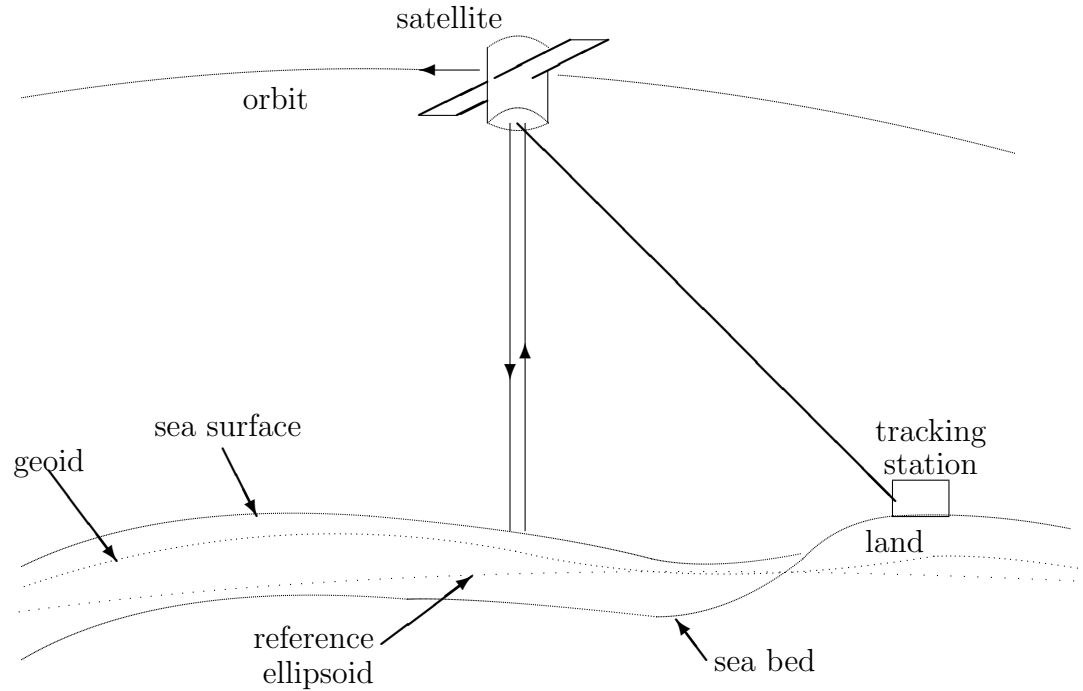
GEOS 1	launched in 1965	$\pm 0.6$ m
SEASAT	launched in 1978	$\pm 0.1$ m
ERS-1	launched in 1991	$\pm 0.07$ m

Land is not a good reflector of radar waves and height measurement is much less accurate over land.

The area of the sea from which the radar signal is reflected is known as the *footprint* of the satellite. The altimeter measures the average height over the area of the footprint. In the case of GEOS, the footprint was 15 km wide. The measurements of the height of the satellite above the sea surface have to be corrected for atmospheric conditions (temperature, pressure, humidity), tides and ocean swells.

**SATELLITE ALTIMETRY - FIGURE**

The figure below illustrates the relation between the satellite height, the sea surface, the geoid and the reference ellipsoid.



## INTERPRETATION OF GRAVITY ANOMALIES

Gravity interpretation can vary from (a) the broad interpretation of gravity maps through (b) rough models of gravity profiles and anomalies to (c) detailed modelling of profiles and anomalies. In case (a) modern methods of processing gravity maps can be very effective in bringing out particular geological trends and features, such as faults and batholiths. The gravity map of Great Britain illustrates this (see later). If some idea of the depth of sediments in a basin or the anomalous mass of an ore body is required, as in case (b), it may be sufficient to derive the estimates from well-tried rules of thumb or simple models. Case (c) entails complex model fitting. This course deals only with large scale gravity features and the phenomenon of isostasy. Cases (b) and (c) belong in the Exploration Geophysics course.

### SOME BASIC RULES FOR GRAVITY INTERPRETATION

- Localised, short wavelength, anomalies can originate only from shallow density inhomogeneities.
- Gravity alone cannot distinguish between a strong density contrast at depth and a more diffuse contrast shallow. Nevertheless large-scale gravity anomalies, such as those registered on maps of the geoid, generally originate from deep-seated variations in density.
- As an aid to interpretation, a *regional* (large scale) gravity variations are usually separated from residual (local) variations. There is no unique way of doing this.
- On maps of Bouguer gravity, gravity lows correspond to low density bodies within denser surrounding rocks; e.g. salt domes, sedimentary basins, granite batholiths in more basic country rock, mountain roots. Conversely gravity highs correspond to higher density bodies.
- Extra information, from seismic surveys or drilling, is necessary to resolve the fundamental ambiguity of detailed gravity interpretation.

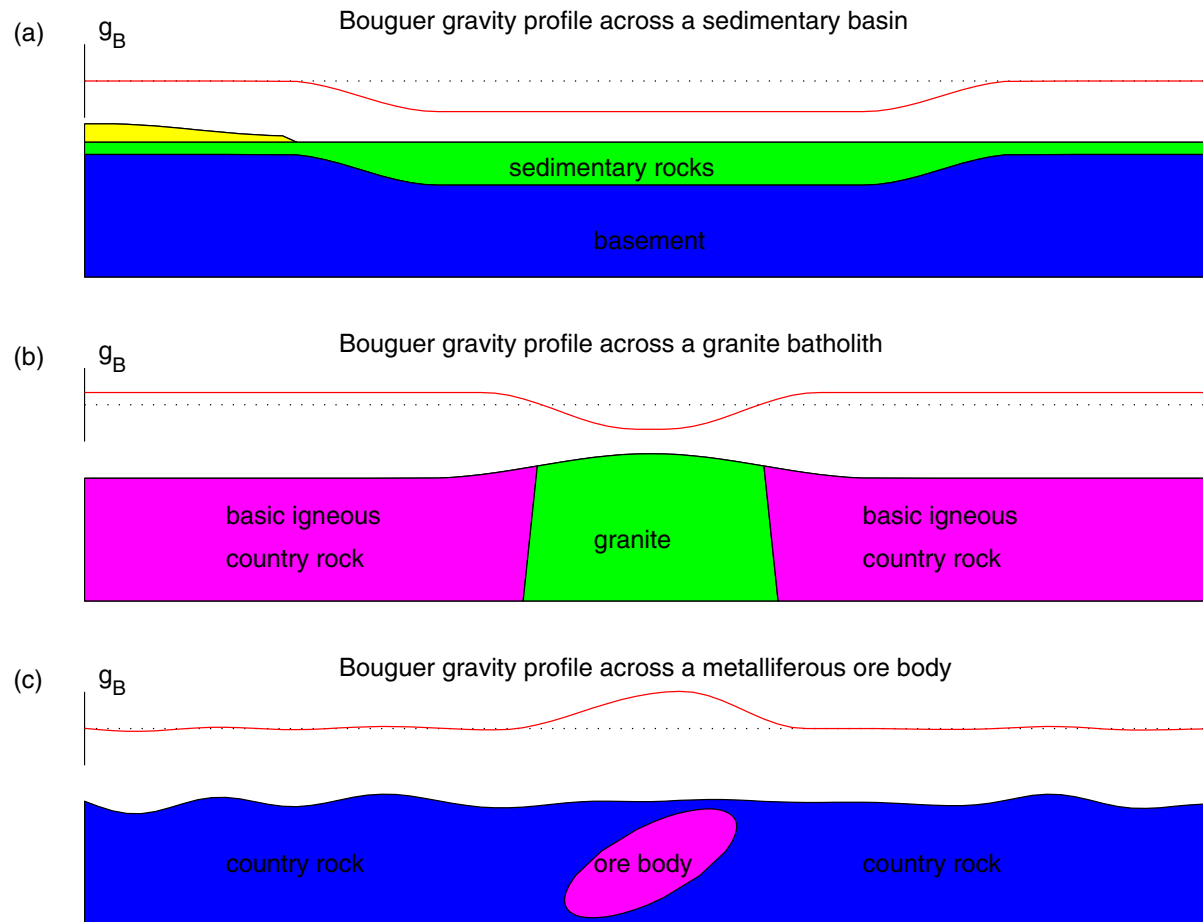
## **EXAMPLES OF BOUGUER GRAVITY PROFILES**

The figure below shows schematic vertical sections and Bouguer gravity profiles across

- (a) a sedimentary basin,
- (b) a granite batholith,
- (c) a metalliferous ore body.

In (a) the basin is filled with sedimentary rocks whose density is less than that of the underlying basement. In (b) the density of the granite batholith is less than that of the surrounding rock. Both give rise to gravity lows. In (c) the density of the metalliferous ore exceeds that of the country rock and the Bouguer profile shows a high. The depth of the sedimentary basin, the extent of the batholith and the size of the ore body can be estimated from the gravity anomalies and the densities of the anomalous bodies and their surroundings.

FIGURES ILLUSTRATING BOUGUER GRAVITY PROFILES





## **GRAVITY MAP OF GREAT BRITAIN**

The following references show a Bouguer gravity map of central Britain in shaded relief and in colour. In shaded relief the gravity map appears as a topography illuminated by the sun. Gravity can also be mapped in colour with red denoting gravity highs and blue gravity lows.

### **REFERENCES:**

Open University, 1990. S339: Understanding the Continents; Tectonic and Thermal Processes of the Lithosphere; Block 1B: Lithospheric Geophysics in Britain.  
M.K.Lee, T.C.Pharaoah & N.J.Soper, 1990. Structural trends in central Britain from images of gravity and aeromagnetic fields, *J.Geol.Soc.London*, **147**, 241–258.

It is instructive to inspect these maps and see how Bouguer gravity highlights regions of

- (i) density deficiency, e.g. the Cheshire basin, granite intrusions in the Lake District;
- (ii) density excess, e.g. basic intrusives in Arran and Mull;
- (iii) density contrast, e.g. the Southern Uplands Fault and Highland Boundary Fault.

## ISOSTASY

Early geodetic and gravity measurements showed that the Andes, Himalayas and Alps did not deflect a plumb bob as much as expected from their exposed mass. The explanation is that mountains have low-density roots beneath them. These roots supply buoyancy that supports the additional mass exposed above mean sea level; that is, variations in surface elevation are hydrostatically supported. This is the principle of *isostasy*: above some depth in the Earth called the *level of compensation*, all columns of rock exert the same pressure. The level of compensation is the depth below which the hydrostatic pressure in the Earth is independent of location (latitude and longitude).

Isostasy applies on a broad scale: individual mountain peaks are not isostatically compensated but most mountain ranges are. So too are mid-ocean ridges. The basic idea is that of flotation. Large scale gravity anomalies indicate that in general the lithosphere is hydrostatically supported, i.e. the rock column 'floats' above the level of compensation. Consequently large scale gravity anomalies reflect the structure of the lithosphere. Some areas of the Earth are not in isostatic balance (see below).

## ISOSTATIC MODELS

The lithological pressure  $P$  exerted at the base of a column of rock of density  $\rho$  and height  $h$  is

$$P = \rho gh$$

If  $h$  is in m,  $\rho$  is in kg/m<sup>3</sup>, and  $g$  is in m/s<sup>2</sup>,  $P$  is in N/m<sup>2</sup> (newtons/sq.m) or Pa (pascals).

1 atmosphere pressure =  $1.013 \times 10^5$  Pa.

According to the principle of isostasy the sum of products  $\rho gh$  down to the level of compensation is a constant. Because of the fundamental ambiguity in interpreting gravity profiles, it is possible to admit an indefinite range of distributions of density with depth in order to explain a particular set of gravity observations. Pratt (1854) and Airy (1855) proposed two rival mechanisms by which surface elevations were isostatically compensated.

- Pratt's hypothesis explains elevations by lateral variations in density: high surface relief corresponds to a low-density column. In Pratt's model these columns all have a common base at the level of compensation.
- Airy proposed constant density columns floating, like blocks of wood, in a denser substratum. The profile of the base of these columns is a magnified mirror image of the surface elevation.

While the existence of mountain roots favours Airy's model, these models are better seen as end-points of a range of possible models. Isostatic modelling generally has elements of both.

## COMMENTS ON ISOSTATIC MODELS

- Because gravity observations cannot be inverted to unique models, isostatic modelling needs to be constrained by information about the geometry of density variations. The usual and best source of information comes from the structure and layering of the subsurface estimated by seismic methods.
- The higher density and submersion of oceanic crust and the lower density and elevation of the continental crust put a Pratt component into the isostatic balance of oceans and continents, whereas the differences in thickness of oceanic (thin) and continental (thick) crust introduce an Airy component.
- Airy's hypothesis is usually more important than Pratt's.

Recall that isostasy is not valid at the scale of localised gravity anomalies.

## ISOSTATIC ANOMALIES

There are some broad regions of the Earth that are not in isostatic balance, notably

- island arcs and ocean trenches, where subduction occurs too fast for isostatic equilibrium to be achieved; and
- Fennoscandia and northern Canada, where the lithosphere is still rebounding from the melting of the ice caps of the last ice age (glacial rebound). The rates of glacial rebound are related to the *viscosity* of the mantle.

*A simple test:* free-air gravity  $\sim$  constant in regions that are in isostatic equilibrium.

The reason for this is that in isostatic equilibrium the excess mass of the mountain above MSL is balanced by the mass deficiency of the lower density mountain root in the crust.

**EXAMPLES OF ISOSTATIC EQUILIBRIUM AND AN ISOSTATIC ANOMALY**

The figure overleaf shows schematic vertical sections through the crust and free-air and Bouguer gravity profiles across

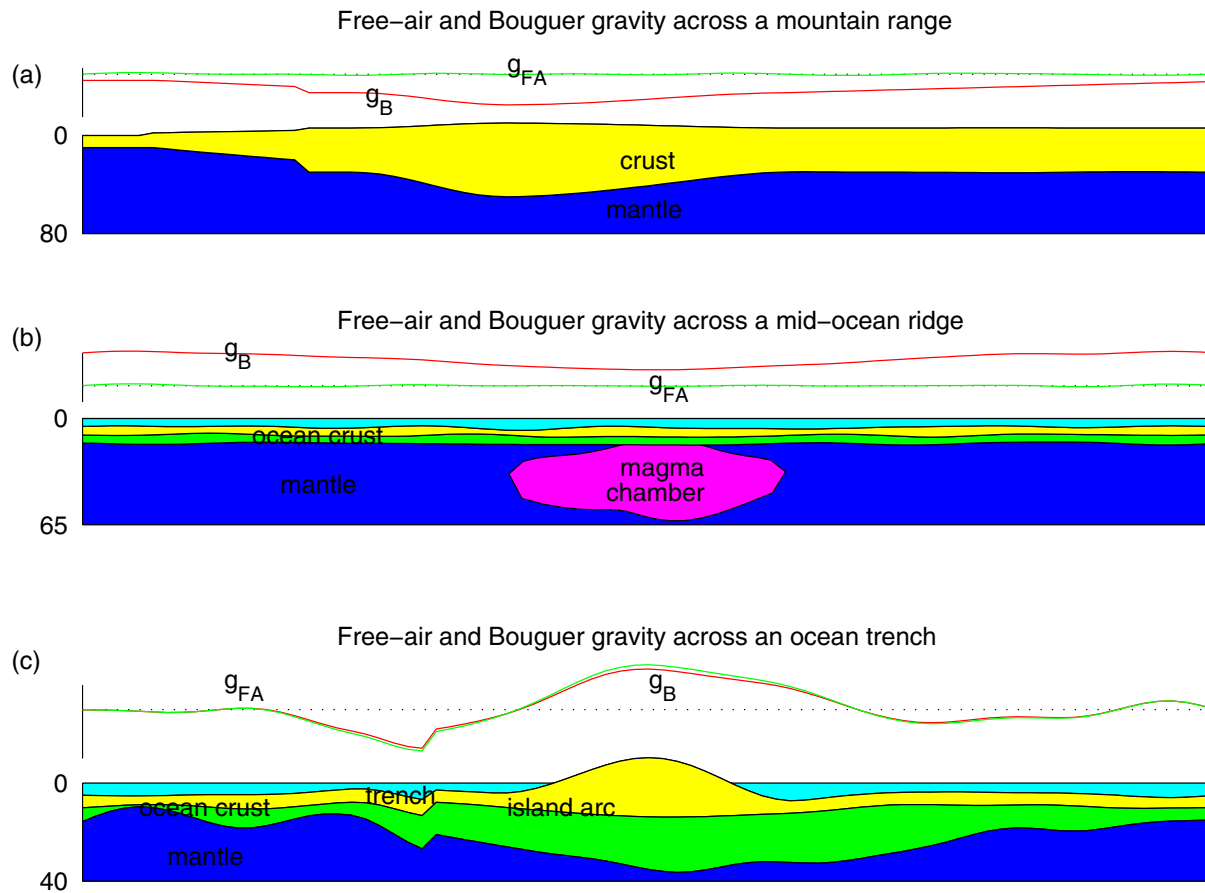
(a) a mountain range,

(b) a mid-ocean ridge,

and (c) an ocean trench and island arc.

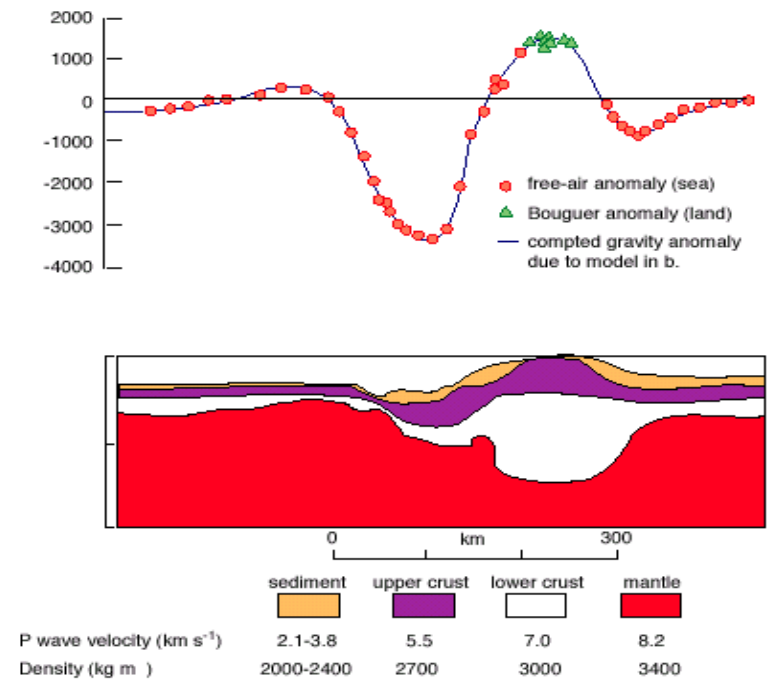
The mountain range and mid-ocean ridge are in isostatic equilibrium. Consequently the free-air profiles are virtually flat. The Bouguer profiles show gravity lows due to the low-density mountain roots in (a) and from the low-density magma chamber in (b). The ocean trench and island arc in (c) are not in isostatic equilibrium because the oceanic plate subducts too fast for equilibrium to be established. Typical values for the deepest gravity lows in these figures are (a)  $-300$  g.u., (b)  $-1000$  g.u., and (c)  $-3000$  g.u.

FIGURES ILLUSTRATING ISOSTATIC EQUILIBRIUM AND AN ISOSTATIC ANOMALY



### GRAVITY PROFILE ACROSS AT AN OCEAN TRENCH

The gravity low at the trench comes from the depth of water (density  $\sim 1.05$  g/cc compared with  $\sim 2.2$  g/cc for sediments). Subduction is too fast for isostatic compensation of this mass deficiency.



## **THE ISOSTATIC ANOMALY FROM GLACIAL REBOUND**

The lithosphere in northern Canada and Fennoscandia (Finland and Scandinavia) is rebounding from the melting of the ice caps of the last ice age. The figure overleaf shows crustal sections and Bouguer gravity profiles across

(a) the crust in isostatic equilibrium before the ice age,

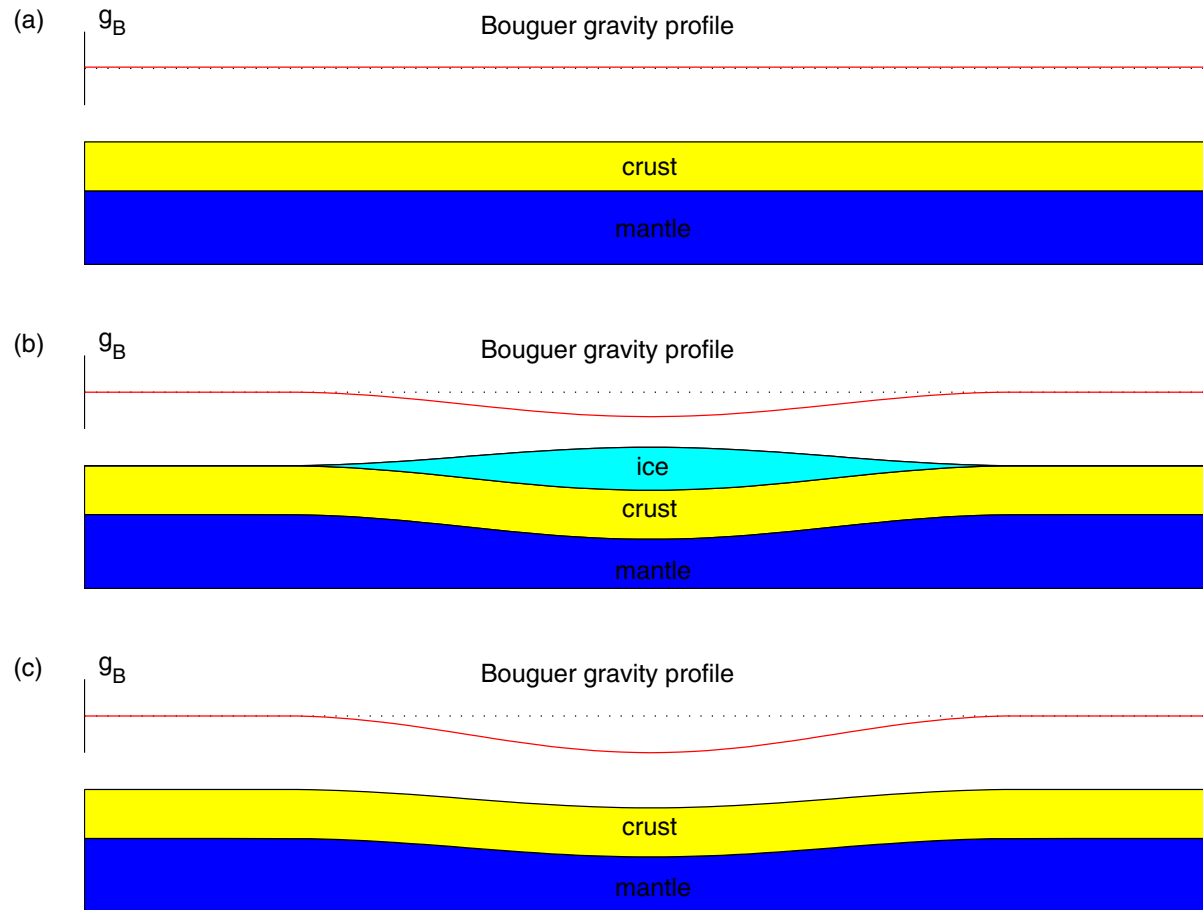
(b) loading of the crust by an ice cap,

and (c) the rebounding crust after the ice has melted.

In (b) the crust sags, forming a root that supports the ice cap. The root is accommodated by an outward flow of mantle material. The root causes a negative Bouguer gravity anomaly. When the ice melts, the crust starts to rebound back to its configuration in (a). For this to happen, mantle material has to flow back into the region. The viscosity of the mantle is the controlling factor in the rate of rebound. This allows the viscosity of the mantle to be estimated from rates of glacial rebound.



FIGURE ILLUSTRATING THE ISOSTATIC ANOMALY FROM GLACIAL REBOUND

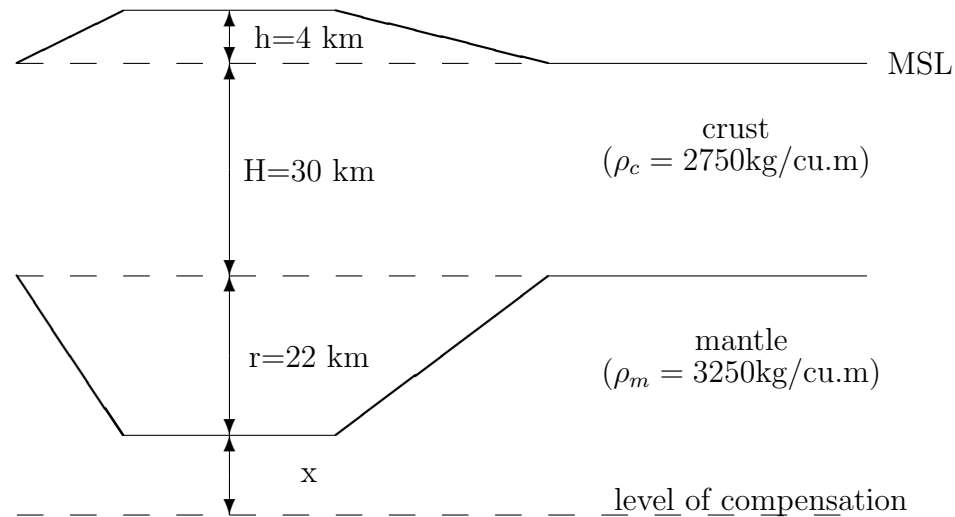


### AIRY ISOSTATIC MODELLING OF MOUNTAIN ROOTS

The figure below represents a profile of the topography across a continent where the land elevation rises steadily from mean sea level to a mountainous plateau  $h = 4.0$  km high after which it decreases steadily and slowly to a broad plain at close to sea level. If Airy's principle of isostatic compensation applies, then the plateau has a root of thickness  $r$  given by

$$r = h \frac{\rho_c}{(\rho_m - \rho_c)}$$

where  $\rho_c$  and  $\rho_m$  are the densities of the crust and upper mantle and  $r$  is measured relative to the base of the crust beneath a region having zero elevation (mean sea level).



**AIRY ISOSTATIC MODELLING OF MOUNTAIN ROOTS (CTD)**

According to Airy's hypothesis for isostatic balance, the buoyancy of the crustal root supports the elevation of the plateau. At isostatic equilibrium the pressure from vertical columns of rock above the level of compensation is the same, whatever the location. Suppose the level of compensation is at a depth  $x$  below the base of the root. Then, at the level of compensation, the pressure beneath the plateau equals that beneath MSL:

$$(h + H + r)\rho_c g + x\rho_m g = H\rho_c g + (r + x)\rho_m g$$

Hence

$$h\rho_c = r(\rho_m - \rho_c)$$

and the formula above follows. Note that the formula does not require the thickness of the crust (given as  $H = 30$  km thick under the plain).

A typical value for the density of the crust is 2750 kg/cu.m and for the upper mantle 3250 kg/cu.m. Given these values, then  $r = 5.5h$  and the thickness of the root of the plateau =  $5.5 \times 4.0 = 22$  km.

*Two questions:*

- (i) What thickness of rock would have to be eroded in order to bring the plateau down to sea level?
- (ii) If in the past a 1.3 km thickness of rock was eroded from the plateau, what was its height before erosion occurred?