Measuring the Earth's Rotation Rate Using a Low-Cost MEMS Gyroscope

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Abstract

Accurate measurements of the Earth's rotation rate can be obtained by using high quality gyroscopes, such as ring laser gyros (RLG) included in tactical grade inertial measurement units. However, such devices are bulky and expensive. As the Micro-Electro-Mechanical System (MEMS) technology has evolved rapidly during the last years, accurate low-cost gyroscopes are now available. Are the new MEMS gyros accurate enough to detect and measure the Earth's rate? In this paper, we describe a method and algorithm that can be used to answer this question. To test the developed method, we used a modern MEMS gyroscope with specified bias stability less than 2 degrees per hour. The bias stability indicates that it is possible to measure the Earth's rate. However, in order to do this, all the external factors that affect the gyro bias need to be carefully taken into account. For example, in order to observe the bias the gyroscope needs to be rotated sequentially. In the paper, we present a sequence of rotations that aims to maximize the signal to noise ratio and minimize the time needed to detect the Earth's rotation rate. Furthermore, the influence of gravity in the bias can be examined with another sequence of rotations. For both cases, we use an accurate stepper motor to switch between different orientations.

With this set up, we have successfully determined the Earth's rotation rate. In addition, by using a Kalman filter we were able to estimate the magnitude of the rate with accuracy better than one degree per hour. The Kalman filter approach was used to improve convergence time and to enable error estimation. The results show that if the factors that affect the bias of the sensor are minimized and correctly modeled, the Earth's rotation rate can be detected and estimated with new MEMS gyroscopes. This level of accuracy makes MEMS gyroscopes suitable for application areas where traditionally more expensive gyro technologies have been exploited.

1.   Introduction

Earth's rotation rate can be measured using various different methods. Modern geodetic techniques include Very Long Baseline Interferometry (VLBI), satellites of the Global Positioning System (GPS) or of the Global Navigation Satellite System (GLONASS), laser ranging to artificial Earth satellites of the Lageos or Starlette (Satellite Laser Ranging), Earth remote sensing satellites like ERS1, ERS2 or TOPEX/POSEIDON and Lunar Laser Ranging (LLR) [1]. Measuring the Earth's rate is also an essential part in the alignment of inertial navigation systems (INSs) [2]. Alignment using Earth's rate requires highly accurate gyroscopes, and only recently, Micro-Electro-Mechanical System (MEMS) gyroscopes have been considered accurate enough to measure it [3, 4]. The ability to measure such slow rotation using low-cost, low-power, and small size sensors is very interesting from both academic and application-wise views.

In this paper we identify the major sources of errors that affect the measurement data of a MEMS gyroscope sensor and also offer solutions for compensating them. This was done in order to maximize the performances of these sensors, which are usually designed as an electronically driven resonator, often made out of a single piece of quartz or silicon. Such gyroscopes operate in accordance with the dynamic theory which states that when an angular rate is applied to a translating body, a Coriolis force is generated [2, 5]. The force is proportional to the applied angular rate, and causes displacement that can be measured capacitively in a silicon instrument or piezoelectrically in a quartz instrument. Theoretically, if the errors of such gyroscopes are properly compensated then it is possible to measure very small angular rates, like the Earth's rotation rate.

The direct measurement of the Earth's rotation rate was performed using methods such as superfluid phase coherence in [6] or by using a MEMS gyroscope in [3, 4]. In the last approach the sensitive axis of the gyroscope was parallel to the Earth's rotation axis. Long term measurements were carried out in order to compensate the external factors that influence the measurement data. In our approach the sensitive axis of the gyroscope is aligned perpendicularly to the local vertical. This means that only about half of the Earth's rate is detected by the sensor (at latitude $61.449°\mathrm{N}$). Bias (including temperature effects) and g-sensitivity compensation was achieved through a mechanical change in the sensor orientation, and the required correction values were estimated using a Kalman filter.

The paper is organized as follows. In Section 2, theoretical background for measuring the Earth's rate is given. Hardware implementation of the measurement setup is provided in Section 3, while Section 4 presents the experimental results. Finally, Section 5 concludes the paper.

2. Theoretical background

According to the World Geodetic System 1984 [7] the angular velocity of the Earth is $\Omega_e = 7292115 \times 10^{-11} \, rad/s \approx 4.178 \times 10^{-3} \deg/s$. The input range of a typical MEMS gyroscope is approximately $\pm 100 \deg/s$ and it is clear that detecting a signal of $0.004 \deg/s$ requires careful error analysis and filter design. Scale factor error does not have large effect with such low signal, and temperature effects can be restricted by keeping the unit at room temperature and allowing the unit to warm up prior the tests. In this paper we will concentrate on the most significant error sources, angle random walk, in-run bias stability, temperature dependant bias, and g-sensitivity. A good measure to characterize the first two errors is the Allan variance.

2.1. Allan variance

The "Allan Variance" was named after Dr. David Allan who worked out a method of characterizing noise and stability in clock systems [8]. It is a method which analyzes a time sequence and pulls out the intrinsic clock noise from a system, as a function of the averaging time. The Allan variance was developed for clocks, but it can easily be adapted for any other type of output [9]. The computation of Allan variance starts by taking a long sequence of data and dividing it into bins based on an averaging time, τ. The equation for the Allan variance is the following:

$$AVAR^2(\tau) = \frac{1}{2 \cdot (n-1)} \sum_i \left(y(\tau)_{i+1} - y(\tau)_i \right)^2. \tag{1}$$

Where:

- $AVAR^2(\tau)$ is the Allan variance as a function of the averaging time;
- y_i is the average value of the measurements in bin i;
- n is the total number of bins.

For short averaging times, the Allan Variance is dominated by the uncorrelated noise in the sensor output. There is a direct correlation between the standard deviation of the output versus time with the slope of the Allan Variance at small τ [9]. In the beginning, the Allan Variance decreases as the averaging time gets longer. As the averaging time increases, the Allan variance typically levels off due to $1/f$ noise [10]. The power of $1/f$ noise is commonly used to define bias instability [11] and thus the minimum value of Allan variance can be used as a measure of bias instability. At some point, the Allan Variance starts to increase again. This is due to bias drift or angular rate random walk error in the sensor output, a process which is clearly non-stationary. Allan variance is a very helpful

tool in MEMS-based navigation error analysis because the $1/f$ and random walk noise components are typically very strong. For example, consider a problem where the gyroscope bias needs to be estimated from the gyroscope output. If the sensitivity axis of the gyroscope is aligned with East-West direction, it is known that the actual input is zero. To estimate the bias, one can then obtain a series of measurements and take the average. This estimate of the constant bias is not necessarily the optimal estimate, as the error process is correlated in time [12], but is suffices for our purpose. The estimated bias will be subtracted from the gyroscope measurements during the „navigation phase", and the resulting series are integrated to obtain the change in orientation (heading, for example) over time. In this study, the aim is to estimate the magnitude of the Earth's rate, and thus we will turn the gyroscope towards North immediately after the bias is estimated. The gyroscope output now includes both Earth's rate and noise, and estimate of the noise variance comes directly from the Equation (1) if the averaging time is set equal to match the bias estimation phase. Thus, Allan variance plot can be used to predict how much of the combination of white noise, $1/f$ noise, and random walk noise will affect our estimate of the Earth's rate.

The necessity of rotations becomes clear if we construct a stochastic model of the gyroscope noise. First, assume that there is a constant (unknown) bias in the gyroscope data. The observation equation is then:

$$z_t = b + \cos(\alpha_t)\omega + \varepsilon_t. \tag{2}$$

where b is the bias, α is the rotation angle (radians from North), ω is the magnitude of the horizontal Earth's rate and ε_t uncorrelated noise of the gyro. Extending this equation to a series of measurements is simple, just define $z = \begin{bmatrix} z_1 & z_2 & ... \end{bmatrix}^T$ and $H = \begin{bmatrix} 1 & \cos(\alpha_1) \\ 1 & \cos(\alpha_2) \\ \vdots & \vdots \end{bmatrix}$,

resulting in a linear model $z = H \begin{bmatrix} b \\ \omega \end{bmatrix} + \varepsilon$ ($z = Hx + \varepsilon$). If there are no rotations, matrix H will be rank deficient, and one cannot solve both b and ω. Using least squares theory, $\operatorname{var}(\hat{z}) = \sigma^2 \begin{bmatrix} H^T & H \end{bmatrix}^{-1}$, one can see that optimal sequence of rotations is equal amount of measurements from North and South directions.

Clearly constant bias model is not adequate (as can be seen from the Allan variance plots). The next step is to add random walk term to the equations. For simplicity, we will neglect the $1/f$ part (that leads to complex state models [13]) and model the gyroscope noise as a combination of white noise (i.i.d, entering the system via Equation 2.) and

random walk. The variances of these noise processes are fitted so that the resulting combination matches with observed Allan variance as closely as possible. In addition, we add first order compensation of temperature, as temperature is also available from the gyro. The full model consists of observation equation

$$z_t = Hx_t + \varepsilon_t,\qquad(3)$$

where $H = \begin{bmatrix} \cos(\alpha_t) & T_t & 1 \end{bmatrix}$, and α_t is the controlled table angle and T_t is the observed gyro temperature. The time update equation is

$$x_t = Ax_{t-1} + w_t,\qquad(4)$$

where A is 3x3 identity matrix,

$$\mathrm{var}(w) = Q = \begin{bmatrix} \delta & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \sigma_{rw}^2 \end{bmatrix},\qquad(5)$$

and 3x1 state vector x consists of Earth rate, temperature coefficient and (random walk of) bias, in this order. In the Equation 5 very small variances δ are added to make the model more realistic. Temperature coefficient very likely changes over time, and Earth rate is not constant either.

3.   Hardware implementation of the measurement setup

For all the experiments provided in this paper we used a prototype version of the SCC1300-D02 MEMS gyroscope sensor, manufactured by VTI Technologies [14]. The hardware components of the measurement setup are presented below:

- SCC1300–D02: VTI Technologies, a combined accelerometer–gyroscope sensor with SPI interfaces [14];
- SPI interface: National Instruments' NI–8451 USB device that provides SPI and I^2C communication interfaces with eight chip select lines [15];
- Power supply: Hewlett Packard E3611A, DC power supply, 0-20 V, 0-1.5 A;
- Voltage regulator: 5 V / 3.3 V voltage regulators. Self-manufactured;
- PC: Used to run the program that reads gyroscope measurements through SPI.

In addition to these components we used the Velmex B5990TS rotary table for positioning the gyroscope sensor in different orientations throughout the experiments. Also, for including the SCC1300–D02 sensor in the measurement setup, a custom PCB was designed. A block diagram depicting the connections between the system's main components is presented in Figure 1.
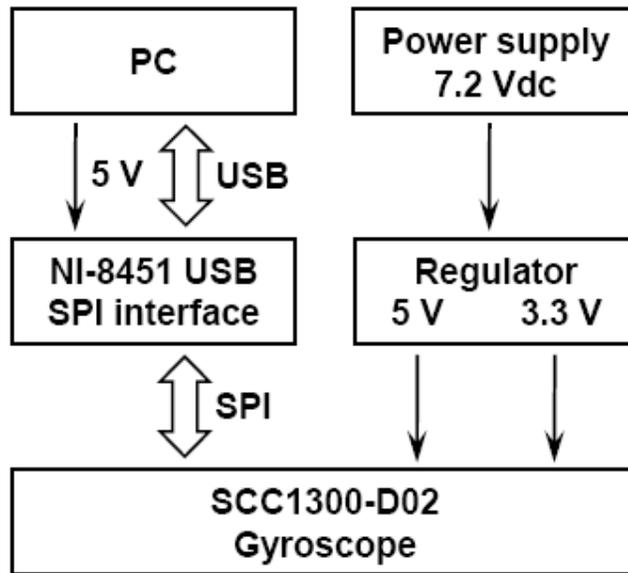
Figure 1: Block diagram showing the connections between the main components

In this block diagram we have used simple arrows for representing the power line connections and up-down arrows for the digital connections. The typical performance values for the SCC1300-D02 gyroscope are listed in Table 1.

Table 1: SCC1300-D02 gyroscope sensor specification

Parameter	Value
Operating range	$\pm 100 \text{ deg/s}$
Noise (RMS)	0.06 deg/s
Sensitivity	$50 \text{ LSB}/(\text{deg/s})$
Offset short term instability	$< 2 \text{ deg/h}$
Quantization	0.05 deg/s
SPI clock rate	$0.1 - 8 \text{ MHz}$

From the gyroscope specifications it can be found that the resolution of the gyroscope is higher than it would be required for measuring the Earth rate. In other words, the Earth's rotation rate lies well below the detection level of the gyroscope. However, the noise of the gyroscope will cause stochastic dithering to the measurement prior the AD-conversion, and thus it will be possible to estimate the angular rate with enough samples [16].

4. Analysis of experimental results and discussions

The experiments that are presented in this chapter were conducted in Tampere (Finland) at latitude $\varphi \approx 61.449°N$. For these measurements the sampling frequency of the gyroscope sensor was approximately 1 kHz. All the data collected from the measurements was saved on a laptop hard drive and was processed using MATLAB 2008 software.

4.1. Bias instability of the SCC1300-D02 gyroscope sensor

In order to determine the bias instability for the SCC1300-D02 gyroscope sensor, a set of measurements were carried out in room temperature. The test was performed in office environment, where natural temperature fluctuations can possibly cause some deterioration of the measurements. Despite the fact that the environment was not perfectly temperature-controlled, the test was expected to yield at least a rough estimate of the bias instability.

During the measurements the gyroscope was stationary on the floor. Its positive sensitive axis was parallel to the local horizontal plane. Total data collection time of the experiment spans to approximately 61 hours. The raw data collected directly from the sensor is shown in Figure 2 as a function of time.
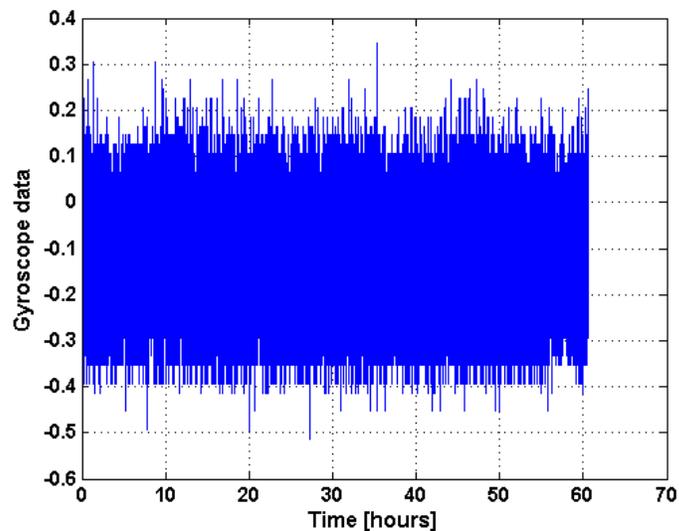

Figure 2: Raw data

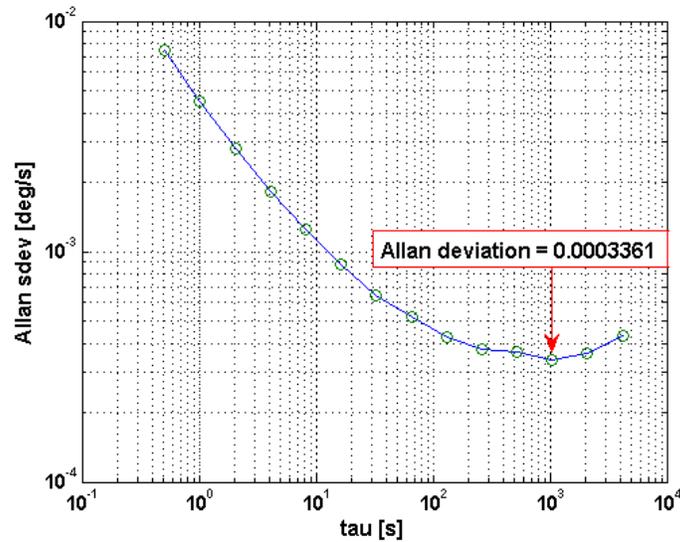The Allan deviation plot derived from the raw data is presented in Figure 3.

Figure 3: Allan deviation of SCC1300-D02 gyroscope

From the Allan variance plot it can be seen that bias instability is approximately $0.00034 \ \mathrm{deg/s}$ which equals about $1.2 \ \mathrm{deg/h}$. The result agrees well with the specifications, taking into account the temperature fluctuations during the measurements. In conclusion, the noise level of the new SCC1300-D02 gyroscope is low enough to theoretically measure the angular velocity of the Earth.

4.2.   Determining the G-sensitivity of the gyroscope sensor

In order to determine the g-sensitivity of the gyroscope, we designed a special experimental setup. The hardware components were presented in Figure 1. In this new setup the sensor was rotated around the Y axis, which was orientated to North. At startup, the sensitive axis was parallel to the local vertical. The measurement positions were at 0°, 45°, 135°, 180°, 225° and 315° from the local vertical. In each position the sensor stayed for approximately 20 minutes. Before rotating the turntable in the next measurement position, the sensitive axis of the gyroscope was orientated for another 20 minutes in a neutral position (east or west). In Figure 4 is presented an average of all the data contained in each of the measurement positions (approximately 5.6 hours/position).
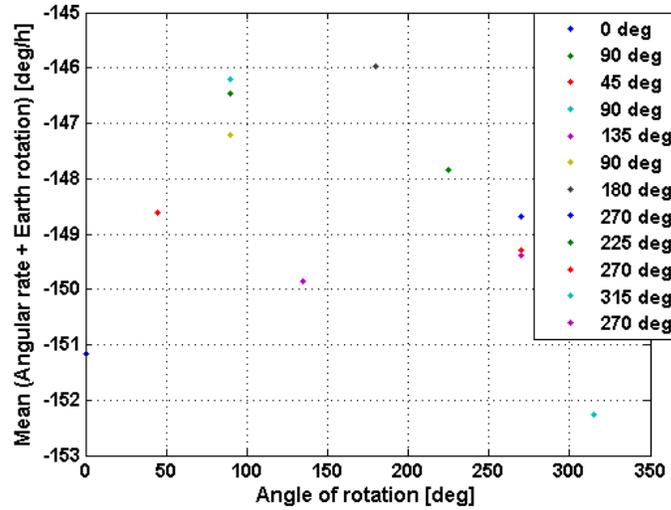
Figure 4: Average of the angular rate data

From this figure the g-sensitivity can't be estimated. This is due to the fact that the data is dependent of the Earth's rotation rate according to the equation:

$$\Omega_{position} = \Omega_e \cos(\beta)\sin(\varphi). \tag{6}$$

Where:

- $\Omega_{position}$ is the theoretical angular velocity sensed by the gyroscope sensor;

- Ω_e is the Earth's rotation rate as defined in WGS84;

- β is the angle between the sensitive axis of the gyroscope and the local vertical;

- φ is our current latitude.

Figure 5 contains the data from the previous plot, without the Earth rotation which was removed according to equation (3).
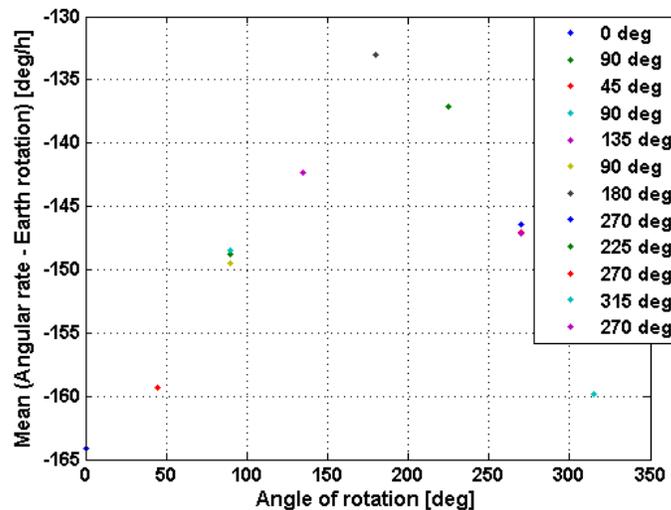
Figure 5: Average of the angular rate data without the Earth rotation

In order to determine the influence of g-sensitivity over the output data of the gyroscope, we used the ordinary least squares method. Using this approach we have determined with 95% confidence that the output of the sensor changes with approximately $15\,\deg/h/g$. The results are presented in Figure 6, where "b" represents the slope of the measurement data.
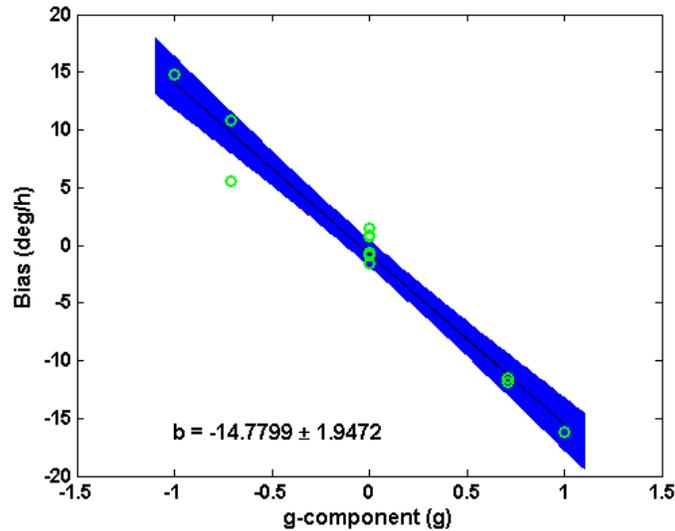

Figure 6: Linear least squares of the g-sensitivity

As demonstrated above, the gravitational force has a large influence over the output data of the gyroscope. Furthermore, the maximum value of the Earth's rotation rate sensed by the sensor can be achieved only when the sensitive axis of the gyroscope is parallel with the Earth's axis, as show in [3, 4]. In these conditions for estimating correctly the Earth's rotation rate a proper compensation of the g-sensitivity must be done. Alternatively, the sensitive axes of the gyroscope can be fixed to be parallel to the local level. Consequently the g is constant in all the measurement positions of the setup, and the compensation is not required. This kind of measurement setup is shown in Figure 7.
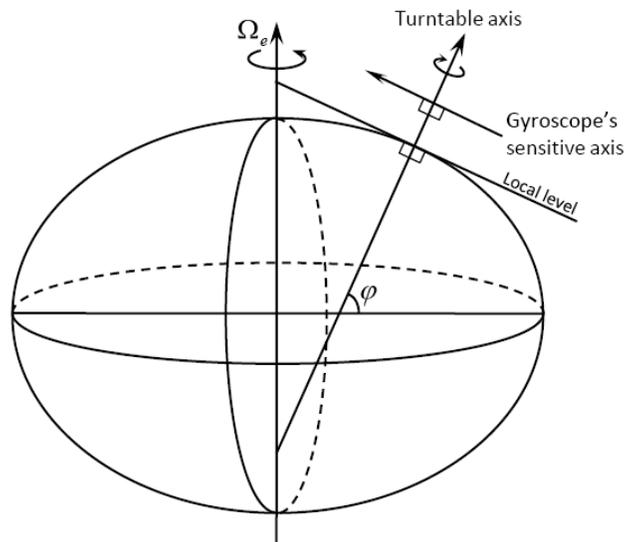
Figure 7: The sensitive axis of the gyroscope is perpendicular to the local vertical while the turntable rotates

4.3. Earth's rotation rate

For measuring the Earth's rotation rate we used the method described in Figure 7. The experiment consists in taking measurements from 4 different orientations, namely: North, East, South and West. Each direction was measured for 5 minutes and then the turntable was automatically rotated into the next measurement position. The process starts in North and then rotates clockwise at each 5 minutes. The measurements take place for several hours. Figure 8 presents the entire experimental setup used for measuring the Earth's rotation rate.
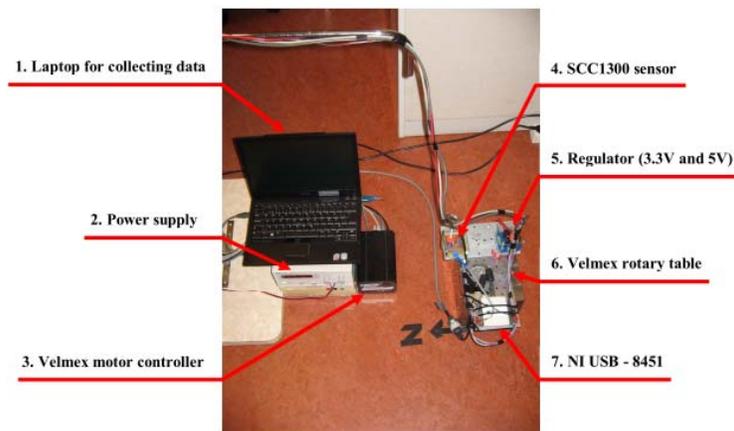
Figure 8: The SCC1300-D02 sensor, parallel with local level

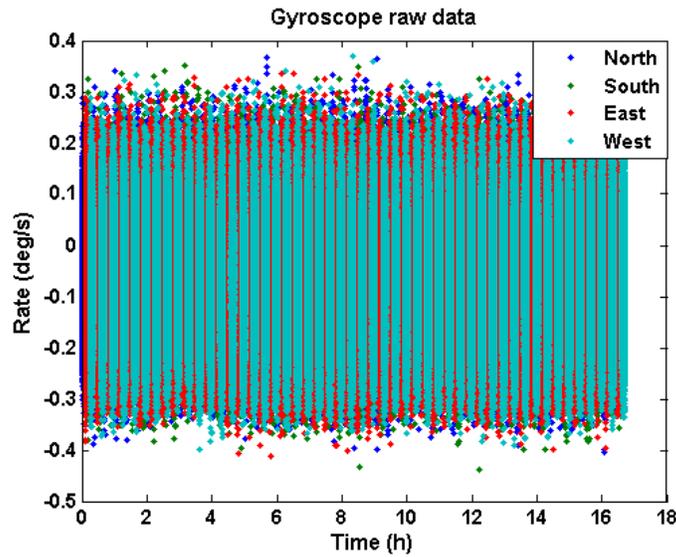The raw data used for measuring the Earth's rotation rate as a function of time is presented in Figure 9.

Figure 9: Raw data used for measuring the Earth's rotation rate

Because the experiment was performed in office environment small temperature fluctuations can possibly cause some deterioration of the measurements. In order to compensate these errors we have collected together with the angular rate data also the temperature data, which was measured with the SCC1300-D02 sensor. The sampling frequency of the temperature sensor was approximately equal to 2 Hz. Figure 10 shows the temperature vs. measured rate (5 minute averages) during this experiment. This figure shows that temperature compensation is useful even though the Earth rate is visible without compensation.
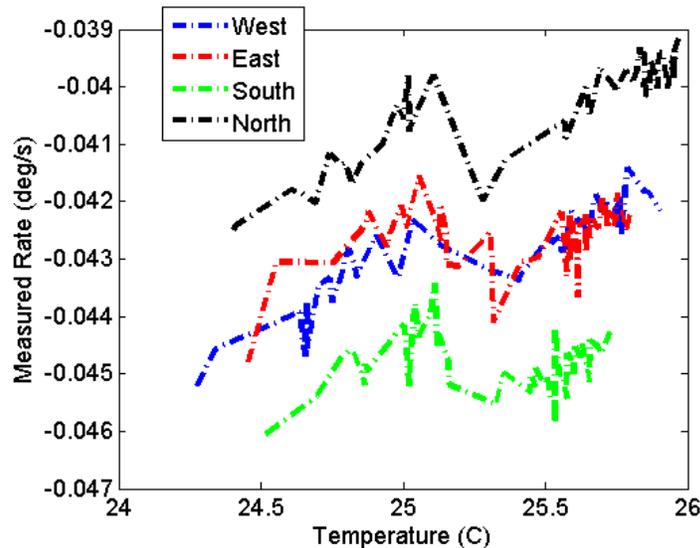

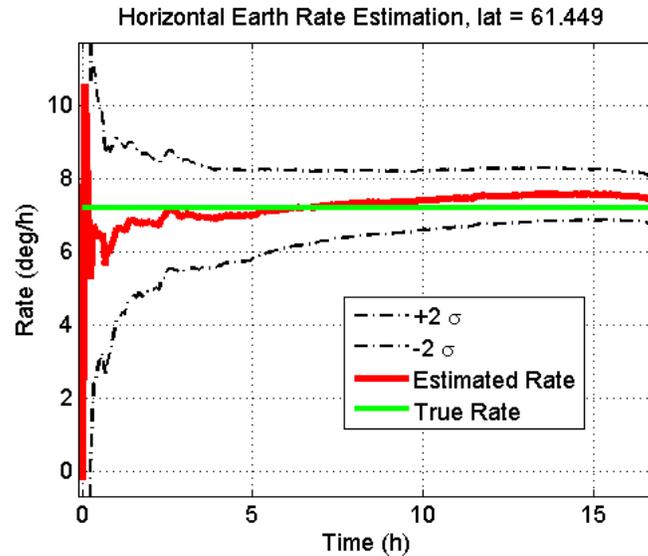Figure 10: Temperature effect over the data

Figure 11: Kalman filter output

To simplify the processing, we used 5-minute averages of the gyro data as input for the Kalman filter described in the Section 2. Covariance matrices were adjusted accordingly. The result of the Kalman filter is shown in Figure 11. Despite the fact that only half ($\cos 61° \approx 0,5$) of the Earth's rotation is sensed in the gyroscope (because the sensing axis is parallel to the local level), the Earth's rotation rate is still detectable without difficulty. As can be seen from Figure 11, the estimated Earth's rotation is very close to the theoretical value, with an error of 0.3 deg /h.

5.   Conclusions

This paper presents an improved method of computing the Earth's rotation rate using a MEMS gyroscope. The main contributions consist of compensating the non-stationary bias errors via sequence of rotations and Kalman filter implementation, and reducing the effect of external factors (temperature, g-sensitivity). This allows the possibility to detect and measure very small angular rates. The experimental setup is quite simple and it doesn't require any complex mechanics. Future work will be focused on developing a low-cost but high accuracy gyrocompass and also on implementing a pedestrian dead reckoning (PDR) based location system.

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